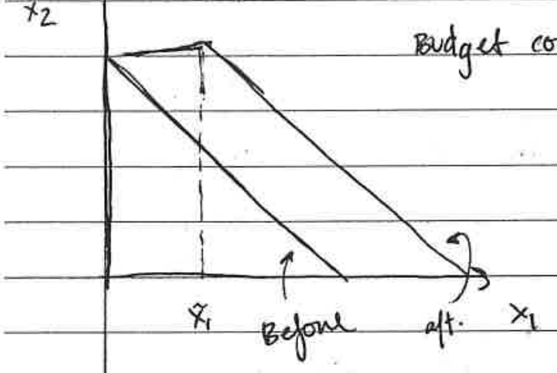
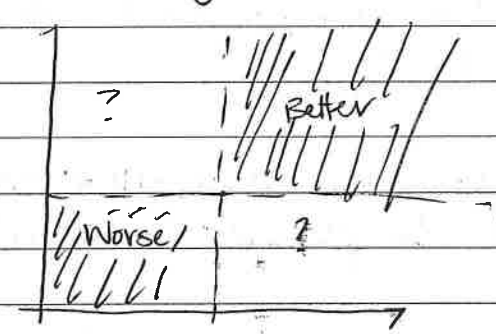
Micro w/ Landry   Aug 21, ~~__~~ Tues



Budget constraint w/ food stamps → $\tilde{x}_1$

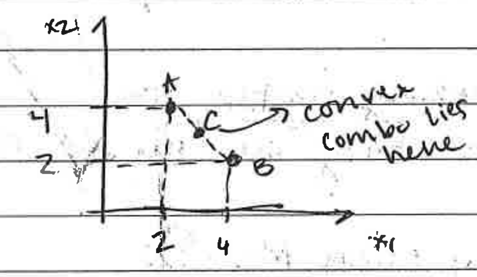
Convexity convex sets vs convex functions



w/ completness
transtiv
+ nonsasiation

need tradeofts for other 2 areas.

Axiom 4: Strict convexity
↳ need to dfn convex combinaton : AKA weighted avg



pt C is ½ A + ½ B (wted avg)

$\{ \frac{1}{2} \cdot 2 + \frac{1}{2} 4, \frac{1}{2} 4 + \frac{1}{2} 2 \}$

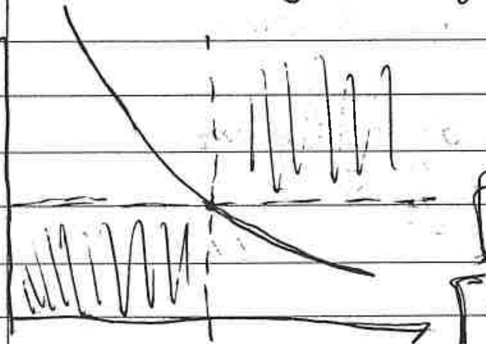
$\{3, 3\}$

* ↑ wt pt B move → B + viceversua

Convex set: any 2 points in a set, convex combo of these points lies w/in set ie → NO DONUTS.

Convex functions: univariate $f(x)$; $f''(x) > 0$ [$\geq$ notstricty convex]
← stricty convex

Strict convexity: "averages prefered to extreams"



indif curve: level sets of $U(x)$

[~~tangent combination~~ convex combo of indif curve will lie on higher or @ least same indif curve] ← words / math

$\left[\forall\ x^A, x^B;\ x^A \sim B^B\ \ x^C = t x^A + (1-t) x^B \succsim x^A, x^B \right]$

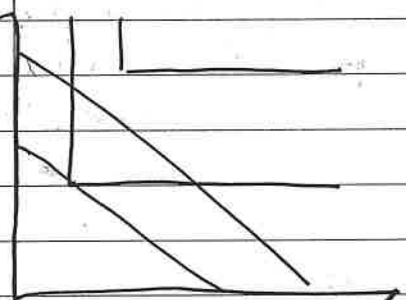
$\forall t\ \ 0 < t < 1$

* upper contour set (anything above indif set) is Convex

[* indif curves → must be shapped ; ie dont violate non satiation ; no curves up
Prelim note ↗]

implies goods are imperfect substitutes; other options
Perfect compliments + perfect substitutes, "bads", nutral goods

"bads" → neg MU, MU<0
nutral goods → MU = 0



perfect compliments w/
perfect subs w/

Dont always assume strict convex → assumes imperfect substitutes.

