Applied Microeconomics: Firm and Household

Lecture 18: Oligopoly

Jason Kerwin

Department of Applied Economics
University of Minnesota

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Outline

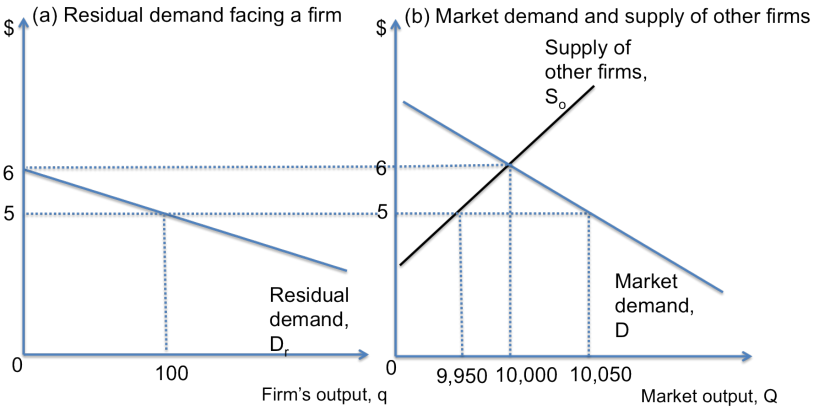
- Digression: Residual Demand Curves
- Cournot Model of Oligopoly
- Stackelberg Model of Oligopoly
- Bertrand Model of Oligopoly
- The Bertrand paradox
    - Capacity constraints
    - Product differentiation
        - Price-setting oligopolies
- The price-leadership game
- Strategic substitutes vs. strategic complements

The residual demand curve

The **residual demand curve** is the demand curve facing a particular firm. A firm sells to consumers whose demand is not met by other firms in the market. Formally,

1. $D_r(p) = D(p) - S_o(p)$
    - $D_r(p)$ is the residual demand
    - $D_r(p)$ is the market demand
    - $S_o(p)$ is the supply of all other firms

Derivation of residual demand curve



Oligopolistic markets

- Oligopolistic markets are characterized by competition among a few firms. Each firm is big enough to affect the industry's price.

The key characteristics of oligopoly are:

- Payoff interdependency: The profits of a firm depend on how much it produces and sells as well as how much its rivals produce and sell.
    - In an oligopolistic market, a firm considers rival firms' behavior when determining its own best actions.
- Strategic competition: Firms make strategic investments in the long run (i.e. capacity) to favorably influence short-run competition (i.e. competition over price or output).

This makes a game-theoretic approach an appealing way to study the problem of oligopolistic competition.

Classic models of oligopoly

The three best known oligopoly models are:

- Cournot model
- Bertrand model
- Stackelberg model

Models of oligopoly mainly differ by:

- Type of actions: price setting vs. quantity setting firms.
- Order of actions: sequential vs. simultaneous moves
- Type of goods: homogeneous vs. differentiated
- Length of time: single-period vs. multi-period games

Oligopolistic games

The common elements of oligopolistic games are:

- There are two or more firms who are strategic decision makers
- Each firm has a set of actions, i.e. a choice of price or quantity
- Each firm is aware that other firms' actions can affect its payoff (payoff interdependency)
- Each firm attempts to maximize its payoff (profits)

The solutions we will look for are Nash equilibria:

- Reminder: A set of strategies is called a **Nash equilibrium** if, holding the strategies of all other firms constant, no firm can obtain a higher payoff by choosing a different strategy.

The Cournot model of duopoly

Main assumptions:

1. Products are homogeneous.
2. Firms choose output.
3. Firms compete with each other just once and they make their production decisions simultaneously (single-period game).
4. There are two firms in the industry and there is no entry by other producers.

The Cournot model of duopoly

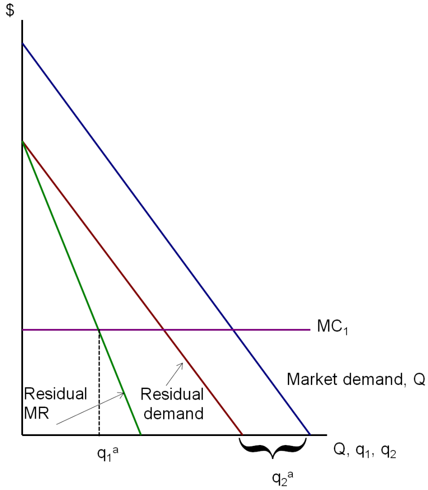
The formal summary of the model is:

- Players: N={firm 1, firm 2}
- Actions: Quantity choice – $q_i \in S_i = [0, \infty)$
- Payoffs: Firm profits – $\pi_i(q_i, q_j)$

