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In[1]:= Quiet[Remove["Global`*"], {Remove::rmnsm}];
Print["Mathematica $Version = ", $Version, ""]; Print["Execution started at = ",
  DateString[DateList[], {"Hour", ":", "Minute", " on ", "DayNameShort", " ", "Day", " ", "MonthNameShort", " ", "Year"}]];
Mathematica $Version = "9.0 for Mac OS X x86 (64-bit) (January 24, 2013)"

Execution started at = 21:18 on Tue 29 Jun 2021

In[2]:= w = 420 × 360 / 127 - 48; h = 297 × 360 / 127 - 48 - 6;
rApproximation = (r ≤ 124);

In[4]:= If[NumericQ[h] && NumericQ[w], Print[N[h / w, 20]]];
0.68958815745947734039

In[5]:= rKnownPoly = Module[{poly},
  poly = Factor[GroebnerBasis[{
    h == 2 r + gR (nR - 1),
    w == 2 gC nSC + 2 r (nC - 2 nSC),
    gR2 + gC2 == 4 r2}, {r}, {gR, gC}][[1]]];
  poly
] /. {nSC → 1, nC → 5, nR → 5};

In[6]:= rKnown = r /. Solve[0 == rKnownPoly, r][[1]]; N[rKnown, 8]
Out[6]:= 122.46582

