**Dynamic Games of Incomplete Information,**
**Signaling Games and Screening Games**

I. Introduction

A. Strategic considerations:

1. In an asymmetric information game at least one player has private information.
2. If a player with private information must make a choice, and if this choice is observed by other players, then the choice may reveal something about the private information.
3. Knowing this, the player with private information has several things to consider when choosing an action:
   a. What are the direct consequences to expected payoffs of choosing a particular action?
   b. What are the indirect consequences to expected payoffs of choosing a particular action that work through changes in other player's beliefs and their consequent choice of actions?
4. The indirect consequences are similar to considerations in dynamic games of complete information where there is an incentive to change conditions in order to influence other players' strategies. Here there is an incentive to influence beliefs in order to influence other players' strategies.

B. Examples of dynamic games of incomplete information and the complicated strategy they can generate

1. Entry deterrence: incumbent firm with private information about its costs facing potential entrant. A high cost incumbent may choose to produce like a low cost incumbent in order to prevent entrant from learning its costs and thereby (perhaps) deterring entry.
2. Education signaling: workers of different types (ability or work effort…) – invest in extra schooling to try to masquerade as a high ability worker ("pooling"), or a high ability worker investing in extra education in order to differentiate themselves ("separating")
3. Reputation games: suppose a player could be either a "tough" type (hard to take advantage of) or a "weak" type (easy to take advantage of) – players may spend effort to develop an appearance of being tough even if they aren't.
   a. In nature: the coral snake is a highly venomous snake that you don't want to mess with. The king snake is a harmless snake – but looks very much like a coral snake (so most animals avoid it too).
   b. Highway restaurants: travelers don't know if the food is good or not. Restaurants can try to signal good food by making the restaurant look attractive (even if the food is lousy).
4. Screening games: design contracts to get players to reveal information or make different choices.
   a. Insurance: how do you offer a menu of insurance coverage without suffering from problem of adverse selection

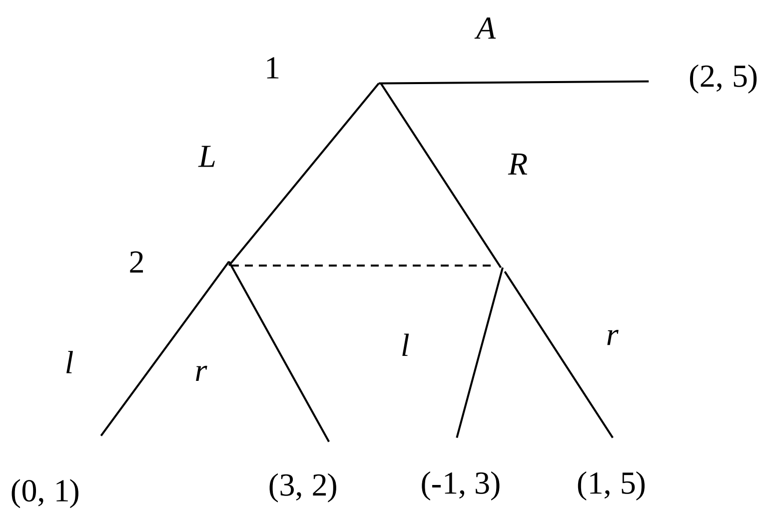
b. Employment contracts: offer employment contracts to screen

5. Large number of issues that dynamic games of incomplete information are relevant. Warning: games get complex and lots of different outcomes

## II. Perfect Bayesian Equilibrium

A. Need a stronger concept than Nash, Bayesian Nash, or subgame perfect equilibrium to analyze dynamic games of incomplete information

1. Example




a. Nash equilibrium: ($A$, $l$), ($L$, $r$). Do these both make sense? "$l$" is an incredible threat – "$l$" is dominated by "$r$".

b. Subgame perfect equilibrium: there is only one subgame which is the entire game. So subgame perfect equilibrium equals Nash equilibrium

c. Bayesian Nash eq: no restriction on what player 2 does – only that they update given Bayes Rule – statement about probabilities but not about best responses.

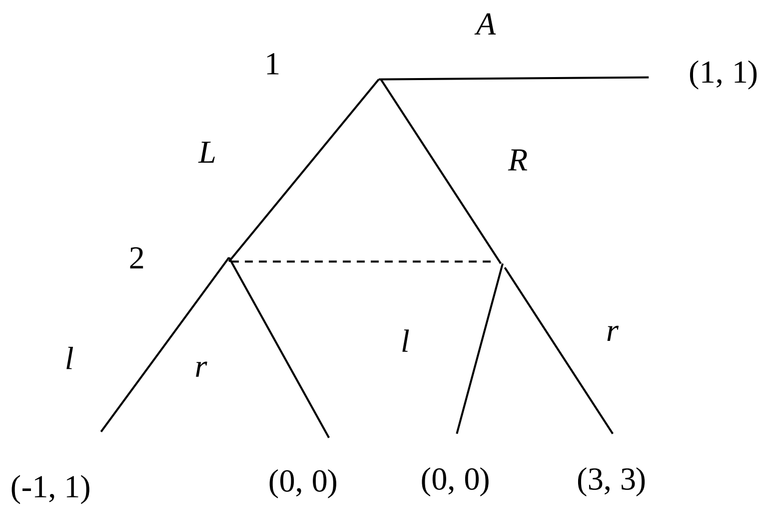
d. Need additional equilibrium concept: Perfect Bayesian equilibrium – combination of subgame perfection (best response given beliefs) along with Bayesian equilibrium

B. Perfect Bayesian Nash equilibrium

1. Definition: a (weak) perfect Bayesian equilibrium is a strategy combination $\boldsymbol{s}$ and a set of beliefs $\boldsymbol{p}$ such that at every information set:
   - For every player $i \in N$, at every information set for player $i$, the strategy for the remainder of the game is a best response given the players' beliefs ($\boldsymbol{p}$) and the strategies of other players (sequential rationality)
   - The beliefs for every player $i$ are rational in the sense that updating of beliefs is consistent with Bayes' Rule.

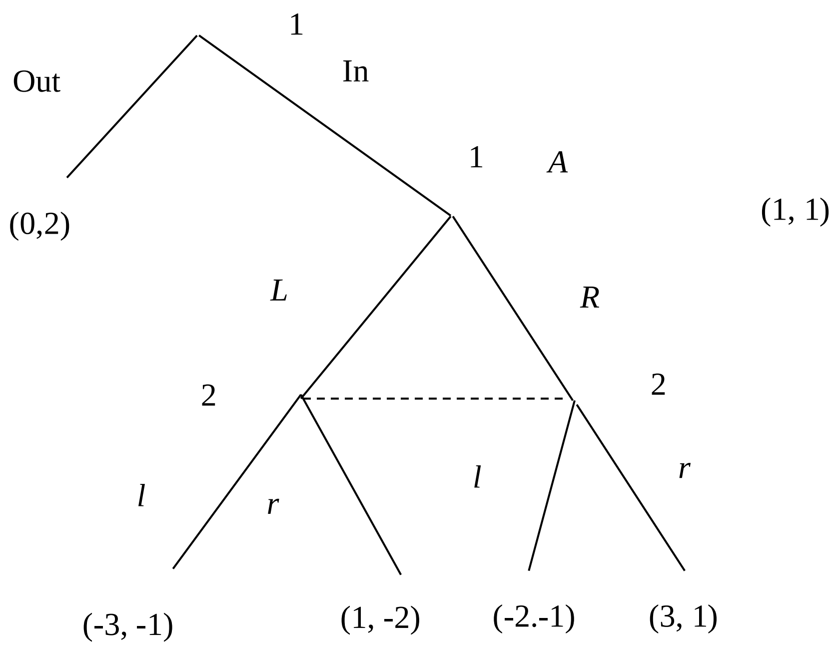
2. Note: alternative definitions exist for perfect Bayesian equilibrium. Mas-Colell et al. discuss imposing stronger conditions to get a perfect Bayesian equilibrium and Gibbons: writes down a large number of conditions. For this class, we will stick with weak perfect Bayesian equilibrium.
3. Sequential rationality: extension of subgame perfection – best response (given beliefs) at each decision node in the game. Rules out incredible threats.
4. Application to example: Player 2's best response is "*r*" and given this the best response for player 1 is *L*. So only perfect Bayesian equilibrium is (*L*, *r*).
5. Bayes' Rule
   a. Let *t* be a possible state or outcome. Example: that player 1 chose *L* so that player 2 is actually in the left-hand node of their information set.
   b. $p(t)$ is the probability of that state or outcome occurring.
   c. Prior beliefs: initial probability
   d. Posterior beliefs: updated probability after observing an action ($a$) that may convey information
   e. Bayes' Rule: $p(t \mid a) = \dfrac{p(a \mid t)\, p(t)}{p(a \mid t)\, p(t) + \sum_{s \in T} p(a \mid s)\, p(s)}$
   f. Note: what if there is no probability of $a$ occurring – $p(a|t) = p(a|s) = 0$? Then Bayes' Rule cannot be applied. Undefined (dividing by zero).
   g. Example:
      - Suppose probability of a disease is $p(d) = .001$; probability of being healthy is $p(h) = .999$ (1 in a thousand chance of being sick)
      - Run a medical test. Let $m$ = positive test result
      - $p(m|d) = 1$ (no false negatives)
      - $p(m|h) = .05$ (5% false positives)
      - Given a positive test result, what is the probability of actually having the disease?
      - $$p(d \mid m) = \frac{p(m \mid d)\, p(d)}{p(m \mid d)\, p(d) + p(m \mid h)\, p(h)}$$
        $$= \frac{1 * 0.001}{1 * 0.001 + 0.05 * 0.999} = 0.0196$$
      - So the correct answer is about 2%.
      - When asked to compute the odds, doctors at Harvard Medical School grossly over-estimated the chance of disease: mode 95%, mean 56%; less than 20% knew the correct answer

C. Refinements of perfect Bayesian equilibrium
1. Even perfect Bayesian equilibrium is not strong enough to rule out some unreasonable equilibrium.
   a. Example:




*L* is dominated by *R* so would expect that player 2 should play *r*. (*R, r*) is a PBE. But (*A, l*) is also a PBE. Given *A*, reaching player 2's information set is a zero probability event. So any beliefs are possible. As long as player 2 puts the probability of being at the left hand node at ≥ 0.75, then *l* is a best response, and then *A* is best response.

