Applied Microeconomics: Firm and Household

Lecture 14: General Equilibrium

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November 3, 2016

Outline

- General Equilibrium Modeling with Two Goods
- A Pure Exchange Economy
- General Equilibrium and Welfare

Partial vs general equilibrium models

Partial equilibrium models provide the economic theory of a single market

- There are many economic agents & they are price-takers
- Given exogenous prices, each agent determines her demand/supply for the good
- The price adjusts to clear the market
- At the market price no agent would desire to change her actions

General equilibrium (GE) takes account of all the interactions between markets, as well as the functioning of individual markets

- All prices are variable
- The equilibrium requires that all markets clear.

We will examine two examples of GE:

- A GE model with two goods
- A pure exchange model

General equilibrium modeling with two goods

Supply-side assumptions

- There are a large number of firms (a single firm cannot impact the market price)
- Firms choose inputs and output to maximize profits
- There are fixed amounts of capital and labor that must be allocated for production of x and y

General equilibrium modeling with two goods

Consider an economy with two inputs, l and k, that are used to produce two outputs, x and y. We seek to answer the following question:

Q: How can we efficiently allocate resources in this economy?

- i.e. all resources are employed in their best uses

First, we will focus on the characterization of **technical efficiency**

- Resources cannot be reallocated to increase the production of one good without decreasing the production of another

Then we will focus on the characterization of **economic efficiency**

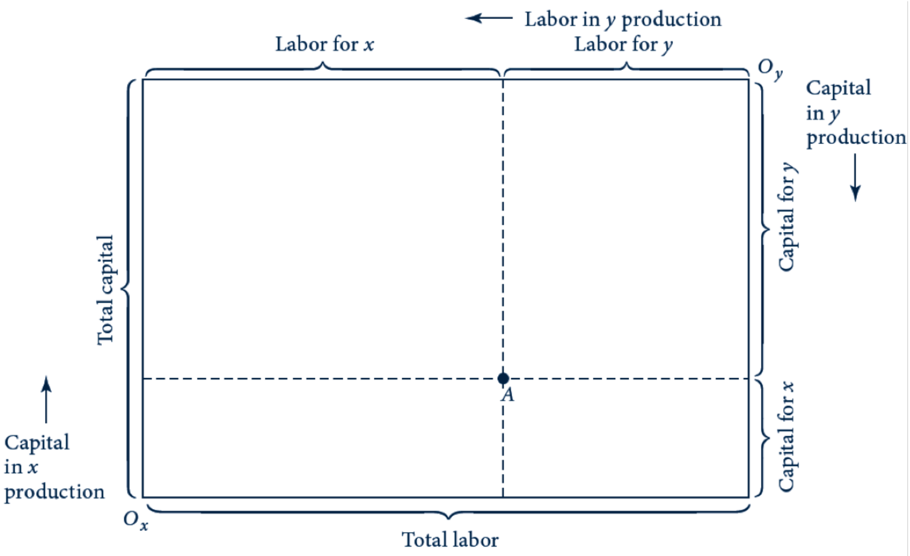
- Resources cannot be reallocated to increase the economic surplus of one agent without decreasing the surplus of another agent

The production possibility frontier (PPF) and Edgeworth box

The **production possibility frontier** shows the alternative combinations of two outputs that can be produced with fixed quantities of inputs if those inputs are employed efficiently.

- To discover efficient allocations of inputs we will use
  - The Edgeworth Box diagram, together with
  - Isoquant maps for x and y.

Edgeworth box



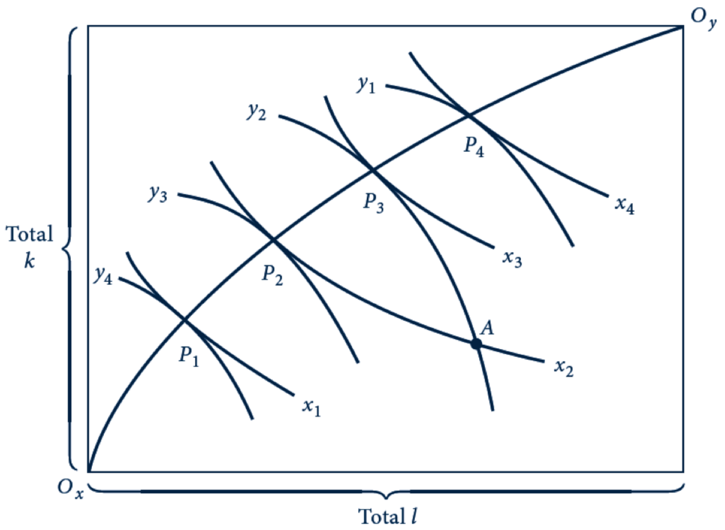
Edgeworth box

Amounts of k and l are fixed

An Edgeworth box shows:

