Exercises

$P \wedge \neg P \vdash Q$

1. $P \wedge \neg P$ data.
2. $\neg Q \to P$ from subcomputation box

2.1 $\neg Q$	assume	$\underline{\underline{P}}$
2.2 P	1., and $\wedge E$	

3. $\neg Q \to \neg P$ from subcomputation box.

3.1. $\neg Q$	assume.	$\neg P$
3.2 $\neg P$	from 1 and $\wedge E$	

4. Q from 2., 3., and $\neg E$.

$P \to (\neg P \to Q)$.

P assume.

$\neg P \to Q$.

$\neg P$ assume

$\neg Q \to P$

$\neg Q \to \neg P$

Q

$P, P \to Q \vdash Q$ then. $P \vdash (P \to Q) \to Q$

~~1. data~~ 1. P data.
2. $P \to Q$ data.
3. Q 1, 2, $\to E$

1. P data.
2. $(P \to Q) \to Q$

2.1	$P \to Q$
2.2	Q

SET list 2.

1. $(P \vee Q) \to R,\ Q \vdash R$.

1. $(P \vee Q) \to R$ data.
2. Q data.
3. $P \vee Q$ 2., $\vee I$.
4. R from 3., 1, $\to E$.

$P \to Q$.

P assumption. $\underline{\underline{Q}}$

⋮

$\underline{\underline{X}}$

Q.

$P \to X$.

Tutorial list 2.

Q6

P works part time.

F works full time.

T plays on the team.

B is busy.

$$P \vee F,\ \neg T \to \neg P,\ T \to B,\ \neg F \vdash B\ ?$$

$P \vee F$

Show $\dfrac{\neg F}{P}$ variant rule.

Show $\neg T \to P$ variant rule.

Show T $\neg E$ with $\neg T \to P$, $\neg T \to \neg P$

Show B $T \to B, T$ modus ponens ($\to E$)

1. $P \vee F$ data.
2. $\neg T \to \neg P$ data.
3. $T \to B$ data.
4. $\neg F$ data.
5. P 1, 4, $\vee E2$. (variant rule)
6. $\neg T \to P$ 5, $\to I2$ (variant rule)
7. T 2, 6, $\neg E$
8. B from 3, 7, $\to E$

All men are mortal. P
Socrates is a man. Q

Socrates is mortal. R

$P, Q \not\models R$.

$\forall x \, [\text{Man}(x) \to \text{mortal}(x)]$
Man(Socrates)

---

mortal(Socrates)

Universal quantification.
Existential quantification

Man(Socrates) → mortal(Socrates)
man(Socrates)

---

mortal(Socrates)

$\Rightarrow \bar{C}$

Man ⇒ Unary

teaches(x, y)
teaches(odinaldo, EIA)

man ⇒ subset of the domain of the elements that have the property of being a man.

teaches ?

Domain: Natural numbers

$<(x,y)$: x is smaller than y.

$=(x,y)$: x is equal to y.

There is a natural number that is the smallest of all natural numbers.

$$\exists x \forall y \left(<(x,y) \vee =(x,y) \right).$$

$\hookrightarrow$ $x=0$ $\Rightarrow$ $\forall y \left[<(0,y) \vee =(0,y) \right]$.

$y=0$.

$$\underbrace{\underset{F}{<(0,0)} \vee \underset{T}{=(0,0)}}_{T}$$

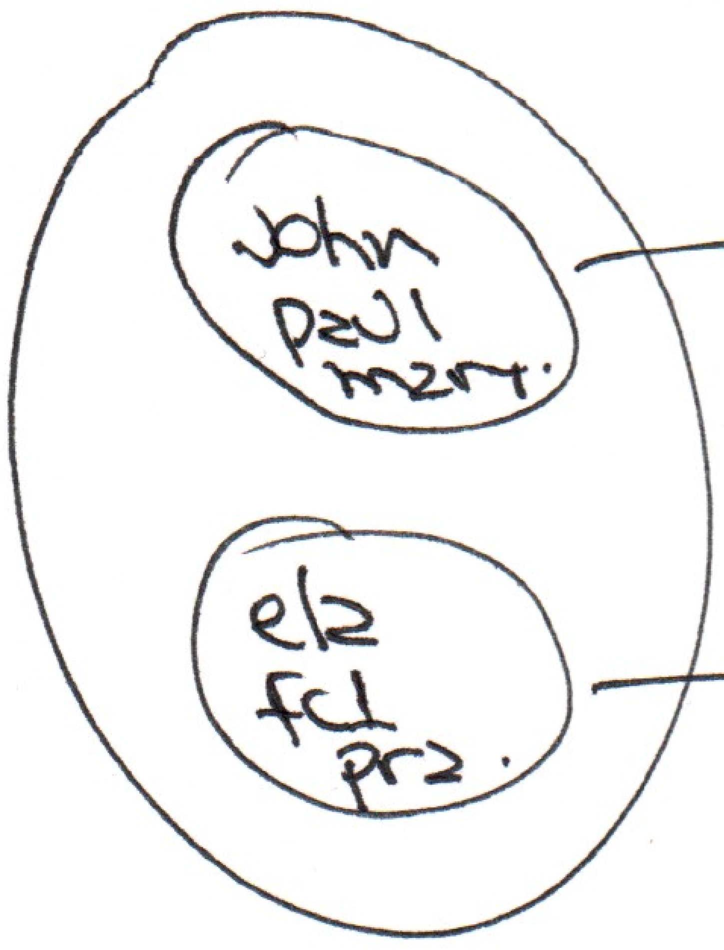
$y=1$.

$$\underbrace{\underset{T}{<(0,1)} \vee \underset{F}{=(0,1)}}_{T}$$

Prolog

Some student went to every lecture.

Domain.



→ Students. $S(x) \Rightarrow x$ is a student.

→ Lectures.

$S(john) \Rightarrow$ True.

$S(ela) \Rightarrow$ false

$$\exists x\,(S(x) \wedge \forall y\,[L(y) \rightarrow A(x,y)])\,.$$

~~Student(john)~~ $S(john)$ $L(ela)$

~~Student(paul)~~ $S(paul)$ $L(fc1)$

~~student(mary)~~ $S(mary)$. $L(pra)$.

All

$A(john, ela)$

$A(john, fc1)$

$A(mary, ela)$

$A(mary, pra)$

$A(paul, ela)$

$A(paul, fc1)$

$A(paul, pra)$.

paul

x → y_1 ela, y_2 fc1, y_3 pra.

$x = paul$

$$\forall y\,(L(y) \rightarrow \exists x\,(S(x) \wedge A(x,y)))\,.$$

Every bird eats every worm.

$\forall x \forall y [B(x) \wedge W(y) \wedge E(x,y)]$. WRONG

$\forall x \forall y [(B(x) \wedge W(y)) \to \underline{E(x,y)}]$.

Some bird do not eat some worms.

No bird is eaten by a worm.

x y.

a.

b.

c.

$\forall x \forall y$.

a – a

a – b

a – c

b – a.

b – b

b – c