2. May wish to impose even further refinements – restrictions on "off the equilibrium path beliefs," which are zero probability events, for which Bayes' Rule doesn't apply
3. If the problem is that any "off equilibrium path beliefs" are possible because these are zero probability events, then one way to impose more discipline is to force there to be some probability of all information sets to be reached. This can be done by using completely mixed strategies that impose some positive probability on all potential actions.
4. Sequential equilibrium: A strategy profile and system of beliefs $(\sigma, \mu)$ is a sequential equilibrium if:
   a. Strategy profile $\sigma$ is sequentially rational given beliefs $\mu$
   b. There exists a sequence of completely mixed strategies $\{\sigma_n\}_{n=1}^{\infty}$, with $\lim_{n\to\infty} \sigma_n = \sigma$, such that $\mu = \lim_{n\to\infty} \mu_n$, where $\mu_n$ are the beliefs derived from $\sigma_n$ using Bayes' Rule.
5. Note: sequential equilibria are subgame perfect whereas perfect Bayesian equilibria need not be (see next example).
6. Example where sequential equilibrium helps prune down set of equilibria.
   a. (weak) perfect Bayesian equilibria:{(Out, R if In), (L)};{(In, R if In), (R)}
   b. First of these equilibria doesn't make much sense
   c. Only the second is a sequential equilibrium because there must be some chance of playing In – which means that player 1 will play R so the best response must be R by player 2 and {(In, R if In), (R)} is then the unique equilibrium




7. In general, there are many suggestions for how to refine perfect Bayesian equilibrium. There is no agreement on which refinements should be used: some refinements work well for certain types of game, but no refinement (to date) works well on all games
8. Will talk about another equilibrium refinement that works well for many games – "Intuitive Criterion" (Cho-Kreps 1987) in signaling games towards the end of the signaling games section.

## III. Signaling Games

A. Introduction
   1. Signaling games are a well-studied class of dynamic games of incomplete information.
   2. Simple signaling game has an informed player move first (signal sender) followed by an uninformed player (signal receiver)
   3. What makes signaling games of particular interest is that they are a relatively simple setting in which to study how players update beliefs based on observed actions (signals) and how players try to strategically reveal or conceal private information through their choice of actions.

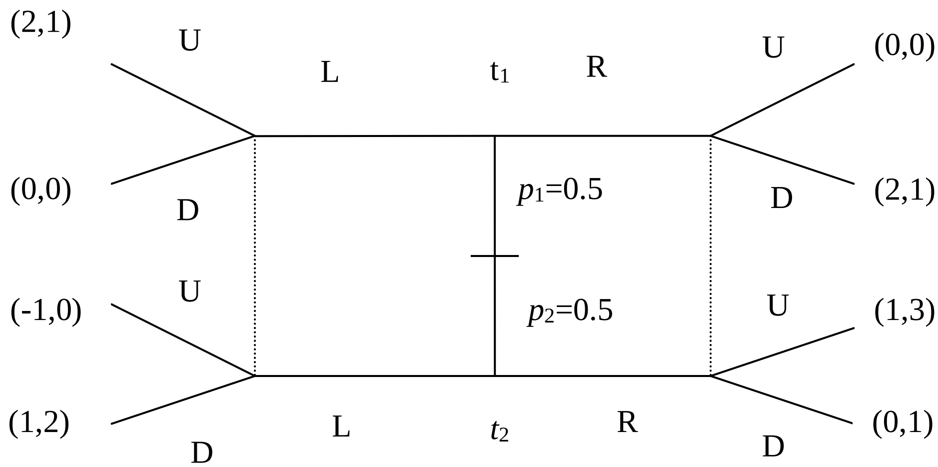
B. Simple two person signaling game: structure of information and order of moves
   1. Initial information structure:
      a. Player 1 (signal sender) can be of several types: $t_i \in T_1$, where $T_1$ is the set of possible types for player 1.
      b. Nature chooses a type for player 1. Let $p(t_i)$ be the probability that player 1 is of type $t_i$.
      c. Player 1 knows their own type: $t_i$.
      d. Player 2 (signal receiver) knows only the probability distribution of types for player 1 but not the actual type.
   2. Player 1 takes an action, $a_1 \in A_1$, which is observed by player 2.
   3. Player 2 updates beliefs based on the observed action using Bayes' Rule. Posterior beliefs that player 1 is of type $t_i$ given action $a_1$ are $p(t_i| a_1)$.
   4. Player 2 takes an action: $a_2 \in A_2$.
   5. Payoffs are realized.

C. Types of perfect Bayesian equilibrium in signaling games

   1. Separating equilibrium: Each different type of player 1 chooses a different action in equilibrium. Therefore, observing the action reveals the type.
   2. Pooling equilibrium: all types for player 1 choose the same action. Therefore, observing the action, $a_1$, reveals nothing about the type $t_i$. In this case, posterior beliefs equal prior beliefs: $p(t_i| a_1) = p(t_i)$ .
   3. Hybrid – semi-pooling, semi-separating – equilibrium: different types for player 1 put positive probability on the same action, but not all types put the same probability on all strategies. A hybrid equilibrium can occur because multiple types play the same strategy while other types play a different strategy, or because at least one type plays a mixed strategy over actions.

D. Example: 2 player, 2 type, 2 action signaling game

Player 1: can choose *L* or *R*
Player 2: can choose *U* or *D*




Warning: there can be numerous perfect Bayesian equilibria in signaling games.

1. Separating
   a. Type 1 plays *R* and type 2 plays *L*.
      - Player 2 beliefs: $p(t_1|\ L) = 0$; $p(t_1|\ R) = 1$.
      - Player 2 strategy: best responses given beliefs: *D* if see *R*; *D* if see *L*.
      - Best response for player 1: type 1 for player 1 is best off choosing *R*, type 2 for player 1 if best off choosing *L*.
      - This constitutes a perfect Bayesian equilibrium (both players are playing best responses given beliefs and beliefs are updated according to Bayes' Rule—where applicable).
      - PBE: {Type 1: *R*, Type 2: *L*, $p(t_1|\ L) = 0$, $p(t_1|\ R) = 1$, *D* if observe *L*, *D* if observe *R*}
   b. Type 1 plays *L* and type 2 plays *R*.
      - Player 2 beliefs: $p(t_1|\ L) = 1$; $p(t_1|\ R) = 0$.
      - Player 2 strategy: best responses given beliefs: *U* if see *R*; *U* if see *L*.
      - Best response for player 1: type 1 for player 1 is best off choosing *L*, type 2 for player 1 if best off choosing *R*.
      - This constitutes a perfect Bayesian equilibrium.
      - PBE: {*L*, *R*, 1, 0, *U*, *U*}
2. Pooling
   a. Both types play *L*.
      - Player 2 beliefs: $p(t_1|\ L) = p(t_1) = 0.5$; $p(t_1|\ R)$ can be anything since this is off-equilibrium and Bayes' Rule does not apply. Suppose that $p(t_1|\ R) < 2/3$.
      - Player 2 strategy: best response given beliefs: *D* if see *L*; *U* if see *R*.
      - Best response for player 1:
        - Type 1: gets a payoff of 0 by playing *L*, but also gets a payoff of 0 playing *R*, so *L* is a best response

        - Type 2: a payoff of 1 by playing *L*, but also gets a payoff of 1 playing *R*, so *L* is a best response.
    - This constitutes a perfect Bayesian equilibrium.
    - PBE: {*L*, *L*, 0.5, p($t_1$| *R*) < 2/3, *D*, *U*}
  b. Both types play *R*.
    - Player 2 beliefs: p($t_1$| *R*) = p($t_1$) = 0.5; p($t_1$| *L*) can be anything since this is off-equilibrium and Bayes' Rule does not apply. Suppose that p($t_1$| *L*) < 2/3.
    - Player 2 strategy: best response given beliefs: *D* if see *L*; *U* if see *R*.
    - Best response for player 1:
        - Type 1: gets a payoff of 0 by playing *R*, but also gets a payoff of 0 playing *L*, so *R* is a best response
        - Type 2: gets a payoff of 1 by playing *R*, but also gets a payoff of 1 playing *L*, so *R* is best response
    - This constitutes a perfect Bayesian equilibrium.
    - PBE: {*R*, *R*, p($t_1$| *L*) < 2/3, 0.5, *D*, *U*}

3. Hybrid
  a. The best response function for player 2:
    - Upon seeing *L*,
        - if p($t_1$| *L*) < 2/3 then *D* is a best response.
        - if p($t_1$| *L*) = 2/3, then anything is a best response
        - if p($t_1$| *L*) > 2/3 then *U* is a best response.
    - Upon seeing *R*,
        - if p($t_1$| *R*) < 2/3 then *U* is a best response.
        - if p($t_1$| *R*) = 2/3, then anything is a best response
        - if p($t_1$| *R*) > 2/3 then *D* is a best response.
  b. The best response function for player 1:
    - Let $q_L$ be the probability that player 2 will play *U* upon seeing *L*, and let $q_R$ be the probability that player 2 will play *U* upon seeing *R*.
    - Type 1's best response is:
        - *L* if $q_L + q_R > 1$
        - *L* or *R* if $q_L + q_R = 1$
        - *R* if $q_L + q_R < 1$
    - Type 2's best response is:
        - *L* if $2q_L + q_R < 1$
        - *L* or *R* if $2q_L + q_R = 1$
        - *R* if $2q_L + q_R > 1$
  c. Because the p($t_1$) = p($t_2$) = 0.5, it must be the case that at either p($t_1$|*L*) < 2/3 or p($t_1$| *R*) < 2/3.
    - Suppose p($t_1$|*L*) < 2/3, p($t_1$|*R*) < 2/3
        - Player 2 will play *D* is they see *L*, *U* if they see *R*.
        - Then $q_L = 0$, $q_R = 1$, so both type 1 and type 2 are indifferent between playing *L* and *R*
        - Any mixed strategies by player 1 are best responses including strategies that induce beliefs p($t_1$|*L*) < 2/3, p($t_1$|*R*) < 2/3
        - This constitutes a perfect Bayesian equilibrium

- Suppose $p(t_1|L) > 2/3$, $p(t_1|R) < 2/3$
  - Player 2 will play $U$ is they see $L$, $U$ if they see $R$.
  - Then $q_L = 1$, $q_R = 1$, so the best response for type 1 is $L$, the best response for type 2 is $R$.
  - These strategies are consistent with inducing beliefs $p(t_1|L) > 2/3$, $p(t_1|R) < 2/3$, and in fact give $p(t_1|L) = 1$ and $p(t_1|R) = 0$.
  - But this is a separating equilibrium.
- Suppose $p(t_1|L) < 2/3$, $p(t_1|R) > 2/3$
  - Player 2 will play $D$ is they see $L$, $D$ if they see $R$.
  - Then $q_L = 0$, $q_R = 0$, so the best response for type 1 is $R$, the best response for type 2 is $L$.
  - These strategies are consistent with inducing beliefs $p(t_1|L) < 2/3$, $p(t_1|R) > 2/3$, and in fact give $p(t_1|L) = 0$ and $p(t_1|R) = 1$.
  - But this is a separating equilibrium

d. In summary, if player 1 mixes such that beliefs induced are that $p(t_1|L) \leq 2/3$ and $p(t_1|R) \leq 2/3$, a hybrid perfect Bayesian equilibrium exists. (Note: if either holds with equality, player 2 still must choose to play $D$ if they see $L$ and $U$ if they see $R$.)

