4CCS1ELA – Elementary Logic with Applications

Small Group Tutorial 1 (week 2)

Propositional Logic

1. [EASY, 5 mins] A police officer has collected witness statements that enables the following assertions, A_1, A_2, A_3 and A_4, to be made from the evidence gathered about four crime suspects Ahmed, Bob, Carol, and Dot:

a_1: *If Ahmed is telling the truth then so is Bob.*

a_2: *Bob and Carol cannot both be telling the truth.*

a_3: *Carol and Dot are not both lying.*

a_4: *If Dot is telling the truth then Bob is lying.*

Represent the information above in propositional logic.

Heuristic for formalising English. Pick the smallest statements without *and, or, if ... then ...* etc. about which you could answer the question 'Is it true or false?'. Using propositional variables to stand for these statements connect them with the relevant logical connectives $\neg$, $\wedge, \vee, \rightarrow$ etc.

2. [EASY, 8 mins] Which of the following formulas are tautologies? Check using truth tables.

(i) $P \vee P$.

(ii) $P \vee (Q \wedge P)$.

(iii) $\neg\neg P \leftrightarrow P$.

(iv) $\neg P \rightarrow \neg P$.

3. [EASY, 9 mins]

(i) If $\neg(P \leftrightarrow Q)$ is true then what can be said about the truth values of $P \wedge Q$ and $P \vee Q$?

(ii) If $P \rightarrow Q$ is false then what can be said about the truth value of $P \wedge \neg Q$?

(iii) If $P \rightarrow Q$ is true then what can be said about the truth value of $P \vee R \rightarrow Q \vee R$?

4. [EASY, 5 mins] Determine whether the following proposition is a tautology, a contradiction, or neither:

$$(((P \rightarrow Q) \wedge (R \rightarrow S) \wedge (\neg Q \vee \neg S)) \rightarrow (\neg P \vee \neg R)).$$

5. Consider the following formula

$$(P \wedge \neg Q) \rightarrow \neg(Q \vee \neg P)$$

(i) [EASY, 3 mins] Draw up a truth table for this formula and determine whether this formula is a tautology, a contradiction or neither.

(ii) [MEDIUM, 2 mins] Read off from the truth table a disjunctive normal form (DNF) of this formula.