Applied Microeconomics: Firm and Household

Lecture 17: Game Theory

Jason Kerwin

Department of Applied Economics
University of Minnesota

November 22, 2016

Outline

- Game Theory in a Nutshell
  - The Prisoner's Dilemma
  - A First Look at the Theory
  - Solution Concepts
  - Nash Equilibriium
- Continuous Action Spaces
  - The Tragedy of the Commons
- Sequential Games
- Repeated Games

The Prisoner's Dilemma

There are two prisoners who are hauled in for a suspected crime. The DA speaks each prisoner separately. She has some evidence to convict them but she offers each of them a deal if they confess.

- **Players:** There are two prisoners.
- **Actions:** Each prisoner has the option to confess or not confess.
- **Timing:** Each prisoner makes decisions without knowing the decision of the other prisoner (this is a simultaneous-move game).
- **Payoffs:**
  - If neither of them confess, the DA has enough evidence to put them both away for a year.
  - If both prisoners confess, both prisoners go to prison for five years.
  - If one prisoner confesses but not the other, the confessing prisoner walks away but the other goes to prison for 15 years.

The Prisoner's Dilemma in Normal Form

Other rules:

- **Static ("one-shot") game:** Both prisoners know that this is a one-time deal. That is, once both decisions are made, the payoffs are realized and the game is over. Their decisions cannot be changed, and the players will never interact again.
- **Common knowledge:** Each prisoner knows that the DA is making an offer to the other prisoner as well. They know every detail about their situation and that fact is commonly known.

The Prisoner's Dilemma in Normal Form

Other rules:

- **Static ("one-shot") game:** Both prisoners know that this is a one-time deal. That is, once both decisions are made, the payoffs are realized and the game is over. Their decisions cannot be changed, and the players will never interact again.
- **Common knowledge:** Each prisoner knows that the DA is making an offer to the other prisoner as well. They know every detail about their situation and that fact is commonly known.

We can represent the prisoner's dilemma game in normal (matrix) form as:

Prisoner 1/ Prisoner 2	Confess	Not Confess
Confess	5, 5	0,15
Not Confess	15, 0	1, 1

Analysis of the Prisoner's Dilemma

Analysis: Note that from the pair's point of view (i.e. using a utilitarian social welfare function) the best outcome is (not confess, not confess).

Analysis of the Prisoner's Dilemma

Analysis: Note that from the pair's point of view (i.e. using a utilitarian social welfare function) the best outcome is (not confess, not confess).

Strategic behavior: However, consider the point of view of prisoner 1, (P1).

1. If P1 thinks that P2 is not going to confess, P1 should confess to go free.
2. If P1 thinks that P2 is going to confess, P1 should confess again to minimize his jail time.
3. So, P1 had better confess *no matter what P2 does*.
4. The same logic runs through P2's mind
5. Thus, the outcome is (confess, confess).

What if the punishment for being ratted out (and staying quiet) is 7 years instead of 15?

Applications of the prisoner's dilemma

1. Settling a dispute (a divorce or labor settlement): Two parties have the option of bringing in a lawyer or not.
    - If they settle without lawyers each get equal share (50, 50)
    - If only one party brings in a lawyer, that party gets more than 50% of the original pie due to better counsel.
    - If both hire lawyers, they still get equal shares, but of a smaller pie due to legal fees.

Applications of the prisoner's dilemma

1. Settling a dispute (a divorce or labor settlement): Two parties have the option of bringing in a lawyer or not.
   - If they settle without lawyers each get equal share (50, 50)
   - If only one party brings in a lawyer, that party gets more than 50% of the original pie due to better counsel.
   - If both hire lawyers, they still get equal shares, but of a smaller pie due to legal fees.
2. Maintaining a Cartel: Firms in a cartel have the option of either abiding by the cartel agreement or cheating.
   - If all firms abide by the agreement, they can share monopoly profits.
   - If none of the firms abides by the agreement, the firms make oligopoly profits, which are less than their share of the monopoly profits.
   - If some firms cheat, the abiding firms will make less than their share of monopoly profits, but the cheating firms would make more than their share of the monopoly profits.

Game theory: a formal definition

Game theory is a formal way of analyzing *interactions* among a group of rational agents who behave *strategically*.

Specifically, **game theory** is formalizes each of the following items:

- *group:* In any game there is more than one decision-maker; each decision maker is referred as a *player*.
- *interaction:* What any one individual player does directly affects at least one other player in the group.
- *strategic:* An individual player accounts for this interdependence in deciding what action to take.
- *rational:* While accounting for this interdependence, each player chooses her best action.

