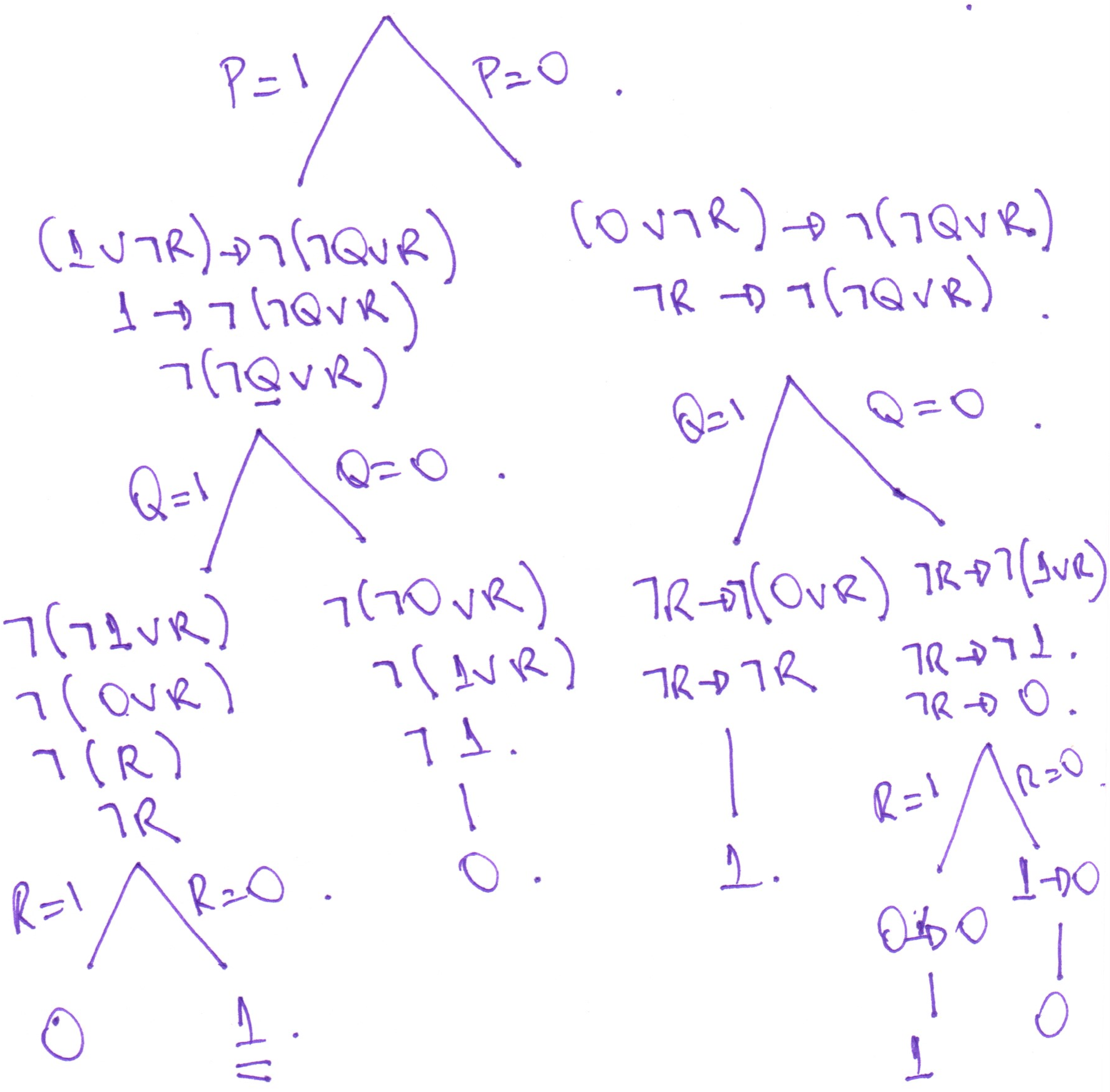
1

10-10-16.

Tutorial 2

Q1.

(ii) $(\underline{P} \vee \neg R) \to \neg(\neg Q \vee R)$.



$(P \wedge Q \wedge \neg R) \vee \ldots$

(2)

P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

$(\neg P \wedge \neg Q) \vee (P \wedge Q)$. DNF

CNF

$\neg[(\neg P \wedge Q) \vee (P \wedge \neg Q)]$.

