AFRE 802
Statistical Methods for Agricultural, Food, & Resource Economists

Wrap-up of Hypothesis Testing & Intro to Stata

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Announcement

- SIRS (Student Instruction Rating System) evaluation forms will be available starting Monday (Nov. 27)
- ***I value your comments and feedback, and take them seriously, so please fill out the SIRS!***
- There is an option to submit open-ended comments at the end >> a great place for you to write specific feedback on what I've done well or suggestions on how to improve the course
- All feedback is completely anonymous
- Thanks in advance! I've enjoyed working with you!

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Reminder

- Ch. 10 HW due next Tuesday (Nov. 28)

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GAME PLAN

- Return graded in-class exercise
- Review
- Wrap-up hypothesis testing (esp. finding type II error and sample size)
- Answer questions on Ch. 10 HW or class material
- Intro to Stata

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Review

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Type I vs. Type II error

		REALITY	
		NULL HYPOTHESIS	
		TRUE	FALSE
STUDY FINDINGS	TRUE		Type II error (β) 'False negative'
	FALSE	Type I error (α) 'False positive'	

- **Type I error: reject H_0 when H_0 is true**
  - Probability: α (significance level)

- **Type II error: fail to reject H_0 when H_0 is false (& H_1 is true)**
  - Probability: β
  - When **computing β**, need to do so for **particular values of α <u>and</u> the target parameter under H_1**

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Calculating the probability of Type II error (β)

<u>General approach</u> for H_0: $\theta=\theta_0$ vs. H_1: $\theta>\theta_0$ for a specific value of the target parameter under H_1 (call it θ_1, where $\theta_1>\theta_0$) and α

<u>Steps</u>

1. Find the cutoff for the RR in terms of Z (**under $\mathbf{H_0}$** and for the **given α**), then express it in terms of the estimator, $\hat{\theta}$. Let k be this cutoff value for $\hat{\theta}$, i.e.:

$$\text{RR } = [k, \infty)$$

2. $P(\text{Type II error}) = P(\text{fail to reject H}_0 \text{ in favor of H}_1 \text{ when H}_0 \text{ is false and H}_1 \text{ is true})$

$$= P(\hat{\theta} \text{ is not in the rejection region when H}_0 \text{ is false and H}_1 \text{ is true})$$

$$= P(\hat{\theta} \le k \text{ when H}_0 \text{ is false and H}_1 \text{ is true, i.e., when } \theta=\theta_1)$$

Find this probability by converting k to a Z-statistic **under $\mathbf{H_1}$**, i.e.:

$$P(z \le Z = \frac{k-\theta_1}{\sigma_{\hat{\theta}}})$$

<u>Note</u>: Will need to reverse signs in steps if H_1: $\theta<\theta_0$

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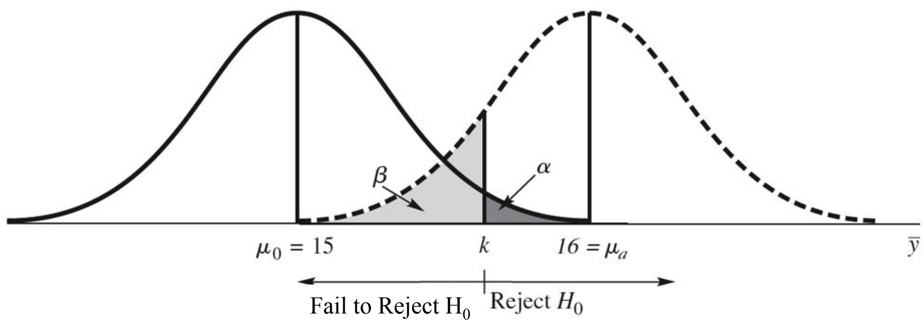
Example 10.8 review

α=P(Type I error)=P(reject H_0 when H_0 is true)

β=P(Type II error)=P(fail to reject H_0 when H_0 is false and H_1 is true given specific values of H_1 and α)

$= P(\overline{Y}$ is <u>not</u> in the rejection region when H_0 is false and H_1 is true)

$= P(\overline{Y} \le k$ when H_0 is false and H_1 is true))



k=15.8225
is the cutoff for the sample mean for our rejection region for H_0: μ=15 vs. H_1: μ > 15 (specifically μ = 16) at the α=0.05 level. That is, we reject H_0 in favor of H_1 if the sample mean is ≥ 15.8225.

Finding the sample size for Z-tests

- In Example 10.8, with N=36 and α=0.05, we calculated that β=0.36 → high P(Type II error)
- A key way to reduce β is to increase the sample size
- The flip side of determining β given N and α is to determine N given desired values of α and β
- Suppose you want to test H_0: $\mu=\mu_0$ vs. H_1: $\mu>\mu_0$ for given values of α and β (and where β is evaluated at specific value $\mu_1>\mu_0$ under H_1). Then:

Sample size for a one-tailed Z-test for μ for given levels of α, β, μ_0 (value of μ under H_0) and μ_1 (value of μ under H_1):

$$N = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_1 - \mu_0)^2} \text{ rounded up to the nearest whole number}$$

Same formula works for H_1: $\mu<\mu_0$. See WMS p. 509 for proof.

Finding the sample size for Z-tests - example

Example 10.9 in WMS

Find the sample size, N, for testing H_0: μ=15 vs. H_1: μ=16 with α=β=0.05. Assume a variance of 9. (Context is the average # of calls/week made by salespeople at a large corporation.)

