APEC 5151: Applied Microeconomics: Firm and Household
Fall 2016 Final Exam SOLUTIONS
December 20, 2016

NAME:

Do not open the test until you are instructed to do so. Everyone will start the test at the same time.

There are 125 possible points on this test, and 15 total questions plus a bonus question. You will have 120 minutes. If you answer a question every seven minutes (or 1.2 points per minute) you will have 15 minutes at the end to check your work. The two bonus points at the end will be added to your score but cannot raise it above 100.

You are allowed to use a simple (non-graphing) calculator during the test, but you should not need a calculator to answer any of the questions. You can leave any fractions in unsimplified form.

If you do not know how to answer a question, write down what you do know, or skip it and come back to it later.

Be sure to answer all parts of each question, as several require multiple answers. Please show all steps of your work. If you use a graph in answering a question, please make sure it is labeled appropriately.

Part A. All firms in a perfectly-competitive market have the following cost function

$$C = 100 + 5q^2 + 10q$$

where C is the total cost of production and q is the firm's level of output.
Inverse demand for the good is given by

$$P_D = 200 - 20Q$$

where Q is the *total* amount of the good in the market.

1. (10 points) Find the shutdown price for firms in this market.

   $MC = 10q + 10$
   $AVC = 10 + 5q$
   $AVC = MC$ for shutdown price
   $10 + 5q = 10q + 10$
   $5q = 10q$
   $0 = 5q$
   $q = 0$
   $P_{shutdown} = 10$

2. (10 points) Find a formula for the long-run equilibrium number of firms in this market, assuming the number can vary continuously (i.e. does not have to be an integer). *(Your answer should be a specific number but you can leave fractions, radicals, exponents, etc. unsimplified).*

$MC = 10q + 10$
$ATC = 100/q + 10 + 5q$
$ATC = MC$ for breakeven price
$100/q + 10 + 5q = 10q + 10$
$100/q + 5q = 10q$
$100/q = 5q$
$100 = 5q^2$
$20 = q^2$
$q = \sqrt{20} = 2\sqrt{5}$
Breakeven price $= MC(2\sqrt{5}) = 10 * 2\sqrt{5} + 10$
$P_{breakeven} = 20\sqrt{5} + 10$
At long-run equilibrium, $P = P_{breakeven}$ and each of n firms produces $2\sqrt{5}$ units.

$20\sqrt{5} + 10 = 200 - 20Q$
$20\sqrt{5} = 190 - 20n * 2\sqrt{5}$
$40n\sqrt{5} = 190 - 20\sqrt{5}$
$n = (190 - 20\sqrt{5})/(40\sqrt{5})$
$n = 190/(40\sqrt{5}) - 1/2$
$n = 19/(4\sqrt{5}) - 1/2$

3. (10 points) Solve for the long-run equilibrium price and quantity of the good produced in this market. *(Your answers should be a specific number but you can leave fractions, radicals, exponents, etc. unsimplified).*

At long-run equilibrium, $P = P_{shutdown}$ and each of n firms produces $2\sqrt{5}$ units.

$Q = nq$ $Q = (19/(4\sqrt{5}) - 1/2)(2\sqrt{5})$
$Q = 19/2 - \sqrt{5}$

Part B. Gormel foods produces CRAM and has the following cost fuction:

$$C = 6Q^2 + 12$$

where C is the total cost of production, Q is the number of cans of CRAM produced. Jason is the only consumer in this market. His demand for CRAM by

$$Q = 12 - P/4$$

where P is the price of a can of CRAM.

1. (10 points) Suppose Gormel operates as a single-price monopolist. Solve for the price it charges and the quantity it sells, and Gormel's profits.

$MC = 12Q$
$P/4 = 12 - Q$
$P = 48 - 4Q$
$MR = 48 - 8Q$
$48 - 8Q = 12Q$
$48 = 20Q$
$Q = 48/20$
$Q = 12/5$
Plug into inverse demand:
$P = 48 - 48/5$
$P = (4 * 48)/5$
$P = 192/5 = 38.4$
Profits:
$\pi = (12/5) * 192/5 - 6 * (12/5)^2 - 12$
$19 * (12/5)$

2. (10 points) Suppose that Gormel knows Jason's full demand curve and resale of CRAM is impossible because he is just one person. If we relax the assumption that Gormel is a single-price monopolist, suggest an alternative pricing policy that Gormel could use to earn the maximum conceivable level of profits in this market. What are Gormel's profits under this policy? What is Jason's level of consumer surplus?