$\neg(\neg P \wedge Q) \wedge \neg(P \wedge \neg Q)$

$(\neg\neg P \vee \neg Q) \wedge (\neg P \vee \neg\neg Q)$

$(P \vee \neg Q) \wedge (\neg P \vee Q)$.

$\neg(A \vee B \vee C)$.

$\neg(A \vee (B \vee C))$.

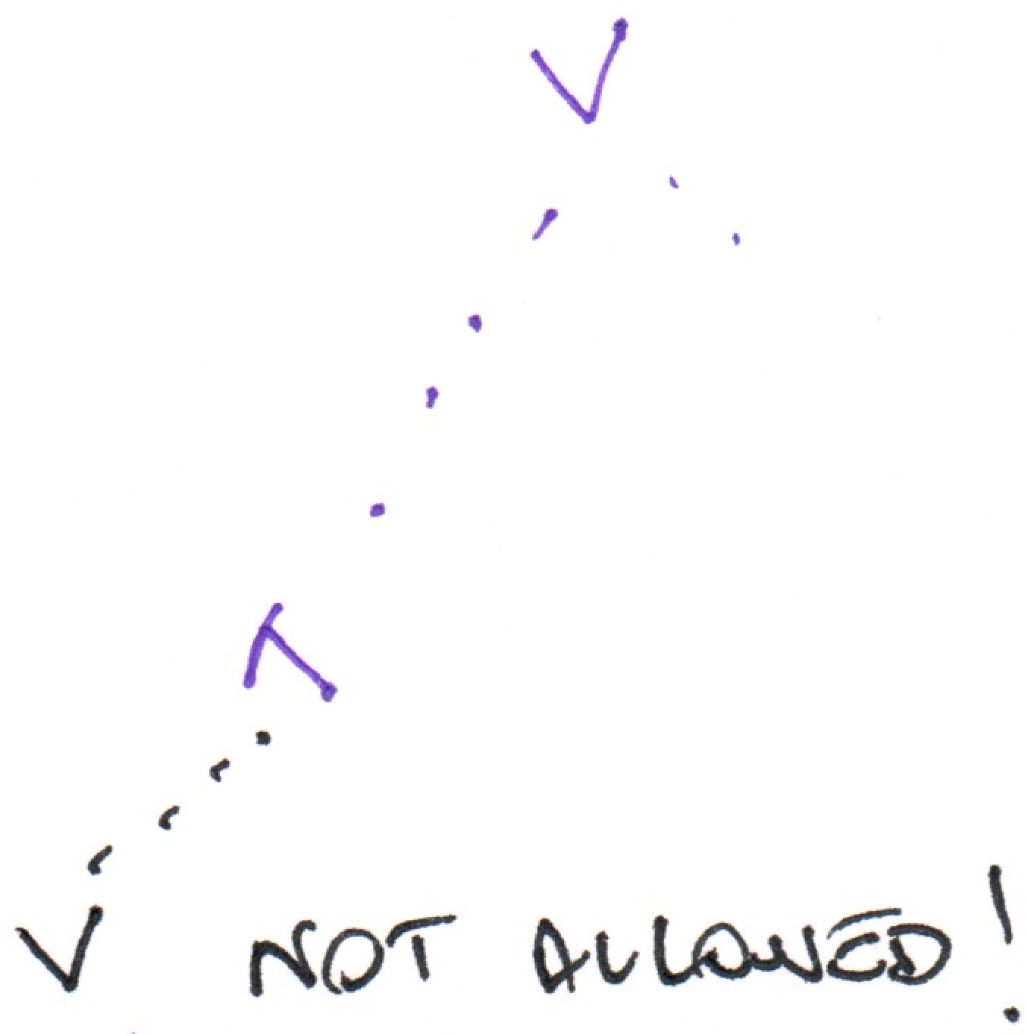
$\neg A \wedge \neg(B \vee C)$.

$\neg A \wedge \neg B \wedge \neg C$.

(3)

CNF

DNF



$P \wedge (Q \vee R)$. CNF

$P \wedge (Q \vee (R \wedge S))$ X CNF

$P \wedge Q \wedge R \wedge S$.

P
Q
R
S .

Let $\mathcal{I}$ be the set of all interpretations.

