

①

3-10-16

logic Puzzle.

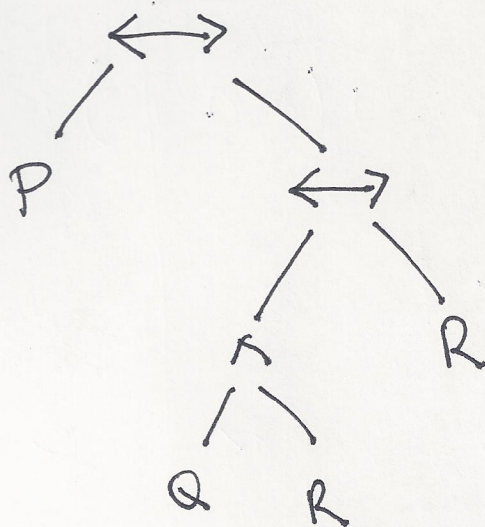
$$A = P \leftrightarrow ((Q \wedge R) \leftrightarrow R)$$

$$v_1(P) = 0$$

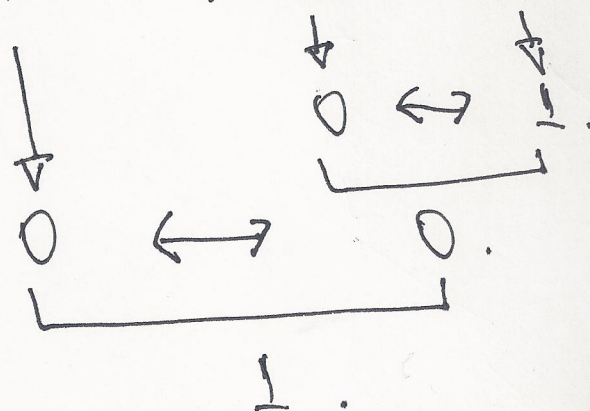
$$v_1(Q) = 0$$

$$v_1(R) = 1$$

$$v_1(A) = ?$$



$$P \leftrightarrow ((Q \wedge R) \leftrightarrow R)$$



$$v_1(A) = 1$$

(2)

A	B	$A \rightarrow B$	$\neg A \vee B$	$(A \rightarrow B) \leftrightarrow (\neg A \vee B)$
0	0	<u>1</u>	<u>1</u>	<u>1</u>
0	1	<u>1</u>	<u>1</u>	<u>1</u>
1	0	0	0	<u>1</u>
1	1	<u>1</u>	<u>1</u>	<u>1</u>

If $A \rightarrow B \equiv \neg A \vee B$, then
 $(A \rightarrow B) \leftrightarrow (\neg A \vee B)$ is a tautology.

$$P \vee \underline{1} \equiv \underline{1}.$$

$$P \vee (Q \vee \neg Q) \equiv \underline{1}.$$

$$\underbrace{((A \rightarrow B) \leftrightarrow (C \vee D))}_{\text{like your ?}} \vee (E \vee \neg E)$$

$$\downarrow$$

$$\underline{1}.$$

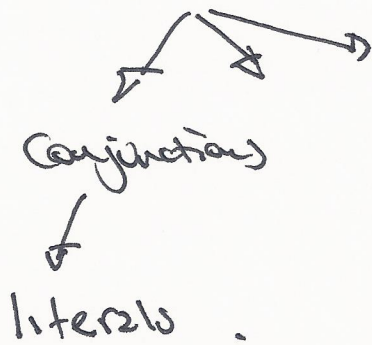
$$\equiv \underline{1}.$$

(3)

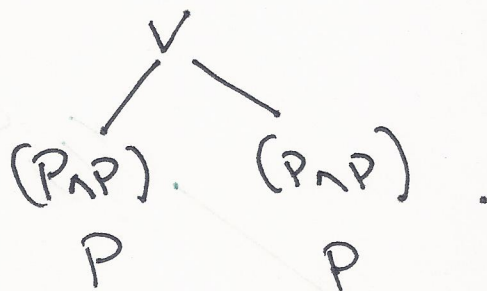
DNF

disjunctive normal form.

disjunctions.



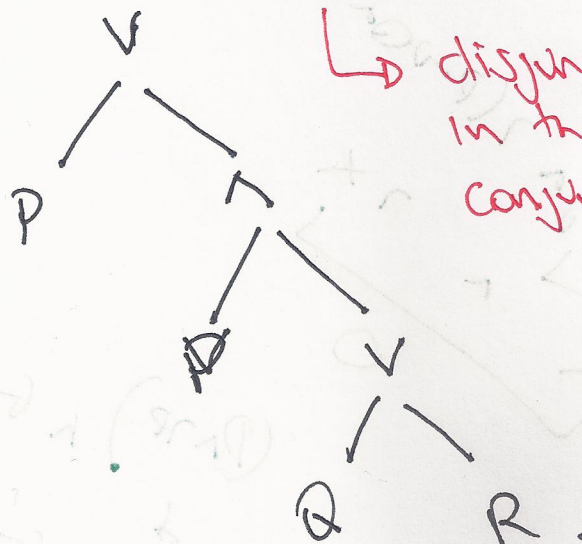
$P \Rightarrow$ is in DNF.



$P \vee P$

\Rightarrow conjunctions in the scope of disjunctions ✓

$P \vee (P \wedge (Q \vee R))$ NOT DNF.