- every possible way the existing k and l might be used to produce x and y
- quantities of resources devoted to x production are measured from origin O_x; quantities devoted to y are measured from origin O_y
- any point in the box represents a fully employed allocation of the available resources to x and y

Efficient allocations



Efficient allocations

Many allocations in the Edgeworth box are technically inefficient:

- It is possible to produce more x and more y by shifting capital and labor around.

With isoquant maps such that

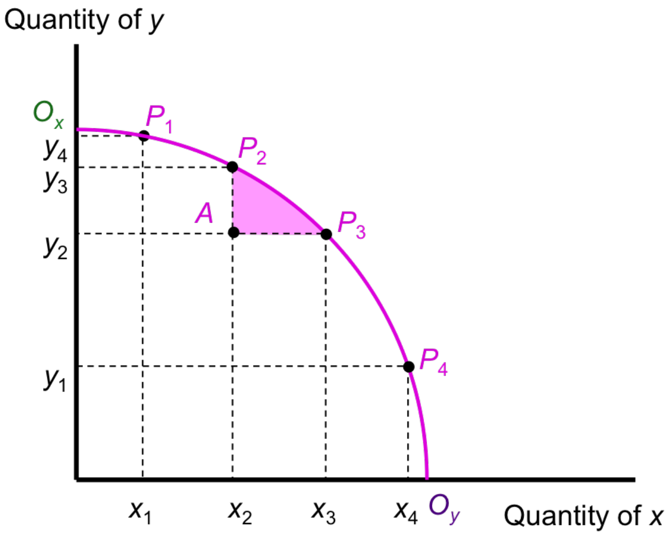
- the isoquant map for good x uses Ox as the origin, and
- the isoquant map for good y uses Oy as the origin,

the efficient allocations occur where the isoquants are tangent to one another.

Efficient allocations

- The locus of the technically efficient points in the Edgeworth box diagram can be used to derive the **production possibility frontier (PPF)**
- The additive inverse of the slope of the PPF shows the trade off between the outputs, i.e., the **rate of product transformation, RPT**.
    - The RPT tells us how much y production should be reduced to produce one more unit of x while continuing to keep the available productive inputs efficiently employed
    - Along the PPF, producing more of one good necessitates lowering the production of the other good (the **opportunity cost** of production)

The Production Possibility Frontier Graph



The rate of product transformation

Let $C(x, y)$ denote the total cost of producing x and y

- Note that $C(x, y)$ is constant along the PPF
    - BBecause the total amount of resources employed in production of x and y are fixed along the PPF

By total differentiation

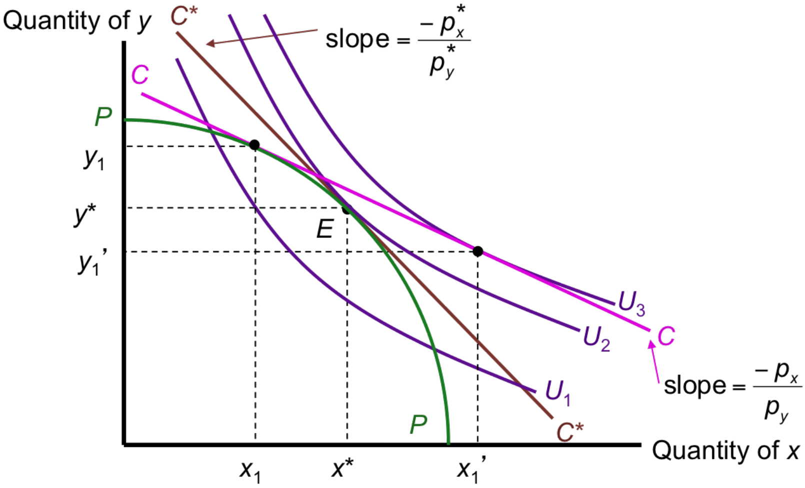
- $\underbrace{dC(x,y)}_{=0} = \underbrace{\frac{\partial C(x,y)}{\partial x}}_{MC_x} dx + \underbrace{\frac{\partial C(x,y)}{\partial y}}_{MC_y} dy$
- $-\frac{dy}{dx} = \frac{MC_x}{MC_y}$
- $RPT = \frac{MC_x}{MC_y}$

