Applied Microeconomics: Firm and Household

Lecture 11: Cost Minimization

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Outline

- Cost concepts
  - Economic vs. Accounting costs
  - Total, marginal and average costs
  - Fixed, variable and sunk costs
- Cost minimization model
  - The optimality condition
  - The firm's expansion path
  - The conditional demand functions

Economic vs accounting costs

Economic cost is the cost of keeping factors of production in their present employment. In other words, it is the remuniration a factor would receive in its best alternative employment (i.e., opportunity cost).

Accounting cost is the total out-of-pocket expenses.

- Generally, sunk costs are the main distinction.

For example, a one-time license fee for a business is not an economic cost. Once paid, the fee is not recoverable, therefore it should not affect the firm's subsequent production decisions. However, it is an accounting cost.

Production costs

Production cost is the total value of inputs used to produce a given level of output. To be able to make the best business decisions firms need to know about their production costs.

We can generalize the type of costs in two categories:

- Fixed costs
- Variable costs

These type of costs are associated with the following three cost concepts to describe the relationship between output and cost:

- Total cost
- Average cost
- Marginal cost

Fixed vs variable costs

We can broadly generalize firm's costs into two categories

1. **Fixed Costs:** costs that do not vary with output.
    - Monthly rent for land, buildings (chicken coops), machinery (combines).

   It is important for business decisions to distinguish between two types of fixed costs. These are

    - **Sunk costs:** The portion of fixed cost thats are not recoverable. Once incurred sunk costs should not affect any subsequent business decisions.

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   - **Avoidable fixed costs:** Fixed costs that are not incurred at zero production are called avoidable costs. For example, if a firm exits the market then the value it would get from selling assets such as machinery or buildings are avoidable costs.

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2. **Variable Costs:** Costs that vary with output.
   - Labor, seed, fertilizer, electricity, etc.

Total cost function

Total Cost (TC): A firm's total cost is the cost of all the factors of production the firm uses. It equals the sum of fixed and variable costs.

- Total Cost = Total Fixed Cost + Total Variable Cost
- $TC = TFC + TVC$

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Formally, we can use a **cost function** to represent a firm's total cost,

- $C = c(w, y)$

where w is a vector input prices and y is the target output level

The cost function

The cost function is the *minimum* cost achievable at any given levels of output and factor prices, denoted as:

- $C = C(y, w_1, ...w_n)$

where y is the output level and $w_1, ..., w_n$ denote the prices of factors employed in production of y, i.e., $x_1, ...x_n$.

- Note that the cost function specifies the total cost of producing any given level of output. Thus, output, y, enters as a parameter in the cost function.

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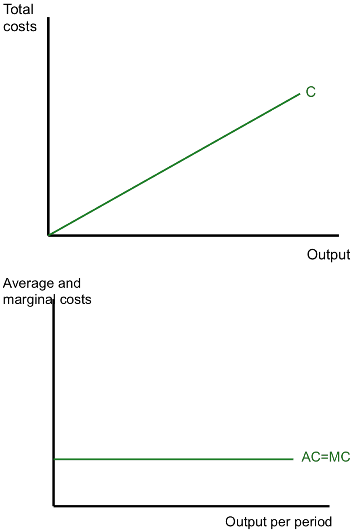
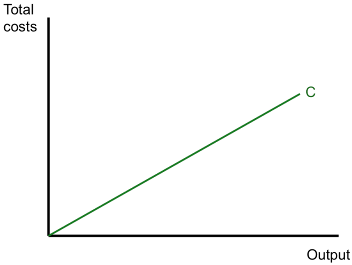
Marginal Cost Function: The rate of change of total cost for a change in output.

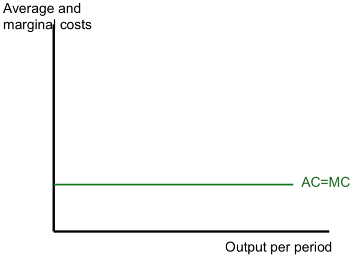
- $MC(w, y) = \frac{\partial C^*(w_1,...,w_n,y)}{\partial y}$

Average Cost Function: Cost per unit of output.