In[7]:= (* rKnown concurs with answer of "122.466" reported by the PostScript software. E.g.: *)
Map[Print[Hyperlink["http://www.jdawiseman.com/" <> StringTake[#, 4] <> "/" <> # <> ".pdf"]] &,
  {"20191121_Rebello_Valente", "20191023_Messias", "20190611_1994s", "20190501_Broadbent",
    "20180605_Eights", "20180508_Sixties", "20180410_1963s", "20171010_Gould_Campbell", "20170608_1967s"}];

```

[http://www.jdawiseman.com/2019/20191121\\_Rebello\\_Valente.pdf](http://www.jdawiseman.com/2019/20191121_Rebello_Valente.pdf)

[http://www.jdawiseman.com/2019/20191023\\_Messias.pdf](http://www.jdawiseman.com/2019/20191023_Messias.pdf)

[http://www.jdawiseman.com/2019/20190611\\_1994s.pdf](http://www.jdawiseman.com/2019/20190611_1994s.pdf)

[http://www.jdawiseman.com/2019/20190501\\_Broadbent.pdf](http://www.jdawiseman.com/2019/20190501_Broadbent.pdf)

[http://www.jdawiseman.com/2018/20180605\\_Eights.pdf](http://www.jdawiseman.com/2018/20180605_Eights.pdf)

[http://www.jdawiseman.com/2018/20180508\\_Sixties.pdf](http://www.jdawiseman.com/2018/20180508_Sixties.pdf)

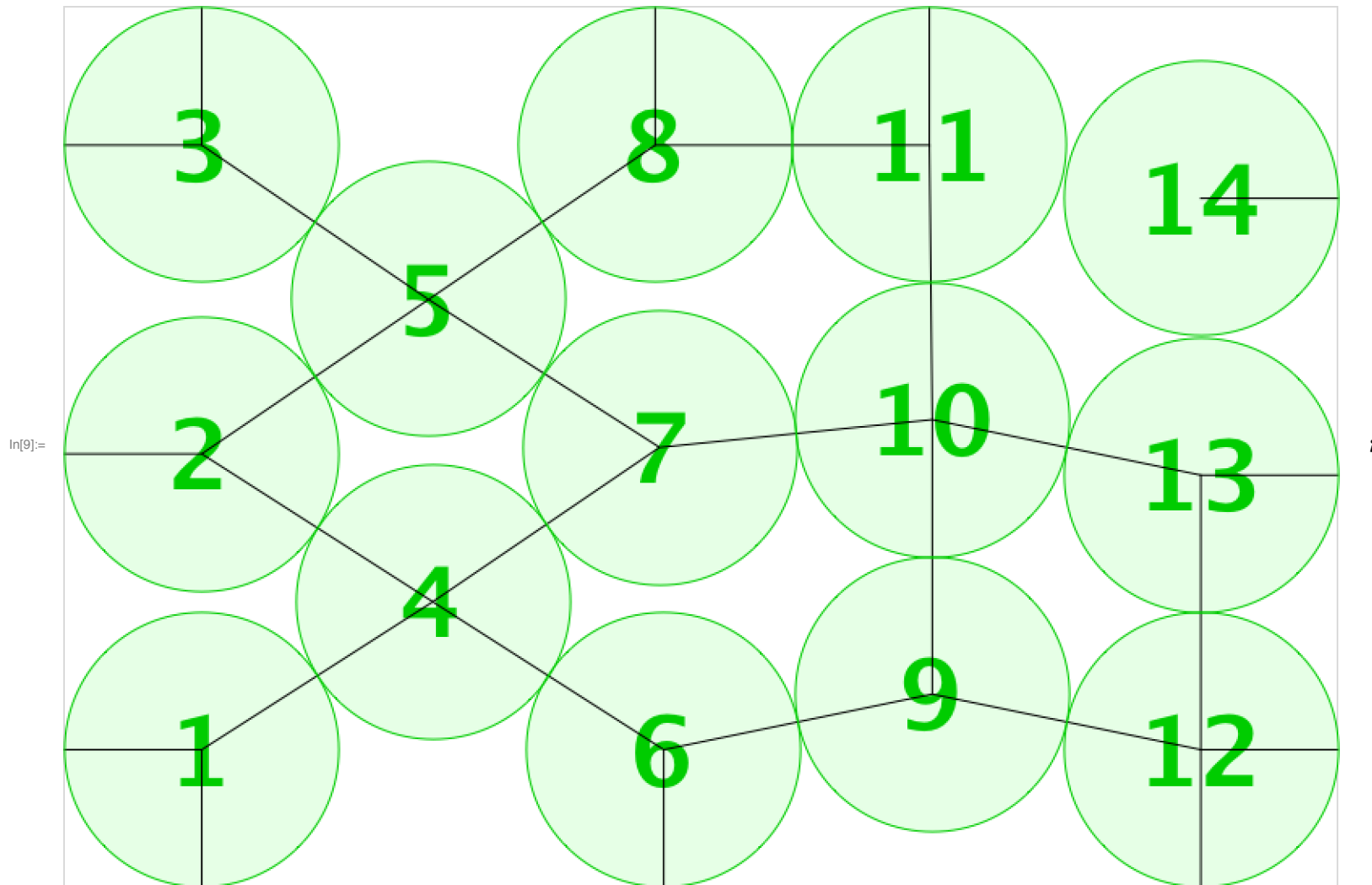
[http://www.jdawiseman.com/2018/20180410\\_1963s.pdf](http://www.jdawiseman.com/2018/20180410_1963s.pdf)

[http://www.jdawiseman.com/2017/20171010\\_Gould\\_Campbell.pdf](http://www.jdawiseman.com/2017/20171010_Gould_Campbell.pdf)

[http://www.jdawiseman.com/2017/20170608\\_1967s.pdf](http://www.jdawiseman.com/2017/20170608_1967s.pdf)

In[8]:= **Hyperlink**["[http://hydra.nat.uni-magdeburg.de/packing/crc\\_700/crc14\\_0.700000000000.html](http://hydra.nat.uni-magdeburg.de/packing/crc_700/crc14_0.700000000000.html)"]

Out[8]= [http://hydra.nat.uni-magdeburg.de/packing/crc\\_700/crc14\\_0.700000000000.html](http://hydra.nat.uni-magdeburg.de/packing/crc_700/crc14_0.700000000000.html)



In[10]:= posns =

r	r
r	y2
r	h - r
x4	(y2 + r) / 2
x5	(y2 + h - r) / 2
2 x4 - r	r
x4 + x5 - r	h / 2
2 x5 - r	h - r
w / 2 + x4 - r	y9
w / 2 + x4 - r	y9 + 2 r
2 x5 + r	h - r
w - r	r
w - r	3 r
w - r	5.0170243 r (* rattler *)

;

In[11]:=

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touchers = {
  {1, 4},
  {2, 4},
  {2, 5},
  {3, 5},
  {4, 6},
  {4, 7},
  {5, 7},
  {5, 8},
  {6, 9},
  {7, 10},
  {8, 11},
  {9, 10},
  {9, 12},
  {10, 11} ,
  {10, 13},
  {12, 13}
};
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In[12]:= vars = Variables[Flatten[posns]];
$Assumptions = Apply[And, Map[# ∈ Reals && # > 0 &, vars]]

Out[13]:= r ∈ Reals && r > 0 && x4 ∈ Reals && x4 > 0 && x5 ∈ Reals && x5 > 0 && y2 ∈ Reals && y2 > 0 && y9 ∈ Reals && y9 > 0

In[14]:= conditionsProto = Map[Simplify[Apply[Plus, (posns[[#1]] - posns[[#2]])^2] == 4 r^2] &, touchers];