Demand in general equilibrium

Next, we will discuss general equilibrium demand. We make the following assumptions about the demand side:

- There is a large number of consumers
- Consumers maximize their utility subject to their budget constraint
- Consumers may also supply of factors of production to generate income (in this case income is endogenous)
- Consumers are price-takers when making their consumption and production decisions
- All consumers have identical preferences

Determination of equilibrium prices



Determination of equilibrium prices

At initial prices p_x and p_y:

- Firms produce x_1 and y_1
- Consumers demand $x_1^{'} > x_1$ and $y_1^{'} < y_1$
    - there is excess demand for x and excess supply for y
    - prices adjust to clear the markets
    - p_x increases, p_y decreases
- At the new equilibrium prices, (p_x^*, p_y^*), supply $=$ demand in both markets

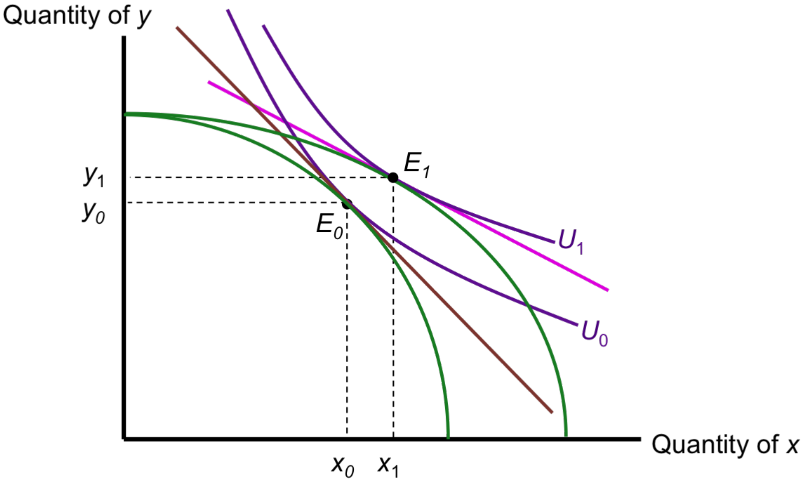
Comparative statics

Q: How do the equilibrium prices and quantities change as production technology changes?

- Changes in technology shift the PPF
  - e.g. technical progress in the production of x would shift out the point where the PPF intersects the x axis

A: The direction of change depends on the relative sizes of the income and substitution effects.

Comparative statics



The pure exchange model

In a **pure exchange model**, *all* of the economic agents are consumers

- each consumer is described by their preferences and the goods they possess at the start (their **initial endowments**)
- consumers trade the goods among themselves according to certain rules
- consumers are assumed to maximize their utility

Q: What is the equilibrium outcome of such a process?

Q: Will there always exist an equilibirum?

Q: What are outcomes of such a process would be desirable?

Q: What allocative mechanisms are appropriate for achieving these desirable outcomes?

The pure exchange model

Suppose there are m consumers and n commodities. Each individual possesses an initial endowment $\bar{x}^i$, which is a vector specifying i's endowments of each of the n commodities. Individuals are price-takers and can trade commodities at market prices p

Our goal is to understand how goods are allocated across agents. Each individual's problem is:

- $\underset{x^i}{\text{Max}} \quad u^i(x^i)$ such that $px_i = p\bar{x}^i$

where

- $x^i = (x^i_1, ..., x^i_n)$ is agent i's consumption bundle
- $p = (p_1, ..., p_n)$ is vector of market prices for n goods
- $\bar{x}^i = (\bar{x}^i_1, ..., \bar{x}^i_n)$ is agent i's initial endowment

That is, the consumer maximizes her utility subject to a budget constraint equaling the market value of her initial endowment (wealth).

Walrasian equilibrium

- $\underset{x^i}{\text{Max}} \quad u^i(x^i)$ such that $px^i = p\bar{x}^i$

The solution to this problem is the consumer demand function:

- $x^i(p, p\bar{x}^i)$

Definition: (p^*, x^*) is a **Walrasian equilibirum** if

- $\sum_i^m x^i(p^*, p^*\bar{x}^i) \leqslant \sum_i^m \bar{x}^i$

That is, p^* is a Walrasian equilibrium if there is no good for which there is positive excess demand.