$mod(A) \subseteq \mathcal{I}$

$\mathcal{I} / mod(A) = mod(\neg A)$

$S \models F$ $\quad mod(S) \subseteq mod(F)$.

If $S \models F$, then $mod(S) \cap mod(\neg F) = \emptyset$

$mod(S \cup \{F\}) = mod(S) \cap mod(F)$

$mod(S \cup \{\neg F\}) = mod(S) \cap mod(\neg F) = \emptyset$

(5)

$\underset{A}{P \vee Q}, \underset{B}{\neg P} \models Q$. ✓

$\neg P \to Q, \neg P \models Q$

P	Q	$P \vee Q$ (A)	$\neg P$ (B)
0	0	0	1
0	1	1	1
1	0	1	0
1	1	1	0

Relevant line.

P	Q	$P \vee Q$	$\neg P$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \to Q$
0	0	0	1	0	1
0	1	1	1	1	1
1	0	1	0	0	1
1	1	1	0	0	1

tautology.

$\mathrm{mod}(\{P \vee Q, \neg P, \neg Q\}) = \emptyset$

Unsatisfiable.

Tutorial 2

Q4

P	Q	P|Q	P|P	X
1	1	0	0	1
1	0	1	0	0
0	1	1	1	0
0	0	1	1	0

$P | P \equiv \neg P$

$|, P, Q \Rightarrow P \wedge Q.$

$P \vee Q.$

$P \to Q.$

Tutorial 3

Q1

S ⇒ Users can save new files.

(1) U ⇒ system being upgraded.

A ⇒ users can access the filesystem

$U \to \neg A$

(2) $A \to S$

(3) $\neg S \to \neg U$

Specification = $\{U \to \neg A,\ A \to S,\ \neg S \to \neg U\}$.

Tutorial 2

P	Q	$P \mid Q$	$P \mid P$	$\neg P$	$P \wedge Q$	$(P \mid Q) \mid (P \mid Q)$
1	1	0	0	0	1	1
1	0	1	0	0	0	0
0	1	1	1	1	0	0
0	0	1	1	1	0	0

$\equiv$ $\equiv$

$P \vee Q$, $P \to Q$.

$\neg(\neg P \wedge \neg Q) \equiv P \vee Q$ $\quad$ $P \mid Q \equiv \neg(P \wedge Q)$.

$\downarrow$

$(P \mid P) \mid (Q \mid Q) \equiv P \vee Q$.

$P \to Q = ?$ $\quad$ $\neg P \vee Q \equiv \neg(P \wedge \neg Q)$

$\neg P \vee \neg\neg Q$

$\neg P \vee Q$

$P \to Q$.

$\neg(P \wedge \neg Q) \equiv$

$P \mid (Q \mid Q)$

$\{ \mid \}$ $\Rightarrow$ complete!

Tutorial 3 Q1 continued

Specification = $\{U \to \neg A,\ A \to S,\ \neg S \to \neg U\}$.

suppose $v(U) = 0$ $v(U \to \neg A) = 1$.

$v(\neg U) = 1$

$v(\neg S \to \neg U) = 1 \Rightarrow v(S) = 0$

$v(A \to S) = 1$.

$v(S) = 0$.

$v(A) = 0$

$v(A \to S) = 1$.

$v(U) = 0$	$v(U \to \neg A) = 1$.
$v(A) = 0$	$v(A \to S) = 1$.
$v(S) = 0$.	$v(\neg S \to \neg U) = 1$.

$\Rightarrow$ v satisfies Specification therefore the Specification is consistent.

Q2 Tutorial 3

(i) $P, P \to Q \models Q$.

$\underline{v(P)=1}$ $\underline{v(P \to Q)=1}$ $\Rightarrow$ $v(Q)=1$.

every interpretation that makes P and $P \to Q$ true also makes Q true!

(ii) $P \to Q \not\models Q \to P$

$v(P \to Q)=1$ but $v(Q \to P)=0$

$v(Q)=1$

$v(P)=0$.

countermodel

(iii) $P \vee \neg Q \not\models P$.

$v(P)=0$ $v(Q)=0$ $v(\neg Q)=1$.

$v(P \vee \neg Q)=1$.

(iv) $P \wedge \neg P \models Q$.

$\Downarrow$

$\mathrm{mod}(P \wedge \neg P) \subseteq \mathrm{mod}(Q)$.

$\emptyset \subseteq \mathrm{mod}(Q)$