Introduction:
Economics of Strategic Interactions and Incomplete Information
Introduction to Game Theory
Elements of a Game

1. Introductory Comments: "The imperfect portion of micro theory"
    a. Much of the first part of micro theory deals with perfect situations
        i. perfect competition
        ii. perfect information
        iii. Pareto efficient outcomes…
        iv. "the best of all possible worlds"
        v. Rationale: analyze simple cases first and show why markets generate efficient outcomes
        vi. Downside: not many real world examples of perfect markets
    b. This part of micro deals with more realistic imperfect situations
        i. Imperfect competition: monopoly, oligopoly – few sellers that realize they influence price through their actions; bilateral trade – bargaining/negotiation
        ii. Incomplete information: economic agents do not all have full information about other economic agents ("asymmetric information")
        iii. Imperfect information and/or incomplete information often yield Pareto inefficient outcomes
        iv. These situations are quite common and economists have developed powerful set of tools and insights into analysis of these situations
        v. Analysis of strategic interactions – main tool is game theory
    c. Purpose of this segment of microeconomic theory is to learn game theory and information

2. Game theory analyzes situations in which each player's payoff depends upon the choices of other players. "Strategic interaction."
    a. Game theory presents much more difficult problems to solve than engineering problems (or perfect competition).
        i. In an engineering problem the environment is a constant and one optimizes given the constant environment.
        ii. In game theory, other players can change their strategy and what one player wants to do may depend on what others do.
        iii. Since the strategies' of other players affects the optimal strategy for a player, each player must have a way to predict what others will do.
        iv. These predictions should be correct or a chosen strategy won't really be optimal for the player.
        v. Game theory isn't rocket science - ***its way harder*** (and much more fun)

3. Major branches of game theory
    a. Cooperative vs. Non-cooperative Game Theory

i. **Cooperative game theory**: assume a set of axioms (e.g., Pareto optimality, players cannot be made worse off than what they can guarantee themselves, equity…) and solve the game give those axioms. Ex: Nash bargaining solution.

ii. **Non-cooperative game theory**: what we will study this semester. Players act in their own best interests. Solve for equilibrium given the assumption that each player pursues self-interest. Ex: Nash equilibrium.

iii. Note: you can come to cooperative *solutions* via non-cooperative play as we will see when we study repeated games and the "folk theorem." Cooperative vs. Non-cooperative is not about outcomes *per se* but about motivation of players and methods for solving games.

b. Standard vs. Behavioral Game Theory

i. Standard approach: how players *should* play the game to maximize own utility.
   - Following standard economics, classical game theory assumes rational players: players take actions that maximize their own expected utility.
   - Game theory requires more rationality than traditional decision theory. In game theory, a player must predict other players' choices, which depends on understanding their expected utility maximization problems, which, of course, depends on their view of other player's expected utility maximization problems, which, depends upon…
   - Standard approach to game theory is a *normative* approach: prediction about how rational players *should* play to maximize their expected utility.

ii. Behavioral approach: ample experimental evidence that players DO NOT play according to predictions of standard game theory.
   - People are human beings not calculating machines – they make mistakes ("irrational" play)
   - Behavioral economics tries to explain how people actually behave (in both strategic and non-strategic situations)
   - In game theoretic situations, actual play may not accord with standard game theory predictions because
     - People may not fully understand the rules of the game
     - People may fail to accurately predict how others will play
     - People may be subject to bounded rationality (complexity of game overwhelms cognitive abililty)
     - They are "irrational" – fail to take actions that are in their own best interest

iii. Example of irrational decision-making

- Experiments on **framing** by Kahneman and Tversky (Tversky, A. and D. Kahneman. 1981. The framing of decisions and the psychology of choice. *Science* 211: 453-458)
  - Choice problem (1) - choose (a) or (b):
    (a) get $100 with probability 1, or
    (b) get $0 with probability .75, get $400 with probability .25.

  Most people choose (a).
  - Choice problem (2): I give you $400. Choose (c) or (d):
    (c) lose $300 with probability 1, or
    (d) lose 0 with probability .25 and lose $400 with probability .75.

