Solutions for Problem Set #4: Partial Equilibrium

A. (25 Points) Elasticity Problems
1. Computing elasticities: Buschena and Perloff (1991) estimate the following demand function for coconut oil: $Q = 1,200 - 9.5p + 16.2p_p + 0.2Y$. where Q is the quantity of coconut oil demanded in thousands of metric tons per year, p is the price of coconut oil in cents per pound, p_p is the price of palm oil in cents per pound, and Y is average consumer income in thousands of dollars per year. What are the own- and cross-price elasticities of demand for coconut oil when p is 45 cents per pound, p_p is 30 cents per pound, and income is $20,000 per year?

The own-price elasticity of demand is $\epsilon = \frac{\partial Q}{\partial p}\frac{p}{Q}$. $p = 45$, $\frac{\partial Q}{\partial p} = -9.5$, and $Q = 1,200-(9.5)(45)+(16.2)(30)+(0.2)(20) = 1,200-427.5+486+4 = 1262.5$, so the own-price elasticity of demand for coconut oil is $\epsilon = \frac{(45)}{(1262.5)}(-9.5) \approx -0.34$.

The cross-price elasticity of demand is $\epsilon = \frac{\partial Q}{\partial p_p}\frac{p_P}{Q}$. $p_p = 30$, $\frac{\partial Q}{\partial p_p} = 16.2$, and $Q = 1,200 - (9.5)(45) + (16.2)(30) + (0.2)(20) = 1,200 - 427.5 + 486 + 4 = 1262.5$, so the cross-price elasticity of demand for coconut oil with respect to the price of palm oil is $\epsilon = \frac{(30)}{(1262.5)}(16.2) \approx 0.38$.

2. Using elasticities to estimate the impact of price changes: The own-price elasticity of demand for coffee is -0.8 and the cross-price (with respect to the price of tea) elasticity of demand for coffee is 0.2. Starting from a coffee price of $6 per kilogram, a tea price of $5 per kilogram, and a quantity demanded of 40,000 kilograms per month, if the price of tea increases to $7.50 per kilogram, does the quantity of coffee demanded increase or decrease? By how much?

Tea prices are what changes here, so the relevant elasticity is the cross-price elasticity of demand for coffee with respect to the price of tea, 0.2. The price of tea rises by $2.50, from $5 to $7.50, for a 50% increase. The percentage change in the quantity of coffee demanded is $(0.2)(50\%) = 10\%$. Thus, the quantity of coffee demanded increases by 10% (or 4,000 Kg per month) from 40,000 Kg per month to 44,000 Kg per month.

3. Using observed point elasticities to estimate demand functions: Suppose the own-price elasticity of demand is -0.75 at a price of 10 and a corresponding quantity of 400. Suppose the demand function is linear, with the form $Q = b - mp$ where b and m are constants. What are the values of b and m that are consistent with the given information?

From the linear functional form, we know that $\frac{\partial Q}{\partial p} = -m$ and by the elasticity formula evaluated at the known point, $-0.75 = \frac{\partial Q}{\partial p}\frac{p}{Q} = \frac{(10)}{(400)}(-m)$ or $m = 30$. Thus $Q = b - 30p$. Using the known datapoint, 400 = b - 30(10) or $b = 700$. The unique linear demand function consistent with our original information is $Q = 700 - 30p$.

B. (25 points) Consider a set of firms with identical cost functions $c(q) = 90 + 20q + 0.1q^2$ for $q > 0$, and $c(0) = 0$, where q is firm an individual firm's output. Suppose aggregate demand is $D(p) = 800 - 10p$.

1. What is the supply function for a single firm?

   Note that since $c(0) = 0$ there are no sunk costs. That means that all costs are relevant for the decision about whether to shut down. Thus the shutdown price is the minimum value of the ATC curve, which occurs at $MC = ATC$. MC is $\frac{\partial c(q)}{\partial q} = 20 + (0.2)q$ for $q > 0$. ATC is $\frac{c(q)}{q} = 90/q + 20 + (0.1)q$ for $q > 0$. Setting $MC = ATC$ and solving for q, we obtain $q^2 = 900$, or $q = 30$ as the output at which average cost is minimized. The average cost at $q = 30$ is $ATC(30) = (90/30) + 20 + (0.1)(30) = 26$. For $p < 26$ firm cannot cover its non sunk costs, and it is optimal to produce nothing, $s(p) = 0$. If $p = 26$ exactly, the firm could just cover its costs by producing output 30, but output 0 is just as good, so $s(26) = 0$ or 30. For $p > 26$, we must solve $p = MCq$ for q to find the optimal output. Setting $p = 20 + (0.2)q$ and solving for q, we obtain $s(p) = 5p - 100$ for $p > 26$. Thus, the supply function for a single firm is:

$$s(p) = \begin{cases} 0, & \text{if } p < 26; \\ 0 \text{ or } 30 & \text{if } p = 26; \\ 5p - 100 & \text{if } p > 26. \end{cases}$$

2. If there are 10 firms, what is the competitive equilibrium price and quantity? What is consumer surplus? What is each firm's producer surplus (profit)?