An example from sports: random drug testing

- The *group* is made of competitive athletes and the International Olympic Committee (IOC).
- The *interaction* is between athletes, who make decisions about how much to train and whether or not to use drugs, and the IOC, which needs preserve the reputation of the sport.
- *Rational strategic* play requires the athletes to make decisions based on their chances of winning and, if they dope, their chances of getting caught. Similarly, it requires the IOC to determine drug testing procedures and punishments on the basis of testing costs and the value of the compeition having a clean reputation.

An example from economics: Pharmaceutical R&D

It is estimated that the average cost of developing a new drug is around $350 million dollars. Companies are concerned about issues such as: which product lines to invest research dollars in; how high to price a new drug; and how to reduce the risks associated with a new drug's development

- The *group* is the set of drug companies.
- The *interaction* arises because the first developer of a drug makes the most profits (via patents). So the amount of R&D investment of one company affects the others' decisions about how much to invest.
- R&D expenditures are *rational & strategic* if they are chosen to maximize the profits from developing a new drug, given inferences about competitors' commitments to this line of drugs (that is, each company tries to maximize its profits taking into account the actions of all other companies).

The formal structure of games

Every game is played by a set of rules which have to specify four things:

1. **who** is playing – the group of players that strategically interact.
2. **what** they are playing with – the alternative actions or choices, and hence the strategies that each player has available.
3. **when** each player gets to play (in what order)
4. **how much** each player gains (or loses) from choices made in the game.

Types of games

- Simultaneous vs. Sequential
  - whether all players choose their actions at the same time, or whether some players can observe the actions of others before they make their decisions.
- Pure vs. Mixed strategy games
  - whether players can choose only a single action, or if they can randomize across two or more actions with positive probability on each
- Single-period vs. Repeated games
  - whether players play the game just for one period or two or more periods.
- Complete vs. Incomplete information (Bayesian games)
  - whether some players have private information that is not known by others.

Common knowledge

Throughout our analysis we will maintain the important assumption that *the rules of a game are common knowledge*.

Common knowledge means every player knows something, and the fact that they know it is commonly known.

Common knowledge simply means that if any two players in a game were asked a question about who, what, when and how much, they would give the same answer, and also that each player knows that other players will give the same answer.

Common knowledge does not mean that players are equally influential. Common knowledge is also domain-specific: we assume that players are equally well-informed about the rules of the game, but there could be other factors that are part of a game that are not common knowledge. For example, when you buy a used car both you and the seller know that the seller is better-informed about the condition of the car. But the condition of the car itself is not common knowledge.

Normal and extensive form of games

So far we have described games verbally, which is imprecise and often tedious. We can describe the rules of a game more compactly using one of the two principal representations of a game:

- Normal (or strategic, or matrix) form
- Extensive form

Normal-form games

A normal-form game is specified by three objects:

- the list of players in the game
- the set of strategies available to each player
- the payoffs associated with any strategy combination

All these objects are summarized in a matrix.

Normal-form games

Again, consider the prisoner's dilemma game:

		Suspect 2	
		Fink	Silent
Suspect 1	Fink	$u_1 = 1, u_2 = 1$	$u_1 = 3, u_2 = 0$
	Silent	$u_1 = 0, u_2 = 3$	$u_1 = 2, u_2 = 2$

Normal-form games

Again, consider the prisoner's dilemma game:

		Suspect 2	
		Fink	Silent
Suspect 1	Fink	$u_1 = 1, u_2 = 1$	$u_1 = 3, u_2 = 0$
	Silent	$u_1 = 0, u_2 = 3$	$u_1 = 2, u_2 = 2$

- In this version, there are two players (Suspects 1 and 2)
- Each player has two actions, Fink (F) or Silent (S)
- There are 4 strategy combinations: (F, F), (F, S), (S, F), and (S, S)
- Each strategy combination has a payoff: (1, 1), (0,3), (3, 0), (2, 2)

Notation

We use the following notation for the three components of strategic form games.

- Players are $i = 1, 2, ..., N$
- S_i denotes player i's set of potential strategies.
- s_i, s_i^* or $s_i^{'}$ denotes a specific player i's strategy
- s_{-i} denotes the strategy choices of all other players besides player i
- u_i denotes player i's payoff
- $(s_1^*, s_2^*, ..., s_N^*)$ denotes a combination of strategies, one strategy for each player
- $u_i(s_1^*, s_2^*, ..., s_N^*)$ denotes player i's payoff when $(s_1^*, s_2^*, ..., s_N^*)$ is the set of strategies played

Best response

To find the solutions for games we adopt the concept of a **Nash equilibrium**

- A Nash equilibrium is a set of strategies for all players, where each player's strategy is the *best choice* for each player *given* the others' equilibrium strategies.

