ApEc 8001
Applied Microeconomic Analysis: Demand Theory

Lecture 1: Preference and Choice (MWG, Chap. 1)

I. Introduction

Much of applied economic research focuses on the behavior of individuals, and the behavior of small groups of individuals (e.g. households). Economists have developed a theoretical framework to analyze how individuals make decisions, which depend on their **preferences** and on the **possible alternatives** (possible choices) available to them. This lecture presents the foundation of this theoretical framework.

To be as general as possible, the theoretical framework is very abstract, which may make it a bit boring. But it will be less abstract in some parts of this course, and much less abstract in other Ph.D. level economics courses you will take. So please be patient.

There are **two ways to model individual choice** behavior. The first begins with the decision maker's **preference relations**: how individuals rank different choices. The second starts with the decision maker's **choices**. The second is more abstract, but it has the advantage that it begins with something that can be observed. This lecture covers both approaches.

II. Preference Relations

You have seen in previous economics classes that the starting point for understanding individuals' behavior is their utility functions. A somewhat more abstract theory is the preference-based approach, which starts by modeling the objectives or goals of the decision maker as **preference relations** that can be assigned to pairs of alternatives (pairs of choices).

The **preference relation** is denoted by $\succsim$. This is a **binary** relationship that compares two alternatives. Let X denote the set of all possible alternatives, and let x and y be two alternatives in this set ($x, y \in X$).

If $x \succsim y$ then "x is at least as good as y".

We can use $\succsim$ to define two other important relations:

1. **Strict preference relation**, denoted by $\succ$:

   $$x \succ y \quad \Leftrightarrow \quad x \succsim y \text{ but not } y \succsim x.$$

   In words, we can say "x is preferred to y".

2. **Indifference relation**, denoted by $\sim$:

   $$x \sim y \quad \Leftrightarrow \quad x \succsim y \text{ and } y \succsim x.$$

   In words, we can say "x is indifferent to y".

Economists often claim that people are "rational". This may or may not be true (to be discussed more in Lecture 12), but for now let's define this concept. Basically, we say that the preference relation $\succsim$ is rational if it satisfies "completeness" and "transitivity".

Definition: The preference relation $\succsim$ is **rational** if it has the following two properties:

1. **Completeness**: For all $x, y \in X$, either $x \succsim y$ or $y \succsim x$ (or both).

2. **Transitivity**: For all $x, y, z \in X$, if $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

Completeness simply means that, for all alternatives available, the decision maker can compare them and decide which is preferred. In other words, the decision maker is "decisive" and "always knows what he or she wants".

Transitivity is quite intuitive and appears reasonable; if people make "mistakes" that imply that their preferences are not transitive than such behavior does seem to be "irrational".

One can show that rational preference relations have implications for strict preference and indifference:

Proposition 1.B.1: If $\succsim$ is rational then:

1. $\succ$ is both **irreflexive** ($x \succ x$ cannot hold) and transitive (if $x \succ y$ and $y \succ z$, then $x \succ z$).

2. $\sim$ is **reflexive** ($x \sim x$ for all x), transitive (if $x \sim y$ and $y \sim z$, then $x \sim z$) and **symmetric** (if $x \sim y$ then $y \sim x$),

3. If $x \succ y$ and $y \succsim z$, then $x \succ z$

While all of these seem pretty reasonable, in real life there are examples where people do not follow rational behavior. See the discussion in Mas-Colell et al., pp.7-8. Example: A person's "tastes" change.

Even if people are individually rational there may be instances where "group behavior" is irrational. An example of this is Condorcet's voting paradox:

Person 1: $A \succ B \succ C$
Person 2: $B \succ C \succ A$
Person 3: $C \succ A \succ B$

If this group decides by "majority rule" their voting on pairs of choices will be $A \succ B$, $B \succ C$ and $C \succ A$.

From Preferences to Utility Functions

Almost all applied economic research that is based on a model of individual (or household) decision making starts with a utility function, not with preference relations. It turns out that utility functions and preference relations are almost identical. This subsection explains how they are related.

