4CCS1ELA – Elementary Logic with Applications

Small Group Tutorial 3 (week 6)

1. [MEDIUM, 8 mins] Rewrite the following propositional formula in (i) a logically equivalent *conjunctive normal form*, and (ii) a logically equivalent *disjunctive normal form*:

$$\big(P \to (Q \wedge R)\big) \to S.$$

2. [HARD, 15 mins] Formalise the following argument in propositional logic and demonstrate its validity using natural deduction.

> *"If I graduate this semester, then I will have passed physics. If I do not study physics for 10 hours a week, then I will not pass physics. If I study physics for 10 hours a week, then I cannot play volleyball.*
> *Therefore, I will not graduate this semester if I play volleyball.*

3. [MEDIUM, 15 mins] Consider the set of natural numbers $\mathbf{N} = \{0, 1, 2, \ldots\}$ with the predicate $<$ and the function $+$ with their usual interpretation in arithmetic.

Express the following first-order sentences in English and determine which of these sentences are true.

(a) $\forall x \exists y (x < y)$
(b) $\forall y \exists x (x < y)$
(c) $\exists x \forall y (x < y)$
(d) $\forall x \forall y (x < y)$
(e) $\exists x \exists y (x < y)$
(f) $\forall x \forall y ((x < y) \to \exists z (x = y + z))$
(g) $\forall x \forall y \exists z (x = y + z)$

4. [MEDIUM, 15 mins] Let $Country(x)$ denote "x is a country; $Plane(x, y)$ denote the fact that one can travel from country x to country y by plane;

$Train(x, y)$ denote the fact that one can travel from country x to country y by train; and $Boat(x, y)$ denote the fact that one can travel from country x to country y by boat.

Let $france$, uk, $germany$, $ireland$ and $switzerland$ be the constants interpreted as France, UK, Germany, Ireland and Switzerland, respectively.

(a) Using the dictionary defined above, represent the following in predicate logic.

1. One can travel from France to the United Kingdom by air, by train and by boat.
2. There is at least one country that can be reached by train from the United Kingdom.
3. Any country that can be reached by plane from France can also be reached by plane from the United Kingdom.

(b) Translate the following sentences into equivalent English statements.

1. $\exists x(Country(x) \land Train(germany, x) \land \neg Train(ireland, x))$.
2. $\neg\exists x(Country(x) \land Boat(switzerland, x))$.
3. $\forall x(Country(x) \rightarrow \forall y(Country(y) \land Plane(x, y) \rightarrow Boat(x, y)))$.