E. Spence: "Job market signaling game" QJE 1973

1. Rules of the game

a. Two types of workers
   - High productivity: $t = H$, $p(t = H) = p$
   - Low productivity: $t = L$, $p(t = L) = 1- p$

b. Workers choose an education level $e \geq 0$ (signal)

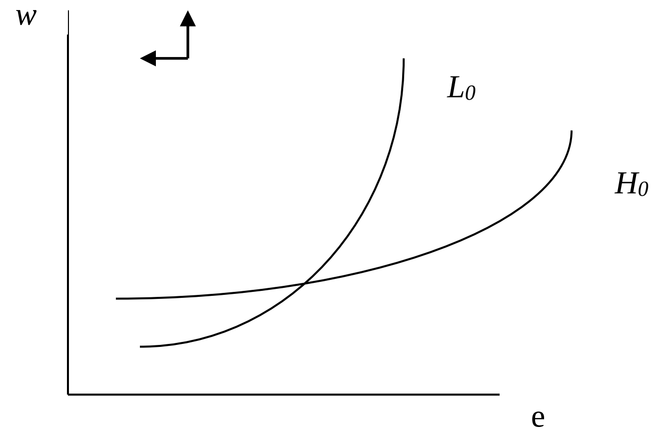
c. Employer observes education signal and offers a wage based on expected productivity given the education signal, $w(e)$.

d. Payoffs
   - output: $y(t, e)$
   - wage: $w$
   - cost of education: $c(t, e)$
   - Payoff for worker: $w - c(t, e)$
   - Payoff for employer: $y(t, e) - w$

e. Will assume there are multiple identical firms so that firms earn zero profit: firms will pay workers a wage equal to the expected productivity: $w(e) = p(H \mid e)y(H,e) + (1 - p(H \mid e))y(L,e)$, where $p(H|e)$ is the conditional probability of the high type given observed level of education $e$.

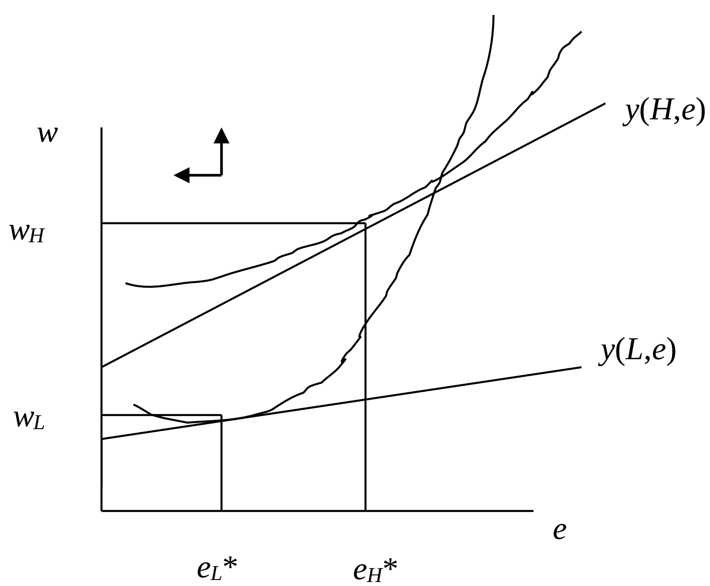
f. Critical assumption: $\frac{\partial c(L,e)}{\partial e} > \frac{\partial c(H,e)}{\partial e}$, for all $e$. In words, the marginal cost of education is higher for a low productivity worker than for a high productivity worker. This assumption is known as the "single-crossing condition."

Indifference curves for low and high productivity types




g. To make things concrete, we can do a specific example with specified functional forms along with the general analysis. For the specific example:
   - $y(L,e) = 2 + 2e$ ; $y(H,e) = 4 + 4e$
   - $c(L,e) = e^2$; $c(H,e) = e^2/2$
   - $p = 0.5$

2. First-best outcome
   a. Max $y(t, e) - c(t, e)$
   b. FOC e*: $\dfrac{\partial y(t,e)}{\partial e} = \dfrac{\partial c(t,e)}{\partial e}$, for all $t = H,L$
   c. $t = L$: $2 = 2e_L^*$; $e_L^* = 1$; $y(L, e_L^*) = 2 + 2e_L^* = 4$; $w_L = 4$
   d. $t = H$: $4 = e_H^*$; $e_H^* = 4$; $y(H, e_H^*) = 4 + 4e_H^* = 20$; $w_H = 20$



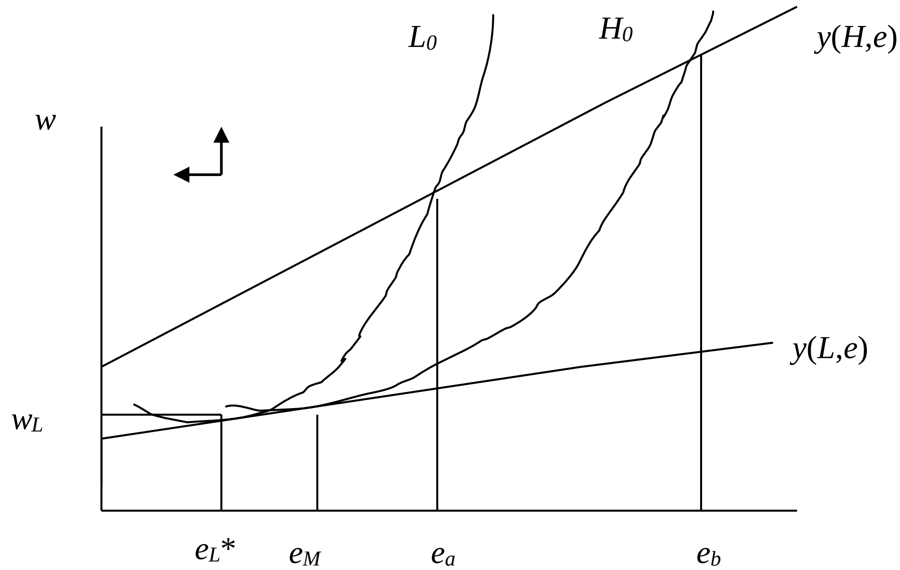
e. Given the indifference curves in the graph, can the first best outcome be implemented as an equilibrium outcome? Answer: No. The low type would do better (get to a higher indifference curve) by mimicking the high productivity type: $w_H - c(L, e_H^*) > w_L - c(L, e_L^*)$. Without knowledge of worker types, the employer cannot get low productivity workers to choose $e_L^*$ for a wage $w_L$ if there is also a possibility of choosing $e_H^*$ for a wage $w_H$.

f. Specific example
   - Payoff for a low productivity worker if they choose $e_L^*$ with a wage $w_L$: $e_L^* = 1$; $w_L = 4$; payoff $= w_L - e_L^2 = 4 - 1 = 3$
   - Payoff for a low productivity worker if they choose $e_H^*$ with a wage $w_H$: $e_H^* = 4$; $w_H = 20$; payoff $= w_H - e_H^2 = 20 - 16 = 4$
   - The low productivity worker would do better to mimic the high productivity worker

3. Perfect Bayesian equilibrium in the job market signaling game
   a. For a set of strategies and beliefs to be a perfect Bayesian equilibrium the following must hold:
      - "Incentive compatibility constraints"
        - If the equilibrium strategy for $t = H$ is $e_H$, then $w_H - c(H,e_H) \geq w(e) - c(H,e)$, for all $e \neq e_H$
        - If the equilibrium strategy for $t = L$ is $e_L$, then $w_L - c(L,e_L) \geq w(e) - c(L,e)$, for all $e \neq e_L$
      - Note: it may also be possible for a worker to "opt out" and choose an alternative. Suppose that a worker of type $t$ ($t = H, L$) could get utility of $\bar{u}_t$ in this alternative. Then, there is an "individual rationality constraint" that utility from being employed is better than the alternative:
        - If the equilibrium strategy for $t = H$ is $e_H$, then $w_H - c(H,e_H) \geq \bar{u}_H$ .
        - If the equilibrium strategy for $t = L$ is $e_L$, then $w_L - c(L,e_L) \geq \bar{u}_L$ .
        - In what follows, we will ignore individual rationality constraints but will pay close attention to incentive compatibility constraints.
      - Updating of beliefs must be consistent with Bayes' Rule
      - Employer will set wages equal to expected productivity:
        - $w(e) = p(H|e)y(H,e) + (1-p(H|e))y(L,e)$
   b. Separating equilibrium: a range of separating equilibria exist
      - Worker strategy:
        - $t = H$: $e = e_s$
        - $t = L$: $e = e_L$
      - Employer beliefs:
        - $p(H|\, e_s) = 1$
        - $p(H|\, e_L) = 0$

- the probability of high productivity type for e $\neq e_s$, $e_L$ can be anything since it is off the equilibrium path so Bayes' Rule does not apply. Suppose that p($H$| $e$) = 0 for all e $\neq e_s$, $e_L$

- Employer will then set wages:
  - for $e = e_s$: $w = y(H, e_s)$
  - for e $\neq e_s$: $w = y(L, e)$
- To be an equilibrium, we must satisfy the incentive compatibility constraints:
  - For type $H$: $y(H, e_s) - c(H, e_s) \geq y(L, e) - c(H, e)$, for all e $\neq e_s$
  - For type $L$: $y(L, e_L) - c(L, e_L) \geq y(L, e) - c(L, e)$, for all e $\neq e_s$, $e_L$, and $y(L, e_L) - c(L, e_L) \geq y(H, e_s) - c(L, e_s)$ for $e = e_s$
- Note: typically, the incentive compatibility constraint on type $L$ is the constraint that might bind (note that it is this constraint that prevents first best from being obtained). Therefore, it is unlikely that $e_s = e_H$*, but we can have $e_L = e_L$*. If type $L$ will be paid $y(L, e)$ in equilibrium, then they would choose $e_L = e_L$*. Set $e_L = e_L$*.
- Range of possible values of $e_s$: $e_s \in [e_a, e_b]$
  - $e_a$ is defined by: $y(H, e_a) - c(L, e_a) = y(L, e_L^*) - c(L, e_L^*)$
  - $e_b$ is defined by: $y(H, e_b) - c(H, e_b) = y(L, e_M) - c(H, e_M)$, where $e_M$ is defined by the tangency of the low type productivity and high type indifference curve: $\dfrac{\partial y(L, e_M)}{\partial e} = \dfrac{\partial c(H, e_M)}{\partial e}$
  - the range of $e_s$ is illustrated in the following figure




- Solving for $e_a$ and $e_b$ in the specific example
  - $e_a$: $y(H, e_a) - c(L, e_a) = y(L, e_L^*) - c(L, e_L{}^*)$