We can solve for the Cournot-Nash equilibrium using best-response functions:

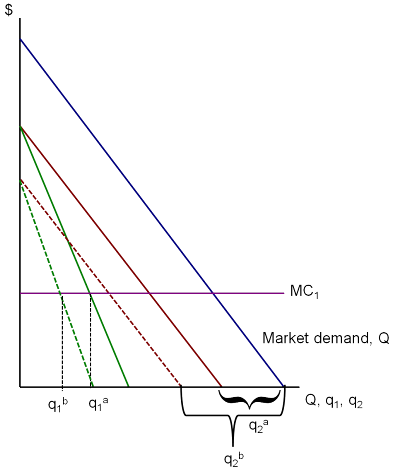
- <u>Reminder:</u> The **best response function**, $b_i(q_j)$, describes firm *i*'s optimal reaction to the output choice of firm *j*. It is given by:
    - $b_i(q_j) = \max_{q_i} \ \pi_i(q_i, q_j)$
- A Cournot-Nash equilibrium is a solution to the system of equations:
    - $q_1^c = b_1(q_2^c)$
    - $q_2^c = b_2(q_1^c)$

Graphical analysis of best-response functions



- Firm 1 believes that firm 2 will sell q_2^a units
- Firm 1 calculates its residual demand provided that $q_2 = q_2^a$
- Firm 1's best response is to behave like a monopoly on its residual demand.
- Thus it produces q_1^a where its MC is equal to the residual MR.

Graphical analysis continued



- Firm 1 believes firm 2's output will be q_2^b units
- In this case firm 1's residual demand shifts further left, thus it produces less: $(q_1^b < q_1^a)$.
- Note that for any q_2, firm 1 has a best response, q_1. At the extreme, if firm 1 believes $q_2 = 0$, then it behaves like a monopoly on the market demand curve.

Analyzing Cournot competition

The profit function of firm i is:

- $\pi_i(q_i, q_j) = q_i P(q_i + q_j) - c_i(q_i)$

Assuming π_i strictly concave in q_i and twice differentiable, the FOC is

- $\pi_i'(q_i, q_j) = \underbrace{P(q_i + q_j)}_{1} - \underbrace{c_i'(q_i)}_{2} + \underbrace{q_i P'(q_i + q_j)}_{3} = 0$
- 1 & 2 yield profitability of an extra unit of output
- 3 is the effect of the extra unit on the profitability of the inframarginal ones, i.e., the extra units create a decrease in P which affects the q units already produced.

There is a *negative externality between the firms:* each firm takes account the adverse effect of market price on its own q, but not on aggregate Q.

- each firm will tend to choose an output level that exceeds the optimal level that maximizes the industry profits (which is the monopoly output)

The Lerner index revisited

We can rewrite the FOC as

- $L_i = \frac{\alpha}{\epsilon}$

where

- $L_i \equiv= \frac{P-c_i^{'}}{P}$ is the Lerner index
- $\alpha_i = q_i/Q$ is the firm's market share
- $\epsilon = -\frac{P^{'}}{P}Q$ is the elasticity of demand.

The Lerner Index is proportional to market share and inversely proportional to the price elasticity of demand. This index is positive: firms sell at a price exceeding marginal cost.

- Cournot equilibrium is not socially efficient

A numerical example

Consider a linear market demand given by:

- $Q = 1,000 - 1,000P$

where $Q = q_1 + q_2$.

Also, assume that firms' costs of production are symmetric and given by:

- $C = 0.28q_i, \quad i = 1, 2$

We want to derive the best-response functions and solve for the Cournot equilibrium. To this end, we derive the inverse demand as:

- $P(Q) = 1 - 0.001Q$, or
- $P(q_1, q_2) = 1 - 0.001(q_1 + q_2)$

Residual marginal revenue

Firm 1 faces the following optimization problem:

- $\underset{q_1}{\text{Max}}\ \pi_1(q_1, q_2) = P(q_1, q_2)q_1 - C(q_1)$, or
- $\underset{q_1}{\text{Max}}\ \pi_1(q_1, q_2) = (1 - 0.001(q_1 + q_2))q_1 - 0.28q_1$

Assuming π_1 strictly concave in q_1 and twice differentiable, the FOC is

- $\pi_1'(q_1, q_2) = \underbrace{1 - 0.001q_2 - 0.002q_1}_{RMR} - \underbrace{0.28}_{MC} = 0$

