# Incentive Contracting

Individuals forced to endure risks & the tradeoff between incentive & risk takes center stage.

- Principal - Agent Models to study above mentioned problem:
  risk neutral (Principal), risk averse (Agent)

- Certainty Equivalent - risk averse decision maker accept certain outcome instead of enduring L outcome.

- Risk premium - (Expected outcome from L - CE)

- CARA - Utility function to study IC :-

$$u(x) = A - e^{-rx}$$

where, $r \geq 0$ is Arrow - Pratt coefficient of absolute risk aversion.

$$\text{i.e.; } R_A = r = \frac{-u''(x)}{u'(x)}$$

If $r=0$, risk neutral.
$r$ increases, more risk averse.
$R_A \perp$ initial wealth.

- Arrow - Pratt approximation for the risk premium -

$$RP = \frac{r\,Var(x)}{2}$$

Effort directly observable

**Model** — Effort will not be observable

$e$ = Agent exert effort
$C(e)$ = Agent suffers cost ← st. convex
$P(e)$ = Principal payoff ← st. concave

## ⓐ Effort can be directly observable (Full/Complete information)

Principal chooses $e^*$ to

$$\max_{e} \; p(e) - w(e)$$ , $w(e)$ = wage fn of effort

IR constraint of Agent → where, $w(e) - c(e) \geq \underline{u}$ ; $\underline{u}$ = Reservation utility (Best outside option)

FOC wrt. $e$ gives:-

$e^*$ = Agent's effort

Optimal wage $(w(e^*)) = c(e^*) + \underline{u}$

Principal's profit $(\pi^*) = P(e^*) - c(e^*)$

## ⓑ Effort will not be observable

$$z = e + x \quad \text{be observable}$$

$z$ ↑ Output/realized sales; $x$ ↑ Random demand noise term & $E(x) = 0$

Let demand in other markets $= y$ that is correlated with $x$ and $E(y) = 0$.

∵ effort not observable, with noise agent could exert low effort & blame lousy demand $(x)$.

So, principal can provide incentive based on output $(z)$ & other information of $y$.

Wage contract ~~s~~ schedule :-

$$w = \underset{\text{Salary}}{\alpha} + \underset{\text{incentive}}{\beta}(\underset{\text{sales}}{z} + \underset{\text{Adjustment to sales}}{\gamma} \underset{\text{Demand in other markets}}{y})$$

∴ Effort unobservable, maximize CE of both agent & Principal.

CE(agent) = CE of $w$ net of effort costs

$$\max_{e} \ \alpha + \beta e - c(e) - \frac{r\beta^2 \mathrm{var}(x + \gamma y)}{2}$$

FOC wrt. e gives :-

$$\beta = c'(e) \leftarrow$$ Agent's marginal cost of effort (incentive)

& CE(Principal) = net of wage

$$= P(e) - \alpha - \beta e$$

Efficient contract maximizes sum of CE(agent) & CE(Princ) subject to contract feasibility (meaning agent effort will maximize his CE) $\longrightarrow \beta = c'(e)$

$$\max_{e} \ CE(\text{agent}) + CE(\text{Principal}) \quad s.t. \ \beta = c'(e)$$

$$\cancel{\alpha} + \cancel{\beta e} - c(e) - \frac{r\beta^2 \mathrm{var}(x+\gamma y)}{2} + P(e) - \cancel{\alpha} - \cancel{\beta e}$$

$$\max_{e} \ P(e) - c(e) - \frac{r\beta^2 \mathrm{var}(x + \gamma y)}{2} \quad s.t. \ \beta = c'(e)$$

$\boxed{\text{Plug } \beta = c'(e)}$ & choose $\gamma = \gamma^*$

$$\max_{e} \ P(e) - c(e) - \frac{r(c'(e))^2 \mathrm{var}(x + \gamma^* y))}{2}$$

Note: If $x$ & $y$ correlated, $\gamma = 0$ in order to reduce the agent's risk.

FOC w.r.t. $e$ gives :-

$e^*$

$\beta^*$ after plugging values for $c'(e)$

Optimal wage schedule $w^*(x)$

FOC of CE(agent) : $\beta = c'(e)$

Ratio of $\Delta\beta$ to $\Delta e$ $= c''(e) = \dfrac{\beta' - \beta}{e' - e}$

& so $\dfrac{1}{c''(e)} = \dfrac{e' - e}{\beta' - \beta}$ responsiveness of effort to incentives.

- $c''(e)$ high $\rightarrow$ want lower incentive $\beta \rightarrow$ agent less responsive
- $c''(e)$ low $\rightarrow$ want higher $\beta \rightarrow$ agent stronger response.

$$\beta^* = \frac{P'(e)}{1 + r c''(e)\, \mathrm{Var}(x + \gamma^* y)}$$

Depends on four factors :-

a) $P'(e)$ $\rightarrow$ incremental profits (+ve effect).

b) $\mathrm{Var}(x + \gamma^* y)$ $\rightarrow$ precision with which desired activities are assessed (−ve effect).

c) $r$ $\rightarrow$ tolerance for risk (−ve effect)

d) $\dfrac{1}{c''(e)}$ $\rightarrow$ Agent's responsiveness to incentives.