    $e_a$ is the larger of the two roots that solves

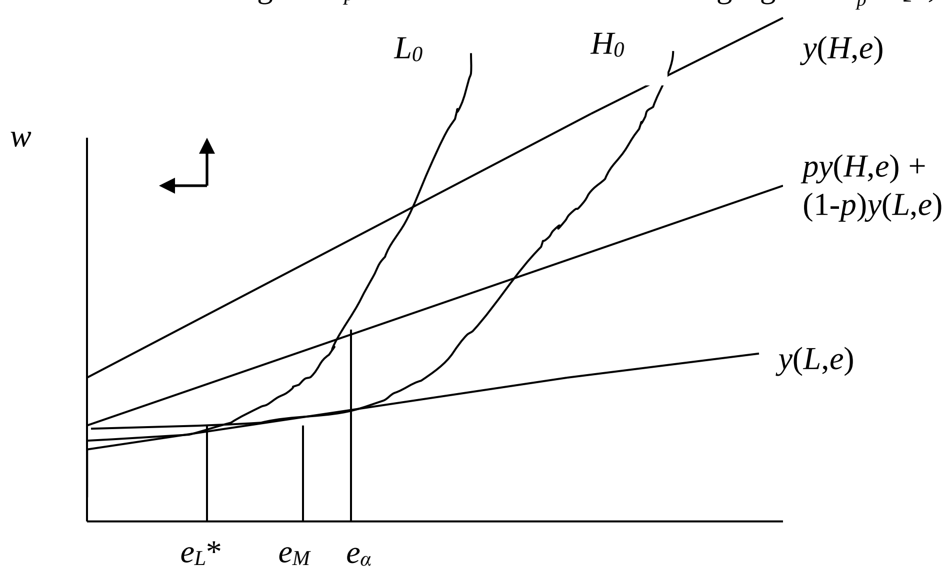
    $4 + 4e_a - e_a^2 = 2 + 2\, e_L^* - (e_L^*)^2 \quad \{e_L^* = 1\}$

    $-e_a^2 + 4e_a + 4 = 3$

    $e_a = 4.236$

- $e_b$: $y(H, e_b) – c(H, e_b) = y(L, e_M) – c(H, e_M)$
  $e_b$ is the larger of the two roots that solves
  $4 + 4e_b – e_b^2/2 = 2 + 2\, e_M - (e_M)^2/2$ $\{e_M = 2\}$
  $e_b = 8$
- Summary: $e_s \in [4.236, 8]$

c. Pooling equilibria: a range of equilibria exist
   - Worker strategy:
     - $t = H$: $e = e_p$
     - $t = L$: $e = e_p$
   - Employer beliefs:
     - $p(H| e_p) = p$
     - beliefs for $e \neq e_p$ can be anything since it is off the equilibrium path so Bayes' Rule does not apply. Suppose that $p(H| e) = 0$ for all $e \neq e_p$
   - Employer will then set wages:
     - for $e = e_p$: $w(e_p) = py(H, e_p) + (1\text{-}p)y(L, e_p)$
     - for $e \neq e_p$: $w(e) = y(L, e)$
   - To be an equilibrium, we must satisfy the incentive compatibility constraints:
     - For type $H$: $py(H, e_p) + (1\text{-}p)y(L, e_p) – c(H, e_p) \geq y(L, e) - c(H, e)$, for all $e \neq e_p$
     - For type $L$: $py(H, e_p) + (1\text{-}p)y(L, e_p) – c(L, e_p) \geq y(L, e) - c(L, e)$, for all $e \neq e_p$
   - Range of possible values of $e_p$: $e_p \in [Max(0, e_\beta), e_\alpha]$
     - $e_\alpha$ is defined by: $py(H, e_\alpha) + (1\text{-}p)y(L, e_\alpha) – c(L, e_\alpha) = y(L, e_L^*) – c(L, e_L^*)$
     - $e_\beta < e_M$ is defined by: $py(H, e_\beta) + (1\text{-}p)y(L, e_\beta) – c(H, e_\beta) = y(L, e_M) - c(H, e_M)$. $e_\beta$ may be negative, in which case pooling equilibrium extends down to 0.
   - The range of $e_p$ is illustrated in the following figure: $e_p \in [0, e_\alpha]$

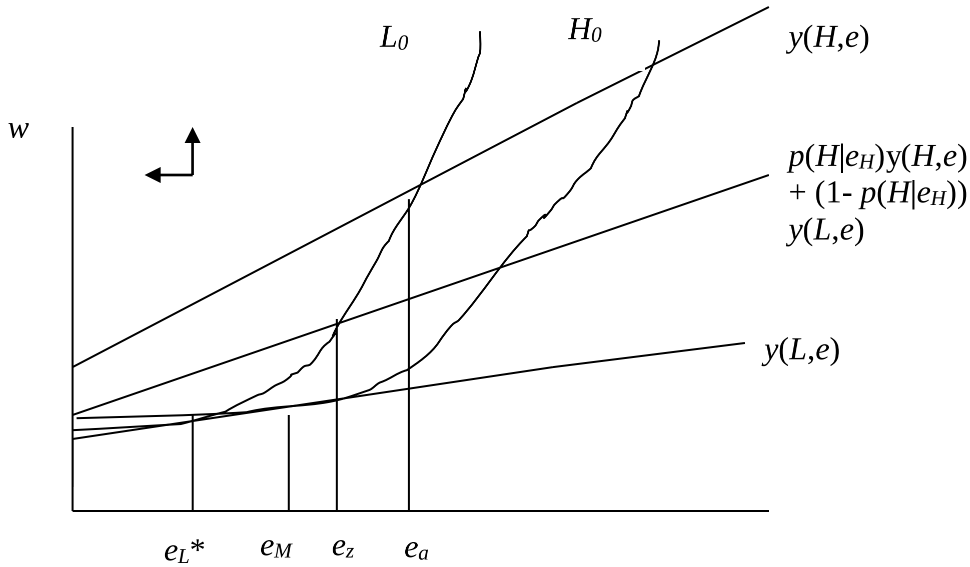


- Solving for $e_\alpha$ and $e_\beta$ in the specific example
  - $e_\alpha$: $py(H, e_\alpha) + (1\text{-}p)y(L, e_\alpha) - c(L, e_\alpha) = y(L, e_L^*) - c(L, e_L^*)$
    $e_a$ is the larger of the two roots that solves
    $(0.5)(4 + 4e_\alpha) + (0.5)(2 + 2e_\alpha) - e_\alpha^{\,2} = 2 + 2\,e_L^* - (e_L^*)^2$
    $-\,e_\alpha^{\,2} + 3e_\alpha + 3 = 3$
    $e_\alpha = 3$
  - $e_\beta$: $py(H, e_\beta) + (1\text{-}p)y(L, e_\beta) - c(H, e_\beta) = y(L, e_M) - c(H, e_M)$
    $e_y$ is the smaller of the two roots that solves
    $(0.5)(4 + 4\,e_\beta) + (0.5)(2 + 2\,e_\beta) - e_\beta^{\,2}/2 = 2 + 2\,e_M - (e_M)^2/2$
    $-\,e_\beta^{\,2}/2 + 3\,e_{\beta\,y} + 3 = 4$
    $e_\beta = 0.042$
  - Summary: $e_p \in [0.042, 3]$

d. Hybrid equilibria: range of hybrid equilibria exist
- In hybrid equilibrium, the high productivity type, the low productivity type, or both, could mix. The following presents an example where the low productivity type mixes.
- Worker strategy:
  - $t = H$: $e = e_H$
  - $t = L$: $e = e_H$ with probability $q$
    $e = e_L^*$ with probability $(1 - q)$
- Employer beliefs:
  - $$p(H \mid e_H) = \frac{p(e_H \mid H)\,p(H)}{p(e_H \mid H)\,p(H) + p(e_H \mid L)\,p(L)}$$
    $$= \frac{1^*\,p}{1^*\,p + q(1-p)}$$
  - Note that if:
    $q = 0$, $p(H|e_H) = 1$ (separating)
    $q = 1$, $p(H|e_H) = p$ (pooling)
    $0 < q < 1$, $p < p(H|e_H) < 1$ (hybrid)
  - $e = e_L^*$, $p(H|\,e_L^*) = 0$
  - beliefs for $e \neq e_L^*$ or $e_H$ can be anything since it is off the equilibrium path so Bayes' Rule does not apply. Suppose that $p(H|\,e) = 0$ for all $e \neq e_L^*$, $e_H$
- Employer will then set wages:
  - for $e = e_H$: $w(e_H) = p(H|e_H)y(H,e_H) + (1\text{-}\,p(H|e_H))y(L,e_H)$
  - for $e \neq e_H$: $w(e) = y(L, e)$
- Since the low productivity type is mixing between $e_L^*$ and $e_H$, the low productivity type must be indifferent between playing these two strategies:
  - $p(H|e_H)y(H,e_H) + (1\text{-}\,p(H|e_H))y(L,e_H) - c(L,e_H) = y(L, e_L^*) - c(L, e_L^*)$
- Range of possible values of $e_H$: $e_H \in [e_z, e_a]$
  - $e_a$ is defined by: $y(H, e_a) - c(L, e_a) = y(L, e_L^*) - c(L, e_L^*)$

- $e_z$ is defined by: $py(H,e_z) + (1- p(H))y(L,e_z) – c(L,e_z) = y(L, e_L^*) – c(L, e_L^*)$

- A hybrid equilibrium for a particular value of $q$ is shown. The range of hybrid equilibrium follow the $L_0$ line from complete pooling to complete separating.

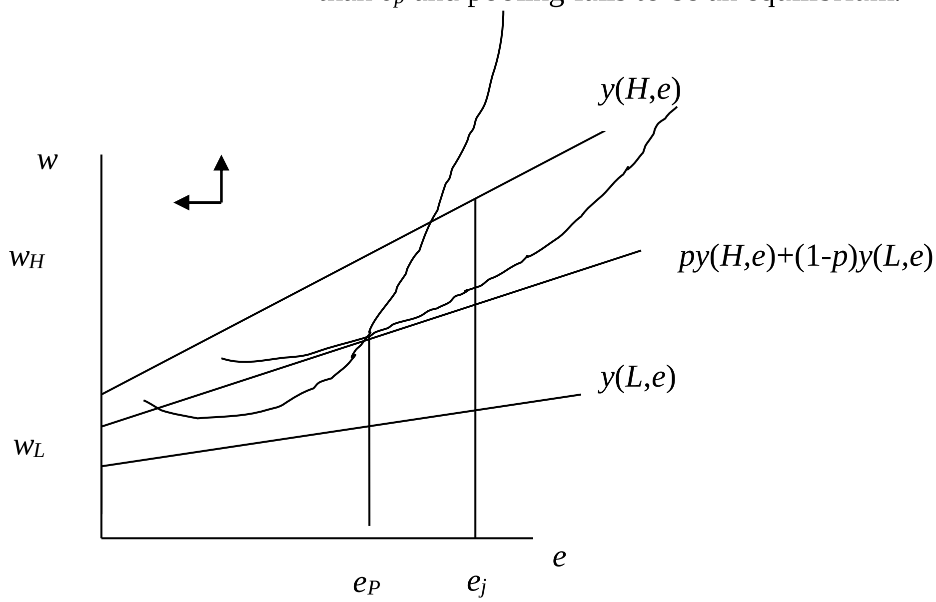



e. Applying the Intuitive Criterion to the Spence job market signaling game
- One problem with perfect Bayesian equilibrium as applied to the signaling game is that there are a large number of possible equilibria. Because Bayes' Rule cannot be applied to non-equilibrium path actions, beliefs are pinned down for virtually many possible actions. This latitude in assigning beliefs opens the door for a large number of candidate equilibria.
- One method for pruning down the number of equilibria is to apply the Intuitive Criterion. In the job market signaling game, the Intuitive Criterion works quite well and results in a unique equilibrium prediction (which is, in fact, an intuitive prediction for the outcome of the game!).
- The Intuitive Criterion defined
  - **Equilibrium dominated**: Given a perfect Bayesian equilibrium in a signaling game, an action $a_i \in A_i$ is *equilibrium dominated* for type $t_i \in T$ if $u^*(t_i) > Max_{a_k \in A_k} u(t_i, a_i, a_k)$, where $u^*(t_i)$ is type $i$'s equilibrium payoff, $a_k$ specifies the actions of rival players and $A_k$ is the set of possible actions by rival players
  - In other words, $a_i$ is equilibrium dominated if the payoff in equilibrium is higher than it is playing $a_i$ no matter what rival players choose to do.