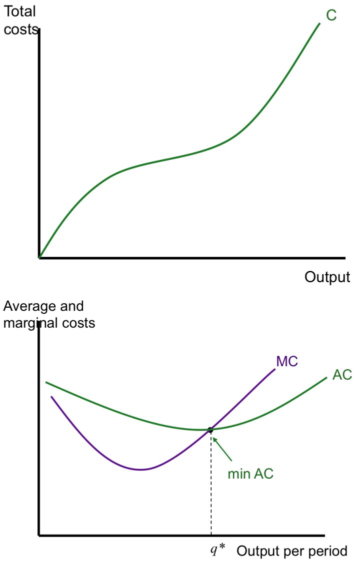
- $AC(w, y) = \frac{C^*(w_1,...,w_n,y)}{y}$

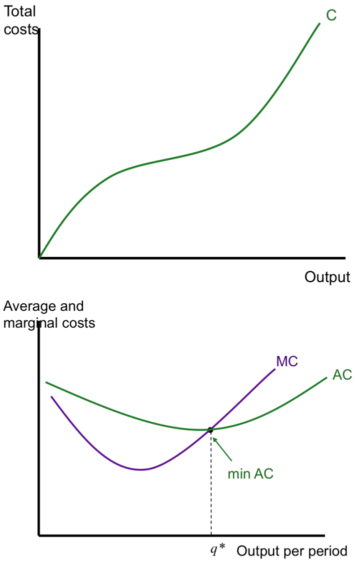

Total
costs
C
Output
Average and
marginal costs
AC=MC
Output per period





- TC increases at a constant rate as output increases (a linear cost function).
- AC and MC are equal and constant for all output levels.


Total
costs
C
Output
Average and
marginal costs
MC
AC
min AC
q^*
Output per period



- Suppose that the total cost function is cubic. It starts out as concave and then becomes convex as output increases
- AC and MC , average and marginal cost curves will be U-shaped
- the marginal cost curve passes through the minimum of the average cost curve at output level q^*

Average and marginal costs

Q: Why are the MC and AC curves U-shaped?

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A: Because of the **law of diminishing returns**: As a firm uses more of a variable input, with a level quantity of fixed inputs, the marginal product of the variable input eventually decreases – hence the marginal cost of producing another unit with the variable input increases assuming a fixed payment for the input.

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Highlights:

1. Due to scale economies marginal cost is decreasing at initial levels of production but eventually increases.
2. By definition AC always decreases (increases) whenever MC is below (above) AC.
3. Due to 2, it must be always true that MC intersects AC at its minimum.

Cost minimization

All cost functions (curves) are derived from the **cost minimization** model:

- $\underset{x}{min}\ C = \sum_i^N w_i x_i$

subject to

- $y = f(x_1, ..., x_n)$

the model asserts that firms minimize the total cost of producing a target output, y, by choosing input levels, x.

As we did with the profit maximization model, our next step is to derive the empirical implications of the cost minimization model.

Notes:

- This optimization problem is a **constrained optimization problem**.
- The target output level represents the firm's constraint for optimization.

The Lagrangian function

Without loss of generality, consider the following two-variable case of the cost minimization model:

- $\underset{x_1, x_2}{\text{Min}}\ C = w_1 x_1 + w_2 x_2$

subject to

- $y = f(x_1, x_2)$

The Lagrangian for this problem is:

- $\underset{x_1, x_2, \lambda}{\text{Min}}\ L = \underbrace{w_1 x_1 + w_2 x_2}_{\text{objective function}} + \lambda \underbrace{(y - f(x_1, x_2))}_{\text{constraint function}}$

where λ is an added variable referred as **lagrange multiplier**?

Q: How do we interpret λ?

Cost minimization

- $\underset{x_1, x_2, \lambda}{\text{Min}} \ L = w_1 x_1 + w_2 x_2 + \lambda(y - f(x_1, x_2))$

Once we set up the Lagrangian our analysis of this problem proceeds the same way as for expenditure minimization (Note the similarities between cost and expenditure minimization problems). We want to:

- derive and interpret the first- and second-order conditions
- define the solutions to this problem in implicit form
- discuss comparative statics.

