Applied Microeconomics: Firm and Household

Lecture 3: Preferences and Utility

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September 13, 2016

Rationality

The theory of choice is based on the assumption of “rationality”.

- To start an analysis of consumer choices we first specify a basic set of postulates (axioms) that characterize “rational behavior”.
- These axioms impose a structure on individual preferences.

Rationality assumption: If consumer preferences are **complete** and **transitive** then they are **rational**.

Axioms of rational choice

Consider m alternative bundles of n goods denoted as
$x^1 = (x_1^1, ..., x_n^1), ..., x^m = (x_1^m, ..., x_n^m)$,

- Completeness: the consumer can always decide on one of the following mutually exclusive situations:
  - x^i is preferred to x^j
  - x^j is preferred to x^i
  - x^i and x^j are equally preferred (indifference)
- Transitivity: if the consumer prefers x^i to x^j, and prefers x^j to x^k, then she must also prefer x^i to x^k.
  - i.e., individual choices are internally consistent

Additional assumptions

- Continuity: if a consumer prefers a sequence of bundles to x^j the limit of the sequence must also be preferred to x^j. In other words, if the consumer prefers x^i to x^j then bundles suitably close to x^i must also be preferred to x^j.
  - This is a technical assumption that facilitates the use of calculus in analysis of consumer decisions.
- Monotonicity: if x^i includes more of one good and no less of the any other good than x^j, then x^i is preferred to x^j.

Most "violations" of rationality are violations of these ancillary assumptions!

Utility Functions

- If the rationality and continuity assumptions hold, then there exists a utility function, $u = u(x_1, x_2, ..., x_n)$ representing preferences, where x_i refers the quantities of goods.
  - Utility functions allow us to examine consumer behavior using calculus rather than set theory.
  - i.e., we attach numbers to utility rankings that accurately reflect the an individual's original preference ordering and represent the same set of choices.
  - If $u(.)$ is representing preferences, $u(x^i) \geqslant u(x^j)$ if and only if x^i is weakly preferred to x^j

Properties of Utility Functions: Non-uniqueness

Non-uniqueness: Our notion of utility is defined up to an order-preserving (monotonic) transformation. That is, utility is defined as a strictly *ordinal* index of preferences. A cardinal measure of utility does not provide any additional refutable implications beyond what an ordinal measure gives us (usually).

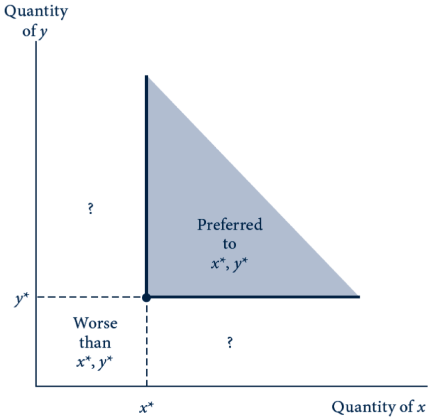
- If $u(.)$ is representing preferences, the monotonic transformations of u, such as $ln(u)$ and u^2, represent the same set of preferences.
- Suppose there are two choices A and B:
  - $U(A) = 2$ and $U(B) = 1$, or
  - $U(A) = 2,000,000$ and $U(B) = 0.0001$.

  In either case the numbers imply that the preference ordering is the same: A is preferred to B.
- Non-uniqueness also implies that it is not possible to compare utilities of different people.

Properties of Utility Functions: Nonsatiation

Nonsatiation: More is always preferred to less.

- Note that this is the same property as monotonicity of preferences.
- In terms of utility functions the implication of this property is that marginal utility is always positive:
  - $MU_x = \frac{\partial u(x)}{\partial x} > 0, \forall x.$



Properties of Utility Functions: Substitution

Substitution: Refers to the notion of trade offs between goods. It asserts that, at any point, the consumer is willing to give up some of one good to get additional increment of some other good.

