2016 Micro Prelim

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1.1

Consider the preferences represented by the utility function $U(x_1, x_2) = ln(x_1) + 4ln(x_2)$, facing market prices $p_1, p_2 > 0$. Let the consumer's wealth be denoted by $I > 0$. Set up a Lagrangian for maximizing utility subject to the budget constraint and solve for the Marshallian (uncompensated) demand functions for x_1 and x_2.

SOLUTION:

$$\begin{gathered}\underset{x_1,x_2}{\text{Max}} \qquad ln(x_1) + 4ln(x_2) \\ \text{s.t.} p_1x_1 + p_2x_2 = I\end{gathered} \tag{1}$$

$$\mathcal{L} = ln(x_1) + 4ln(x_2) + \lambda(I - p_1x_1 - p_2x_2) \tag{2}$$

FOC:

$$\begin{aligned}\frac{\partial\mathcal{L}}{x_1} &= \frac{1}{x_1} - \lambda p_1 = 0 \\ \frac{\partial\mathcal{L}}{x_2} &= \frac{4}{x_2} - \lambda p_2 = 0 \\ \frac{\partial\mathcal{L}}{\lambda} &= I - p_1x_1 - p_2x_2 = 0\end{aligned} \tag{3}$$

Solving for x_1, x_2:

$$\begin{aligned}\frac{\frac{1}{x_1}}{\frac{4}{x_2}} &= \frac{\lambda p_1}{\lambda p_2} \\ \frac{x_2}{4x_1} &= \frac{p_1}{p_2} \\ x_2 &= \frac{4x_1p_1}{p_2} \\ x_1 &= \frac{p_2x_2}{4p_1}\end{aligned} \tag{4}$$

Plugging this into our constraint:

$$
\begin{aligned}
p_1x_1 + p_2(\frac{4x_1p_1}{p_2}) &= I \\
p_1x_1 + 4x_1p_1 &= I \\
5p_1x_1 &= I \\
x_1^M &= \frac{I}{5p_1}
\end{aligned} \tag{5}
$$

By symmetry, we then know: $x_2^M = \frac{4I}{5p_2}$.

1.2

How do you interpret the Lagrange multiplier in the utility maximization problem?

SOLUTION:

Generally, the Lagrange multiplier for a constrained maximization problem is the shadow value of relaxing the constraint (i.e. it gives the rate of change of the solution to the constrained maximization problem as the constraint varies). In this case, λ is the shadow value that tells us how much more utility the consumer can get by changing his/her budget.

1.3

Find the Indirect Utility function. State and demonstrate three properties of the Indirect Utility Function.

SOLUTION:

$$
\begin{aligned}
V(p_1, p_2, I) &= ln(x_1^M) + 4ln(x_2^M) \\
&= ln(\frac{I}{5p_1}) + 4ln(\frac{4I}{5p_2}) \\
&= ln(I) - ln(5p_1) + 4ln(4) + 4ln(I) - 4ln(5p_2) \\
&= 5ln(I) + 4ln(4) - ln(5) - ln(p_1) - 4ln(5) - 4ln(p_2) \\
V(p_1, p_2, I) &= 5ln(I) + 4ln(4) - 5ln(5) - ln(p_1) - 4ln(p_2)
\end{aligned} \tag{6}
$$

Three properties of the Indirect Utility Function:

1. Homogeneous of degree zero.

$$
\begin{aligned}
V(\alpha p_1, \alpha p_2, \alpha I) &= 5ln(\alpha I) + 4ln(4) - 5ln(5) - ln(\alpha p_1) - 4ln(\alpha p_2) \\
&= 5ln(\alpha) + 5ln(I) + 4ln(4) - 5ln(5) - ln(\alpha) - ln(p_1) - 4ln(\alpha) - 4ln(p_2) \\
&= 5ln(\alpha I) + 4ln(4) - 5ln(5) - ln(\alpha p_1) - 4ln(\alpha p_2) \\
&= 5ln(\alpha) + 5ln(I) + 4ln(4) - 5ln(5) - ln(\alpha) - ln(p_1) - 4ln(\alpha) - 4ln(p_2) \\
&= 5ln(I) + 4ln(4) - 5ln(5) - ln(p_1) - 4ln(p_2) = V(p_1, p_2, I)
\end{aligned} \tag{7}
$$

2. Strictly increasing in I, and non-increasing in p_l for any l good.

$$\begin{aligned} \frac{\partial V(p_1,p_2,I)}{\partial I} &= \frac{5}{I} > 0 \\ \frac{\partial V(p_1,p_2,I)}{\partial p_1} &= -\frac{1}{p_1} < 0 \\ \frac{\partial V(p_1,p_2,I)}{\partial p_2} &= -\frac{4}{p_2} < 0 \end{aligned} \tag{8}$$

3. Continuous in p and I.

   This property arises from the functional form of $V(.)$, which in our case is logarithmic and therefore clearly continuous in prices and wealth.

