4CCS1ELA: Tutorial list 5

1. Assume that $\exists x \forall y P(x, y)$ is true. Which of the following formulas must also to be true? If the formula is true, explain. Otherwise, give a counterexample.

(i) $\forall x \forall y P(x, y)$.

(ii) $\forall x \exists y P(x, y)$.

(iii) $\exists x \exists y P(x, y)$.

2. Consider the formula $\mathcal{F} = \neg \forall x \exists y P(x, y)$. Determine which of the following formulas is logically equivalent to $\mathcal{F}$ and which is not. If the formula is equivalent to $\mathcal{F}$, then show it using successive equivalences.

(i) $\exists x \neg \forall y P(x, y)$.

(ii) $\forall x \neg \exists y P(x, y)$.

(iii) $\exists x \forall y \neg P(x, y)$.

(iv) $\exists x \exists y \neg P(x, y)$.

3. Give a reason based on interpretations and the meaning of quantifiers why

(i) the following first-order sentence is valid:
$\exists x P(x) \wedge \forall y (P(y) \to Q(y)) \to \exists z Q(z)$

(ii) the following first-order sentence is not valid:
$\exists x P(x) \wedge \forall y (P(y) \to Q(y)) \to \forall z \neg Q(z)$

4. Use successive equivalences, showing your work, to show that the formula

$\neg \exists x \forall y (\neg P(x) \wedge (Q(y) \to R(x, y)))$ is logically equivalent to $\forall x (P(x) \vee \exists y (Q(y) \wedge \neg R(x, y)))$.

5. Determine whether the formula $\mathcal{F}$

$$\exists x \forall y (P(x) \to x = y)$$

is true or false under each of the following interpretations over the domain $D = \{a, b\}$.

(i) both $P(a)$ and $P(b)$ are true;

(ii) both $P(a)$ and $P(b)$ are false;

(iii) $P(a)$ is true and $P(b)$ is false.