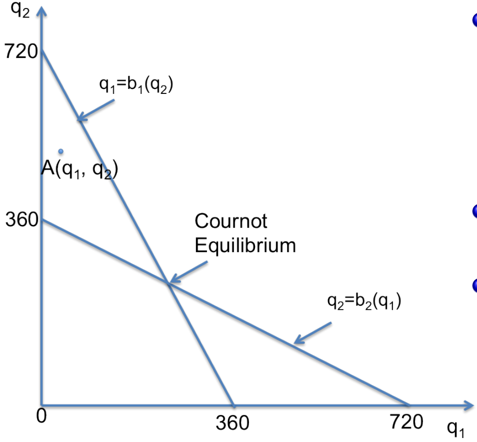
The best response function of firm 1 is:

- $q_1 = b_1(q_2) = 360 - \frac{q_2}{2}$

Similarly, the best response function of firm 2 can be derived as:

- $q_2 = b_2(q_1) = 360 - \frac{q_1}{2}$

Graph of best response functions



- Can point $A(q_1, q_2)$ be an equilibrium?
- From the definition of a best-response function we know that given other firm's production level each firm has an incentive to be on its best-response curve.
- Point A cannot be an equilibrium.
- Equilibrium requires that both firms are simultaneously on their best-response functions. This occurs at the intersection of the curves.

Cournot-Nash equilibrium

1. $q_1 = 360 - \frac{q_2}{2}$
2. $q_2 = 360 - \frac{q_1}{2}$

The Cournot-Nash equilibrium is the solution to (1) and (2)

- $q_1 = 360 - \frac{1}{2}\left(360 - \frac{q_1}{2}\right)$
- $q_1 = 360 - 180 + \frac{q_1}{4}$
- $q_1 = 240$

Because the firms are identical the second firm produces the same amount. To verify this, substitute $q_1 = 240$ into (2):

- $q_2 = 360 - \frac{240}{2} = 240$

Cournot-Nash equilibrium

The Cournot-Nash equilibrium is:

- $(q_1^c, q_2^c) = (240, 240)$

The equilibrium market outcomes are:

Aggregate output:

- $Q^c = q_1^c + q_2^c = 480$

Market price:

- $P^c = 1 - 0.001Q^c = 1 - 0.001(480) = 0.52$

Profits of each firm; i=1,2:

- $\pi_i^c = (P^c - MC)q_i^c = (0.52 - 0.28)(240) = (0.24)(240) = 57.6$

Cournot vs. perfect competition and monopoly

Let's compare the Cournot outcome to the perfect competition and monopoly outcomes.

The market price under perfect competition is: $P^* = MC = 0.28$. Industry output is:

- $1 - 0.001Q = 0.28$
- $Q^* = (1 - 0.28)(1000) = \underline{720}$

The monopoly quantity is determined by $MR = MC = 0.28$:

- $1 - 0.002Q = 0.28$
- $Q^m = (1 - 0.28)(500) = \underline{360}$

Thus, the monopoly price is

- $P^m = 1 - 0.001(360) = \underline{0.64}$

Cournot vs. perfect competition and monopoly

Now let's compare each of the outcomes

- $(Q^c, P^c) = (480, 0.52)$
- $(Q^*, P^*) = (720, 0.28)$
- $(Q^m, P^m) = (360, 0.64)$

Total output of Cournot duopoly is greater than the output of monopolized industry but less than that of a perfectly-competitive industry.

- $Q^* > Q^c > Q^m$

The Cournot-Nash equilibrium price is less than the monopoly price, but greater than the perfectly-competitive price

- $P^* < P^c < P^m$

Cournot competition vs. collusion/cartel

Imagine that firms collude and act like monopoly. Each firm's output is:

- $q^d = 0.5Q^m = 180$

Since total output is the monopoly output, the market price is the monopoly price:

- $P^m = 0.64$

Each firm makes half of the monopoly profits:

- $\pi^d = 0.5(0.64 - 0.28)360 = (0.36)180 = 64.8$

Note that

- $\pi^d = 64.8 > \pi^c = 57.6$

So each firm makes more profits if they cooperate. Is this collusive outcome sustainable?

Cournot vs. collusion (Cartel)

To find the answer we need to look at whether the firms have an incentive to deviate from the agreed-upon output levels.

Suppose that firm 2 abides by the agreement and produces

- $q_2^d = Q^m/2 = 180$

Using firm 1's best-response function we can find its best response:

- $q_1 = 360 - \frac{1}{2}(180) = 270$

That is if firm 2 abides to agreement firm 1 has an incentive to cheat and increase its output. Because given that firm 2 produces half of the monopoly output, firm 1's best response is to produce more than half of the monopoly output.

