Static Games of Complete Information

I. Introduction
   A. First (and simplest) type of game that we will cover in the course
      1. Will introduce us to the workhorse equilibrium concept – Nash equilibrium
      2. Other equilibrium concepts are also reasonable and we will discuss some of these as well (e.g. dominant strategy equilibrium)
   B. Definitions and representation
      1. Static: all players play simultaneously one time (single-shot game)
      2. Complete information: symmetric information (all players have the same information)
      3. Because there is no order of moves and no issues with asymmetric information the normal form representation of the game contains a complete description of the rules of the game
   C. Notation: rules of the game
      1. Players: $i = 1, 2, \ldots, n;\ i \in N$
      2. Strategies: s_i is a pure strategy (action) for player i, $s_i \in S_i$ is set of possible pure strategies for player i. $s = (s_1, s_2, \ldots, s_n)$, $s \in S$ where $S = X\, S_i$
      3. Utility (preferences): $U_i(s)$, describes player i's ranking of various possible outcomes. $U = \{U_1, U_2, \ldots, U_n\}$
      4. **{*N, S, U*}** are the rules of the game
      5. We assume that the rules of the game are common knowledge - every player knows the rules of the game, and knows that other players know the rules on the game…

   C. Example: Two players: i = 1, 2. Player 1 (rows players) can choose *U*, *M*, or *D*. Player 2 (column player can choose *L*, *C*, or *R*. Payoffs given in the matrix

Player 2 / Player 1	L	C	R
U	20, 15	25, 5	30, 10
M	10, 5	40, 20	15, 30
D	15, 0	45, 30	10, 40

What strategy should players choose? What would you predict to be the outcome of this game?

II. Solution concepts:

A. Apply a solution concept to "solve" the game – make a prediction about how rational players whose goal is to maximize utility should play the game
B. There is no single right answer to what is the right solution concept. Game theorists have settled on some solution concepts that seem reasonable and have desirable properties. Analogy to econometrics: why is least-squares the estimator of choice? (Good properties – best linear unbiased estimator)
C. Solution concepts for static games of complete information:
    1. Nash equilibrium
    2. Dominant strategy equilibrium
    3. Iterated strict dominance
    4. Rationalizable strategies (not actually a solution concept but a way of eliminating strategies)
    5. Maxi-min solution

III. Nash equilibrium

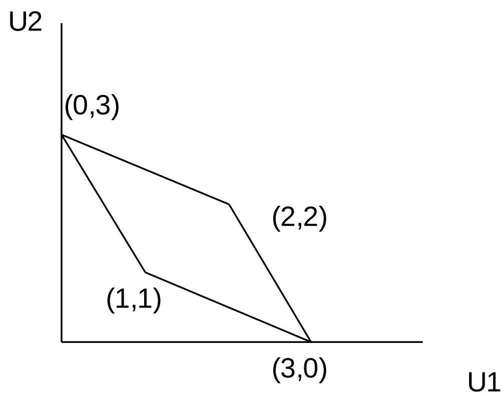
A. Definition and discussion
    1. Core concept of game theory. Most applications in economics use Nash equilibrium or refinement of Nash equilibrium.
    2. Basic idea: equilibrium means that no player has an incentive to change strategy – "no profitable deviation." Find a combination of strategies where all players cannot do any better than what they are doing, given what the other players are doing.
    3. Let s_i represent the strategy of player $i \in N$ and let $s_{\sim i}$ represent the strategies of all players $j \neq i \in N$
    4. Definition 1: $s^* \in S$ is a **pure strategy Nash equilibrium** if for all players $i \in N$, $U_i(s_i^*, s_{\sim i}^*) \geq U_i(s_i, s_{\sim i}^*)$ for all $s_i \in S_i$. {No profitable unilateral deviations}
    5. Definition 2:
        a. A **best response** function for player i:
        $br_i(s) = \{s_i \in S_i$: $U_i(s_i, s_{\sim i}) \geq U_i(s_i', s_{\sim i})$ for all $s_i' \in S_i\}$
        b. Best response correspondence:
        $br(s) = X\ br_i(s)$
        c. $s \in S$ is a pure strategy Nash equilibrium if $s \in br(s)$ {strategy is a best response to itself}
    6. Note: definition 2 is useful for existence proofs. Nash equilibrium is a fixed point of best response correspondence. To prove existence use proofs of existence of fixed point. However, existence is not usually the problem with Nash equilibrium. Multiplicity of equilibria is more often the problem.

