

## 4CCS1ELA: Tutorial list 3

1. Represent the following system specification in propositional logic and determine whether this specification is consistent.

*Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded.*

2. Which of the following are true? Explain your answers carefully.

- (i)  $P, P \rightarrow Q \models Q$ ;
- (ii)  $P \rightarrow Q \models Q \rightarrow P$ ;
- (iii)  $P \vee \neg Q \models P$ ;
- (iv)  $P \wedge \neg P \models Q$ .

3. Determine which of these arguments are valid. If an argument is valid, indicate the rule of inference being used. If it is not, what logical error occurs?

- (i) If  $n$  is a real number such that  $n > 1$ , then  $n^2 > 1$ .  
Suppose,  $n^2 > 1$ . Then  $n > 1$ .
- (ii) If  $n$  is a real number such that  $n > 3$ , then  $n^2 > 9$ .  
Suppose,  $n^2 \leq 9$ . Then  $n \leq 3$ .
- (iii) If  $n$  is a real number such that  $n > 2$ , then  $n^2 > 4$ .  
Suppose,  $n \leq 2$ . Then  $n^2 \leq 4$ .

4. Let  $\mathcal{A}, \mathcal{B}$  be propositional formulas. Demonstrate that if there exists a propositional formula  $\mathcal{C}$  such that  $\mathcal{A}$  is a logical consequence of  $\mathcal{C}$  and  $\mathcal{B}$  is a logical consequence of  $\neg \mathcal{C}$ , then formula  $\mathcal{A} \vee \mathcal{B}$  is a tautology.

5. Suppose we have the two propositions (with symbols to represent them):

*It is raining ( $R$ ) or I work in the yard ( $W$ ).*  
*It is not raining ( $\neg R$ ) or I go to the library ( $L$ ).*

What conclusion can we draw from them?

6. Determine whether this argument is valid, using natural deduction:

Lynn works part time or full time.

If Lynn does not play on the team, then she does not work part time.

If Lynn plays on the team, she is busy.

Lynn does not work full time.

**Therefore**, Lynn is busy.

7. Show that the following hold using natural deduction:

1.  $P \rightarrow Q, \neg Q \vdash \neg P$
2.  $(P \rightarrow Q) \rightarrow Q, Q \rightarrow P \vdash P$
3.  $\neg(P \wedge \neg Q) \vdash P \rightarrow Q$