4CCS1ELA: Tutorial list 4

1. Formalising scenarios. (The ambiguity of natural languages.)

Let	$S(x)$	represent	'x is a student'
	$L(x)$	represent	'x is a lecture'
	$A(x, y)$	represent	'x attended y'

Formalise the following sentence:

'*At least one student attended every lecture.*'

2. Let $B(x)$ mean "x is a bird", let $W(x)$ mean "x is a worm", let $E(x, y)$ mean "x eats y". Using these predicates, represent in first-order logic each of the following statements:

(i) *Every bird eats every worm.*

(ii) *Some birds do not eat some worms.*

(iii) *No bird is eaten by a worm.*

(iv) *Some worms do not get eaten by birds.*

(v) *Only birds eat worms.*

3. Identify which occurrences of variables in the formulas below are free and which occurrences are bound. Justify you answers.

1. $y \geq 0 \ \wedge \forall x(N(x) \rightarrow x \geq y)$
2. $x \geq 0 \ \wedge \forall x(N(x) \rightarrow x \geq y)$
3. $\forall x(N(x) \rightarrow \exists y(N(y) \wedge x \geq y))$

Here N is a unary predicate symbol, $\geq$ is a binary predicate symbol in infix notation, and $x \geq y$ is an atom in infix notation.

4. Let $\mathcal{F}$ be a wff interpreted over D and $d \in D$. Then $\mathcal{F}(x/d)$ denotes the wff obtained from $\mathcal{F}$ by replacing all **free** occurrences of x by d.

Compute the following substitutions and determine the meaning (the truth-values) of the resulting sentences over natural numbers.

Here $N(x)$ denotes "x is a natural number", predicates $\geq$ and $>$ have their usual interpretation

1. $\big(y \geq 0 \ \wedge \forall x(N(x) \rightarrow x \geq y)\big)(y/3)$
2. $\big(x \geq 0 \ \wedge \exists y(N(y) \wedge x \geq y)\big)(x/3)$
3. $\big(\forall x(N(x) \rightarrow \exists y(N(y) \wedge x > y))\big)(x/3)$
4. $\big(\forall x(N(x) \rightarrow \exists y(N(y) \wedge y > x))\big)(y/3)$