Golf-Sport: Managing Operations
A Case Study in Optimization

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1 Introduction

This problem examines a manufacturing optimization case, in which a company is looking to create an optimal two-month business plan. The case study seeks to develop a production plan that will optimize profit over this production period. The two-month period is set, with no information given as to the going concern of this company. This two-month optimization can serve as a baseline, and could later be expanded to more periods if desired. This particular problem appears to lend itself to integer programming as the ideal optimization method. The problem will be coded and solved using MATLAB's optimization toolbox functionality.

1.1 Background & Prior Work

Optimizing profit through production in processes is studied within the realm of the operations management. As the global market continues to grow and expand, businesses are focusing on remaining competitive. In a survey of optimization techniques being used within operations management, the authors broken the process into three distinct parts. These can be identified as input management, facilities management, and output management. Stated more simply, these correspond to scheduling, production, and vehicle routing, respectively. This Golf-Sport problem falls squarely within the facilities management/production sector. The authors discovered that for large-scale production optimizations, heuristics were necessarily utilized. Additionally, the most common solution approaches utilize branch-and-cut or branch-and-price [1].

Several approaches for operations management use genetic algorithms, especially when focusing on lot sizing within material requirements, aggregate and assortment planning, and facility layout problems [2]. Additionally, particle swarm optimization methods have been successfully implemented into operations research problems. Nearchou expands on this by stating that "A possible reason for this absence is that, PSO was introduced as global optimizer over continuous spaces, while a large set of POM problems are of combinatorial nature with discrete decision variables" [3]. In short, he is declaring that the large scale nature of such production problems are indeed appropriate for PSO algorithms. While this problem is not large enough to necessitate utilizing such algorithms (a global optimal solution can be determined through linear programming), these implementations are interesting approaches.

Generally speaking, many optimization problems can be converted into a linear programming model. The drawback to this, however, can be extremely long and computationally expensive runtimes, depending on the solution space. Klotz and Newman conducted research into this, and while they concluded that many operations management/production problems can be solved using linear programming, it may not be the best approach. However, by adjusting settings and parameters in software-based solvers, many large-scale problems can be effectively solved in this way [4].

2 Description of Problem

Golf-Sport is a golf club manufacturing company with three plants in Arizona. Each plant has an attached retail store and is treated as its only supplier. Each location has predicted minimum and maximum sales of each item type per period. The goal of this optimization is to maximize profit over a two-month period by determining the optimal number of each product to make at each location. In this problem, two types of constraints exist: local & global. Local constraints are unique to each plant/store combination, while the global constraints refer to company-wide resources. Initially, the problem will be modeled and optimized adhering stictly to the provided constraints and bounds.

3 Mathematical Formulation & Model

This problem is set up as a mixed integer programming problem, with a single objective function to be maximized (representing profit). The decision variables are set to be the production of each product item, indexed by location and period. In determining the maximum profit given the constraints, the solution will also yield the optimal production plan for Golf-Sport.

3.1 Assumptions

Two main assumptions were made in order to create an effective model for this problem. First, this model represents the company's production as if it were not a going-concern. The model is constructed to optimize profit over two months, thereby not building up any additional inventory for carryover into a third month. By default, the model will not have any inventory left at the end of the second month. If further information were to arise along with a modeling need for a third month, this could be accomplished by expanding the current model.

The second assumption used is the carryover of graphite stock between months. Advertising and graphite allotments are companywide in this problem, and it is clearly stated that the amount of money earmarked for advertising each month will not carryover into the next month. Graphite seems to have no such restriction, so it appears reasonable to incorporate the possibility of carryover of graphite stock.

