4CCS1ELA: Tutorial list 1 – Sample Solutions

1. Let P and Q be the propositions

P: it is below freezing;

Q: it is snowing.

(a) Write the following propositions using P and Q and logical connectives.

Sample solution:

1. It is below freezing and snowing.

   $P \wedge Q$

2. It is below freezing but not snowing.

   $P \wedge \neg Q$

3. It is either snowing or below freezing (or both).

   $P \vee Q$

4. It is not snowing if it is below freezing.

   $P \to \neg Q$

5. That it is below freezing is necessary for it to be snowing.

   $Q \to P$

6. That it is below freezing is sufficient for it to be snowing.

   $P \to Q$

7. That it is below freezing is necessary and sufficient for it to be snowing.

   $P \leftrightarrow Q$

8. If it is freezing, it is also snowing.

   $P \to Q$

(b) Express each of the following propositions as an English sentence.

Sample solution:

P: it is below freezing;
Q: it is snowing.

1. $\neg P$

   It is not below freezing.

2. $P \rightarrow Q$

   If it is below freezing, then it is snowing.

3. $P \vee Q$

   It is either below freezing or snowing (or both).

4. $P \wedge Q$

   It is below freezing and snowing.

5. $\neg\, Q \rightarrow \neg P$

   If it is not snowing, then it is not below freezing.

6. $P \leftrightarrow Q$

   It is below freezing if and only if it is snowing.

7. $\neg P \wedge \neg\, Q$

   It is not below freezing and it is not snowing.

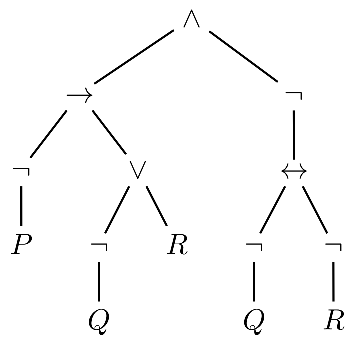
8. $\neg P \vee (P \wedge Q)$

   It is not below freezing, or it is both below freezing and snowing.

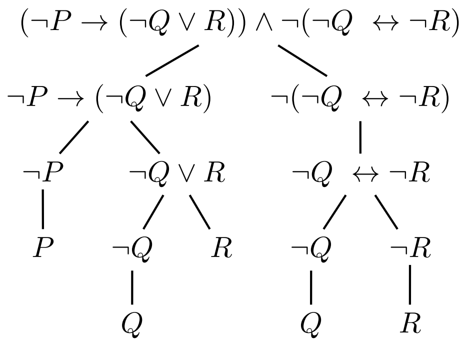
2. Construct the syntactic decomposition tree (or syntax tree) of the following propositional formula:

$$(\neg P \to (\neg Q \vee R)) \wedge \neg(\neg Q \ \leftrightarrow \neg R)$$

<u>Sample solution:</u>



Equivalently,



3. Consider the propositional formula $(P \vee \neg Q) \to \neg(Q \vee \neg P)$.

- Draw up a truth-table for the formula
- Determine whether this formula is a tautology, a contradiction or neither (a *contingency*), giving your reason.

Sample solution:

P	Q	$P \vee \neg Q$	$\to$	$\neg(Q \vee \neg P)$
1	1	1	0	0
1	0	1	1	1
0	1	0	1	0
0	0	1	0	0

Neither a tautology (because principal column contains a 0) nor a contradiction (because principal column contains a 1).

4. A variety of terminology is used to express conditional proposition $P \to Q$ (e.g. see Rosen, 6th edition, page 6):

"if P, then Q"
"P is sufficient for Q"
"Q if P"
"Q when P"
"a necessary condition for P is Q"
"Q unless $\neg P$"
"P implies Q"
"P only if Q"
"a sufficient condition for Q is P"
"Q whenever P"
"Q is necessary for P"
"Q follows from P"

Write each of the following statements in the form "if P, then Q":

Sample solution:

1. Winds from the south imply a spring thaw.

   If the winds blow from the south, then a spring thaw takes place.

2. A sufficient condition for the warranty to be good is that you bought the computer less then a year ago.

   If you bought the computer less then a year ago, then the warranty would be good.

3. Lenny gets caught whenever he cheats.

   If Lenny cheats, then he gets caught.

4. You can access the website only if you pay a subscription fee.

   If you can access the website, then you have payed a subscription fee.

5. It is necessary to have a valid password to log on to the server.

   If you have logged on to the server, then you have a valid password.

6. Jan will go swimming unless the water is too cold.

   If the water is not too cold, then Jan will go swimming.

7. Finding a good job follows from learning discrete mathematics.

   If you learn discrete mathematics, then you will find a good job.

5. Consider the following atomic propositions P_1, P_2, P_3 and P_4:

P_1**:** Galileo was born before Descartes. (**true**)

P_2**:** Descartes was born in the sixteenth century. (**true**)

P_3**:** Newton was born before Shakespeare. (**false**)

P_4**:** Einstein was a contemporary of Galileo. (**false**)

Given that P_1 and P_2 are true and P_3 and P_4 are false, determine the truth-value of the following sentence:

> *If Einstein was not a contemporary of Galileo then either Descartes was not born in the sixteenth century, Newton was born before Shakespeare, or Galileo was not born before Descartes.*

Sample solution:

This sentence is formalized by the following propositional formula:

$$\neg P_4 \to (\neg P_2 \ \vee \ P_3 \ \vee \ \neg P_1).$$

Given that $I(P_1) = I(P_2) = 1$ and $I(P_3) = I(P_4) = 0$, it follows that $I(\neg P_4) = 1$, $I(\neg P_1) = I(\neg P_2) = 0$. Then $I(\neg P_2 \ \vee \ P_3 \ \vee \ \neg P_1) = 0$.

Since the premise of the implication is true and the conclusion is false, we conclude that the formula is false under the given truth-value assignment.

6. **(a)** The proposition P *NAND* Q is true when either P or Q, or both are false; and it is false when both P and Q are true. (The proposition P *NAND* Q is denoted by $P \mid Q$, the connective $\mid$ is called *the Sheffer stroke*).

- Construct a truth-table for the logical connective *NAND*.
- Show that $P \mid Q$ is logically equivalent to $\neg(P \wedge Q)$.

(b) The proposition P *NOR* Q is true when both P and Q are false, and it is false otherwise. (The proposition P *NOR* Q is denoted by $P \downarrow Q$, the connective $\downarrow$ is called *the Pierce arrow*).

- Construct a truth-table for the logical connective *NOR*.
- Show that $P \downarrow Q$ is logically equivalent to $\neg(P \vee Q)$.

Sample solution:

The Sheffer stroke

P	Q	$P \mid Q$	$\neg(P \wedge Q)$
1	1	0	0
1	0	1	1
0	1	1	1
0	0	1	1

The Pierce arrow

P	Q	$P \downarrow Q$	$\neg(P \vee Q)$
1	1	0	0
1	0	0	0
0	1	0	0
0	0	1	1