B. Simple examples

1. Prisoner's Dilemma

Player 2 / Player 1	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

a. Unique Nash equilibrium is for both players to defect (*D*, *D*): denote Nash equilibrium by *strategies*, not by payoff.
b. Can see that (*D*, *D*) is a Nash equilibrium by either showing that there are no profitable deviations from (*D*, *D*), definition (1), or by showing that *D* is a best response for each player, definition (2).
c. A sure fire way to find Nash equilibria:
   i. Choose a strategy for player *j* and ask what is player *i*'s best response to this strategy and underline it.
   ii. For example, if player 2 players *C*, the best response for player 1 is to play *D*.
   iii. Repeat for all strategies for player *j*
   iv. Repeat the process but reversed: choose a strategy for player *i* and ask what player *j*'s best response to this strategy is and underline it. Do this for all strategies of player *i*.
   v. If you have a pair of strategies with underlines this constitutes a Nash equilibrium because the pair of strategies is a best responses to itself.
d. Nash equilibrium and efficiency: (*D*, *D*) is the only pure strategy combination that is not Pareto optimal. Game theory solutions do not always satisfy fundamental welfare theorems ("a competitive equilibrium is Pareto efficient). Why not? Case of an externality: the actions of one player directly affect the payoffs of another.



2. Battle of the Sexes

Husband \ Wife	Football	Ballet
Football	2, 1	0, 0
Ballet	0, 0	1, 2

a. Two pure strategy Nash equilibria: (*F*, *F*), (*B*, *B*).
b. General problem: with multiple equilibria, what is the prediction of the game? How do we know that players will even get to equilibrium?
c. Need a refinement
    i. Schelling: focal points. Meet in NYC tomorrow. Grand Central Station at noon.
    ii. Pareto dominance: may work. In the following game, we would expect players to choose (*U*, *L*)

Player 1 \ Player 2	Left	Right
Up	2, 2	0, 0
Down	0, 0	1, 1

    iii. But, it may not. What would you predict for this game? Might think that players will choose (*D*, *R*) because of fear that they might get stuck with a payoff of 0. Risk averse strategies (*D*, *R*)

Player 1 \ Player 2	Left	Right
Up	9, 9	0, 8
Down	8, 0	7, 7

3. Matching Pennies

Player 2 / Player 1	Heads	Tails
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

a. There does not exist a pure strategy Nash equilibrium.
b. There does, however, exist a mixed strategy Nash equilibrium {How would you play this game?}. Will show an existence proof for Nash equilibrium once we have introduced mixed strategies.

C. Mixed Strategy Nash Equilibrium

1. Definition of Mixed Strategies

$\sigma_i(s_i)$ = probability that player i will play pure strategy s_i.

$\sigma_i(S_i)$ is a **mixed strategy** (tells the probability of playing each pure strategy)

Σ_i is the set of mixed strategies for player i

$$\Sigma_i = \{\sigma_i(S_i) : \sum_{s_i \in S_i} \sigma_i(s_i) = 1, \sigma_i(s_i) \geq 0, \text{for all } s_i \in S_i\}$$

2. Mixed Strategy Nash Equilibrium

a. Definition: A mixed strategy profile σ^* is a Nash equilibrium if, for all players $i \in N$, $U_i(\sigma_i^*, \sigma_{\sim i}^*) \geq U_i(s_i, \sigma_{\sim i}^*)$ for all $s_i \in S_i$.
b. Note: if $U_i(\sigma_i^*, \sigma_{\sim i}^*) \geq U_i(s_i, \sigma_{\sim i}^*)$ then it is also the case that $U_i(\sigma_i^*, \sigma_{\sim i}^*) \geq U_i(\sigma_i, \sigma_{\sim i}^*)$

3. Example: Battle of the Sexes

Wife / Husband	Football	Ballet
Football	2, 1	0, 0
Ballet	0, 0	1, 2

Recall that in this game there were two pure strategy Nash equilibria for this game. Is there also a mixed strategy Nash equilibrium?