Q: Will there always exist a price vector where all markets clear?

Existence of Walrasian equilibria

Next, we will analyze the question of the existence of Walrasian equilibria. To this end, define the aggregate excess demand: function

- $z(p) = \sum_i^m (x^i(p, p\bar{x}^i) - \bar{x}^i)$
  - Note that Walrasian equilibrium is $z(p) \leqslant 0$

The aggregate excess demand function $z(p)$ has the following propertiies:

- Homogeneous of degree zero
  - We know x^i is HD0 in prices, (i.e., $x_i(p, p\bar{x}^i) = x_i(tp, tp\bar{x}^i)$). Since the sum of homogeneous functions are also homogeneous of the same degree, $z(p)$ is also HD0 in prices.
- Continuous
  - If all x^i are continuous then $z(p)$ is also continuous.
- Satisfies **Walras' law**

Walras' law

Walras' law: For any price vector p, we have $pz(p) \equiv 0$; i.e., the value of excess demand is identically zero.

Proof: By the definition of excess demand

- $pz(p) = p\left[\sum_i^m (x^i(p, p\bar{x}^i) - \bar{x}^i)\right]$
- $pz(p) = \sum_i^m (px^i(p, p\bar{x}^i) - p\bar{x}^i) = 0$

because $x^i(p, p\bar{x}^i)$ must satisfy budget constraint $px^i = p\bar{x}^i \;\; \forall i$ ■

Walras' law simply says that if each individual satisfies her budget constraint the value of the individual excess demand is zero.

Then, the value of the sum of the excess demands must also be zero.

Implications of Walrasian Eequilibirum and Walras Law

The following implications emerge from the definition of Walrasian equilibrium and Walras' law:

Market clearing: If demand equals supply in $n - 1$ markets and $p_n > 0$, then demand equals supply in the n^{th} market.

Free goods: If p^* is a Walrasian equilibrium and $z_j(p^*) < 0$, then $p_j^* = 0$.

- If some good is in excess supply at a Walrasian equilibrium it must be a free good.

Desirability: Good j is desirable if $z_j(p) > 0$ when $p_j = 0$.

- In words, if there is excess demand for a free good then it must be desirable.

Implications of Walrasian Eequilibirum and Walras Law

Equality of Demand and Supply: If all goods are desirable and p^* is a Walrasian equilibrium (i.e., $z(p^*) \leqslant 0$), then in fact $z(p^*) = 0$.

That is, all we require for equilibrium is that

- there is no excess demand for any good, and if some good is in excess supply then its price must be zero.

Therefore, if all goods are desirable, the equilibrium will in fact be characterized by the equality of demand and supply in every market.

Existence of an equilibrium

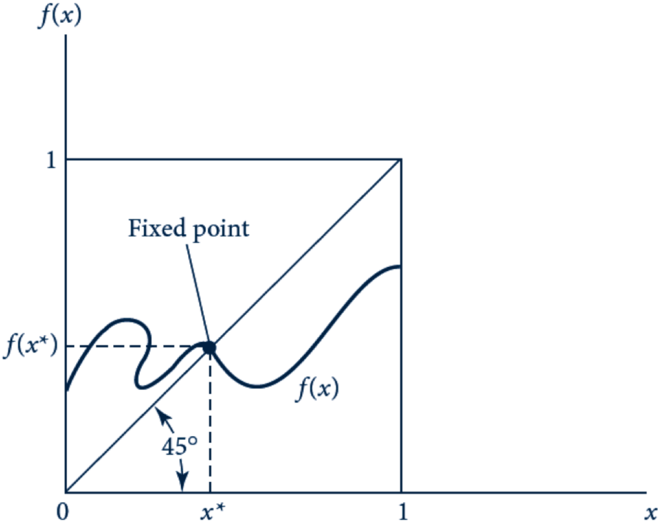
So the analysis of the question of the existence of an equilibrium mostly boils down to the following question:

- Does a vector of p^* exist such that $z(p^*) = 0$?

To answer the question we will

- express demands in terms of relative prices
  - $p_j^{'} = \frac{p_j}{\sum_k^n p_k}$
  - Because $\sum_j^n p_j^{'} = 1$ we can restrict our attention to an $n-1$ dimensional unit simplex.
- introduce Brouwer's fixed point theorem
  - If $f : S^{k-1} \rightarrow S^{k-1}$ is a continuous function from the unit simplex to itself, there is some x in S^{k-1} such that $x = f(x)$.

