CASE 1

Help, I'm Not Getting Any Younger!

Profile of a university professor:

- 45-year-old formerly athletic male
- 215 pounds
- 71 inches tall
- Exercises no more than once a week. Walks 0.5 miles daily to and from the car while carrying a 10-pound briefcase.
- Family history of adult diabetes.

I need help! My diet is terrible, and I have been gaining weight and feeling more tired.

I heard that Professor George Dantzig of Stanford once used linear programming to construct a diet. It would be great if *you* could tell me what to eat during each day. So, because I'm a firm believer in mathematical models, I want you to use linear programming to determine a reasonable diet for me to eat during a week. It is your job to collect data for use in the model.

I have the following requirements for the diet:

- I like variety. You cannot prescribe a diet in which I eat just one food during the entire week (like 10 boxes of Total cereal). I would like to eat at least 15 different foods during the week.
- You have to give me something from each of the four basic food groups (dairy, fruit and vegetable, meat, and grains)—not Mcfood, frozen food, pizza food, or food on a stick.
- I like nutrition. You cannot prescribe a diet that does not meet minimum daily requirements for essential minerals and vitamins. You cannot prescribe a diet in which I gain a lot of weight. I could stand to lose a few pounds.
- I hate Brussels sprouts, sweet potatoes, pears, and organ meats such as liver and kidney.
- Forget about any canned fruits or vegetables. Yuck.
- I do not eat any pork or pork products.
- I am not a big fan of frozen dinners, no matter how nutritious or convenient they are.
- I don't drink milk with any meal except breakfast.
- I work for the university, so I have a limited budget for food. Try to keep costs less than $100 per week (the lower the better).
- I might consider taking vitamin pills to get nutritional requirements, but I would rather eat food.

Key Questions

- What should I eat at each meal?
- If I allowed less variety, would your recommendation change?
- If I allowed more than $100 per week, would your recommendation change? How?
- What key minerals and vitamins constrain the solution?

CASE 2

Solar Energy for Your Home

As our ability to extract and process fossil fuels decreases, many people are looking to renewable resources to meet their energy needs. In particular, solar energy is becoming an advanced technology that has economic promise. In areas with large solar insolations, there can be enough energy to power an entire home. The amount of solar energy reaching the earth each year is many times greater than worldwide energy demand; it varies, of course, with location, time of day, and season. Sunlight is also a widespread resource and can be captured from virtually anywhere on earth.

There are two categories of home solar systems: passive and active. In a passive system, the solar energy heats a material that is used in a productive manner. For example, in Arizona it is common to use a passive solar system to heat swimming pools and the water used in the home. Every building has some of its heating requirements met by solar energy. Sunlight passing through windows is a source of heat, and the value of passive solar heating is enhanced by proper building insulation. A well-insulated building requires less energy for heating; thus, much of the heating load can be met by passive solar features. Optimum passive solar design begins with the layout of a building lot; a house must be oriented so that it can take full advantage of available solar energy.

Active systems are more complex and generally involve converting the solar energy to electrical energy. Photovoltaic (PV) cells use the energy of the sun to produce electricity. They produce none of the greenhouse or acid gas emissions that are commonly associated with the use of fossil fuels to generate electricity. The main barrier to increased use of this technology is cost. A common semiconducting material used in PV cells is single crystal silicon. Single crystal silicon cells are generally the most efficient type of PV cells, converting as much as 23% of incoming solar energy into electricity. The main problem with them is their production costs. Polycrystalline silicon cells are less expensive to manufacture but less efficient than single crystal cells (15% to 17%). Thin films (0.001–0.002 mm thick) of amorphous or uncrystallized silicon are another PV alternative. These thin films are inexpensive and may be easily deposited on materials such as glass and metal, thus lending themselves to mass production. Amorphous silicon thin-film PV cells are widely used in commercial electronics, powering watches and calculators. These cells, however, are not especially efficient—12% in the lab, 7% for commercial cells—and they degrade with time, losing as much as 50% of their efficiency with exposure to sunlight.

Solar power is an intermittent source of electricity. If PV cells are your only source, then the storage of electricity may be necessary. Electricity for a home can be stored in batteries, which can be expensive. Also, to generate sufficient electricity, you need a large area of collectors on your roof or somewhere on your property. The amount of solar energy captured depends on the surface area of the collectors and their conversion efficiency.

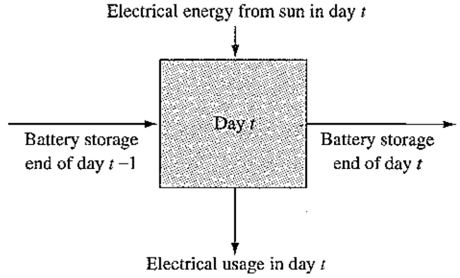
A solar energy system can often be looked at as a conservation system. Figure 1 depicts one way to look at the daily flow of energy.

Your job is to design an active solar system for a home in your area. For the analysis, you will have to collect data on:

- system cost and efficiency,
- daily solar insolation in your area (usually measured in watts/meter^2; this information can be found locally where weather data are stored and collected), and
- typical daily power requirements for a home in your area.

The costs of the system generally include a fixed component and variable components that depend on the total area of the PV collectors, the type of material used in the collector (usually only material is chosen), and the amount of battery storage needed. Your analysis should cover at least 6 months of data (12 months would be better because you would like your design to be appropriate for the entire year). You should assume that *all* energy requirements for the home will

FIGURE 1
Daily Energy Flow



be met by this system (no natural gas will be used for heating or cooking, for example).

Your design should include the following:

- the area of the PV collectors and the amount of battery storage that you need,
- an estimate of the cost of the system (you may include any tax advantages that accrue from the purchase of solar energy systems),
- a profile of the battery storage levels at the end of each day for a six-month period,
- an estimate of cost savings (or loss) over buying your electrical power from the local utility company.

CASE 3

Golf-Sport: Managing Operations

Golf-Sport is a small-sized company that produces high-quality components for people who build their own golf clubs and prebuilt sets of clubs. There are five components—steel shafts, graphite shafts, forged iron heads, metal wood heads, and metal wood heads with titanium inserts—made in three plants—Chandler, Glendale, and Tucson—in the Golf-Sport system. Each plant can produce any of the components, although each plant has a different set of individual constraints and unit costs. These constraints cover labor and packaging machine time (the machine is used by all components); the specific values for each component–plant combination are given in Tables 1–3. Note that even though the components are identical in the three plants, different production processes are used, and therefore the products use different amounts of resources in different plants.