In[15]:= conditions = Apply[And, conditionsProto // DeleteDuplicates // FullSimplify // DeleteDuplicates] // Simplify;
(* http://mathematica.stackexchange.com/questions/146292/ *)

In[16]:= Print[vars, " = vars;\n",
  Length[vars], " = Length[vars];\n",
  Length[touchers], " = Length[touchers];\n",
  Length[conditionsProto], " = Length[conditionsProto]\n",
  Length[conditions], " = Length[conditions]."]
]

{r, x4, x5, y2, y9} = vars;
5 = Length[vars];
16 = Length[touchers];
16 = Length[conditionsProto]
7 = Length[conditions].

In[17]:= easyBounds = (
  r > rKnown && rApproximation
  && y2 ≥ 3 r && y2 ≤ h - 3 r
  && x4 > r && x4 ≤ r (Sqrt[3] + 1) && x4 < w / 2
  && x5 > r && x5 ≤ r (Sqrt[3] + 1) && x5 < w / 2
  && y9 > r && y9 ≤ r (Sqrt[3] + 1) && y9 < h / 2
);

In[18]:= (* Calc time ≈ 4½ minutes on a 2.7 GHz Intel Core i5 *)
solution = FindInstance[conditions && easyBounds, vars, Reals][[1]];

In[19]:= Print[solutionN = N[solution, 30]]; posnsNew = posns /. solutionN; rBest = r /. solutionN;
{r → 123.024542592996512906027189627, x4 → 330.309804633144305248629269789,
  x5 → 326.488110592899628218843767612, y2 → 388.150719812910558563238967901, y9 → 172.780369766975963048528414029}

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In[20]:= distanceSq = Flatten[Table[{j, i,  $\frac{(\text{posnsNew}[[i, 1]] - \text{posnsNew}[[j, 1]])^2 + (\text{posnsNew}[[i, 2]] - \text{posnsNew}[[j, 2]])^2}{r^2}$ }] /. solutionN,
    {i, 1, Length[posns]}, {j, 1, i - 1}], 1];

In[21]:= (* Errors, number too close *)
Length[Select[distanceSq, Last[#] < 4 &]]

Out[21]= 0

In[22]:= (* Touchers *)
Length[Select[distanceSq, Abs[Last[#] - 4] < 10-8 &]]

Out[22]= 16

In[23]:= (* Percentage improvement *)
{rBest / rKnown - 1, Log[rBest / rKnown]} * 100

Out[23]= {0.456225932857837905900900336, 0.455188376884860357055269969}

In[24]:= (* mm improvement in diameter *)
2 (rBest - rKnown) 127 / 360

Out[24]= 0.3942085910685544225068871078

In[25]:= (* How slight is the asymmetry? *)
{(y2 /. solutionN) / h, (h / 2 - y2 /. solutionN) 127 / 360}

Out[25]= {0.492645973658727998016543232430, 2.044051621556552951301808546}

In[26]:= (* Rattler to nearest neighbours, in radii *)
Map[{#[[1]], #[[2]], Sqrt#[[3]]}] &, Select[distanceSq, #[[2]] == 14 && Last[#] < 7 &]] // MatrixForm

Out[26]//MatrixForm=

$$\begin{pmatrix} 10 & 14 & 2.5371 \\ 11 & 14 & 2.01702 \\ 13 & 14 & 2.01702 \end{pmatrix}$$


In[27]:= (* Where should the rattler have been placed? *)
((y14 / r) /. Solve[(y14 - 3 r)2 == (2 x5 + 2 r - w)2 + (h - r - y14)2, y14][[1]]) /. solutionN

Out[27]= 5.0170243091291929686885508759

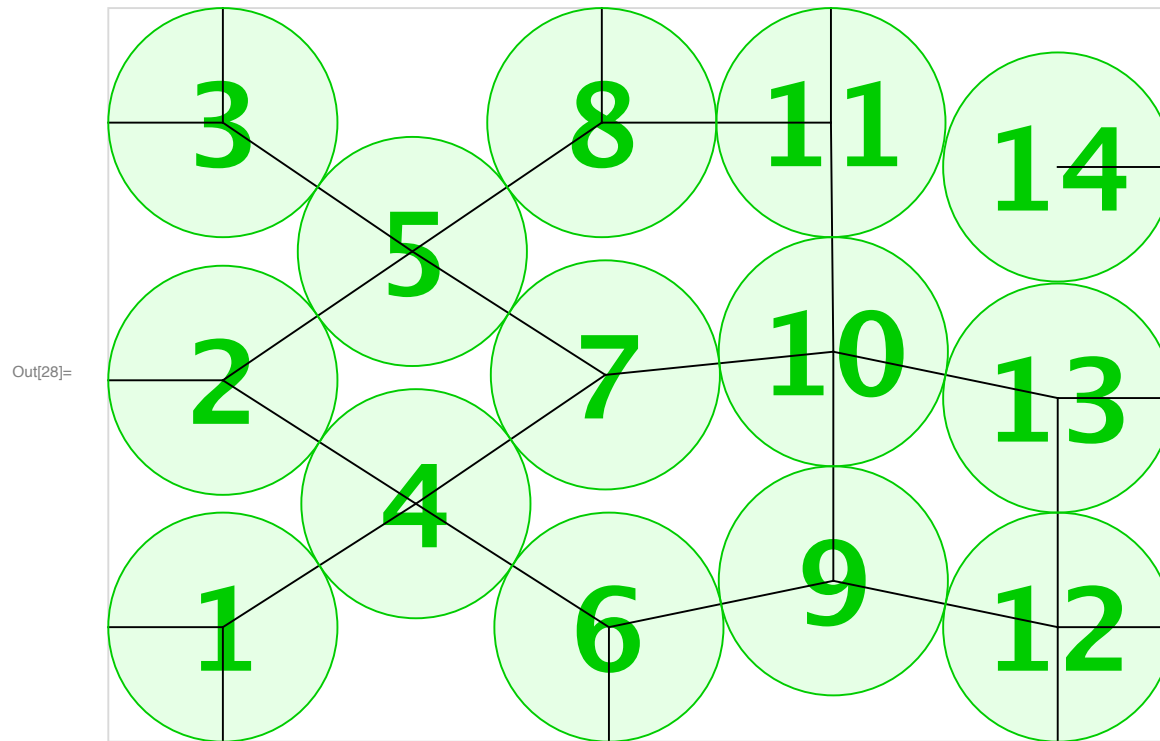
```