3.2 Variables & Indices

Variable	**Description**
x	Production of Product i in Period j
e	Graphite Stock Carryover

Index i	**Product**
s	Steel Shafts
g	Graphite Shafts
r	Forged Iron Heads
w	Metal Wood Heads
h	Titanium Insert Wood Heads
v	Set, Steel Shafts, Metal Heads
u	Set, Steel Shafts, Insert Heads
y	Set, Graphite Shafts, Metal Heads
z	Set, Graphite Shafts, Insert Heads

Index j	**Period Description**
1	Produced & Sold in Period 1
2	Produced & Sold in Period 2
3	Produced in Period 1 & Sold in Period 2

3.3 Objective

The objective of this model is to maximize profit, represented as the difference between sales revenue and cost. For the purposes of this function let *R, C & I* represent revenue, product cost, and inventory cost, respectively. The function can be expressed as:

$$f(x) = \sum_{j=1}^{3} Rx_i - \sum_{j=1}^{3} Cx_i - \sum Ix_{i,3}$$

The total revenue from all products produced and sold, less the cost of manufacture and inventory, is represented in this function. While designed for a two-period optimization, the base function can be easily expanded to accomodate for more periods of production and sales.

3.4 Constraints

In construction of this model, the following constraints were constructed within the parameters set forth by the problem:

$$\sum c_i x_{i,1} + \sum c_i x_{i,3} \leq L_1 \tag{1}$$

$$\sum c_i x_{i,2} \leq L_2 \tag{2}$$

$$\sum c_i x_{i,1} + \sum c_i x_{i,3} \leq P_1 \tag{3}$$

$$\sum c_i x_{i,2} \leq P_2 \tag{4}$$

$$\sum c_i x_{i,1} + \sum c_i x_{i,3} \leq A \tag{5}$$

$$\sum c_i x_{i,2} \leq A \tag{6}$$

$$\sum c_i x_{i,1} + \sum c_i x_{i,3} + e \leq G_1, i = g, y, z \tag{7}$$

$$\sum c_i x_{i,2} \leq G_2 + e, i = g, y, z \tag{8}$$

$$\sum c_i x_{i,1} + \sum c_i x_{i,3} \leq U_1, i = v, u, y, z \tag{9}$$

$$\sum c_i x_{i,2} \leq U_2, i = v, u, y, z \tag{10}$$

$$Min_1 \leq x_{i,1} \leq Max_1 \tag{11}$$

$$Min_2 \leq x_{i,2} + x_{i,3} \leq Max_2 \tag{12}$$

$$x_{i,j} \geq 0, integer \tag{13}$$

$$e \geq 0 \tag{14}$$

Constraints 1-4 represent the labor and packing limitations by plant with cost by product. The first month's costs include the items manufactured and sold that month, as well as the items manufactured for carryover into the second month. Constraints 5-8 represent the global constraints in this problem: advertising budget and graphite stock. The advertising budget is a fixed amount for each month, with no carryover. The graphite stock has a consistent amount per month as well, but excess from the first month can be carried over into the next month. This carryover amount is represented by e. Constraints 9 & 10 represent the unique assembly time for the sets of clubs, again varying by plant location.

The final constraints (11-14) represent variable bounds. Constraint 11 ensures that all items produced and sold in period 1 are within the minimum and maximum demand. Similarly, inequality 12 ensures that the sum of items produced in period 2 and items carryed over to period 2 are within that month's demand bounds. Constraints 13 & 14 further bound all items to be non-negative.

4 Implementation

4.1 Hardware & Software

The problem was coded and solved in MATLAB R2017b on a personal computer. The code does not appear to be backward-compatible due to the use of the *optimproblem* implementation. The small scale of this optimization does not necessitate a computer beyond average specifications.

4.2 Algorithm

This problem was solved using the *intlinprog* function in MATLAB. This function conducts a linear relaxation on the problem, in order to set the upper bound. Then, the algorithm iterates through a branch-and-cut approach to solve for the optimal integer solution. By default, the function will iterate through 1,000,000 iterations prior to terminating. These iterations check the feasible solutions at the nodes along the tree created through branch-and-bound. Additionally, the function applies basic heuristics in the form of Gomory cuts. Gomory cuts are defined as an added constraint along a hyperplane to a vertex. They are commonly implemented in most commercial integer programming solvers as an initial heuristic. In optimization software implementations, these cuts tend to be accurate overall and invalid cuts are rarely made [5]. Given this knowledge, it is safe to assume that these cuts will not negatively affect the solution to this problem.

