Econ 8010 HW2
Solutions

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1. Consider a quasilinear utility function $u : \mathbb{R}^L_+ \times \mathbb{R} \to \mathbb{R}$ given by

$$u(x, m) = \phi(x) + m$$

Suppose that ϕ is twice continuously differentiable and strictly concave: its Hessian derivative matrix $D^2\phi(x)$ is continuous and negative definite for all x.

(a) Express Walrasian demand for x as an **implicit** function of prices p and wealth w.

- $\nabla\phi(x(p, w)) = \frac{1}{p_{L+1}} p_L$.

(b) Express Hicksian demand for x as an **implicit** function of prices p and required utility level $\bar{u}$.

- $\nabla\phi(h(p, \bar{u})) = \frac{1}{p_{L+1}} p_L$.

(c) Use the implicit function theorem to compute the $L \times L$ Slutsky matrix for the L non-numeraire goods. (Hint: Negative definite matrices are invertible.) What has to be true about the Hessian derivative matrix $D^2\phi(x)$ for each of the L non-numeraire goods to be net substitutes? What about net complements?

- From the implicit function theorem,

$$D_{p_L} h(p, \bar{u}) = \frac{1}{p_{L+1}} (D^2\phi(h(p, \bar{u})))^{-1}$$

So the inverse Hessian $(D^2\phi(h(p,\bar{u})))^{-1}$ must have nonnegative off-diagonal elements for the non-numeraire goods to be net substitutes, and nonpositive off-diagonal elements for them to be net complements.

2. (exercise by Dan Quint[1]) Let $u(x) = x_1^{\alpha}(x_2 + x_3)^{1-\alpha}$.

   (a) Is utility homothetic? What does that tell you about the change in Walrasian demand as wealth increases?

   - Homotheticity is pretty obvious. Thus, income effects do not depend on wealth.

   (b) If $p_2 > p_3$, which goods will the consumer demand? If $p_3 > p_2$?

   - The consumer will demand the cheaper of x_2 and x_3.

   (c) Solve the UMP and find Walrasian demand:

      i. when $p_2 > p_3$

$$x_1(p,w) = \frac{\alpha w}{p_1},\ x_2(p,w) = 0,\ x_3(p,w) = \frac{(1-\alpha)w}{p_3}$$

      ii. when $p_3 > p_2$

$$x_1(p,w) = \frac{\alpha w}{p_1},\ x_2(p,w) = \frac{(1-\alpha)w}{p_2},\ x_3(p,w) = 0$$

      iii. when $p_2 = p_3$

$$x_1(p,w) = \frac{\alpha w}{p_1},\ x_2(p,w) + x_3(p,w) = \frac{(1-\alpha)w}{p_3}$$

   (d) Solve the EMP and find Hicksian demand:

      i. when $p_2 > p_3$

$$h_1(p,\bar{u}) = \bar{u}\left(\frac{\alpha p_3}{(1-\alpha)p_1}\right)^{1-\alpha},\ h_2(p,\bar{u}) = 0,\ h_3(p,\bar{u}) = \bar{u}\left(\frac{(1-\alpha)p_1}{\alpha p_3}\right)^{\alpha}$$

      ii. when $p_3 > p_2$

$$h_1(p,\bar{u}) = \bar{u}\left(\frac{\alpha p_2}{(1-\alpha)p_1}\right)^{1-\alpha},\ h_2(p,\bar{u}) = \bar{u}\left(\frac{(1-\alpha)p_1}{\alpha p_2}\right)^{\alpha},\ h_3(p,\bar{u}) = 0$$

[1] I have added part (d) to the original exercise.

iii. when $p_2 = p_3$

$$h_1(p,\bar{u}) = \bar{u}\left(\frac{\alpha p_3}{(1-\alpha)p_1}\right)^{1-\alpha}, \; h_2(p,\bar{u}) + h_3(p,\bar{u}) = \bar{u}\left(\frac{(1-\alpha)p_1}{\alpha p_3}\right)^{\alpha}$$

(e) Compute the indirect utility function $v(p,w)$ and the expenditure function $e(p,\bar{u})$.

$$\begin{aligned} v(p,w) &= w\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{p_1^\alpha(\min\{p_2,p_3\})^{1-\alpha}}, \\ e(p,\bar{u}) &= \bar{u}p_1^\alpha(\min\{p_2,p_3\})^{1-\alpha}\left(\left(\frac{1-\alpha}{\alpha}\right)^\alpha + \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\right) \end{aligned}$$

3. A consumer has Cobb-Douglas utility

$$u(x_1,x_2) = \frac{1}{3}\log x_1 + \frac{2}{3}\log x_2$$

and wealth $w = 18$.

Suppose prices change from $(p_1,p_2) = (1,2)$ to $(3,1)$.

(a) What is her consumption bundle at the original price vector? How much utility does this yield? How much wealth will she need to afford her old consumption bundle after the price change?

- Original bundle=$(6,6)$, Utility=$\log 6$, $w' = 24$

(b) Calculate the (total, not infinitesimal) Slutsky substitution effect of the price change.

- Demand at $w' = 24$ and $p = (3,1)$ is $(\frac{8}{3},16)$. So Slutsky effect is $(-\frac{10}{3},10)$

(c) Calculate the (total, not infinitesimal) Hicks substitution effect of the price change.

- We calculated Cobb-Douglas Hicksian demand in (2). So Hicksian demand at $\bar{u} = \log 6$ is $(6^{1/3},6^{4/3})$. So Hicks effect is $(6^{1/3}-6, 6^{4/3}-6)$.