Cournot vs. collusion (Cartel)

Note that in this case the total output is

- $Q = 180 + 270 = 450$

the market price is

- $P = 1 - 0.001(450) = 0.55$

Thus, firm 1's profit is:

- $\pi_1 = (P^d - MC)(270) = (0.55 - 0.28)(270) = 72.9$

Compared to the collusive outcome, $\pi^d = 64.8$, firm 1 is able to make more profits by cheating. Therefore the collusive outcome is NOT sustainable. To summarize:

- The problem of collusion for Cournot firms is a prisoner's dilemma.
- Duopolists/prisoners both are better of if they collude.
- But given that one firm colludes, the other has an incentive to cheat.
- Therefore, both firms/prisoners are worse off when each responds to their private incentives.

An example: Cournot competition with N firms

We will analyze the same model with the following market demand and cost functions:

- $Q = 1,000 - 1,000P$

where $Q = Nq$, because the industry now has N firms that are assumed to be identical.

The firms' costs of production are symmetric:

- $C = 0.28q_i, \quad i = 1, ..., N$

As in the case of duopoly, the goral is to derive residual demand functions, best-response functions, and solve for the Cournot equilibrium. To this end, we derive the inverse demand as:

- $P(Q) = 1 - 0.001Q$, or
- $P(q_1, ..., q_N) = 1 - 0.001(Nq)$

N-firms Cournot model

A representative firm faces the following optimization problem:

- $\underset{q_i}{\text{Max}} \;\; \pi_i(q_1, \ldots, q_N) = (1 - 0.001[(N-1)q + q_i])q_i - 0.28q_i$

Assuming π_1 is strictly concave in q_1 and twice differentiable, the FOC is:

- $\pi_i'(q_1, \ldots, q_N) = \underbrace{1 - 0.001(N-1)q - 0.002q_i}_{RMR} - \underbrace{0.28}_{MC} = 0$

The best response function of firm 1 is:

- $q_i = 360 - \frac{(N-1)q}{2}$

The other firms have similar best response functions (why?):

- $q = 360 - \frac{(N-1)q}{2}$

N-Firms Cournot-Nash equilibrium

- $q = 360 - \frac{(N-1)q}{2}$

solving for q yields the Cournot equilibrium quantity of each firm:

- $q = \frac{720}{N+1}$

We can now calculate the equilibrium market outcomes:

Aggregate output: $Q^c = \frac{720N}{N+1}$

Market price: $P^c = 1 - 0.001Q^c = \frac{1+0.28N}{N+1}$

Profits of each firm; i=1,2,...,N:

- $\pi_i^c = (P^c - MC)q_i^c = \frac{518.4}{(N+1)^2}$

Industry Profits: $\pi^c = \frac{(518.4)N}{(N+1)^2}$

Cournot-Nash equilibrium with few and many firms

			Firm		Industry	
	Number of Firms	Price	Output	Profit ($)	Output	Profit ($)
Monopoly	1	64	360	129.60	360	129.60
	2	52	240	57.60	480	115.20
	3	46	180	32.40	540	97.20
	4	42.4	144	20.74	576	82.94
	5	40	120	14.40	600	72.00
	6	38.3	102.9	10.58	617.1	63.48
	7	37	90	8.10	630	56.70
	8	36	80	6.40	640	51.20
	9	35.2	72	5.18	648	46.66
	10	34.5	65.5	4.28	654.5	42.84
	15	32.5	48	2.30	675	32.26
	20	31.4	34.3	1.18	685.7	23.51
	50	29.4	14.1	0.20	705.9	9.97
	100	28.7	7.1	0.05	712.9	5.08
	500	28.1	1.4	0.002	718.6	1.03
	1000	28.1	0.7	0.001	719.3	0.52
Competition	∞	28	~ 0	0.00	720	0.00

The Stackelberg (leader-follower) model

The only difference of Stackelberg model from Cournot is that firms move sequentially.

- The leader firm picks its output level
- Other firms (followers) observe the leader firm's output level, then pick their optimal quantity.
- In some industries, historical, institutional, or legal factors determine which firm is the first mover
    - An innovative firm has a natural first-mover advantage.

The Stackelberg (leader-follower) model

The main assumptions of the Stackelberg model are:

1. There are two identical firms (firm 1 and firm 2)
2. Products are homogeneous
3. Firms choose output
4. Firm 1 chooses its output first, then firm 2 chooses its output knowing the output level of firm 1 (a sequential-move game)
5. Firms compete with each other just once (a single-period game)
6. There is no entry by other firms

The formal summary of the model is:

- Players: N={firm 1, firm 2}
- Actions: Quantity choice – $q_i \in S_i = [0, \infty)$
- Timing: Firm 1 moves first, then firm 2
- Payoffs: Firm profits – $\pi_i(q_1, q_2)$

The follower's decision

As always, the first step in finding the Nash equilibrium is to find the best response functions of each player.