- We refer to the rate at which one good is substituted for another as the **marginal rate of substitution**, MRS.
- Substitution provides an important measure of “value”. Value is measured by what people are willing to give up in order to obtain one unit of a good – the **opportunity cost** they are willing to pay.

Properties of Utility Functions: Diminishing MRS

Diminishing marginal rate of substitution: The marginal value of any good, x, decreases as more of that good is consumed. That is, people are progressively less willing to trade away another good, y, to get more x, consequently MRS_{yx} diminishes.

- In the case of two-goods, the diminishing MRS implies that the indifference curves are convex to the origin.

Indifference Curves

An **indifference curve** shows a set of consumption bundles over which the individual is indifferent.

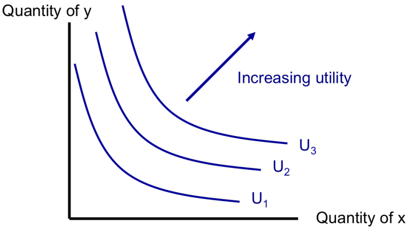
The slope of an indifference curve at some point (times negative one) measures the **MRS** at that point.

- $MRS = -\frac{dy}{dx}|_{u=u_0}$
- The fact that the indifference curve is sloped means that the individual is willing to make tradeoffs: give me enough x and I will give up one y.

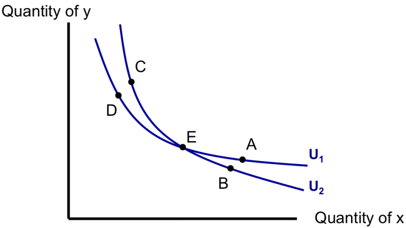
What does an indifference curve look like for someone who is unwilling to make tradeoffs?

Example Indifference Curves

Nonsatiation:

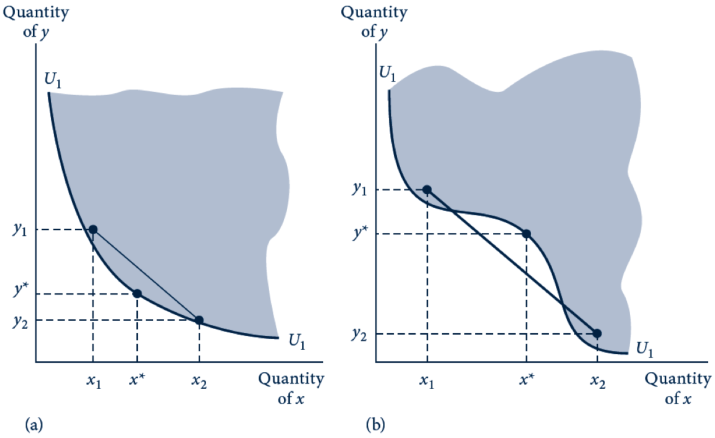


Violation of transitivity:



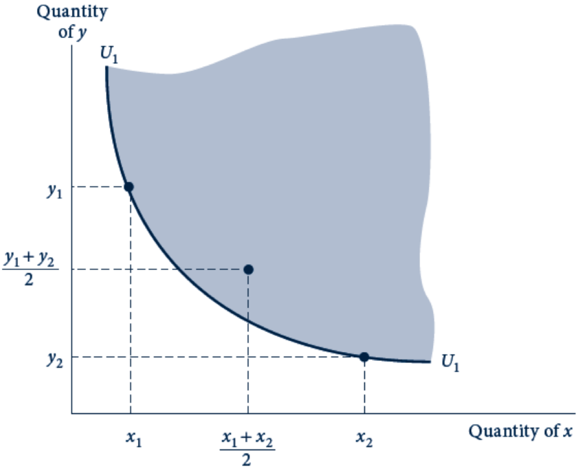
Indifference Curves: Convexity

Standard indifference curves are convex to the origin (diminishing MRS)



Indifference Curves: Balanced Consumption

Convexity of indifference curves implies that individuals prefer to balance their consumption, all else equal



Formal Analysis of Indifference Curves

Deriving indifference curves: Given an arbitrary differentiable utility function: $u = u(x_1, x_2)$, we can derive the indifference curves by solving for x_2.