- **The Intuitive Criterion**: If the information set following $a_i$ is off the equilibrium path and $a_i$ is equilibrium dominated for type $t_i$, then $p(t_i| a_i) = 0$ as long as $a_i$ is not equilibrium dominated for all types in $T$.
- In other words, if it never makes sense for type $t_i$ to play $a_i$ but it might make sense for some other type to play $a_i$, then after observing $a_i$ we should set $p(t_i| a_i) = 0$, (i.e., it's not type $t_i$).

- The Intuitive Criterion applied
  - Pooling:

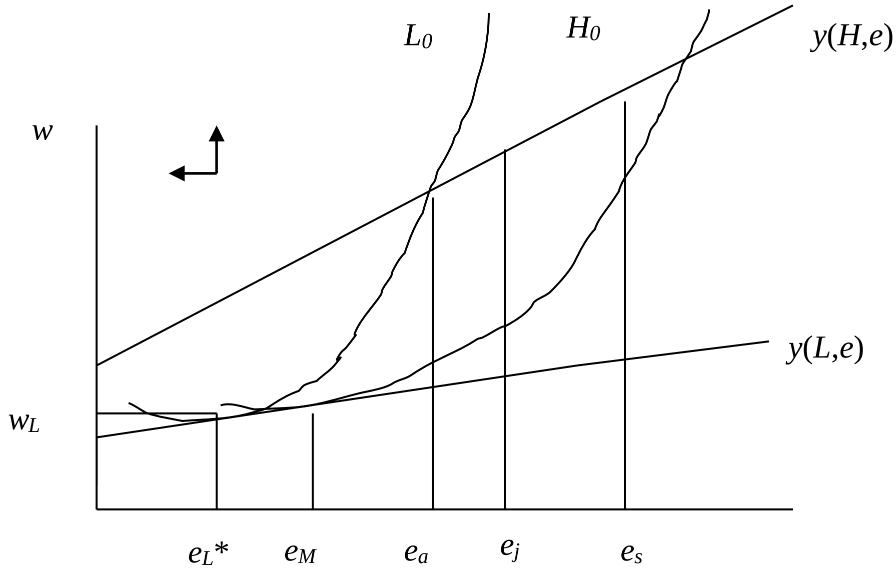
    Because of the single-crossing condition, for some levels of $e_j > e_p$, we must have:
    $py(H,e_p) + (1-p)y(L,e_p) – c(L,e_p) > y(H, e_j) - c(L, e_j)$
    $py(H,e_p) + (1-p)y(L,e_p) – c(H,e_p) < y(H, e_j) - c(H, e_j)$
    But then the employer should believe $p(H| e_j) =1$.
    But then the high type would be better off playing $e_j$ rather than $e_p$ and pooling fails to be an equilibrium.




  - Hybrid: showing that hybrid equilibrium fails works in the same fashion as showing that pooling fails
  - Separating:

    For any separating equilibrium strategy $e_s > e_a$, we can find an $e_j$ such that $e_s > e_j > e_a$ where
    $y(L,e_L{*}) – c(L, e_L{*}) > y(H, e_j) - c(L, e_j)$
    $y(H,e_s) – c(H,e_s) < y(H, e_j) - c(H, e_j)$
    But then the employer should believe $p(H| e_j) =1$.
    But then the high type would be better off playing $e_j$ rather than $e_s$ and separating equilibrium with $e_s > e_a$ fails to be an equilibrium.

However, for $e_s = e_a$ , there is no way to have $e_s > e_j > e_a$ and the above argument does not apply. $e_s = e_a$ satisfies the Intuitive Criterion.




- Summary

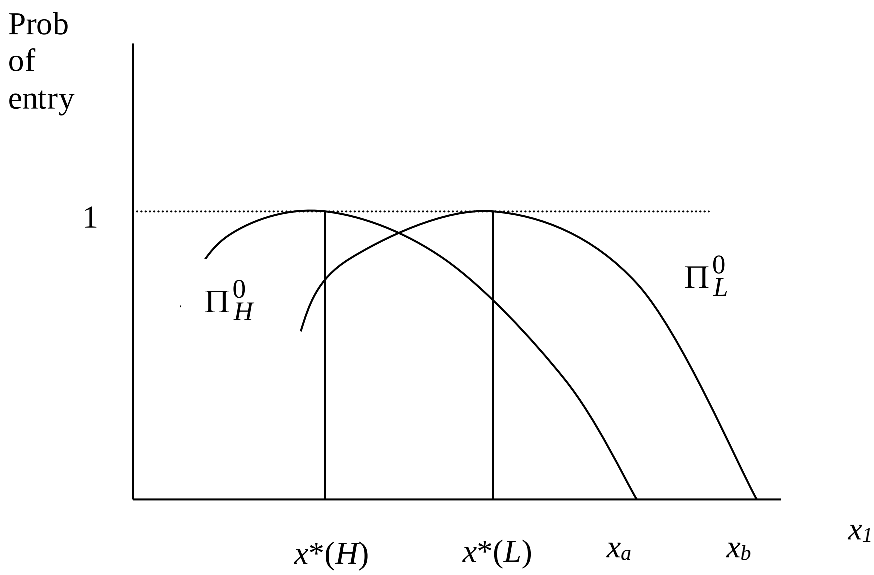
  The unique equilibrium that satisfies the Intuitive Criterion is for the high productivity type to set education just high enough for the low productivity type not to mimic and for the low productivity type to choose its optimal education level ($e_L$*).

4. Comments on the Spence job market signaling game
   a. The job market signaling model illustrates the multiplicity of perfect Bayesian equilibria – including separating, pooling and hybrid equilibria.
   b. Application of the Intuitive Criterion (or other refinements) reduces the set of equilibria by pinning down what are reasonable off-the-equilibrium path beliefs. In this model, the Intuitive Criterion picks out a unique (separating) equilibrium prediction for the game.
   c. It also illustrates the inefficiency that can occur with asymmetric information. In order to separate from low-productivity types, the high productivity types often need to choose levels of education that exceeds the efficient level of education.
   d. In a model where there is no productivity gain from education, high productivity types might still choose positive levels of education to separate themselves from low productivity types. In this case, the only function of education is signaling.
   e. In general, signaling models show the strategic importance of information and controlling the flow of information.

F. Milgrom and Roberts "Limit Pricing and Entry under Incomplete Information" Econometrica 1982

1. Rules of the game
   a. Incumbent firm can be of two types:
      - High cost: $t = H$, $pr(t = H) = p$
      - Low cost: $t = L$, , $pr(t = H) = 1 - p$
   b. Incumbent chooses a quantity to produce, $x_1(t)$
   c. After observing $x_1(t)$, a potential entrant decides whether or not to enter
      - If entry occurs, the incumbent and entrant compete as duopolists
      - If entry does not occur, the incumbent acts as a monopolist
   d. Payoffs:
      - Define:
        - $M(t, x_1(t))$ represents monopoly payoffs when the incumbent of type $t$ plays $x_1(t)$
        - $M(t)$ represents maximum monopoly payoffs when the incumbent is of type $t$
        - $D_I(t)$ represents duopoly profits for the incumbent of type $t$
        - $D_E(t)$ represents duopoly profits for the potential entrant when facing an incumbent of type $t$; assume that $D_E(H) > 0 > D_E(L)$
        - $\delta$ is the discount factor
      - Incumbent:
        - With entry: $M(t, x_1(t)) + \delta D_I(t)$
        - Without entry: $M(t, x_1(t)) + \delta M(t)$
      - Potential entrant:
        - With entry: $D_E(t)$
        - Without entry: 0
2. Equilibria in the limit pricing game
   a. Separating equilibrium
      - Incumbent strategy: initial strategy choice is $x_1(H)$ if $t = H$ and $x_1(L)$, if $t = L$, and $x_1(H) \neq x_1(L)$,
      - Potential entrant beliefs: update according to Bayes' Rule. Observing $x_1(t)$ reveals the type, so:
        - $p(H| x_1(H)) = 1$
        - $p(H| x_1(L)) = 0$
        - for $x_1 \neq x_1(H), x_1(L)$, Bayes'Rule does not apply and the entrant can believe anything. Suppose that $p(H| x_1) = 1$ for $x_1 \neq x_1(H), x_1(L)$
      - Entrant's best response given beliefs:
        - Do not enter if observe $x_1(L)$
        - Enter upon observing $x_1(H)$ or any strategy $x_1 \neq x_1(L)$
      - A high cost incumbent will set $x_1(H)$ equal to the static monopoly strategy and get monopoly profits in the initial period (since entry is going to occur, why not get maximum first period payoffs).

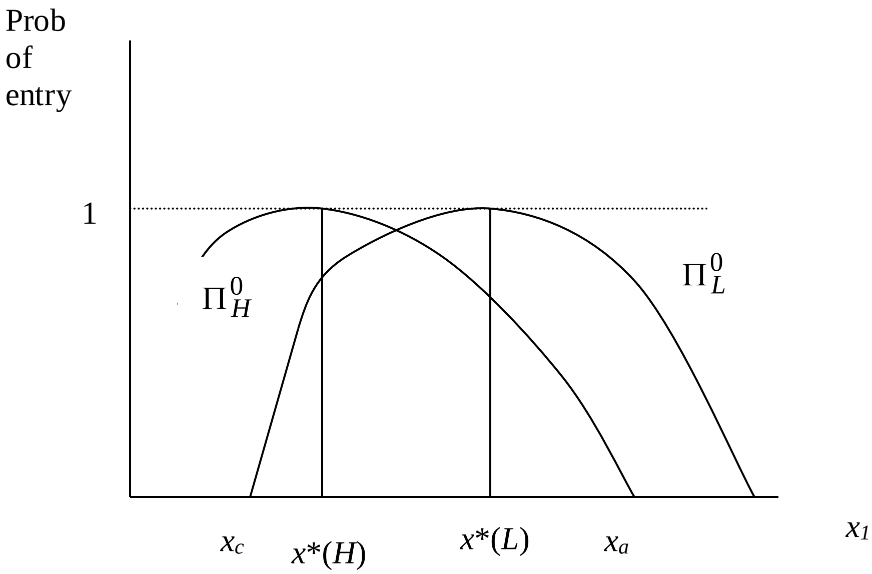
- Payoffs:
  - Incumbent of type $H$: $M(H) + \delta D_I(H)$
  - Incumbent of type $L$: $M(L, x_1(L)) + \delta M(L)$
  - Entrant: $D_E(H)$ if do not observe $x_1(L)$ and enter; 0 if observe $x_1(L)$ and do not enter
- To be an equilibrium: a) must satisfy best responses given beliefs and rival's strategy (sequential rationality, incentive compatibility constraints), b) updating of beliefs is consistent with Bayes' Rule. Already checked (a) for the entrant, and (b). Still need to check (a) for the incumbent. Incentive compatibility constraints:
  - Incumbent of type $H$: $M(H) + \delta D_I(H) \geq M(H, x_1(L)) + \delta M(H)$ {Q: what about any other strategy $x_1 \neq x_1(H), x_1(L)$? Since entry will occur anyway, this will give lower payoffs than playing monopoly strategy in period 1}
  - Incumbent of type $L$: $M(L, x_1(L)) + \delta M(L) \geq M(L) + \delta D_I(L)$
- Figure shows iso-profit contours in $x_1$ and probability of entry for each type. Iso-profit contours have zero slope at the static profit maximizing level.




