

Econometrics Qualifying Exam Retake  
July 20, 2016

**Answer all questions.** Show work fully and write neatly. Good luck.

1. For the model  $y = X\beta + \varepsilon$ ,
  - a. list all the assumptions that make it the classic linear model.
  - b. of those assumptions, explain which one if violated causes the fewest problems and how would you address the violation of that assumption (provide full details).
  - c. of those assumptions, explain which one if violated causes the most or hardest problems and how would you address the violation of that assumption (provide full details).
2. You want to estimate a demand model for a consumer good with several substitutes. You have data on the quantity purchased each week for three years for the good to be modeled, along with prices for that good and four substitutes. You also have consumer income data. Economic theory tells us that the sum of the own and cross price elasticities plus the income elasticity should be zero (i.e., if all prices and incomes double, demand is unchanged). Describe in detail how to specify a demand model and test this restriction implied by economic theory.
3. An experiment was performed in the Georgia State Prison in which inmates in one cell block were randomly assigned to a vocational program or not. In another cell block inmates were allowed to voluntarily enroll in the vocational program. The inmates were followed for three years after release and those inmates in the mandatory vocational program had similar recidivism rates as those in the general prison population whereas those in the voluntary vocational program had much lower rates of recidivism. Explain.
4. Consider the estimated regression model  $y = X\hat{B} + \hat{u}$  where  $y$  is  $(n \times 1)$ ,  $X$  is  $(n \times k)$ ,  $\hat{B}$  is  $(k \times 1)$  and  $\hat{u}$  is  $(n \times 1)$ . Given that  $y_0$  is the (scalar) value to be taken by  $y$  given  $x_0$  ( $1 \times k$ ),  $\hat{y}_0 = x_0 \hat{B}$  is the predicted value of  $y$  given  $x_0$ , and  $\hat{u}_0 = y_0 - \hat{y}_0$ :
  - a) Show that  $E[\hat{u}_0] = 0$ ,
  - b) Derive  $\text{Var}[\hat{u}_0]$ ,
  - c) Identify any necessary assumptions and then show how the above can be used to construct a  $(1-\alpha)$  confidence interval for  $y_0$ ,
  - d) Identify the value of  $x_0$  at which that confidence interval would be the narrowest.
5. Suppose that the regression model is:  $y_i = \mu + u_i$ , where  $y_i$  and  $u_i$  are random variables,  $\mu$  is a constant parameter,  $E[u_i | x_i] = 0$ ,  $\text{cov}[u_i, u_j | x_i, x_j] = 0$  for  $i \neq j$ , and  $\text{var}[u_i | x_i] = \sigma^2 x_i^2$ ,  $x_i > 0$ .
  - a) Given a sample of observations on  $Y_i$  and  $X_i$ , what is the most efficient estimator of  $\mu$ ? What is its variance?
  - b) What is the OLS estimator of  $\mu$ ? What is its variance?
  - c) Prove that the estimator in part a) is at least as efficient as the estimator in part b).
  - d) Discuss and compare the asymptotic properties of these two alternative estimators of  $\mu$ .