- $u(x_1, x_2) = u^0$, where u^0 is fixed utility level
- $x_2 = u^{-1}(x_1, u^0)$

Marginal Rate of Substitution (MRS): MRS negative one times the slope of the indifference curve. We can derive this by totally differentiating the utility function:

- $du = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2$
- $0 = \frac{\partial u}{\partial x_1} dx_1 + \frac{\partial u}{\partial x_2} dx_2$
- $-\frac{dx_2}{dx_1} = \frac{u_1}{u_2} = MRS$

Summary: Properties of Utility Functions

To summarize, the preceding properties of utility function assert that

- All consumers possess utility function $u = u(x_1, ..., x_n)$ that are differentiable everywhere
- $u(.)$ is non-unique
- $u(.)$ is strictly increasing
- $u(.)$ has a diminishing marginal rate of substitution

Example: Cobb-Douglas utility

Suppose $u(x_1, x_2) = x_1 x_2$. Derive the equation of an indifference curve and calculate the MRS. Verify that preferences are convex.

By setting $u(x_1, x_2) = u^0$, the indifference curve is:

- $x_2 = \frac{u^0}{x_1}$

The MRS is given as:

- $MRS = -\frac{dx_2}{dx_1} = \frac{u^0}{x_1^2}$

Alternatively,

- $MRS = \frac{u_1}{u_2} = \frac{x_2}{x_1} = \frac{u^0/x_1}{x_1} = \frac{u^0}{x_1^2}$

The derivative of MRS is

- $\frac{d}{dx_1} MRS = -2\frac{u^0}{x_1^3} < 0$

Hence, the utility function exhibits diminishing MRS $\forall x$

Cobb-Douglas utility

A Cobb-Douglas (CD) utility function is given by

- $u(x_1, x_2) = x_1^{\alpha} x_2^{\beta}$, for $\alpha > 0$ and $\beta > 0$

The CD indifference curve is:

- $x_1^{\alpha} x_2^{\beta} = u_0$
- $x_2 = x_1^{\alpha/\beta} u_0^{1/\beta}$

The marginal utilities are:

- $u_1 = \alpha x_1^{\alpha-1} x_2^{\beta}$ and $u_2 = \beta x_1^{\alpha} x_2^{\beta-1}$

The MRS is:

- $MRS = \frac{u_1}{u_2} = \frac{\alpha x_2}{\beta x_1}$

Perfect substitutes

Suppose an individual is buying food for a party. She wants enough food for her guests and considers two hot dogs to be equivalent to one hamburger. These preferences can be represented as

- $u(x_1, x_2) = x_1 + 2x_2$, where x_1 is hot dogs, x_2 is hamburger

In general form, the perfect substitute preferences is represented as:

- $u(x_1, x_2) = \alpha x_1 + \beta x_2$

The indifference curve is:

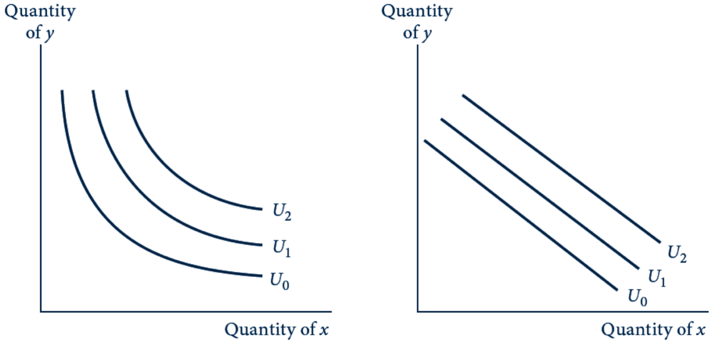
- $x_2 = u_0/\beta - \frac{\alpha}{\beta} x_1$

