4CCS1ELA - Elementary Logic with Applications
Programming with Logic

Small Group Tutorial 5 Solutions

Question 1

Solution

$\neg(p \rightarrow (\neg q \rightarrow s))$
$\neg(\neg p \vee (\neg q \rightarrow s))$ *(R1)*
$\neg(\neg p \vee (\neg\neg q \vee s))$ *(R1)*
$\neg(\neg p \vee (q \vee s))$ *(R5)*
$\neg\neg p \wedge \neg(q \vee s)$ *(R3)*
$p \wedge \neg(q \vee s)$ *(R5)*
$p \wedge (\neg q \wedge \neg s)$ *(R3)*
$p \wedge \neg q \wedge \neg s$

Question 2

Solution

1. $\neg((P \wedge G) \rightarrow \neg M)$

$\neg(\neg(P \wedge G) \vee \neg M)$
$\neg(\neg P \vee \neg G \vee \neg M)$
$P \wedge G \wedge M$
$\rightarrow P, \rightarrow G, \rightarrow M$

2. $M \rightarrow (P \rightarrow (Q \rightarrow R))$

$\neg M \vee (P \rightarrow (Q \rightarrow R))$
$\neg M \vee (\neg P \vee (Q \rightarrow R))$
$\neg M \vee (\neg P \vee (\neg Q \vee R))$
$\neg M \vee \neg P \vee \neg Q \vee R$
$M, P, Q \rightarrow R$

3. $\neg(P \wedge G) \vee Q$

$(\neg P \vee \neg G) \vee Q$
$\neg P \vee \neg G \vee Q$
$P, G \rightarrow Q$

Question 3

Solution

$\neg(\forall x \forall y\, P(x, y) \rightarrow \exists y\, (G(y) \vee F(y)))$
$\neg(\neg \forall x \forall y\, P(x, y) \vee \exists y\, (G(y) \vee F(y)))$
$\neg\neg \forall x \forall y\, P(x, y) \wedge \neg \exists y\, (G(y) \vee F(y))$
$\forall x \forall y\, P(x, y) \wedge \neg \exists y\, (G(y) \vee F(y))$
$\forall x \forall y\, P(x, y) \wedge \forall y\, \neg(G(y) \vee F(y))$
$\forall x \forall y\, P(x, y) \wedge \forall y\, (\neg G(y) \wedge \neg F(y))$
$\forall x \forall y\, P(x, y) \wedge \forall z\, (\neg G(z) \wedge \neg F(z))$
$\forall x \forall y \forall z\, P(x, y) \wedge \neg G(z) \wedge \neg F(z)$

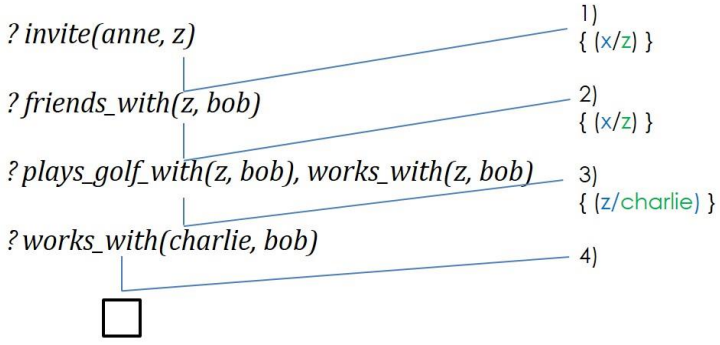
Question 4

Solution

a) The program is as follows (use of universal quantifiers and implication connectives is optional):
   1) $\forall x\, friends\_with(x, bob) \rightarrow invite(anne, x)$
   2) $\forall x\, plays\_golf\_with(x, bob), works\_with(x, bob) \rightarrow friends\_with(x, bob)$
   3) $\rightarrow plays\_golf\_with(charlie, bob)$
   4) $\rightarrow works\_with(charlie, bob)$

   Query (use of existential quantifier is optional): $?\exists z\, invite(anne, z)$.

b)



Query succeeds.