- Range of $x_1(L)$ for separating equilibrium: $x_1(L) \in [x_a, x_b]$. For $x_1(L)$ in this range, the high type would rather play $x^*(H)$ and face entry than try to mimic the low type by playing $x_1(L)$. Beyond $x_b$, the low type would rather play $x^*(L)$ and face entry.

b. Pooling equilibrium
- A necessary condition for pooling equilibrium to exist:
  - $pD_E(H) + (1-p)D_E(L) < 0$
  - If this condition does not hold then entry will occur after having seen the pooling strategy played. But if entry will occur anyway, then each type should play its static monopoly strategy in the first period.

- Range of potential pooling equilibria. Incentive compatibility constraints:
  - Incumbent of type $H$: $M(H, x_p) + \delta M_I(H) \geq M(H) + \delta D_I(H)$
  - Incumbent of type $L$: $M(L, x_p) + \delta M(L) \geq M(L) + \delta D_I(L)$




  - Incentive compatibility constraints are satisfied for $x_p \in [x_c, x_a]$
  - Below $x_c$, the low type would rather play $x^*(L)$ and face entry.
  - Above $x_a$, the high type would rather play $x^*(H)$ and face entry.

3. Specific numerical example
   a. Assumptions
      - Let the inverse demand function in periods 1 and 2 be given by:
        - $P(x, y) = 24 - x - y$, where $x$ is the quantity of the incumbent and $y$ is the quantity of the potential entrant
      - Costs of production
        - High cost incumbent: $c(H, x) = 12x$
        - Low cost incumbent: $c(L, x) = 6x$
        - Entrant: $c(y) = 12y + 12E$, where $E = 1$ with entry and 0 otherwise (fixed cost of entry)
      - No discounting: $\delta = 1$
      - Probabilities: $p = 0.5$
      - Static monopoly profits:
        - High type: Max $(24-x)x - 12x$; $x^*(H) = 6$; $M(H) = 36$
        - Low type: Max $(24-x)x - 6x$; $x^*(L) = 9$; $M(L) = 81$
      - Monopoly profits as a function of x:
        - High type: $M(H, x_1(H)) = (24 - x_1(H))\, x_1(H) - 12\, x_1(H)$
          $= (12 - x_1(H))\, x_1(H)$
        - Low type: $M(L, x_1(L)) = (24 - x_1(L))\, x_1(L) - 12\, x_1(L)$
          $= (18 - x_1(L))\, x_1(L)$

- Duopoly profits:
  - When $t = H$:

    Max $(24-x-y)x - 12x$ | Max $(24-x-y)y - 12y$
    $x_D(H) = 4$ | $y(H) = 4$
    $D_I(H) = 16$ | $D_E(H) = 16$

  - When $t = L$:

    Max $(24-x-y)x - 6x$ | Max $(24-x-y)y - 12y$
    $x_D(L) = 8$ | $y(L) = 2$
    $D_I(H) = 64$ | $D_E(H) = 4$

b. Separating equilibria
- Incentive compatibility constraint for the high type:
  - $M(H) + \delta D_I(H) \geq M(H, x_1(L)) + \delta M(H)$

    $36 + 16 \geq (12 - x_1(L))\, x_1(L) + 36$

    This will be true for $x_1(L) \geq 10.47$, or $x_1(L) \leq 1.53$
- Incentive compatibility constraint for the low type:

  $M(L, x_1(L)) + \delta M(L) \geq M(L) + \delta D_I(L)$

  $(18 - x_1(L))\, x_1(L) + 81 \geq + 81 + 64$

  This will be true for $4.88 \leq x_1(L) \leq 13.12$
- Separating equilibria exist over the range $x_1(L) \in [10.47, 13.12]$

c. Pooling equilibria
- Incumbent of type $H$:
  - $M(H, x_p) + \delta M(H) \geq M(H) + \delta D_I(H)$

    $(12 - x_p)\, x_p + 36 \geq 36 + 16$

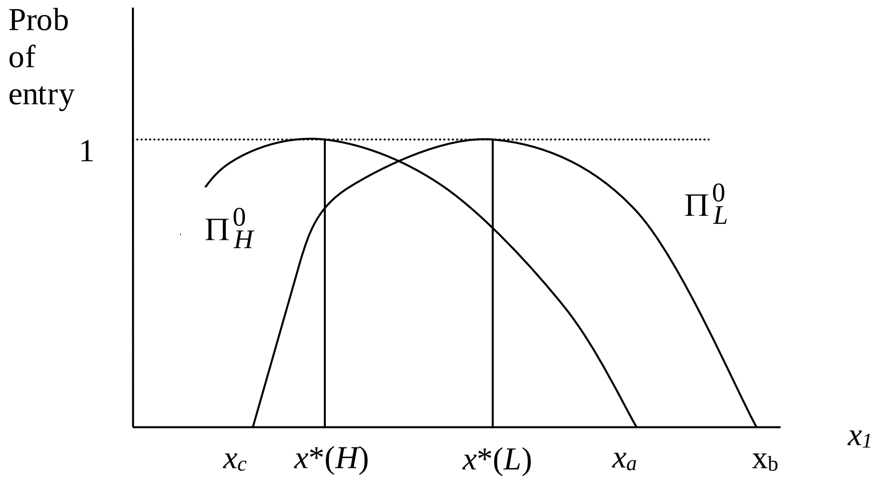
    This will be true for $1.53 \leq x_p \leq 10.47$
- Incumbent of type $L$:
  - $M(L, x_p) + \delta M(L) \geq M(L) + \delta D_I(L)$

    $(18 - x_p)\, x_p + 81 \geq + 81 + 64$

    This will be true for $4.88 \leq x_1(L) \leq 13.12$
- Pooling equilibria exist over the range $x_p \in [4.88, 10.47]$

4. Application of the intuitive criterion

a. Do any separating equilibria satisfy the Intuitive Criterion?
- Consider a separating equilibrium where $x_1(H) = x^*(H)$, and $x_1(L) = x_L$, with $x_a < x_L < x_b$.
- For some strategy $x_a < x_z < x_L$, $x_z$ will be equilibrium dominated for type $H$ but not for type $L$. Therefore, $p(H|\, x_z) = 0$. But then type $L$ would rather play $x_z$ than $x_L$, which means that $x_L$ is not an equilibrium that satisfies the Intuitive Criterion.
- The only separating equilibrium that satisfies the Intuitive Criterion is $x_1(H) = x^*(H)$, and $x_1(L) = x_a$; the entrant does not enter upon observing $x_a$ but does enter upon observing $x^*(H)$.




b. Do any pooling equilibria satisfy the Intuitive Criterion?
   - Consider a pooling equilibrium where $x_1(H) = x_1(L) = x_p < x^*(L)$.
     - Since $x^*(L)$ maximizes payoffs for type $L$ when there is no entry, $x^*(L)$ cannot be equilibrium dominated for type $L$.
     - However, $x^*(L)$ can be equilibrium dominated for type $H$. Equilibrium domination of $x^*(L)$ for type $H$ occurs when the payoff from playing the equilibrium pooling strategy ($x_p$) exceeds the payoff from playing $x^*(L)$ no matter what the entrant does. To check whether payoffs are higher playing $x_p$ than playing $x^*(L)$ no matter what the entrant does (including no entry) involves checking whether monopoly profits for type $H$ are higher with $x_p$ or $x^*(L)$. In the numerical example, $x^*(L) = 9$, which generates profits of 27. Monopoly profits for type $H$ are higher than 27 for any $x \in (3,9)$, which includes the entire range of pooling equilibrium less than $x^*(L)$.
     - Since $x^*(L)$ is equilibrium dominated for type $H$ but not type $L$, the entrant should believe $p(H|\ x^*(L)) = 0$. But then the entrant should not enter when observing $x^*(L)$. Therefore, type $L$ should play $x^*(L)$, which means that $x_p$ is not a pooling equilibrium.
   - Consider a pooling equilibrium where $x_1(H) = x_1(L) = x_p \geq x^*(L)$.
     - Now there are no strategies that are equilibrium dominated for type $H$ that are not also equilibrium dominated for type $L$. In this case, the Intuitive Criterion does not restrict off-equilibrium path beliefs to be other than $p(H|\ x^*(L)) = 1$, which does not reduce the set of pooling equilibria. Therefore, pooling equilibria for which $x_p \geq x^*(L)$ satisfy the Intuitive Criterion.

c. In sum, the Intuitive Criterion rules out many perfect Bayesian equilibria but unlike the Spence education signaling game, it does not produce a unique equilibrium prediction for this game. There are still a range of pooling equilibria and a separating equilibrium that survives the application of the Intuitive Criterion.

## IV. Screening Games

A. Introduction

1. In signaling games, the informed player moved first and the uninformed player can make an inference about the informed players' type by observing the action chosen.
2. In screening games, the uninformed player moves first followed by the informed player. The uninformed player doesn't know what type of player they face and so cannot condition their play on this information. Instead, they must choose an action (or menu of choices) from which the informed player can choose. The uninformed player must anticipate what types will choose which options.
3. Problem of **adverse selection**: actions taken by the informed player adversely affect the uninformed player. Example: trading game between buyer and seller where only the seller knows the quality of a good. When offering a price to all sellers, only those sellers who have low quality goods may find it advantageous to sell ("Market for Lemons" Akerlof *QJE* 1970).
4. Examples of screening games:
   a. Employer-employee: wage offer given to workers who can be of various skill levels but where the employer does not know the skill level of an individual worker
   b. Insurance contracts: insurance firm does not know whether a given individual is high or low risk
   c. Used car market (or trading games in general): buyer makes an offer on a used car which may be a good car (peach) or a bad car (lemon)

B. Simple screening model: competitive model of wages and employment

1. Basic model assumptions
   a. Many identical (perfectly competitive) risk-neutral firms can hire workers.
   b. Each worker hired produces output. Workers differ by skill level (type). The productivity of a worker is determined by their skill. Let $\theta$ be the amount of output produced by a worker, with $\theta \in [\underline{\theta}, \overline{\theta}]$. The cumulative density function is $F(\theta)$.
   c. The skill level of the worker is private information to the worker. Firms know only the distribution of types but not the type of any individual worker.
   d. A worker of type $\theta$ who does not work for a firm can work at home and earn $r(\theta)$. Assume $r'(\theta) \geq 0$.
   e. Output is sold in the market for a price of 1 per unit. The firm has no other costs besides labor costs. Since firms are perfectly competitive they will pay wages equal to the expected productivity of workers.
   f. Timing: firms offer wage ($w$). Workers decide which firm to work for or to work at home.