Two-part tariff. Set P equal to MC, charge entry fee equal to CS.
$MC = 12Q$
$P/4 = 12 - Q$
$P = 48 - 4Q$
$12Q = 48 - 4Q$
$16Q = 48$
$Q = 3$
$P = 3 * 12 = 36$
Profits equal entire social surplus:
Consumer side (captured by entry fee) is $(1/2) * (48 - 36) * 3 = 0.5 * 12 * 3 = 6 * 3 = 18$
Producer side (captured by price is $(1/2) * (36 - 0) * 3 = 18 * 3 = 54$
Profits are \$72.
CS is zero.

3. (5 points) Replace Gormel's cost function with the following function:

$$C = 6bQ^2 + 12$$

where b is a positive parameter. Assume that Gormel is a single-price monopolist. Find the elasticity of Gormel's profits with respect to b, $\frac{\partial \pi}{\partial b}\frac{b}{\pi}$.

$MR = MC$
$48 - 8Q = 12bQ$
$\pi = TR - TC = (48 - 4Q) * Q - 6bQ^2 + 12$
$48 - 8Q - 12bQ = 0$
$48 = (8 + 12b)Q$
$Q = 48/(8 + 12b)$
$\pi = (48 - 4(48/(8 + 12b))) * 48/(8 + 12b) - 6b(48/(8 + 12b))^2 + 12$
$\pi = 48^2/(8 + 12b) - 4(48^2/(8 + 12b)^2) - 6b(48/(8 + 12b))^2 + 12$
$\pi = 48^2(8 + 12b)/(8 + 12b)^2 - (4 * 48^2)/(8 + 12b)^2 - 6b(48/(8 + 12b))^2 + 12$
$\pi = (48^2(8 + 12b) - 4 * 48^2 - 6 * b * 48^2)/(8 + 12b)^2 + 12$
$\pi = 48^2((8 + 12b) - 4 - 6 * b)/(8 + 12b)^2 + 12$
$\pi = 48^2(8 + 12b - 4 - 6 * b)/(8 + 12b)^2 + 12$
$\pi = 48^2(4 + 6b)/(8 + 12b)^2 + 12$
$\pi = 48^2(1/2)/(8 + 12b) + 12$
$\pi = (24 * 48)/(8 + 12b) + 12$
$\partial\pi/\partial b = (-1) * (12) * (24 * 48)/(8 + 12b)^2$
$\partial\pi/\partial b = (-13824)/(8 + 12b)^2 < 0$
$\varepsilon_{\pi,b} = \partial\pi/\partial b(b/\pi) = (-13824)/(8 + 12b)^2 * b/((24 * 48)/(8 + 12b) + 12)$

Part C. Nehanda and Shaka live together on an island, where they get coconuts (C) and fish (F) for free each day. Akbar's initial endowments are given by $(\bar{C}_N, \bar{F}_N)$. Shaka's initial endowments are given by $(\bar{C}_S, \bar{F}_S)$. Nehanda's utility function is $u^N(C_N, F_N)$ and Shaka's utility function is $u^S(C_S, F_S)$.

1. (10 points) Draw an Edgeworth box representing the economy. Be sure to indicate the following:

   (a) The two agents' initial endowments.

   (b) The two agents' initial indifference curves.

   (c) A curve representing *all* Pareto-optimal allocations in this economy, sometimes called the "contract curve".

   (d) The set of Pareto-optimal endowments that are feasible outcomes of this market, sometimes called the "core".

   See the lecture slides for what an Edgeworth box looks like.

2. (10 points) What conditions must hold for an equilibrium allocation in this market? Make sure to list all of them, and be as specific as possible.

   Two conditions

   1. All agents must optimize simultaneously.
   2. Markets must clear - consumed quantities equal total endowments, everyone faces same

prices.
In this case:
Same prices for all agents implies that MRS for each agent equals same price ratio, so the two MRSs must be equal.
$u_C^N/u_F^N = u_C^S/u_F^S$
Market clearing implies that total equilibrium consumption of C and F equals initial endowments:
$(C_N^* + C_S^*, F_N^* + F_S^*) = (\bar{C}_N + \bar{C}_S, \bar{F}_N + \bar{F}_S)$

Part D. Sam and Tiffany are playing a game where Sam chooses L or R and Tiffany chooses a or b. Their payoffs are written (Sam's payoff, Tiffany's payoff) If Sam chooses L and Tiffany chooses a, the payoffs are (10,3). If Sam chooses L and Tiffany chooses b, the payoffs are (2,-1). If Sam chooses R and Tiffany chooses a, the payoffs are (1,3). If Sam chooses R and Tiffany chooses b, the payoffs are (4,7).