The wife prefers F to B when
$U_W(F) > U_W(B) \Rightarrow$
$1\sigma_H(F) + 0(1 - \sigma_H(F)) > 0\sigma_H(F) + 2(1 - \sigma_H(F)) \Rightarrow$
$\sigma_H(F) > 2/3$
The wife prefers B to F when $\sigma_H(F) < 2/3$
When $\sigma_H(F) = 2/3$, the wife is indifferent between B and F

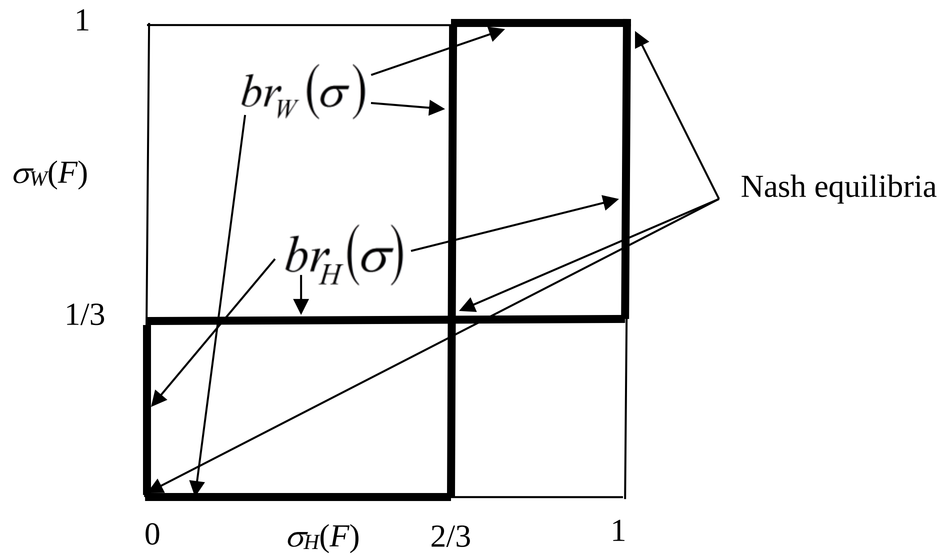
The wife's best response function can be written as

$$br_W(\sigma) = \begin{cases} \sigma_W(F) = 0, & \text{for } 2/3 > \sigma_H(F) \geq 0 \\ \sigma_W(F) \in [0,1], & \text{for } \sigma_H(F) = 2/3 \\ \sigma_W(F) = 1, & \text{for } 1 \geq \sigma_H(F) > 2/3 \end{cases}.$$

The husband prefers F when
$U_H(F) > U_H(B) \Rightarrow$
$2\sigma_W(F) + 0(1 - \sigma_W(F)) > 0\sigma_W(F) + 1(1 - \sigma_W(F)) \Rightarrow$
$\sigma_W(F) > 1/3$.
The husband prefers B to F when $\sigma_W(F) < 1/3$
When $\sigma_W(F) = 1/3$, the husband is indifferent between B and F.
The husband's best response function can be written as

$$br_H(\sigma) = \begin{cases} \sigma_H(F) = 0, & \text{for } \sigma_W(F) < 1/3 \\ \sigma_H(F) \in [0,1], & \text{for } \sigma_W(F) = 1/3 \\ \sigma_H(F) = 1, & \text{for } \sigma_W(F) > 1/3 \end{cases}.$$

The figure below illustrates the best response functions and shows that there is indeed a mixed strategy Nash equilibrium where the wife chooses football with probability 1/3 and the husband chooses football with probability 2/3 (as well as two pure strategy equilibria.



