① . 14-11-16 .

$\forall x P(x) \to \exists x P(x)$ tautology.

$\exists x P(x) \to \forall x P(x) \Rightarrow$ contingency

$D = \{a, b\}$.

$\exists x P(x) \to \forall x P(x)$.

T F

$P(a) \Rightarrow \exists x P(x) = T$.

For example. say $P(a)$ is true.

$P(b)$ is false.

$P(b)$ is false. $\Rightarrow \neg P(b)$.

$\exists x \neg P(x)$.

$\hookrightarrow \forall x P(x)$ is false.

$\exists x P(x)$ is true

$\exists x P(x) \to \forall x P(x) \Rightarrow$ false.

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$\exists x (P(x) \wedge \neg P(x)) \Rightarrow$ contradiction.

$\exists x P(x) \wedge \exists x \neg P(x)$ ? $\Rightarrow$ contingency!

2

$$\neg\forall x F \equiv \exists x \neg F$$

$$\neg\forall x F \equiv \exists x F$$

$$\exists x \neg F \equiv \neg\forall x F$$

$$\neg\forall x (C(x) \to T(x)) \equiv$$
$$\neg\forall x (\neg C(x) \vee T(x)) \equiv$$
$$\exists x \neg(\neg C(x) \vee T(x)) \equiv$$
$$\exists x (\neg\neg C(x) \wedge \neg T(x))$$
$$\exists x (C(x) \wedge \neg T(x)).$$

$$\underset{\equiv}{\underline{\forall x}} F \equiv \neg\exists x \neg F$$

$$\neg\forall x F \equiv \neg\neg\exists x \neg F \equiv \exists x \neg F.$$

$$\neg\exists x (C(x) \wedge \neg T(x)) \equiv$$
$$\neg\neg\forall x \neg (C(x) \wedge \neg T(x)) \equiv$$
$$\forall x \neg (C(x) \wedge \neg T(x)) \equiv$$
$$\forall x (\neg C(x) \vee \neg\neg T(x)) \equiv$$
$$\forall x (\neg C(x) \vee T(x)) \equiv$$
$$\forall x (C(x) \to T(x))$$

$$\neg\forall x (\neg L(x) \vee \neg V(x))$$
$$\neg\neg\exists x \neg (\neg L(x) \vee \neg V(x))$$
$$\exists x \neg (\neg L(x) \vee \neg V(x))$$
$$\exists x (\neg\neg L(x) \wedge \neg\neg V(x))$$
$$\exists x (L(x) \wedge V(x)).$$

③

Show that

$$\exists x(P(x) \rightarrow Q(x)) \equiv \forall x P(x) \rightarrow \exists x Q(x).$$

$$\Downarrow$$

$$\exists x(\neg P(x) \vee Q(x)) \equiv$$

$$\underbrace{\exists x}_{\Downarrow} \neg P(x) \vee \exists x Q(x) \equiv$$

$$\underbrace{\neg \forall x \neg} \neg P(x) \vee \exists x Q(x)$$

$$\neg \forall x P(x) \vee \exists x Q(x).$$

$$\forall x P(x) \rightarrow \exists x Q(x).$$

Tutorial 04
Q2 (iii)

No bird is eaten by a worm.

$$\forall x(B(x) \rightarrow \forall y(W(y) \rightarrow \neg E(y,x))) \equiv$$

$$\neg \exists x \exists y(B(x) \wedge W(y) \wedge E(y,x))$$

$$\forall x \forall y(B(x) \rightarrow (W(y) \rightarrow \neg E(y x))).$$

$$\forall x \forall y(\neg B(x) \vee (W(y) \rightarrow \neg E(y,x))).$$

$$\forall x \forall y(\neg B(x) \vee \neg W(y) \vee \neg E(y,x))$$

$$\neg \exists x \neg \neg \exists y \neg(\neg B(x) \vee \neg W(y) \vee \neg E(y,x))$$

$$\neg \exists x \exists y(\neg \neg B(x) \wedge \neg \neg W(y) \wedge \neg \neg E(y,x))$$

$$\neg \exists x \exists y(B(x) \wedge W(y) \wedge E(y,x)).$$

(4)

All penguins are birds.
No penguin can fly.
Tweety is a penguin.

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Some birds cannot fly.