Cost minimization

- $\underset{x_1, x_2, \lambda}{\text{Min}}\ L = w_1 x_1 + w_2 x_2 + \lambda(y - f(x_1, x_2))$

The FOCs are:

1. $L_1 = \frac{\partial L}{\partial x_1} = w_1 - \lambda f_1 = 0$
2. $L_2 = \frac{\partial L}{\partial x_2} = w_2 - \lambda f_2 = 0$
3. $L_\lambda = \frac{\partial L}{\partial \lambda} = y - f(x_1, x_2) = 0$

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From FOCs 1 and 2:

- $\lambda = \frac{w_1}{f_1} = \frac{w_2}{f_2}$

Therefore, at the optimum it must be the case that

- $\frac{w_1}{w_2} = \frac{f_1}{f_2} = RTS$

Cost minimization: graphical interpretation

- $\frac{w_1}{w_2} = \frac{f_1}{f_2}$

The condition for an optimum states that a cost-minimizing firm employs inputs at a level where the ratio of their prices equals to the ratio of their marginal products.

Cost minimization: graphical interpretation

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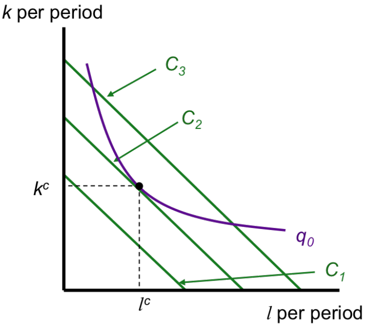

Figure: Graph of cost minimization

The firm's expansion path

The firm can determine

- The cost-minimizing combinations of x_1 and x_2 for every level of output
- If input costs (w_1 and w_2) remain constant for all amounts of x_1 and x_2 we can trace the locus of cost-minimizing choices
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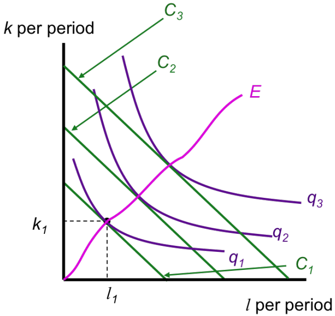

Figure: Firm's Expansion Path

The second-order sufficient condition (SOSC)

Recall that the matrix of second partials of a constrained optimization problem is the **bordered Hessian.** In this case, the second-order sufficient condition (SOSC) for an interior minimum is that the determinant of the bordered Hessian must be negative. To form the bordered hessian we derive the second partials:

- $L_{11} = -\lambda f_{11}, L_{12} = -\lambda f_{12}, L_{1\lambda} = -f_1$
- $L_{21} = -\lambda f_{21}, L_{22} = -\lambda f_{22}, L_{1\lambda} = -f_2$
- $L_{\lambda 1} = -f_1, \quad L_{\lambda 2} = -f_2, \quad L_{\lambda\lambda} = 0$

Using the second partials, the SOSC is given as:

- $H = \begin{vmatrix} -\lambda f_{11} & -\lambda f_{12} & -f_1 \\ -\lambda f_{21} & -\lambda f_{22} & -f_2 \\ -f_1 & -f_2 & 0 \end{vmatrix} < 0$

The conditional demand and cost functions

So long as the second order condition holds (and $H \neq 0$), we can solve the system of equations of FOCs to obtain:

- $x_1 = x_1^*(w_1, w_2, y)$
- $x_2 = x_2^*(w_1, w_2, y)$
- $\lambda = \lambda^*(w_1, w_2, y)$
- $C = C^*(x_1^*, x_2^*) = C^*(w_1, w_2, y)$

x_1^* and x_2^* are the conditional factor demands (conditional on the level of output), and C^* is the optimal total cost.

Note that, again, the solutions to the model variables are functions of model parameters.

Next we seek to investigate the properties of x_1^*, x_2^*, C^* and analyze the connection between the cost minimization and profit maximization models.