Applied Microeconomics: Firm and Household

The Almost Ideal Demand System

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Summary

- Estimating Demand Systems
- Almost Ideal Demand System

Demand systems for empirical work in economics

Demand system estimation plays an important role in empirical economics. Knowing the demand system facilitates:

- analyzing incentives facing firms in a given market
  - pricing decisions
  - alternative investments (i.e., advertising, new products)
  - understanding cross-price elasticities (defining/delineating markets)
- welfare analysis
  - e.g., value of innovation, introduction of a new product
- regulation/policy analysis
  - allow mergers? increase taxes (e.g., soda tax)? subsidize innovation?

Approaches to demand estimation

- representative agent model model
  - aggregate demand is derived from a single utility function
- heterogeneous agent model
  - aggregate demand is derived from the distribution of consumer characteristics

the analysis could be performed in either

- product space
  - consumers are assumed to have preferences over products
- characteristic space
  - consumers are assumed to have preferences over product characteristics

Digression: The expenditure function

Let $M(p, u)$ denote an expenditure function (the optimized value of the expenditure minimization problem):

- $M(p, u) \in R = px^h(p, u); \ \forall x^h \in x(p, u)$

where x^h is the optimal bundle derived from the expenditure minimization problem. The properties of an expenditure function are:

- Homogeneous of degree 1 and concave in p
- Non–decreasing in u and p
- Continuous in p and u

Almost Ideal Demand System (AIDS)

Consider the following expenditure function

- $ln(M(p_1, ...p_n, u)) = a(p_1, ...p_n) + ub(p_1, ...p_n)$

where

- $a(p_1, ...p_n) = \alpha_0 + \sum_i \alpha_i \ln p_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j$
- $b(p_1, ...p_n) = \beta_0 \prod_j p_j^{\beta_j}$

We impose the following restrictions from economic theory:

1. $\sum_i \alpha_i = 1 \quad \sum_i \gamma_{ij} = 0 \quad \sum_i \beta_i = 0 \quad (adding - up)$
2. $\sum_j \gamma_{ij} = 0 \quad (homogeneity)$
3. $\gamma_{ij} = \gamma_{ji} \quad (symmetry)$

The AIDS model

The first derivative of this expenditure function w.r.t. p_i gives the expenditure share of good i, denoted as s_i

- $\frac{\partial \ln M(p,u)}{\partial \ln p_i} = \underbrace{\frac{\partial M}{\partial p_i}}_{x_i} \frac{p_i}{M}$

- $\frac{\partial \ln M(p,u)}{\partial \ln p_i} = \frac{x_i p_i}{M} = s_i$

The AIDS Model

- $\ln(M(p,u)) = \alpha_0 + \sum_i \alpha_i \ln p_i + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln p_i \ln p_j + u\beta_0 \prod_j p_j^{\beta_j}$

Using Shephard's Lemma, the share equations are:

- $\frac{\partial \ln(M(p,u))}{\partial lnp_i} = \alpha_i + \sum_j \gamma_{ij} \ln p_j + p_i \left(\beta_i u \beta_0 p_i^{\beta_i - 1} \prod_{i \neq j} p_j^{\beta_j}\right)$
- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i u \underbrace{\beta_0 \prod_j p_j^{\beta_j}}_{b(.)}$
- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i u b(.)$

The AIDS Model

We can express the term u in terms of money income M as

- $u = \frac{lnM - a(.)}{b(.)}$

we obtain the share equations by substitution for u

- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i \frac{lnM - a(.)}{b(.)} b(.)$
- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i (lnM - a(.))$
- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\left(\frac{M}{\mathbf{P}}\right)$

where $ln\mathbf{P} = a(.)$ is a price index.

The LA/AIDS Model

Dealing with the "exact" price index can be tricky as it makes the model non-linear.

Deaton and Muelbauer argue that if prices are highly collinear, which we might expect since goods are assumed to be closely related, **P** can be approximated by Stone Price Index to obtain a linear approximate version of AIDS model

Moschini (1995) argues that "corrected" Stone's price index provides better approximation. These price indices are

- Stone's Price index: $ln\mathbf{P} = \sum_i s_i \ln p_i$
- Corrected Stone's Price index: $ln\mathbf{P} = \sum_i s_i \ln(p_i/\bar{p}_i)$

where $p_i/\bar{p}_i$ are normalized prices with their respective averages.

Estimation of the LA/AIDS Model

We must omit one of the share equations in estimation, because the sum of the shares equals one. So we estimate $n-1$ equations rather than n.

For example, consider the following LA/AIDS model:

- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \sum_j \mu_{ij} lnZ_j + \beta_i ln\frac{M}{P}, \quad i =$ beef, pork, chicken

where Z_j denotes the matrix of demand shifters such as demographic variables and/or advertising.

Estimation of the LA/AIDS Model

- $s_i = \alpha_i + \sum_j \gamma_{ij} lnp_j + \beta_i ln\frac{M}{P}, \quad i =$ beef, pork, chicken
- we get the adding-up restrictions for free when we omit one of the share equations
  - hence the adding-up restrictions cannot be tested statistically
- homogeneity and symmetry restrictions can be imposed via parameter restrictions and *can* be tested statistically.

Parameter estimates can be obtained by 3SLS.

Three-Stage Least Squares (3SLS)

- OLS estimation on price and quantity data doesn't identify anything (draw on board)
- Solution: 2SLS. Find a demand shifter to identify supply curve, or a supply shifter to identify a demand curve.
- But 2SLS pretends that the demand for each good is independent. That's wrong!
- 3SLS = 2SLS, plus a step to correct for correlations across equations

"Identify" here means to find a consistent estimate of a parameter in the model. There are other meanings, e.g. having enough equations to solve for a set of unknowns. The latter meaning sometimes is used in structural estimation, causing confusion.

Next Class

We will start our discussion of producer theory. Please read

NS Chapter 9: Production Functions