D. Existence Theorem for finite games:
Every finite game has a Nash equilibrium in the game with mixed strategies. (Nash 1950)

Sketch of Proof (a more complete proof is given in Fudenberg and Tirole, pp. 29-30): Application of a fixed point theorem. A Nash equilibrium is a fixed point of the best response correspondence: $\sigma \in br(\sigma)$.

Kakutani's Fixed Point Theorem: sufficient conditions for a correspondence $br(\sigma)$ to have a fixed point $\sigma \in \Sigma$ are:

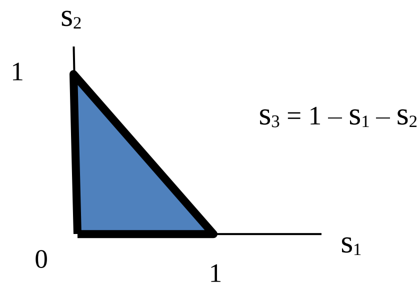
i) Σ is a compact, convex and nonempty subset of a (finite-dimensional) Euclidean space
ii) $br(\sigma)$ is nonempty for all σ.
iii) $br(\sigma)$ is convex for all σ.
iv) $br(\sigma)$ is upper hemi-continuous ("continuous").

Not difficult to show that each condition is satisfied:
(i) Player i's strategy space Σ_i is compact (closed and bounded), convex (if *x* and *x*' are contained in *X* then λ*x*+(1-λ)*x*' is contained in *X*), and non-empty. Player i's strategy space is:

$$\Sigma_i = \{\sigma_i(S_i) : \sum_{s_i \in S_i} \sigma_i(s_i) = 1, \sigma_i(s_i) \geq 0, \text{for all } s_i \in S_i$$

If player i has X strategies, then Σ_i is an $X - 1$ dimensional simplex.

Example: $X = 3$.



A finite simplex is compact, convex, and non-empty. Since each player's strategy space satisfies the desired properties, then $\Sigma_i = X_{i \in N} \Sigma_i$ satisfies the desired properties.

(ii) Each player's payoff is linear in strategies (expected utility is linear in probabilities) and is therefore continuous. A continuous correspondence obtains a maximum (minimum) over a compact space so a best response exists for any given mixed strategy.

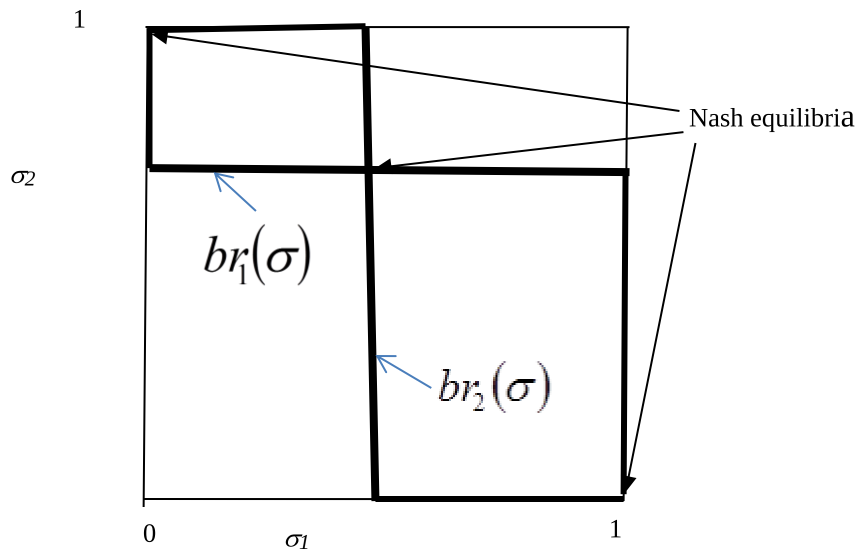
(iii) Expected utility is linear in probabilities so that convexity of the best response correspondence is assured.

(iv) Expected utility is continuous in probabilities so the best response correspondence will be continuous in strategies.