1. (10 points) Suppose Sam and Tiffany move simultaneously. Write down this game in normal form. Identify all the Nash equilibria of the game.

See the lecture slides for what the normal form of a game looks like.

Nash equilibria are (L,a) and (R,b)

2. (10 points) Now suppose Sam moves first. Write down the game in extensive form. Identify *all* subgames of this game.

   See the lecture slides for what the extensive form of a game looks like.

   There are three subgames: (i) the entire game, (ii) everything after the node where Tiffany plays after Sam plays L, and (iii) everything after the node where Tiffany plays after Sam plays R.

3. (10 points) Identify all the subgame perfect Nash equilibria of the sequential-move version of the game.

   Subgame perfection eliminates one of the two Nash equilibria. Tiffany could claim that

   her strategy is to play b no matter what. This would make Sam's best response to play R. But if Tiffany sees L played she will play a — this is not a credible threat. Knowing this, Sam will play L and the outcome will be (L,a). This is the unique subgame perfect Nash equilbrium of the game.

Part E Individual questions:

1. (5 points)A consumer has an indirect utility function given by

$$AI\frac{1}{4}^{\frac{1}{4}}\frac{3}{4}^{\frac{3}{4}}p_x^{-\frac{1}{4}}p_y^{-\frac{3}{4}}$$

   where A is a constant, I is income, p_x is the price of good x and p_y is the price of good y. Find a function for the consumer's demand for good y.

   By Roy's identity, $x_i^* = -\frac{\partial u^*/\partial p_i}{\partial u^*/\partial I}$

   So $y^* = -\frac{\partial u^*/\partial p_y}{\partial u^*/\partial I}$
   $y^* = -\frac{-AI\frac{1}{4}^{\frac{1}{4}}\frac{3}{4}^{\frac{7}{4}}p_x^{-\frac{1}{4}}p_y^{-\frac{7}{4}}}{A\frac{1}{4}^{\frac{1}{4}}\frac{3}{4}^{\frac{3}{4}}p_x^{-\frac{1}{4}}p_y^{-\frac{3}{4}}}$
   $y^* = -(-I\frac{3}{4}p_y^{-1})$
   $y^* = \frac{3I}{4p_y}$

2. (5 points) Your friend tells you that they are going to quit their job and start a small business where they will be the owner-operator and not pay themself a salary. Their business plan shows that they will earn a large accounting profit. Explain why their plan greatly overstates the *economic* profit they will earn.

   In a perfectly-competitive market, if your friend kept their job they would earn a salary equal to the excess accounting profit they would earn by running their own company.

3. (5 points) Consider the following hypothetical expenditure function:

$$E(p_1, p_2, u_0) = \frac{p_1 p_2}{u_0}$$

Show that this is not a legitimate expenditure function for a rational consumer.

Expenditure functions are homogeneous of degree 1 in prices. But if we double all prices in this function then expenditure rises by a factor of 4.

4. (5 points) Compare the optimization problem for the first-mover in a Stackelberg game to the optimization problem for a Cournot competitor. How are the two optimization problems different? Why does this difference exist?

A Stackelberg leader's optimization problem includes their competitor's Cournot reaction

function instead of their competitor's quantity. This happens because the Stackelberg leader knows that their competitor has no choice but to choose their optimal quantity conditional on the leader's choice and so will act as a monopolist on the residual demand curve. In contrast, under Cournot competition the competitor can choose whatever quantity they want.

5. BONUS (2 points) We have repeatedly discussed the "money illusion" in this course. Explain in a sentence or two what the money illusion is and why it does not affect the behavior of agents in a neoclassical rational choice model.

The money illusion is the idea that the numbers we put on prices and income do not

matter, only how they compare to one another, so creating new dollars that are each worth 10 old dollars will have no impact on anyone's behavior. The money illusion does not affect neoclassical agent's behavior because it does not change the tangency conditionsn they face at all, and because their budget constraints and isocost curves remain unchanged once you divide by the scale factor.