Definition: A function u: $X \rightarrow \mathbb{R}$ is a **utility function representing the preference relation** $\succsim$ if, for all $x, y \in X$:

$$x \succsim y \iff u(x) \geq u(y)$$

In fact, there are many utility functions that could satisfy this definition, so **the utility function is not unique**. More specifically, for any strictly increasing function f( ): $\mathbb{R} \rightarrow \mathbb{R}$, $v(x) \equiv f(u(x))$ also represents the preference relation $\succsim$. This implies that the utility function u( ) is **ordinal**, not **cardinal**.

Proposition 1.B.2: A preference relation $\succsim$ can be represented by a utility function only if $\succsim$ is rational.

Proof: This is relatively easy to prove given the above definition of a utility function. This can be done by first showing that the existence of a utility function implies completeness, and then showing that it implies transitivity.

Completeness. Since u( ) is a real-valued function defined for all x in X, it must be that for any x, y $\in$X, either u(x) $\geq$ u(y) or u(y) $\geq$ u(x). By the definition of u( ) this implies that either x $\succsim$ y or y $\succsim$ x.

Transitivity. Suppose that x $\succsim$ y and y $\succsim$ z. Since u( ) represents $\succsim$, we must have u(x) $\geq$ u(y) and u(y) $\geq$ u(z). This implies that u(x) $\geq$ u(z), which in turn implies x $\succsim$ z, which demonstrates transitivity.

Technically speaking, this proposition does **not** imply that all rational preference relations can be represented by a utility function; we will see a rational preference relation that cannot be represented by a utility function in Lecture 4. On the other hand, if X is finite then any rational preference relation can be represented by a utility function.

III. Choice Rules

The second approach to the theory of decision making takes decision makers' choices themselves as the foundation of ("primitive" object of) the theory. In formal (mathematical) terms, this choice behavior is represented by a **choice structure**, which can be denoted by two "components", $\mathcal{B}$ and C( ):

1. $\mathcal{B}$ is a "family" (a set) of nonempty subsets of X. In other words, every element of $\mathcal{B}$ is a set $B \subset$ X. You can think of each *B* as a "budget set": what a consumer can afford given his or her budget. More generally, each *B* is a set of all possible options (all possible choices) available to the decision maker.

2. C( ) is a **choice rule** (mathematically, a correspondence) that assigns a nonempty set of chosen elements of *B*, i.e. C(*B*) $\subset$ *B* for every budget set in $\mathcal{B}$. In the simplest case C(*B*) is one element of *B*. Yet it is **possible for C(*B*) to contain multiple elements of *B***, in which case the decision maker may choose (in repetitions of the same decision problem) any of those multiple elements. [We could say that they all have the same "value" to the decision maker, but we don't say this because the whole point is to focus on the choices without referring to underlying preferences.]

To make this clearer, here are **two examples**.

[**Write these on the whiteboard.]**

Let X = {x, y, z} and $\mathcal{B}$ = {(x, y}, {x, y, z}}.

1. One possible choice structure is $(\mathscr{B}, C_1(\ ))$, where the choice rule $C_1(\ ))$ is $C_1(\{x, y\}) = \{x\}$ and $C_1(\{x, y, z\}) = \{x\}$. Here the decision maker chooses x over both y and z.

2. Another possible choice structure is $(\mathscr{B}, C_2(\ ))$, where the choice rule $C_2(\ ))$ is $C_2(\{x, y\}) = \{x\}$ and $C_2(\{x, y, z\}) = \{x, y\}$.

The second example may not seem very "rational", so it is useful to set some "reasonable" restrictions on choice structures. The **weak axiom of revealed preference** does exactly that:

Definition: The choice structure $(\mathscr{B}, C(\ ))$ satisfies the **weak axiom of revealed preference** if the following property holds:

> If for some $B \in \mathscr{B}$, with $x, y \in B$, we have $x \in C(B)$, then for any $B' \in \mathscr{B}$, with $x, y \in B'$, and $y \in C(B')$, we must also have $x \in C(B')$.