To find the Nash equilibrium first we need to identify each player's best strategy for each potential strategy chosen by the other players.

Definition: A strategy s_i is a best response to a strategy vector s_{-i} of the players if

- $u_i(s_i, s_{-i}) \geqslant u_i(s_i^{'}, s_{-i}) \quad \forall\, s_i^{'} \in S_i$.

Nash equilibrium

Note that for an equilibrium to hold there needs to be a condition to ensure that player i is correct in his conjecture that the other players are going to play s_{-i}. Similarly, the condition should ensure that the other players correct in their conjectures. This takes us to the concept of *Nash equilibrium*.

Definition: The strategy vector $s^* = s_1^*, ..., s_N^*$ is a Nash equilibrium if

- $u_i(s_i^*, s_{-i}^*) \geqslant u_i(s_i, s_{-i}^*)$ for all s_i and all i.

That is,

- At the equilibrium each player must be playing a best response against the conjectured strategies of her opponents.
- The conjecture must be correct.
  - No one has an incentive to change their strategy s_i^*. Thus, s^* is stable.

Dominant strategies

Definition: Strategy $s_i^{'}$ **strictly dominates** all other strategies of player i if the payoff to $s_i^{'}$ is strictly greater than the payoff to any other strategy, regardless of which strategy is chosen. Formally:

- $u_i(s_i^{'}, s_{-i}) > u_i(s_i, s_{-i}) \forall s_{-i}$

where s_{-i} is the strategy vector of players other than i.

Dominant strategies

Definition: Strategy $s_i^{'}$ **strictly dominates** all other strategies of player i if the payoff to $s_i^{'}$ is strictly greater than the payoff to any other strategy, regardless of which strategy is chosen. Formally:

- $u_i(s_i^{'}, s_{-i}) > u_i(s_i, s_{-i}) \forall s_{-i}$

where s_{-i} is the strategy vector of players other than i.

Consider a game with two players (1 and 2) and two strategies (a and b). If, for example s_1^b is a dominant strategy for player 1, then it must be the case that:

1. $u_1(s_1^b, s_2^a) > u_1(s_1^a, s_2^a)$
2. $u_1(s_1^b, s_2^b) > u_1(s_1^a, s_2^b)$

Dominant strategies

Definition: Strategy $s_i^{'}$ **strictly dominates** all other strategies of player i if the payoff to $s_i^{'}$ is strictly greater than the payoff to any other strategy, regardless of which strategy is chosen. Formally:

- $u_i(s_i^{'}, s_{-i}) > u_i(s_i, s_{-i}) \forall s_{-i}$

where s_{-i} is the strategy vector of players other than i.

Consider a game with two players (1 and 2) and two strategies (a and b). If, for example s_1^b is a dominant strategy for player 1, then it must be the case that:

1. $u_1(s_1^b, s_2^a) > u_1(s_1^a, s_2^a)$
2. $u_1(s_1^b, s_2^b) > u_1(s_1^a, s_2^b)$

Definition: A combination of strategies is a **dominant strategy solution** if each player's strategy is a dominant strategy.

The Prisoner's Dilemma

		Suspect 2	
		Fink	Silent
Suspect 1	Fink	$\underline{u_1 = 1}, \underline{u_2 = 1}$	$\underline{u_1 = 3}, u_2 = 0$
	Silent	$u_1 = 0, \underline{u_2 = 3}$	$u_1 = 2, u_2 = 2$

The Prisoner's Dilemma

		Suspect 2	
		Fink	Silent
Suspect 1	Fink	$\underline{u_1 = 1}, \underline{u_2 = 1}$	$\underline{u_1 = 3}, u_2 = 0$
	Silent	$u_1 = 0, \underline{u_2 = 3}$	$u_1 = 2, u_2 = 2$

- $b_1(P2 = F) = F$, $b_1(P2 = S) = F$
- $b_2(P1 = F) = F$, $b_2(P1 = S) = F$

- Solution: Each player has a dominant strategy, which is F. Therefore (F, F) is the dominant strategy solution.

