4CCS1ELA - Elementary Logic with Applications
Programming with Logic II:
Predicate Definite Clause Programming

Tutorial List 7 Solutions

Question 1:

Solution:

See slides

Question 2:

Solution:

1)

$\forall x\ P(x) \rightarrow \exists x\ Q(x)$

$\neg\forall x\ P(x) \vee \exists x\ Q(x)$ (step1)

$\exists x\ \neg P(x) \vee \exists x\ Q(x)$ (step 2)

$\exists x\ \neg P(x) \vee \exists y\ Q(y)$ (step3)

$\exists x\ \exists y\ \neg P(x) \vee Q(y)$ (step4)

The matrix $\neg P(x) \vee Q(y)$ is already in CNF. It contains one clause, but it is **not** a definite clause as the prefix contains existential quantifiers. So we cannot derive definite rules.

2)

$\exists x\ P(x) \rightarrow \forall x\ Q(x)$

$\neg\exists x\ P(x) \vee \forall x\ Q(x)$ (step1)

$\forall x\ \neg P(x) \vee \forall x\ Q(x)$ (step 2)

$\forall x \neg P(x) \lor \forall y\, Q(y)$ (step3)

$\forall x \forall y \neg P(x) \lor Q(y)$ (step4)

The matrix $\neg P(x) \lor Q(y)$ is already in CNF and contains one clause.

This is a definite clause as: 1) prefix contains only universal quantifiers quantifying over all variables in matrix and 2) matrix contains exactly one positive atom.

It can be represented as the definite rule $\mathbf{\forall x \forall y\, P(x) \rightarrow Q(y)}$

3)

$(\forall y\, H(y) \rightarrow \exists z\, W(z, y)) \rightarrow \exists z\, G(z)$

$\neg (\forall y\, H(y) \rightarrow \exists z\, W(z, y)) \lor \exists z\, G(z)$

$\neg (\neg \forall y\, H(y) \lor \exists z\, W(z, y)) \lor \exists z\, G(z)$

$\neg (\exists y \neg H(y) \lor \exists z\, W(z, y)) \lor \exists z\, G(z)$

$(\neg \exists y \neg H(y) \land \neg \exists z\, W(z, y)) \lor \exists z\, G(z)$

$(\forall y\, H(y) \land \forall z \neg W(z, y)) \lor \exists z\, G(z)$

$(\forall y\, H(y) \land \forall z \neg W(z, y)) \lor \exists x\, G(x)$

$\forall y \forall z \exists x\, (H(y) \land \neg W(z, y)) \lor G(x)$

The matrix is not in CNF. So we need to transform it in CNF using distributivity:

$\forall y \forall z \exists x\, (H(y) \lor G(x)) \land (\neg W(z, y) \lor G(x))$

The two clauses are not definite clauses and as such cannot be represented as definite rules, because the prefix contains an existential quantifier.

4)

$\forall x\, (P(x) \rightarrow (F(x) \land G(x)))$

$\forall x\, (\neg P(x) \lor (F(x) \land G(x)))$ (step1)

There is no need to apply steps 2, 3 or 4. We have a PNF formula, but the matrix is not in CNF. So we use distributivity:

$\forall x\, (\neg P(x) \lor F(x)) \land (\neg P(x) \lor G(x))$

Now the matrix contains two definite clauses that can be represented as the definite rules:

$\mathbf{\forall x\, P(x) \rightarrow F(x)}$ and $\mathbf{\forall x\, P(x) \rightarrow G(x)}$

Question 3:

Solution:

1) $\exists x\, (T(x) \wedge L(x)) \rightarrow \forall x\, (T(x) \rightarrow L(x))$

(if there is some x which is both a train and it is late, then it is true that for all objects, if the object is a train then it is late)

2) $\neg\exists x\, (T(x) \wedge L(x)) \vee \forall x\, (\neg T(x) \vee L(x))$

   $\neg\exists x\, (T(x) \wedge L(x)) \vee \forall y\, (\neg T(y) \vee L(y))$

   $\forall x\, \neg(T(x) \wedge L(x)) \vee \forall y\, (\neg T(y) \vee L(y))$

   $\forall x\, (\neg T(x) \vee \neg L(x)) \vee \forall y\, (\neg T(y) \vee L(y))$

   $\forall x \forall y\, (\neg T(x) \vee \neg L(x) \vee \neg T(y) \vee L(y))$

3) $\forall x \forall y\, (\neg T(x) \vee \neg L(x) \vee \neg T(y) \vee L(y))$

   The matrix contains one definite clause, which can be represented as the definite rule:

   $\boldsymbol{\forall x \forall y\; T(x), L(x), T(y) \rightarrow L(y)}$

   If x is a late train and y is a train, then y is late!