# 4CCS1ELA: Tutorial list 2

**1.** Consider the formula

$$(P \vee \neg R) \rightarrow \neg(\neg Q \vee R)$$

(i) Build a conjunctive normal form for this formula from its truth table.

(ii) Transform this formula in a logical equivalent disjunctive normal form (DNF) using the rewrite rules.

(iii) Find a disjunctive normal form for this formula using Quine's tree.

**2.** Rewrite the following propositional formula in (i) a logically equivalent conjunctive normal form, and (ii) a logically equivalent disjunctive normal form:

$$(P \rightarrow Q) \ \wedge \ \neg(S \ \rightarrow \ R).$$

**3.** Which of the following propositional formulas are substitution instances of the formula

$$P \rightarrow (Q \rightarrow P) \ ?$$

If a formula is indeed a substitution instance, give the formulas substituted for $P, Q$.

(i) $\neg R \rightarrow (R \rightarrow \neg R)$

(ii) $\neg R \rightarrow (\neg R \rightarrow \neg R)$

(iii) $\neg R \rightarrow (\neg R \rightarrow R)$

(iv) $(P \wedge Q \rightarrow P) \rightarrow ((Q \rightarrow P) \rightarrow (P \wedge Q \rightarrow P))$

(v) $((P \rightarrow P) \rightarrow P) \rightarrow ((P \rightarrow (P \rightarrow (P \rightarrow P)))) \ ?$

**4.** Let $P|Q$ be defined as the wff with $P$ and $Q$ having the truth-table below:

| $P$ | $Q$ | $P\|Q$ |
|---|---|---|
| 1 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 0 | 1 |

Define $\wedge$, $\vee$, $\neg$ and $\Rightarrow$ using $|$.

**Hint:** Look at the truth-table for $P|P$ too!