2. Competitive equilibrium
   a. Full information case: pay each worker $w = \theta$. Workers work for firms if and only if $w \geq r(\theta)$, and work at home if $w < r(\theta)$. This outcome is Pareto efficient.
   b. Incomplete information case.
      i. Worker choice: set of workers who will work for a firm $A(\theta; w) = \{\theta \mid r(\theta) \leq w\}$
      ii. Firms will want to hire workers if the expected productivity is equal or greater than the wage (but not otherwise): $E[\theta \mid \theta \in A(\theta; w)] \geq w$
      iii. Competitive equilibrium: a wage rate, $w^*$, and a set of worker types who work for firms, $A(\theta; w^*)$, such that $A(\theta; w^*) = \{\theta \mid r(\theta) \leq w^*\}$ and $E[\theta \mid \theta \in A(\theta; w^*)] = w^*$.
      iv. In competitive equilibrium, workers are paid their expected productivity and only those workers who earn an equal or higher wage at a firm than their earnings at home work for firms.
      v. A competitive equilibrium may not exist. (Akerlof Lemons model)
   c. Simple example where incomplete information causes the market to unravel: no workers work for firms except type $\theta = \underline{\theta}$ .
      i. Suppose that $\theta \sim Uniform[0,1]$; $F(\theta) = \theta$
      ii. Suppose that $r(\theta) = (2/3)\theta$,
      iii. Since $r(\theta) \leq \theta$ it is efficient for all workers to work for firms rather than work at home
      iv. Suppose firms set wage at $w$
      v. Solving for the set of workers who choose to work for firms at wage $w$, $A(\theta, w)$ are all types $\theta$ such that $r(\theta) \leq w$:

$$r(\theta) = (2/3)\theta \leq w$$

$$\theta \leq (3/2)w$$

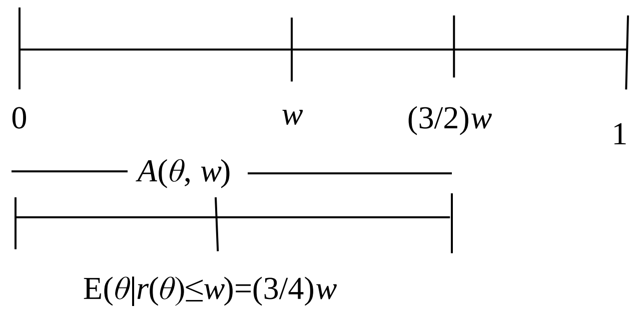
      vi. The expected productivity is:

$$\int_{\underline{0}}^{(3/.2)w} \theta f(\theta) d\theta$$

$$= \int_{\underline{0}}^{(3/.2)w} \theta \frac{1}{(3/2)w} d\theta$$

$$= \frac{\theta^2}{2}\Bigg|_0^{(3/2)w} \frac{1}{(3/2)w}$$

$$= \frac{[(3/2)w]^2}{2(3/2)w} = \frac{3}{4}w$$




vii. So, firms lose money by offering any $w > 0$. The only equilibrium strategy is to set $w = 0$.

viii. This example demonstrates the results found by Akerlof in the "Market for lemons" (Akerlof QJE 1970)

d. General case of competitive equilibrium: a wage rate, $w^*$, and a set of worker types who work for firms, $A(\theta; w^*)$, such that $A(\theta; w^*) = \{\theta \mid r(\theta) \le w^*\}$ and $E[\theta \mid \theta \in A(\theta; w^*)] = w^*$

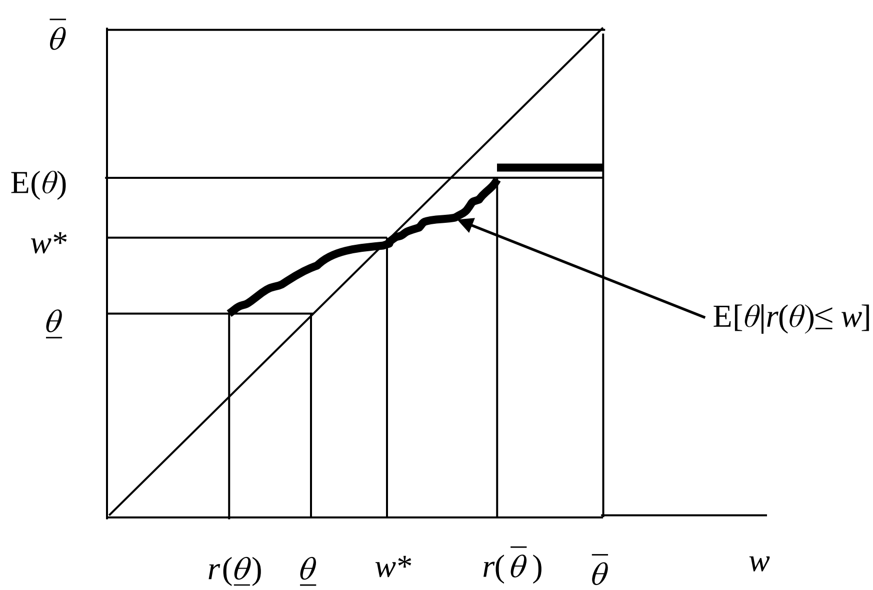
i. To solve for competitive equilibrium, first plot the expected productivity of workers who choose to work for firms for various levels of wages.

ii. No worker works for $w < r(\underline{\theta})$. At $w = r(\underline{\theta})$, $E[\theta \mid r(\theta) \le w = r(\underline{\theta})] = \underline{\theta}$

iii. For high wages, $w \ge r(\bar{\theta})$, all workers will work for firms so that $E[\theta \mid r(\theta) \le w \text{ when } w \ge r(\bar{\theta})] = E(\theta)$

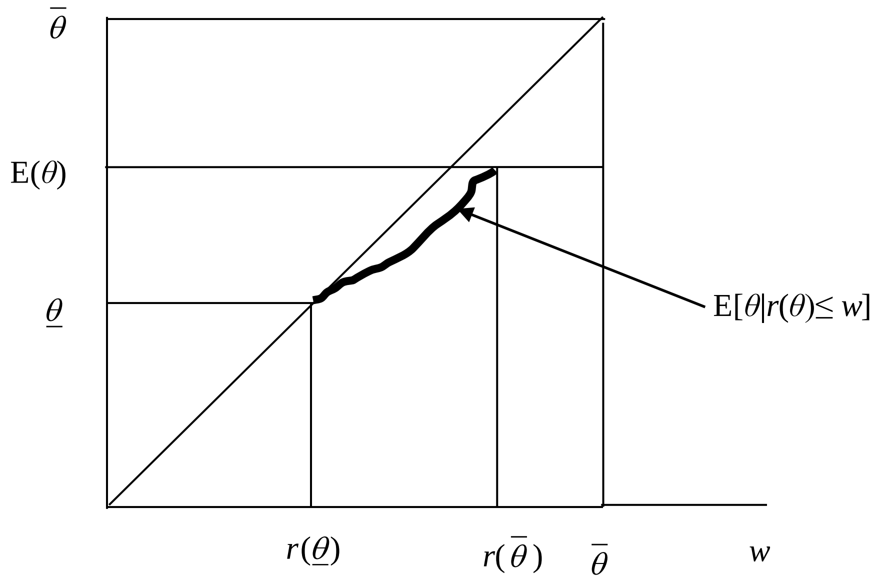
iv. As wage rises from $w = r(\underline{\theta})$ to $w = r(\bar{\theta})$, more workers will choose to work so that $E[\theta \mid r(\theta) \le w]$ rises.

v. Competitive equilibrium occurs where $E[\theta \mid r(\theta) \le w^*] = w^*$



e. There is no guarantee that the world is as well-behaved as shown in the prior diagram

i. Unraveling case




ii. Can also have multiple equilibrium – there is nothing that restricts the slope of $E[\theta \mid r(\theta) \leq w]$ as $w$ increases

f. The competitive equilibrium in the incomplete information case is typically Pareto inefficient: do not necessarily get the right workers working for firms. Incomplete information may cause the market to fail entirely (Market for lemons).

3. This model is extremely simple: workers have only one choice to make – they either work for a firm or they work at home. There is no choice of education, or effort level, or anything else. We next consider a screening model in which workers choose two things, education and whether to work for a firm (as in the Spence signaling model).

C. Screening model with education and wages

1. In Spence's education signaling model, workers (informed player) moved first and chose education, $e$, and then firms (uninformed player) offered wages based on education signal, $w(e)$
2. Suppose instead that firms (uninformed player) move first by offering menus of employment contracts ($w$, $e$) and then workers choose their most preferred contract
3. Assume everything else from the Spence model remains the same. Rules of the game

a. Two types of workers
   - High productivity: $t = H$, $p(t = H) = p$
   - Low productivity: $t = L$, $p(t = L) = 1 - p$

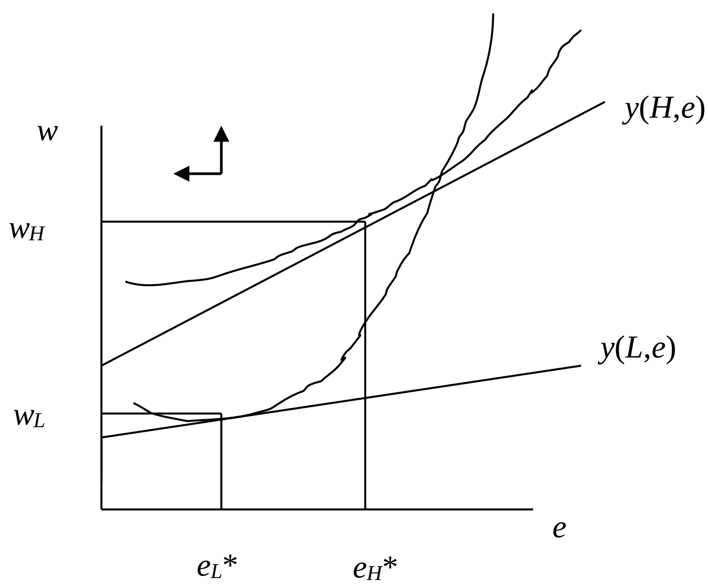
b. Payoffs
   - output: $y(t, e)$

- wage: $w$
- cost of education: $c(t, e)$
- Payoff for worker: $w - c(t, e)$
- Payoff for employer: $y(t, e) - w$

