5CCS2FC2: Foundations of Computing II

Tutorial Sheet 3

2.5 (i) Show that the language A_{TM} is recursively enumerable by constructing a sound and complete algorithm that recognises all words $\langle M, w\rangle$, where M encodes a TM that accepts w.

(ii) Hence, or otherwise, show that its complement $\overline{\mathsf{A}_{TM}}$ is *not* recursively enumerable.

2.6 *(Tricky!)*

(i) Show that the language $\overline{\mathsf{EQ}_{TM}}$ is not recursively enumerable by reducing A_{TM} to its complement EQ_{TM}. (In other words, that EQ_{TM} is not co-recursively enumerable.)

(ii) Show that the language $\overline{\mathsf{EQ}_{TM}}$ is also not co-recursively enumerable by reducing A_{TM} to $\overline{\mathsf{EQ}_{TM}}$. (In other words, that EQ_{TM} is not recursively enumerable.)

(It follows that $\overline{\mathsf{EQ}_{TM}}$ and EQ_{TM} are 'harder' than any recursively enumerable or co-recursively enumerable problem. There are not even any sound-and-complete algorithms for either problem)

3.1 Determine whether the following are true or false?

(i) $10^{15}n \in O(n)$,

(ii) $n^2 \in O(n)$,

(iii) $n^2 \in O(n \log n)$,

(iv) $n^2 \in O(n \log^2 n)$,

(v) $n \log n \in O(n^2)$,

(vi) $2^{(2n+1)} \in O(4^n)$,

(vii) $n^{\log \log n} \in O(n^{10})$.

3.2 For each of the following formulas F construct a graph G_F and choose an integer k such that

$$F \text{ is satisfiable} \quad \Longleftrightarrow \quad G_F \text{ contains a clique of size } k$$

(i) $F = (P \vee \neg Q \vee \neg S) \wedge (Q \vee \neg R \vee S) \wedge (\neg Q \vee R \vee S)$

(ii) $F = (P \vee \neg Q \vee \neg R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)$

(iii) $F = (P \vee Q \vee \neg R) \wedge (P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R)$

Use this property to identify which of the above formulas are satisfiable.

3.3 Construct a propositional formula that is satisfiable if and only if the following graph $G = (V, E)$ can be coloured using only two colours, where

$$\begin{aligned} V &= \{1, 2, 3, 4, 5\} \\ E &= \{(1,2), (1,3), (2,3), (2,4), (3,5)\} \end{aligned}$$