5 Solution

The model was successfully optimized to the same solution over several runs, yielding the following optimal solution with constraints as laid out in the original problem above:

Optimal Chandler Production			
Item	**Period 1**	**Period 2**	**Period 1 Carryover**
Steel Shafts	0	0	251
Graphite Shafts	100	2000	0
Iron Heads	200	0	200
Metal Wood Heads	35	1	30
Titanium Insert Wood Heads	2000	1299	701
Set, Steel, Metal Heads	0	0	0
Set, Steel, Titanium Heads	0	0	0
Set, Graphite, Metal Heads	0	0	0
Set, Graphite, Titanium Heads	0	35	84

Optimal Glendale Production			
Item	**Period 1**	**Period 2**	**Period 1 Carryover**
Steel Shafts	0	0	0
Graphite Shafts	100	100	0
Iron Heads	200	199	1
Metal Wood Heads	36	30	6
Titanium Insert Wood Heads	2000	1289	711
Set, Steel, Metal Heads	0	0	0
Set, Steel, Titanium Heads	0	0	0
Set, Graphite, Metal Heads	0	0	0
Set, Graphite, Titanium Heads	0	68	83

Optimal Tucson Production			
Item	**Period 1**	**Period 2**	**Period 1 Carryover**
Steel Shafts	1	0	2
Graphite Shafts	431	1265	2
Iron Heads	100	0	100
Metal Wood Heads	339	0	139
Titanium Insert Wood Heads	2000	814	1186
Set, Steel, Metal Heads	0	0	0
Set, Steel, Titanium Heads	0	0	0
Set, Graphite, Metal Heads	0	0	0
Set, Graphite, Titanium Heads	0	92	92

This optimal production scenario generates a maximized profit of $940,366 for Golf-Sport as a company in the two month period. The linear relaxation yielded a global optimal profit of $941,110, so this IP solution is valid (does not violate the Representation Theorem). The problem does not provide a current company profit, so it is not known how much of an increase in potential profit this soution would yield.

6 Further Analysis

The initial model and code provides a valid, optimal solution. While the solution is valid, there are several other points to be addressed to create a more meaningful model. Hence, the problem will be expanded upon further in two separate directions.

First, while the original analysis and model provides a valid optimal integer solution for the profit function, the actual production plan does not appear very practical for a business. The existing parameters seem to not account for maintaining a workforce; the lower bounds are set too low. This analysis will adjust some of these parameters and constraints to produce a more actionable business plan.

Second, the problem will be analyzed and adjusted to explore several range sensitivity topics. This can help direct the business' focus and direction in order to further maximize profit potential. Specifically, advertising dollars and graphite stock range sensitivity will be explored. These are both areas in which Golf-Sport has considered contributing extra resources. Additionally, a proposed advertising plan that could potentially double the maximum demand will be explored within current model parameters.

7 Added Practicality Constraints

7.1 Modifications to Existing Model

Upon further examination and analysis of the model & results, the current parameters do not appear to be as complete as possible. While it exists only as a case study, several further parameters will be added in order to present a more realistic model and solution. The added parameters will include:

- Addition of minimum labor constraints
- Addition of basic start-up cost constraints
- Increasing of minimum demand for items with currently a zero minimum
- Marginal increase in per-item revenue for period two to help offset for cost increases
- Incorporate parameters to reflect items lost in quality control checks

The addition of these parameters is necessary to model a more complete production problem. First, the current model yields very uneven production amounts between periods. While perhaps optimal, it would be unrealistic to recommend that a company lay off its workers if unnecessary. Therefore, labor minimums will be set to 25% of the current maximums. Second, any production facility will incur fixed costs per month, regardless of production. This will be modeled as:

$$z_{ij} = F_i, z \in 0, 1$$

This start-up operating cost will be modeled as $7,500 per plant per period. While the model will include this as a binary equation, it will effectively always be included due to the minimum labor constraints. The addition of the binary constraint, however, allows for ease of possible analysis without minimum production.

Next, the minimum for items with a current minimum demand of zero will be increased to 5% of the current maximum demand. For retail store purposes, even items with underwhelming sales should be stocked on the store shelves. By increasing this minimum demand slightly, it will ensure that each location produces a full catalog of items. Additionally, the increase in cost and inventory carryover cost is not currently reflected in item revenue for the second month. As costs increases significantly, at least some of that cost should be passed along to the customer. Therefore, the modifed model will include a 6% revenue (price) increase per item for the second month.