In words, this axiom states that if x is ever chosen when y is available, then there can be no budget set containing both alternatives for which y is chosen but x is not chosen.

Question: Is it also true that $y \in C(B)$?

A somewhat simpler statement of the weak axiom of revealed preference can be obtained by a "revealed preference" relation, $\succsim^*$, from the observed choice behavior given by C( ):

Definition: Given a choice structure ($\mathscr{B}$, C( )), the **revealed preference relation**, $\succsim^*$, is defined by:

$$x \succsim^* y \iff \text{there is some } B \in \mathscr{B} \text{ such that } x, y \in B \text{ and } x \in C(B)$$

We read x $\succsim^*$ y as "x is revealed at least as good as y".

Note that the revealed preference relation $\succsim^*$ **need not be either complete or transitive**.

This definition of the revealed preference relation allows one to "restate" the weak axiom of revealed preference as:

"If x is revealed at least as good as y, then y cannot be revealed preferred to x."

[Mas-Colell et al. "informally" define "x is revealed preferred to y" if for some $B \in \mathscr{B}$ such that x, y $\in B$, x $\in$ C(B) and y $\notin$ C(B); that is the choice rule always selects x and never y when both are feasible.]

Recall the two choice structure examples at the top of page 8. Do they satisfy the weak axiom (of revealed preference)?

For the first one $C_1(\ )$, the choices imply that $x \succsim^* y$ and $x \succsim^* z$, but they do not tell us anything about the relationship between y and z. This structure satisfies the weak axiom (trivially, because y and z are never chosen).

For the second one, the choice rule $C_2(\ )$ that leads to $C_2(\{x, y, z\}) = \{x, y\}$ implies that $y \succsim^* x$, (and $x \succsim^* y$, $x \succsim^* z$ and $y \succsim^* z$). But since $C_2(\{x, y\}) = \{x\}$, x is revealed preferred to y. Therefore the choice structure $(\mathscr{B}, C_2(\ ))$ violates the weak axiom. Alternatively, this *directly* contradicts the weak axiom.

So what is the difference or relationship between preference relations and choice rules? This is examined in the next (which is the last) section.

IV. The Relationship between Preference Relations and Choice Rules

To see how preference relations and choice rules are related, we will end this lecture by considering two questions about their relationship:

1. If a decision maker has a rational preference ordering $\succsim$, do his or her decisions when facing choices from the "budget" sets in $\mathscr{B}$ generate a choice structure that satisfies the weak axiom?

2. If a decision maker's choice behavior for a family of budget sets $\mathscr{B}$ takes the form of a choice structure ($\mathscr{B}$, C( )) that satisfies the weak axiom, does it follow that there is a rational preference structure relation that is consistent with these choices?

The answer to the first question is "yes", but the answer to the second question is "maybe" (or "it depends").

First Question

Start with a person who has a rational preference relation $\succsim$ for a set of alternatives (choices) denoted by X. If this person faces a (nonempty) subset of alternatives, $B \subset X$, his/her preference-maximizing behavior is to choose one of the elements in the set:

$$C^*(B, \succsim) = \{x \in B: x \succsim y \text{ for every } y \in B\}$$

Note: $C^*(B, \succsim)$ is *defined* as the choice behavior that corresponds to $\succsim$. It can have more than one element.

That is, the elements in the set $C^*(B, \succsim)$ are the decision makers "most preferred" alternatives (choices) in B.

In theory, it is possible that this set is a null set, but it is not a null set if B is finite or if certain continuity assumptions hold, so do not worry about this.

More precisely, we will consider only preferences $\succsim$ and "families" of budget sets $\mathscr{B}$ such that $C^*(B, \succsim)$ is a nonempty set for all possible B in $\mathscr{B}$. That is, the rational preference relation **generates** the choice structure $(\mathscr{B}, C^*(B, \succsim))$ for any B in $\mathscr{B}$.

The following proposition says that any choice structure generated by rational preferences necessarily satisfies the weak axiom:

Proposition 1.D.1: Suppose that $\succsim$ is a rational preference relation. Then the choice structure generated by $\succsim$, which is denoted by $(\mathscr{B}, C^*(\cdot, \succsim))$, satisfies the weak axiom of revealed preference.