Sample size for a one-tailed Z-test for μ for given levels of α, β, μ_0 (value of μ under H_0) and μ_1 (value of μ under H_1):

$$N = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_1 - \mu_0)^2}$$ rounded up to the nearest whole number

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The "power" of statistical tests

- We have discussed β=P(Type II Error) =P(fail to reject H_0 when H_0 is false and H_1 is true)
- The **"power" of a statistical test is 1-β**, i.e., the probability that we do reject H_0 when H_0 is false and H_1 is true. **More power is better than less power!**
  - As with β, the power of a test depends on the parameter value specified under H_1 (θ_1)
- *How does β change as N increases?*
- *So how does power change as N increases?*

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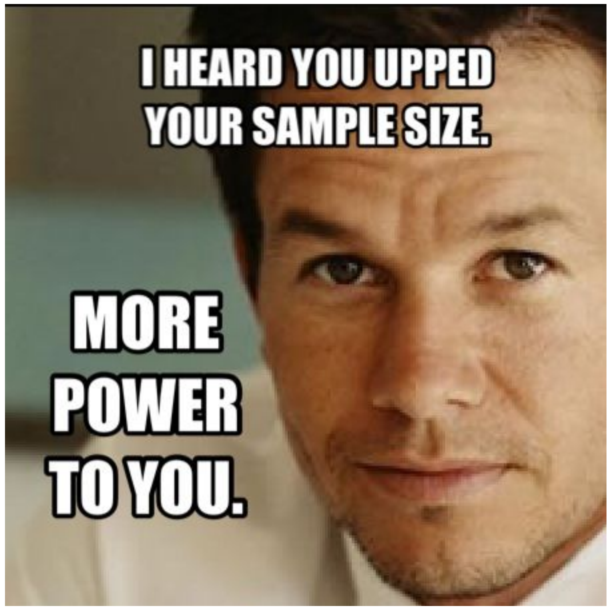

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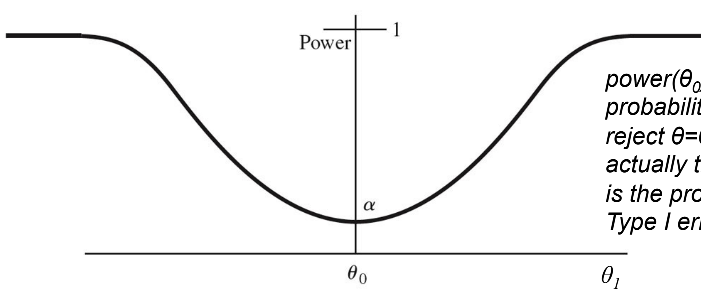
The "power" of statistical tests (cont'd)

- Final note on power:
  - *Do you think statistical tests have more power for parameter values under H_1 (θ_1) that are close to or farther away from the value under the H_0 (θ_0) ? Why?*
  - It is easier to detect that H_0 is false (more power) when θ_1 is **farther** from θ_0



power(θ_0)=α b/c the probability that we reject $\theta=\theta_0$ when it is actually true that $\theta=\theta_0$ is the probability of Type I error (α)

Figure: A typical power curve for the test of $H_0 : \theta=\theta_0$ vs. $H_1 : \theta=\theta_1$ for various values of θ_1

Summary

- In Chapters 8 and 9, we talked about **how to estimate** numerical values of target parameter **θ**
  - Point estimates & confidence intervals (CIs)
  - Desirable properties of estimators (consistency, unbiasedness, efficiency, low MSE)
  - Methods of estimation (MOM, MLE, least squares)
- In Chapter 10, we talked about:
  - Testing hypotheses related to θ for large samples, and for μ for small samples
  - The relationship between hypothesis testing and CIs
  - p-values
  - Probabilities of Type I (α) and Type II (β) errors, and the power of a statistical test (1-β) → these probabilities tell us how 'good' our inferences are (i.e., how much faith we can put in the results of our hypothesis tests)
  - Computing the sample size for Z tests

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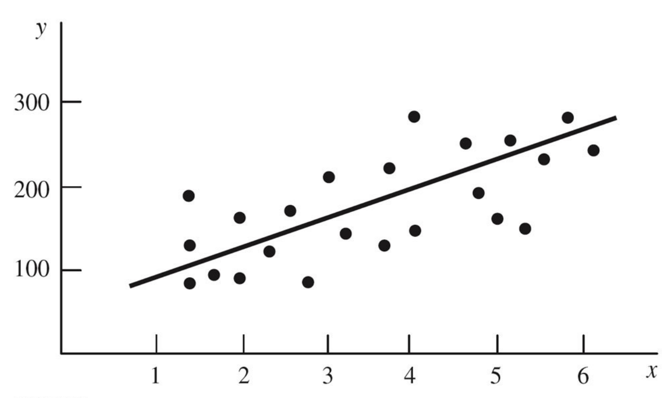
After Thanksgiving: Simple Linear Regression

- Theory and Stata implementation
- Hurray!!!



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Questions on Ch. 10 HW or class material?

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Getting Started with Stata

- Do-file and dataset are on D2L >> "Stata data- and do-files, and other resources" >> Lecture 1 - frequencies, histograms, & summary stats (We didn't have time for this the 1st day of class.)
- Go through do-file
- (Time-permitting) Go through some topics in "Wooldridge – Rudiments of Stata" document. PDF & data file (WAGE1.DTA) are in "Stata data- and do-files, and other resources" on D2L

Homework:

- **All Ch. 10 HW is due on Tuesday, Nov. 28

Remaining lectures – only 4 left – time flies!

- 4 classes after Thanksgiving break: introduction to OLS (hurray!) and course wrap-up

Reading for Tuesday after break

- Optional: WMS Ch. 11 (sections 11.1-11.3)
- Required: Wooldridge *Introductory Econometrics* (2003) pp. 22-37 – on D2L

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