Q: How do the firms calculate their best response functions?

The Follower:

1. The follower knows that it moves after the leader. So it is best for the follower to behave like a monopoly on the residual demand that is not met by the leader.
2. So, the follower needs to calculate its residual demand (thus, best response) for any level of output that the leader might choose.
3. The problem is identical to that of a Cournot firm. Therefore, the follower chooses its output using its Cournot best response function.

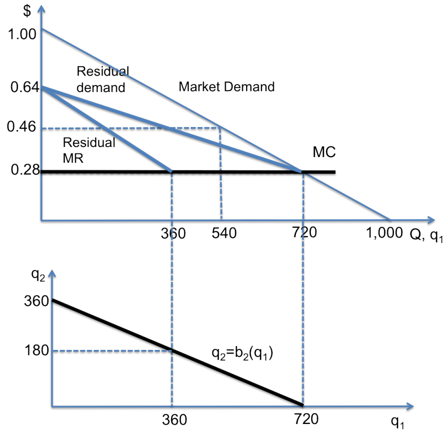
The leader's decision

The optimal behavior of the leader is also to find its residual demand and behave like a monopoly on its residual demand. But, leader is moving first, so how does it calculate its residual demand?

Analysis of Leader's decisions:

1. The leader expects the follower to choose its output using the Cournot best-response function.
2. Consequently, the leader knows how much the follower will produce at any output level that it chooses. Therefore, it can calculate its residual demand.
3. Note that the leader does not calculate its residual demand based on any level of the follower's output, it calculates based on the follower's best response output.
4. Thus, the leader's residual demand is equal to the market demand minus the follower's Cournot best response.

The Stackelberg equilibrium



- Leader (firm 1) calculates its residual demand provided that follower (firm 2) chooses its output using its Cournot best response, $q_2 = b_2(q_1)$.
- The leader's best response is to behave like a monopoly on its residual demand.
- Thus, at $MC = RMR$, the outcome is $q_1 = 360$, $q_2 = 180$.

Stackelberg competition: a numerical example

Consider the same market demand that we used for Cournot competition example. Assume that firm 1 is the leader and firm 2 is the follower.

- $Q = 1,000 - 1,000P$

where $Q = q_1 + q_2$.

Also, assume that firms' costs of production are symmetric and given by

- $C = 0.28q_i, \quad i = 1, 2$

The inverse demand function is:

- $P(Q) = 1 - 0.001Q$, or
- $P(q_1, q_2) = 1 - 0.001(q_1 + q_2)$

We want to derive the best-response functions of both firms and find the Stackelberg outcome.

The follower's Cournot best-response function

The best-response function of the follower is the same as the best-response function of a Cournot firm.

So without going through the same algebra that we did before, we can write the follower's Cournot best-response as:

- $q_2 = 360 - \frac{q_1}{2}$

The leader's residual demand function

The leader faces the following optimization problem:

- $\underset{q_1}{\text{Max}} \;\; \pi_1(q_1, q_2) = P(q_1, q_2)q_1 - C(q_1)$, or
- $\underset{q_1}{\text{Max}} \;\; \pi_1(q_1, q_2) = (1 - 0.001(q_1 + \left(360 - \frac{q_1}{2}\right)))q_1 - 0.28q_1$

Assuming π_1 is strictly concave in q_1 and twice differentiable, the FOC is

- $\pi_1'(q_1, q_2) = \underbrace{0.64 - 0.001q_1}_{RMR} - \underbrace{0.28}_{MC} = 0$

Note: The leader's residual demand is *not* a function of the follower's quantity.

Solving for the Stackelberg equilbrium

Leader's optimal quantity is:

- $RMR_1 = 0.64 - 0.001q_1 = 0.28$
- $0.001q_1 = 0.36$
- $q_1 = \underline{360}$

given that $q_1 = 360$, the follower's optimal quantity is:

- $q_2 = 360 - \frac{360}{2}$
- $q_2 = \underline{180}$

Characteristics of the Stackelberg equilbrium

The Stackelberg equilibrium is:

- $(q_1^s, q_2^s) = (360, 180)$

We can now calculate the equilibrium market outcomes:

Aggregate output:

- $Q^s = q_1^s + q_2^s = 540$

Market price:

- $P^s = 1 - 0.001Q^s = 1 - 0.001(540) = 0.46$

Profits of each firm; i=1,2:

- $\pi_1^s = (P^s - MC)q_1^s = (0.46 - 0.28)(360) = (0.18)(360) = 64.8$
- $\pi_2^s = (P^s - MC)q_2^s = (0.46 - 0.28)(180) = (0.18)(180) = 32.4$
- Total Profits = $64.8 + 32.4 = 97.2$

Cournot vs. Stackelberg equilbrium

The Stackelberg equilibrium

- $(q_1^s, q_2^s) = (360, 180)$
- $Q^s = 540$
- $P^s = 0.46$
- $\pi_1^s = 64.8$
- $\pi_2^s = 32.4$
- Total Profits: $\pi^s = 97.2$

The Cournot equilibrium

- $(q_1^c, q_2^c) = (240, 240)$
- $Q^c = 480$
- $P^c = 0.52$
- $\pi_1^c = 57.6$
- $\pi_2^c = 57.6$
- Total Profits: $\pi^c = 115.2$

- $Q^s > Q^c$, $P^s < P^c$, $\pi_1^s > \pi_1^c$, $\pi_2^s < \pi_2^c$, , $\pi^s < \pi^c$
- Bottom line: The advantage of moving first and knowing how its rival will behave allows the leader to profit at the follower's expense.

The Bertrand model of oligopoly

The main aspects of Bertand competition are the following:

1. Firms set prices rather than output.
2. If consumers have complete information and realize that firms produce identical products, they buy the one with the lowest price.
3. Each firm believes its rival's price is fixed. Thus, by a slight price cut the firm is able to capture all its rival's business.

The Bertrand model of oligopoly

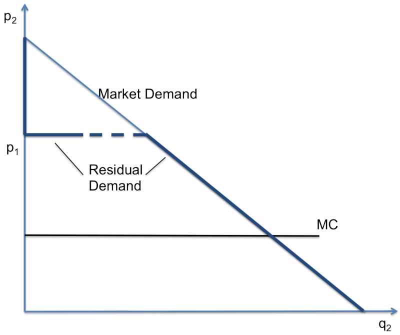
Main assumptions:

1. Products are homogeneous
2. Firms choose price (“firms compete on price”)
3. Firms compete with each other just once and make their pricing decisions simultaneously (a single-period simultaneous-move game)
4. There are two identical firms in the industry and there is no entry by other producers

The formal summary of the model is:

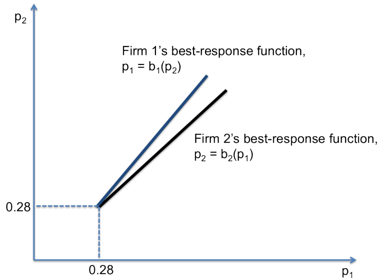
- Players: N={firm 1, firm 2}
- Actions: Price choice – $p_i \in S_i = (0, \infty)$
- Timing: Firms choose their actions simultaneously
- Payoffs: Firm profits – $\pi_i(p_1, p_2)$

The residual demand facing a Bertrand competitor



- Suppose firm 1 charges p_1
- All consumers buy from firm 2 if $p_2 < p_1$ (& vice versa)
- If $p_1 = p_2$ consumers are indifferent; firms are assumed to split the total market demand
- Thus, the residual demand facing Firm 2 is:
  - $q_2(p_1, p_2) =$
    $$\begin{cases} 0, & \text{if } p_1 < p_2 \\ 0.5Q(p_2) & \text{if } p_1 = p_2 \\ Q(p_2) & \text{if } p_1 > P_2. \end{cases}$$

Bertrand best-response functions



- MC = 0.28
- Best response functions intersect at MC

- Given any price p_1, Firm 2 will set its price p_2 slightly lower than p_1.
- The same is true for Firm 1. That is, each firm's best strategy is to undercut its rival's price.
- But, neither firm is willing to undercut below MC.
- The only possible Bertrand-Nash equilibrium is $p = MC = 0.28$
- This is efficient outcome, i.e. perfect competition.

Comparison of oligopoly outcomes

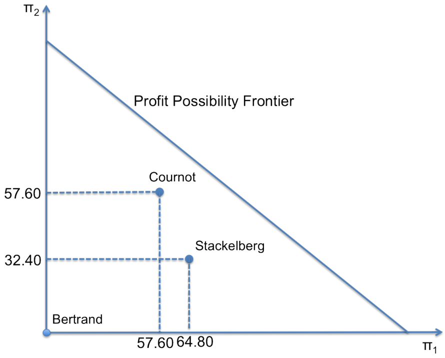
Now let's compare all the various oligopoly model outcomes, using the same example we have been using.

- Cournot Duopoly: Each firm makes \$57.60
- Stackelberg Model: The leader makes \$64.80 while the follower makes \$32.40
- Bertrand Model: Neither firm makes positive profits.
- Cartel: If firms can sustain a cartel they make the highest collective profits, totaling = \$129.60

Definition: The **profit possibility frontier** shows the highest profit one firm could earn holding the profit of other firm constant.

