Assignment 1 Stats

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1 Introduction

2-14:
P(65‖40)= (P(40‖65)*P(65))/P(40)= P(65)/P(40)= 0.66/0.97=0.68
2-26:
P(U‖F)=P(F‖U)*P(U)/P(F)=(0.1*0.8)/0.12=(2/3)
2-34:
To Prove Independence:
P(A card being chosen—A suit)=P(A card being chosen) and vice versa.
P(4‖Heart)=P(4 and Heart)*P(Heart)=(1/52)/(1/4)= (4/52)
P(4)=4/52 Therefore P(4‖Heart)=P(4)
This is the definition of independence.
2-41:
P(Y=1)=0.3 and P(X=a)=0.4
a.) P(Y=1 and X=a) $\neq$ P(Y=1)P(X=a) Therefore not independent
b.) P(Y=2 and X=a)/P(Y=2)=0.2/0.4=0.5
c.) P(X=a and Y=2)/P(Y=2)=0.5
d.) P(X=a and Y $>$ 2)/ P(Y $>$ 2)=(3/7)

2-42:

	1	2	3	fx(X)
a	0.12	0.16	0.12	0.4
b	0.12	0.16	0.12	0.4
c	0.06	0.08	0.06	0.2
fy(Y)	0.3	0.4	0.3	1

2-R1:
a.)

	fx(X)	fy(Y)
1	0.2	0.2
2	0.4	0.4
3	0.4	0.4

b.) P($\|x-y\|$ =1)

P($\|x-y\|$ =1)

P(X=1 and Y=2)	0
P(X=2 and Y=1)	0
P(X=2 and Y=3)	0.2
P(X=3 and Y=2)	0.2
Total	0.4

c.) P(Y—X=3)= 0.4
d.) Yes
e.) No
f.)

P(X+Y)	
2	0.1
3	0
4	0.4
5	0.4
6	0.1

2-R5:
P(X=0)=0.5
P(Y=0)=3/8
P(Y=0 and X=0)=1/4
P(X=0)*P(Y=0)=3/16
1/4 does not equal 3/16 Therefore not independent
b.) not independent (see a) and they are not exchangeable. P(X=2—Y=0)=1/8; P(Y=2—X=0)=0 Therefore not exchangeable

3-10:

u(M)= 0; M < 9,000
1; M > 9,000

E[u(M)]=	1/3	Do the Exchange
E[u(M)]=	1/9	Do the Exchange
E[u(M)]=	$\sqrt{1/3}$	Do the Exchange

3-11:

E[f(g—R)]=	1/3	Do the Exchange
E[u(M)]=	1/9	Do the Exchange
E[u(M)]=	$\sqrt{1/3}$	Do the Exchange

3-19:
E(pro score)=2(.02)+3(.16)+4(.68)+5(.13)+6(.01)=3.95
E(Member Score)=5.53
Var(Pro Score)=0.64
Var(Member Score)=1.37

3-21:
Mean Deviation=E($\|X-\mu\|$)
$= \sum_{i=1}^{n}(\|X_i-\mu\|)*f(x)$
since $f(x)$ is a probability function all values of $f(x)$ are between 0 and 1
$= \sum_{i=1}^{n}(\|X_i-\mu\|)*f(x) = \sum_{i=1}^{n}(\|X_i*f(x)-\mu*f(x)\|$
$= \sum_{i=1}^{n}(\|X_i*f(x)-\mu*f(x)\| < \sum_{i=1}^{n}(X_i*f(x)-\mu*f(x))^2$
Standard Deviation = $\sqrt{E(X^2)-\mu^2}$

$=\sqrt{\sum_{i=1}^{n}((X_i^2 * f(x)) - \mu^2)}$
$=\sqrt{\sum_{i=1}^{n}((X_i^2 * f(x)) - \mu^2)} < \sum_{i=1}^{n}((X_i^2 * f(x)) - \mu^2)$
therefore if M.A.D is $\leq$ Standard Deviation then
E($\|X-\mu\|$) $< \sqrt{E(X^2)-\mu^2}$
Therefore
$\sum_{i=1}^{n}(X_i * f(x) - \mu * f(x))^2 \leq \sum_{i=1}^{n}((X_i^2 * f(x)) - \mu^2)$
$\sum_{i=1}^{n}(X_i^2 * f(x)^2 - 2X_i\mu f(x)^2 + \mu * f(x)^2 \leq \sum_{i=1}^{n}((X_i^2 * f(x)) - \mu^2)$
since $f(x) \leq 1 f(x)^2 \leq f(x) and < iff(x) < 1$
therefore it holds that M.A.D is $\leq$ S.D

3-23:

a.) cov(X,Y)= E(XY)-E(X)E(Y)=
(1*1*.1+2*1*.2+2*1*.3+4*.2)+(1.4)(1.5)=-0.2
b.)0.3
c.) $var(X)+var(Y)+2cov(X,Y) =$
$(1-1.3)^2*.6+(2-1.3)^2*.4+(0-1.5)^2*.2+(.5^2)*.3+0.5^2*.5-2*(-0.2)$
=1.925
d.) $var(Y\|X=1) = (0-.7)^2 * .2 + (1-.7)^2 * .1 + (2-0.7)^2 * .3$
$= 0.534$

