Chapt 10 → Multicollinearity

multicollin:

- if concerned key variable not stat signif? → check multicoll.

Perfect multicollin:

- perfect linear corrl b/w 2 or more indep var.
  - ↳ dont leave out one dum etc
  - ↳ const indep var
- present? OLS cant produce parameter or se est
- This is a mistake → included Redundant info → respecify model

severe multicollinearity:

- Some degree of correl expected; becomes severe when corrl is high and interferes w/ estimation of parameters @ desired level of certainty
  - ↳ if se low enough + all est are signif; dont worry

Symptoms of multicollin:

- indep var that are considered critical are not statisticly signif.
- high R^2; signif F test but few/no signif t test (particularly w/ small obs set)
- Param est drastically Δ when some indep var left out

Detecting multicol:

- conduct artificial regressions → use one IV as dv
  - ↳ VIF → $1/(1-R_j^2)$ → R^2 of artificial model
  - → calcs inflation on $V[\hat{B}_j]$ → no multicol → =1 [3-4 expected]
    - → se is $\sqrt{\ }$ of v effect
    - → vif = 2 → $V[\hat{B}_j]$ is double
- condition # on x matrix
  - ↳ as corrl ↑ ratio of high to low eigenvalue of $X'X$ ↑
  - ↳ condition # is $\sqrt{\text{ratio}}$
  - → higher than 100 → PROBLEM
  - → can be skewed w/ scale
- others → Theil's multicollenearity effect
  Determinant of ~~corl~~ corrl matrix of IV

Addressing Multicoll:

- good to be aware of severe multicoll → hard to fix
  - ↳ can help justify why you left something in that wasnt signif.
- add obs / new sample will help lessen
- exclude IV that cause problem? → might be what you want to see
- ** REMEMBER → leave out rel IV will make OLS BIAS ***
- Modify model specification → Nom v. Real; reciprocal v. poly nomial in nonlinear functional forms
- usually some corrl but as long as statistical inferences not dissipointing all is good

Chapt 11 → Heteroskedicity

Hetero skedicity:

→ occurs when error term → (anything y) don't have a const variance across observations

→ ols param still unbias BUT SE est are BIAS

↳ means any statistical test w/ se are incorrect on average

↳ ols param. no longer most efficient

↳ have to use: $V[\hat{\beta}] = (x'x)^{-1} x' E[VV'] x (x'x)^{-1}$

Test for heterosk:

White test:

- Regress square OLS resid as dv; vs. explan var, their squares + xproducts.
- AUX Regression: include entercept exclude any R + sid redundant var
- Ho: Homosk; $n \cdot R^2_{AR} \sim \chi^2(p-1)$ → perameters
- * dont square dummy var.
- Want lower #
- ** key point: what kind of a? → really don't want to fail to reject incorrect ↳ use high a → 25%
- can be fooled → misspecific of random component (exclude rel IV or incorrect functional form) or corrl b/w IV + error = rejection Ho
- Must be confident OLS Assump hold.

Breusch-Pagan Test:

- Specific Ha: $\sigma_t^2 = h(z, \gamma)$

  h: any non linear function of z_t, γ → can have weird functional form not accounted for...

  z_t = vector of suspects → w/ entercept

  γ = vector of param.
- Ho: $\gamma^* = \emptyset$ → γ^* = perameters except for entercept
- if error ~ N; $BPTS = ESS_{AR}/2 \sim \chi^2(s-1)$

  → AR: $\hat{u}^2/\tilde{\sigma}^2$ is dv

  $\tilde{\sigma}^2 = \Sigma \hat{u}_i^2 / n$ → $\frac{ESS}{n}$
- Advantage: targeted to heterosk
- Disadvant: Ha more restricted; req. Normal error

Dealing w/ Heterosk

- Single suspect → WLS
  - Before OLS → all data multiplied by set of wts
  - wts: set of n values that when multiplied by error → give const var.
  - Just one var → use dif functions of it as wt.
  - R^2 not valid → y is transformed but B's still comparable

Test two → Oct 27

Chapt 11 → Heterosked.

Dealing w/ heterosked

- more than one var → use predictions from AR to obtain est for error variances for each obser.
  - ↳ since dv is $\hat{U}_i^2$; predictions from it are proportional est for σ_i^2 might need to be transformed to be correct wts.
- acceptable but NOT optimal

Generilized least squares:

- general method to est regression models where error term is not iid.

$$var[U] = \sigma^2 \Psi \neq \sigma^2 I$$

↳ covar matrix

- Based on fuct that Ψ is (+) definate, SYMMETRIC, matrix and there is another matrix (P) → $P\Psi P' = I$
- so: $var[U] = \sigma^2 \Psi$; $var[PU] = P\,var[U]\,P' = \sigma^2 P \Psi P' = \sigma^2 I$

* $P^{-1}(P')^{-1} = \Psi$ + $\Psi^{-1} = P'P$

- $P \Psi P' = I_n$
- $PY = PXB + PU \rightarrow Y^* = X^* B + U^*$ (multiply all by P)
- ols used will produce efficient param est + unbiasse
- $\tilde{B} = (X'P'PX)^{-1} X'P'PY = (X'\Psi^{-1}X)^{-1} X'\Psi Y$
- $V[\tilde{B}] = \sigma^2 (X'P'PX)^{-1} = \sigma^2 (X'\Psi^{-1}X)^{-1}$

Dealing w/ Heterosked:

- if inefficiency of ols is not concern → use → var[V] not unbias but consist.
  - Heterosked-consist se → $(X'X)^{-1} X'\Omega X (X'X)^{-1}$
    - ↳ Ω is matrix w/ square OLS resid in diag + 0s elsewhere
- yeilds consist. estimates → can be considered correct + used for hypo test if sample is large
- also est $\hat{\Psi}$ then apply formula

Chapt 12 → Auto correl

Intro: indep distrib → Autocorrel

- ↳ means when errors are organized in particular order (usually time) U_t not depend on U_{t-1}
  - Auto corr is when they are correl (comod prices

AR(1) → $U_t = \rho U_{t-1} + V_t$

- $V_t \sim N(0, \sigma_V^2)$
- as long as ρ; $-1 < \rho < 1$ → comes back to 0
  - ↳ closer to 1 → more clumps

Test two → oct 27

chapt 12 → Auto correl:

varance + autocorrl f(·) for AR1; Ω Ψ matrix

$Var[U_t] = \sigma_t^2 = E[U_t^2] = E[(\rho U_{t-1} + v_t)^2] = E[\rho^2 U_{t-1}^2 + v_t + 2\rho U_{t-1} v_t] =$

$= \rho^2 E[U_{t-1}^2] + E[v_t^2] + \rho E[\cancel{U_{t-1} v_t}]$ uncorl

$\sigma_U^2 = \rho^2 \sigma_U^2 + \sigma_v^2 + \phi$

$\sigma_U^2 = \frac{\sigma_v^2}{1-\rho^2}$

$\sigma_{t,t-1} = E[U_t U_{t-1}]$

$= E[(\rho U_{t-1} + v_t)(\rho U_{t-2} + v_{t-1})]$

$= E[\rho^2 U_{t-1} V_{t-2} + \underbrace{\rho U_{t-1} v_{t-1}}_{\text{corrl}} + \underbrace{\rho \cancel{U_{t-2}} v_t}_{0} + \underbrace{v_t \cancel{U_{t-1}}}_{0}]$

$\sigma_{t,t-1} = \rho \underbrace{\sigma_{t1,t2}}_{\sigma_{t,t-1}} + \rho E[U_{t-1} v_{t-1}]$

$\rho E[U_{t-1} v_{t-1} = \rho E[((\rho U_{t-2} + v_{t-1}) U_{t-1}]$

$= \rho E[\underbrace{\rho \cancel{U_{t-1}}}_{0} + v_{t-1} + v_{t-1}^2$

$\rho E[U_{t-1}, v_{t-1}] = \rho E[v_{t-1}]^2$

cov b/w U_{t-1} + U_{t-2} → $\rho \sigma_{t,t-1}^2 + \rho \sigma_v^2$

---

corrl b/w v_t + U_{t-1} → $\rho \frac{\sigma_v^2}{\frac{\sigma_v^2}{1-\rho^2}}$

* $Cov[V_t, U_{t-2}] = \frac{\rho^2 \sigma_v^2}{(1-\rho^2)}$

* $corrl[U_t, U_{t-2}] = \rho^2$

* $corrl[U_t, U_{t-k}] = \rho^k$

T = total # obs.

* $\Omega = \begin{bmatrix} \frac{\sigma_v^2}{1-\rho^2} & & & \\ \frac{\rho\sigma_v^2}{1-\rho^2} & \ddots & & \\ \frac{\rho^2\sigma_v^2}{1-\rho^2} & \frac{\rho\sigma_v^2}{1-\rho^2} & \ddots & \\ \frac{\rho^{T-1}\sigma_v^2}{1-\rho^2} & & & \frac{\sigma_v^2}{1-\rho_2} \end{bmatrix}$

$= \underbrace{\frac{\sigma_v^2}{1-\rho^2}}_{\sigma_U^2} \underbrace{\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \rho^T \\ \rho & 1 & & & \\ \rho^2 & & 1 & & \\ \rho^3 & & & 1 & \\ \rho^{T-1} & & & & 1 \end{bmatrix}}_{\Psi}$

Test two → oct 27

MA → moving average

$U_t = \theta V_{t-1} + V_t \qquad V_t \sim N(0, \sigma_v^2)$

$E[U_t] = 0; \ var[U_t] = \sigma_v^2$

$var[U_t] = \sigma_U^2$
$= E[U_t^2]$
$= E[(\theta V_{t-1} + V_t)^2]$
$= E[\theta^2 V_{t-1}^2 + V_t^2 + 2\theta \cancel{V_{t-1} V_t}]$ → 0
$= \theta^2 \sigma_v^2 + \sigma_v^2$
$= \sigma_v^2 (1-\theta^2)$ Diagonal

MA → seasons

allows cyclical w/in cyclical

cov. $U_t U_{t-1} = E[(\theta V_{t-1} + V_t)(V_{t-2} + V_{t-1})]$

$= E[\theta^2 \cancel{V_{t-1} V_{t-2}} + \underbrace{\theta V_{t-1}^2}_{\theta \sigma_v^2} + \cancel{V_t \theta V_{t-2}} + \cancel{V_{t-1} V_t}]$ → θ, ∅, θ

$= \theta \sigma_v^2$

corr $U_t U_{t-1} = \dfrac{\theta \sigma_v^2}{\sigma_v^2 (1-\theta^2)} = \dfrac{\theta}{1-\theta^2}$

cov $U_t U_{t-2} = \emptyset$

ARMA:

ARMA(P,Q) = $U_t = P_1 U_{t-1} + P_2 U_{t-2} + \ldots + P_p U_{t-p} + \theta_1 V_{t-1} + \theta_2 V_{t-2} + \ldots + \theta_q V_{t-q} + V_t$

Random walk:

- nonstationary
- mean/var not stable over time
- $\rho = 1$ → no var/covar.

Quiz Nov 1 – Nov 8
Auto correlation:

Consequences:

exact same as heterosked → param unbias but no longer most efficient
↳ use GLS
→ OLS stand errors are bias
→ f test also invalid

Autocorrelation | diagnostics:

- Visual inspection of OLS Resid. vs. time: prelim diagnostic tool
  - ↳ groups of pos. then groups of neg → (+) corrl.
  - ↳ clear alteration (−) to (+) is neg. corrl (Rare)
- Formal test: Durbin-Watson (tests for 1st order autocorrl) → corrl. b/w previous + present resid.

$$d^* = \sum_{t=2}^{T} (\hat{u}_t - \hat{u}_{t-1})^2 \Big/ \sum_{t=1}^{T} \hat{u}_t^2$$

- No 1st order coorl? d^* will be close to 2; <2 = pos. autocorrl; >2 = neg autocorrl
- Ho: no autocorrlel; Ha: autocorrl (Pos.) [ONE TAIL ALT]
  - if $d^* > du$; conclude Ho ($d^* < 4-du$)
  - if $d^* < dl$; conclude Ha ($d^* > 4-dl$)
  - otherwise → inconclusive

  use upper as threshold to be safe

  * 2 tail? Same table; just double α
- Durbin-Watson only detects autocorrl when no lagged dependent var. included
  - ↳ if lagged included d^* close to 2 even w/ autocorrl.

→ soln. → d-w h stat

$$h^* = [1-(d^*/2)]\sqrt{T/[1-T(\hat{v}(\hat{B}))]}$$

- $\hat{v}(\hat{B})$ is square of the estimated standard error of the perameter of the lagged ~~ta~~ endogenous var. + T is # of observations
- Ho: no autocorrl; $h^* \sim N$ w/ unit var. → use stand normal table
- ** Ha: pos autocorrl? level of signif is α/2
- Need modified version if $T[\hat{v}[\hat{B}]] > 1$ → can't have $\sqrt{\ }$ of ∅ or (−)

- Important to note Model selection
- ~~Bartlt~~ Bartlett test: Ho: $\rho_k = 0$ [v_t + v_{t-1} uncorrl]
  - then $\sqrt{T}\,\hat{\rho}_k \sim z(0,1)$
  - if large reject
- Box-Pierce Test → Joint test
  - $T\sum_{k=1}^{K} \hat{\rho}_k^2 \rightarrow \sim \chi^2_{(k)}$

    y_t rejects easily
    v_t does not.
  - Ho: $\rho_1 = \rho_2 = \rho_3 = \rho_k = 0$

Auto corrl Quiz cont:

Side Notes:

- $Var[v_t] = \sigma_v^2 = \frac{\sigma_v^2}{1-\rho^2}$
- $cov[v_t, v_{t-1}] = \rho\sigma_v^2/(1-\rho^2)$
- $corrl[v_t, v_{t-1}] = \rho$

Durbin-watson

$v_t = \rho v_{t-1} + v_t$

$H_0 = \rho = 0$

$H_a = \rho > 0$

$$\sum_{t=2}^{T} (\hat{v}_t - \hat{v}_{t-1})^2 = \sum_{t=2}^{T} (\underbrace{\hat{v}_t^2}_{\approx \sum v_t^2} + \underbrace{\hat{v}_{t-1}^2}_{\approx \sum v_t^2} - \underbrace{2\hat{v}_t \hat{v}_{t-1}}_{= \emptyset \text{ if uncorr}})$$

No corrl: $d^* \approx \frac{2\sum^T \hat{v}_t^2}{2 v_t^2} \approx 2$

Max Liklihood

- $E[\hat{\sigma}^2] \neq \sigma^2$
- $\hat{\mu} = \sum y_i/n$
- $\hat{\sigma}^2 = \frac{\sum(y_i - \hat{\mu})^2}{n}$ *not n-1

$$Pdf = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(y_i - \mu)^2}{\sigma^2}}$$

$$\hookrightarrow \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2}\frac{\sum(y_i-\mu)^2}{\sigma^2}}$$

Tests for Random walk

Augmented Dickey-fuller } comput. calcs.
Phillips-Perron unit root

$v_t = \rho v_{t-1} + v_t \rightarrow$ no intercept

$H_0: \rho = 1$
$H_a: \rho \neq 1$

$v_t = B_1 + \rho v_{t-1} + v_t \rightarrow$ intercept

$v_t = B_1 + B_2 t + \rho v_{t-1} + v_t \rightarrow$ intercept + not centered @ $\emptyset$

*stat < P → Dont Reject unit root

*stat > P → Reject Ho (unit Root)

→ fix w/ 1st diff.

Structure v. time series:

- Pure time series → no explan var
- structure: systematic and time series component

AIC → Akaike information criteria

SBC → Schwartz Baysian Criterion

$$\Omega = \underbrace{\frac{\sigma_v^2}{1-\rho^2}}_{\sigma_v^2} \underbrace{\begin{bmatrix} 1 & & & \\ \rho & 1 & & \\ \rho^2 & \rho & 1 & \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}}_{\psi}$$

$\psi^{-1} = P'P \rightarrow$ get P

$\hookrightarrow \hat{B} = (x'\psi^{-1}x)^{-1}x'\psi^{-1}y$

$var[\hat{B}] = (x'\psi^{-1}x)^{-1}$

~~Oct~~ QUIZ NOV 1 - NOV 8

Auto corrl:

correcting for:

- same as heterosked → Δ original regression equation into one w/ non corrl error term
  - ↳ $u_t = \rho u_{t-1} + v_t$ → 1st order auto regression; ρ is corrl coef b/w errors$_t$ + errors$_{t-1}$
- if ρ known:

$$\rho y_{t-1} = \rho B_1 + \rho B_2 x_{2t-1} + \dots + \rho B_k X_{kt-1} + \rho u_{t-1}$$

$$y_t - \rho y_{t-1} = (1-\rho)B_1 + B_2(x_{2t} - \rho x_{2t-1}) + \dots + B_k(x_{kt} - \rho x_{kt-1}) + u_t - \rho u_{t-1}$$

OR

$$Y^*_t = (1-\rho)B_1 + B_2 x^*_{2t} + \dots + B_k x^*_{kt} + v_t \text{ (use OLS on this)}$$

- x_s are a type of 1st difference; v_t satisfies ~~all~~ OLS ASSUMP
- will yeild BLUE param

EGLS Estimation

- use est of ρ ($\hat{\rho}$)
  - ↳ $\hat{\rho} = \sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1} / RSS$ [Cochrane-Orcutt prodecure]
- OR regress $\hat{u}_t$ on $\hat{u}_{t-1}$ (w/o intercept) + use param est as est of ρ
- OR $\hat{\rho} = 1 - .5d^*$ → d^* = d-w test stat
- use previous procedure in iterative manner:
  - → est model by OLS
  - → compute resid → obtain 1st est ρ
  - → param est via EGLS
  - → EGLS resid → 2nd $\hat{\rho}$
  - → cont until $\hat{\rho}$ stops Δ
- Not clear if iteration helps obtain more efficient param est
- None of previous is prefered method :(
- MOST EFFICIENT → min sum of squared residuals of transformed model
  - ↳ only solved via numerical search process

only go up to ~ AR(3)

$$= \text{Min}(U'P'PU) = \text{Min}(U'\psi^{-1}U) \rightarrow \text{Min}[(y - x\hat{B})'\psi^{-1}(y - x\hat{B})]$$

NLS Estimation by simple search:

non linear least Δ

- large # of possible ρ used to est transformed model → est w/ lowest RSS selected; can be narrowed once neighboorhood found
  - ↳ usually evenly spaced b/w -1 & 1
- ψ involves only 1 or 2 param.

- Alt to GLS/NLS → Maximum Likli hood
- → ~~this~~ method envolves numerical optimization
  - → if error normal GLS/NLS & ML yeild asymptotically equiv. results.

* key: value of pdf is a measure of liklihood

↳ Always consistant; can be bias

Quiz Nov 8-10

Maximum Likelihood:

Maximum Likelihood estimation:

- Most general method to estimate econometric models
- can be used to est w/ non linear in perameters + non iid u + any type of heterosKid/autocorrl

- $(y_i) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(y_i-\mu)^2}{\sigma^2}}$ (Liklihood f(·) OR Joint prob f(·))

add Σ for joint

- $\hat{\mu} = \Sigma \frac{y_i}{n}$
- $\hat{\sigma}^2 = \frac{\Sigma(y_i-\mu)^2}{n}$ * BIAS But consistant

- if pop error term is iid normal, the probabity that the ith observaton occurs for any given set of peram is given by the normal prob dense f(·)
- Since ui is indep distrib; Probab. of whole sample under a set of peram is the product of the single obs probab.
- this f(·) (measures probability of occurance of whole sample for each set of peram IS LIKELIHOOD FUNCTION

$$LF(u_i/B_1,\dots,B_K,\sigma^2) = (2\pi\sigma^2)^{-\frac{n}{2}} e^{-\frac{1}{2}\Sigma\frac{u_i^2}{\sigma^2}}$$

- Max Likelyhood est. $B_1,\dots,B_K$ + σ^2 ~~given~~ from $\hat{B}s$ + $\hat{\sigma}^2$ that max LF given our n obs on y + x

- usual prodedure in Max Like est is to max ln(LF) → log liklihood f(·) enstead of original

- often easier + gives same result since ln is monotonik ↑ transfom

$$LLF(u_i/B_1,\dots,B_K,\sigma^2) = -\frac{1}{2}\Sigma[\ln(\sigma^2)+(u_i^2/\sigma^2)]$$

* lose 1 $-\frac{n}{2}\ln(2\pi)$ b/c its a constant.

- as in OLS; obs are constant and peram are variablse to be maximized
- when system being estimated is linear → estimates b/w OLS + ML are same; but if ui not normal ML more efficient

- $\hat{\sigma}^2 = S^2 = \Sigma u_i^2/n$ → BIAS but CONSISTANT
- under general ML conditions peramerts are consistant + asymtotically most efficient as long as Model specification (and thus likel. funct) are correct

- Minus the expected value of the matrix of 2nd deriv of LLF wRT K+1 Peram eval @ LLF maximiz values is known as the enformaton matrix

- the enverse of enform. matrix is consistent est for covar. matrix
  - ↳ thus Diag. = statist sound est for var of estimators: $(B_1,\dots,B_K,\sigma^2)$

$$I = -E\begin{bmatrix} \frac{dLLF^2}{d\sigma^{2^2}} & \frac{dLLF^2}{d\sigma^2 dB_2} & \rightarrow & \frac{dLLF^2}{d\sigma^2 dB_K} \\ \frac{dLLF^2}{dB_2\sigma^2} & \frac{dLLF^2}{d^2B_2} & & \\ \downarrow & & \ddots & \\ dLLF/dB_K d\sigma^2 & & & \frac{dLLF^2}{d^2B_K} \end{bmatrix} \rightarrow I^{-1} = \text{covar matrix} \rightarrow \sqrt{\text{cov diag}} = Se$$

* use Numerics of not iid stand N

Quiz Nov 8-10

Max Liklihood:

Max Liklihood est:

→ Special programs developed to find param val that max via Numerics.

- have start values + will num est.

Liklihood Ratio tests:

- Hypo tests for ML → se (w/o Hessian) may be unreliable)
- Ho: restricted
- Ha: unrestrict
- LRTS = 2 (Max unrest log like func - Max Restricted LLF)
  - ↳ negs not an issue as long as dif is (+)
  - ↳ distrib $\chi^2_{(g)}$ → # restrictions
- if calc val exceeds χ^2 value @ desired stat level → reject Ho in favor Ha → @ least 1 restriction not justifiable
- Max UR LLF ≈ Max RLLF implies liklihood for either is ~ same
  - ↳ restricted model nearly adiquite
- Linear Normal model; F test is good
- Ratio tests for when using more general models
- F test & Ratio are asymptotically equiv.
- w/ OLS model w/ Low n; f test more prompt to reject
- Also legrange multiplier test (asymtot equiv test)

Dont give up the ship

Qualitative Choice Model:

Intro: Models where the dep var takes qualitative measure

Binary-choice model:

- used when depend var takes one of two mutually exclusive values
- used to quantify impact of dif factors on probabil. of dep var taking one value over the other
- used to predict the prob. that the depend var is in one catig. or other given set of values taken by expl var

Linear probab. Model:

- most elementary binary model: $Y = XB + U$
  - → U is indep distrib rv w/ 0 mean BUT Dont assume Normal
  - → linear prob model is est by ols → yeilds inefficient param est. + bias se
- Y takes prob → $E[Y_i] = 1(P_i) + 0(1-P_i) = P_i$ **

  $E[U_i] = 0$ → $P_i = E[Y_i] = E[XB]$ → Disadvantage → can take numbers outside of 0-1

- Y is Bernoulli r.v.
- $X_i\hat{B} = \hat{P}_i$

Quiz Nov 8-10

qualitative models

Linear prob model

- $P_i = \begin{cases} x_i\hat{B} & \text{when } 0 < x_i\hat{B} < 1 \\ 1 & \text{when } x_i\hat{B} \geq 1 \\ 0 & \text{when } x_i\hat{B} \leq 0 \end{cases}$

- u_i is shown to be heteroskedastic
  ↳ lower var when P_i is close to 0 or 1 + highest when .5
- Models predictions are bias

Logit + Probit models:

- Predicted probs are unbias (w/ respect model) and in 0-1 interval
- Requires $\underbrace{E[y_i]}_{P_i}$ expressed as non linear function of x_iB, specifically a cumulative dens fcn (cdf)
- cdf only takes values b/w 0 + 1 regardless of value of argument ($x_i\hat{B}$)
- use stand Normal CDF (Probit) or logistic CDF (LOGIT)
- Probit:

$$P_i = NCDF(x_iB) = \left(\frac{1}{\sqrt{2\pi}}\right)\int_{-\infty}^{x_iB} e^{-z^2/2}\,dz$$

*z is Normal random var w/ 0 mean + unit var

- as x_iB ↑ prob y takes value of 1 (P_i) also ↑

- Logit:

$$P_i = LCDF(x_iB) = 1/(1 - e^{-x_iB})$$

- same as probit but fatter tail → also simpler math

- Best est for either is ML method
- Both: prob y takes val of 1 is P_i + 0 is $(1-P_i)$
- Pdf for given obs:

$$P_i^{y}(1-P_i)^{1-y_i}$$

- Liklihood joint:

$$\prod_{i=1}^{n} P_i^{y_i}(1-P_i)^{1-y_i}$$ also joint PDF

- LLF to max:

$$\Sigma[y_i \ln(P_i) + (1-y_i)\ln(1-P_i)]$$

Quiz Thurs 15 → Thurs 1

Reduced Form Model:

reduced form model: consists of 1 equation for each endog var in structural model, which is only a function of the structural models param + predetermined var.

$P_t = \Pi_{11} + \Pi_{12} Y_t + \Pi_{13} P_{t-1} + v_t$

$Q_t = \Pi_{21} + \Pi_{22} Y_t + \Pi_{23} P_{t-1} + v_{2t}$

* reduced form model param are f(·) of structural model param + vice versa

IDentification Problem:

*: Reduced form param can be consistently est w/ OLS

↳ sometimes not possible to calc αs + β from Π

→ this estimation method is called INDIRECT LEAST SQUARES

→ exactly identified only if a single set of param val for that structural model

↳ over IDed if more than one; not IDed if ∅

$$\left.\begin{aligned} Q_t &= \alpha_1 + \alpha_2 P_t + \varepsilon_t \\ Q_t &= \beta_1 + \beta_2 P_t + v_t \end{aligned}\right\} \text{cant ID}$$

↳ end up w/ 2Π + 4 param :(

- w/ previous model → p^* + Q^* remain the same through time → only thing shifting is error → More info needed

- w/ original model: y_t Δ D → ID supply
  P_{t-1} Δ S → ID demand

*But add another struct. param to supply? Demand over ID → vice versa
ie add wealth to demand: 7β + 8Π

ORDER CONDITION: for an equation to be identified → the # of predetermined var excluded from the equation must be equal ~~to~~ or greater than the # of exog var included in the Rt-side of structure eq.

- if = → soln IDed
- if greater: over ID but fixable

- order condition is necessary but not sufficient → possible in large systems to be satisfied but no ID
  → use Rank condition if this occurs.

Consistant param Estimation:

- Problem w/ ILS → if over ID have to not use some info :(
- use instrumental var instead:

$B_{IV} = (Z'X)^{-1} Z' Y$ Z is nxk of instrumental var ~~[illegible]~~ corr w/ endog var + not w/ error

* Z can be based on X only w/ endog replaced

- can use predeter var in system but not in equation w/ IV
  → corr w/ endog

* if over IDed → multiple choices for Z → diff param est. → IV + ILS same if IDed

Quiz: Thurs 15 → Dec

Qualitive choice Models:

Multipul choice Models:

- Some ~~choices~~ qual choice ex. the dep var can rep 2 or more mutually exclusive choices
- indiv. face a set of more than 2 choices
- simple multipul choice → unranked alts. → types of irrigation systems
- there are multipul choice (multinomial) versions of Probit & logit
- more sophisticated multipul choice: ordered probit / logit & Poisson count Regressions
  - depend var alt have clear rank. → crop insurance levels
- Poisson Regressions: count var → ordered & almost numerical
  → negitive binomial regresson when counts are correlated
- Tobit model: when dep var is continuous but truncated (Grades)
  - higher the truncation less ols works.

Intro to multiple Equation models

Intro:

- Single equation model: assumes no feed back
- multipul simultaneous equation models account for interrelations w/in set of jointly determined dep var as well as exog var.

* Model: a set of equations stating a known interrelation b/w endogenous var; however each equation can be estimated seperatly

ex: Mkt supply/demand:

Simultanious systems

- $Q_t^S = \alpha_1 + \alpha_2 P_t + \alpha_3 P_{t-1} + \varepsilon_t$
- $Q_t^D = B_1 + B_2 P_t + B_3 Y_t + U_t$
- $Q_t^S = Q_t^D = Q_t^*$

– endog var: values determined in the system

– predetermined var: help cause movement of endog var w/in syst

[* P_{t-1} → lagged endog var → not really exog var (pre determined)

* Y_t is exog → value is determined outside system]

- The endogenity of P_t & Q_t → graphs are easier but Δ en ε_t or U_t would affect both Q_t & P_t

- B/c of this applying ols to est demand/supply = bias & inconsist param est
- can be est w/. instrumental variables, 2 & 3 stage least squares, max. likihood
  → these get you consistant est.
- IV & 2SLS are single eq. est models: applied in any instance the Rt hand var is corr w/ error → over comes measurment error

Re[illegible]ed form:

- Previous model is structural model → endog var on left & (if simultaneous) endog & exog var on Rt side

  * – equations in a simultaneous ~~model~~ structural model can be solved for each endog var as a f(·) of predeterned var only *

Quiz Dec 1

2 Stage Least Squares:

- used when system over IDed w/out loss of info

① est Π → OLS

② obtain est for $\hat{Q}^*$ + $\hat{P}^*$ → predict endog var

③ replace observed endog var w/ predicted from step 2

④ est structural param w/ OLS

* in large sample size should be indep of error → consistent est.

→ IV, ILS, 2SLS same if exact ID but 2SLS better if over ID

- all 3 impossible if not ID
  ↳ endog var predicton will be perfectly corrl w/ predeterman var in unIDed equation
- * remember some equations in syst can be IDed where others are not.

Math for Structure to Reduced form:

$$Q_t^S = Q_t^D$$

$$\alpha_1 + \alpha_2 P_t + \alpha_3 P_{t-1} + \varepsilon_t = B_1 + B_2 P_t + B_3 y_t + v_t$$

$$\alpha_2 P_t - B_2 P_t = (B_1 - \alpha_1) + B_3 y_t - \alpha_3 P_{t-1} + (v_t - \varepsilon_t)$$

$$\cancel{(\alpha_2 - B_2)} P_t = \underbrace{\frac{(B_1 - \alpha_1)}{(\alpha_2 - B_2)}}_{\Pi_{11}} + \underbrace{\frac{B_3}{(\alpha_2 - B_2)}}_{\Pi_{12}} y_t - \underbrace{\frac{\alpha_3}{\alpha_2 - B_2}}_{\Pi_{13}} P_{t-1} + \underbrace{\frac{v_t - \varepsilon_t}{(\alpha_2 - B_2)}}_{v_{1t}}$$

Final write out: Econometrics

INTRO

Regression analysis: used when in particular var + how it is affected by other variables

→ Basis for econometrics

REGRESSION MODEL:

- Regression analysis: starts by conceptualizing a behavioral relation based on economic theories or reasoning
  - → Behavioral Relation includes: indep + dep var:

$$Y = B_1 + B_2 X_2 + B_3 X_3 + B_4 X_4 + u$$

  - → Bs will be estimated
  - → u (error) takes into account other factors that affect y
    - Rel unemp explan var
    - random measurement error in y
    - chance
  - → Model seeks to capture essentials of economic process under review → not a perfect representation
  - → Main uses:
    - est magnitude of effects
    - obtain predictions for y

Population v. Estimated:

- Systematic: y, B, x
- random: u
- B is unknown but can find $\hat{B}$

ORDINARY LEAST SQUARES:

- Min Resid. sum of squares
- Squared so as to penilize larger distance
- Min unexpectedness

Min RSS: $\text{Min} \sum \hat{u}_i^2$

$\text{Min} \sum (y_i - \hat{y})^2$

$\text{Min} \sum (Y_i - \hat{B}_1 - \hat{B}_2 x_i)^2$

* See side Page for written out math formulas

- to find: partial derives:

$$dRSS/d\hat{B}_1 = \sum Y_i - \hat{B}_1(n) - \hat{B}_2(\sum x_i) = 0$$

$$dRSS/d\hat{B}_2 = \sum Y_i x_i - \hat{B}_1(\sum x_i) - \hat{B}_2(\sum x_i^2) = 0$$

- Mathmatical properties of OLS model
  - $\sum \hat{u}_i = 0$
  - $\sum \hat{Y}_i = \sum Y_i$
  - always passes through $(\bar{Y}, \bar{x})$
  - $\hat{u}_i$ uncorrl w/ indep var + dep predictions $(\hat{y}_i)$

OLS: ordinary least squares

Overview:

→ Wants to min the square residuals → ie the smallest amount of unexplained variaton from the mean.

$$\left[\begin{aligned} \text{Min RSS} &= \min \sum_{1}^{n} \hat{u}_i^2 \\ &= \min \sum (y_i - \hat{y}_i)^2 \\ &= \min \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i)^2 \end{aligned}\right] \text{ Math behind}$$

Requirements:

→ Mathmatical properties
- $\sum \hat{u}_i = 0$
- $\sum \hat{y}_i = \sum y_i$
- always passes through pt $(\bar{y}, \bar{x})$
- residuals not correlated w/ ~~[illegible]~~ indep var; + dep variabls

→ OLS Requirements
- OLS can only be used on models linear in perameters
- # obs > # k → 4 obs: every k
- some level of variability in sample values of explan variables
- can't be perfect multicollinearity

→ OLS Assumptions
- values taken by explanitory variables are fixed in repeated sampling or if random; are not correl w/ error term
- expected value of $u = \emptyset$
- no auto correlaton
- no heteroskedasticity

Process:

- $RSS = \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 x_{i2} - \ldots - \hat{\beta}_k x_{ik})^2$

$$\left.\begin{aligned} \frac{dRSS}{d\beta_1} &= 0 \\ \frac{dRSS}{d\beta_2} &= 0 \\ &\vdots \\ \frac{dRSS}{d\beta_k} &= 0 \end{aligned}\right\} \text{should be solved simultaneously}$$

Other Math

- $\hat{\beta} = (x'x)^{-1} x'y$
- $v[x_i] \cdot E[(x_1 - E[x_1])]$

  $\hat{\sigma}_1^2 = \dfrac{\sum (x_{i1} - \bar{x}_1)^2}{n-1}$
- $covar = E[(x_1 - E(x_1))(x_2 - E[x_2])]$
- $\hat{\sigma}_{12}^2 = \dfrac{\sum (x_{i1} - \bar{x}_1)(x_{i2} - \bar{x}_2)}{n-1}$
- $se = \sqrt{var}$
- $var[\hat{\beta}] = \sigma^2 (x'x)^{-1}$

Misc:

- errors must be iid (Assump 3/4) or no se (correct ones @ least) → will be bias

OLS

· Technical Requirements:

- · OLS only used to est models linear in peramenters
- · obs # > # peram → 4 obs / peram
- · some level variation in explan variables
- · No perfect multicollin

OLS Assumptions:

- · values of explan var fixed in repeated sampling or; if random not corrl w/error
  - ~~if happens:~~ possible to occur if:
    - · explan var is proxy or measured w/error
    - · dep var has feedback
- · expected value of error is ∅: this requires:
  - · No key element left out
  - · Functional form correcty specified

unBias:

~~$E[\hat{B}|x]$~~ $\lim_{x \to \infty} \text{Prob}\left(|B-\hat{B}| > \gamma\right) = \emptyset$

- · No autocorrl: errors indep distributed → error from dift obs uncorrl.
- · No Heteroskidasticity → error term vare. are same regardless of # taken by var.

* Math for unbiasand consistent:

$\hat{B} = (x'x)^{-1}x'y$

$\hat{B} = (x'x)^{-1}x'(xB+U)$

$= \underbrace{(x'x)^{-1}x'x}_{I}B + (x'x)^{-1}x'U$

$E[\hat{B}] = E[B] + \underbrace{(x'x)^{-1}x'}E[U]$

const. not incld. in E[ ]

should = ∅

$E[\hat{B}] = B$

Covariance Matrix:

- · $V[x_i] = E[(x_i - \overset{\mu_i}{E[x_i]})^2] = \sigma_i^2 \sim \hat{\sigma}_i^2 = \frac{\Sigma(x_{i1} - \bar{x}_i)^2}{n-1}$
- · $\text{covar}(x_1, x_2) = E[(x_i - E[x_i])(x_2 - E[x_2])]$
  $\hookrightarrow \hat{\sigma}_{12}^2 = \frac{\Sigma(x_{11}-\bar{x}_1)(x_{12}-\bar{x}_2)}{(n-1)}$
- · $V[\omega_1] = \sigma^2 I$ if error iid

Standard errors:

- · B is random var b/c relies on other random var → x
- · $Se_i = \sqrt{V[x_i]}$
- · each est has SE → measure of how precise OLS is
- · true peram w/in ± 2SE
- · $V[\hat{B}_2] = \hat{\sigma}^2 / \left\{(1-r_{23}^2)\sum_i(x_{2i}-\bar{x}_2)^2\right\}$

OLS

Properties of OLS estimators:

- unbias: on avg est expected to equal the values of unknown coef (pop)
- unbias requires assump 1&2
- 1-4 hold: est has min variance among all possible unbias est that are LINEAR f(·) of y
  - ↳ OLS most likely to yeild est for B that is close to actual param
  - BLUE → Best linear unbias est.

Measures of Goodness of fit:

- Blue ≠ How well can best fit model predict y
- use R^2
- R^2
  - concept based on notion that each Y obs can be decomposed into total, explained, + residual variation
    - Total: $(Y_i - \bar{y})^2 \rightarrow TSS = \Sigma(y_i - \bar{y})^2$
    - explain: $(\hat{y}_i - \bar{y})^2 \rightarrow ESS = \Sigma(\hat{y}_i - \bar{y})^2$
    - resid: $(y_i - \hat{y}_i)^2 \rightarrow RSS = \Sigma(y_i - \hat{y}_i)^2$
    - TSS = RSS + ESS

MATH:

TSS = RSS + ESS

$$\Sigma(y_i - \bar{y})^2 = \Sigma[(\hat{y}_i - \bar{y}) + (y_i - \hat{y}_i)^2]$$

$$= \Sigma[(\hat{y}_i - \bar{y})^2 + (y_i - \hat{y}_i)^2 + 2(\hat{y}_i - \bar{y})(y_i - \hat{y})]$$

$$= \underbrace{\Sigma(\hat{y}_i - \bar{y})^2}_{ESS} + \underbrace{\Sigma(y_i - \hat{y})^2}_{RSS} + \underbrace{2\Sigma(\hat{y} - \bar{y})(y_i - \hat{y})}_{\text{Has to} = 0}$$

$$= 2\Sigma(\hat{y} - \bar{y})(y_i - \hat{y}_i)$$

$$= \Sigma\hat{y}_i y_i - \Sigma\hat{y}_i^2 - \Sigma\bar{y}y_i - \Sigma\bar{y}\hat{y}_i \rightarrow 0$$

$$= \Sigma\hat{y}_i(\hat{y}_i + u) - \Sigma\hat{y}_i^2 - \bar{y}[\Sigma(\cancel{y_i - \hat{y}_i})]$$

$$= \cancel{\Sigma\hat{y}_i^2} + \underbrace{\Sigma u_i\hat{y}_i}_{= 0 \text{ if uncorrl}} - \cancel{\Sigma\hat{y}_i^2}$$

$R^2 = \Sigma(\hat{Y}_i - \bar{y})^2 / \Sigma(y_i - \bar{y})^2 \rightarrow \frac{ESS}{TSS}$ → Measures proportion of total variation explained

OR

$= 1 - (RSS/TSS)$ → measures total variation not explained

- R^2 measures portion of total y variation explained by model
- is a ratio → NO UNITS
- $R^2 = 0$ = no fit
- $R^2 = 1$ → perfect fit → higher R^2 better fit
- time series data: R^2 80 or ↑, cross sectional 50 or ↑
- $R2$ is measure of models capacity to predict y → if low R^2 precise param est serve other purpous.

* careful: addition of x will always ↓ R^2

Measures of Best fit

Adj R^2: $\bar{R}^2$

- used to asses if addition of indep var likely to ↑ predictions of y
  - $\bar{R}^2 = 1 - [(RSS/TSS)([n-1)/[n-k])]$
    - always less than R^2 unless $k=1$ or $R^2=1$
    - penilizes more ks.

Warning:

- if var has direct noatable effect on y → add!
  - ↳ exclusion = bias/incosist for other param
- irrel var = less serious than missing ones.

Normal error models:

Normal-Error model → additional error-term assumptions:

- $Cov[U, X_j] = 0$
- $E[U_i] = 0$
- $Var[U_i] = \sigma^2$
- $cov[U_i, U_j] = 0 \quad i \neq j$
- error assumed Normal
- $E[\hat{B}] = B$; $Plim[\hat{B}] = B$
- $var[\hat{B}] = \sigma^2(x'x)^{-1}$
- if all OLS assum hold → error norm distrib
- $\hat{B} \sim N(B, \sigma^2(x'x)^{-1}) \to B$ linear $f(\cdot)$ U

confidence intervals:

- $z = (\hat{B}_j - B_j)/se[\hat{B}_j] \to N(0,1)$
  - $\to se[\hat{B}_j] = \sqrt{var[\hat{B}_j]}$
- $t = (\hat{B}_j - B_j)/\hat{se}[\hat{B}_j] \sim t(n-k)$
- ci:

$$Pr(t_{(\alpha/2, n-k)} < t < t_{(\alpha/n-k)}) = 1-\alpha$$

so

$$Pr(\hat{B}_j - t_{(\alpha/2, n-k)} \cdot \hat{se}[\hat{B}_j] < B_j < \hat{B}_j + t_{(\alpha/2, n-k)} \cdot \hat{se}[\hat{B}_j]$$

Hypothesis tests: T test

- do t test w/ $B = C \to t = (\hat{B}_j - C/\hat{se}[\hat{B}_j]$
  - $H_0: B_j = C$
  - $H_a: B_j \neq C$
- $|t| < t^*$ cannot Reject ; $|t| > t^*$ Reject
- OLS must hold! + error ~N or large N
- can test if B statistcly Δ from 0 → $H_0: B_j = 0$ $H_a: B_j \neq 0$
- 2 tail v. 1 tail does not Δ t stat but Δ α → values 1/2 α

ci for y_0:

$\hat{y}_0 \pm t \, \hat{se}[\hat{U}_0]$

Tests

R^2

side math

$ESS = \sum(\hat{y}_i - \bar{y})^2$

$TSS = \sum(y_i - \bar{y})^2$

$RSS = \sum(y_i - \hat{y}_i)^2$

- Tests for how well est regression fits y data

$R^2 = \sum(\hat{y}_i - \bar{y})^2 / \sum(y_i - \bar{y})^2 \rightarrow ESS/TSS$

OR

$= 1 - (RSS/TSS)$

- Ratio; proportion of total variation on Y that is explained; higher the better
- Time series >.8 ; crosssectional >.5

Adj R^2

- assess if adding indep var likely to ↑ ability to predict y

$\bar{R}^2 = 1 - [(RSS/TSS)([n-1]/[n-k])]$

confidence interval

- $z = (\hat{B}_j - B_j)/se[B_j] \sim N(0,1) \rightarrow$

$t = (\hat{B}_j - B_j)/\hat{se}[\hat{B}_j] \sim t(n-k)$

$Pr(\hat{B}_j - t_{(\alpha/2, n-k)} \cdot \hat{se}[\hat{B}_j] < B_j < \hat{B}_j + t_{(\alpha/2, n-k)} \cdot \hat{se}[\hat{B}_j]) = 1 - \alpha$

- confident B b/w x + y (get %)

Hypo tests:

t test:

- $t = (\hat{B}_j - B_j)/\hat{s}[\hat{B}_j] \sim t(n-k)$
- can test how likely a bj is to come from t distrib
  - Ho: $B_j = c$; Ha: $B_j \neq$
  - find t^* from table $\rightarrow |t| > t^*$ reject

* usually used to test if Bj statistically Δ from Ø

Ci y_0:

$\hat{y}_0 \pm t_{(n-k)} \hat{se}[\hat{v}_0]$

$E[y_0]$ " " $\hat{se}[\hat{y}_0]$

F test:

- $F^* = (ESS/k-1) / (RSS/N-K)$
- tests joint effects. → Ho: $\cancel{B_1} = B_2 = B_3 \cdots = B_K = Ø$

Ha: @ least one not = Ø

* F test impervious to multicolin

Joint F (several coef)

$F^* = [(RSS_R - RSS_{UR})/q] / [(RSS_{UR}/(n-k)]$

- tests subgroup → restricted is w/o subgroup
- if Ho correct RSS Δ small

F test (linear functions of Regression coef)

- $F^* = [(RSS_R - RSS_{UR})/q] / [(RSS_{UR})/(n-k)]$
- test cets; Ho: $B_2 - B_3 = 0$ etc

same coef

GLS: Generalized least squares

overview:

- general method to est regressions when error not iid
- $var[U] = \sigma^2\Psi \rightarrow \Psi$ is a pos symetric ~~[illegible]~~ difinite matrix

Requirements:

- same a OLS but U can be not iid

Process:

- $var[U] = \sigma^2\Psi \neq \sigma^2 I$
- $var[U] = \sigma^2\Psi$; $var[PU] = P\,var[U]P' = \sigma^2 P\Psi P' = \sigma^2 I_n$
- $\Psi = P^{-1}(P')^{-1}$ or $\Psi^{-1} = P'P$
- multiply all by P matrix that makes $U \rightarrow \sigma^2 I$
- $\tilde{B} = (x'P'Px)^{-1}x'P'Py \rightarrow (x'\Psi^{-1}x)^{-1}x'\Psi^{-1}y$
- $V[\hat{B}] = \sigma^2(x'P'Px)^{-1} = \sigma^2(x'\Psi^{-1}x)^{-1}$

EGLS:

overview → use est of ρ ($\hat{\rho}$) to impliment
→ aprox GLS estimator

Requirements: same as above

Process:

- $\hat{\rho} = \sum_{1}^{T} \hat{U}_t \hat{U}_{t-1} / RSS$

  or

- Regress $\hat{U}_t$ on $\hat{U}_{t-1}$ + use perm est for ρ

  or

- $\hat{\rho} = 1 - .5d^*$

  ↳ calc durbin-watson stat

- est by OLS
- compute resid + obtain 1st est of ρ
- obtain perm est using EGLS
- recompute resid + obtain ρ
- cont until ρ stops Δ

* use ρ like P matrix

NLS (Non Linear Least)

overview:

used Numerical search

Requirement: See above

Process: ρ est by brute force → one w/ lowest RSS selected

Maximum likelihood:

overview: use of Pdfs:

Require: Pdfs and probabilities

Pdf for given obs → $P_i^{Y_i}(1-P_i)^{1-y_i}$

→ likli hood $f(\cdot) \rightarrow \prod P_i^{y_i}(1-P_i)^{1-y_i}$

→ likely hood $f(\cdot)$ to be maximized → $\sum[Y_i \ln(P_i) + (1-Y_i)\ln(1-P_i)]$

→

Corrections for issues:

Multicollinearity:

- Not issue just unfortunate unless perfect/severe
- Fixes:
  - Just check dummies → leave one out?
  - indiv var is const?
- Why issue: cant get param or se → or great enterference

Symptom:

- key variable not stat signif.
- High R^2, ~~low~~ good f test; low ts.
- Param est Δ ALOT w/ loss of some indep var.

Test → VIF

Fixes: sometimes don't need to; just be aware; help justify why left varables in

- More obs/new sample
- exclude prob. IV [Remember issues w/ exclud import IV]
- Δ Model specification

Heteroskidic

- param still unbias but SE bias → screws all stat. test/ci
- Param no longer most efficient

Tests: White/B/W

- Fixes:
  - use WLS of one variable
  - use AR predictions for se → w/ $\hat{u}$ as dep var → proportional to σ_i^2
  - use GLS
  - Fix just error:
    Heterosked consist se → $(x'x)^{-1}x'\Omega x(x'x)^{-1}$

Auto corrl

- error no longer indep → past error has effect
- $U_t = \rho U_{t-1} + V_t$
- Param not most efficient; se ~~not~~ bias ∴, F test shot as well
- Fixes: transform orig. regression eq w/ autoregressive error → so can use OLS
  - ↳ multiply through by ρ
  - → EGLS

Log-Lin

$\ln y = B_1 + B_2X_2 + \ldots + B_kX_k$

↳ $B \times 100 \to \% \Delta y$ for unit Δx

Lin Log

$Y = B_1 + B_2 \ln x_2 + \ldots + B_k \ln_k$

↳ $B \times 100 \to$ unit Δy for every $\% \Delta x$

Reciprocal Specification:

$Y = B_1 + B_2(1/x_2) + \ldots + B_k(1/x_k)$

↳ slope is opsign of B

Log Reciprocal:

$\ln y = B_1 + B_2(1/x_2) + \ldots + B_k(1/x_k)$

↳ S shape

Polynomial:

$\hat{Y}_i = \hat{B}_1 + \hat{B}_{21}X_{i2} + \hat{B}_{22}X_2^2 + \ldots$

↳ can possibly employ reversal of relation eventually

↳ can have multical prob

dummy variables:

$Y = B_1 + D_{12}X_2 + B_{13}X_3 + B_{14}X_4 + B_5X_5$ (intercept)

or

$B_{13}X_3X_5$ (slope)

↳ allows for alt slop/intercept for various qualitative inputs

AR(1) / MA(1) / ARMA

$U_t = \rho U_{t-1} + V_t$

↳ errors corr (AR)

↳ MA → relation b/w V_t

↳ ARMA → has both

Binary choice Model:

—Looks same @ Reg → dift interp.

↳ quant impact of diff factors on probab that dep var takes 1 val over another

Linear Probab model:

→ type of binary choice

↳ Probabilities

↳ may have to truncate

Probit + Logit

logit: $P_i = 1/(1-e^{-x_iB})$ Probit: $P_i = (1/\sqrt{2\pi})\int_{-\infty}^{x_iB} e^{-z^2/2}\,dz$

↳ uses CDF

Multinomial choice

↳ more than 2 mutually exclusive models

Tobit

↳ truncated ends

Mutiple equations:

simultaneous → D/S $f(\cdot)$

↳ 2SLS

IVS

IV