Proof: Suppose that, for some $B \in \mathscr{B}$, there exists x, y $\in B$ and x $\in C^*(B, \succsim)$. This implies that x $\succsim$ y. Now suppose that, for another budget set $B' \in \mathscr{B}$, with x, y $\in B'$, the following holds: y $\in C^*(B', \succsim)$.

This means that y $\succsim$ z for all z $\in B'$. We have already seen that x $\succsim$ y. Thus transitivity implies that x $\succsim$ z for all z $\in B'$, which in turn implies that x $\in C^*(B', \succsim)$. Therefore the choices generated by the preference relation $\succsim$ satisfy the weak axiom.

Second Question

To answer the second question, we need to start with a definition:

Definition: Given a choice structure ($\mathscr{B}$, C( )), the rational preference relation $\succsim$ **rationalizes** C( ) for the family of budget sets $\mathscr{B}$ if:

$$C(B) = C^*(B, \succsim)$$

for all $B \in \mathscr{B}$, that is if $\succsim$ generates the choice structure ($\mathscr{B}$, C( )).

To put this into words, the rational preference relation $\succsim$ rationalizes the choice rule C( ) on $\mathscr{B}$ if the optimal choices generated by $\succsim$ (which are denoted by $C^*(B, \succsim)$) are exactly the same as the choices generated by C( ) for all budget sets in $\mathscr{B}$. **That is, we can interpret the decision maker's choices as if he or she were maximizing a set of preferences.**

Note: It is possible that there is more than one rationalizing preference relation for a given choice structure ($\mathscr{B}$, C( )).

However, the **weak axiom** by itself is **not sufficient** to guarantee the existence of a rationalizing preference relation. This is seen in a simple example:

Example: Let X = {x, y, z}, $\mathscr{B}$ = {{x, y}, {y, z}, {x, z}}, C({x, y}) = {x}, C({y, z}) = {y}, and C({x, z}) = {z}. You should be able to show that this choice structure satisfies (does not contradict) the weak axiom. But there is no set of rationalizing preferences that explain this behavior, because C({x, y}) = {x} implies x $\succ$ y and C({y, z}) = {y} implies y $\succ$ z, and these two preference relations imply x $\succ$ z. Yet C({x, z}) = {z} implies z $\succ$ x, which contradicts x $\succ$ z. Thus this choice structure does not have a rationalizing preference relation.

The intuition behind this example is that the more budget choices there are in $\mathscr{B}$, the more the weak axiom restricts the decision maker's behavior. Adding more budget sets to this example will help. More specifically, you can show that adding the budget set {x, y, z} to $\mathscr{B}$ implies that the choices made in this example violate the weak axiom.

More generally, if $\mathscr{B}$ includes all budget sets with up to three elements, then there exists a rational preference relation that rationalizes any choice structure that satisfies the weak axiom. This was shown by Arrow (1959). More formally, we have:

Proposition 1.D.2: If ($\mathscr{B}$, C( )) is a choice structure that meets the following two requirements:

1. The weak axiom of revealed preference is satisfied
2. $\mathscr{B}$ includes all subsets of X with up to three elements

then there exists a rational preference relation $\succsim$ that rationalizes C( ) for the family of budget sets $\mathscr{B}$. That is, C(B) = C*(B, $\succsim$) for all $B \in \mathscr{B}$. In addition, this rational preference relation is the **only** preference relation that rationalizes C( ) for budget sets in $\mathscr{B}$.

The proof is given on pp.13-14 of Mas-Colell et al. This is optional.

It turns out that there are some cases in which this requirement of $\mathscr{B}$ is not met. But this can be "fixed" by appealing to the "strong" axiom of revealed preference, which we will discuss in a later lecture.

For applied economic research a theory of choice that satisfies a revealed preference axiom can be represented by rational preferences, which in turn can be represented by a utility function. Thus the focus of this course will be on utility functions and functions derived from the maximization of utility functions.