Battle of the Sexes (B=ballet, O=boxing)

Player 1 (Wife) \ Player 2 (Husband)	Ballet	Boxing
Ballet	2, 1	0, 0
Boxing	0, 0	1, 2

A husband and wife are deciding whether to go to a ballet or a boxing match. The husband prefers boxing while the wife prefers ballet. However, each of them would rather go with the spouse than go alone.

Battle of the Sexes (B=ballet, O=boxing)

		Player 2 (Husband)	
		Ballet	Boxing
Player 1 (Wife)	Ballet	2, 1	0, 0
	Boxing	0, 0	1, 2

A husband and wife are deciding whether to go to a ballet or a boxing match. The husband prefers boxing while the wife prefers ballet. However, each of them would rather go with the spouse than go alone.

Neither player has a dominant strategy. For example O, is not a dominant strategy for the husband because it does not do as well as B if the wife chooses B. That is, $u_h(s_h^o, s_w^b) < u_h(s_h^b, s_w^b)$. Similarly, $u_h(s_h^b, s_w^o) < u_h(s_h^o, s_w^o)$. The same logic applies to the wife's strategies. Thus there is no dominant strategy solution.

Battle of sexes (B=ballet, O=boxing)

		Player 2 (Husband)	
		Ballet	Boxing
Player 1 (Wife)	Ballet	$\underline{2}, \underline{1}$	0, 0
	Boxing	0, 0	$\underline{1}, \underline{2}$

Battle of sexes (B=ballet, O=boxing)

		Player 2 (Husband)	
		Ballet	Boxing
Player 1 (Wife)	Ballet	2, 1	0, 0
	Boxing	0, 0	1, 2

- $b_h(w = B) = B$ and $b_h(w = O) = O$
- $b_w(h = B) = B$ and $b_w(h = O) = O$

Battle of sexes (B=ballet, O=boxing)

		Player 2 (Husband)	
		Ballet	Boxing
Player 1 (Wife)	Ballet	2, 1	0, 0
	Boxing	0, 0	1, 2

- $b_h(w = B) = B$ and $b_h(w = O) = O$
- $b_w(h = B) = B$ and $b_w(h = O) = O$

Note that if both players play B, neither has an incentive to change their choice. This is also true if both play O. Therefore, there are two Nash equilibria:

- Nash equilibrium: (B, B) and (O, O).

Bertrand pricing

Consider the following game in which there are two firms producing the same product. Each firm can chose to price high (H), medium (M), or low (L). The strategic form of the game is as following:

Firm 1 / Firm 2	H	M	L
H	6, 6	0, 10	0, 8
M	10, 0	5, 5	0, 8
L	8, 0	8, 0	4, 4

Bertrand pricing

Consider the following game in which there are two firms producing the same product. Each firm can chose to price high (H), medium (M), or low (L). The strategic form of the game is as following:

Firm 1 / Firm 2	H	M	L
H	6, 6	0, 10	0, 8
M	10, 0	5, 5	0, 8
L	8, 0	8, 0	4, 4

The best responses of F1 for each strategy of F2 are

- $b_1(H) = M$, $b_1(M) = L$, and $b_1(L) = L$

The best responses of P2 for each strategy of P1 are

- $b_2(H) = M$, $b_2(M) = L$, and $b_2(L) = L$

Bertrand pricing

Consider the following game in which there are two firms producing the same product. Each firm can chose to price high (H), medium (M), or low (L). The strategic form of the game is as following:

Firm 1 / Firm 2	H	M	L
H	6, 6	0, 10	0, 8
M	10, 0	5, 5	0, 8
L	8, 0	8, 0	4, 4

The best responses of F1 for each strategy of F2 are

- $b_1(H) = M$, $b_1(M) = L$, and $b_1(L) = L$

The best responses of P2 for each strategy of P1 are

- $b_2(H) = M$, $b_2(M) = L$, and $b_2(L) = L$

Thus, the only Nash equilibrium in this game is (L, L). At (L, L) neither player has an incentive to change their choice.