Besides component sales, the company takes the components and manufactures sets of golf clubs. Each set requires 13 shafts, 10 iron heads, and 3 wood

TABLE 1
Product-Resource Constraints: Chandler

Products	Resources		
	Labor (Minutes/Unit)	Packing (Minutes/Unit)	Advertising ($/Unit)
Steel shafts	1	4	1.0
Graphite shafts	1.5	4	1.5
Forged iron heads	1.5	5	1.1
Metal wood heads	3	6	1.5
Titanium insert heads	4	6	1.9
Monthly availability (minutes)	12,000	20,000	—

TABLE 2
Product-Resource Constraints: Glendale

Products	Resources		
	Labor (Minutes/Unit)	Packing (Minutes/Unit)	Advertising ($/Unit)
Steel shafts	3.5	7	1.1
Graphite shafts	3.5	7	1.1
Forged iron heads	4.5	8	1.1
Metal wood heads	4.5	9	1.2
Titanium insert heads	5.0	7	1.9
Monthly availability (minutes)	15,000	40,000	—

TABLE 3
Product-Resource Constraints: Tucson

Products	Resources		
	Labor (Minutes/Unit)	Packing (Minutes/Unit)	Advertising ($/Unit)
Steel shafts	3	7.5	1.3
Graphite shafts	3.5	7.5	1.3
Forged iron heads	4	8.5	1.3
Metal wood heads	4.5	9.5	1.3
Titanium insert heads	5.5	8.0	1.9
Monthly availability (minutes)	22,000	35,000	—

heads. All of the shafts in a set must be the same type (steel or graphite), and all of the wood heads must be the same type (metal or metal with inserts). Assembly times for the sets at each plant are shown in Table 4.

Each plant of Golf-Sport has a retail outlet to sell components and sets, and the specific plant is the only supplier for its retail outlet. The minimum and maximum amount of demand for each plant–product pair is given in Table 5. Note that, although the minimums must be satisfied, you do not need to satisfy demand up to the maximum amount.

This planning problem is for two months. The costs in Table 6 increase by 12% for the second month, and production times are stationary. Inventory costs are based on end-of-period inventory for each product set and cost out at 8% of the cost values in Table 6. Table 7 lists the revenue generated by each product. Initially, there is no inventory.

The corporation controls the capital available for expenses; the cash requirements for each product are given in the last column of Tables 1–3. There is a total of $20,000 available for advertising for the entire system during each month, and any money not spent in a month is not available the next month. The corporation also controls graphite. Each shaft requires 4 ounces of graphite; a total of 1,000 pounds is available for each of the two months.

TABLE 4

Plant	Time (Minutes per set)	Total Time Available (Minutes)
Chandler	65	5,500
Glendale	60	5,000
Tucson	65	6,000

TABLE 5
Minimum and Maximum Product Demand per Month

Products	Store (or Plant) Chandler	Glendale	Tucson
Steel shafts	[0, 2,000]	[0, 2,000]	[0, 2,000]
Graphite shafts	[100, 2,000]	[100, 2,000]	[50, 2,000]
Forged iron heads	[200, 2,000]	[200, 2,000]	[100, 2,000]
Metal wood heads	[30, 2,000]	[30, 2,000]	[15, 2,000]
Titanium insert heads	[100, 2,000]	[100, 2,000]	[100, 2,000]
Set: Steel, metal	[0, 200]	[0, 200]	[0, 200]
Set: Steel, insert	[0, 100]	[0, 100]	[0, 100]
Set: Graphite, metal	[0, 300]	[0, 300]	[0, 300]
Set: Graphite, insert	[0, 400]	[0, 400]	[0, 400]

Your job is to determine a recommendation for the company. A recommendation must include a plan for production and sales. In addition, you should also address the following sensitivity-analysis issues in your recommendation:

- If you could get more graphite or advertising cash, how much would you like, how would you use it, and what would you be willing to pay?
- At what site(s) would you like to add extra packing machine hours, assembly hours, and/or extra labor hours? How much would you be willing to pay per hour and how many extra hours would you like?
- Marketing is trying to get Golf-Sport to consider an advertising program that promises a 50% increase in their maximum demand. Can we handle this with the current system or do we need more resources? How much more is the production going to cost if we take on the additional demand?

TABLE 6
Material, Production, and Assembly Costs ($) per Part or Set

Products	Plants Chandler	Glendale	Tucson
Steel shafts	6	5	7
Graphite shafts	19	18	20
Forged iron heads	4	5	5
Metal wood heads	10	11	12
Titanium insert heads	26	24	27
Set: Steel, metal	178	175	180
Set: Steel, insert	228	220	240
Set: Graphite, metal	350	360	370
Set: Graphite, insert	420	435	450

TABLE 7
Revenue per Part or Set ($)

Products	Plants Chandler	Glendale	Tucson
Steel shafts	10	10	12
Graphite shafts	25	25	30
Forged iron heads	8	8	10
Metal wood heads	18	18	22
Titanium insert heads	40	40	45
Set: Steel, metal	290	290	310
Set: Steel, insert	380	380	420
Set: Graphite, metal	560	560	640
Set: Graphite, insert	650	650	720

CASE 4

Vision Corporation: Production Planning and Shipping

Vision is a large company that produces video-capturing devices for military applications such as missiles, long-range cameras, and aerial drones. Four different types of cameras (differing mainly by lens type) are made in the three plants in the system. Each plant can produce any of the four camera types, although each plant has its own individual constraints and unit costs. These constraints cover labor and machining restrictions, and the specific values are given in Tables 8–10. Note that even though the products are identical in the three plants, different production processes are used and thus the products use different amounts of resources in different plants. The corporation controls the material that goes into the lenses; the material requirements for each product are given in the last column of Tables 8–10. A total of 3,500 pounds of material is available for the entire system during the planning period.

TABLE 8
Product-Resource Constraints: Plant 1

Products	Resources Labor (Hours/Unit)	Machine (Hours/Unit)	Material (Lb./Unit)
Small	3	8	1.0
Medium	3	8.5	1.1
Large	4	9	1.2
Precision	4	9	1.3
Total available	6,000	10,000	—

TABLE 9
Product-Resource Constraints: Plant 2

Products	Resources Labor (Hours/Unit)	Machine (Hours/Unit)	Material (Lb./Unit)
Small	3.5	7	1.1
Medium	3.5	7	1.0
Large	4.5	8	1.1
Precision	4.5	9	1.4
Total available	5,000	12,500	—

TABLE 10
Product-Resource Constraints: Plant 3

Products	Resources Labor (Hours/Unit)	Machine (Hours/Unit)	Material (Lb./Unit)
Small	3	7.5	1.1
Medium	3.5	7.5	1.1
Large	4	8.5	1.3
Precision	4.5	8.5	1.3
Total available	3,000	6,000	—

Transport has 3 major customers (RAYco, HONco, and MMco) for its products. The maximum sales for each customer–product pair is given in Table 11. Product sales prices are given in Table 12, and the shipping costs from each plant to each customer are detailed in Table 13. Table 14 contains the production costs for each product–plant pair.

All shipping from plants 1 and 2 that goes to RAYco or HONco must go through a special inspection. These units are sent to a central site, inspected, and then sent to their destination. The capacity of this special inspection site is 1,500 pieces.

Your job is to determine a recommendation for the company. A recommendation must include a plan for production and shipping as well as the cost and revenue generated from each plant. In addition, you should address the following potential issues in your recommendation:

- If you could get more material, how much would you like? How would you use it? What would you be willing to pay?
- If you could get more inspection capacity, how much would you like? How would you use it? What would you be willing to pay?

TABLE 11
Maximum Product Sales ($) per Unit

Products	Customers RAYco	HONco	MMco
Small	200	400	200
Medium	300	300	400
Large	500	200	300
Precision	200	400	300

TABLE 12
Product Sales Price ($) per Unit

Products	Customers RAYco	HONco	MMco
Small	17	16	16
Medium	18	18	17
Large	22	22	23
Precision	29	26	27

TABLE 13
Shipping Costs ($) per Unit

Plant	Customers RAYco	HONco	MMco
1	1.0	1.6	1.1
2	1.2	1.5	1.0
3	1.4	1.5	1.3

TABLE 14
Production Costs ($) per Unit

Products	Plant 1	2	3
Small	14	13	14
Medium	16	17	15
Large	18	20	19
Precision	26	24	23

- At what plant(s) would you like to add extra machine hours? How much would you be willing to pay per hour? How many extra hours would you like?
- Marketing is trying to get RAYco to consider a 50% increase in its demand. Can we handle this with the current system or do we need more resources? How much more money can we make if we take on the additional demand?

CASE 5

Material Handling in a General Mail-Handling Facility†

For more than 200 years, the United States Postal Service (USPS) has delivered mail across the country. Daily delivery goes to some 137 million households; in 2001, the USPS processed and delivered more than 207 billion pieces of mail to a delivery network that grew by 1.7 million new addresses. Clearly, the USPS is the largest material handler (in terms of pieces) in the world. Statistics recorded by Pricewaterhouse Coopers show that 94% of first-class mail destined for next-day delivery received overnight service—and this was a record performance for a second straight year. Despite the high volume, the USPS managed to cut costs by $900 million in 2001 while maintaining record service performance and high levels of customer satisfaction.

To process mail quickly, one must use advanced mechanization. Mail-sorting machines can process 10 letters per second (we are long past the days of hand sorting in front of a large set of post boxes). Sorting using the zip+4 standard can result in a mail sort down to an individual carrier's walk sequence, which saves significant carrier time.

Five major operations can be performed on each letter, and each operation has its own machine:

Automatic facer and canceller (AFC) This machine cancels the stamp and orients all of the letters so that the stamp is in the upper-right corner. This machine also separates mail into one of three streams—automation, mechanization, or manual.

Letter sorting machine (LSM) This machine is semi-automated and helps human operators sort mail. The operator reads the address and then types in a destination code. The machine then routes the letter to the appropriate bin.

Optical character reader (OCR) This machine reads handwritten or typed addresses and then prints a machine-readable barcode on the envelope.

Barcode sorter (BCS) This machine reads the barcode on the letter (either printed by the OCR or by the sender's equipment) and then sorts it to a bin.

†Based on work done jointly with Ron Askin and Sanjay Jagdale, 1994.

Delivery barcode sorter (DBCS) This machine does a two-pass sort that uses barcodes with the zip+4 code and sorts down almost to a walk sequence. When using the DBCS, the result is such that a carrier requires little to no processing at the carrier station to deliver the mail.

Items known as *flats* (for example, magazines and 8.5 × 11–inch envelopes) also are processed through the system. A flat-sorting machine (FSM) is used in a semiautomated process. An operator loads a piece onto the machine and keys in a code based on the flat's address; the machine then routes the piece to an appropriate bin.

A letter that enters a general mail facility (GMF) follows a routing that depends on the machine readability of the address and the presence of an existing barcode. Although many routes are possible, the major ones are given in Figure 2. Once letters go through the AFC, they are stored in cardboard or plastic trays that hold approximately 400 letters. The letters are moved in these trays throughout the facility, and each machine sorts the mail into different trays.

As part of quality improvement, the post office is always looking for ways to cut costs while expediting mail processing. To meet goals for overnight delivery and three-day cross-country delivery, letters that arrive by 6 p.m. from box pickup must be processed that night and be on planes or trucks for the next destination. Because the sorting and character-reading machines cost millions of dollars each, increasing machine use and saving the purchase of even one machine in a GMF is a significant achievement.

FIGURE 2
Processing Routes for Letters

AFC → OCR → BCS → DOCK
AFC → BCS → DBCS → DOCK
AFC → OCR → LSM
AFC → LSM → DOCK
AFC → OCR → DOCK
AFC → BCS → DOCK
BCS → DBCS → DOCK

One of the keys to faster processing and increasing utilization is an effective material-handling and data system. Each tray has a barcode that describes the salient characteristics of its mail. When a sorting machine is ready for operation (say, for example, that we are going to sort down to a group of 10 zip codes), a call goes out to bring all trays with appropriate mail to the appropriate BCS machine. The data system must (1) know where those trays are located, (2) go and get them, (3) bring the trays to the machine-input area, and (4) exit the area. The faster this can be done, the better.

The network for our GMF is given in Figure 3. Each machine has an input and output point. For example, nodes 1 and 28 are the inputs and output, respectively, for AFC 1. For this application, the material-handling system is an overhead monorail. Carriers that hold one tray circulate around the system to pick up and deliver trays; they rest in the parking lot when not in use. The arcs in the diagram are the links of the monorail; all links are one-way. The dotted lines on Figure 3 represent links that are above the machine level and offer shortcuts across the facility. The facility also contains switches (nodes 29, 18, and 32) that allow carriers to change directions. Node 34 is the link to the shipping dock, and all trays enter and exit the system at this point.

The carriers travel at approximately 1 mile per hour, and there must be 15 feet between carriers on the same link. For the purpose of this study, assume that there is a bypass at each node so a carrier can pass other carriers that are stopped for loading and unloading operations. Also assume that the switches operate quickly relative to the speed of the vehicles so that collisions do not occur and the switch capacity is not constraining. Figure 3 is drawn approximately to scale. The facility is approximately 220 feet long by 160 feet wide. At 1 mile per hour, it takes a carrier approximately 2.5 minutes to run the length of the facility (from node 14 to node 1, for example).

Table 15 contains the tray movement loads for the peak hour. Each load has an origin node, a destination node, and the number of trays that must be moved. Each load is a leg in the route for a particular tray of mail. The system must have capacity to move an empty carrier from the parking lot to the origin, load the tray, move the tray to the destination, unload the tray, and then return to the parking lot. All carriers are dispatched from the parking lot because this simplifies the logic of the scheduler. By capacity, there must be a sufficient number of vehicles and the capacity on each link between nodes cannot be overloaded. As-

FIGURE 3
General Mail Facility: Track Layout

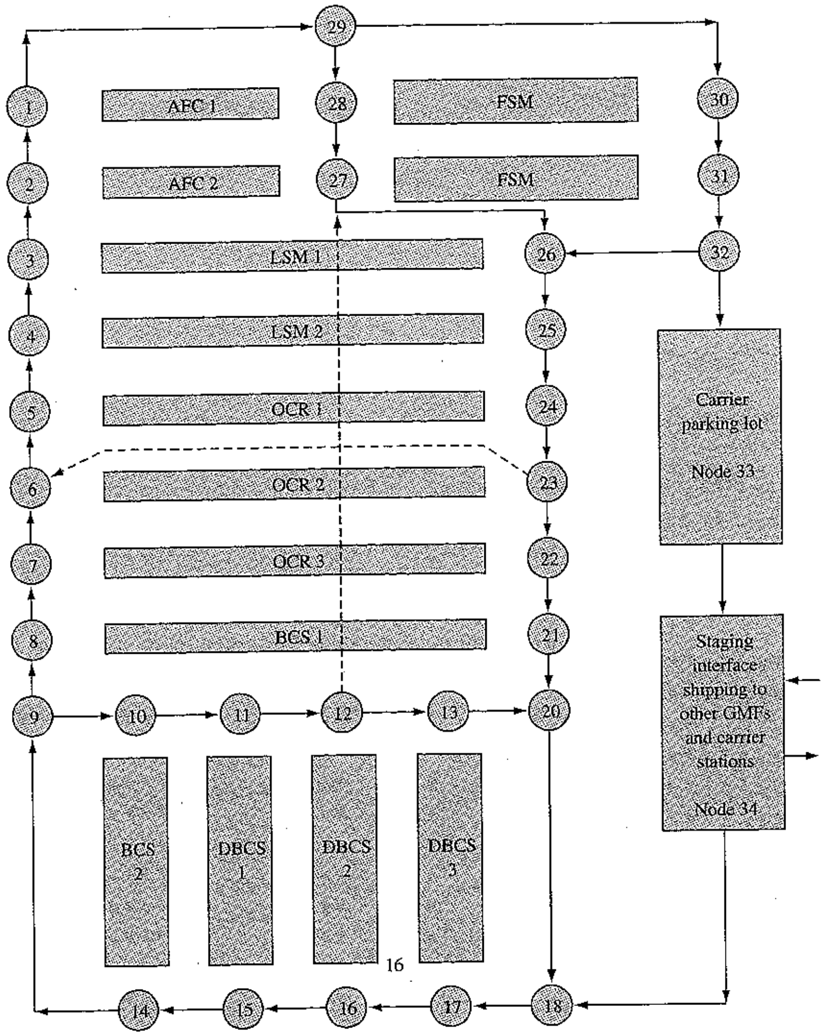


TABLE 15
Load Data for the Peak Hour

Load Number	Origin Node	Destination Node	Number of Trays
1	33	1	15
2	28	4	20
3	22	14	30
4	10	33	30
5	24	8	15
6	21	33	30
7	24	4	5
8	25	33	15
9	27	17	15
10	13	33	40

sume that it takes one minute to perform a loading or unloading operation.

Your jobs are to:

- Determine if this system has enough material-handling capacity for moving the trays in the peak hour (we generally design for peak hour so that we are sure that the system will not get bogged down when demand is high).
- Suggest where we might add extra track to relieve capacity congestion. This should be minimized because track cost is high.
- Determine the flows of trays through the network during the peak hour. Which routes are chosen for each load?

- Determine places in the network that are risky—that is, if a link goes down, machines can be cut off from the material-handling system.
- Estimate the total carrier travel distance during the peak hour.
- Investigate the effect of reducing the intercarrier spacing. You need space between carriers to prevent carrier collisions. If we put better sensors on the front of the carriers, they can stop more quickly and we can have less spacing.

Some tips:

- This is a difficult problem. Be patient and try not to become discouraged.
- Do not forget the empty carrier movements to the origin and from the destination.
- Compute the capacity on each arc. Initially, assume a single lane. To increase capacity, consider multiple lanes between nodes or consider adding arcs to give more paths between origin nodes and destination nodes.
- A precise formulation of this problem can be larger than most problems you have seen. You should not undertake to solve a large-sized formulation unless you have software that can handle large problems. Some approximate formulations are more manageable, but they still can require hundreds of variables and constraints.

CASE 6

Selecting Corporate Training Programs†

Introduction

Training has become a major cost of doing business. A 1995 survey of all U.S. businesses with 100 or more employees revealed that approximately $52 billion was being spent on training; it has been estimated that $90 to $100 billion is being spent for training overall. Developing strategies to implement cross-training is a current topic in the operations research (OR) literature. Management consultants advocate aggressive education and professional development to remain competitive in the global and local markets. Employees now expect job and skill growth to be a major component of their duties.

Increased training costs have occurred for many reasons. Employees view training in the form of formal degrees and documented technical skills as important for job security. Technology is changing at a rapid pace. It has been claimed that high schools and universities are not producing the skills needed by industry, so industry must train and reeducate recent graduates. For high school graduates, this may include training in technology-based skills; for college graduates, this may include developing nontechnical skills such as leadership, communications, interpersonal relations, and ethics.

Problem Environment

For a corporation, the primary purpose of training is to ensure that employees have the key skills needed to effectively manage and operate the business. There are many options for providing training. For example, to train staff members in computer skills, a corporation may use any of the following strategies:

- hiring an outside consultant to develop and present an on-site training course,
- using corporate personnel to develop and present an on-site training course,
- purchasing a training course and having employees use it for self-study,
- contracting with a local college or university to provide training, or
- sending employees to an off-site training seminar.

The above possibilities are for a single skill. The purpose of many training programs, however, is to give employees a broad set of skills. Often the skill sets of two or more programs partially overlap. When this happens, the corporation must choose the set of programs that give employees the required skills for their jobs and the appropriate employees for each training program. In any case, training decisions made in an ad-hoc "pay-as-you-go" manner will be inefficient and generally result in additional expense.

To give the decision problem structure, the following assumptions are appropriate:

- We have a known study period—for example, the next 3 or the next 5 years—over which we need to plan training. The study period should fit with the overall business strategy and enable accurate estimates for training needs and available resources.
- There is a known set of skills that employees need. Among others, these may include technical, interpersonal, communication, and management skills.

†Based on work done jointly with John V. Farr, and David A. Thomas at USMA, 1995.

TABLE 16
Skills List

No.	Skill	No.	Skill
1	New employee orientation	22	Stress management
2	Performance appraisals	23	Computer programming
3	Personal computer apps	24	Diversity
4	Leadership	25	Data processing/MIS
5	Sexual harassment	26	Planning
6	Team building	27	Public speaking and presentation
7	Safety	28	Strategic planning
8	Hiring and selection process	29	Writing skills
9	New equipment operation	30	Negotiating skills
10	Training the trainer	31	Finance
11	Product knowledge	32	Marketing
12	Decision making	33	Substance abuse
13	Listening skills	34	Ethics
14	Time management	35	Outplacement and retirement
15	Conducting meetings	36	Creativity
16	Quality imiprovements	37	Purchasing
17	Delegation skills	38	Smoking cessation
18	Problem solving	39	Financial and business literacy
19	Goal setting	40	Reengineering
20	Managing change	41	Foreign language
21	Motivation		

TABLE 17
Salary and Skills Required for Each Job Classification

Person	Salary ($)	Skills 1–20
Senior Manager	250,000	0 1 0 1 0 0 0 0 0 0 0 1 1 1 0 1 0 1 1 0
Project Manager	200,000	0 1 0 0 1 0 0 0 0 0 1 1 1 1 1 0 0 1 0 0
Professional	150,000	1 1 0 0 1 1 1 0 1 0 0 0 1 0 1 0 0 0 0 1
Sales	150,000	1 1 0 0 0 0 0 0 1 1 0 0 1 0 0 0 1 0 0 1
Technician	100,000	1 1 1 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1
Administrative Assistant	80,000	1 1 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Person	Skills 21–41
Senior Manager	1 1 1 1 0 1 1 0 1 0 1 1 0 1 0 1 1 1 0 0 0
Project Manager	0 1 1 1 0 1 1 1 1 0 1 0 1 0 0 1 1 1 1 0 1
Professional	0 0 0 0 1 1 0 0 0 1 0 0 1 0 1 0 0 1 1 0 1
Sales	0 0 0 0 1 1 0 0 0 1 0 0 1 0 1 0 0 1 1 1 1
Technician	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
Administrative Assistant	0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 1 1 0

TABLE 18
Enrollment Cost and Skills of Each Program

Program	Enrollment Cost ($)	Skills 1–20	Skills 21–41
Program 1	500	1 1 0 1 0 0 0 0 0 0 1 1 1 1 1 0 1 0 0 0	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
Program 2	300	0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0	0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
Program 3	500	0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 1 0 0 0 1	1 1 1 1 1 1 1 0 1 0 1 0 0 0 0 1 0 0 0 0 0
Program 4	575	0 1 0 1 0 0 0 0 1 1 0 0 1 0 0 0 1 0 0 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 1
Program 5	800	0 1 1 1 0 0 0 0 0 1 0 0 1 0 0 0 1 0 0 0	0 0 0 1 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 1 0
Program 6	400	0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1	0 0 0 0 0 0 0 0 1 0 0 1 1 0 0 1 1 0 0 0 0
Program 7	200	0 0 0 0 1 1 0 0 0 0 0 0 1 0 0 0 0 0 0 1	0 1 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0
Program 8	1,000	0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 0 0 1 0	1 1 1 1 0 0 1 0 1 1 0 0 0 0 0 0 1 0 0 0 0
Program 9	200	1 1 1 1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0	0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0
Program 10	500	0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 1 1 0 0 0
Program 11	700	0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0	0 1 1 0 0 0 1 0 1 1 0 1 0 0 1 1 0 0 0 0 0
Program 12	600	0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0	0 0 0 0 0 0 0 1 0 1 1 0 0 0 0 0 0 1 1 1 0
Program 13	400	0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 1 1 1 0	1 0 1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0
Program 14	900	1 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1 0 1 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 1
Program 15	700	1 0 1 0 1 0 1 1 1 0 0 0 0 0 0 0 0 0 1 0	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 1

- Employees are divided into classes. In each class, we have estimates for (1) the number of employees, (2) the employee hourly wage, (3) the number of employees that require each particular skill, and (4) the maximum time available for training employees in each class during the study period.
- There is a list of training programs. For each program, we assume we have (1) the set of skills taught, (2) the cost, (3) the development time, (4) the completion time for an employee, and (5) the maximum number of employees who can participate per decision cycle.
- Training is equally effective for all people, thus we are concerned with which programs to offer and which employees in each class to assign to each program. If we know the quality of the training for individual skills for individual classes, then we can relax this assumption.

Potential Corporate Setting

Your job is to develop models to aid businesses and corporations in determining the appropriate training programs to use. The type of model and issues often depend on the size of the corporation and the potential

TABLE 19
Interfering Programs

Program Number (Days Long)	Programs that Interfere			
Program 1 (2)	3	5	8	
Program 2 (2)	3	7	10	
Program 3 (4)	1	2	12	
Program 4 (3)	6	7	14	
Program 5 (2)	1	9	12	
Program 6 (3)	4	7	11	14
Program 7 (5)	2	4	6	
Program 8 (2)	1	10	13	
Program 9 (3)	5	15		
Program 10 (3)	2	8		
Program 11 (2)	6	12	15	
Program 12 (4)	3	5	11	
Program 13 (3)	8	14		
Program 14 (4)	4	6	13	
Program 15 (3)	9	11		

uses of the models. For large corporations, there are many employees in each class, so it is not necessary to model and schedule down to the individual employee. Concentrate instead on the assignment of classifications to programs and ignore the assignment of specific individuals to programs. Also, sufficient resources exist to develop internal training programs, hence you should consider program development costs as well as employee costs (lost work time, travel, lodging, meals, course materials) in the objective. A large corporation can use the model to plan the development of courses. This will help determine (1) program-development costs so that in-house programs are cost-effective and (2) appropriate programs for each employee classification so that, on average, there is sufficient time to complete the assigned programs within the available time.

For small businesses, the focus is often different. Typically, these companies do not develop in-house programs because they do not train enough employees to justify development costs. Because the number of employees is small, it is important to model down to the employee level and schedule employees so that both training and job tasks can be completed.

Your OR consulting firm has been hired to design the training program for a small company. There are no in-house classes, and vendors provide all training. The company has determined 41 skills that are important for its employees; these are listed in Table 16. There are six employees; the salary level and skills required for each person are given in Table 17. You can assume that there are 250 working days per year. There are 15 programs available for use; Table 18 contains the cost per person and the skills covered for each program. In Table 18, a 1 in the row for program p and the column for skill s implies that program p contains skill s. Table 19 lists the programs that conflict in time with other programs (for example, programs 3, 5, and 8 conflict with program 1). An employee cannot take two programs simultaneously. It is company policy that each employee is limited to 15 days for training per year.

Key Questions

Your job is to develop a recommendation for the company for addressing its training needs. In particular, you should address the following key questions:

- Which training programs should we be using? What is the assignment of personnel to those programs?
- Identify programs with heavy use that may justify the development of an in-house course. How much would you be willing to pay for that development if you could use the program for the next three years?
- We have the opportunity to negotiate prices for programs. Which programs would you suggest are candidates for negotiation?
- What skills are especially expensive for us to cover? If we were to develop our own programs, what skills should be covered?
- Would your recommendation change if we allowed more days of training per year?

CASE 7
BestChip: Expansion Strategy

BestChip (BC) is a large nationwide corporation that produces low-fat snack products for an expanding market (pun intended). Basically, BC takes materials (corn, wheat, and potatoes) and turns them into two types of snacks: chips (regular and green onion) and party mix (one variety). BC is expanding into the western United States and is considering sites for locating production facilities.

BC currently has eight candidate sites. Table 20 shows the sites' purchase prices and the purchase and shipping cost per ton of each material to each site.

The purchase cost represents the yearly amortized cost of opening and operating the site (exclusive of

TABLE 20
Site Information and Material Shipping Cost

Site Location	Purchase Cost ($/Year)	Material Shipping Cost ($/Ton)		
		Corn	Wheat	Potato
Yuma, AZ	125,000	10	5	16
Fresno, CA	130,000	12	8	11
Tucson, AZ	140,000	9	10	15
Pomona, CA	160,000	11	7	14
Santa Fe, NM	150,000	8	14	10
Flagstaff, AZ	170,000	10	12	11
Las Vegas, NV	155,000	13	12	9
St. George, UT	115,000	14	15	8

TABLE 21
Demand Information

Company	Location	Demand		
		Regular	Green Onion	Party Mix
Jones	Salt Lake City	1,300	900	1,700
YZCO	Albuquerque	1,400	1,100	1,700
Square Q	Phoenix	1,200	800	1,800
AJ Stores	San Diego	1,900	1,200	2,200
Sun Quest	Los Angeles	1,900	1,400	2,300
Harm's Path	Tucson	1,500	1,000	1,400

shipping costs). Each site may produce as many as 20,000 tons of product per year.

BC has six major customers, and all demand is shipped by truck from the plant to the customer warehouse. The shipping cost depends on the tonnage and distance and comes to $0.15 per ton-mile. The customers, their location, and their yearly demand in tons for each product are listed in Table 21. You must meet demand.

The makeup of the products does not depend on the production plant. Table 22 gives the product-ingredient mix data. The company requires that we consolidate our business, so we cannot locate plants in more than two states.

For this analysis, ignore the differences in property and income tax rates between the states (this is usually critical, but it gets us far afield of the key issue of math programming). In addition, many critical factors actually determine locations; for example, the method of financing the site purchase will also be a major factor in the decision—but we will ignore that also.

Your job is to determine how we should expand into the west and develop alternatives. Questions you should answer include:

TABLE 22
Product-Ingredient Mix

Product	Ingredient		
	Corn	Wheat	Potato
Regular chips	70	20	10
Green onion chips	30	15	55
Party mix	20	50	30

- What sites should be selected? How should the customers be served?
- If gasoline gets more expensive and our trucking costs change, then how is the recommendation affected?
- If rail freight costs for material shipping increase, then how is the recommendation affected?

Please consider other sensitivity-analysis issues that you feel might be important for management's decision-making process.

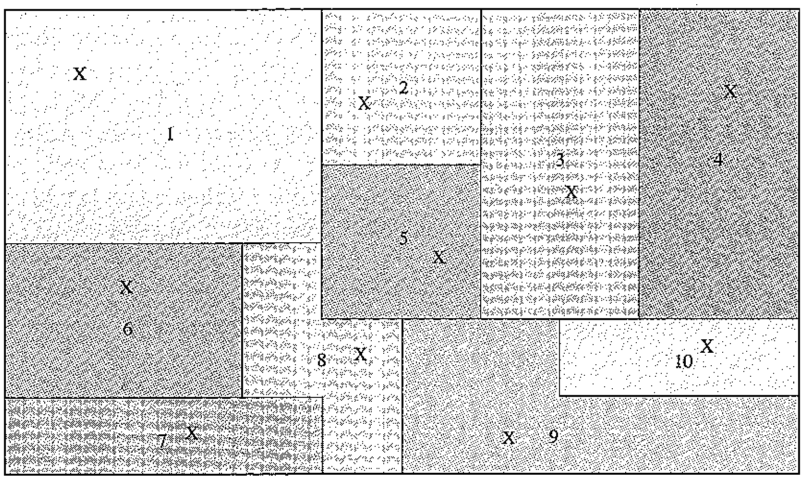
CASE 8
Emergency Vehicle Location in Springfield

You are the logistics manager for the Springfield Fire Department. You are to develop a recommendation for providing emergency service to Springfield. The department's resources include engine trucks, ladder trucks, and paramedic vehicles. The budget suggests a total of 15 vehicles are fundable in the coming year. Currently, seven engines, three ladder trucks, and five paramedic vehicles are in operation. This system runs 24 hours per day.

The city has been divided into 10 zones (see Figure 4). The map is drawn to scale. For each zone, the department has estimated the number of fire calls, the number of false alarms, and the number of medical calls per 12-hour day. These data are listed in Table 23. Currently, there are five fire stations in the city; these are listed in Figure 4. Each existing station costs $20,000 per year to operate. The yearly costs for each potential station (including the amortized cost of construction) are also listed in Table 23. Each station can hold two vehicles at most.

For fire calls, an engine and a ladder truck must respond. For medical calls, a paramedic vehicle always responds and an engine also goes when one is available and closer than the nearest paramedic vehicle. On average, fire calls take 2.5 hours, false alarms 10 minutes, and medical calls 45 minutes.

FIGURE 4
Map of the City (17 miles by 11 miles)



X marks the spot of the existing sites and the possible sites in each zone

TABLE 23
Demand Information per Year

Zone	Fire Calls	False Alarms	Medical Calls	Base Cost ($/Year)
1	100	200	1000	40,000
2	50	100	450	Existing
3	75	100	600	35,000
4	120	75	1300	50,000
5	150	100	1400	Existing
6	300	150	1000	50,000
7	200	100	800	Existing
8	250	175	1000	Existing
9	100	25	900	Existing
10	75	50	650	35,000

When vehicles are dispatched on a call, the closest idle vehicle is dispatched first. If no vehicles are idle, then the call must be sent to a private provider; these responses cost the city \$5,000 per medical call, \$15,000 per fire call, and \$200 per false alarm. There is no queueing. The street network is largely rectangular, and the fire department estimates that the cost per mile for travel is \$1.50 per mile for engines and ladders and \$0.75 per mile for paramedic units.