1.4

Find the consumer's Expenditure function in the simplest way possible.

SOLUTION:

By Duality, we can take the intermediate demands found above in the UMP and plug them into the EMP constraint to derive the Hicksian demands for goods 1 and 2.

Recall, the intermediate demands for good 2: $x_2 = \frac{4x_1p_1}{p_2}$. And, the constraint for the EMP is $ln(x_1) + 4ln(x_2) = \bar{U}$.

1.5

State and determine three properties of the Expenditure Function.

SOLUTION:

1.6

Derive the Hicksian (compensated) demand functions for x_1 and x_2.

SOLUTION:

$$\begin{aligned} \underset{x_1,x_2}{\text{Min}} \quad & p_1x_1 + p_2x_2 \\ & \text{s.t.} U(x_1,x_2) \geq \bar{U} \end{aligned} \tag{9}$$

Lagrangian:

$$\mathcal{L} = p_1x_1 + p_2x_2 + \lambda(\bar{U} - ln(x_1) - 4ln(x_2)) \tag{10}$$

FOC:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_1} &= p_1 - \frac{\lambda}{x_1} = 0\\ \frac{\partial \mathcal{L}}{\partial x_2} &= p_2 - \frac{4\lambda}{x_2} = 0\\ \frac{\partial \mathcal{L}}{\partial \lambda} &= \bar{U} - ln(x_1) - 4ln(x_2) = 0\end{aligned} \tag{11}$$

Intermediate demands:

$$\begin{aligned}\frac{p_1}{p_2} &= \frac{x_2}{4x_1}\\ x_2 &= \frac{4p_1x_1}{p_2}\\ x_1 &= \frac{p_2x_2}{4p_1}\end{aligned} \tag{12}$$

Demand for x_1:

$$\begin{aligned}ln(x_1) + 4ln(\frac{4p_1x_1}{p_2}) &= \bar{U}\\ ln(x_1) + 4ln(4) + 4ln(p_1) + 4ln(x_1) - 4ln(p_2) &= \bar{U}\\ 5ln(x_1) &= \bar{U} - 4ln(4) - 4ln(p_1) + 4ln(p_2)\\ x_1^5 &= e^{\bar{U}} + 4^{-4} + p_1^{-4} + p_2^4\\ x_1^H &= e^{\frac{\bar{U}}{5}} + 4^{\frac{-4}{5}} + p_1^{\frac{-4}{5}} + p_2^{\frac{4}{5}}\end{aligned} \tag{13}$$

Demand for x_2:

$$\begin{aligned}ln(\frac{p_2x_2}{4p_1}) + 4ln(x_2) &= \bar{U}\\ ln(p_2) + ln(x_2) - ln(4) - ln(p_1) + 4ln(x_2) &= \bar{U}\\ 5ln(x_2) &= \bar{U} - ln(p_2) + ln(4) + ln(p_1)\\ x_2^5 &= e^{\bar{U}} + p_2^{-1} + 4 + p_1\\ x_2^H &= e^{\frac{\bar{U}}{5}} + p_2^{\frac{-1}{5}} + 4^{\frac{1}{5}} + p_1^{\frac{1}{5}}\end{aligned} \tag{14}$$

2.1

Consider a pure exchange economy with 2 people (A and B) and 2 commodities denoted x_1 and x_2. The preferences of individuals A, and B are represented as:

$$U^A(x_1^A, x_2^A) = x_1^A + 2x_2^A \qquad U^B(x_1^B, x_2^B) = min\{2x_1^B, x_2^B\}$$

where $\delta > 1$. Solve for the competitive equilibrium of this economy assuming initial endowments are:

$$e_1^A = 10, e_2^A = 10, \quad e_1^B = 10, e_2^B = 10$$

You do not have to show any work, but you must box the following (a) the setup of all optimization problems, (b) the demand functions for each agent, and (c) the final answer for the Wlarasian equilibrium. Clearly and precisely illustrate the equilibrium, and the set of all Pareto Efficient Allocations.

SOLUTION: