AFRE 802
Statistical Methods for Agricultural, Food, & Resource Economists

Hypothesis Testing – Part 3 of 3
(WMS Ch. 10.4, 10.10, 10.12)
November 16, 2017

Nicole Mason
Michigan State University
Fall 2017

GAME PLAN

- Housekeeping issues
  - Ch. 10 HW due Tuesday, 11/28
- Review
- Graded in-class exercise
- Hypothesis testing – Part 3 of 3
  - Hypothesis testing with χ^2 and F statistics
  - Calculating Type II error probabilities and finding the sample size for Z tests
  - The "power" of statistical tests
  - Wrap-up of Chapter 10

1

Review: Hypothesis testing

- Steps essentially the same for large sample hypothesis testing and small sample hypothesis testing about μ but *key difference* is:
  - **Large sample tests: can invoke CLT & use Z-stat ~ N(0,1)**
  - **Small sample tests for μ: use T-stat ~ t with N-1 d.f.; data need to be from approximately normal distribution**

Review: Relationship b/w CIs & hypothesis testing

- **If testing H_0: $\theta=\theta_0$ vs. H_1: $\theta\neq\theta_0$**
  - **Fail to reject** H_0 in favor of H_1 at the α level **if θ_0 lies inside the 100(1-α)% two-sided CI; o.w. reject H_0**
- **If testing H_0: $\theta=\theta_0$ vs. H_1: $\theta>\theta_0$ (implicitly H_0: $\theta\leq\theta_0$)**
  - **Fail to reject** H_0 in favor of H_1 at the α level **if θ_0 lies inside the 100(1-α)% lower one-sided CI; o.w. reject H_0**
- **If testing H_0: $\theta=\theta_0$ vs. H_1: $\theta<\theta_0$ (implicitly H_0: $\theta\geq\theta_0$)**
  - **Fail to reject** H_0 in favor of H_1 at the α level **if θ_0 lies inside the 100(1-α)% upper one-sided CI; o.w. reject H_0**

SEE HANDOUT FROM LAST CLASS FOR DETAILS

Review: p-values

- *Definition?*
  - **p-value** = the smallest α for which the data suggest the null hypothesis should be rejected (in favor of the alternative)
  - The probability of observing a test "statistic as extreme as we did if the null hypothesis is true" (Wooldridge 2003, p. 129)
- *Which is better if want to reject H_0 – small or large p-value?*
  - The smaller the p-value, the stronger is the evidence against the null (in favor of the alternative)
- *How to find the p-value for a test statistic?*
  - Follow the usual hypothesis testing steps but rather than picking α and identifying the rejection region, determine the significance level of your test statistic (keeping the alternative hypothesis in mind and thus whether you are dealing with an "α" or "α/2" situation)
  - EX) **2-sided alternative** and Z-stat: **p=2*P(z > |Z-stat|)**
  - EX) **1-sided alternative** and Z-stat: **p=P(z > |Z-stat|)**
  - Similar for T (with appropriate D.F.) because also symmetric
- *Suppose you are conducting a hypothesis test at a given α level. What do you conclude if p≤α? What if p>α?*

Graded in-class exercise

Hypothesis testing with χ^2 and F statistics

- We've now worked a lot with ***Z* and *t* statistics** but we don't have enough time in this course to do more hypothesis testing that involves **χ^2 and *F* statistics**
- But you'll encounter these more next semester & beyond
- **Examples of hypothesis tests with test statistics ~ χ^2**
  - Testing hypotheses about the **variance of one normal RV**
  - The Jarque-Bera **test for normality**
  - Ljung-Box Q **test for autocorrelation**
  - **Likelihood ratio tests** (hypothesis testing for MLE)
- **Applications of *F* distributions**
  - Testing hypotheses about the **variances of two normal RVs**
  - **Joint hypothesis testing in regression analysis**, e.g.,

$$y=\beta_0+\beta_1 x_1+\beta_2 x_2+u$$

$$H_0: \beta_1=\beta_2=0 \quad \text{vs.} \quad H_1: \beta_1\neq 0 \text{ and/or } \beta_2\neq 0$$

Calculating the probability of Type II error (β)

- Very difficult for some statistical tests but pretty straightforward for the large-sample tests we covered (Z-stat-based)
- *Review: what is type II error?*
  - **Failing to reject H_0 (in favor of H_1) when H_0 is false**
- When calculating β=P(Type II error), must do so for **specific values** of the target parameter **under H_1**
  - E.g., if testing H_0: μ=5 vs. H_1: μ > 5, need to pick a specific value of μ > 5 (e.g., 6, 100, whatever)
- Let's work through an example then go over some general rules for finding β=P(Type II error)

6

Calculating the probability of Type II error (β)

Example 10.8 in WMS

Suppose we tested H_0: μ=15 vs. H_1: μ > 15 at the α=0.05 level using data from a random sample of size N=36 with sample mean 17 and sample standard deviation 3. (Context is the average # of calls/week made by salespeople at a large corporation.)

a. *We obtain Z=4. $z_{\alpha=0.05}$ = 1.645 so do we reject or fail to reject H_0 in favor of H_1? What is the p-value for our test?*

b. Now suppose we want to know β=P(Type II error) for testing H_0: μ=15 vs. H_1: μ = 16 given α=0.05.

Steps:

1. Find the cutoff for the RR in terms of Z (**under H_0** and for the **given α**), then express it in terms of $\bar{Y}$. Let k be this cutoff value for $\bar{Y}$.
2. $P(\text{Type II error}) = P(\text{fail to reject } H_0 \text{ in favor of } H_1 \text{ when } H_0 \text{ is false and } H_1 \text{ is true})$

$$= P(\bar{Y} \text{ is } \underline{\text{not}} \text{ in the rejection region when } H_0 \text{ is false and } H_1 \text{ is true})$$

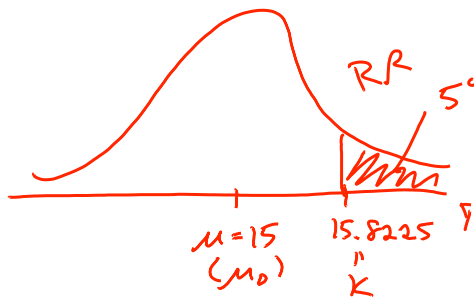
$$= P(\bar{Y} \leq k \text{ when } H_0 \text{ is false and } H_1 \text{ is true})$$

$\Rightarrow$ Find this probability by converting $\bar{Y} = k$ to a Z-statistic **under H_1**.

Step 1: Find the cutoff in terms of Z under H_0, then express in terms of $\bar{Y}$ (and call this "K")

$$Z_\alpha \text{ or } Z_{cutoff} = 1.645 = \frac{\bar{Y} - \mu_0}{\sigma/\sqrt{n}}$$

Solve for $\bar{Y}_{cutoff} = 15.8225 \equiv K$



8

Step 2: Find P(Type II error)

= P($\bar{Y}$ is not in the RR when H_0 is false and H_1 is true)

= P($\bar{Y} \leq 15.8225$ when $\mu = \mu_1 = 16$)
("K")

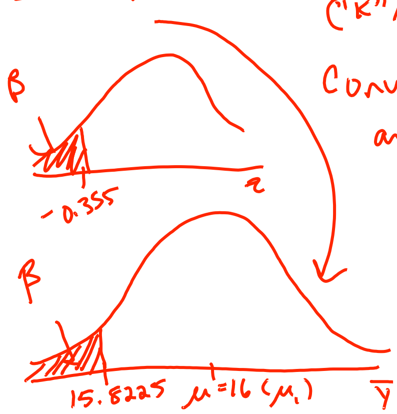
Convert to Z under H_1 and find the probability.

$$Z = \frac{\overset{\text{"K"}}{\bar{Y}_{cutoff}} - \mu_1}{\sigma/\sqrt{n}} = \frac{15.8225 - 16}{3/\sqrt{36}}$$

$= -0.355$

$P(Z \leq -0.355)$

$= 0.3613 = \beta$



9

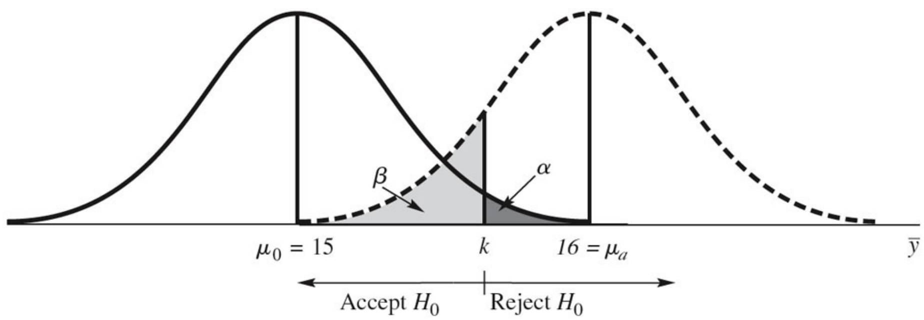
Example 10.8 (cont'd)

α=P(Type I error)=P(reject H_0 when H_0 is true)

β=P(Type II error)=P(fail to reject H_0 when H_0 is false and H_1 is true given specific values of H_1 and α)

$= P(\bar{Y}$ is not in the rejection region when H_0 is false and H_1 is true)

$= P(\bar{Y} \leq k$ when H_0 is false and H_1 is true))



© Cengage Learning

k=15.8225
is the cutoff for the sample mean for our rejection region for H_0: μ=15 vs. H_1: μ > 15 (specifically μ = 16) at the α=0.05 level. That is, we reject H_0 in favor of H_1 if the sample mean is ≥ 15.8225.

10

Calculating β=P(Type II error)

General approach for H_0: $\theta=\theta_0$ vs. H_1: $\theta>\theta_0$ for a specific value of the target parameter under H_1 (call it θ_1, where $\theta_1>\theta_0$)

1. Find the cutoff for the RR in terms of Z (**under $\mathbf{H_0}$** and for the **given α**), then express it in terms of the estimator, $\hat{\theta}$. Let k be this cutoff value for $\hat{\theta}$, i.e.:

$$\text{RR} = [k, \infty)$$

2. $P(\text{Type II error}) = P(\text{fail to reject H}_0 \text{ in favor of H}_1 \text{ when H}_0 \text{ is false and H}_1 \text{ is true})$

$$= P(\hat{\theta} \text{ is not in the rejection region when H}_0 \text{ is false and H}_1 \text{ is true})$$

$$= P(\hat{\theta} \leq k \text{ when H}_0 \text{ is false and H}_1 \text{ is true, i.e., when } \theta=\theta_1)$$

Find this probability by converting k to a Z-statistic **under $\mathbf{H_1}$**, i.e.:

$$P(z \leq Z = \frac{k-\theta_1}{\sigma_{\hat{\theta}}})$$

Note: Will need to reverse signs in the steps above if H_1: $\theta<\theta_0$

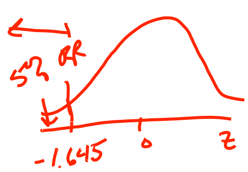
11

Another example

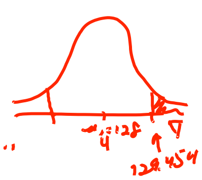
Suppose N=40, sample mean = 128.6, and sample standard deviation is 2.1. Find the probability of type II error for testing H_0: μ=130 vs. H_1: μ = 128 given α=0.05.

$H_0: \mu = 130$, $H_1: \mu < 130$ $(\mu_1 = 128)$

Step1: Find RR in terms of Z, then express in terms of $\bar{Y}$



$Z_{cutoff} = -1.645 = \dfrac{\bar{Y}-\mu_0}{\sigma/\sqrt{n}}$



⇒ Solve for $\bar{Y}$

⇒ $\bar{Y}_{cutoff} = 129.454 \equiv$ "K"

Step 2: Find P(Type II error)

P($\bar{Y}$ is not in RR when H_0 is false & H_1 is true)

P($\bar{Y} \geq K$ (129.454) when $\mu = \mu_1 = 128$)

Convert to Z under H_1: $Z = \dfrac{129.454-128}{2.1/\sqrt{40}} = 4.378$

β is b/w 0.0000312 and 0.00000340

Finding the sample size for Z-tests

- In Example 10.8, with N=36 and α=0.05, we calculated that β=0.36 → high P(Type II error)
- A key way to reduce β is to increase the sample size
- The flip side of determining β given N and α is to determine N given desired values of α and β
- Suppose you want to test H_0: $\mu=\mu_0$ vs. H_1: $\mu>\mu_0$ for given values of α and β (and where β is evaluated at specific value $\mu_1>\mu_0$ under H_1). Then:

Sample size for a one-tailed Z-test for μ for given levels of α, β, μ_0 (value of μ under H_0) and μ_1 (value of μ under H_1):

$$N = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_1 - \mu_0)^2} \text{ rounded up to the nearest whole number}$$

Same formula works for H_1: $\mu<\mu_0$. See WMS p. 509 for proof.

14

Finding the sample size for Z-tests - example

Example 10.9 in WMS

Find the sample size, N, for testing H_0: $\mu=15$ vs. H_1: $\mu=16$ with $\alpha=\beta=0.05$. Assume a variance of 9. (Context is the average # of calls/week made by salespeople at a large corporation.)

Sample size for a one-tailed Z-test for μ for given levels of α, β, μ_0 (value of μ under H_0) and μ_1 (value of μ under H_1):

$$N = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_1 - \mu_0)^2} \text{ rounded up to the nearest whole number}$$

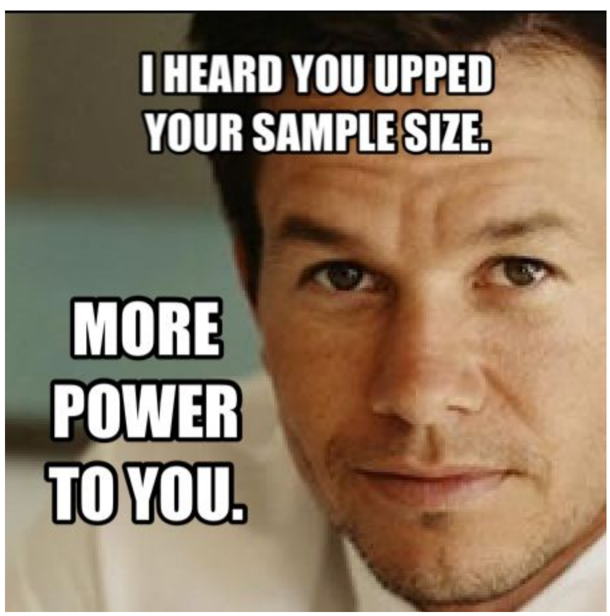
15

MICHIGAN STATE UNIVERSITY

The "power" of statistical tests

- We have discussed β=P(Type II Error) =P(fail to reject H_0 when H_0 is false and H_1 is true)
- The **"power" of a statistical test is 1-β**, i.e., the probability that we do reject H_0 when H_0 is false and H_1 is true. **More power is better than less power!**
  - As with β, the power of a test depends on the parameter value specified under H_1 (θ_1)
- *How does β change as N increases?*
- *So how does power change as N increases?*

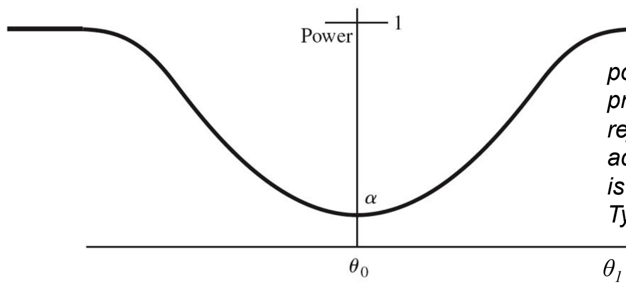
MICHIGAN STATE UNIVERSITY



17

The "power" of statistical tests (cont'd)

- Final note on power:
  - *Do you think statistical tests have more power for parameter values under H_1 (θ_1) that are close to or farther away from the value under the H_0 (θ_0) ? Why?*
  - It is easier to detect that H_0 is false (more power) when θ_1 is **farther** from θ_0



power(θ_0)=α b/c the probability that we reject $\theta=\theta_0$ when it is actually true that $\theta=\theta_0$ is the probability of Type I error (α)

Figure: A typical power curve for the test of $H_0 : \theta=\theta_0$ vs. $H_1 : \theta=\theta_1$ for various values of θ_1

Summary

- In Chapters 8 and 9, we talked about **how to estimate** numerical values of target parameter **θ**
  - Point estimates & confidence intervals (CIs)
  - Desirable properties of estimators (consistency, unbiasedness, efficiency, low MSE)
  - Methods of estimation (MOM, MLE, least squares)
- In Chapter 10, we talked about:
  - Testing hypotheses related to θ for large samples, and for μ for small samples
  - The relationship between hypothesis testing and CIs
  - p-values
  - Probabilities of Type I (α) and Type II (β) errors, and the power of a statistical test (1-β) → these probabilities tell us how 'good' our inferences are (i.e., how much faith we can put in the results of our hypothesis tests)
  - Computing the sample size for Z tests

19

Homework:

- WMS Ch. 10 (cont'd):
  - Type II error probabilities & sample size for Z tests: 10.38, 10.39, 10.41, 10.42
- **All Ch. 10 HW is due on Tuesday, Nov. 28

Remaining lectures – only 5 left – time flies!

- Tuesday: Review, answer your questions; tie up Ch. 10 loose ends
- 4 classes after Thanksgiving break: introduction to OLS (hurray!) and course wrap-up

Reading for Tuesday after break

- Optional: WMS Ch. 11 (sections 11.1-11.3)
- Required: Wooldridge *Introductory Econometrics* (2003) pp. 22-37 – on D2L

20

MICHIGAN STATE UNIVERSITY

Table 8.1 Expected values and standard errors of some common point estimators

Target Parameter θ	Sample Size(s)	Point Estimator $\hat{\theta}$	$E(\hat{\theta})$	Standard Error $\sigma_{\hat{\theta}}$
μ	n	$\overline{Y}$	μ	$\frac{\sigma}{\sqrt{n}}$
p	n	$\hat{p} = \frac{Y}{n}$	p	$\sqrt{\frac{pq}{n}}$
$\mu_1 - \mu_2$	n_1 and n_2	$\overline{Y}_1 - \overline{Y}_2$	$\mu_1 - \mu_2$	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ *†
$p_1 - p_2$	n_1 and n_2	$\hat{p}_1 - \hat{p}_2$	$p_1 - p_2$	$\sqrt{\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}}$ †

Square root of variance of estimator

*σ_1^2 and σ_2^2 are the variances of populations 1 and 2, respectively.

†The two samples are assumed to be independent.

© Cengage Learning