Your job is to design a system for the fire department. The questions that should be considered are these:

- What sites should be selected and how should the vehicles be distributed?
- If travel gets more expensive, how is the recommendation affected?
- If the cost of using the private provider increases, then how should the system be changed?
- Is all of this equipment needed to serve the public?
- How much more demand can be handled with the full complement of vehicles?

Your write-up should include a description of your models and any assumptions made in model formulation. You will have to make simplifying assumptions because this problem has details that may be difficult to model. There are *many* ways to model parts of this system, and you can use different approaches to answer different questions. You may use Excel or LINDO, or you may use heuristics. Your call will depend on your modeling approaches.

Hint: This case is less specific and has vague components; simplify as a first approach and then get more complex. If you try to include everything, you will become frustrated because this does not fit any standard modeling paradigm.

CASE 9

System Design: Project Management†

System Design (SD) is a small corporation that contracts to manage systems and industrial engineering projects. In this case it must manage the design and construction of a power plant's data-processing and data-collection system. SD's role in the project is to hire subcontractors, ensure each task is completed within specification, determine how much labor to assign to each task, and generally ensure the project's success.

SD is really a subcontractor within the larger project of building the power plant. Table 24 details SD's plant-construction and data-system-design tasks. SD is directly in charge of tasks 2, 6, 7, 10, 14, 15, and 19. The remaining tasks in Table 24 interact with those in SD's charge. Assume that the remaining tasks (1) will start whenever their predecessor tasks are complete and (2) will finish exactly after their duration.

To shorten the seven SD tasks, you must pay additional labor and overhead costs. Table 25 lists the functions you can use to compute the cost of changing each task duration to a new value. (*Note:* t_j is the original duration of task j; d_j is the minimum duration of task j.) The table also lists the lowest possible task-duration value. You may not increase the duration of any task.

The revenue that SD obtains from the project depends not only on its tasks but also on when the total project is completed. The project is due at day 900, and SD receives the contract price of \$600,000 if the project is done then. If the project is finished x days early, then SD receives a total of $\$600,000 + \$15,000x^{0.7}$ in revenue. If SD finishes x days late, then it receives $\$600,000 - \$20,000x^{1.4}$ in revenue.

Expediting tasks can be profitable and necessary to meet deadlines, although employees do not really like it. Task completion quality is a function of the task completion time, and we would like to have a high quality. This may conflict with our objective of maximizing profit. Because quality affects future revenues, it is difficult to estimate the dollar impact of poor quality. If t_j is the original duration of task j and x_j is the expedited duration time for task j, then quality, measured on a scale of 0–100 (with 100 being the best), can be represented by the function

$$100 - \min\,[100, (t_j - x_j)^{2.2}]$$

This is only the quality for a task. It is unclear how one might quantify the quality of the project.

Your job is to determine how we should proceed with our tasks. Your analysis should answer some of the following questions:

- What tasks are critical to project completion? What tasks will you expedite?
- How are you measuring system quality, and how does your recommendation measure up relative to that objective?

†This material is expanded from a homework problem in *Applied Mathematical Programming* by Wayne Winston.

TABLE 24
Task Information

Task No.	Task Name	Task Duration (Days)	Immediate Predecessor Tasks
1	Preliminary system description	40	—
2	Develop specifications	100	1
3	Client approval	50	2
4	Develop input-output summary	60	2
5	Develop alarm list	40	4
6	Develop log formats	40	3, 5
7	Software definition	35	3
8	Hardware requirements	35	3
9	Finalize input-output summary	60	5, 6
10	Analysis performance calculations	70	9
11	Automatic turbine startup analysis	65	9
12	Boiler guides analysis	30	9
13	Fabricate and ship	200	10, 11, 12
14	Software preparation	80	7, 10, 11
15	Install and check	130	13, 14
16	Termination and wiring lists	30	9
17	Schematic wiring lists	60	16
18	Pulling terminals and cables	60	15, 17
19	Operational test	125	18
20	First firing	1	19

TABLE 25
Expediting Costs ($1,000) and Limits

	Duration (Days)		
Task No.	Current t_j	Minimum d_j	Cost to Decrease t_j by x Days
2	100	70	$1.5x^{1.8}$
6	40	20	$2x^{2.0}$
7	35	20	$1x^{2.0}$
10	70	40	$1.8x^{1.9}$
14	80	60	$1.9x^{1.6}$
15	130	120	$0.95x^{2.7}$
19	125	80	$0.9x^{2.9}$

- How much money do you make on the project?
- What will you give the decision maker to help with the decision?
- If we can move minimum duration days to lower values, then which values would you like to reduce?
- If you could control additional tasks by paying more money, which ones would you like to take, and how much would you be willing to pay for control?

CASE 10

Modular Design for the Help-You Company

The Help-You Company is in the business of manufacturing first-aid kits for cars, hikers, campers, sports teams, and scouting groups. The company is located in Tucson, Arizona, and all materials must be sent to Tucson and then shipped to customers' warehouses. The company has done extensive market surveys of its customers; Table 26 shows its estimates for the demand for its kits in the coming year.

Each kit contains the individual items shown in Table 27, which are listed with their base sizes in pounds. Help-You, for example, can buy packs of Acetaminophen extra-strength caplets. Each pack contains 12 tablets and costs Help-You $1.50.

Help-You buys the individual items and then assembles kits based on the requirements for each part in each kit as shown in Table 28.

These are minimum requirements in that the customer expects at least the listed quantity of each item in each specific kit. For example, in the kit for campers, there must be at least four blankets and at least three cold units (six cold packs) as well as the other items.

There are two strategies available for assembly of the kits:

- In *direct assembly,* the exact requirements are put into each kit.
- In *modular assembly,* one or more standard modules are developed that can be assembled and combined into a kit with enough modules so that the minimum requirements for each item are met. A graphic of the approach is detailed in Figure 5.

If you design a module, for example, that has two units of Band-Aids (as well as the other items) and place three of these modules in a scouting kit, then the kit will have 3*2 = 6 units of Band-Aids; this will meet the requirement of four units of Band-Aids for scouting kits. In this example, there is an "overage" of two extra units of Band-Aids that costs

$$2 * \$1 \text{ per unit} = \$2$$

per each scout kit demanded. Also, the total unit content in a module cannot weigh more than 15 pounds (an assembly requirement).

Direct assembly meets requirements exactly, although it usually has higher labor costs than modular assembly. For storing inventory, modules are easier to use because they tend to be smaller than kits.

