Solutions to Problem Set #2: Consumer Theory Problems

A. Please solve the following end-of-chapter questions:

1. (25 points) 5.12: Quasi-linear utility (revisited)

   a. We apply the Lagrange method and derive $x = \frac{I-p_x}{p_x}$ and $y = \frac{p_x}{p_y}$.
      - Income effect for x: $-x\frac{\partial x}{\partial I} = \frac{p_x - I}{p_x^2}$.
      - Income elasticity for x: $\epsilon_{x,I} = \frac{\partial x}{\partial I}\frac{I}{x} = \frac{I}{I-p_x}$.
      - Income effect for y: $-y\frac{\partial y}{\partial I} = 0$.
      - Income elasticity for y: $\epsilon_{y,I} = \frac{\partial y}{\partial I}\frac{I}{y} = 0$.

   b. Now we need to find the compensated demand functions for both goods. First we find the indirect utility function: $V = \frac{I-p_x}{p_x} + lnp_x - lnp_y$. So the expenditure function is $E = p_x(V - lnp_x + lnp_y + 1)$. Using Shephard's Lemma,
      - $x^c = \frac{\partial E}{\partial p_x} = V - lnp_x + lnp_y$.
      - $y^c = \frac{\partial E}{\partial p_y} = p_x/p_y$.
      - The substitution effect for x is: $\frac{\partial x^c}{\partial p_x} = -1/p_x$.
      - The compensated own-price elasticity for x is $\epsilon_{x^c,p_x} = \frac{\partial x^c}{\partial p_x}\frac{p_x}{x^c} = \frac{1}{lnp_x - lnp_y - V}$.
      - Substitution effect for y: $\frac{\partial y^c}{\partial p_y} = -p_x/p_y^2$.
      - The compensated own-price elasticity for y is: $\epsilon_{y^c,p_y} = \frac{\partial y^c}{\partial p_y}\frac{p_y}{y^c} = -1$.

   c. For the Slutsky equation, we also need the own-price effects of the demand functions: $\frac{\partial x}{\partial p_x} = -\frac{I}{p_x^2}$ and $\frac{\partial y}{\partial p_y} = -\frac{p_x}{p_y^2}$. For the elasticity version of the Slutsky equation, we need the own-price elasticity of demand for each good: $\epsilon_{x,p_x} = \frac{I}{p_x - I}$ and $\epsilon_{y,p_y} = -1$.

   Now, put all the terms into the Slutsky equation to verify that the Slutsky equation holds for both goods. Repeat the exercise for the elasticity version of the equation. For example, for good x verify that:

   - $\frac{\partial x}{\partial p_x} = \frac{\partial x^c}{\partial p_x} - x\frac{\partial x}{\partial I}$ and $\epsilon_{x,p_x} = \epsilon_{x^c,p_x} - s_x\epsilon_{x,I}$

2. (25 points) 5.13: The almost ideal demand system

   - Please see the brief solutions provided at the end of the book.

B. Consider the following strictly concave utility function for a representative consumer:

$$U(x_1, x_2; z)$$

where x_1, x_2 are food goods, and z is the consumer's health awareness index. z is a parameter of the model, and in the context of this problem $\frac{\partial U}{\partial z}$ has little meaning. Let M denote total expenditure and p_i denote good i's price.

1. (5 points) Set up the consumer's constrained utility maximization problem.

   - $\underset{x_1,x_2;}{\text{Max}}\ u(x_1, x_2; z)$ s.t. $p_1x_1 + p_2x_2 = M$
   - $\underset{x_1,x_2,\lambda}{\text{Max}}\ L = u(x_1, x_2; z) + \lambda(M - p_1x_1 + p_2x_2)$

2. (10 points) Derive and interpret the first order conditions.

   - $\frac{\partial L}{\partial x_1} = L_1 = \frac{\partial u(x_1,x_2;z)}{\partial x_1} - \lambda p_1 = u_1(x_1, x_2; z) - \lambda p_1 = 0$
   - $\frac{\partial L}{\partial x_2} = L_2 = \frac{\partial u(x_1,x_2;z)}{\partial x_2} - \lambda p_2 = u_2(x_1, x_2; z) - \lambda p_2 = 0$
   - $\frac{\partial L}{\partial \lambda} = L_\lambda = M - p_1x_1 + p_2x_2 = 0$

   From the FOCs 1 and 2 we can derive that $\frac{u_1}{u_2} = MRS = \frac{p_1}{p_2}$. That is, the consumer's optimal mix of x_1 and x_2 is the one where their marginal rate substitution equals their price ratio.

3. (5 points) Write down the optimal solutions in implicit form.

   - $x_1^* = x_1(p_1, p_2, M, z)$
   - $x_2^* = x_2(p_1, p_2, M, z)$
   - $\lambda^* = \lambda(p_1, p_2, M, z)$

4. (15 points) Let $u^*(.)$ denote the optimal utility. <u>In words</u>, what does $\frac{\partial^2 u^*}{\partial x_1 \partial z} < 0$ mean?

   - The negative sign indicates that marginal utility of x_1 decreases as the consumer's health awareness index increases.

5. (15 points) Intuitively (or formally if you prefer), explain why it must be true that if $\frac{\partial^2 u^*}{\partial x_1 \partial z} < 0$ then $\frac{\partial x_1^*}{\partial z} < 0$. Please give an example of a real-world food item where this relation, (i.e., $\frac{\partial x_1^*}{\partial z} < 0$) might hold true.

   - At the optimum, $MRS = \frac{u_1}{u_2} = \frac{p_1}{p_2}$. Note that when z changes, the price ratio remains the same. However, an increase in z decreases u_1 and thus the MRS. That is, per dollar marginal utility from x_1 is lower than per dollar marginal utility from x_2. Therefore, the consumer will reduce the consumption of x_1 and increase the consumption of x_2 to restore the equilibrium again at $MRS = \frac{p_1}{p_2}$. Hence as z rises, x_1 falls.
   - An example of this in the real world is soda consumption - if x_1 is soda and x_2 is other drinks. Soda consumption has fallen in recent years due to increases in health awareness, which have driven down the marginal utility of drinking soda.