   The aggregate supply function is the sum of the supply functions for the individual firms. At $p = 26$, each firm is indifferent between two outputs, so the sum must take into accoutn all of the different possible combinations of optimal choices. Since the firms are identical, for prices other $p = 26$, $S(p) = 10s(p)$. Thus aggregate supply is

$$S(p) = \begin{cases} 0, & \text{if } p < 26; \\ 0 \text{ or } 30 \text{ or } 60 \text{ ... or } 270 \text{ or } 300 & \text{if } p = 26; \\ 50p - 1000 & \text{if } p > 26. \end{cases}$$

   At $p = 26$, D(p) = 540, whereas the maximum of S(26) could be 300. Thus the equilibrium price must exceed 26. At $p > 26$, $S(p) = 50p - 1000 = 800 - 10p = D(p)$. Solving for p yields $p^* = 30$. The corresponding aggregate quantity is $D(p^*) = 800 - 10(30) = 500$. Each firm produces $s(30) = 50$ units of output. Since there are no sunk costs, producer surplus is the same as profit. Each firm's profit is $\pi = (50)(30) - (90 + 20(50) + 0.1(50)^2) = 1500 - 1340 = 160$. (Total surplus is $n\pi = (10)(160) = 1600$). To calculate consumer surplus, we first note that the demand curve intersects the price axis at $0 = 800 - 10p \to p = 80$. Thus $CS = (80 - 30)(500)/2 = 12,500$.

3. In the long run firms are free to enter or leave the industry. What is the long run competitive equilibrium price and quantity? How many firms operate in this industry, and what is each firm's producer surplus?

   In the long run, the only possible equilibrium price is $p = 26$. To see this first note that at $p < 26$, aggregate supply is zero, whereas $D(p) > 0$. Therefore entry by at least one firm is profitable. However, at $p > 26$ firms make positive profits which is not possible in the long run due to free entry by other firms. Thus, the only possible equilibrium price $p = 26$, at which firms earn zero economic profit producing either zero or 30. Since $D(26) = 540$, there must be just the right number of firms, N, producing output 30 so that $30N = 540$. Thus, in the long run the equilibrium price is $p = 26$, aggregate quantity is $Q = 540$, there are $N = 540/30 = 18$ firms (and technically a possibly-infinite number of firms producing zero), with each firm making zero economic profit. (Economic profits are zero since there can't be any sunk costs in the LR, and hence each firm's surplus is equal to their profit, which is zero).

4. Suppose the "90" in each firm's cost function is the salary that must be paid to get a competent manager to work for the the firm and run its production process (Each of the many potential managers have alternative job opportunities that pay 90 per period). Five new potential managers just graduated from an applied economics program in which they learned how to reorganize the production process for a firm in this industry. Any firm with one of these managers will have "$14q + 0.1q^2$" instead of "$20q + 0.1q^2$" in its cost function, and all the firms know this. Unfortunately the information is industry specific, so the managers have the same alternative job opportunities that pay salary 90 per period as all other potential managers. Taking account the competition for managers, what is the new long-run equilibrium in the industry? Do these new managers work in this industry? If so, what are their salaries, and how do their firms' equilibrium output levels and levels of producer surplus compare to those of other firms in the industry?

   If these new managers bring any advantage to a firm, then the firms will compete to hire the mangers, driving the salary for the special managers up until the firms that hire them are back to the normal rate of return, i.e. a producer surplus of zero. (What is special is the managerial ability of these 5 *managers*, not any particular *firm*. Thus the return to the firms that hire a special manager must be no better than for a firm that does not hire one of the special managers, or another firm would pay a slightly higher salary and scoop the manager up. The 5 managers are identical to each other, but are not identical to typical managers. Thus in equilibrium the 5 managers will get the same salary, but that salary can be higher than the 90 per period paid to a typical manager.)