Note: mixed strategies are essential for proof because otherwise the strategy set is not convex.

Intuition: 2x2 game:

- Player 1 best response: gives the payoff maximizing strategy for each (mixed) strategy of player 2. So we must have a response for each strategy of player 2 from the bottom of the box to the top on the box.
- Player 2 best response gives the payoff maximizing strategy for player 2 for each (mixed) strategy of player 1. So we must have a response for each strategy of player 2 from the left of the box to the right on the box
- Best response functions are continuous
- Drawing a continuous line from the bottom to the top must cross a continuous line from the left of the box to the right of the box at least once.
- Where the best response functions cross is a Nash equilibrium.



E. Existence Theorem for continuous games: When S_i are nonempty, compact, convex subsets of Euclidean space, payoffs U_i are continuous in S and quasiconcave in S_i, there exists a pure strategy Nash equilibrium. (Debreu 1952, Glicksburg 1952, Fan 1952)

F. Economic example with continuous strategies and N players: harvesting from a common property resource

1. Players: $i = 1, 2, \ldots, n$.
2. Strategies: effort level by fisherman i (hours of fishing) $e_i \geq 0$. Let $E = \sum_{i=1}^{n} e_i$ represent total effort of all fishermen.

3. Utility (payoffs):

p is price per pound of fish

c is cost of a fishing effort

Total harvest: $Q = \begin{cases} \alpha E - E^2 & \text{for } 0 \leq E \leq \alpha \\ 0 & \text{otherwise} \end{cases}$

Portion of total harvest for fisherman i: e_i/E.

Utility (payoff) for fisherman i:

$$U_i(e_i, e_{\sim i}) = p(\alpha E - E^2)\frac{e_i}{E} - ce_i$$
$$= p(\alpha - E)e_i - ce_i$$

4. Find the best response function for fisherman *i* (assuming that other fishermen's strategy is fixed). Take the first order condition and set it equal to zero:

$$\frac{\partial U_i(e_i, e_{\sim i})}{\partial e_i} = p(\alpha - E - e_i) - c = 0$$

$$(\alpha - E - e_i) - c/p = 0$$

5. Sum over all *N* harvesters:

$$N\alpha - (N+1)E - N(c/p) = 0$$

$$E = \frac{N(\alpha - c/p)}{N+1}$$

6. Use this to solve for e_i:

$$e_i = \frac{(\alpha - c/p)}{N+1}$$

This is the Nash equilibrium harvest strategy for each fisherman.

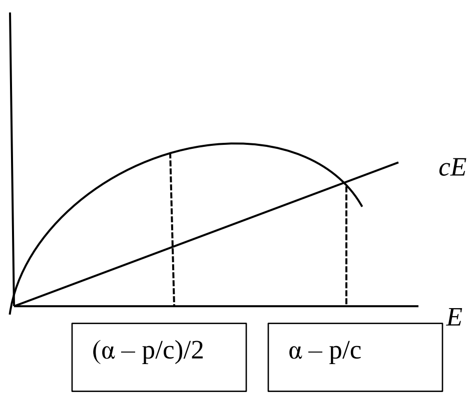
7. What happens as *N* goes to infinity?

$$E \to \alpha - c/p$$

$$U_i(e_i, e_{\sim i}) = p(\alpha - E)e_i - ce_i$$

$$\to p(\alpha - \alpha + c/p)e_i - ce_i = 0$$

As *N* goes to infinity you get the "tragedy of the commons" - total rent dissipation. Efficiency occurs for *N* = 1 when there is no externality to other harvesters.



IV. Dominant Strategy Equilibrium

A. Motivation

1. Nash equilibrium is the dominant equilibrium concept used to make predictions about games. But Nash equilibrium requires making a prediction about how others will play.
2. In some situations one might be able to appeal to simpler reasoning that applies that could make for a more compelling or convincing argument about the equilibrium outcome.
3. One such concept is dominant strategy equilibrium: *if* it exists it is usually fairly compelling as a prediction for the outcome of the game.