- summarizes any combination of firms profits in which the sum of profits is maximum (129.60), e.g., firms can split (64.80, 64.80), or one firm can get 0, other gets the rest.

The profit possibility frontier



- All outcomes lie inside the frontier.
- This gives incentives to firms to collude and increase their profits to reach the frontier.
- The incentive to collude is the strongest under Bertrand competition, because the outcome puts the firms the farthest away from the frontier.

The Bertrand paradox

Under Bertrand competition a firm may increase sales from zero to the entire market by slightly lowering its price (because the residual demand is kinked).

- such sudden shifts are not realistic – and rarely observed in most industries.

Also, the Bertrand model is counter-intuitive:

- so long as there are at least two firms the Bertrand price is the competitive price (marginal cost).

This result is known as the “Bertrand Paradox”

It is a paradox because we would expect that when there are few firms in the market (e.g. just two), each firm should be able to price above its marginal cost, because each is “big” enough to affect the market price.

Reasons for the Bertrand paradox

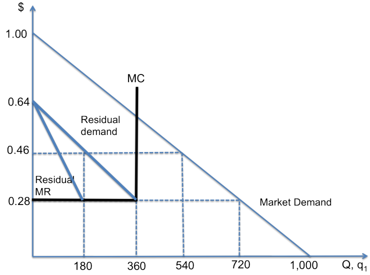
The Bertrand outcome depends on the strong assumptions made by the model. Namely:

1. Any firm can produce as much as it wants at constant marginal cost
2. The products are homogeneous
3. The market lasts for only one period

When at least one of these assumptions is relaxed, the Bertrand outcome is *not* $p = MC$. In reality:

1. firms face capacity constraints
2. products are differentiated
3. markets lasts for many periods (multi-period games)

Bertrand residual demand with limited capacity



- Suppose each firm's maximum output capacity is 360.
- Thus, MC is horizontal up to 360, and vertical at 360.
- Note that Firm 1 can let Firm 2 sell at 0.28 as much as it can.
- Only half of the market could buy from Firm 2.
- Then, Firm 1 can behave like a monopoly on the residual demand and charge $p_1 = 0.46$, and make positive profits.
- Since, each firm has an incentive to move from (0.28, 0.28), p=MC is not a Bertrand-Nash equilibrium when there are capacity constraints.

Differentiated product markets

In real-world markets, firms produce differentiated products. T

- Two products are **differentiated** products if consumers view them partial, but not perfect, substitutes.

There are two types of product differentiation:

- **Horizontal differentiation:** if some consumers buy product 1 and some others buy product 2 when the price is the same (due to consumer preferences) then the products are horizontally differentiated.
  - Examples: Vanilla vs. Chocolate ice cream, Pepsi vs. Coca Cola, Ferrari vs. Porsche
- **Vertical differentiation:** if all consumers prefer product 1 over product 2 at the same price, then the products are vertically differentiated.
  - Ferrari vs. Kia, Intel Core i7 vs. Intel Pentium.

Price setting game

Suppose there are J differentiated products in a market, each of which is produced by a single firm. Thus each firm faces a demand curve of the form:

- $q_i = q_i(p_i, \mathbf{p_k}, Z)$

Firms compete on price. The profit maximization problem of firm i is given as:

- $\max_{p_i} \pi_i = p_i q_i(p) - C(q_i)$

the FOC is

- $\frac{\partial \pi_i}{\partial p_i} = q_i + \frac{\partial q_i}{\partial p_i} p_i - \frac{\partial C}{\partial q_i} \frac{\partial q_i}{\partial p_i} = 0$, or
- $\frac{\partial q_i}{\partial p_i} [p_i - MC_i] + q_i = 0$

Price setting game

We can rewrite the FOC as:

- $p_i - mc_i = -q_i \frac{\partial p_i}{\partial q_i}$, or
- $\frac{p_i - MC_i}{p_i} = -1/\epsilon_i$

where ϵ_i is the own-price elasticity of demand.

We obtain the familiar result that in differentiated-product markets, price-setting firms behave like a monopoly on their product's demand curve: at the optimum, the price-cost margin equals the inverse of the demand elasticity.