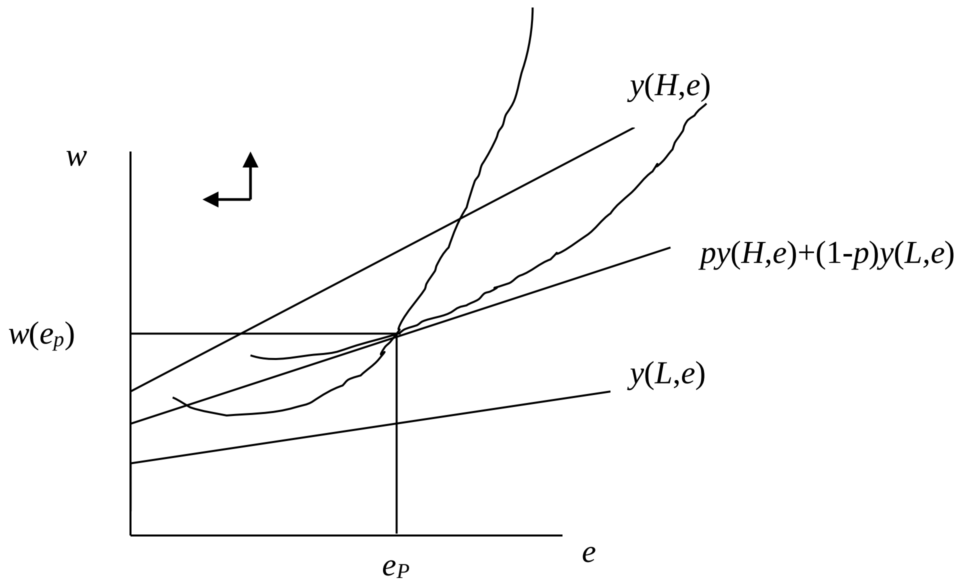
c. Single-crossing condition assumption: $\frac{\partial c(L,e)}{\partial e} > \frac{\partial c(H,e)}{\partial e}$, for all $e$.

d. Competitive firms so that they always earn zero profit: pay wages equal to expected productivity

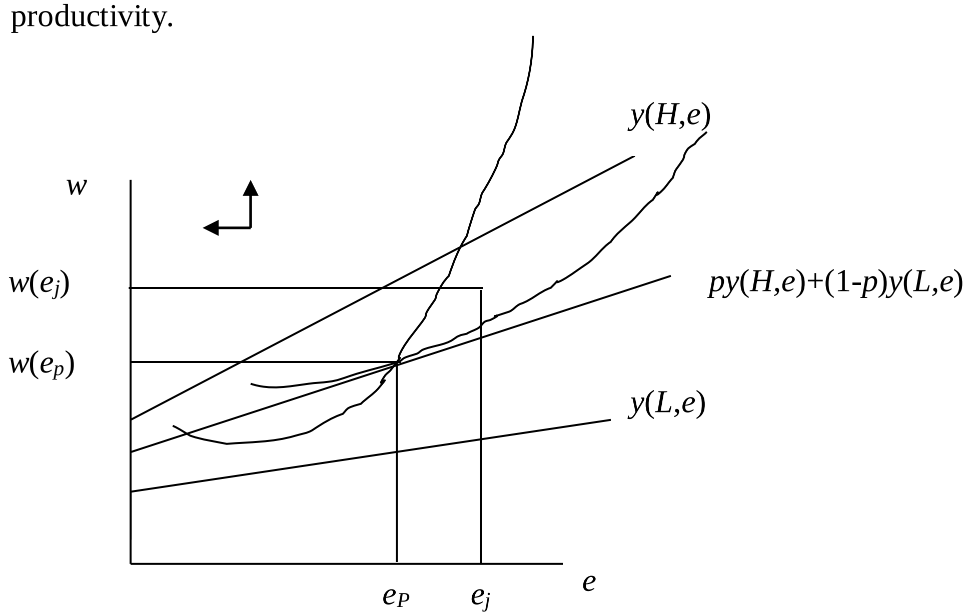
4. Analysis of potential equilibria: solve for subgame perfect Nash equilibrium
   a. Why is SPNE sufficient? Why don't we need perfect Bayesian equilibrium? Workers are fully informed so we can solve for how workers of each type will respond to the menu of contracts. Fold this back to the firm's decision and solve for what contracts should be offered (no Bayesian updating is needed)
   b. Complete information: efficient solution. Offer contract to $L$ type ($w_L$, $e_L$*) and H type ($w_H$, $e_H$*)




   c. Incomplete information.
      i. Efficient solution will not work. Offer to all workers the set of contracts ($w_L$, $e_L$*) , ($w_H$, $e_H$*), then ALL workers would choose ($w_H$, $e_H$*) but then firms would lose money because the expected productivity of workers is $py(H,e) + (1\text{-}p)y(L,e) < y(H,e)$ .
      ii. Look for pooling and separating equilibrium as in the Spence model. Consider all pure strategy equilibria in the Spence model (ignore hybrid equilibria).
      iii. Pooling equilibria: all firms offer the same contract ($w(e_p)$, $e_p$).

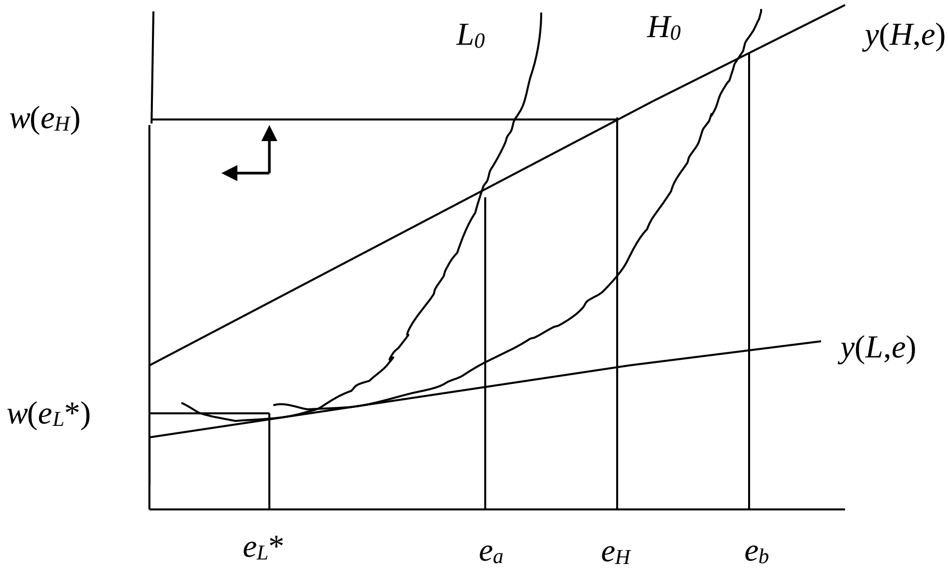



- For this contract, firms break even as workers are paid their expected productivity. And workers achieve higher utility than not working.
- In order for ($w(e_p)$, $e_p$) to be an equilibrium, there must be no profitable deviations, i.e., no firm can offer a different contract and earn positive profit.
- Suppose that one firm offers contract ($w(e_j)$, $e_j$). All high productivity workers prefer this contract to ($w(e_p)$, $e_p$) but no low productivity workers do. Therefore, ($w(e_j)$, $e_j$) will be profitable as the firm pays wages below productivity. The original pooling contract will now be unprofitable as a firm offering this contract will only employ low productivity workers so wage will exceed productivity.

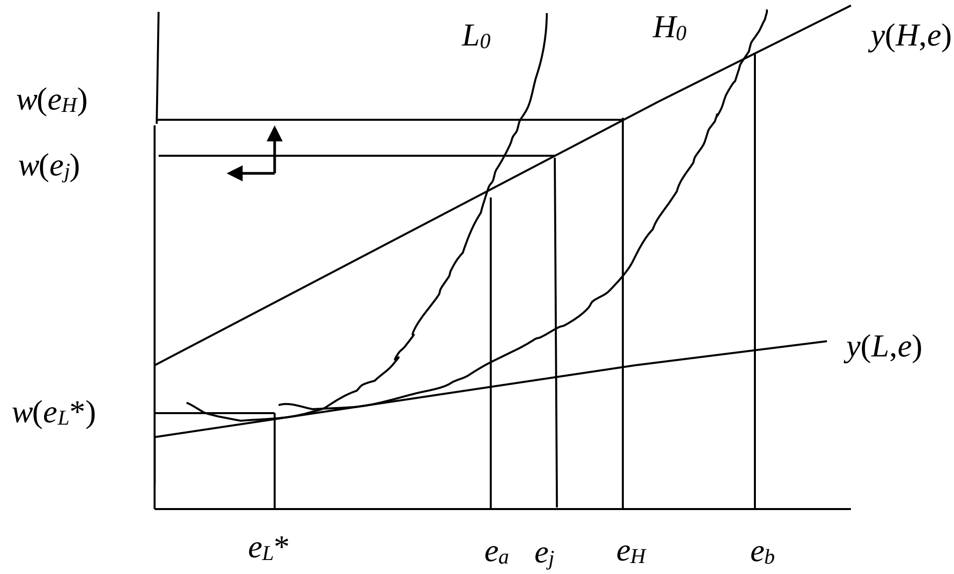


- No pooling equilibria exist.

iv. Separating equilibria: here different contracts are offered to high and low type workers ($w(e_H)$, $e_H$); ($w(e_L)$, $e_L$)




- For the low type: offer $e_L$* and pay $w(e_L*)$. If offer any other contract on the $y(L,e)$ productivity line, low type workers would view this as inferior to $w(e_L*) - c(L, e_L*)$.
- For the high type: offer $e_H$ that has high enough education so that it is preferred by the high type but not by the low type ($e_H \geq e_a$).
- Must make offers such that low type prefers the low type contract to the high type contract and the high type prefers the high type contract to the low type contract (incentive compatibility constraints):
  - For type $H$: $w(e_H) - \mathrm{c}(H, e_H) \geq w(e_L*) - c(H, e_L*)$
  - For type $L$: $w(e_L*) - c(L, e_L*) \geq w(e_H) - \mathrm{c}(L\ e_H)$
- The incentive compatibility constraints are true by construction of the contracts
- Is the separating equilibrium ($w(e_H)$, $e_H$); ($w(e_L*)$, $e_L*$) an equilibrium? What if some firm offers a contract ($w(e_j)$, $e_j$); with ej < eH.
- This new contract is strictly preferred by workers so all high types will abandon the original contact and accept the new contract.
- Therefore, the only pair of contracts that actually are a subgame perfect equilibrium is ($w(e_a)$, $e_a$); ($w(e_L*)$, $e_L*$)




5. Summary
   a. The unique subgame perfect equilibrium in the screening game is the same as the perfect Bayesian equilibrium that satisfies the Intuitive Criterion in the signaling game
   b. Simple switch in timing of moves often gives rise to very different dynamics in games – we may get very different types of outcomes in screening versus signaling games.
   c. Example: Akerlof's Lemons game. If instead of the uninformed player making the first offer, the informed player made the first offer you would not get market unraveling.