B. Definition

1. **Dominant strategy**: for $s_i, t_i \in S_i$,
   i. s_i ***weakly dominates*** t_i if: $U_i(s_i, s_{\sim i}) \geq U_i(t_i, s_{\sim i})$ for all $s_{\sim i} \in S_{\sim i}$;
   ii. s_i ***dominates*** t_i if: $U_i(s_i, s_{\sim i}) \geq U_i(t_i, s_{\sim i})$ for all $s_{\sim i} \in S_{\sim i}$ and $U_i(s_i, s_{\sim i}) > U_i(t_i, s_{\sim i})$ for some $s_{\sim i} \in S_{\sim i}$
   iii. s_i ***strictly dominates*** t_i if: $U_i(s_i, s_{\sim i}) > U_i(t_i, s_{\sim i})$ for all $s_{\sim i} \in S_{\sim i}$.
2. **Dominant strategy equilibrium**: $s \in S$ is a (weakly/strictly) dominant strategy equilibrium if for all $i \in N$, and all $t_i \in S_i$, s_i (weakly/strictly) dominates t_i.

C. Example: Prisoner's Dilemma

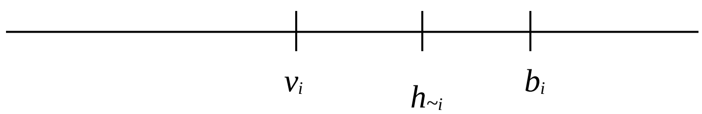
Player 2 Player 1	Cooperate	Defect
Cooperate	2, 2	0, 3
Defect	3, 0	1, 1

Defect is a strictly dominant strategy for each player.
(*D*, *D*) is a strictly dominant strategy equilibrium.

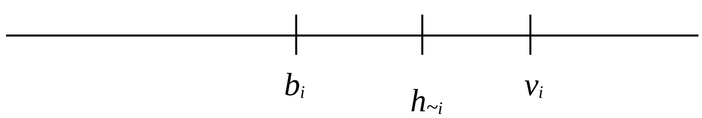
D. Second price sealed bid auctions (strictly speaking, this game is one of incomplete information and should be covered later in the course – but it is a rare economic situation with a dominant strategy equilibrium – so we will cover it here).

1. *N* players i = 1, 2, …, *n*.

2. Each player has a value for a good to be auctioned ($v_1, v_2, \ldots, v_n$). Player i's value is known only by player i.
3. Strategies: players bid simultaneously ($b_1, b_2, \ldots, b_n$).
4. Utility: high bid wins the auction, pays an amount equal to the second highest bid.
5. Consider player i, let $h_{\sim i}$ be the highest bid for all players not i. Payoff for player i: $U_i = \begin{cases} v_i - h_{\sim i} \text{ for } b_i > h_{\sim i} \\ 0 \text{ for } b_i < h_{\sim i} \end{cases}$ (ignore ties)
6. Claim: dominant strategy to bid equal to the value: $b_i = v_i$.
7. Suppose $b_i > v_i$. If $b_i > v_i$ then there is some $h_{\sim i} : b_i > h_{\sim i} > v_i$. In this case, player i "wins" the auction and gets a payoff of $v_i - h_{\sim i} < 0$. In contrast, if player i had bid $b_i = v_i$, then $b_i < h_{\sim i}$ and player i would not have won the auction and would have gotten a payoff 0. For $h_{\sim i}$ outside of this range, bidding $b_i > v_i$ generates the same payoffs as bidding $b_i = v_i$.



8. Suppose $b_i < v_i$. If $b_i < v_i$ then there is some $h_{\sim i} : v_i > h_{\sim i} > b_i$. In this case bidding $b_i < v_i$ means player i doesn't win the auction and gets payoff of 0. If player i had bid $b_i = v_i$ then player i would have won the auction and gotten $v_i - h_{\sim i} > 0$. For $h_{\sim i}$ outside of this range, bidding $b_i < v_i$ generates the same payoffs as bidding $b_i = v_i$.