Lastly, in any production setting, items are lost to defects and quality control audits. Six Sigma production guidelines set a goal of 3.4 defects per million items. For the purposes of this model, it will be assumed that Golf-Sport is far from the Six Sigma efficiency levels. To reflect this, Golf-Sports total revenue formula will be adjusted to 99.5% to account for items lost due to defects.

7.2 Modifed Solution

After factoring in these additional constraints, the problem was optimized to yield $xxxxxx in profit from the following production scheme:

Optimal Modified Chandler Production			
Item	**Period 1**	**Period 2**	**Period 1 Carryover**
Steel Shafts	0	0	251
Graphite Shafts	100	2000	0
Iron Heads	200	0	200
Metal Wood Heads	35	1	30
Titanium Insert Wood Heads	2000	1299	701
Set, Steel, Metal Heads	0	0	0
Set, Steel, Titanium Heads	0	0	0
Set, Graphite, Metal Heads	0	0	0
Set, Graphite, Titanium Heads	0	35	84

Optimal Modified Glendale Production			
Item	**Period 1**	**Period 2**	**Period 1 Carryover**
Steel Shafts	0	0	0
Graphite Shafts	100	100	0
Iron Heads	200	199	1
Metal Wood Heads	36	30	6
Titanium Insert Wood Heads	2000	1289	711
Set, Steel, Metal Heads	0	0	0
Set, Steel, Titanium Heads	0	0	0
Set, Graphite, Metal Heads	0	0	0
Set, Graphite, Titanium Heads	0	68	83

Optimal Modified Tucson Production			
Item	**Period 1**	**Period 2**	**Period 1 Carryover**
Steel Shafts	1	0	2
Graphite Shafts	431	1265	2
Iron Heads	100	0	100
Metal Wood Heads	339	0	139
Titanium Insert Wood Heads	2000	814	1186
Set, Steel, Metal Heads	0	0	0
Set, Steel, Titanium Heads	0	0	0
Set, Graphite, Metal Heads	0	0	0
Set, Graphite, Titanium Heads	0	92	92

8 Range Sensitivity

Range sensitivity and resource increases were examined in regards to this problem. The two main points of focus (as supplied by the case study) are a hypothetical increase in total maximum demand and proposed increases in advertising or graphite budget. Both of these situations will be examined independently, using the initial mathematical model as the basis.

8.1 Advertising Budget v. Graphite Supply

The case study of the Golf-Sport problem presents a situation: which would be more beneficial for the company, more monthly advertising budget or more graphite stock? Since an amount of increase was not given, the situation was modeled with double the resources in the original problem. This allowed for a $40,000 monthly advertising budget or a 2,000 pound monthly graphite stock. As was the case before, advertising budget did not carry over to the next period, but graphite stock was assumed to.

These new parameters were incorporated separately into two different runs of the model. Graphite was determined to be the more valuable resource, as it increases the maximum profit to \$968,831. This profit potential is notably larger than the baseline of \$940,366 and the increased advertising profit of \$940,110. This should help show Golf-Sport the importance of graphite to the operation, as it is a limiting factor on current profit. It should be noted that when graphite is doubled in this case, excess stock exists at the end of the two-month period. This suggests that with current demand and constraints, obtaining more than 2,000 pounds of graphite stock per month is unnecessary, as it will not increase profit beyond this point.

8.2 Doubling the Demand

The Golf-Sport case study also proposes an interesting change: if maximum demand were to double, would the company be able to benenfit and maximize their profit while utilizing the current production system as originally described? The model was adjusted to account for a hypoethetical doubling of maximum demand while maintaining all of the original constraints as set forth in the model. Using this modified model, an integer solution was found. The objective value (maximum profit) is optimized at \$998,330. This is approximately a \$48,000 greater profit than the original demands yielded. This shows that the model does benefit from the increased demand.

9 Conclusions & Implications

9.1 Mathematical Approach

Evaluate our approach, suggest changes in future.

9.2 Company Approach

Discuss optimized solution, range sensitivity, business suggestions

References

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