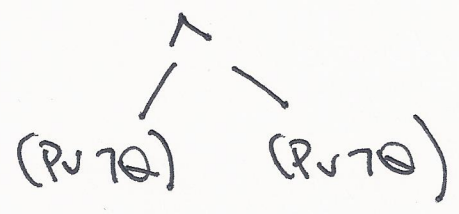


\hookrightarrow disjunction (\vee) in the scope of conjunction (\wedge) ✗

(4)

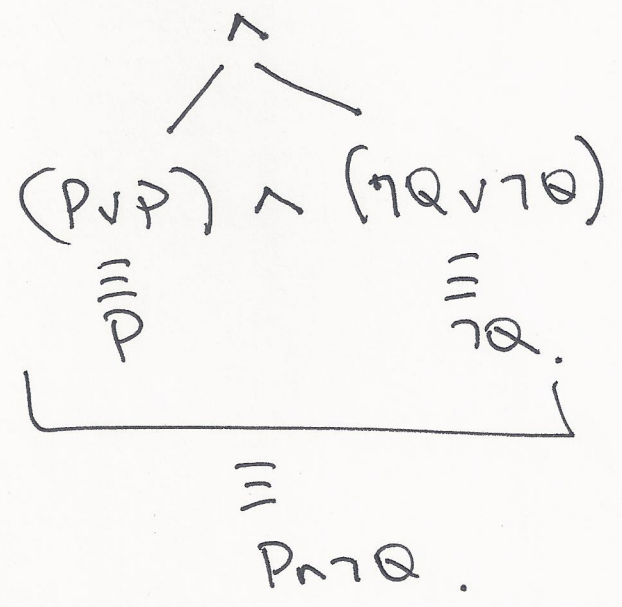
$$P \vee \neg Q$$

CNF



✓ disjunctions ~~in~~
in the
scope of
conjunctions

$$P \wedge \neg Q$$



DNF transformations.

$$A \rightarrow B \equiv \neg A \vee B$$

$$\begin{aligned} (F \rightarrow G) \rightarrow H &\equiv \neg(F \rightarrow G) \vee H \\ &\equiv \neg(\neg F \vee G) \vee H \\ &\equiv (\neg\neg F \wedge \neg G) \vee H \\ &\equiv (F \wedge \neg G) \vee H \end{aligned}$$

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$$(P \rightarrow Q) \wedge (Q \rightarrow P) \equiv P \leftrightarrow Q$$

DNF?

	P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$	$P \leftrightarrow Q$
*A	0	0	1	1	1	1
	0	1	1	0	0	0
	1	0	0	1	0	0
*B	1	1	1	1	1	1

$$\begin{aligned}
 & \text{*A} \Rightarrow \begin{matrix} P=0 & Q=0 \\ (\neg P \wedge \neg Q) \end{matrix} \\
 & \text{*B} \Rightarrow \begin{matrix} P=1 & Q=1 \\ (P \wedge Q) \end{matrix}
 \end{aligned}
 \Rightarrow (\neg P \wedge \neg Q) \vee (P \wedge Q)$$

$$\begin{aligned}
 \neg[(\neg P \wedge Q) \vee (P \wedge \neg Q)] &\equiv \neg(\neg P \wedge Q) \wedge \neg(P \wedge \neg Q) \\
 &\equiv (\neg\neg P \vee \neg Q) \wedge (\neg P \vee \neg\neg Q) \\
 &\equiv (P \vee \neg Q) \wedge (\neg P \vee Q) \\
 &\equiv (Q \rightarrow P) \wedge (P \rightarrow Q) \\
 &\equiv (P \rightarrow Q) \wedge (Q \rightarrow P) \\
 &\equiv P \leftrightarrow Q
 \end{aligned}$$

(6)

Tutorial 1.

(A)

Q5

$$\neg P_4 \rightarrow (\neg P_2 \vee P_3 \vee \neg P_1)$$

$$1 \rightarrow (0 \vee 0 \vee 0)$$

$$P_1 = 1$$

$$P_2 = 1$$

$$P_3 = 0$$

$$P_4 = 0$$

$$1 \rightarrow (0)$$

$$0$$

$P_4 \Rightarrow 1$ the implication will be true.

(B)

P	Q	$P \vee Q$	$P \wedge Q$	$\neg(P \wedge Q)$	$P \oplus Q$	$P \vee Q$	$\neg(P \vee Q)$
0	0	1	0	1	1	0	1
0	1	1	0	1	0	1	0
1	0	1	0	1	0	1	0
1	1	0	1	0	0	1	0

$\xrightarrow{\quad \quad \quad} \equiv$

$\xrightarrow{\quad \quad \quad} \equiv$

$$A \equiv B$$

All rows for A produce the same result as B!

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$$(P \vee \neg R) \rightarrow \neg(\neg Q \vee R)$$

$$\underbrace{(P \vee \neg R)}_A \rightarrow \underbrace{\neg(\neg Q \vee R)}_B \equiv \neg A \vee B$$

$$\neg(P \vee \neg R) \vee \neg(\neg Q \vee R)$$

$$(\neg P \wedge \neg \neg R) \vee (\neg \neg Q \wedge \neg R)$$

$$(\neg P \wedge R) \vee (Q \wedge \neg R)$$

$$\underbrace{(\neg P \wedge R)}_A \vee \underbrace{(Q \wedge \neg R)}_{(B \wedge C)}$$

$$(A \vee B) \wedge (A \vee C)$$

P	Q	R	\rightarrow
0	0	0	\times_1
0	0	<u>1</u>	\times_2
	\vdots		
<u>1</u>	<u>1</u>	<u>1</u>	\times_8

$$\neg(\neg P \wedge \neg Q \wedge \neg R)$$

$$(P \vee Q \vee \neg R) \wedge$$

$$\times_2 \cdot 0$$