   To determine the advantage one of these managers brings to a firm, suppose one of the special managers could be hired for 90 per period. The the firm would have cost function $90 + 14q + 0.1q^2$, and its supply function would be

$$s^*(p) = \begin{cases} 0, & \text{if } p < 20; \\ 0, \text{ or } 30 & \text{if } p = 20; \\ 5p - 70 & \text{if } p > 20. \end{cases}$$

   At $p = 26$, such a firm would produce $s * (26) = 60$ and obtain profit $(26)(60) - [90 + (14)(60) + 0.1(60)^2] = 270$. Note that at price 26, 5 such firms produce output $300 < D(26)$, so all 5 managers can work in this industry in equilibrium. As the firms compete to hire these special mangers, the special managers' salaries are bid up to $90 + 270 =$

360. A firm with one of the special managers at salary 360 per period has cost function $360 + 14q + 0.1q^2$, and its supply function will be

$$s^{**}(p) = \begin{cases} 0, & \text{if } p < 26; \\ 0, \text{ or } 60 & \text{if } p = 26; \\ 5p - 70 & \text{if } p > 26. \end{cases}$$

The increased salary for the special manger has raised the firm's shut down price back up to 26, the same as for a firm with a regular manager at salary 90 per period.

The equilibrium price remains 26, with aggregate quantity $D(26) = 540$ as originally. At price 26, since the firms with special managers produce 60 rather than the 30 produced by firms with regular managers, in the new equilibrium there will be only 13 firms, 5 with special managers and 8 with regular managers. All firms obtain producer's surplus 0. Even though firms are indifferent between hiring regular and special managers (the reduced production cost is exactly counterbalanced by the increased salary), all the special managers must be employed in this industry since they strictly prefer it. Special managers get a salary of 360 per period in this industry, as opposed to only 90 per period at their alternative jobs. Regular managers still get 90 per period.

C. A market is supplied competitively by 50 low-cost firms, each with cost curve $C_l(q) = 350 + 2q + q^2$, and n high-cost firms, each with cost curve $C_h(q) = 400 + 2q + q^2$. Market demand is $Q = 2500 - 10p$. If none of the high-cost firms earns a positive profit, how large is n? How much profit do the low-cost firms make?

For the high-cost firms, price must equal both average and marginal cost: $p = \frac{400}{q} + 2 + q = 2 + 2q$. Solving for q and p, we find that $q = 20$ and $p = \$42$. Hence, $Q = 2,080$. The low-cost firms produce the same output as the high-cost firms, because they face the same price and have the same marginal cost curve. It follows that n is $2,080/20 - 50 = 54$. The low-cost firms each make a profit of \$50.

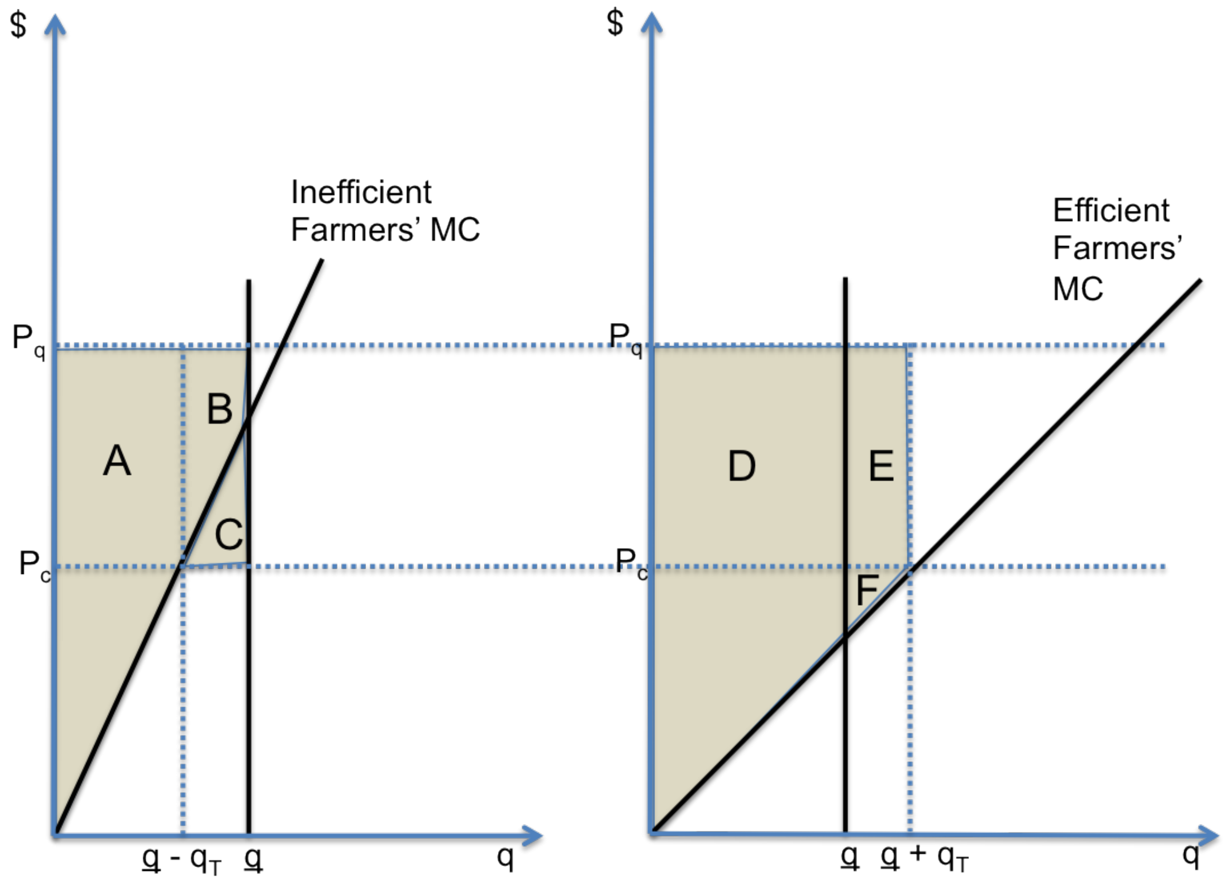
D. (25 Points) Analysis of a government quota with trade:

The government has decided to restrict overall milk output to a maximum level of $\underline{Q}$. It knows that half of the n dairy farmers in the country are efficient, with milk production costs equal to $c(q) = F + q^2$, and half are inefficient, with costs $C(q) = F + 2q^2$. however, government cannot tell which farmers are which. Using the diagram below (or redrawing it yourself),

1. Show that imposing the same output quota $\underline{q} = Q/n$ on all farmers is an inefficient way of achieving the government's objective.

2. What would be the total efficiency gains if farmers were allowed to trade quotas?

3. With tradable quotas, what is the optimum quantity of quotas traded and the price of each quota?


$
Inefficient Farmers' MC
P_q
P_c
A
B
C
q - q_T
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q
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Efficient Farmers' MC
P_q
P_c
D
E
F
q
q + q_T
q

Note: P_q is the market price under the quota, and P_c is the price if the market were perfectly competitive.

If the same output quota $\underline{q}$ is imposed on both efficient and inefficient farmers, the producer surplus of both types combined is given by the area A + B + D in the diagram. If, instead, the inefficient farmers produce only $\underline{q} - q_T$, while the efficient farmers produce $\underline{q} + q_T$, overall output is unaffected, but producer surplus is equal to the area A + D + E + F. Because area E alone is equal to areas B + C, overall producer surplus is higher. (It is in fact maximized, subject to the constraint that overall output is $2\underline{q}$.) Since the government cannot tell which producers are high-cost and which are low-cost, it will need to find a mechanism for achieving this more-efficient outcome. For example, the above-described inefficiency can be reduced or eliminated by allowing farmers to trade their quota assignments with one another. If a competitive market for the quotas develops, inefficient farmers will in equilibrium sell q_T units to efficient farmers, at a price $P_q - P_c$, which is equal to the revenue from last unit of production for both farmers. (Note that the MC of inefficient farmers at $\underline{q} - q_T$ is equal to the MC of efficient farmers at $\underline{q} + q_T$). The net gain from such a market for permits is equal to area C for inefficient farmers and area F for efficient farmers.

For a formal discussion, note that at the equilibrium the revenue from last unit of production for both farmers must be equal. That is,

- $P_q - MC_E(\underline{q} + q_T) = P_q - MC_I(\underline{q} - q_T)$.
- $MC_E(\underline{q} + q_T) = MC_I(\underline{q} - q_T)$
- $2\underline{q} + 2q_T = 4\underline{q} - 4q_T$
- $q_T = \frac{1}{3}\underline{q}$

The revenue from the last unit of production for both types of farmer is the price of a single unit of the quota. For example, take the revenue of efficient firms, $P_q - MC_E(\underline{q} + q_T)$. At equilibirum, $q_T = (1/3)\underline{q}$), the price of one unit of a quota is $P_q - 2\underline{q} - (2/3)\underline{q} = P_q - (8/3)\underline{q}$.

ALTERNATIVE APPROACH: Quotas are priced so that inefficient farmers are indifferent between paying an efficient farmer to produce for them and doing the production themselves, and inefficient farmers are likewise indifferent. Hence:

$$\begin{aligned} P_{quota} &= MC_E(\underline{q} + q_T) \\ &= 2\underline{q} + 2q_T \\ &= 2\underline{q} + \frac{2}{3}\underline{q} \\ &= \frac{8}{3}\underline{q} \end{aligned}$$

It is possible to solve this problem without any reference to actual milk sales, so answers can vary and receive full credit.