Example: Toothpaste as a differentiated product

Let $C(q) = 0$ and demand for product i be given as,

- $q_i = a_i - p_i + \frac{p_j}{2}$

The profit maximization problem for firm i is:

- $\max_{p_i} \pi_i = p_i q_i(p) - C(q_i) = p_i(a_i - p_i + \frac{p_j}{2})$

So the FOC is:

- $\frac{\partial \pi_i}{\partial p_i} = a_i - 2p_i + \frac{p_j}{2} = 0$

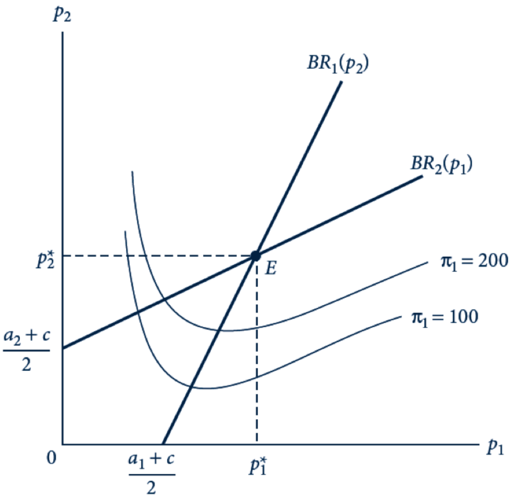
The best-response function for $i = 1, 2$ is:

- $p_1 = \frac{1}{2}\left(a_1 + \frac{p_2}{2}\right)$ and $p_2 = \frac{1}{2}\left(a_2 + \frac{p_1}{2}\right)$

And the Bertrand-Nash equilibrium prices are:

- $p_i^* = \frac{8}{15}a_i + \frac{2}{15}a_j$

Best-response functions for Bertrand competition with differentiated products



The price-leadership game

Consider the same toothpaste market, but in this case assume that firm 1 is the leader and firm 2 is the follower. Using backward induction, firm 2's best response function is given as:

- $p_2 = \frac{1}{2}\left(a_2 + \frac{p_1}{2}\right)$

The profit maximization problem of firm *i* is given as:

- $\max_{p_1} \pi_1 = p_1 (a_1 - p_1 + \frac{\frac{1}{2}\left(a_2 + \frac{p_1}{2}\right)}{2})$

So the FOC is:

- $\frac{\partial \pi_1}{\partial p_1} = a_1 + \frac{1}{4} a_2 + \frac{7p_1}{4} = 0$
- $p_1^* = \frac{4a_1 + a_2}{7}$ and $p_2^* = \frac{1}{2} + \frac{4a_1 + a_2}{28}$

Bertrand vs. price leadership model

Without loss of generality, set $a_1 = a_2 = 1$.

The Bertrand outcome is

- $p_i^* = p_j^* = 0.667$
- $q^i = q_j^* = 0.667$
- $\pi_i^* = \pi_j^* = 0.444$
- Total Profits: $\Pi = 0.888$

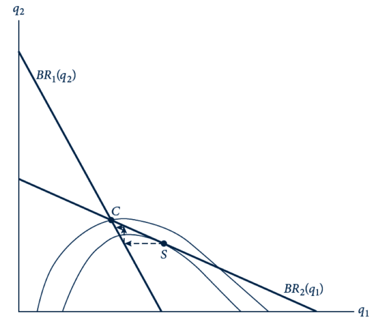
The price leader outcome is:

- $(p_i^*, p_j^*) = (0.714, 0.679)$
- $(q_i^*, q_j^*) = (0.626, 0.678)$
- $(\pi_i^*, \pi_j^*) = (0.446, 0.460)$
- Total Profits: $\Pi = 0.906$

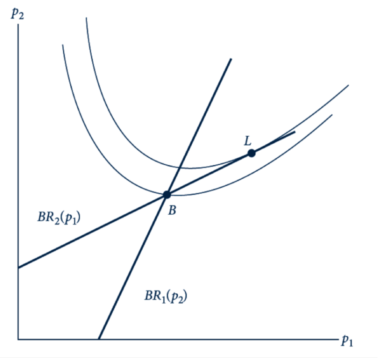
- Player 1 plays a “puppy dog” strategy – it increases its price relative to the simultaneous move game
- In Cournot Stackelberg model, Player 1 plays a “top dog” strategy – it increased its quantity relative to simultaneous move game

Quantity vs. price leaders

Quantities are Strategic Substitutes



Prices are Strategic Complements



Top dog vs. puppy dog strategies

Whether competitor's decisions are **strategic substitutes** or **complements** is determined by whether more "aggressive" (increasing quantity, decreasing price) play by one firm in a market lowers or raises competing firms' marginal profitabilities in that market.

- If decisions are strategic substitutes, the best-response functions are downward-sloping
  - In this case, a firm can induce a rival to respond less aggressively by playing more aggressively (a top dog strategy)
- If decisions are strategic complements, the best-response functions are upward-sloping
  - In this case, a firm can induce a rival to respond less aggressively by playing less aggressively (a puppy dog strategy)