Hawk-Dove ("Chicken")

Suppose there are two players who are racing their cars against each other. If both stay tough (T), they crash. If both concede (C), no one wins but it is better than crashing. If one stays tough while the other concedes the player who stays tough wins. The strategic form is:

Player 1 / Player 2	T	C
T	-1, -1	10, 0
C	0, 10	5, 5

Hawk-Dove ("Chicken")

Suppose there are two players who are racing their cars against each other. If both stay tough (T), they crash. If both concede (C), no one wins but it is better than crashing. If one stays tough while the other concedes the player who stays tough wins. The strategic form is:

Player 1 / Player 2	T	C
T	-1, -1	10, 0
C	0, 10	5, 5

The best responses of P1 for each strategy of P2 are

- $b_1(T) = C$, $b_1(C) = T$

The best responses of P2 for each strategy of P1 are

- $b_2(T) = C$, $b_2(C) = T$

Hawk-Dove ("Chicken")

Suppose there are two players who are racing their cars against each other. If both stay tough (T), they crash. If both concede (C), no one wins but it is better than crashing. If one stays tough while the other concedes the player who stays tough wins. The strategic form is:

Player 1 / Player 2	T	C
T	-1, -1	10, 0
C	0, 10	5, 5

The best responses of P1 for each strategy of P2 are

- $b_1(T) = C$, $b_1(C) = T$

The best responses of P2 for each strategy of P1 are

- $b_2(T) = C$, $b_2(C) = T$

There are two Nash equilibria: (T, C) and (C, T). At these equilibria neither players has an incentive to change their choice.

Continuous of action spaces

In most economic models, agents' choice variables are continuous:

- Cournot model: firms choose quantities
- Bertrand model: firms choose prices

Continuous of action spaces

In most economic models, agents' choice variables are continuous:

- Cournot model: firms choose quantities
- Bertrand model: firms choose prices

For games with a continuum of actions we find the Nash equilibrium as:

1. set up the optimization problem of each of the *n* agents
2. find the FOC for each optimization problem
   - the FOCs can be rearranged as **best-response** functions
3. the solution to the system of *n* equations (i.e. best-response functions) is the Nash equilibrium

Note that the basic approach to find NE is the same (first find each player' s best response, then determine the strategy set at which the best responses intersect).

The Tragedy of the Commons

A **common-pool resource** is a good with the following properties:

- **non-excludable:** anyone can use the good
- **rival:** when someone uses the good its value decreases for all

The Tragedy of the Commons

A **common-pool resource** is a good with the following properties:

- **non-excludable:** anyone can use the good
- **rival:** when someone uses the good its value decreases for all

We can analyze the problem of overuse of common-pool resources using game theory. Consider the following game:

1. There are two herders
2. Each herder decides how many sheep to graze on the commons
3. The commons is quite small and can rapidly succumb to overgrazing
4. Herders make a decision just once and simultaneously (a single-period game).
5. Herders receive a fixed per-sheep value of grazing on the commons

Tragedy of commons: formal description

The formal summary of the model is:

- Players: N={herder i, herder j} $i, j = 1, 2$
- Actions: choice of number of sheep - $q_i \in S_i = [0, \infty)$
- Timing: simultaneous
- Payoffs: $\pi_i(q_i, q_j) = q_i v_i(q_i, q_j)$,
  - where $v_i(q_i, q_j) = 120 - (q_i + q_j)$ is value of grazing each sheep

Tragedy of commons: formal description

The formal summary of the model is:

- Players: N={herder i, herder j} $i, j = 1, 2$
- Actions: choice of number of sheep - $q_i \in S_i = [0, \infty)$
- Timing: simultaneous
- Payoffs: $\pi_i(q_i, q_j) = q_i v_i(q_i, q_j)$,
    - where $v_i(q_i, q_j) = 120 - (q_i + q_j)$ is value of grazing each sheep

We can solve for the Nash equilibrium using best-response functions:

- The **best-response function**, $b_i(q_j)$ describes herder i's optimal reaction to the output choice of herder j:
    - $b_i(q_j) = \max\limits_{q_i} \ \pi_i(q_i, q_j)$
- Nash equilibrium, (q_i^*, q_j^*) is the solution to:
    - $q_i = b_i(q_j)$ and $q_j = b_j(q_i)$

Social optimum: tragedy of the commons

Let's first look at the social optimum:

- When herder *i* increases his herd size, the value of grazing decreases for both herders
- This is a negative externality imposed on the competitor's payoff
  - That is, when calculating their profits herders do not account for the portion of the costs of their decisions that is incurred by their competitors.
  - Not accounting for the full costs of production leads to overuse of the common-pool resource.