Brouwer's fixed point theorem



Existence of an equilibrium

Now consider the following price equation that defines a new set of prices as a function of the initial set of prices

- $p_1 = f(p_0) = p_0 + kz(p_0)$

where

- p_0 is initial price
- $z(p_0)$ is excess demand at p_0
- $k > 0$ is a positive constant
- p_1 is a new price whenever $z(p_0) \neq 0$
  - Note that $p_1 = p_0$ if (and only if) there is no excess demand or supply (i.e., $z(p_0) = 0$)

The equation is analogous to the law of demand. It states that

- if there is excess demand ($z(p_0) > 0$), prices increase: $p_1 > p_0$
- if there is excess supply ($z(p_0) < 0$), prices decrease: $p_1 < p_0$

Existence of an equilibrium

- $p_1 = f(p_0) = p_0 + kz(p_0)$

Note that the two conditions of fixed point theorem holds for this function

1. $f(p_0)$ is continuous because $z(p_0)$ is continuous
2. p_1 and p_0 are normalized prices, so the function maps prices from the unit simplex onto itself

Therefore, by Brouwer's fixed point theorem:

- There exist a fixed point $p = f(p)$. Let p^* be a fixed point. Then,
- p^* is an equilibrium price vector because $z(p^*) = 0$. ■

General equilibrium and welfare

In the pure exchange model, market forces generate the prices that achieve the Walrasian equilibrium.

Questions: What are the welfare consequences of this equilibrium? Is Adam Smith's "invisible hand" hypothesis correct?

- In an economy, producers/consumers make economic decisions (i.e., production, consumption) to maximize their own well being (maximize utility/profits). This creates market forces (the "invisible hand") that operate in a way that maximizes well-being for society as a whole – even though maximizing social welfare is not the main goal of any individual economic agent.
- To discuss the welfare properties of Walrasian equilibrium we first introduce the concept of **Pareto efficiency**.

Pareto efficiency

Definition: A feasible allocation of x is a Pareto efficient allocation if there is no feasible allocation $x^{'}$ such that all agents weakly prefer $x^{'}$ to x and at least one agent strictly prefers $x^{'}$ to x.

- That is, if an allocation is Pareto efficient, it is not possible to devise an alternative allocation in which at least one person is better off and no one is worse off.

Consider the two-good, two-person case. Pareto efficient allocations can be derived by solving

- $\underbrace{\text{Max}}_{x_1, x_2} \; u^1(x_1)$, s.t. $u^2(x_2) \geqslant 0$ and $x_1 + x_2 = \bar{x}^1 + \bar{x}^2$

Later, we will examine the solution to this problem in the Edgeworth box.

Walrasian equilibrium

Let's revisit the definition of a Walrasian equilibrium.

- An allocation-price pair (x, p) is a Walrasian equilibrium if:
  1. the allocation is feasible
  2. each agent is making an optimal choice from her budget set.

  Formally,
  1. $\sum_i^m x^i = \sum_i^m \bar{x}^i$
  2. if x'^i is preferred by agent i to x^i, then $px'^i > p\bar{x}^i$

This definition is the same as the previous one so long as the desirability assumption is satisfied (i.e. there are no free goods).

The first theorem of welfare economics

The first theorem of welfare economics says that if (x, p) is a Walrasian equilibrium, then x is Pareto efficient.

Proof: Assume the opposite is true. Let x' be a feasible allocation that all agents prefer to x. Then, from the 2^{nd} property of a Walrasian equilibrium we have:

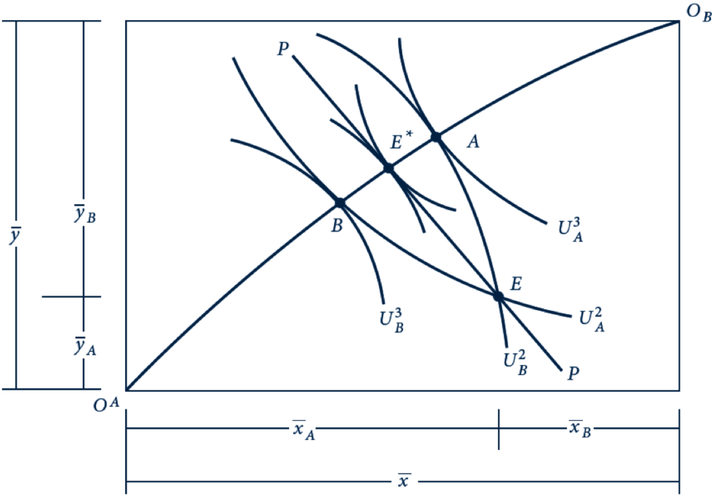
- $px'^i > p\bar{x}^i$ for $i = 1, ..., m$, and so
- $p\sum_i^m x'^i > \sum_i^m p\bar{x}^i$

