

Aug 30 → Sept 2

$$\text{Var}[\hat{\beta}] = (x'x)^{-1} x' \cdot \overset{\text{const.}}{\sigma^2} \underset{\text{error term var}}{I} (x'x)^{-1}$$

$$= \sigma^2 (x'x)^{-1} x' x (x'x)^{-1}$$

$$\text{Var}[\hat{\beta}] = \sigma^2 (x'x)^{-1}$$

* How computer will call unless otherwise specified

* don't know σ^2

$$\begin{aligned} \hat{\sigma}^2 &= \sum_{i=1}^n \frac{\hat{u}_i^2}{n-1} = \frac{1}{n-1} \left(\hat{u}' \hat{u} \right) \\ &= \sum_{i=1}^n \frac{(\hat{u}_i)^2}{n-1} \end{aligned}$$

$$= \frac{RSS}{n-k}$$

• can derive matrix by scalar
• inverse of matrix = division in scalar

So

$$\widehat{\text{Var}}[\hat{\beta}] = \hat{\sigma}^2 (x'x)^{-1}$$

$$\widehat{\text{Var}}[\hat{\beta}] = \begin{bmatrix} \widehat{\text{Var}}[\hat{\beta}_1] & \text{cov} \\ \text{cov} & \widehat{\text{Var}}[\hat{\beta}_k] \end{bmatrix}$$

Standard error est:

Var($\hat{\beta}$)

* we now have $\hat{\beta}$ & standard error est. w/ $\hat{\beta}$

- $\text{Var} \hat{\beta}_i = E[\hat{\beta}_i - E[\hat{\beta}_i]]^2$
- expected squared deviation of $\hat{\beta}_i$ from mean $\hat{\beta}_i \rightarrow \beta_i$
- $\hat{\beta}$ is linear function of y
- Dispersion of $\hat{\beta}_i$ from β_i
- measure of precision

Covariance matrix:

$$V[x_1] = E[(x_1 - E(x_1))^2] = \sigma_1^2$$

→ "measure of dispersion"

$$\hat{\sigma}_{12}^2 = \frac{\sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2)}{n-1} \rightarrow E[\hat{\sigma}_{12}^2] = \sigma_{12}^2$$

• unbiased estimator

$$\text{Cov}(x_1, x_2) = E[(x_1 - E(x_1))(x_2 - E(x_2))]$$

equation

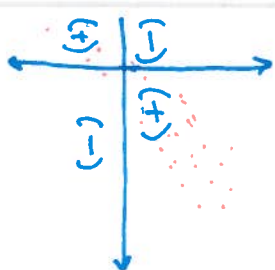
\downarrow
 μ_1

\downarrow
 μ_2

→ deviation of x_1 from mean and x_2 from mean

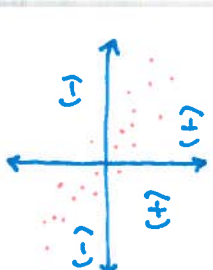
• cov. of itself = variance

$$\hat{\sigma}_{12}^2 = \frac{1}{n-1} \sum_{i=1}^n (x_{1i} - \bar{x}_1)(x_{2i} - \bar{x}_2) \rightarrow E[\hat{\sigma}_{12}^2] = \sigma_{12}^2$$



- if most values in \pm are pos. expected value of deviation is pos.

- if negative correlation → negative covariance



- units are x_1 unit $\cdot x_2$ unit

↳ So → $4/10 + 4/10 \rightarrow 4/10^2$

value of cov depend on x_1/x_2 units the value itself isn't meaningful

[Correlation fixes this!]

$$\text{Cov}[x_1, x_2] \cdot \frac{\sigma_{12}^2}{\sigma_1 \sigma_2} = \frac{\sigma_{12}^2}{\sqrt{\sigma_1^2 \sigma_2^2}} = \rho_{12}$$

$$\text{Est}_{\hat{\rho}_{12}}: \frac{\hat{\sigma}_{12}^2}{\sqrt{\hat{\sigma}_1^2 \hat{\sigma}_2^2}} \rightarrow E[\hat{\rho}_{12}] = \rho_{12}$$

* Same sign as covariance
* gets rid of measurement [no units]

Correlation coef does not Δn units.

→ Bounded w/in 1 & -1



Measures dispersion of x_1 & x_2 from line

→ more dispersed closer to 0

→ No pattern → $\rho = 0$

Apply

Cov Matrix:

$\text{Var}[U] \rightarrow$ $n \times n$ covariance matrix for

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

$$\text{Var}[U] = E[(U - E[U])(U - E[U])']$$

(n x 1) (1 x n)

So

$$E \begin{bmatrix} u_1 - E[u_1] \\ u_2 - E[u_2] \\ \vdots \\ u_n - E[u_n] \end{bmatrix} \begin{bmatrix} u_1 - E[u_1] & u_2 - E[u_2] & \dots & u_n - E[u_n] \end{bmatrix}$$

gives you

$$\text{Var}[U] = E \begin{bmatrix} \sigma_{u_1}^2 & \text{cov}[u_1, u_2] & \dots & \text{cov}[u_1, u_n] \\ \text{cov}[u_2, u_1] & \sigma_{u_2}^2 & & \\ \vdots & & \ddots & \\ \text{cov}[u_n, u_1] & & & \sigma_{u_n}^2 \end{bmatrix}$$

*symmetric matrix

*No auto correlation? → all covar are 0

*Yes homoskedasticity? → all var are same

$$\begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} \rightarrow \sigma^2 I \text{ if error term is iid}$$

iid: independent & identically distributed
(no autocor) (no heterosked)

Var of v : error term cov matrix

- Var on diag: oft diag is covar
- iid: var same + cov is 0

Calc OLS standard error

- Cov Matrix $\hat{\Sigma} \cdot \hat{v} []$

- $\hat{\Sigma}$ is Random var

$$\rightarrow \hat{\Sigma} = (x'x)^{-1}x'u \rightarrow y = x\beta + u \rightarrow \text{error term is a random var; } x \text{ is fixed;}$$

$\Sigma = (k \times k)$: so y is random so $\hat{\Sigma}$ is Random
 \rightarrow dif clark, dif $\hat{\Sigma}$

- Variance of $\hat{\beta} = E[(\hat{\beta} - E[\hat{\beta}])(\hat{\beta} - E[\hat{\beta}])']]$

$$\text{Var } \hat{\beta} = E \left[\begin{bmatrix} \hat{\beta}_1 - E[\hat{\beta}_1] \\ \hat{\beta}_2 - E[\hat{\beta}_2] \\ \vdots \\ \hat{\beta}_k - E[\hat{\beta}_k] \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 - E[\hat{\beta}_1] & \hat{\beta}_2 - E[\hat{\beta}_2] & \dots & \hat{\beta}_k - E[\hat{\beta}_k] \end{bmatrix}' \right] \quad \text{*symmetric}$$

$$\text{Var } \hat{\beta} = E \left[\begin{matrix} \text{Var } \hat{\beta}_1 & \text{Cov } \hat{\beta}_1 \hat{\beta}_2 \\ \text{Cov } \hat{\beta}_2 \hat{\beta}_1 & \text{Var } \hat{\beta}_2 \\ \vdots & \vdots \\ \text{Cov } \hat{\beta}_k \hat{\beta}_1 & \text{Cov } \hat{\beta}_k \hat{\beta}_2 & \dots & \text{Var } \hat{\beta}_k \end{matrix} \right]$$

Really interested in variances:

$$\text{Var } \hat{\beta} = E[(\hat{\beta} - E[\hat{\beta}])(\hat{\beta} - E[\hat{\beta}])']$$

* WHERE DOES U COME FROM?

$$\hat{\beta} = (x'x)^{-1}x'y$$

$$E[\hat{\beta}] = \beta$$

$$\hat{\beta} - \beta = (x'x)^{-1}x'u$$

$$\text{Var } \hat{\beta} = E[(x'x)^{-1}x'u u'x(x'x)^{-1}]$$

$$\text{Var } \hat{\beta} = (x'x)^{-1}x' E[uu']x(x'x)^{-1}$$

- \rightarrow Var $U \rightarrow$ if don't know U from stock
- if violate 3/4 \rightarrow have to plug in whole matrix
- if meet assumptions $\rightarrow \text{Var } U = \sigma^2 I$

Properties of OLS estimators

- if all OLS ~~est.~~ assump. hold \rightarrow estimator is BLUE

Best linear unbiased Estimator

- Best in sense it is min variance
- linear unbiased in that is only best among all possible unbiased est. \rightarrow also linear functions of y

- However \rightarrow if y_i is Normally distributed \rightarrow OLS estm. has min var. among all possible unbiased est.

TOPIC III: Measures of goodness of fit

- Measures of goodness of fit

- OLS method \rightarrow Best fitting model as in it has smallest possible ESS + possibly BLUE
- HOWEVER \rightarrow how ~~ever~~ well can this best fitting model predict the behavior of y remains unanswered \rightarrow use R^2 to quantify

The R^2

- Based on notion that each y observation (y_i) can be decomposed into total, explained, and residual variation (wrt \bar{y})

- Total variation: Square of the difference between observed y (y_i) and mean of y (\bar{y})
 $\rightarrow [(y_i - \bar{y})^2]$

- Explained variation: Square difference between the predicted value of y (\hat{y}_i) and mean of y (\bar{y})
 $\bar{y} \rightarrow [(\hat{y}_i - \bar{y})^2]$

- Residual variation: Square of difference between the observed y (y_i) and predicted value of y (\hat{y}_i)
 $\rightarrow [y_i - \hat{y}_i]^2$

- Specifically: $TSS + ESS = \sum (y_i - \bar{y})^2$

- Measures total amount of variation in observed values and dependent variable
- numerator of variance
- $ESS: (ESS = \sum (\hat{y}_i - \bar{y})^2)$

- Measures how much of total variation is being explained by the estimated regression model

$$RSS: (RSS = \sum (y_i - \hat{y}_i)^2) = \sum (\hat{u}_i)^2$$

- Measures how much of total amount of variation is not explained by estimated model

$$\begin{aligned} TSS &= ESS + RSS \Rightarrow \sum (y_i - \bar{y})^2 = (\hat{y}_i - \bar{y})^2 + (y_i - \hat{y}_i)^2 \\ &= \sum [(y_i - \bar{y})^2] = \sum [(\hat{y}_i - \bar{y})^2 + (y_i - \hat{y}_i)^2 + 2(\hat{y}_i - \bar{y})(y_i - \hat{y}_i)] \\ &= \sum (y_i - \bar{y})^2 + \sum (y_i - \hat{y}_i)^2 + 2 \sum (\hat{y}_i - \bar{y})(y_i - \hat{y}_i) \end{aligned}$$

$\xrightarrow{\text{has to equal 0}}$

$$0 = 2 (\hat{y}_i - \bar{y}) (y_i - \hat{y}_i)$$

$$= \sum \hat{y}_i y_i - \sum \hat{y}_i^2 - \sum \bar{y} y_i + \sum \bar{y} \hat{y}_i$$

$$= \sum \hat{y}_i y_i - \sum \hat{y}_i^2 - \bar{y} \sum y_i + \bar{y} \sum \hat{y}_i$$

$$= \sum \hat{y}_i (y_i - \bar{y}) - \sum \hat{y}_i^2 + \bar{y} \sum \hat{y}_i$$

$$= \sum \hat{y}_i (y_i - \bar{y})$$

$$= \sum \hat{y}_i (y_i - \bar{y})$$

$$= \sum \hat{y}_i (y_i - \bar{y})$$

* Mathematical Properties

have to hold

→ if $R^2_1 = 0 \Rightarrow 1st$

normal equation doesn't hold

unrelated
SD = 0

• R^2 calculated as:

$$R^2 = \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} = \frac{ESS}{TSS} \rightarrow \text{Proportion of total variation in dep var explained by model}$$

$$R^2 = 1 - \frac{\sum \hat{y}_i^2}{\sum (y_i - \bar{y})^2} = 1 - \frac{RSS}{TSS} \rightarrow 1 - \text{the proportion that is not explained by model}$$

• SO: R^2 measures ~~variation~~ Portion of total variation in y that is explained by Model

• R^2 is ratio → no units, so Δ units y is measured in ~~units~~ DOES NOT Δ R^2

• Takes only value b/w 0+1

• $R^2 = 0 \rightarrow (RSS = TSS, ESS = 0) \rightarrow$ No fit @ all

• $R^2 = 1 \rightarrow (RSS = 0, ESS = TSS) \rightarrow$ Perfect fit

• ↑ R^2 = better fit

• Value of R^2 must be assessed in light of type of data being analyzed

• time series data: $R^2 > .80$

• cross sectional data: $R^2 > .5$

• R^2 is only a ^{Measure} of model's capacity to predict $y \rightarrow$ can have low R^2 but precise parameters

• * R^2 is only valid if model includes intercept $\rightarrow \sum y_i \neq \sum \hat{y}_i \rightarrow$
TSS \neq RSS + ESS \therefore

• Disadvantage of $R^2 \rightarrow R^2 \uparrow$ as indep var are added (even if var does not affect y)

• When OLS est ~~est~~ coef \rightarrow any added indep var = smaller RSS \rightarrow ~~more~~ $R^2 \uparrow$

• An increase in R^2 as a result of adding indep var \neq expanded model is better or that var. affects y

• Adjusted R^2

• Denoted $\bar{R}^2 \rightarrow$ better to assess ~~the~~ if the addition of an indep var likely to ↑ model's ability to predict y

Adj R²:

$$\bar{R}^2 = 1 - \hat{\text{var}}(u) / \hat{\text{var}}(y) = 1 - \frac{[(RSS/TSS)(n-1)/(n-k)]}{\text{...}}$$

is less \bar{R}^2 unless $k=1$ or $\bar{R}^2=1$
 No model
 $k \geq 2$
 $\frac{RSS}{TSS} = 0$
 Penalty for # of expl. var
 These two terms have to override to Δ

$\frac{RSS}{TSS} = 0$
 $-\frac{n-1}{n-k}$ wouldn't matter

\bar{R}^2 does not have same straight forward interpretation as R^2 ; can be (-) [BAD]

Some argue \bar{R}^2 provides better measure of goodness of fit when model is est w/ lots of variables + few observations

Final Note on Model Specification

Any var. believed to DIRECTLY and NOTABLY affect y , and does not hold const. value should be included in model

excluding this var cause est. of remaining parameters to be bias + inconsistent $\rightarrow E[u] \neq 0$
 *The tradeoff \rightarrow always favor putting in

Potential concept of includ irrelevant var in model is less serious

if var affect y indirectly through another indep var; doesn't have to be included

Topic 4/5 \rightarrow Normal Error model

- Normal error model:

- in addition to normal error term assump:

- $\text{cov}[u, x_i] = 0$
- $E[u_i] = 0$ for all i
- $\text{var}[u_i] = \sigma^2$ for all i
- $\text{cov}[u_i, u_j] = 0$ for all $i \neq j$

Normality

error term is also normally distributed $\rightarrow u \sim N(0, \sigma^2)$

Supposedly supported by central limit theorem

central limit theorem: sum of any large set r.v. [if iid] will be normal

if not iid have to be indep.

Recall Assum 1+2 \rightarrow unbiased + consist

in addition of u_i in iid; $\text{var}[u] = \sigma^2 \cdot 1 + \text{var}[\hat{\beta}] = \sigma^2 (x'x)^{-1}$

of all assume hold + error term normal distrib:

- $\hat{\beta} \sim N(\beta, \sigma^2 (x'x)^{-1})$ thus.
- $\hat{\beta}_j \sim N(\beta_j, \text{var}[\hat{\beta}_j])$ $j=1 \dots k$ where $\text{var}[\hat{\beta}_j] = \text{diagonal element of } \sigma^2 (x'x)^{-1}$

[Hypothesis testing: t-tests]

* 95% CI \rightarrow 5% chance that the B is out of trust range

$$t = (\hat{\beta}_j - \beta_j) / SE(\hat{\beta}_j)$$

- only follows t distribution if guess correct β_j
 - \rightarrow more β_j value departs from true β_j the more the t-ratio differs from
- if compute t over several acts? Sets of t follow t distribution w/ $n-k$ degrees of freedom

$$\begin{cases} \text{Null Hypo: } H_0 \Rightarrow \beta_j = c \text{ (const value)} \\ \text{Alt Hypo: } H_a \Rightarrow \beta_j \neq c \end{cases}$$

- ascertain how likely is it that a particular t value is drawn from a t-distro
 - \rightarrow pick $\alpha \rightarrow$ usually .05 \rightarrow prob of being wrong if reject H_0 \rightarrow we know nothing about fail to reject
 - \rightarrow find t value for that level of signif. [t*]
 - \rightarrow if $|t| > t^*$ reject H_0 ; favor $H_a \rightarrow \beta_j \neq w$ & prob. of being mistaken
 - \rightarrow if $|t| \leq t^*$ cannot reject H_0 BUT probability that H_0 is correct is unknown

[Statistical Hypothesis testing]

- t test only holds if 4 assumptions hold AND error term is normally distrib
- HOWEVER if sample size is large (100+) t test can be used even if v not Normal
 - \rightarrow w/ large sample \rightarrow convergence to t distrib
- Most common use of t-stat: test if parameter is statistically different from 0 \rightarrow implies x will affect y
 - Pay attention to \pm and situation
 - $H_0: \beta_j = 0$ (x has no effect)
 - $H_a: \beta_j \neq 0$ (x has effect)
 - two tail \rightarrow pos or neg: allows for pos or neg effect only
- Rule of thumb: Ratio $|\hat{\beta}_j / SE(\hat{\beta}_j)| > 12$ tend to reject H_0
- α of .1, .05, .01 usually \rightarrow mean 3 levels of statistical certainty when rejecting H_0 : 90, 95, 99 respectively

- p-value: exact prob that for which H_0 can be rejected in favor of TWO TAILED HYPOTHESIS $\rightarrow H_a: \beta_j \neq 0$

1-tail makes easier to reject H_0

- ONE tail tests usually have knowledge that effect isn't in one of the tails \rightarrow can't have neg effect or something
- Assume all assumptions hold; if take a test out have to recalc; assume all units are similar
- if opt for one tail don't have to recalc t-value \rightarrow critical table value Δ by $\alpha/2$ for idea that one of the tails is left out
- if same critical value is used $\alpha \rightarrow \alpha/2$

Statistical Hypothesis testing

(SE
(SE) * used on signif level

- When reporting results → report SE or t-value
- also include test of each model parameter
- Identify include p-value → state 1 or 2 tail

[Confidence interval for y_0]

- x_0 = ~~the~~ 1xk vector of particular x 's
- y_0 = value taken long y when x_0 y_1 → **observed**
- $\hat{y}_0 = x_0 \hat{\beta}$ predicted value given x_0
- $\hat{u}_0 = y_0 - \hat{y}_0$; predicted error $\in x_0$
- can be shown that $E[\hat{u}_0] = 0$
- can be shown that $\text{var}[\hat{u}_0] = \sigma^2 + \sigma^2 x_0 (x'x)^{-1} x_0'$
- * & if \hat{u} normal distribn then \hat{u}_0 is usually normal
- thus $\hat{u}_0 | \text{SE}[\hat{u}_0] \sim N(0,1) + \hat{u}_0 / \text{SE}[\hat{u}_0] \sim t_{n-k}$
- confidence interval for $y_0 \Rightarrow \hat{y}_0 \pm t_{n-k} \text{SE}[\hat{u}_0]$
- ci for $E[y_0] = x_0 \beta$?
- also normal so w/ $\text{SE}[\hat{y}_0] \rightarrow t_{n-k}$
- with $E[y_0] + \text{SE}[\hat{y}_0]$ makes ci narrower
- $E[y_0] = \hat{y}_0 \pm \text{SE}[\hat{y}_0] t_{(k/2, n-k)}$

$$E[\hat{u}] = 0 \rightarrow E[y_0 - \hat{y}_0] = E[y_0] - E[\hat{y}_0] = 0$$

$$E[x_0 \hat{\beta}] - E[\hat{y}_0] = 0$$

$$E[\hat{u}_0] = 0$$

$$E[x_0 \hat{\beta}]$$

$$\text{var}[\hat{u}_0] = \text{var}[y_0 - \hat{y}_0]$$

$$= \text{var}[y_0] + \text{var}[\hat{y}_0]$$

$$= \text{var}[x_0 \hat{\beta} + u_0] + \text{var}[x_0 \hat{\beta}]$$

$$= \text{var}[x_0 \hat{\beta}] + \text{var}[u_0] + x_0 \text{var}[\hat{\beta}] x_0'$$

$$= \sigma^2 + x_0 \sigma^2 (x'x)^{-1} x_0'$$

$$\hat{u}_0 = y_0 - \hat{y}_0 = x_0 \beta + u_0 - x_0 \hat{\beta}$$

$$= x_0 \beta + u_0 - x_0 (x'x)^{-1} x' y$$

$$= x_0 \beta + u_0 - x_0 (x'x)^{-1} x' (x \beta + u)$$

$$= x_0 \beta + u_0 - x_0 (x'x)^{-1} x' x \beta - x_0 (x'x)^{-1} x' u$$

$$= x_0 \beta - x_0 \beta - x_0 (x'x)^{-1} x' u$$

$$\hat{u}_0 = u_0 - x_0 (x'x)^{-1} x' u$$

BUIT Sept 20 - Oct 3

Regression through origin:

- usually used when theory dictates $y = 0$ if all $x = 0$

↳ ie price consistency

- estimated using OLS by excluding 1st column of 1s

- will Δ slope parameter estimated

- R^2 not useable \rightarrow no longer bounded @ 0 \rightarrow TSS \neq ESS + RSS

\rightarrow can use "exam" R^2 [or (Y^2)] but interpret not the same

- only use through 0 if there is STRONG evidence

Scaling & units of Measure:

- Δ in scaling of any explanatory var by a fixed multiplicative factor of w_x only Δ value of [corresponding parameter] + [se by a factor] of [$1/w_x$]

- Δ in scaling of dependent var by fixed multiplicative factor w_y changes value of all Model Parameters + correspond se by factor of [w_y]

Standard Regression:

- Regression w/ all var ($y+x$) being standardized to mean 0 and variance 1

- each var: subtract μ & divide by σ

$$y_i^* = \frac{y_i - \bar{y}}{s_y}$$

$$\beta_2^* = \frac{s_{dx}}{s_{dy}} \cdot \hat{\beta}_2$$

- Parameter est in this regression are "Beta coef"

- B/c all var have mean of 0; intercept is 0

- interpretation of β coef follows from both y & x are now measured in stand. dev.

\rightarrow ie measure the sd Δ in y when x Δ by 1 sd

- no unit of measure

- used for relative importance of x s but doesn't account for

- a larger β means the x contributes to explaining var in y more

$$y_i^* = \frac{y_i - \bar{y}}{s_y} - \frac{\bar{y}}{s_y}$$

$$w_y x = \frac{1}{s_y}$$

$$\frac{1}{w_x} = s_x$$

Lagged variables

- Sometimes the value of $y \rightarrow$ depends on value of x from previous time period.

- Can use OLS \rightarrow must rearrange data so that y_t corresponds x_{t-1}

- lose one var observation

- The Δ in value from one period to the next

$$y_t = \beta_1 + \beta_2(x_t - x_{t-1}) + u_t \text{ or } y_t = \beta_1 + \beta_2 \Delta x_t + u_t$$

- What affects y ? value of x or $\Delta \ln x$

- lose one observation

- used to forecast

- will $\Delta \beta_s$ so do this before OLS

Alt Model specifications:

- so far only looked at linear models

- ARE x & y linearly related? \rightarrow OLS: linear in parameters but not in explain/dep var

LOG LINEAR:

- AK-A: log log or double log

- can accommodate: \uparrow deer rate; \uparrow deer rate + \downarrow deer rate

$$\hat{y}_i = e^{\hat{\beta}_1} x_2^{\hat{\beta}_2} x_3^{\hat{\beta}_3} \dots x_k^{\hat{\beta}_k}$$

- $\hat{\beta}_j = 0$; as $x_j \uparrow$; $y \downarrow$ @ deer rate

- $\hat{\beta}_j = 1$; as $x_j \uparrow$; $y \uparrow$ @ const rate

- $0 < \hat{\beta}_j < 1$; as $x_j \uparrow$; $y \uparrow$ @ deer rate

- $\hat{\beta}_j > 1$; as $x_j \uparrow$; $y \uparrow$ @ deer rate

$\ln y = \ln e^{\hat{\beta}_1} \ln x_2^{\hat{\beta}_2} \ln x_3^{\hat{\beta}_3} \dots$

$\ln y = \hat{\beta}_1 + \hat{\beta}_2 \ln x_2 + \hat{\beta}_3 \ln x_3 \dots + u$

So use normal OLS but all apply to $\ln x_i$ & y

- disadvantage: all x_i - y relations assumed non lin

- all x & y val must be POS $\rightarrow \ln(-)$ not defn \rightarrow can have 0 but not recommended

- $\hat{\beta}_j$ directly measures elast of y wrt $x_j \rightarrow \% \Delta \ln y$ when $x_j \Delta 1\%$

- slope varies but elasticity is const

- since $\ln y \rightarrow R^2$ not the same

- use $r_{\ln y}$

- Better good ness of fit \rightarrow square of the correlation coeff b/w observed & predicted val of y

- Note: ~~est~~ predictions from orig. equation not est model

$$\text{corr}[x_1, x_2] = \frac{\sigma_{12}^2}{\sigma_1 \sigma_2} = (R_{12})$$

- use $r_{\ln y}$

- Better good ness of fit \rightarrow square of the correlation coeff b/w observed & predicted val of y

- Note: ~~est~~ predictions from orig. equation not est model

- use $r_{\ln y}$

Summ log: log lin

- log of dep. var but explain in original form

- β measure rel $\Delta \ln y$ when $x \Delta$ log 1 unit

\rightarrow if $\cdot 100 \rightarrow \% \Delta y$ for 1 unit Δx

- as $x \uparrow y \uparrow$ @ const rate ($\beta_j > 0$) or \downarrow @ deer rate ($\beta_j < 0$)

- y can't be neg

Semi log: $\ln \log$

- log of explain var but not dep. var
- β_j measure unit Δy for Δx rel Δx
 - $\neq 100\% \rightarrow$ unit Δy for $x \Delta 1\%$
- as $x \uparrow y \uparrow$ @ deer rate ($\beta_j > 0$) deer @ deer rate ($\beta_j < 0$)
- y can be neg but not x

Reciprocal Specification

$$y_i = \hat{\beta}_1 + \hat{\beta}_2 \left(\frac{1}{x_i}\right) + \dots$$

- $\hat{\beta}_2 > 0 \rightarrow y \downarrow$ @ deer rate (thus asymptote)
 - as $y \uparrow$ approach intercept
- if $\hat{\beta}_2$ neg $\rightarrow y \uparrow$ @ deer rate; intercept is max val
- Slope op. sign of $\hat{\beta}_2$
- slope: $\frac{d\hat{y}_i}{dx_i} = -\hat{\beta}_2 \left(\frac{1}{x_i}\right)^2$
 - dif slope dep on value of x
- can have reciprocal + original in same model

Note on OLS Estimation

- all ($\log \ln$; $\ln \log$; $\log \log$; + reciprocal) can be set w/ normal OLS
- Make sure all logs + reciprocals taken + used before est of β

Log Reciprocal:

- $\ln y + \frac{1}{x}$
- get 3 shape
- \uparrow @ max then unc @ deer
- $(-)\beta_2 \nearrow$
- $(+)\beta_2$: deer @ deer rate

Polynomial Specification:

$$\hat{y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i2}^2 + \hat{\beta}_3 \dots + \hat{\beta}_k x_{i2}^k$$

- deer @ deer rate or deer @ deer rate
- deer @ deer rate or deer @ deer rate
- very flexible
- Might eventually imply reversal of relationship :-
- can combine linear & non linear
- used to test non linearity of x/y
- if parameter of x_{ij}^2 is not stat dif. than 0; it is linear cannot be rejected
- can have multicollinearity problem

Choice of functional form:

- usually the theory or knowledge of subject

Also consider:

- signs of parameters is according to theory
- value of ~~significance~~ significance of param. in other models
- R^2 or \bar{R}^2 as long as dep var same across model choices

Residual analysis:

compute $\hat{u}_i = u_1 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4}^2 + \beta_5 x_{i5}^2 + \beta_6 x_{i5} x_{i3} + \dots + u_i$

+ term
interaction

- don't want param to be statistically significant \rightarrow if yes? missed explain var
- value \square s to pick up pattern; need @ least those
- $H_0: \sigma_2 = \sigma_3 = \sigma_4 = \dots = 0$ } jointly = 0
- $H_1: @$ least one not 0

F test:

Recall: $TSS: \sum (y_i - \bar{y})^2 \rightarrow n-1$ df

$ESS: \sum (\hat{y}_i - \bar{y})^2 \rightarrow k-1$ df

$RSS: \sum (y_i - \hat{y}_i)^2 \rightarrow n-k$ df

Bygones RSS from explain var is unexplained

$SS: n y_i - 1 \bar{y} \rightarrow n-1$

$ESS: k \hat{y}_i - 1 \bar{y} \rightarrow k-1$

$RSS: n y_i - k \hat{y}_i = n-k$

\rightarrow # to call

df \rightarrow # of indep ex var

recall

- F test: used to eval H_0 that all model param (w/ exception of intercept) are 0

- for reg residual analysis: want to not reject

- Means none of the var shown have effect only

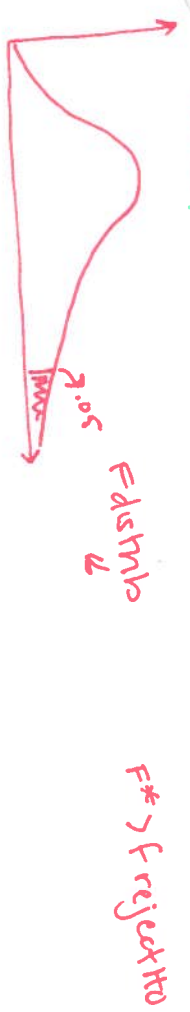
- if H_0 not reject model as a whole cannot be said to be useful to explain

- H_1 : AT LEAST ONE not equal to 0

$F^* = (ESS / (k-1)) / (RSS / (n-k))$

- Under H_0 : w/ 4016 ~~param~~ assumpt + u is normal; F^* follows f_{distn} w/ $k-1$ & $n-k$ df

- some param F^* significant to critical F table



Other types of F test:

- joint test on several coef \rightarrow subset
- test involve linear function of the regression coef
- test involve equality of coef of diff regression equations

Joint test

- eval subgroup of x

\rightarrow live examples

- unrestricted Model: $y = \beta_1 + \beta_2 x_2 + \dots + \beta_k x_k + u$

- want to test of subset of g of regressed coef jointly = to 0

- Rewrite:

$$y = \beta_1 + \beta_2 x_2 + \dots + \beta_g x_g + \beta_{g+1} x_{g+1} + \dots + \beta_k x_k + u$$

- test of coef all = 0 so:

$$y = \beta_1 + \beta_2 x_2 + \dots + \beta_g x_g + u$$

- $H_0: \beta_{g+1} = \beta_{g+2} = \dots = \beta_k = 0$

- $H_1: \text{at least one not 0}$

- Need to est Rest. + Unrest model

$$SSR \geq RSS_{ue}$$

- if H_0 is correct; drop g var have little effect on explain model
 \rightarrow SSR slight higher than RSS_{ue}

$$F^* = \frac{RSS_{ue} - RSS_{ur}}{g}$$

$$\frac{RSS_{ur}}{n-k}$$

- Not equivalent to indiv t test

- B/c multi collinearity possible all test are insignificant joint F shows rejection H_0
b/c F is impact of mult collin

test of linear functions of regression coef

- Recall in Cobb Douglas production function sum of exponential coef indicates returns to scale of production $\rightarrow ex: \hat{y} = \alpha \hat{\beta}_1 x_1 \hat{\beta}_2 x_2 \hat{\beta}_3$

- Production economists often want to test if CETS so: $\beta_2 + \beta_3 = 1 = CETS$

$$H_0: \beta_2 + \beta_3 = 1 \quad OR \quad \beta_2 = 1 - \beta_3$$

$$get: \hat{y} = \hat{\beta}_1 + 1 - \hat{\beta}_3 \ln x_2 + \hat{\beta}_3 \ln x_3$$

$$OR \quad \ln y = \hat{\beta}_1 + \hat{\beta}_3 (\ln x_3 - \ln x_2)$$

- Another example is $\beta_2 - \beta_3 = 0$ OR $\beta_3 = \beta_2$

$$UR: y = \beta_1 + \beta_2 x + \beta_3 x \dots$$

$$E: y = \beta_1 + \beta_2 (x_2 + 1)$$

OR $\beta_2 (x_2 - x_3)$ which is $\beta_3 = -\beta_2$ * look @ signs

- F test is same as before:

$$F^* = (RSS_E - RSS_U / q) / (RSS_U / (n - k))$$

\downarrow restrictions

- can also have linear functions of more than 2 coef but each function = restriction not even parameters

F test for equality of coef of dif. Regression eq:

- want to test if same model work for 2 data sets or if need new model
- "chow" test
- can have dif # observations; closely related dep. var + same indep var (but can take dif var)
- HO: two models identical
- HA: @ least 1 par of coef not the same
- $RSS_U = RSS_1 + RSS_2$
- df: $n_1 + n_2$
- $RSS_E = \text{merged model}$
- $F^* = ((RSS_E - RSS_U) / k) / (RSS_U / (n + m - 2k))$
- df $(RSS_E) - df(RSS_U) = (n + m - k) - ((n - k) + (m - k)) = n + m - k - n - m + 2k = k = \# \text{ rest imposed}$
- if the rejected data cant be pooled
- dummy vars are same

Use of dummy var:

- Some indep var are qualitative \rightarrow values w/ no natural/lexical ordering
- modeled w/ dummy var
- set of dv created for each category var \rightarrow # dv = # categories
- Make sure to exclude one category \rightarrow avoid multicollinearity
- B_1 = intercept for excluded cat; param = dif b/w B_2 categ + that category
- Use t-test to see if B_i is statistically dif
- to see if cotton yields statist. dif. under varieties 1 & 2; est 2nd version of model w/ dif. exclud. cat.

- w/ just intercept var \rightarrow parallel shift "intercept shifters"

- slope shifts: B_2 Dgi: x_i \rightarrow $B_4 + B_5$ is slope x_i when var 2 present

- when 2 or more indep var in a regression model are highly correlated to each other it is difficult to determine if var have indep effect + the magnitude of said effect

Multi collinearity

- Math: occurs w/c, among many CP, bc assoc w/ given obs param est will be higher if corresp 1 & 2 are more highly correlated w/ IV
- $\hat{\sigma}^2[\tilde{B}_2] = \hat{\sigma}^2 / [(1 - r_{23}^2) \sum (x_{2i} - \bar{x}_2)^2]$
 \rightarrow perfect cor = 1 have to divide by 0 ... not possible
 $(x'x)^{-1}$ doesn't work
- higher correl = \uparrow var \rightarrow var double of .5; se effect = $\sqrt{V_{effect}}$
- could be reason why reg IV are not statist. signif w/ t-test
- not a mistake or violation of OLS \rightarrow just undesirable
- more common in time series data

Perfect Multi collinearity:

- occurs when perfect linear correl between two IV
 \rightarrow dummy var
 \rightarrow IV = const value
- if ~~if~~ present: OLS cannot produce param. or stand est: THIS IS A MISTAKE
 \rightarrow redundant info?

Severe multi collin:

- multicol that correl is high \rightarrow intercepts w/ est. of parameters @ desired Statist. level

Symptoms of multicoll:

- key indep var not statistically signif according to t-test
- high R^2 , high signif F-test; few/no stat signif t-test
- Paradox Δ [B16] when exclude some indep var

Detecting Multicollin:

- conduct "artificial" regression w/in each indep var + remaining indep var.
- Variance Inflation Factors (VIF)

$$VIF_j = 1/(1 - R_j^2) \rightarrow R_j^2 \text{ is artificial regressed } R^2 \text{ w/ } j \text{ is.}$$

\rightarrow calculates inflation on $\sqrt{[B_j]}$ caused by multicoll.; if none $VIF=1$

- $VIF=2 = \sqrt{[B_j]}$ twice of no multicoll. · what % ΔX_2 is already accounted for in others
- $VIF > 10$ is clear evidence $\rightarrow 3-4$ is expected
- also condition # of X matrix
 - \rightarrow if 14 are over correl. ratio of highest + lowest eigen value of $X'X$ gets larger
 - \rightarrow condition # is square root of this
- if > 100 = problem
- good: gives overall read
- Bad: not immune to scale Δ

- others: Theil's multicollin. effect; determinant of corrd matrix of 14

(1 = none; 0 = perfect

Addressing multicoll:

- beware of presence \rightarrow help justify why you left something in w/ "not signif" t
- new sample / more obsv will \downarrow multicoll
- exclude 14 trust cause problem (but might be of interest... effy)

* * * even if not the cause: recall omitting rel. indep var = OLS est B1 B2 B3 B4 B5 B6 B7 B8 B9 B10 B11 B12 B13 B14 B15 B16 B17 B18 B19 B20 B21 B22 B23 B24 B25 B26 B27 B28 B29 B30 B31 B32 B33 B34 B35 B36 B37 B38 B39 B40 B41 B42 B43 B44 B45 B46 B47 B48 B49 B50 B51 B52 B53 B54 B55 B56 B57 B58 B59 B60 B61 B62 B63 B64 B65 B66 B67 B68 B69 B70 B71 B72 B73 B74 B75 B76 B77 B78 B79 B80 B81 B82 B83 B84 B85 B86 B87 B88 B89 B90 B91 B92 B93 B94 B95 B96 B97 B98 B99 B100 B101 B102 B103 B104 B105 B106 B107 B108 B109 B110 B111 B112 B113 B114 B115 B116 B117 B118 B119 B120 B121 B122 B123 B124 B125 B126 B127 B128 B129 B130 B131 B132 B133 B134 B135 B136 B137 B138 B139 B140 B141 B142 B143 B144 B145 B146 B147 B148 B149 B150 B151 B152 B153 B154 B155 B156 B157 B158 B159 B160 B161 B162 B163 B164 B165 B166 B167 B168 B169 B170 B171 B172 B173 B174 B175 B176 B177 B178 B179 B180 B181 B182 B183 B184 B185 B186 B187 B188 B189 B190 B191 B192 B193 B194 B195 B196 B197 B198 B199 B200 B201 B202 B203 B204 B205 B206 B207 B208 B209 B210 B211 B212 B213 B214 B215 B216 B217 B218 B219 B220 B221 B222 B223 B224 B225 B226 B227 B228 B229 B230 B231 B232 B233 B234 B235 B236 B237 B238 B239 B240 B241 B242 B243 B244 B245 B246 B247 B248 B249 B250 B251 B252 B253 B254 B255 B256 B257 B258 B259 B260 B261 B262 B263 B264 B265 B266 B267 B268 B269 B270 B271 B272 B273 B274 B275 B276 B277 B278 B279 B280 B281 B282 B283 B284 B285 B286 B287 B288 B289 B290 B291 B292 B293 B294 B295 B296 B297 B298 B299 B300 B301 B302 B303 B304 B305 B306 B307 B308 B309 B310 B311 B312 B313 B314 B315 B316 B317 B318 B319 B320 B321 B322 B323 B324 B325 B326 B327 B328 B329 B330 B331 B332 B333 B334 B335 B336 B337 B338 B339 B340 B341 B342 B343 B344 B345 B346 B347 B348 B349 B350 B351 B352 B353 B354 B355 B356 B357 B358 B359 B360 B361 B362 B363 B364 B365 B366 B367 B368 B369 B370 B371 B372 B373 B374 B375 B376 B377 B378 B379 B380 B381 B382 B383 B384 B385 B386 B387 B388 B389 B390 B391 B392 B393 B394 B395 B396 B397 B398 B399 B400 B401 B402 B403 B404 B405 B406 B407 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B2002 B2003 B2004 B2005 B2006 B2007 B2008 B2009 B2010 B2011 B2012 B2013 B2014 B2015 B2016 B2017 B2018 B2019 B2020 B2021 B2022 B2023 B2024 B2025 B2026 B2027 B2028 B2029 B2030 B2031 B2032 B2033 B2034 B2035 B2036 B2037 B2038 B2039 B2040 B2041 B2042 B2043 B2044 B2045 B2046 B2047 B2048 B2049 B2050 B2051 B2052 B2053 B2054 B2055 B2056 B2057 B2058 B2059 B2060 B2061 B2062 B2063 B2064 B2065 B2066 B2067 B2068 B2069 B2070 B2071 B2072 B2073 B2074 B2075 B2076 B2077 B2078 B2079 B2080 B2081 B2082 B2083 B2084 B2085 B2086 B2087 B2088 B2089 B2090 B2091 B2092 B2093 B2094 B2095 B2096 B2097 B2098 B2099 B2100 B2101 B2102 B2103 B2104 B2105 B2106 B2107 B2108 B2109 B2110 B2111 B2112 B2113 B2114 B2115 B2116 B2117 B2118 B2119 B2120 B2121 B2122 B2123 B2124 B2125 B2126 B2127 B2128 B2129 B2130 B2131 B2132 B2133 B2134 B2135 B2136 B2137 B2138 B2139 B2140 B2141 B2142 B2143 B2144 B2145 B2146 B2147 B2148 B2149 B2150 B2151 B2152 B2153 B2154 B2155 B2156 B2157 B2158 B2159 B2160 B2161 B2162 B2163 B2164 B2165 B2166 B2167 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Heteroskedasticity:

- Ours when error term (transform y) don't have const variance across observations
- OLS parameter est still unbiased but OLS SE are bias → $\text{var } v \neq \sigma^2 I$
 ↳ can't do $\text{var } B = \sigma^2 (X'X)^{-1}$
 ↳ get $\text{var } B = (X'X)^{-1} X' E(uu') X (X'X)^{-1}$
 ↳ will skew test
- means any test that uses SE ~~est~~ is wrong on avg.
- OLS parameter est no longer most efficient (min variance even if error is normal)

Test for heteroskedasticity:

- [White test] is most common
- Process: Regress the squared OLS residuals on the depend var. vs. explaining var.

* this aux. Regression must include an intercept & exclude any redundant R² and side variables

- Under H₀: Homoskedasticity (good): $n \cdot R^2$ of aux regression is distrib as a χ^2_{n-p}
- degrees freedom → $p = \# \text{ parameters in aux regression} - 1$
 ↳ excludes intercept

- want a lower # [fail to reject]

- careful w/ dummy var → don't need the square
- ** Key point: don't want to fail to reject in correctly (think you're good but you're not)
 ↳ what kind of α ? → high α will ↓ critical value

- Disadvantage: can be other reasons we reject null w/ white test
- if misspecification of random component, incorrect functional form; correlated indep var + u → will trigger rejection of H₀
- Assump H2

- Only use white test if confident all OLS Assump hold ***

[Breusch-Pagan]: older legacy multiplier test

- specific H₀: $\sigma^2_{\epsilon} = h(z, \gamma)$ where h is any possible nonlinear function of z, γ
- and z is (1x5) vector of suspect var + intercept; γ is vector of parameters
- can have weird convex functional form that is not accounted for

- since the intercept (1) is included in z the null hypothesis of homosked.
- is equiv to $H_0: \gamma^* = 0$ where γ^* includes all other param in γ
- all except for intercept are 0

2012 → Oct 11 → 15
Tests for heteroskedasticity:
[Breusch-Pagan test]

$$Z_L = \begin{bmatrix} 1 & z_1 & z_2 & z_3 & z_5 \end{bmatrix} \leftarrow \text{subset of suspects } S = 4$$

New parameters
(x var on u)

- $x_1 + x_2 z_2 + x_3 z_3 + x_4 z_5$ linear function of suspect variables

- if error term is normally distributed; under H_0 ; $BPTS = ESS_{n/2}$ follows $\chi^2_{(n-1)}$ where ESS is in Expt. Sum of squares of regression w/ \hat{u}_i^2 / σ^2 as dep var and $n-1$ as df.

Suspect variables in team explain var

- compare computed statistic w/ table value of $\chi^2_{(3-1)}$ at desired α
- $H_0 = \text{normal dist.}$

* error has to be *
Normal or χ^2 is
not exact

* $\chi^2_{250/n} \rightarrow$
 $\sigma^2 = \frac{0.55}{n-k}$

$H_0 = \text{Homo skd.}$

- Advantage: ~~Not~~ more targeted to heterosked (not as easily tricked)

- disadvantage: the model restricted \rightarrow requires $u \sim \text{Normal}$

Dealing w/ interviews:

- if single suspect variable \rightarrow can resolve w/ weighted least squares estimator

- Before on all data multiplied by set of weights; where the weights

and as if n values that when multiplied by error term make it have a constant variance

$$\text{var}[v_i] = \sigma_i^2 \quad w_i = \frac{1}{\sigma_i}$$

$$Y = \beta_0 + \beta_1 \frac{1}{\sigma_i} x_i + \frac{v_i}{\sigma_i}$$

$$Variance \quad Var[V_i] = \sigma_i^2$$

- if only one variable try to use dif. functions of it? (ie. $w_i = z_i^2, z_i = \frac{1}{z_i}$ etc.)

- must retest residuals against to see if problem is fixed

- use white test to see if model is better
- P2 not valid: y is transformed but B are still comparable

- More than one var: Use predictions from mux regression to obtain t -st for β_1

error term variances for each of the docs

→ if some are (-) just add lowest to = 0
→ if some are (+) just add lowest to = 0

→ Since dep var of PR is y_i , predictions are proportional est for σ_i^2

2020

→ predictions might need to be transformed in order to obtain appropriate wfs to

convert ketanostol

WTS and parameters

Dealing w/ Heterosked

- iid: $\text{var}(u) = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{bmatrix} = \sigma^2 I$

Not iid:

$$\text{var}(u) = \Sigma = \begin{bmatrix} \text{var } u_1 & \text{cov.} & \dots & \text{cov.} \\ \text{cov.} & \text{var } u_2 & \dots & \text{cov.} \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov.} & \text{cov.} & \dots & \text{var } u_n \end{bmatrix} = \sigma^2 \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & d_3 & \\ & & & d_n \end{bmatrix}$$

$d \neq 1 \rightarrow$ heterosked

* factor out const. \rightarrow "variance"
 \rightarrow diagonal is not the same or 1
 "corr. matrix"

Generalized least squares:

- general method to est regression model w/ error term not iid

$$\rightarrow \text{var}[u] = \sigma^2 \Psi \neq \sigma^2 I_n$$

if: Ψ is positive definite symmetric matrix there is another matrix P such that $P\Psi P' = I_n$

$$\text{Therefore of } \text{var}[u] = \sigma^2 \Psi; \text{var}[Pu] = P\text{var}[u]P' = \sigma^2 P\Psi P' = \sigma^2 I_n$$

$$\text{Also } \Psi^{-1}(P\Psi)^{-1} \rightarrow \Psi^{-1} = P^{-1}P'$$

$$\begin{aligned} - P\Psi P' &= I_n \\ - P^{-1}P\Psi P'(P\Psi)^{-1} &\Rightarrow P^{-1}I_n(P\Psi)^{-1} \\ - \Psi &= P^{-1}(P\Psi)^{-1} \end{aligned}$$

$P\Psi P' = I_n$
 $\Psi = P^{-1}(P\Psi)^{-1}$
 $\Psi^{-1} = P^{-1}P'$
 $P^{-1}I_n(P\Psi)^{-1} = P^{-1}P\Psi P'(P\Psi)^{-1}$

- End up w: $PY = PX\beta + PV$ or $Y^* = X^*\beta + U^*$ $\rightarrow U^*$ is iid
 - if OLS used on transform model \rightarrow do get efficient param est & unbiased est.

- OLS to this model: GLS estimator

$$\hat{\beta} = (X'P'PX)^{-1}X'P'PY = (X'\Psi^{-1}X)^{-1}X'\Psi^{-1}Y$$

* - this is an application of GLS

$$P = \begin{bmatrix} \frac{1}{\sqrt{z_1}} & & \\ & \frac{1}{\sqrt{z_2}} & \\ & & \ddots \\ & & & \frac{1}{\sqrt{z_n}} \end{bmatrix} \rightarrow P'P = \begin{bmatrix} \frac{1}{z_1} & & \\ & \frac{1}{z_2} & \\ & & \ddots \\ & & & \frac{1}{z_n} \end{bmatrix} \leftarrow \text{inverse} \rightarrow \begin{bmatrix} z_1 & & \\ & z_2 & \\ & & \ddots \\ & & & z_n \end{bmatrix} \cdot \sigma^2 = \Omega = \sigma^2 \Psi$$

$$\text{Why: } \Psi^{-1} = P^{-1}P + (P^{-1}P)^{-1} = Y$$

Dealing w/ Heterosked

- if inefficiency of OLS param. est. not a concern; simplifying of dealing w/ Heterosked

to est Heterosked-consist SE:

$$HCS E = (x'x)^{-1} x' \Omega x (x'x)^{-1}; \Omega = \text{matrix of } \text{square} \text{ as square of resid on}$$

diagonal & 0s elsewhere

- yields consistent estimates for SE; can be considered correct & confidently used for
type testing & building CI if sample size is large (>100)

Topic 4 → Normal Error Model

- Normality results from OLS estimator being a linear function of $Y + U$ in Normally distributed

• only holds asymptotically (large samples) if x s are random in small samples $\hat{\beta}$ is approx. normal

$$\left. \begin{aligned} \cdot \hat{\beta} &= \frac{(x'x)^{-1}x'y}{M} \\ \cdot \hat{\beta} &= \beta - \frac{(x'x)^{-1}x'u}{M} \end{aligned} \right\} \begin{array}{l} \text{linear function of } y + u + \text{linear function} \\ \text{of Normal var and normal} \end{array}$$

• Means: of multiple samples taken & OLS obtains multiple est ($\hat{\beta}$) then one can think $\hat{\beta}_j$ comes from Normal distrib w/ mean β_j & var $\rightarrow \text{var}[\hat{\beta}_j]$

- $1 \text{ se}[\hat{\beta}_j] \rightarrow 68\% \text{ prob}$
- $2 \text{ se}[\hat{\beta}_j] \rightarrow 95\% \text{ prob}$
- $3 \text{ se}[\hat{\beta}_j] \rightarrow 99.8\% \text{ prob}$

Confidence Interval

• since $\hat{\beta}_j \sim N(\beta_j, \text{var}[\hat{\beta}_j])$

$$\rightarrow Z = (\hat{\beta}_j - \beta_j) / \text{se}[\hat{\beta}_j] \sim N(0,1) \text{ where } \text{se} = \sqrt{\text{var}[\hat{\beta}_j]}$$

• Recall: σ^2 for $\text{se}[\hat{\beta}_j]$ is unknown \rightarrow est by $\hat{\sigma}^2 \rightarrow$ means one only has to est for $\text{se}[\hat{\beta}_j] \rightarrow \hat{\text{se}}[\hat{\beta}_j]$

• shown that: $t = (\hat{\beta}_j - \beta_j) / \hat{\text{se}}[\hat{\beta}_j] \sim t_{(n-k)}$

• + distribution $n-k$ degree of freedom
 $k \rightarrow$ # parameters

• from previous: $\Pr(-t_{(n-k)} < t < t_{(n-k)}) = 1 - \alpha$

$$\downarrow$$

$$(\hat{\beta}_j - \beta_j) / \hat{\text{se}}[\hat{\beta}_j]$$

• multiply all elements by $\hat{\text{se}}[\hat{\beta}_j]$, subtract $\hat{\beta}_j$ & multiply by -1

$$\rightarrow \Pr(\hat{\beta}_j - t_{(n-k)}(\hat{\text{se}}[\hat{\beta}_j]) < \beta_j < \hat{\beta}_j + t_{(n-k)}(\hat{\text{se}}[\hat{\beta}_j])) = 1 - \alpha$$

Test one:

[Topic one: The basics]

Intro: Regression analysis used when interested when a particular var is affected by our variables

[What is a Regression Model]

- Start by conceptualizing a behavioral relation based on economic theories/knowledge or postreason
 - includes a depend var (y) + 1 or more indep var (x)
 - makes mathematical model:
$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + u$$
 - Born const coeff \rightarrow true parameters; estimated using data on y+x
 - \rightarrow random error term recognizes relation b/w x+y is not exact
- V: account for other var any \rightarrow we can't explain var
 - \rightarrow random error in y measurement

* Models seek to capture essentials; not to be a perfect representation of the process

Main uses \rightarrow estimate magnitude of explanatory var only [est. parameters]

\rightarrow obtain predictions for dep. var

Population v. est:

- Population β unknown
- X (observed) (within var)
- Assume population model generates y_i for observe x_i

[Topic 2: OLS Estimation]

- OLS: ordinary least squares
- Method: minimize $ESS = \min_{\hat{\beta}_1} \sum_{i=1}^n \hat{u}_i^2$
 - $= \min \sum_{i=1}^n (y_i - \hat{\beta}_1)^2$
 - $= \min \sum_{i=1}^n (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2$
- Min ESS; squared to penalize large u
- Take derivatives to find β values
$$\frac{dESS}{d\hat{\beta}_1} = \frac{\partial}{\partial \hat{\beta}_1} \left(\sum_{i=1}^n (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2 \right) / \frac{\partial}{\partial \hat{\beta}_1} = 0$$
$$= \sum_{i=1}^n (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i) = 0$$
- end up w/ normal equations:
 - $n \hat{\beta}_1 + (\sum x_i) \hat{\beta}_2 = \sum y_i \rightarrow \hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$
 - $(\sum x_i) \hat{\beta}_1 + (\sum x_i^2) \hat{\beta}_2 = \sum x_i y_i \rightarrow \hat{\beta}_2 = \frac{\sum (x_i - \bar{x}) y_i}{\sum (x_i - \bar{x})^2}$

[test one]

Topic 2 \rightarrow OLS

Mathematical properties of OLS:

- $\sum \hat{u}_i = 0 \rightarrow \sum (\hat{y}_i - \hat{y}_i) = \sum \hat{y}_i - \sum \hat{y}_i = 0$
- $\sum \hat{y}_i = \sum y_i \rightarrow -2 \sum (y_i - \hat{B}_1 - \hat{B}_2 x_i) = 0 = \sum y_i - \sum (\hat{B}_1 - \hat{B}_2 x_i) = 0 = \sum (y_i) = \sum (\hat{y}_i)$
- always passes through pt $(\bar{x}, \bar{y}) \rightarrow$ we est model through mean data
- Residuals are not correlated w/ values of x s or \hat{y}_i

[OLS Requirements]

- OLS is only used to est models that are linear in parameters
- # of observations $>$ # parameters [not in rule of thumb]
- Some level of variation in x
- can't be Perfect multicollinearity \rightarrow if in $(x'x)^{-1}$ doesn't exist \rightarrow can't est \hat{B}

[OLS Assumptions]

- values taken by x are fixed or random + uncorrel w/ error term
- can be corr if: proxy measured w/ error
dep var has feedback

1 + 2 hold?

B is unbiased & consistent

- $E[u] = 0$
- No key rel x left out
- functional form correctly specified (linear)

- No auto correlation $\rightarrow u$ is indep distributed
 \rightarrow errors are uncorr w/ constant
- No heteroskedasticity $\rightarrow \text{var}[u_i] = \sigma^2$
- error term variances are same regardless of x s

[Covariance Matrix]

$$\text{var}[u] = E[(u - E[u])(u - E[u])']$$

- Matrix gives $n \times n$ matrix w/ var on diagonal + covar on off diag
- No auto correlation: all covar = 0 independently
- No heteroskedasticity: all var are same identically distributed

$$\hat{B} = (x'x)^{-1} x'y$$

$$= (x'x)^{-1} x'(xB + u)$$

$$= \frac{1}{n} \frac{(x'x)^{-1} x'x B + (x'x)^{-1} x'u}{n}$$

$$= B + (x'x)^{-1} x'u$$

$$E[\hat{B}] = B + (x'x)^{-1} x' E[u]$$

$$E[\hat{B}] = B \quad \text{if } (unbiased)$$

[consistent]

$$\lim_{n \rightarrow \infty} \text{prob}(|B - \hat{B}| > \epsilon) = 0$$

OLS standard error:

- $\hat{\beta}$ is random var b/c it is a function of v & v is random var
- covariance Matrix of $\hat{\beta} \rightarrow$ symmetric

$$\text{var}[\hat{\beta}] = E[(\hat{\beta} - E[\hat{\beta}])(\hat{\beta} - E[\hat{\beta}])']$$

$$= E[(\hat{\beta} - E[\hat{\beta}])(\hat{\beta} - E[\hat{\beta}])']$$

$$= E[(x'x)^{-1}x'u(x'x)^{-1}x'u]$$

$$= \frac{1}{n-k} (x'x)^{-1}x'E[uu']x(x'x)^{-1}$$

$$= (x'x)^{-1}x'\sigma^2 I (x'x)^{-1}x'$$

$$= \sigma^2 (x'x)^{-1}x'x(x'x)^{-1}$$

$$= \sigma^2 (x'x)^{-1}$$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n (\hat{u}_i - \bar{\hat{u}})^2}{n-k} \quad \text{S.E. } \sqrt{\widehat{\text{var}}(\hat{\beta}_1)}$$

$$= \frac{\sum_{i=1}^n (\hat{u}_i)^2}{n-k}$$

$$= \frac{\text{RSS}}{n-k}$$

$$= \frac{\hat{u}'\hat{u}}{n-k}$$

$$= \frac{\hat{u}'\hat{u}}{n-k}$$

- Provide a degree of precision measure
- Rule of thumb \rightarrow true parameter w/in ± 2 S.E. of OLS est

Note on S.E.:

$$\widehat{\text{var}}[\hat{\beta}_2] = \hat{\sigma}^2 / [(1 - r_{23}^2) \sum (x_{2i} - \bar{x}_2)^2]$$

\rightarrow D of corr'd b/w x_2 & x_3
 \rightarrow sum of D deviation of x_2 & \bar{x}_2

- $\hat{\sigma}^2 \uparrow$; S.E. \uparrow
- $r_{23} \uparrow$; S.E. \uparrow
- $\sum (x_{2i} - \bar{x}_2)^2 \uparrow$; S.E. \downarrow

Properties of OLS estimators

- unbiased; on average est expected to equal values of unknown pop. model
- requires Assump 1+2
- of 1-4 hold estimator has min. variance among all possible unbiased estimators that are linear
- OLS mostly yield of est for B
- if v is normal \rightarrow OLS is min var of all unbiased estimators
- BLUE: Best linear unbiased estimator

Test one

Topic 3 → goodness of fit

- OLS doesn't measure how well can this best-fitting model predict y behaviour
- use R^2

R^2

- each y_i can be decomposed into explained + residual variation

total: $(y_i - \bar{y})^2$

explained: $(\hat{y}_i - \bar{y})^2$

residual: $(y_i - \hat{y}_i)^2$

$\left. \begin{array}{l} \text{explained: } (\hat{y}_i - \bar{y})^2 \\ \text{residual: } (y_i - \hat{y}_i)^2 \end{array} \right\} \sum \text{ in front} = \text{TSS, ESS, ESS}$

$\text{TSS} = \text{ESS} + \text{RSS}$

$\sum (y_i - \bar{y})^2 = \sum [(\hat{y}_i - \bar{y})^2 + (y_i - \hat{y}_i)^2]$

$= \sum (\hat{y}_i - \bar{y})^2 + \sum (y_i - \hat{y}_i)^2 \rightarrow \sum 2(\hat{y}_i - \bar{y})(y_i - \hat{y}_i)$

$= 0$

$0 = 2 \sum (\hat{y}_i - \bar{y})(y_i - \hat{y}_i)$

$= \sum (\hat{y}_i y_i) - \sum (\hat{y}_i^2) - \sum (\bar{y} y_i) + \sum (\bar{y} \hat{y}_i)$

$= \sum (\hat{y}_i (\hat{y}_i + \hat{u}_i)) - \sum \hat{y}_i^2 - \bar{y} \sum (y_i) + \bar{y} \sum (\hat{y}_i)$

$= \cancel{\sum \hat{y}_i^2} + \sum \hat{y}_i \hat{u}_i - \cancel{\sum \hat{y}_i^2} - \bar{y} \sum (y_i - \hat{y}_i)$

$= \sum \hat{y}_i \hat{u}_i - \bar{y} \sum (y_i - \hat{y}_i)$

$0 = \sum \hat{y}_i \hat{u}_i + \bar{y} \sum (y_i - \hat{y}_i)$

R^2 calculated as:

$R^2 = \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} = \text{ESS} / \text{TSS} \rightarrow \text{proportion of total variation explained by model}$

OR

$R^2 = 1 - \frac{\sum \hat{u}_i^2}{\sum (y_i - \bar{y})^2} = 1 - \text{RSS} / \text{TSS} \rightarrow \text{proportion not explained by model}$

So R^2 measures proportion of var explained by model

- Same regardless of units

- only take values b/w 0 + 1

$R^2 = 0 = \text{no fit}$

$R^2 = 1 = \text{perfect fit}$

$R^2 \uparrow = \text{better fit}$

$R^2 = \frac{\sum (y_i x_i)^2}{\sum (y_i)^2 \sum (x_i)^2} \rightarrow \text{square of correl coef b/w } y \text{ + } x$

~~Explain~~ evaluate R^2 w/ type of data

time series? ~~$R^2 \geq .8$~~ $R^2 \geq .8$

cross sectional? $R^2 \geq .5$

low R^2 still have value \rightarrow precise parameter est

ONLY V MIDE MODEL HAS INTERCEPT

Bad: can't add more x added

Test one

Topic 3 \rightarrow measure of best fit

[Adj R^2]

assess if add x will \uparrow predictability of y

$$\bar{R}^2 = 1 - \left[\frac{ESS}{TSS} \cdot \frac{n-1}{n-k} \right]$$

always $< R^2$

not same interp as R^2 ; can be neg

\bar{R}^2 better for model w/ lots var + few obs

Final notes:

any var believed to have direct notable effect on $y \rightarrow$ include

having extr var prefer to leaving out $\rightarrow E[u] \neq 0$ in

indirect effect? leave out

Topic 4 \rightarrow Normal error model + E_i | hypothesis test:

[Normal-Error model]

Assumptions: $\text{cov}[u, x_j] = 0$

$E[u_i] = 0$

$\text{var}[u_i] = \sigma^2$

$\text{cov}[u_i, u_j] = 0$ for all $i \neq j$

error term is assumed normal

if

$\text{cov}[u, x_j] = 0$ all obs $\rightarrow \hat{\beta} \sim N(\beta, \sigma^2(x'x)^{-1})$

$E[u] = 0$ $\rightarrow \hat{\beta}_j \sim N(\beta_j, \text{var}[\hat{\beta}_j])$

if independent u is iid

$\beta | C \rightarrow$ ols estimator is linear function of u

so β_c w/in $\beta_j \pm SE \hat{\beta}_j \rightarrow 68\%$

$2SE \rightarrow 95\%$

$3SE \rightarrow 99\%$

[Confidence intervals]

$t = \hat{\beta}_j - \beta_j / SE[\hat{\beta}_j]$

vs. $z_{model} \beta_j - \beta_j / SE[\hat{\beta}_j]$

$Pr[-t(1/2, n-k) < t < t(1/2, n-k)] = 1 - \alpha$

$Pr[\hat{\beta}_j - t(1/2, n-k) SE[\hat{\beta}_j] < \beta_j < \hat{\beta}_j + t(1/2, \dots) SE[\hat{\beta}_j]] = 1 - \alpha$

[Hypothesis]

$H_0: \beta_j = c$ vs $H_1: \beta_j \neq c$

↳ compute $t = (\hat{\beta}_j - c) / \text{se}[\hat{\beta}_j]$

• How likely that particular t val is to have been pulled from t_{n-k}

- select α
- find table t^*
- if $|t| > t^*$ reject the inference that $\beta_j \neq w$ & > possibility of being mistaken
- if $|t| < t^*$ one can't reject H_0 ; prob H_0 is correct is unknown

Statistical Hypothesis testing

- t test only valid if σ s assumed hold / σ known
- if sample size large σ doesn't have to be known
- use t test to test if β is statistically different from $\rho \rightarrow$ x affecting
- $H_0: \beta_j = 0 \rightarrow$ no effect
- $H_1: \beta_j \neq 0 \rightarrow$ x has effect
- two tail to see $\beta_j(-)$ or $\beta_j(+)$
- t test reduce to $\hat{\beta}_j / \text{se}[\hat{\beta}_j]$ → 2 reject
- ↳ rule of thumb $\hat{\beta} / \text{se}[\hat{\beta}] \rightarrow$ 2 reject
- P-value: exact lowest α at which $H_0: \beta_j = 0$ can be rejected
- ↳ in favor of 2 tail
- opt for 1 tail → t val not alt but critical value $\alpha \rightarrow$ fill all α in one

[CI for γ_0]

- $\hat{\gamma}_0 \pm t \cdot \text{se}[\hat{\gamma}_0] = \gamma_0$
- $\hat{\gamma}_0 \pm t \cdot \text{se}[\hat{\gamma}_0] = E[\hat{\gamma}_0] \rightarrow$ narrower

* $\frac{RSS}{df} = 0 - 2x'y + (x'x + (x'y)^2) \times \hat{\beta}$

$= -2x'y + 2(x'y)\hat{\beta}$

$= 2(x'y)\hat{\beta} = 2x'y$

$(x'y)'(x'y)\hat{\beta} = x'y \quad (x'y)^{-1}$

$\hat{\beta} = (x'y)^{-1}x'y$

Remember \uparrow

$$U'U = RSS$$

$$(y - x\hat{\beta})(y - x\hat{\beta})'$$

$$y'y - 2y'x\hat{\beta} - \hat{\beta}'x'x\hat{\beta}$$

Test TMO \rightarrow Oct 2 +
 Chapt 8 \rightarrow F test

Other F test
 \rightarrow F test w/ Linear Function of Regression Coef

- $F^* = ((RSS_E - RSS_U) / r) / ((RSS_U) / (n - k))$
- can be extended to restrictions involving linear functions.
- each linear function implies 1 restriction regardless of # of coef involved

\rightarrow F test for equality of coef of dif regressions

- see if model applies to same / somewhat dif dataset +
- "Chow" test
- can have dif # obs; but dep var and dif but closely related; \pm x var the same
- H0: identical $B_1 = b_1, B_2 = b_2; H_A \rightarrow$ no pair (at least) not identical
- UP $\rightarrow RSS_1 + RSS_2 \rightarrow$ df sum of df from the two models $\rightarrow (n - k) + (m - k)$
 $m + n - 2k$
- R \rightarrow RSS merge
- $F^* = ((RSS_E - RSS_U) / r) / (RSS_U / (n + m - 2k))$
- to rejected? can't pool data \rightarrow or use dummy var to bridge.

Chapt 9 \rightarrow Dummy Var \rightarrow Regression

- qualitative variables \rightarrow no natural order
 \rightarrow can be modeled w/ dummy var
- set of dummies for each categorical IV
- leave out one dum. on set \rightarrow avoid multicol.
- intercept dum \rightarrow just parameter and variable: $D_{21} + B_2$
- intercept in var for excluded cat.
- slope dum \rightarrow param; dum IV + reg IV.
 \rightarrow param on reg IV in which left out cat is used

Chapt 10 \rightarrow Multicollinearity

Multicollinearity:

- when two or more indep variables in a regression model are highly correl. to each other \rightarrow difficult to tell ~~which~~ if each indiv affect + quantity effect
- occurs w/ (many thin CP) \rightarrow SE assoc w/ given OLS Parameter est will be higher if IV is cor (highly) w/ other indep models
 \rightarrow larger error always SE \rightarrow correl. 5? double var; $\sqrt{\text{effect on var}} = \text{SE effect}$
 \rightarrow leads to key var w/ not signif t test
- Not mistake or violation; just undesirable
- More common in time series data

Chapter 4 → Regression Stuff.

Regression through the origin:

- used when theory dictates intercept = 0 → when all explain var are 0 y ~~est~~ should = 0.
- estimated easily by using 1st col of Y .
- changes parameter estimates
- R^2 not appropriate; no longer bounded b/w 0 & 1; $TSS \neq RSS + ESS$

→ can use " R^2_{adj} " → not the same as R^2

→ $(\text{corr}[y, y])^2$

→ ONLY recommended if STPON's evidence that true pop val of intercept is zero

Scaling & units of measurement:

- a change in scaling of any x var by fixed factor (viz) only Δ parameters + SE for that x by factor of $\frac{1}{W_x}$
- a change in scale of y → $W_y W_1$; Δ value for all model parameters + SE by factor of W_1

Standard Regression:

- regression w/ all var (x, y) standardized to mean 0 & variance of one
- for each var: subtract mean + divide by stand. deviation
- Parameters of this are "beta coefficients"
- intercept would be 0
- Good enterp: $y + x$ measured in sd → sol Δ in y if $x \Delta$ by one sd
- elim unit of measure → used for rel import of x on explaining y BOI

doesn't say anything about FATE of flux in x

$$W_x = \frac{1}{S_y} \quad \frac{1}{W_x} = S_x$$

$$y_i^* = \frac{y_i}{S_y} - \frac{\bar{y}}{S_y}$$

$$x_i^{**} = \frac{x_i}{S_x} - \frac{\bar{x}}{S_x}$$

Lagged variables:

$$y_t = B_1 + B_2 x_{t-1} + u_t \quad (t=2)$$

lagged

- can also be more than one lag.
- lag one ~~var~~ for every lag obs

first differences variable:

→ change from $t \rightarrow t+1$

$$\rightarrow Y = B_1 + B_2 (x_t - x_{t-1}) + u_t \quad (t=2)$$

* do this before running OLS

$$Y_t = B_1 + B_2 \Delta x_t + u_t$$

- need to believe $\Delta \ln x$ affects value taken by y in linear fashion

Alt model specifications

- Regression models so far assume relation b/w y & indep var is linear

* OLS req for linear in param; not req for non-linear. In expl var. / dep. var.

log-linear:

$$\hat{y}_i = \hat{\beta}_1 \hat{x}_1 \hat{\beta}_2 \hat{x}_2 \dots \hat{\beta}_k \hat{x}_k \rightarrow \ln(\hat{y}_i) = \ln(\hat{\beta}_1 \hat{\beta}_2 \dots \hat{\beta}_k) + \ln(\hat{x}_1) + \ln(\hat{x}_2) + \dots + \ln(\hat{x}_k)$$

• $\ln x$ @ $\ln y$ rate

* Take lns before OLS

• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rate• $\ln x$ @ $\ln y$ rateSemi log: log ~~linear~~ linear

• log dep var; all rest are normal

• $\hat{\beta}_1 \rightarrow$ multiplied by 100 \rightarrow % Δy when x Δ 1 unit.• $\beta_j > 0$; $x \uparrow$; $y \uparrow$ @ $\ln y$ rate• $\beta_j < 0$; $x \uparrow$; $y \downarrow$ @ $\ln y$ rate• y can't be neg; x canSemi log model: $\ln y$ log• log of expl var is taken; y left alone• $\beta_j \rightarrow$ unit Δy for % $\Delta x \rightarrow$ divide by 100• $\beta_j > 0$; $x \uparrow$; $y \uparrow$ @ $\ln y$ rate• $\beta_j < 0$; $x \uparrow$; $y \downarrow$ @ $\ln y$ rate

Chapter 6:

Alt model specifications:

Reciprocal: $\frac{1}{x_i}$

• $B_j > 0; x_T; y \uparrow @ \text{deer rate}$

• $B_j < 0; x \uparrow y \uparrow @ \text{deer rate}$

• has asym tope

• slope depends on x

* do all model manipulation before OLS

log reciprocal:

• $\ln y + \frac{1}{x}$

• get S curve  or deer deer rate 

Polynomial:

• include x_i^2

• as $x \uparrow$: y cant go @ inner or deer rate (very flexible)
↳ might lead to reversal of relation

• can combine models + easily test for nonlinearity
↳ assume others that alter x not y

• if $H_0: B_1 = 0$ cant be rejected \rightarrow linearity cant be disproved

• can create multi adin problem

choice of functional form:

• underlying theory will help decide

• Also consider: sign of parameter in ET theory

• statistical sign of param in alt model

• R^2 or adj R^2 on long indep var same across choices

• Residual analysis!

Chapt 8 \rightarrow F test:

F test:

• Recall $TSS = \sum (y_i - \bar{y})^2 \rightarrow n-1 \text{ df}$

$ESS = \sum (\hat{y}_i - \bar{y})^2 \rightarrow k-1 \text{ df}$

$RSS = \sum (y_i - \hat{y}_i)^2 \rightarrow n-k \text{ df}$

• greater the RSS to ESS \rightarrow more unexplained

• F test used to evaluate the null hypothesis: all model parameters w/ exception of intercept are zero

↳ $H_0: B_2 = B_3 = \dots = B_k = 0$

↳ Means none of the indep var chosen have any effect on y

* if has something to do w/ # of indep var
var ~~test~~ unexplained calculation

F-test:

- If H_0 is not rejected → model as a whole cannot be said to be useful explaining
- H_0 : at least one indep var affects y
- use var dep var? → don't want to reject

$$F^* = (ESS / (k-1)) / (RSS / (n-k))$$

- 4 OLS Assump hold + u in $\sim N$ (small samples); F^* follows F distrib w/ $(k-1)$ & $(n-k)$ df
- $F^* \rightarrow p$ val for F ; exact probab w/ which we allow to reject

Other F-tests:

→ Joint test w/ several coef:

- eval subgroup as whole
- UR (unrestricted) Model: $y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 \dots \beta_k x_k + u$
- want to test subset g of regression coef if = (jointly) to zero
- Rewrite Model:

$$y = \beta_1 + \beta_2 x_2 + \dots \beta_k x_k + u$$

Restricted Model →

$$H_0: \beta_{k-g+1} = \beta_{k-g+2} = \dots = \beta_k = 0$$

→ g = # restrictions

- estimate R + UR ; $RSS_R \geq RSS_{UR}$
- if H_0 correct; drop var have little effect on $RSS \rightarrow RSS_R$ slightly higher RSS_{UR}

$$F^* = ((RSS_R - RSS_{UR}) / g) / ((RSS_{UR}) / (n-k))$$

→ H_0 correct: follow F w/ g df (num) & $n-k$ (denom)

- not the same as indiv t test *
- Need OLS Assump. + $u \sim N$
- can get insignif t test & still have F test that rejects the (un)perous to multivars

→ F-test w/ linear function of regression coef

→ in Cobb Douglas → sum of exponents = RTS

→ test CRTC

$$UR: \ln \hat{y}_i = \hat{\beta}_1 + \beta_2 \ln x_2 + \beta_3 \ln x_3$$

$$Restriction: \beta_2 + \beta_3 = 1 \text{ or } \beta_2 = 1 - \beta_3$$

$$R: \ln y = \beta_1 + 1 - \beta_3 \ln x_2 + \beta_3 \ln x_3$$

→ Pay attention to sign → $\beta_2 + \beta_3 = 0$ NS.

$$\ln y - \ln x_2 = \beta_1 + \beta_3 (\ln x_3 - \ln x_2)$$

$\beta_2 + \beta_3 = 1$