4CCS1ELA: Tutorial list 1

"Understanding and application are interdependent. Application without understanding is blind, and quickly leads to ghastly errors. On the other hand, comprehension remains poor without practice in application. In particular, you do not understand a definition until you have seen how it takes effect in specific situations: positive examples reveal its range, negative example show its limits. It also takes time to recognize when you have really understood something, and when you have done no more than recite the words, or call upon it in hope of blessing.

For this reason, doing exercises is a indispensable part of the learning process. That is part of what is meant by the old proverb 'there is no royal road in mathematics'. It is also why we give so many problems and provide sample answers to some. Skip them at your peril: no matter how simple and straightforward a concept seems, you will not fully understand it unless you practice using it. So, even when an exercise is accompanied by a solution, you will benefit a great deal if you place a sheet over the answer and first try to work it out for yourself. That requires self-discipline and patience, but it brings real rewards."

David MAKINSON
Sets, Logic and and Maths for Computing
Preface for the Student
Springer-Verlag London Ltd (October 2008)

1. Let P and Q be the propositions

P: it is below freezing;
Q: it is snowing.

(a) Write the following propositions using P and Q and logical connectives.

1. It is below freezing and snowing.
2. It is below freezing but not snowing.
3. It is either snowing or below freezing (or both).
4. It is not snowing if it is below freezing.
5. That it is below freezing is necessary for it to be snowing.
6. That it is below freezing is sufficient for it to be snowing.

7. That it is below freezing is necessary and sufficient for it to be snowing.
8. If it is freezing, it is also snowing.

(b) Express each of the following propositions as an English sentence (given that P denotes 'it is below freezing' and Q denotes "it is snowing").

1. $\neg P$
2. $P \rightarrow Q$
3. $P \vee Q$
4. $P \wedge Q$
5. $\neg\, Q \rightarrow \neg P$
6. $P \leftrightarrow Q$
7. $\neg P \wedge \neg\, Q$
8. $\neg P \vee (P \wedge Q)$

2. Construct the syntactic decomposition tree (or syntax tree) of the following propositional formula:

$$(\neg P \rightarrow (\neg Q \vee R)) \wedge \neg(\neg Q \;\leftrightarrow \neg R)$$

3. Consider the propositional formula $(P \vee \neg Q) \rightarrow \neg(Q \vee \neg P)$.

- Draw up a truth-table for the formula
- Determine whether this formula is a tautology, a contradiction or neither (a *contingency*), giving your reason.

4. A variety of terminology is used to express conditional proposition $P \to Q$ (e.g. see Rosen, 6th edition, page 6):

"if P, then Q"	"P implies Q "
"P is sufficient for Q"	"P only if Q"
"Q if P"	"a sufficient condition for Q is P"
"Q when P"	"Q whenever P"
"a necessary condition for P is Q"	"Q is necessary for P"
"Q unless $\neg P$"	"Q follows from P"

Write each of the following statements in the form "if P, then Q":

1. Winds from the south imply a spring thaw.
2. A sufficient condition for the warranty to be good is that you bought the computer less then a year ago.
3. Lenny gets caught whenever he cheats.
4. You can access the website only if you pay a subscription fee.
5. It is necessary to have a valid password to log on to the server.
6. Jan will go swimming unless the water is too cold.
7. Finding a good job follows from learning discrete mathematics.

5. Consider the following atomic propositions P_1, P_2, P_3 and P_4:

P_1**:** Galileo was born before Descartes. (**true**)

P_2**:** Descartes was born in the sixteenth century. (**true**)

P_3**:** Newton was born before Shakespeare. (**false**)

P_4**:** Einstein was a contemporary of Galileo. (**false**)

Given that P_1 and P_2 are true and P_3 and P_4 are false, determine the truth-value of the following sentence:

> *If Einstein was not a contemporary of Galileo then either Descartes was not born in the sixteenth century, Newton was born before Shakespeare, or Galileo was not born before Descartes.*

6. **(a)** The proposition P *NAND* Q is true when either P or Q, or both are false; and it is false when both P and Q are true. (The proposition P *NAND* Q is denoted by $P \mid Q$, the connective $\mid$ is called *the Sheffer stroke*).

- Construct a truth-table for the logical connective *NAND*.
- Show that $P \mid Q$ is logically equivalent to $\neg(P \wedge Q)$.

(b) The proposition P *NOR* Q is true when both P and Q are false, and it is false otherwise. (The proposition P *NOR* Q is denoted by $P \downarrow Q$, the connective $\downarrow$ is called *the Pierce arrow*).

- Construct a truth-table for the logical connective *NOR*.
- Show that $P \downarrow Q$ is logically equivalent to $\neg(P \vee Q)$.