However, from the 1^{st} property:

- $\sum_i^m \bar{x}^i = \sum_i^m x'^i$, or
- $p\sum_i^m \bar{x}^i = p\sum_i^m x'^i$,

which is a contradiction. ■

The first theorem of welfare economics, graphically



The first theorem of welfare economics

Highlights:

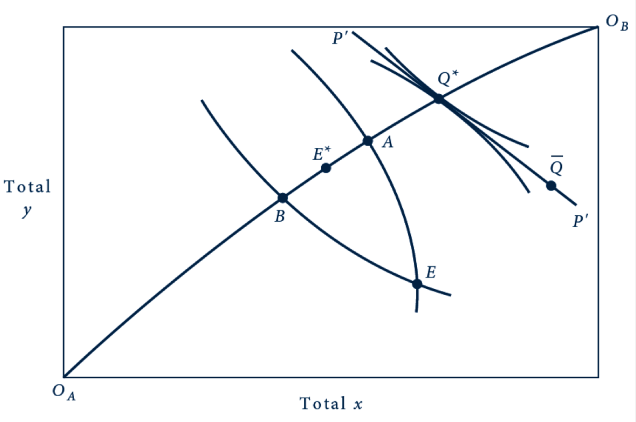
- Under our set of behavioral assumptions, the market is efficient.
- The outcome entirely depends on the original distribution of endowments.
- There are some Pareto optimal allocations that cannot be attained through voluntary transactions.
- The theory does not say anything about the *distribution* of welfare.
- Whether the outcome is "optimal" (or "fair") in any ethical sense would require some further ethical criterion to choose among the efficient allocations (a **social welfare function**).

The second theorem of welfare economics

The second theorem of welfare economics:
Suppose that x^* is a Pareto efficient allocation. Suppose further that a competitive equilibrium exists given the initial endowments $\bar{x}^i = x^{*i}$; denote it as $(p^{'}, x^{'})$. Then, in fact $(p^{'}, x^*)$ is a Walrasian equilibrium.

- In words. if a competitive equilibrium exists from a Pareto efficient allocation, then that Pareto efficient allocation is itself a competitive equilibrium.
- Note that in the definition of the theory the initial endowment is allowed to adjust.
- That is, a Pareto optimal allocation is also a Walrasian equilibrium so long as the initial endowments are adjusted accordingly.

The second theorem of welfare economics, graphically



Welfare maximization

Pareto efficiency is only concerned with efficiency and has nothing to say about distribution

- Even if we agree that we should be at *some* Pareto efficient allocation, we don't know *which* one we should be at

One way of picking is to use a "social welfare" function that aggregates individual utility functions:

- Social Welfare = $SW[U_1(x_1), U_2(x_2), , U_m(x_m)]$

This function may represent the preferences of a "social decision maker"

The social planner's problem is to choose allocations of goods among the *m* individuals in the economy in a way that maximizes SW

Social welfare functions

Alternative functional forms for the social welfare function provide alternative ethical criteria for social welfare. For example,

- $SW[U_1(x_1), U_2(x_2), \ldots, U_m(x_m)] = U_1 + U_2 + \ldots + U_m$

This is a “utilitarian” function. The social planner chooses the allocation with the highest aggregate sum of utilities among all feasible allocations. The social planner does not care about how welfare is distributed.

- $SW[U_1(x_1), U_2(x_2), \ldots, U_m(x_m)] = min[U_1, U_2, \ldots, U_m]$

This is a “maximin” function. The social planner chooses the allocation which has the highest minimum among all feasible allocations.

> *The second principle of justice states that social and economic inequalities are to be arranged so that they are to be of the greatest benefit to the least-advantaged members of society (Rawls, A Theory of Justice, 1971)*