The marginal utilities and MRS are:

- $u_1 = \alpha$ and $u_2 = \beta$
- $MRS = \frac{\alpha}{\beta}$

Important special classes of preferences

The indifference curves are convex to the origin (diminishing MRS) for Cobb-Douglas but not for perfect substitutes



(a) Cobb-Douglas (b) Perfect substitutes

Perfect complements (Leontief utility)

Suppose an individual consumes a hamburger patty with two slices of bread. If she has 3 patties and 9 slices of bread, then the last 3 slices are worthless. Similarly, if any extra patties without two slices of bread are worthless too.

- $u(x_1, x_2) = min\{2x_1, x_2\}$, where x_1 are patties, x_2 are slices of bread

In general, perfect complements (Leontief) preferences is represented as:

- $u(x_1, x_2) = min\{\alpha x_1, \beta x_2\}$

The marginal utilities and MRS are:

- $u_1 = \alpha$ and $u_2 = 0$ if $\alpha x_1 < \beta x_2$
  - $MRS = \frac{\alpha}{0} = \infty$
- $u_1 = 0$ and $u_2 = \beta$ if $\alpha x_1 > \beta x_2$
  - $MRS = \frac{0}{\beta} = 0$

Constant Elasticity of Substitution (CES)

The CES utility function is given by

- $u(x_1, x_2) = \frac{x_1^\delta}{\delta} + \frac{x_2^\delta}{\delta}$, for $\delta \neq 0$
- $u(x_1, x_2) = ln(x_1) + ln(x_2)$, for $\delta = 0$

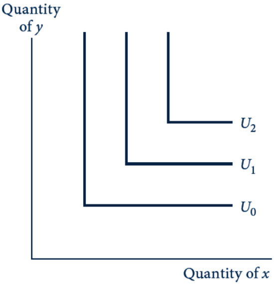
- As $\delta \to 0$, the CES approximates Cobb-Douglas (why?)
- As $\delta \to 1$, the CES approximates perfect substitutes
- As $\delta \to -\infty$, the CES approximates perfect complements

The marginal utilities and MRS are:

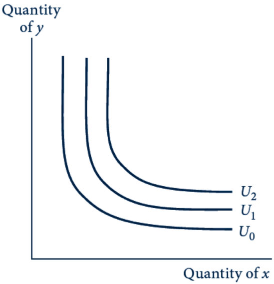
- $u_1 = x_1^{\delta-1}$ and $u_2 = x_2^{\delta-1}$
- $MRS = \frac{u_1}{u_2} = \frac{x_2^{1-\delta}}{x_1^{1-\delta}}$

What Leeontief and CES indifferennce curves look like

Indifference curves are convex, implying that individuals prefer balance their consumption



(c) Perfect complements



(d) CES

Homothetic Preferences

A utility function exhibits **homothetic preferences** if MRS only depends on the ratio of the amounts of the two goods, not the total.

Properties of homothetic functions:

- Indifference curves exhibit the same curvature at the same ratio of goods
    - i.e., slopes depend on x_1/x_2 not on how far out is the indifference curve
- MRS is constant along any ray line, i.e., MRS is a function of x_1/x_2

Quasi-Linear preferences

An individual has quasi-linear preferences if they can be represented by a utility function of the form

- $u(x_1, x_2) = v(x_1) + x_2$
  - Quasi-linear preferences are linear in x_2
  - These preferences are often used to analyze goods which constitute a small part of an individual's income.
  - Consider x_2 as general consumption (a.k.a. income)

The MRS is equal to

- $MRS = \frac{v'(x_1)}{1} = v'(x_1)$
  - MRS depends only on x_1
  - These preferences are not homothetic.