Econ 8010 HW6

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Nathan Yoder

University of Georgia

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1. Consider the following extensive form game of imperfect information played by players $i \in \{1, 2, 3\}$. The game takes place in five periods $t \in \{0, 1, 2, 3, 4\}$ and proceeds as follows.

   - At $t = 0$, Nature chooses a **state of the world** $\theta \in \{H, L\}$ according to the probability distribution ρ_0, where

     $$\rho_0(H) = \frac{1}{2}, \; \rho_0(L) = \frac{1}{2}.$$

     This state of the world corresponds to whether an investment will yield a **high** payoff (state H) or a **low** payoff (state L). It is not observed by any of the players.

   - At $t = 1$, Nature chooses a **signal** $r_1 \in \{h, \ell\}$ according to the probability distribution $\rho(\cdot|\theta)$, where

     $$\begin{aligned} \rho(h|H) &= \frac{3}{4}, & \rho(\ell|H) &= \frac{1}{4}, \\ \rho(h|L) &= \frac{1}{4}, & \rho(\ell|L) &= \frac{3}{4}. \end{aligned}$$

     This signal is observed by player 1 (only). Then, player 1 chooses an action $a_1 \in \{\textit{Invest, Don't Invest}\}$. This action is observed by players 2 and 3.

- At $t = 2$, Nature chooses another signal $r_2 \in \{h, \ell\}$ according to $\rho(\cdot|\theta)$. This signal is observed by player 2 (only). Then, player 2 chooses an action $a_2 \in \{\textit{Invest, Don't Invest}\}$. This action is observed by player 3.
- At $t = 3$, Nature chooses yet another signal $r_3 \in \{h, \ell\}$ according to $\rho(\cdot|\theta)$. This signal is observed by player 3 (only). Then, player 3 chooses an action $a_3 \in \{\textit{Invest, Don't Invest}\}$.
- At $t = 4$, the game ends. Each player who chose *Invest* receives a payoff of 1 if $\theta = H$ and $-\frac{9}{10}$ if $\theta = L$. Each player who chose *Don't Invest* receives a payoff of zero, regardless of the state of the world.

Note that players' beliefs are only relevant insofar as they describe the probability placed on each state of the world. Thus, instead of writing beliefs as the probabilities of each decision node (e.g., $\mu_3(H, \ell, \textit{Don't Invest}, h, \textit{Invest}, \ell)$), we can simply write them as the probability of θ given the current information set (e.g., $\mu_3(H|\textit{Don't Invest}, \textit{Invest}, \ell)$).

(a) What are the unique Bayesian beliefs $\mu_1(H|h), \mu_1(H|\ell)$ for player 1 after $r_1 \in \{h, \ell\}$?

(b) In any weak sequential equilibrium, what action will player 1 take after signal h? After ℓ?

(c) What are the unique Bayesian beliefs

$$\mu_2(H|\textit{Invest}, h), \mu_2(H|\textit{Invest}, \ell), \mu_2(H|\textit{Don't Invest}, h), \mu_2(H|\textit{Don't Invest}, \ell)?$$

(d) In any weak sequential equilibrium, what action will player 2 take after $(\textit{Invest}, h)$? After $(\textit{Invest}, \ell)$? After $(\textit{Don't Invest}, h)$? After $(\textit{Don't Invest}, \ell)$?

(e) Solve for all weak sequential equilibria. (You only need to solve for beliefs about the state, not about individual decision nodes that follow the same state.)

(f) In any weak sequential equilibrium, what is the probability of a history (a_1, a_2) occurring after which

   i. player 3 invests, no matter what her signal r_3 is?

ii. player 3 does not invest, no matter what her signal r_3 is?

iii. player 3 plays the "correct" action (i.e., *Invest* if the state is H or *Don't Invest* if the state is L), no matter what her signal r_3 is?

iv. player 4 plays the "incorrect" action (i.e., *Don't Invest* if the state is H or *Invest* if the state is L), no matter what her signal r_3 is?

2. Consider the following extensive form game of imperfect information played by two players: a pharmaceutical firm (player 1) and a regulator (player 2). Play proceeds as follows.

- **First,** nature chooses whether the firm's new drug has high effectiveness (θ_h) or low effectiveness (θ_ℓ) with equal probability. The firm observes the effectiveness of this choice, but the regulator does not.

- **Second,** the firm chooses whether to conduct a costly test of the drug's effectiveness (T) or not (N). If the firm declines to test, the game ends and both players get a payoff of zero. If the firm tests, it costs the firm \$1 million.