  Most people choose (d).

  Expected utility maximizers: if choose (a) then must choose (c), if choose (b), must choose (d). Typical pattern of choice is a violation of expected utility maximization. Irrational choice: choice should not depend upon irrelevant framing of the choice.
- Experiments on **anchoring** by Prelec, Loewenstein and Ariely (Ariely *Predictably Irrational*, Chapter 2 *The Fallacy of Supply and Demand*)
  - Asked students to write down the last two digits of their social security number.
  - Then asked students how much would they be willing-to-pay for a certain items (e.g., bottle of wine, box of chocolates, cordless keyboard and mouse, etc.)
  - Found that students anchored on their reported social security number. Top 20% of SSN bid 216 to 346% higher than the bottom 20% of SSN.

  For rational behavior, the irrelevant information about last two digits of the social security number should not influence the bid, but they did. People use cues to construct bids rather than having necessarily having a coherent set of preferences in their heads.

iv. Much of the interesting work in game theory these days is in trying to understand how people actually play games and explain patterns of play
   - Under what conditions do people play according to standard game theory ("rational behavior")
   - Under what conditions to they deviate from standard game theory and do they do so in a predictable manner
   - What explains the deviations from rational behavior?
     - Irrational play of some form (as in examples above)
     - Or more complex utility function

- In experiments, payoffs are **money** payoffs. But game theory (both standard and behavioral) is about **utility** payoffs. Some apparent differences between actual play and predicted play can be explained by correctly understanding the utility function. Is utility just monetary returns? What about the role of emotions, notions of fairness, reciprocity, image…. Need a translation from money payoffs to utility payoffs – which requires further understanding of players psychology (we will cover some notions of fairness, reciprocity, image towards the end of the semester)

4. Structure of Games: "Rules of the Game"
    a. Players ("who")
    b. Actions ("who can do what when")
    c. Information ("who knows what when")
    d. Payoffs ("who gets what")
    e. Specification of (a) – (d) constitute the **rules of the game**.
    f. We assume the rules of the game are **common knowledge**. Common knowledge: All players know the rules of the game. Further, all players know that all players know the rules of the game. And all players know that all players know that all players know the rules of the game…

5. What are strategies?
    a. A strategy for a player is a complete plan on how the player will play the game: it specifies what action the player will choose at every instant in the game where the player has a choice of actions.
    b. In very simple games where each player makes only a single choice, then a strategy equals an action. In more complex games with multiple choices, a strategy is a vector of actions.
    c. Strategies are the basic concept that we will work with in analyzing games.
6. Static vs. Dynamic Games
    a. **Static games**: all players move **simultaneously** a single time (there is no "when")
    b. **Dynamic games**: there is **sequential** play ("when" matters)
        i. They may play one after the other
        ii. They may move simultaneously multiple times…

7. Complete versus Incomplete Information Games
    a. **Complete information**: every player knows the payoff functions of all other players (players can predict what other players will get as a function of the combination of strategies chosen).
    b. **Incomplete information**: at least one player knows information that affects payoffs that rival players do not (**private information** or **asymmetric information**). Ex: firm 1 may have high or low cost of

production so that firm 2 cannot know the profit of firm 1 just by observing quantity choices.

c. **Perfect information** versus **imperfect information**: with perfect information predictions of payoffs are fully accurate (no uncertainty by any player at any point in the game). Imperfect information – there is some uncertainty.
   i. Incomplete information is necessarily imperfect
   ii. Imperfect information may be either complete or incomplete information

8. Normal Form Representation of a Game
   a. Normal form is also called the matrix form because it can be represented in a payoff matrix. Good for finding solutions for static games. Caution: must be very careful with normal form in dynamic games.
   b. Notation:
      i. Players: $i = 1, 2, \ldots, n;\ i \in N$
      ii. Strategies: s_i is a pure strategy (action) for player i, $s_i \in S_i$ is set of possible pure strategies for player i.
      $s = (s_1, s_2, \ldots, s_n)$, $s \in S$ where $S = X\, S_i$
      iii. Utility (preferences): $U_i(s)$, describes player *i's* ranking of various possible outcomes. $U = \{U_1, U_2, \ldots, U_n\}$
      iv. Normal form: $\{N, S, U\}$ (usually presented in the form of a matrix, see below)
   c. For static games, $\{N, S, U\}$ *are* the rules of the game. Normal form is a complete representation of the rules of the game.
   d. Example: 3*x*3 simultaneous game
      i. Players: {1, 2} – rows player and column player
      ii. Strategies: Rows player {U, M, D}
      Column player {L, M, R}
      iii. Payoffs: normal form payoffs (payoff matrix)

	L	C	R
U	20, 15	25, 5	30, 10
M	10, 5	40, 20	15, 30
D	15, 0	45, 30	10, 40

9. Extensive Form Representation of a Game

  a. To fully represent all elements of a game we need to go beyond the normal form to the extensive form. Extensive form represents, players (who), actions/strategies (who can do what when), information (who knows what when) and payoffs (who gets what).
  b. To describe a game in the extensive form, we must describe the tree. A tree consists of three essential elements:
     i. A set of nodes (*V*)
     ii. A set of branches (*A*)
     iii. A root (*r*).
  c. A branch connects two distinct nodes. A path is a sequence of nodes connected by branches. Any two nodes are connected by exactly one path. The root is where the tree begins. The root also provides a sense of direction and dictates the ends to the tree. A branch to a node that is on the path to the root is incoming. If it is not on the path to the root it is outgoing. Nodes without outgoing branch are called terminal nodes and represent an end to the tree and game. Nodes with outgoing branches are called non-terminal or decision nodes. A branch can then be thought of as a particular choice or action.
  d. Figure 1 shows an example of a tree.
     i. The nodes are v_1, v_2, v_3, v_4, v_5, v_6, and v_7.
     ii. v_1, v_2, v_3 are decision nodes
     iii. v_4, v_5, v_6, v_7 are terminal nodes
     iv. Branches can be identified by the nodes they connect: (v_1, v_2), (v_1, v_3), (v_2, v_4), (v_2, v_5), (v_3, v_6), and (v_3, v_7).
  e. Figure 2 shows an example that is not a tree because node v_1 and v_5 are connected by more than one path: v_1, v_2, v_5 and v_1, v_3, v_5.

Figure 1: A Basic Tree

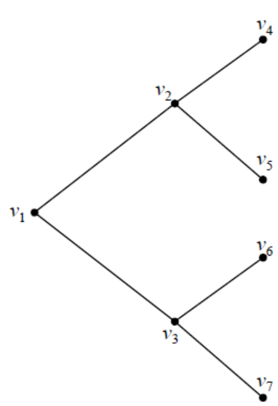
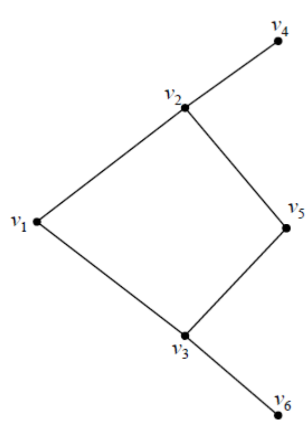


Figure 2: Not a Tree



  f. Figure 2 shows an example that is not a tree because node v_1 and v_5 are connected by more than one path: v_1, v_2, v_5 and v_1, v_3, v_5.

g. Describing elements of the game in the game tree

i. Describe the players: $N = \{0, 1, 2,\ldots, n\}$ where player 0 is a special player referred to as chance or Nature.

ii. Partition the non-terminal nodes in the tree to the players: $P_0, P_1,.., P_n$.

iii. Assign actions or choices to each branch of the tree.

iv. Assign probability distributions over outgoing branches for each node in P_0

v. Partition the nodes into $k(i)$ information sets for each player i: $I_1^i, I_2^i, \ldots, I_{k(i)}^i$. Information sets are used to describe what players know when. A player cannot distinguish between nodes in the same information set.

vi. Describe an n-dimensional vector, $U(t) = (U^1(t), U^2(t), \ldots, U^n(t))$ of payoffs for each terminal node t.

h. Example: the game of matching pennies.

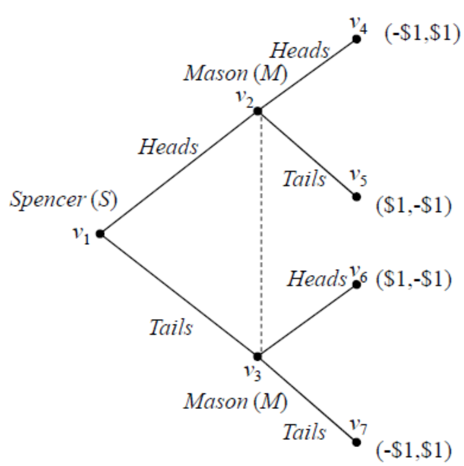
i. The rules of the game: The game has two players {*Mason, Spencer*}. Players simultaneously choose either *Heads* or *Tails*. *Mason* wins when both make the same choice. *Spencer* wins when both make different choices. The winner pays the loser \$1, so this is an example of a special class of games called zero sum games.

ii. Normal form representation

Spencer *Mason*	*Heads*	*Tails*
Heads	1, -1	-1, 1
Tails	-1, 1	1, -1

iii. Extensive form representation

FIGURE 3: Traditional Matching Pennies Game

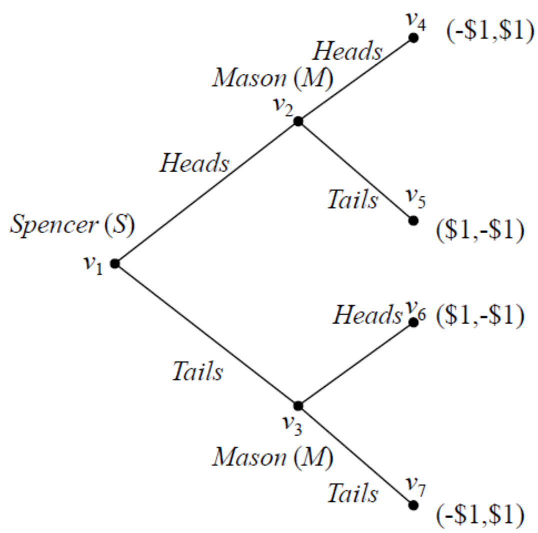


- Players: $N = \{M, S\}$
- Partition of nodes: $P^0 = \emptyset, P^S = \{v_1\}, P^M = \{v_2, v_3\}$
- Assign actions each branch of the tree
  $(v_1, v_2) = (v_2, v_4) = (v_3, v_6) = Heads$
  $(v_1, v_3) = (v_2, v_5) = (v_3, v_7) = Tails$

- Assign probability distributions over outgoing branches for each node in P_0 {Not relevant for this game}
- Partition the nodes into $k(i)$ information sets for each player i:
  $I_1^M = \{v_2, v_3\}, I_1^S = \{v_1\}$
- Describe an n-dimensional vector of payoffs:
  $U(t) = (U^S(t), U^M(t))$
  $U(v_4) = U(v_7) = (-1,1)$
  $U(v_5) = U(v_6) = (1,-1)$

i. Variant on matching pennies
   i. Suppose that Spencer moves first followed by Mason, and that Mason gets to see Spencer's choice.

FIGURE 4: Mason's Preferred Version of the Game



   ii. Notation:
   - Players: $N = \{M, S\}$
   - Partition of nodes: $P^0 = \varnothing, P^S = \{v_1\}, P^M = \{v_2, v_3\}$
   - Assign actions each branch of the tree
     $(v_1, v_2) = (v_2, v_4) = (v_3, v_6) = Heads$
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   - Assign probability distributions over outgoing branches for each node in P_0 {Not relevant for this game}
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     $I_1^M = \{v_2\}, I_2^M = \{v_3\}, I_1^S = \{v_1\}$
   - Describe an n-dimensional vector of payoffs:
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     $U(v_4) = U(v_7) = (-1,1)$
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