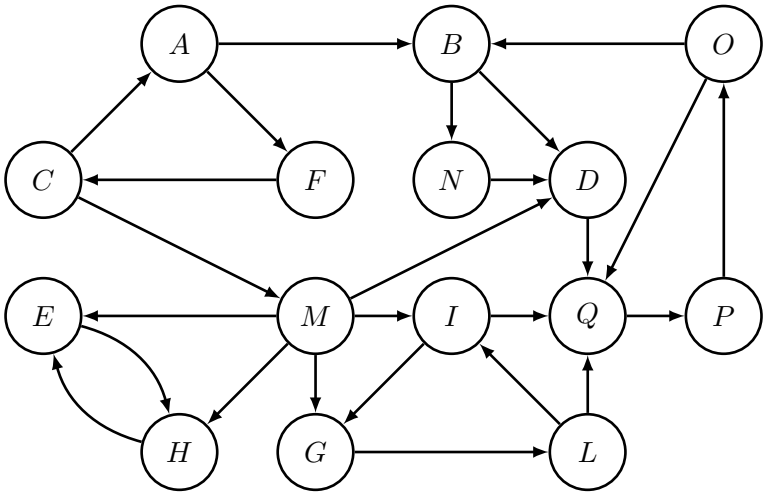
5CCS2FC2: Foundations of Computing II

Tutorial Sheet 5

5.1 Consider the following graph $G = (V, E)$ describing a set which tasks that must be completed before or concurrently with which others:



(i) Use the algorithm described in class to identify which tasks must be performed concurrently.

(ii) Draw the component graph G^{SCC}.

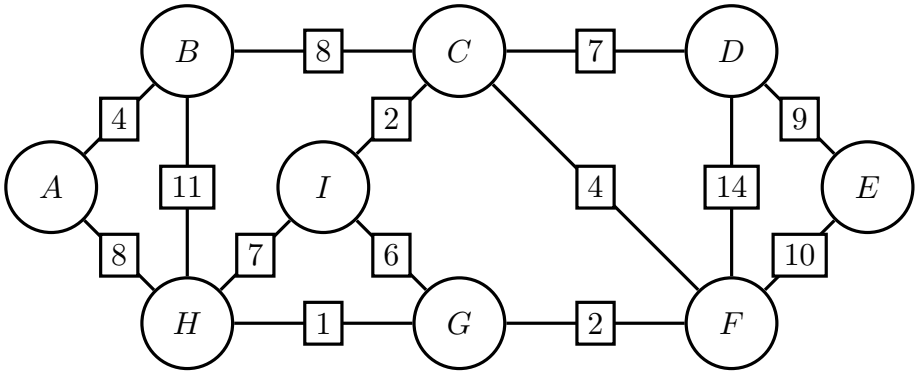
(iii) Perform a topological sort on the component grap G^{SCC} to identify an order in which the task can be scheduled?

(iv) How many ways can the number of number of strongly connected components change if a single additional edge is added to G?

5.2 Consider the following naïve algorithm for the minimum spanning tree problem

> **Step 1)** If the graph contains at most one edge, return this as the minimum spanning tree.
>
> **Step 2)** Otherwise, divide the graph G into two subgraphs G_1 and G_2 that differ in size by at most 1 vertex.
>
> **Step 3)** Identify a minimum spanning tree T_1 for G_1 and a minimum spanning tree T_2 for G_2. (Do this by recursively applying this algorithm to each subgraph)
>
> **Step 4)** Connect the two spanning trees T_1 and T_2 with the lightest edge between then.

(i) Apply this algorithm to the following graph:



(ii) Does the algorithm always return a spanning tree?

(ii) Does the algorithm always return a *minimum* spanning tree?