9. Therefore, setting $b_i = v_i$ dominates all other strategies for player i, $i = 1, 2, \ldots, N$. Each player bidding their value is a dominant strategy equilibrium.
10. Appeal of second price auction: "truth-telling" is a dominant strategy. Not the case in first price auction where want to lower bid to reduce payment. Here payment is not tied to own bid.

E. Dominant strategy equilibrium typically fail to exist. Not many cases in economics where dominant strategies exist.

V. Iterated Strict Dominance

A. Dominant strategies may fail to exist but might use the idea of dominance to generate a prediction.

B. No rational player should ever play a strictly dominated strategy. Reasonable prediction of play would throw out strictly dominated strategies. But then more strategies may become strictly dominated. Continue to toss out strictly dominated strategies until process ceases: iterated strict dominance.

C. Only discard strategies that are strictly dominated: $U_i(s_i, s_{\sim i}) > U_i(t_i, s_{\sim i})$ for all $s_{\sim i} \in S_{\sim i}$.

D. If process of tossing out strictly dominated strategies results in a single outcome, we say the game is solvable by iterated strict dominance.

E. Example:

	L	*C*	*R*
U	4, 3	5, 1	6, 2
M	2, 1	8, 4	3, 6
D	3, 0	9, 6	2, 8

C is strictly dominated by *R* for player 2.

	L	*R*
U	4, 3	6, 2
M	2, 1	3, 6
D	3, 0	2, 8

M and *D* are strictly dominated by *U*.

	L	*R*
U	4, 3	6, 2

R is strictly dominated by *L*.
Iterated strict dominance prediction: (*U*, *L*)

F. Is iterated strict dominance as “reasonable” as dominant strategy equilibrium? Not really. It relies on a chain of logic that depends upon the reasonableness of play of the rival player. What if the other player is capable of making “mistakes.

G. Example:

	L	*R*
U	8, 10	-100, 9
D	7, 6	6, 5

For player 2, *L* strictly dominates *R*. Therefore, player 1 should just look at the payoffs with *L* and should then choose *U*. *U, L* is iterated strict dominance prediction and Nash equilibrium. But – how do you think real players would play this game. Risk averse strategy: play *D*, avoid the chance of getting -100.

H. If players have limited reasoning ability, perhaps because they are in a novel situation or because the game is complex, it might not be reasonable to assume that they can apply a complex iterated chain of logic. Even if others are sophisticated, if there are doubts about the sophistication of rivals this might be enough for players to choose not to apply iterated dominance. Example: Beauty contest game. Experimental evidence: players use limited steps of iterated reasoning: mean number of iterations is 2 (but can learn to do more).

I. Iterated strict dominance may fail to yield a prediction (Ex: Battle of the Sexes).

VI. Rationalizable Strategies

A. What are all the strategies that a rational player would play? Rule out strategies that are never a best response to rival's strategies.

B. Strategy σ_i is a best response for player *i* when rival players play $\sigma_{\sim i}$ if $U_i(\sigma_i, \sigma_{\sim i}) \geq U_i(\sigma_i', \sigma_{\sim i})$ for all $\sigma_i' \in \Sigma_i$. Strategy σ_i is **never a best response** if there is no $\sigma_{\sim i}$ for which σ_i is a best response.

C. Restrict beliefs to be rational in the sense that there should be zero probability weight on rival's strategies that are never a best response.

D. Players can then iterate on set of remaining strategies and eliminate all strategies that are never a best response to the remaining strategies.

E. The set of strategies that remain after successive elimination of strategies is the set of **rationalizable strategies**.

F. In two person games, rationalizability is the same as iterated strict dominance. In *N* person games, these concepts need not be the same.

VII. Maxi-min Solution

A. If a player is fearful that rival players might make mistakes, the player may wish to be cautious in how they play in order to avoid bad outcomes. This is the basic idea behind a maxi-min strategy. Alternatively, if the player is paranoid (the other players are out to get me) then the player should find the strategy that guarantees the highest possible amount, which is the maximum minimum value or maxi-min.