Social optimum: tragedy of the commons

Let's first look at the social optimum:

- When herder *i* increases his herd size, the value of grazing decreases for both herders
- This is a negative externality imposed on the competitor's payoff
  - That is, when calculating their profits herders do not account for the portion of the costs of their decisions that is incurred by their competitors.
  - Not accounting for the full costs of production leads to overuse of the common-pool resource.
- A social optimum can be achieved by internalizing this externality
  - Herders can cooperate to determine the optimal herd size that maximizes the value of the commons.
- $\underset{q_1, q_2}{\text{Max}} \ (q_1 + q_2)(120 - q_1 - q_2)$

The solution to this problem is

- $q_1^s = 30$ and $q_2^s = 30$

Best-response functions

Herder 1 faces the following optimization problem:

- $\underset{q_1}{\text{Max}} \;\; \pi_1(q_1, q_2) = q_1(120 - q_1 - q_2)$

Assuming π_1 strictly concave in q_1 and twice differentiable, the FOC is

- $\pi_1'(q_1, q_2) = 120 - 2q_1 - q_2 = 0$

Best-response functions

Herder 1 faces the following optimization problem:

- $\underset{q_1}{\text{Max}} \ \pi_1(q_1, q_2) = q_1(120 - q_1 - q_2)$

Assuming π_1 strictly concave in q_1 and twice differentiable, the FOC is

- $\pi_1'(q_1, q_2) = 120 - 2q_1 - q_2 = 0$

The best-response function of herder 1 is:

- $q_1 = b_1(q_2) = 60 - \frac{q_2}{2}$

By symmetry, the best-response function of herder 2 is:

- $q_2 = b_2(q_1) = 60 - \frac{q_1}{2}$

Best response functions

1. $q_1 = b_1(q_2) = 60 - \frac{q_2}{2}$
2. $q_2 = b_2(q_1) = 60 - \frac{q_1}{2}$

The Nash equilibrium (q_1^*, q_2^*) is the pair of strategies that satisfies 1 and 2:

- $q_1 = 60 - \frac{1}{2}\left(60 - \frac{q_1}{2}\right)$
- $3q_1/4 = 30$
- $q_1^* = 40$, similarly $q_2^* = 40$

Compare the total herd Nash equilibrium solution (q_1^*, q_2^*) to the social optimum (q_1^s, q_2^s)

The sequential-move prisoner's dilemma game

Recall the normal form of the simultaneous move prisoners' dilemma:

Prisoner 1 / Prisoner 2	C	N
C	5, 5	0, 15
N	15, 0	1, 1

The sequential-move prisoner's dilemma game

Recall the normal form of the simultaneous move prisoners' dilemma:

Prisoner 1 / Prisoner 2	C	N
C	5, 5	0, 15
N	15, 0	1, 1

Now consider the following sequential version of the game

- **timing**: P1 moves first. P2 sees P1's move, then moves.
- Players, actions and payoffs are the same as before.
- In this case P2 has four pure strategies.

The sequential-move prisoner's dilemma game

Recall the normal form of the simultaneous move prisoners' dilemma:

Prisoner 1 / Prisoner 2	C	N
C	5, 5	0, 15
N	15, 0	1, 1

Now consider the following sequential version of the game

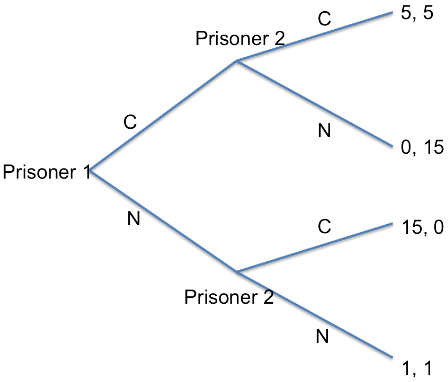
- **timing**: P1 moves first. P2 sees P1's move, then moves.
- Players, actions and payoffs are the same as before.
- In this case P2 has four pure strategies.

The normal form of this game is:

Prisoner 1 / Prisoner 2	CC	CN	NC	NN
C	5, 5	5, 5	0, 15	0, 15
N	15, 0	1, 1	15, 0	1, 1

The extensive form

Extensive form: A pictorial representation of the game. The main pictorial form is called the *game tree*, which is made up of a root and branches arranged in order.



The extensive form

- The game tree starts from a *root*, at this point the first of the players has to make a choice.
- The various actions (choices) available to this player are represented as *branches* emanating from the root.
- At the end of each branch the next player gets to play (decision node).
- When the tree ends, the payoffs are realized.

Representation of the rules:

The extensive form

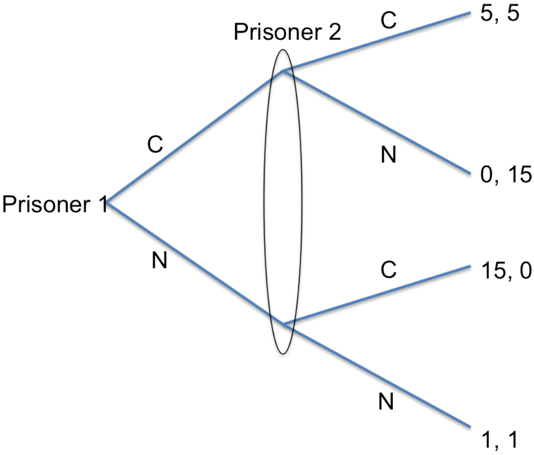
- The game tree starts from a *root*, at this point the first of the players has to make a choice.
- The various actions (choices) available to this player are represented as *branches* emanating from the root.
- At the end of each branch the next player gets to play (decision node).
- When the tree ends, the payoffs are realized.

Representation of the rules:

- **who** – any individual who has a decision node in the game tree is a player.
- **what** – the branches that come out of a decision node represent actions available at that point.
- **when** – the player at the decision node that is closer to the root moves first (what if it is a simultaneous-move game?)
- **how much** – the payoffs are listed at each terminal node

The extensive form for a simultaneous-move game

The above extensive form permits only one player to move at a time. The simultaneous move game can be represented as:



Information set and strategies

An **information set** is a group of decision nodes at which

- the same player moves
- the deciding player cannot tell which node she is at when called upon to make her decision

Information set and strategies

An **information set** is a group of decision nodes at which

- the same player moves
- the deciding player cannot tell which node she is at when called upon to make her decision

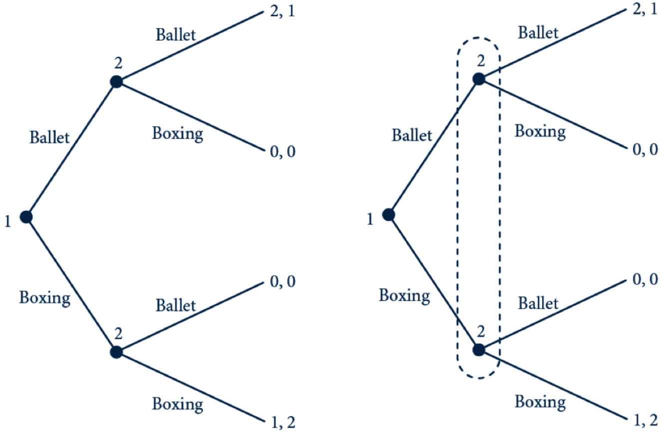
A **strategy** for a player is a complete contingent plan of action – what she would do at each potential information set.

- In any game, a collection of strategies, one for each player, will determine which branch of game tree actually gets played out.

For example, in the *sequential*-move prisoner's dilemma game, P1's strategies are (C) and (C). P2's strategies are (CC), (CN), (NC) and (NN).

Note that these are just collections of strategies; they are not the "optimal" strategy.

Extensive form for the battle of the sexes



(a) Sequential version

(b) Simultaneous version

Non-credible threats

Consider the following normal form game

Player 1 / Player2	L	R
A	0, 2	0, 2
B	-3, -1	2, 1

Non-credible threats

Consider the following normal form game

Player 1 / Player2	L	R
A	0, 2	0, 2
B	-3, -1	2, 1

In this game the Nash equilibria are *BR* and *AL*. Describe this game in extensive form and note that

- AL does not seem “credible”. Player 2 “threatens” to choose *L* if called upon to move in the extensive from game. This induces player 1 to play A.
- We can eliminate non-credible threats by **backwards induction.**

Backwards induction and subgame perfection

Backwards induction is used to eliminate non-credible threats. It

- starts at the end of an extensive form game
- works backwards through the game tree
- actions taken at any node have to be optimal *from that point on*
- the same principle applies at all following nodes

What is a subgame?

- start at an information set containing a single node
- include everything that follows from that node
- there cannot be any partial information sets in what follows (subgames always leave information sets intact – why?)

