Econ 8010 HW5
Solutions (partial)

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1. Two firms $i \in \{1, 2\}$ produce identical commodities and choose quantities $q_i \geq 0$. The inverse demand for their product is given by

$$P(Q) = \max\{20 - Q, 0\}$$

where $Q = q_1 + q_2$. Firm 1 and firm 2 each produce at constant marginal cost of 4. Thus, their payoffs when they play (q_1, q_2) are

$$\pi_1(q_1, q_2) = q_1 \max\{20 - q_1 - q_2, 0\} - 4q_1$$
$$\pi_2(q_1, q_2) = q_2 \max\{20 - q_1 - q_2, 0\} - 4q_2$$

(a) First, suppose that the firms make decisions simultaneously. Find the pure strategy Nash equilibrium of the resulting normal form game.

- **Solution:** $q_1 = q_2 = \frac{16}{3}$.

(b) Now, suppose that firm 1 chooses its quantity q_1 first. After observing q_1, firm 2 chooses its quantity q_2. Find the subgame perfect equilibrium of the resulting extensive form game.

- **Solution:** This time, we plug firm 2's best response function into firm 1's objective before maximizing it. We get $q_1 = 8, q_2(q_1) = 8 - \frac{q_1}{2}$.

2. (Exercise written by Bill Sandholm) Arthur and Beatrix compete in a race. At the start of the race, both players are 6 steps away from the finish line. Who gets the first turn is determined by a toss of a fair coin; the players then alternate turns, with the results of all previous turns being observed before the current turn occurs. During a turn, a player chooses from these four options:

   (I) Do nothing at cost 0;

   (II) Advance 1 step at cost 2;

   (III) Advance 2 steps at cost 7;

   (IV) Advance 3 steps of at cost 15.

   The race ends when the first player crosses the finish line. The winner of the race receives a payoff of 20, while the loser gets nothing. Finally, there is discounting: after each turn, payoffs are discounted by a factor of δ, where δ is less than but very close to 1.

   (a) Find all subgame perfect equilibria of this game. (Hint: In all subgame perfect equilibria, a player's choice at a decision node only depends on the number of steps he has left and on the number of steps his opponent has left. To help take advantage of this you might want to write down a table.)

   (b) Suppose that Arthur wins the coin toss. Compare his equilibrium behavior with his optimal behavior in the absence of competition. Provide intuition for any similarities or differences you find.

   - This question was based on Harris and Vickers (1985) and uses payoffs from Dutta (1999). I won't repeat Prof. Sandholm's solution here, but you can get an idea of the solution technique from reading Harris and Vickers (1985).