- If the firm chose to test the drug, Nature chooses whether the test succeeds (s) or fails (f) according to the probability distribution $\rho(\cdot|\theta)$, where

$$\rho(s|\theta_h) = \beta, \qquad \rho(f|\theta_h) = 1 - \beta,$$
$$\rho(s|\theta_\ell) = .05, \qquad \rho(f|\theta_\ell) = .95.$$

  for $\beta \in (.5, 1)$.

- After seeing whether the test is a success or failure, the regulator decides whether to approve the drug or deny approval. If the regulator approves the drug, the firm receives \$5 million in monopoly profits from selling it.

  In making its decision, the regulator works to maximize consumer welfare. If the regulator approves a drug which has high effectiveness, consumers are \$1 million better off. If the regulator approves a drug which has low effectiveness, consumers are \$1 million worse off.

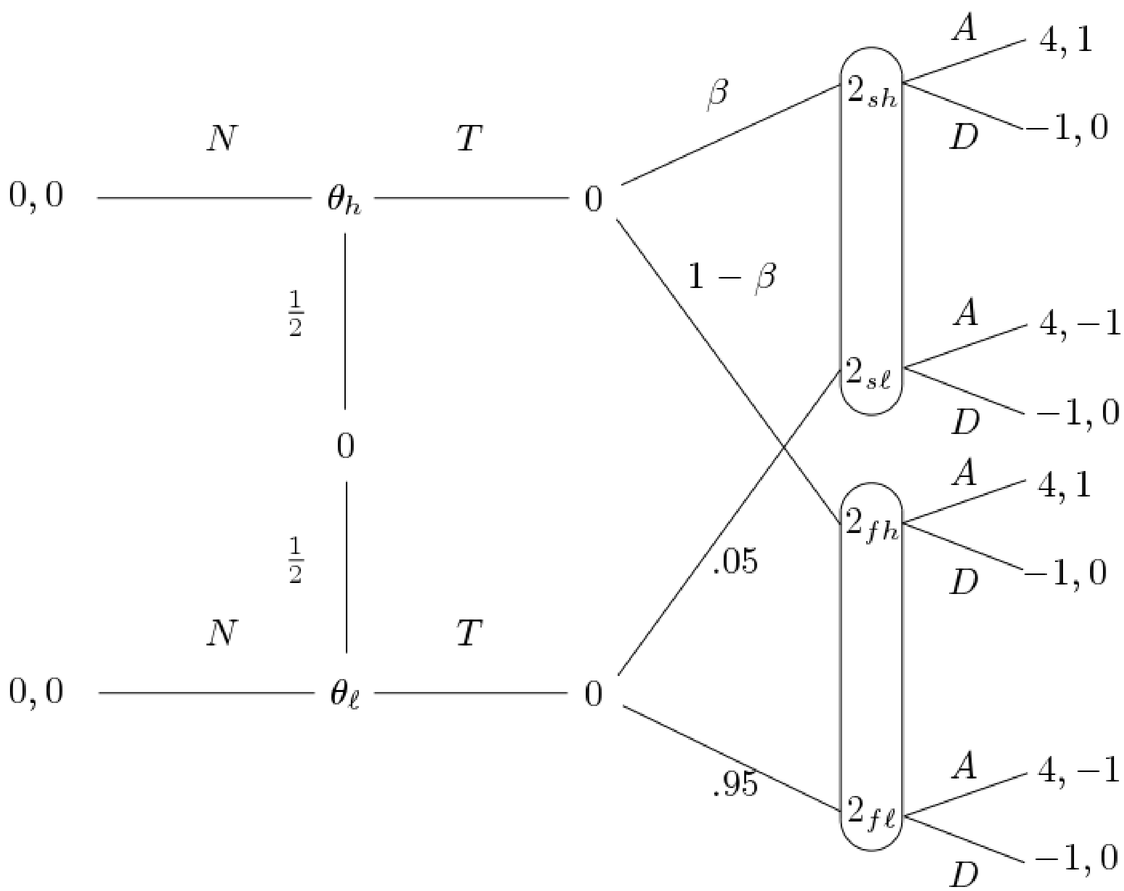

Figure 1: The game tree for the game described in Exercise 2.

This game can be thought of as a signaling game (with messages T and N) where the receiver receives additional information about the sender's type after T.

(a) Find all sequential equilibria of the game. (Hint: like standard signaling games, all weak sequential equilibria of this game are sequential equilibria.)

(b) Explain how a change in β affects

  i. the equilibrium probability that a high-effectiveness drug is accepted;

  ii. the equilibrium probability that a low-effectiveness drug is accepted;

  iii. the regulator's expected payoff in equilibrium.

(c) Explain how the "intuitive criterion" of Cho and Kreps should be applied to this game. Which of the sequential equilibria satisfy it?