Solutions for Problem Set #5

A.1 Please see the lecture notes and textbook for definitions.

2.Using the given utility functions, we first show that $MRS^1_{12} = x_{21}$ and $MRS^2_{12} = x_{22}$. The contract curve can be derived from the equation $MRS^1_{12} = MRS^2_{12}$, which implies that $x_{21} = x_{22}$. Note that the total endowment of the second commodity is equal to three: $\omega_{21} + \omega_{22} = 3$. Hence by market clearing $x_{21} = 3 - x_{22}$, and so the contract curve is $x_{21} = x_{22} = \frac{3}{2}$.

3. Let $p_1 \equiv 1$ and $p_2 = p$. The consumers' budget constraints are:

- $m_1 = p_1 x_{11} + p_2 x_{21} = 1 + 2p$
- $m_2 = p_1 x_{21} + p_2 x_{22} = 2 + p$

Note that the utility maximization problem for consumer i is

- $\underset{x_{1i}, x_{2i}}{\text{Max}}\ u_i = x_{1i} + ln x_{2i}$ s.t. $m_i = p_1 x_{1i} + p_2 x_{2i}$

From this we can set up the Lagrangian and derive the FOCs. The first two FOCs imply that the optimality condition is $MRS^i_{12} = \frac{p_1}{p_2}$. The third FOC just gives us back the budget constraint. The budget constraint and the MRS condition can be combined to give us the demand equations for consumer 1:

- $MRS^1_{12} = \frac{p_1}{p_2}$
- $x_{21} = \frac{1}{p}$

Substitute this in the budget constraint

- $p_1 x_{11} + p_2 x_{21} = 1 + 2p$
- $1 * x_{11} + p * \frac{1}{p} = 1 + 2p$
- $x_{11} = 2p$

Similar algebra gives us the following demand functions for consumer 2: $x_{12} = 1 + p$ and $x_{22} = \frac{1}{p}$.

Now we want to solve for the equilibrium prices. Market clearing implies that at the optimum, aggregate demand must be equal to aggregate supply. The total endowment of good 2 is equal to 3. Aggregate demand is $x_{21} + x_{21} = \frac{1}{p} + \frac{1}{p} = \frac{2}{p}$. Hence, the equilibrium price is $p^* = 2/3$. We can use this price to solve for the equilibrium allocation. At $(p_1, p_2) = (1, 2/3)$, the quantities demanded are $(x^*_{11}, x^*_{21}) = (\frac{4}{3}, \frac{3}{2})$ for consumer 1 and $(x^*_{12}, x^*_{22}) = (\frac{5}{3}, \frac{3}{2})$ for consumer 2.

4. See the lecture notes for a graphical representation of a similar equilibrium.

B.1. The monopolist sets its per-unit price at $p = MC$, and charges a participation fee to capture 100% of the consumer surplus. The inverse demand curve is $p(q) = 500 - 0.75q$ for

$q \leq 2000/3$. Setting $p = MC$ and solving for q, we find that $q^* = 320$. At this quantity the monopolist, sets the per-unit price at $p^* = p(q^*) = 500 - 0.75(320) = 260$. Aggregate consumer surplus at a price of 260 is $CS = 0.5(500 - 260)320 = 38400$, and each of the 100 identical consumers has a surplus of $38400/100 = 384$. Thus, the optimal lump-sum fee $F^* = 384$, which reduces consumers surplus to zero.

2. The monopolist receives revenue of (320)(260) from unit sales and (100)(384) from lump-sum fees, for a total of 121,600. It costs are $c(320) = 25,000 + 100(320) + 0.25(320)^2 = 25,000 + 32,000 + 25,600 = 82,600$. Its profits are $\pi = 121,600 - 82,600 = 39,000$. The DWL is 0; the monopolist has the correct incentive to produce the surplus-maximizing level of output because it gets all surplus as profit.

C.1. We can find the optimal price discrimination strategy directly from the optimality condition, $MR_1(q_1) = MR_2(q_2) = MC(Q)$:

1. $MR_1 = MC(Q) \to 1500 - q_1 = 60 + 0.2(q_1 + q_2)$
2. $MR_2 = MC(Q) \to 700 - 4q_2 = 60 + 0.2(q_1 + q_2)$

From (1):

- $1500 - q_1 = 60 + 0.2(q_1 + q_2)$
- $1440 - 0.2q_2 = 1.2q_1 \to q_1 = 1200 - (1/6)q_2$.

Substitute $q_1 = 1200 - (1/6)q_2$ into (2)

- $700 - 4q_2 = 60 + 0.2((1200 - (1/6)q_2) + q_2)$
- $700 - 4q_2 = 60 + 240 + (1/6)q_2 \to 4q_2 + (1/6)q_2 = 400$
- $(25/6)q_2 = 400 \to q_2^* = 96$.

And hence $q_1^* = 1200 - (1/6)q_2^* = 1200 - (1/6)(96) = 1184$. Using the inverse demand functions for each market, the equilibrium prices are $p_1^* = 908$ and $p_2^* = 508$.

2. To solve for the non-price discriminating solution, we need to find the aggregate demand function, $D_p = q_1(p) + q_2(p)$

$$D(p) = \begin{cases} 3000 - 2p, & \text{if } 700 < p \leq 1500; \\ 3350 - 2.5p & \text{if } 0 \leq p \leq 700. \end{cases}$$

The inverse demand curve is:

$$P(Q)) = \begin{cases} 1500 - 0.5Q, & \text{if } 0 \leq Q \leq 1600; \\ 1340 - 0.4Q & \text{if } 1600 \leq Q \leq 3350. \end{cases}$$

And the marginal revenue curve is:

$$MR(Q) = \begin{cases} 1500 - Q, & \text{if } 0 < Q \leq 1440; \\ 1340 - 0.8Q & \text{if } 1600 \leq Q \leq 1675. \end{cases}$$

Now we can set $MR(Q) = 1500 - Q = 60 + 0.2Q = MC(Q)$, and find that $Q = 1200$. At $Q^* = 1200$, $P^* = 900$. Since $900 \in (700, 1500]$ this must be the correct solution. Only customers on the first segment of the demand curve buy the product.

In this case, a price-discriminating monopolist will increase the price in the first segment from 900 to 908, and sell less. While the customers on the second segment will purchase the product under price discrimination, they don't make any purchases if there is no price discrimination. With price discrimination, customers on the first segment are worse off, but customers in the second segment are better off.

3. Under price discrimination, we need to calculate the welfare measures in each market separately and then sum them up to find the totals. For the no-price-discrimination case, we calculate the welfare measures under the monopoly in the aggregate (combined) market. Note that these are standard calculations of consumer & producer surplus and DWL. To obtain the DWL for each case you also need to find the outcome if the markets were perfectly competitive: $p^c = 486.7,\ Q^c = 2133.4,\ q_1^c = 2026.7,\ q_2^c = 106.7$. We can compare these results to verify that the monopolist is able to increase its profits via third-degree price discrimination. However, this is not bad for society, because the monopolist is increasing its profits by moving production upward & closer to the social optimum, and therefore improving the overall efficiency of the market.

4. $MC = AC = 316$. The monopolist wants to find the optimal two-part tariff when facing two types of consumers. This is an example of second degree price discrimination. The monopolist knows that there are two types of customers (just as in 3rd-degree price discrimination). However, it cannot identify who is who, and therefore cannot set a separate per-unit price in each submarket). In this case, we can derive the optimum two-part tariff by setting the lump-sum fee equal to the less eager consumers' surplus and solve for the optimal per-unit price that maximizes profits $\pi = 2F + \text{Profit from Sales (S)}$. The two demand curves imply that the customers in the second market are less eager, because their demand curve lies below the demand curve of customers in the first market for all Q. So the monopolist sets the first part of the tariff based on the second market: $F = CS_2 = 0.5(700 - p)q_2$. Since $S = (p - MC)Q$, the profit equation is:

- $\pi = (700 - p)(350 - 0.5p) + (p - 316)(3350 - 2.5p)$

The FOC is

- $\frac{\partial \pi}{\partial p} = -700 + p + 4140 - 5p$
- $p^* = 860$

However, $p^* \notin (0, 700]$ so this cannot be a solution. That is, monopolist cannot find an optimum two-part tariff that maximizes its profits and serves both markets. So, the monopolist will only sell to the more-lucrative market, which is market 1. In this case $p^* = MC = 316$ and $NF^* = 0.5(1500 - 316) * 2368$, or $F^* = 1401856/N$, where N is number of customers.

5. This question is the same as (4) with $MC = AC = 12$. The profit equation is given as:

- $\pi = (700 - p)(350 - 0.5p) + (p - 12)(3350 - 2.5p)$

FOC is

- $\frac{\partial \pi}{\partial p} = -700 + p + 3380 - 5p$
- $p^* = 670$

Since $p^* \in (0, 700]$, this is the solution. In this case $F^* = 225/N$, $q_1^* = 1660$ and $q_2^* = 15$. The monopolist's profits are:

- $\pi^* = 450 + (670 - 12) * 1675 = 1102600.$

Even though the monopolist can sell to both markets, it may still be better off if it chooses to sell to only one of them. To check this, find out optimal price and fee if sells only to market 1 (the more-lucrative one). At $MC = 12$ monopolist can set $p^* = 12$ and $NF^* = 0.5 * (1500 - 12) * (3000 - 2 * 12) = 2214144$. Thus the monopolist is still better off if it only sells to the first market.

6. In this case, the monopolist is better off under 3rd-degree than 1st-degree price discrimination because it has more information. This is different from the usual comparison: we assumed that the monopolist could not differentiate between the consumers when setting a two-part tariff, whereas usually 1st-degree price discrimination implies that it can tell the consumers apart. In both cases the monopolist knows that there are different types of consumers. However, under 2nd-degree price discrimination the monopolist does not know each consumer's type, whereas under 3rd-degree price discrimination the monopolist can identify the different types of consumers and charge per-unit prices accordingly. The ability to identify the different types of consumers helps the monopolist to increase its profits.