Backwards induction and subgame perfection

Backwards induction is used to eliminate non-credible threats. It

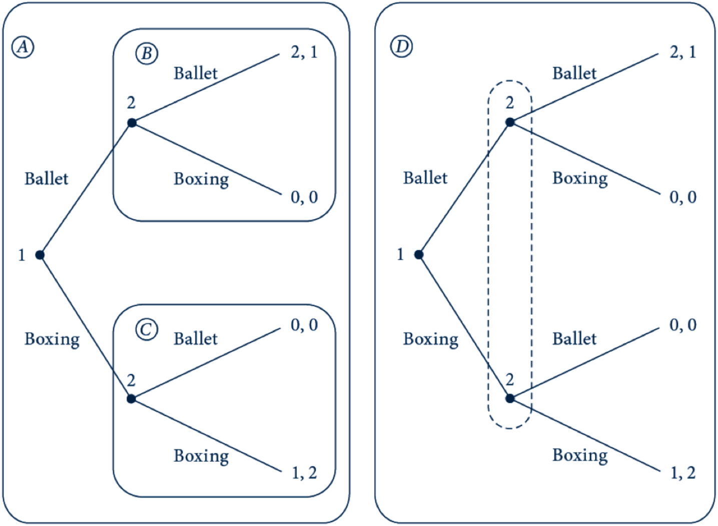
- starts at the end of an extensive form game
- works backwards through the game tree
- actions taken at any node have to be optimal *from that point on*
- the same principle applies at all following nodes

What is a subgame?

- start at an information set containing a single node
- include everything that follows from that node
- there cannot be any partial information sets in what follows (subgames always leave information sets intact – why?)

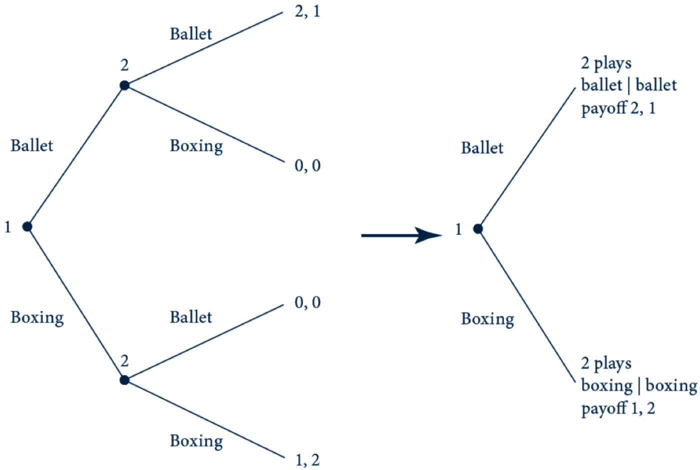
A **subgame perfect Nash Equilibrium** is a Nash equilibrium that induces a Nash equilibrium in every subgame

Subgames of the battle of the sexes



(a) Sequential (b) Simultaneous

Subgame perfect Nash equilibrium of the battle of the sexes



Infinitely-repeated games

Players can sustain cooperation in infinitely repeated games by retaliating

- the retaliation must be severe enough to deter deviation
- it must also be credible, i.e. subgame perfect

Suppose that the prisoner's dilemma game is played infinitely

		Suspect 2	
		Fink	Silent
Suspect 1	Fink	$u_1 = 1, u_2 = 1$	$u_1 = 3, u_2 = 0$
	Silent	$u_1 = 0, u_2 = 3$	$u_1 = 2, u_2 = 2$

Infinitely repeated prisoner's dilemma

Suppose that the prisoners decided to remain silent every period. Let δ denote the discount factor

- that is, δ is the value of payoff of 1 if it is received one period in the future rather than today

Suppose that each player uses the following "grim trigger" strategy

- continue to be silent if no one finks. Fink forever afterwards if anyone finks

Infinitely repeated prisoner's dilemma

Suppose that the prisoners decided to remain silent every period. Let δ denote the discount factor

- that is, δ is the value of payoff of 1 if it is received one period in the future rather than today

Suppose that each player uses the following "grim trigger" strategy

- continue to be silent if no one finks. Fink forever afterwards if anyone finks

- If both players are silent every period, the present discounted value of payoffs:
  - $V^{eq} = 2 + 2\delta + 2\delta^2 + \ldots = 2/(1-\delta)$
- If a player deviates and then the other finks every period, that players payoff is
  - $V^{dev} = 3 + \delta + \delta^2 + \ldots = 3 + \delta/(1-\delta)$

Infinitely repeated prisoner's dilemma

Trigger strategies will form a cooperative solution (i.e., a subgame perfect equilibrium) if $V^{eq} \geqslant V^{dev}$

- $2/(1-\delta) \geqslant 3 + \delta/(1-\delta)$
- $2 \geqslant 3 - 2\delta$
- $\delta \geqslant 1/2$

That is,

- cooperation can be an equilibrium outcome so long as the future payoffs are not discounted too highly. In this case, a cooperative equilibrium is attained with $\delta \geqslant 1/2$.

Exercise: What else do we need to check to guarantee this strategy works?