Econ 8010 Midterm
No Notes, No Calculators
100 Points, 75 Minutes

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1. (30 points) A hospital is looking to fill **up to** three open positions in its residency program. There are three doctors who might apply: Alice (a), Bob (b), and Claire (c). Thus, its set of alternatives is the set of possible hiring decisions:

$$X = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\}\}.$$

For any $Y \subseteq \{a,b,c\}$, define the **power set of** Y as

$$2^Y \equiv \{Z \mid Z \subseteq Y\}.$$

2^Y is the set of hiring decisions that the hospital can make when it receives applications from the doctors in Y.

The hospital's budget sets $B \in \mathcal{B}$ are the sets of hiring decisions it can make after receiving applications from some combination of Alice, Bob, and Claire:

$$\mathcal{B} = \{2^Y \mid Y \subseteq \{a,b,c\}\}$$

- When it receives applications from Alice and Bob, it will choose to hire Bob (and not Alice):
$$C(2^{\{a,b\}}) = \{b\}$$
- When it receives applications from Bob and Claire, it will choose to hire Claire (and not Bob):
$$C(2^{\{b,c\}}) = \{c\}$$

(a) (15 points) What restrictions does the weak axiom place on the hospital's hiring decision $C(2^{\{a,b,c\}})$ when it receives applications from Alice, Bob, and Claire?

(b) (15 points) What restrictions does the weak axiom place on the hospital's hiring decision $C(2^{\{a,c\}})$ when it receives applications from Alice and Claire?

2. (35 points) A widget manufacturing company produces a single output, widgets q. In doing so, it uses two inputs: machines m and sprockets s. These inputs must be consumed in whole (integer) quantities: $m, s \in \mathbb{Z}_+$.

When two sprockets are fed into a machine, it will produce one widget. More formally, the firm's production function $f(m,s)$ is given by

$$f(m,s) = \min\{m, 2s\}$$

(a) (15 points) Show that f is supermodular.

(b) (10 points) If the price of sprockets increases, will the firm use more or fewer machines? Why?

(c) (10 points) If the price of widgets increases, will the firm use more or fewer machines? Sprockets? Why?

3. (35 points) A consumer's preferences are described by a utility function that is homogeneous of degree two: For all $\alpha > 0$ and $x \in \mathbb{R}^L_+$, $u(\alpha x) = \alpha^2 u(x)$.

(a) (7 points) Are this consumer's preferences homothetic? Show that they are or give a counterexample.

(b) (7 points) Show that this consumer's Walrasian demand is multiplicatively separable in prices and wealth: $x(p,w) = y_0(w)y_1(p)$ for some $y_0 : \mathbb{R}_+ \to \mathbb{R}_+$ and $y_1 : \mathbb{R}^L_+ \to \mathbb{R}^L_+$. What is $y_0(w)$?

(c) (7 points) Show that this consumer's indirect utility is multiplicatively separable in prices and wealth: $v(p,w) = v_0(w)v_1(p)$ for some $v_0 : \mathbb{R}_+ \to \mathbb{R}_+$ and $v_1 : \mathbb{R}^L_+ \to \mathbb{R}_+$. What is $v_0(w)$?

(d) (7 points) Show that this consumer's Hicksian demand is multiplicatively separable in prices and required utility level: $h(p,\bar{u}) = g_0(\bar{u})g_1(p)$ for some $g_0 : \mathbb{R}_+ \to \mathbb{R}_+$ and $g_1 : \mathbb{R}^L_+ \to \mathbb{R}^L_+$. What is $g_0(\bar{u})$?

(e) (7 points) Show that for all p, $g_1(p)v_1(p) = y_1(p)$.