608 Tunneling transmission coefficient

We have $E = 2eV, V_0 = 4eV$

(a) $k_{II}a = 0.72$, and

$$T_{6-49} = 0.62, \qquad T_{6-50} = 0.94$$

(b) $k_{II}a = 65.21$, and

$$T_{6-49} = 9.33 \cdot 10^{-57}, \qquad T_{6-50} = 9.33 \cdot 10^{-57}, \quad \frac{|T_{6-49} - T_{6-50}|}{T_{6-49}} \approx 10^{-57}$$

(c) $k_{II}a = 7.24$, and

$$T_{6-49} = 2.02 \cdot 10^{-6}, \qquad T_{6-50} = 2.02 \cdot 10^{-6}, \quad \frac{|T_{6-49} - T_{6-50}|}{T_{6-49}} \approx 10^{-6}$$

609 Rectangular potential barrier

(a) The opacity of a barrier is proportional to $2mV_0a^2/\hbar^2$ and therefore the lower mass particle, i.e. proton, has the higher probability of getting through.

(b) With $V_0 = 10MeV, E = 3MeV, a = 10^{-14}m$, it follows that

$$16\frac{E}{V_0}(1 - \frac{E}{V_0}) = 3.36.$$

The required masses are $m_p = 1.673 \times 10^{-27}kg, m_d \approx 2m_p$. For the proton $k_2a = 5.803$ and, using the approximate formula, i.e. eq 6.50,

$$T_p = 3.36e^{-2\times 5.803} = 3.06 \times 10^{-5}.$$

Since $m_d \approx 2m_p$, as noted above, $k_2a = 8.207$. Hence, for the deuteron

$$T_d = 3.36e^{-2\times 8.207} = 2.5 \times 10^{-7}.$$

616 Square well potential

Please refer to Appendix H of E & R.

The combination is $V_0 a^2$.

In the limit that the strength of the well is small, i.e. in the limit that the radius of the quarter circle defined (in the Appendix) as $q^2 + \xi^2$ is small, the number of admissible bound levels is 1. In general, irrespective of the value of $V_0 a^2$, it intersects p at least once, see figure H-1, since tan is an add function.

618 Potential well

We have

$$\psi_1 = Ae^{ik_1 x} + B^{-ik_1 x}, \qquad x < 0$$

$$\psi_2 = Fe^{-ik_2 x} + Ge^{ik_2 x}, \qquad 0 < x < a$$

$$\psi_3 = Ce^{ik_3 x} + De^{-ik_3 x}, \qquad x > a$$

here $D = 0$ since there is no wave coming from the right, and

$$\frac{\hbar^2 k_1^2}{2m} = 9V_0 - V_0 = V_0, \qquad \frac{\hbar^2 k_2^2}{2m} = 9V_0 - 0 = 9V_0, \qquad \frac{\hbar^2 k_3^2}{2m} = 9V_0 - 5V_0 = 4V_0.$$

That is $k_2 = 3k_1, k_3 = 2k_1$.

Matching ψ and the current $\frac{d\psi}{dx}$ at $x = 0$ gives

$$A + b = F + G, \qquad A - B = 3(G - F).$$

Similarly at $x = a$ the condition yields

$$Fe^{-ik_2 a} + Ge^{ik_2 a} = Ce^{ik_3 a}, \qquad -k_2 Fe^{-ik_2 a} + k_2 Ge^{ik_2 a} = k_3 C^{ik_3 a}$$

Solving the above equations, we find

$$A = \frac{10 - e^{i6k_1 a}}{6e^{ik_1 a}} C,$$

thus, the transmission probability is

$$T = \left| \frac{j_3}{j_1} \right| = \frac{|C|^2 \hbar k_3 / m}{|A|^2 \hbar k_1 / m} = \frac{72}{101 - 20\cos(6k_1 a)}$$

623 Particle in a box

(a) The energy levels are given by

$$E_n = E_1 n^2$$

here $E_1 = \frac{\pi^2 \hbar^2}{2ma^2}$. Therefore,

$$\frac{\Delta E_n}{E_n} = \frac{E_{n+1} - E_n}{E_n} = \frac{2n+1}{n^2}$$

(b) In the classical limit, i.e. $n \to \infty$, we find

$$\lim_{n \to \infty} \frac{\Delta E_n}{E_n} = 0$$

that is, since the energy levels become so close, they are indistinguishable. Hence, quantum effects are not apparent.

630 Simple harmonic oscillator potential

The zero point energy is

$$E_0 = \frac{1}{2} h\nu = \frac{1}{2} h \cdot \frac{1}{2\pi} \sqrt{\frac{C}{m}}$$

Therefore,

$$E_0 = 0.051 eV.$$

631 Simple harmonic oscillator potential

(a) Using $E_0 = 0.051eV$, the level spacing will be

$$\Delta E = E_{n+1} - E_n = h\nu = 2E_0 = 0.102eV$$

(b) The energy of the emitted photon E_γ is $E_\gamma = E_1 - E_0 = \Delta E = 0.102eV$.

(c) Since $E_\gamma = h\nu_\gamma = \Delta E = h\nu$. Therefore, $\nu_\gamma = \nu$. Thus,

$$\nu_\gamma = \frac{\Delta E}{h} = 2.5 \times 10^{13} Hz.$$

(d) Photons of this frequency are in the infrared spectrum.

634 Simple harmonic oscillator potential

We have

$$\psi_2 = A_2(1-2u^2)e^{-u^2/2}, \qquad E_2 = \frac{5}{2}h\nu = \frac{5}{2}\hbar\left(\frac{C}{m}\right)^{1/2},$$

here $u = \frac{(Cm)^{1/4}}{\hbar^{1/2}}x$

To verify we evaluate the derivatives

$$\frac{d\psi}{dx} = \frac{(Cm)^{1/4}}{\hbar^{1/2}}A_2(2u^3-5u)e^{-u^2/2}$$

$$\frac{d^2\psi_2}{dx^2} = \frac{(Cm)^{1/2}}{\hbar}A_2(11u^2-2u^4-5)e^{-u^2/2}$$

Substituting the above result $\frac{d^2\psi_2}{dx^2}$ into Schrodinger equation one gets

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} + \frac{C}{2}x^2\psi_2 = \frac{5}{2}\hbar\left(\frac{C}{m}\right)^{1/2}\psi_2,$$

which is $E_2\psi_2$.