$P(x) \Rightarrow$ x is a penguin.
$B(x) \Rightarrow$ x is a bird.
$F(x) \Rightarrow$ x can fly.
tweety $\Rightarrow$ constant.

$$\forall x\,(P(x) \rightarrow B(x))$$
$$\forall x\,(P(x) \rightarrow \neg F(x))$$
$$P(tweety)$$

---

$$\exists x\,(B(x) \wedge \neg F(x)).$$

1. $P(tweety)$ data.
2. $\forall x(P(x) \rightarrow B(x))$ data.
3. $\forall x\,(P(x) \rightarrow \neg F(x))$ data.
4. $P(tweety) \rightarrow B(tweety)$ $\forall$-E ($x = tweety$).
5. $B(tweety)$ 1, 4, $\rightarrow$E
6. $P(tweety) \rightarrow \neg F(tweety)$ $\forall$-E ($x = tweety$).
7. $\neg F(tweety)$ 1, 6, $\rightarrow$E.
8. $B(tweety) \wedge \neg F(tweety)$ 5, 6, $\wedge$I.
9. $\exists x(B(x) \wedge \neg F(x))$ $\exists$-I in 8.

5. All interrupt commands are undesirable. A

B Some control commands are interrupt commands.

Therefore:

C Some control commands are undesirable.

$$\frac{\begin{array}{ll}\forall x(\text{Interrupt}(x) \to \neg\text{desirable}(x)) & \text{A} \\ \exists x(\text{control}(x) \wedge \text{interrupt}(x)) & \text{B}\end{array}}{\exists x(\text{control}(x) \wedge \neg\text{desirable}(x)) \quad \text{C}}$$

1. A (data)
2. B (data)
3. $\text{control}(a) \wedge \text{interrupt}(a)$ — $\exists$-E in 2.
4. $\text{interrupt}(a) \to \neg\text{desirable}(a)$. — $\forall$-E in 1. $(x=a)$
5. $\text{interrupt}(a)$ — $\wedge$-E in 3.
6. $\neg\text{desirable}(a)$ — $\to$E 4, 5.
7. $\text{Control}(a) \wedge \neg\text{desirable}(a)$ — $\wedge$I 6.5, 6.

6.5. $\text{control}(a)$ — $\wedge$E 3.

8. $\exists x(\text{control}(x) \wedge \neg\text{desirable}(x))$. — $\exists$-I in 7.

(6.5 needs to go between 6. and 7.)

Added afterwards!

Tutorial 5, Q3 VALIDITY

VALID ‽

(i) $\exists x P(x) \wedge \forall y (P(y) \to Q(y)) \to \exists z Q(z)$.

(ii) $\exists x P(x) \wedge \forall y (P(y) \to Q(y)) \to \forall z \neg Q(z)$

NOT VALID ! $\exists z Q(z)$

$\downarrow$

$\neg \forall z \neg Q(z)$.

(i) Suppose.

$\exists x P(x) \wedge \forall y (P(y) \to Q(y))$ is true.

(A) (B)

(A) is true.

Suppose $\underline{x = a}$ (P(a) is true for some "a")

From (A) P(a) must be true.

(B) $\forall y (P(y) \to Q(y))$ must be true

$\to P(a) \to Q(a)$ true.

$\to Q(a)$ true.

$\to \exists z Q(z)$ $z = \underline{a}$

TUTORIAL $\underline{\underline{5}}$. $\underline{\underline{Q4}}$

$$\neg \exists x \forall y (\neg P(x) \wedge (Q(y) \to R(x,y))) \equiv \forall x (P(x) \vee \exists y (Q(y) \wedge \neg R(x,y)))$$

$$\neg\neg \forall x \neg \forall y (\neg P(x) \wedge (Q(y) \to R(x,y))) \equiv$$

$$\forall x \neg (\neg P(x) \wedge \forall y (Q(y) \to R(x,y)))$$

$$\forall x (\neg\neg P(x) \vee \neg \forall y (Q(y) \to R(x,y)))$$

$$\forall x (P(x) \vee \neg\neg \exists y \neg (Q(y) \to R(x,y)))$$

$$\forall x (P(x) \vee \exists y (Q(y) \wedge \neg R(x,y))).$$

Q5

(i). $\exists x \forall y (P(x) \to x = y)$

$P(a) = F$

$P(b) = F$ $\qquad D = \{a, b\}$.

$\underline{\underline{x = a}}$ $\Rightarrow$ $\forall y (P(a) \to a = y)$.

$y=a$ $(\underline{\underline{P(a)}} \to a = a) = T$

and $y=b$ $(\underline{\underline{P(a)}} \to a = b) = T$

(ii) $P(a) = T$

$P(b) = F$.

$x = b$ $\Rightarrow$ $\forall y (P(b) \to b = y)$.

$P(b) \to b = a \Rightarrow T$

$P(b) \to b = b \Rightarrow T$.