Develop a strategy for modular assembly. The key costs of the modular system are the overages that occur. Your strategy must include the following:

- the number of modules you are designing (the more modules you have, the closer you can match requirements exactly, although the higher the costs for assembly and inventory);
- the unit content of each module designed; and
- an estimate for the total number of each module required.

Your analysis should consider issues such as:

- the trade-off between the number of different modules designed and the total overage cost (you do not need to try more than five different modules—why?);

TABLE 26
Kit Demand

Type	Number of Kits Sold
Cars	1,000
Hikers	800
Campers	100
Sports teams	200
Scouting groups	300

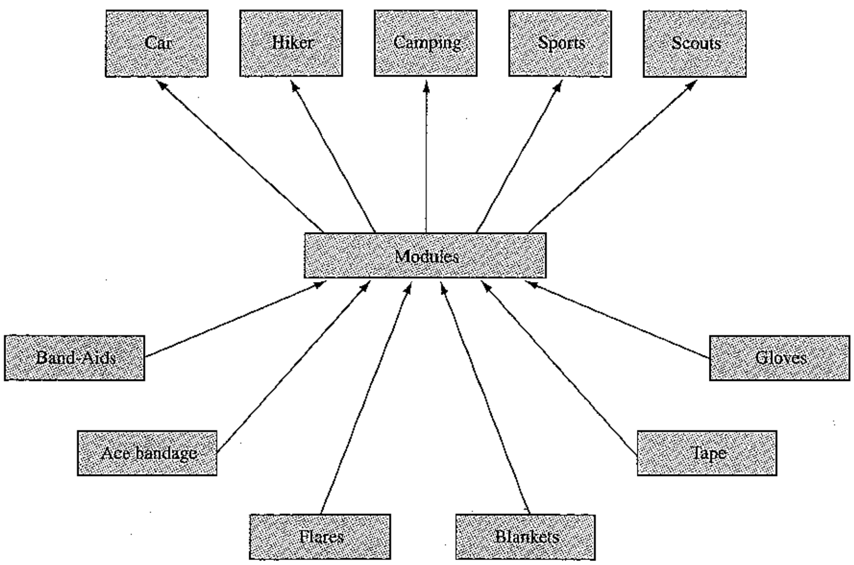
TABLE 27
Item Cost and Base Size

Item	Cost ($)	Base Unit	Base Unit Weight (Lb.)
Adhesive Band-Aids	1	10 per pack	0.20
Ace bandages	2	1 bandage	0.20
Flares	4	3 per pack	1.00
Blankets	15	1 blanket	2.00
Adhesive tape	2.50	1 roll	0.40
Cold packs	4	2 per pack	0.80
Sunburn cream	3.50	1 tube	0.40
Antiseptic cream	2	1 tube	0.50
Acetaminophen extra-strength caplets	1.50	12 tablets	0.30
Rubber gloves	1.50	3 pairs	0.20

TABLE 28
Item to Kit Requirements (Base Units)

Kit Item	Cars	Hikers	Campers	Sports Teams	Scouting Groups
Ace bandages	1	2	4	12	6
Band-Aids	0	2	4	4	4
Flares	2	1	1	0	2
Blankets	1	1	4	2	3
Adhesive tape	2	2	3	6	4
Cold packs	2	2	3	6	3
Sunburn cream	1	2	4	4	5
Antiseptic cream	1	2	3	2	4
Acetaminophen caplets	1	2	4	6	6
Rubber gloves	1	1	2	10	5

FIGURE 5
Modular Assembly



- the sensitivity of your solution to the kit demand estimates (for example, what happens to your recommendations if the number of scouting kits sold changes by 20%?);
- the sensitivity of your recommendations to the unit cost values;
- the sensitivity of your recommendations to the weight limit on the size of each module; and
- discussion about your confidence that you have the optimal solution in light of what you have covered concerning convex functions and sets.

CASE 11

Brite Power: Capacity Expansion

Brite Power is a small power provider in the Finger Lakes region of New York state. Because of California's power shortage in summer 2001, Brite Power's board of directors has decided to commission a study to ensure that the company has sufficient power until 2020. The study requires a time horizon of 16 years; the first decisions can be implemented in January 2005. Even though operations plans for power companies are important, this study is at a higher level.

Our concern is with power capacity during the year and not with day-to-day or hour-to-hour power-usage fluctuations.

By the start of 2005, Brite will have plants with 60 megawatts' worth of production capacity. Estimates of demand for power from the company for the years 2005 to 2020 have been made and are listed in Table 29.

The economics of a coal-fired plant run according to a *power law*—that is, the cost of a new plant in constant dollars (sometimes called "year 0 dollars") follows the following estimation rule:

$$\text{cost of plant with capacity K} = \left[\frac{\text{capacity K}}{\text{capacity of base size plant}}\right]^{0.8} * \text{cost of the base size plant}$$

For this analysis, the base plant's production capacity is 5 megawatts; its cost is $18 million. The company estimates inflation at 4% per year for the duration of the time horizon. The company uses a discount rate of 12% per year; this assumes that actual dollars are used in the analysis [1 actual dollar in year 1 is equivalent to $1/(1.12) = .89$ dollars now in year 0].

The time required for constructing a new plant is two years. The project requires 65% of the cost at the start of the first year; the remaining 35% is spent at the start of the second year. If Brite Power starts a new plant in 2007, for example, then 65% of the costs occur in 2007 and 35% in 2008; the plant then comes online and can be used to satisfy demand in 2009. With this lead time, it is clear that Brite Power needs to do advanced planning.

Brite Power can build plants with 5-, 10-, 15-, and 20-megawatt capacities. If the company invests now in research for new technologies ($3 million per year for 5 years), then it can reduce the exponent in the power model from 0.8 to 0.65.

Besides building new capacity, Brite Power must operate plants; operations costs are based on the amount of capacity used. If demand in year t is D_t megawatts and total capacity in year t is C_t megawatts, then the operations cost in constant dollars for the year is:

$$\left(\frac{C_t}{D_t}\right)^{0.5} * D_t * \$400{,}000$$

By and large, the company must satisfy all demand in the year, although there are opportunities to buy 3 megawatts per year from neighboring power companies at $600,000 per megawatt (in constant dollars).

TABLE 29

Year	Demand (Megawatts)	Year	Demand (Megawatts)
2005	54	2013	87
2006	58	2014	87
2007	63	2015	90
2008	63	2016	90
2009	69	2017	100
2010	75	2018	110
2011	77	2019	110
2012	77	2020	120

Key Questions

- The Brite Board would like to know when to augment capacity. How big should the expansions be? When should they start?
- What are the actual dollars spent over the time horizon to acquire and satisfy demand? What is the discounted value of these expenditures?
- Should the company invest in research to lower the power law coefficient?
- If demand estimates are increased or decreased, then how would the plan change?
- If the $400,000 value in the operations cost changes, then how would the plan change?