3-24:

a.) var(3)+var(x)-2cov(3,X)=1
b.) var(2x)+var(4)-2cov(2x,4)= 4
c.) var(X-Y)= Var(X)-Var(Y)-2Cov(X,Y)=1-1-2(1)=-2
d.) Cov(X,X)=$E(X^2)-E(X)^2 = Var(X) = 1$
e.) Cov(X,X+Y)=$E(X^2+XY)-E(X)E(X+Y) =$
$E(X^2)+E(XY)-E(X)^2+E(Y) =$
$Var(X)+Cov(X,Y) = 2$
f.) Var(4x+Y)= Var(4X)+Var(Y)-2Cov(4X,Y)=
16+1-8(E(X*Y)-E(X)E(Y))=17-8=9

3.25:

8(3.5)=28 which is E(8 Dice)
s.d=1.71 for 1 die Var(Die)=2.917.
S.D(8Dice)=Sqrt(Var(8Dice))=Sqrt($8^2*Var(Die)) = 8*S.D(Die) = 13.683.26 :$

$a.)Var(2X+Y) = 4Var(X)+Var(Y)-2Cov(X,Y) = 4+1 = 5$
$b.)Cov(2X+Y, X-Y)$ =E(2X$^2 - 3XY + Y^2) - (2E(X)^2 - 3E(X)E(Y) + E(Y)^2)$
= 2Var(X)-3Cov(XY)+Var(Y)=2+1=3
ρ(X,Y)=Cov(X,Y)/(Var(X)Var(Y))=0; Because Cov(X,Y)=0
ρ(U,V)=Cov(U,V)/(Var(U)Var(V))=0; Because Cov(U,V)=0

3.27:

a.) $Cov(aX+bY, cX+dY) =$
$E(ac*X^2+(ad+bc)*XY+bd*Y^2)-ac*E(X)^2-(ad+bc)*E(X)E(Y)-bd*E(Y^2)$
$= ac*Var(X)+(ad+bc)*Cov(X,Y)+bd*Var(Y)$

b.)

$$Cov(\sum_i a_i X_i, \sum_j b_j Y_j)$$

=

$$E(\sum_i a_i X_i * \sum_j b_j Y_j) - E(\sum_i a_i X_i) * E(\sum_j b_j Y_j)$$

=

$$\sum_i \sum_j (a_i b_i) * \sum_i \sum_j (X_i Y_i)^2 f(x,y) - \sum_i \sum_j (a_i b_i) E(\sum_i X_i) * E(\sum_j Y_j)$$

=

$$\sum_i \sum_j (a_i b_i) * Cov(X_i, Y_i)$$

17:

P(A‖C)= P(AC)/P(C) and P(B‖C)=P(BC)/P(C)

P(AC)=P(C‖A)*P(A) and P(BC)=P(C‖B)*P(B)

Assume $P(A) > P(B) and P(AC) < P(BC)$ This would say that the Probability

of C given B is higher than P(C‖A)*P(A)/P(B) which could exist.

Therefore this is false, however this is true if C&A and C&B were independent.

18:

P((AB)C)/P(C)== P(AC)P(BC)/P(C) (DeMorgan's Law and Independence of A and B)

Therefore this breaks up into P(A‖C)P(B‖C)

19:

Var(X‖Y)= $E(Y - E(Y\|X)^2\|X)$

$E(Y\|X=1) = 3/8; E(Y\|X=2) = 1$

$Var(Y\|X=1) = (1-3/8)^2 + (2-3/8)^2 = 3.031$

$Var(Y\|X=2) = 0 + (1)^2 = 1$

The $Var(Y\|X) = 3.031 + 1 = 4.031$

20:

Var(X)=

$$\int_0^1 x^3 dx + \int_1^2 2x^2 + x^3 dx - (\int_0^1 x^2 dx - \int_1^2 2x + x^2 dx)^2$$

21.

Var(X)=

$$\int_0^1 \int_0^x 2x^2 dy dx - \int_0^1 \int_0^x 2x dy dx$$

= (1/2)-(4/9)=(1/18)

Cov(X,Y)=

$$\int_0^1 \int_0^x 2xydydx - (\int_0^1 \int_0^x 2xdydx \int_0^1 \int_0^x 2ydydx)$$

=(1/4)-(2/3)=(-5/12)

22:

$Cov(X+Y, XY) = E(X+Y)(XY)+(MX+MY)(MX)(MY) = E(X^2Y)+E(Y^2X) + MX^2My + MY^2MX$

$= Var(X) * E(Y) + Var(Y)E(X) = 2 + 4 = 6$

23:

a.) P(Winning/| Never Switching)= 1/3

P(Winning ‖ Switching)=

P(Winning‖Pick First Time)+P(Winning‖Dont Pick First Time)

=0+2/3=2/3

b.) Probability of winning 4/20 in Strat 1. In Strat 2 15/20

24:

P(H‖C)=P(C‖H)*P(H)/P(C)

P(H‖C)=0.85*.06/.11=0.46

25:

P(2M‖A)=P(A‖2M)*P(2M)/P(A).

therefore if 2M and A are independent then yes it would,

however we are assuming here that its not independent.

We want P(A‖2M) and we need P(A) and P(2M).