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In[28]:= layoutImage = Graphics[Flatten[{
  {{EdgeForm[{AbsoluteThickness[1], LightGray}], FaceForm[None], Rectangle[{0, 0}, {w, h}]}},
  Map[{RGBColor[0.9, 1.0, 0.9], Disk[#, rBest]} &, posnsNew],

  Map[Text[Style[ToString[#], FontSize → 60, Bold, RGBColor[0, 0.8, 0]], posnsNew[[#]] &, Table[i, {i, 1, Length[posnsNew]}]],
  Map[{AbsoluteThickness[1], Black, Line[{#, {0, #[2]}]}]} &, Select[posnsNew, Abs[#[1] - r /. solutionN] < 10-6 &]],
  Map[{AbsoluteThickness[1], Black, Line[{#, {#[1], 0}}]}]} &, Select[posnsNew, Abs[#[2] - r /. solutionN] < 10-6 &]],
  Map[{AbsoluteThickness[1], Black, Line[{#, {w /. solutionN, #[2]}]}]} &,
    Select[posnsNew, Abs[#[1] + r - w /. solutionN] < 10-6 &]],
  Map[{AbsoluteThickness[1], Black, Line[{#, {#[1], h /. solutionN}}]}]} &,
    Select[posnsNew, Abs[#[2] + r - h /. solutionN] < 10-6 &]],
  Map[{AbsoluteThickness[1], Black, Line[{posnsNew[[#[1]]], posnsNew[[#[2]]]}]}]} &,
    Select[distanceSq, Abs[Last[#] - 4] < 10-8 &]],
  Map[{AbsoluteThickness[1], RGBColor[0.0, 0.8, 0.0], Circle[#, rBest]} &, posnsNew],
  Map[{AbsoluteThickness[3], Red, Line[{posnsNew[[#[1]]], posnsNew[[#[2]]]}]}]} &, Select[distanceSq, Last[#] ≤ 4 - 10-8 &]]
}, 1], ImageSize → Scaled[0.6]]

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```
In[29]:= (* Possible element of PackingStyles, for margins precisely as above *)
"[ /Array /Positions " <>
StringJoin[Map["[" <> (#[[1]] // N // ToString) <> " " <> (#[[2]] // N // ToString) <> "]" " &, posnsNew]] <> "]"

Out[29]:= [ /Array /Positions [123.025 123.025] [123.025 388.151] [123.025 664.865] [330.31
255.588] [326.488 526.508] [537.595 123.025] [533.773 393.945] [529.952 664.865] [778.561 172.78]
[778.561 418.829] [776.001 664.865] [1019.53 123.025] [1019.53 369.074] [1019.53 617.217] ]
```



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In[30]:= (* Possible element of PackingStyles, being left-right mirror of previous *)
"[ /Array /Positions " <>
  StringJoin[Map["[" <> (w - #[[1]]) // N // ToString) <> " " <> (#[[2]] // N // ToString) <> "]" " &, posnsNew]] <> "]"
Out[30]:= [ /Array /Positions [1019.53 123.025] [1019.53 388.151] [1019.53 664.865] [812.241
255.588] [816.063 526.508] [604.956 123.025] [608.778 393.945] [612.6 664.865] [363.99 172.78]
[363.99 418.829] [366.55 664.865] [123.025 123.025] [123.025 369.074] [123.025 617.217] ]

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