B. Strategy σ_i* is a maxi-min strategy if it maximizes player i's minimum possible payoff: $\sigma_i{}^* = \max_{\sigma_i}[\min_{\sigma_{\sim i}} U_i(\sigma_i, \sigma_{\sim i})]$

C. Maxi-min solution: each player plays a maxi-min strategy.

D. Example: matching pennies

Player 2 Player 1	Heads	Tails
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

To find a maxi-min strategy for player 1, we first minimize $U_1(\sigma_1(H), \sigma_2(H))$ with respect to $\sigma_2(H)$.

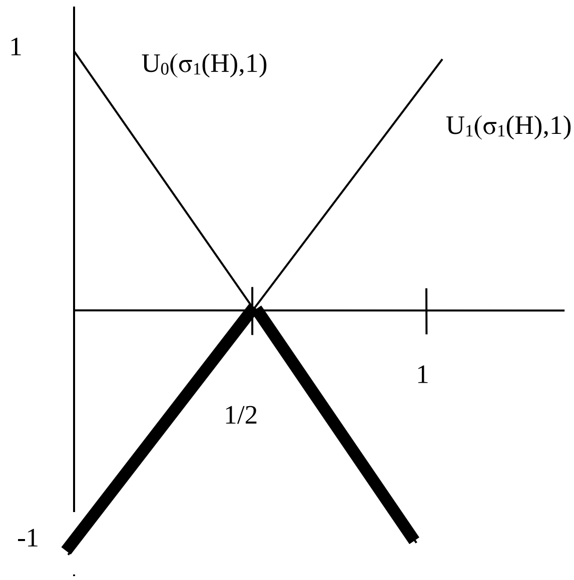
$$
\begin{aligned}
U_1(\sigma_1(H), \sigma_2(H)) &= \sigma_1(H)\sigma_2(H) - \sigma_1(H)(1-\sigma_2(H)) - (1-\sigma_1(H))\sigma_2(H) + (1-\sigma_1(H))(1-\sigma_2(H)) \\
&= 1 - 2\sigma_1(H) - 2\sigma_2(H) + 4\sigma_1(H)\sigma_2(H)
\end{aligned}
$$

$$\frac{\partial U_1(\sigma_1(H), \sigma_2(H))}{\partial \sigma_2(H)} = -2 + 4\sigma_1(H)$$

Note that this expression is positive for $\sigma_1(H) > 1/2$ and negative for $\sigma_1(H) < 1/2$. Since player 2 wants to minimize utility for player 1, player 2 will set $\sigma_2(H) = 0$ for $\sigma_1(H) > 1/2$ and $\sigma_2(H) = 1$ for $\sigma_1(H) < 1/2$.

When $\sigma_2(H) = 0$, $U_1(\sigma_1(H), 0) = 1 - 2\sigma_1(H)$. When $\sigma_2(H) = 1$, $U_1(\sigma_1(H), 1) = 1 - 2\sigma_1(H) - 2 + 4\sigma_1(H) = 2\sigma_1(H) - 1$. These expressions are graphed below. The segments highlighted show the choices that player 2 will make based on $\sigma_1(H)$: $\sigma_2(H) = 0$ for $\sigma_1(H) > 1/2$ and $\sigma_2(H) = 1$ for $\sigma_1(H) < 1/2$. Player 1 then has the choice of anywhere on the highlighted sections and would like to maximize utility. Player 1 will then choose $\sigma_1(H) = 1/2$.

Similar calculations show that maxi-min strategy for player 2 is $\sigma_2(H) = 1/2$.



E. The maxi-min solution was prominent when game theorists primarily studied zero-sum games. Zero-sum games (constant sum games) are games of pure competition – one person's gain is another loss. Matching pennies is an example of a zero-sum game. In zero-sum games, paranoia is justified – they are out to get you (so they get more).

F. In non-zero sum games, maxi-min may not make much sense. Example:

	L	*R*
U	0,0	3,1
D	400,300	100,2

Columns player should be fairly certain that rows player will play *D*, in which case the column player can get 300 by playing *L*.