Department of Agricultural and Applied Economics 2009 PhD Econometrics Qualifying Exam

Please answer each of the following questions thoroughly:

1. For each of the following models, state which of the three classical tests (likelihood ratio test, Lagrange multiplier test, Wald test) would be easiest for testing the null hypothesis stated next to the model. Then show the steps to performing the test and the form of the test that would be used.

a)
$$y = X\beta + e$$

Ho:
$$\beta_3\beta_4 = 1$$

b)
$$y = (X\beta + e)^{\eta}$$
 Ho: $\eta = 1$

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$$\eta = 1$$

c)
$$y = X\beta + Z\gamma + e$$

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$$y = X\beta + Z\gamma + e$$
 Ho: $\gamma_1 = \gamma_2 = \gamma_3 = 0$

- 2. For a simple linear model, $y = X\beta + e$, with $X = (100 \times 6)$ and $y = (100 \times 1)$,
 - a) Show that OLS estimation is inefficient if $var(e_i) = \sigma^2 x_{i3}$. That is, the model has heteroscedasticity related to the third regressor.
 - b) Find the properties of the GLS estimator if you wrongly assume the heteroscedasticity follows the pattern var(e) = $\sigma^2 x_{i4}$.
 - c) Have you made things better or worse?
- 3. Consider the following models:

$$(i)\quad y=\beta_0+X_1\beta_1+X_2\beta_2+\epsilon$$

(ii)
$$y = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + \epsilon$$

(iii)
$$y = \beta_0 + X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + X_4\beta_4 + \varepsilon$$

- a. If you estimate (i) but the correct model is (ii) and X_3 is correlated with X_1 , what are the consequences for your estimates of the coefficients and their variances in model (i)? Would you be comfortable with the results of hypothesis tests based on model i? Why or why not?
- b. If you estimate (i) but the correct model is (ii) and X₃ is uncorrelated with X₁ and X_2 , what are the consequences for your estimates of the coefficients and their variances in model (i)? Would you be comfortable with the results of hypothesis tests based on model i? Why or why not?