Tutorial 2

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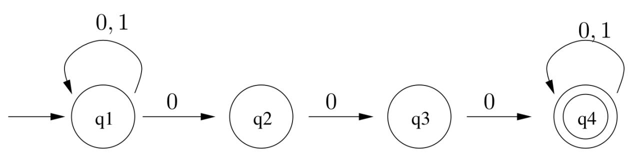
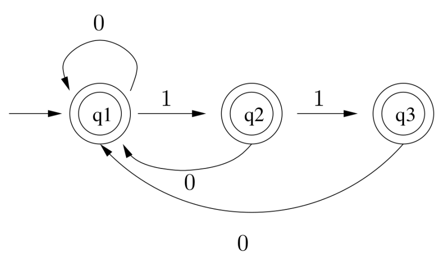
1. Build finite automata with alphabet $\{0, 1\}$ to recognise:
   - the language of strings that have three consecutive 0s;
   - the language of strings that do not have three consecutive 1s.





2. Let A be a finite automaton. Show that the set of subwords (that is, prefixes, suffixes, or any continuous segment) of the words in the language associated with A, $L(A)$, can also be recognised by a finite automaton.

   To show that the language consisting of prefixes of words in $L(A)$ is recognisable by a finite automaton, we can simply build an automaton for it using as a starting point the automaton A. Indeed, to recognise a prefix of a word in $L(A)$, it is sufficient to turn every state in A for which there is a path to a final state into a final state. In this way, we have a finite automaton A' with the same alphabet as A and such that if a word is a prefix (i.e., the initial segment) of a word in $L(A)$, then A' will reach a final state.

   Recognising suffixes is slightly more subtle, but again, starting from A we can build an automaton with the required property by inserting ϵ transitions between the initial state of A and all the other states for which there is a path to a final state. This gives a non-deterministic automaton A'' that, for any suffix (i.e., final segment) of a word in $L(A)$, reaches a final state. Finally, combining both techniques, we can obtain an automaton that recognises any continuous segment of words in $L(A)$.

3. How can a push-down automaton recognise the language

$$\{w\bar{w} | w \text{ is a string of } 0\text{s and } 1\text{s and } \bar{w} \text{ is its mirror image}\}?$$

   Give an informal description of such an automaton, and then build the automaton.

   The idea is to define states that non-deterministically put in the stack the symbols read and also start popping symbols in case we have already reached the middle point in the word.

4. **Challenge:** Use the Pumping Lemma to show that the language L containing all the words of the form $a^n b^n c^n$, for any $n \geq 0$, cannot be recognised by a finite automaton.

If a word $a^n b^n c^n$ is in L, then as a consequence of the Pumping Lemma there is a substring that can be repeated an arbitrary number of times. Therefore L must contains strings where the number of symbols a, b, or c is different, which contradicts the assumptions.