* utility function → $x \in \mathbb{R}_+^n \Rightarrow \mathbb{R}$ ranking function, yay

↓ non unique in that rankings presered w/ monoton. Δ

Marginal utility: Rate of Δ in utility ranking as quant of good Δ

if uf(·) is cont. diffent; then $MU = \frac{\partial U}{\partial x}$

$MU > 0$ "goods"

$MU = 0$ nutral goods no units → qualitative signifigance

$MU < 0$ "bads"

ex: U provides a consistant rank for consm 1 then so will $N = U^2$

Marginal rate of substitution:

dfn of indifference curve: $U(x) = U^0$ implicit dfn of utility
↳ level sets.

take total difeventiall of $U(x_1, x_2)$: $dU = \underbrace{\partial U/\partial x_1}_{\text{Rate } \Delta x_1} \underbrace{dx_1}_{\Delta x_1} + \underbrace{\partial U/\partial x_2}_{\text{Rate } \Delta x_2} \underbrace{dx_2}_{\Delta x_2}$

Landry cont. Aug 21, Tuesday

$dU = \partial U/\partial x_1 \, dx_1 + \partial U/\partial x_2 \, dx_2 = 0$ (along endif curve) add $\partial U/\partial x_i \, dx_i$ as needed for # of dimentions

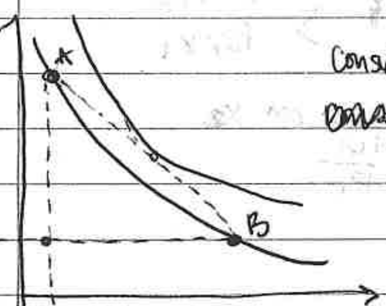
$$\frac{\partial U/\partial x_2 \, dx_2}{\partial U/\partial x_2 \, dx_1} = \frac{-\partial U/\partial x_1 \, dx_1}{\partial U/\partial x_2 \, dx_1}$$

$$\underbrace{\frac{dx_2}{dx_1} = \frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{\Delta x_2}{\Delta x_1}}_{}$$ slope of endif. curve = Marg. rate of subst. ↳ $-\frac{MU_1}{MU_2}$

NOTE THOUGH!

* Remember: utility has no natural units and it can be transformed monotonically; Ratio of utity has usful meaning

Marginal Rate of subst $= \left| -\frac{MU_1}{MU_2} \right|$



Consider point A: steep endif curve $\Rightarrow MU_1 >> MU_2$ @ pt A

~~consider pt B~~ Rich in x_2, poor in x_1 ∴ want x_1 alot more

pt B is the vice versa

HW|

① prove strict convex: neg $-\frac{MU_1}{MU_2}$; slope of endiffenence curve dimin marg rate of subst.

$$d\frac{MU_1}{MU_2} \Big/ dx_1 > 0$$ for strict convex

② eval axioms.

① completeness → does f(·) go from $\mathbb{R} \to \mathbb{R}$ $\forall x_1, x_2$ and/or prove continuous.

② transitive → if maped on cont euclid. space.

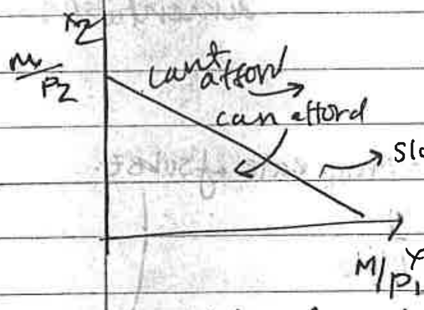
③ non-satiation: if 1 partial is positive for all x_i in $\mathbb{R}^L$

strict monotonic → $\partial U/\partial x_i > 0 \ \forall i = 1, \dots, n$

③ U Max.

$U\max \rightarrow$ optimal value functions $\rightarrow V(p,m)$ indirect U fc.

draw pictures yay consum equilib



$M = p_1x_1 + p_2x_2$

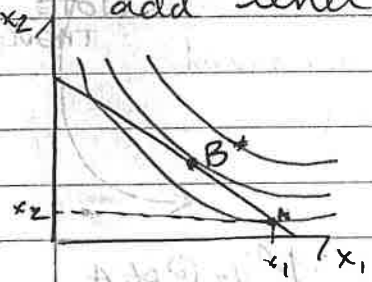
Solve for $x_2 \rightarrow x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$

Slope $= \frac{-p_1}{p_2}$ AKA Price Ratio

Budgetline

financial TOT

add level sets for indif curves + U measure



@ A: $p_1/p_2 > \frac{MU_1}{MU_2} \Rightarrow \frac{MU_2}{p_2} > \frac{MU_1}{p_1}$

util/\$ for x_2 > util/\$ for x_1

so spend more on x_2

@ B^* $p_1/p_2 = \frac{MU_1}{MU_2} \Rightarrow \frac{MU_2}{p_2} = \frac{MU_1}{p_1}$

$\max_{x_1 x_2} U(x_1,x_2)$ st $p_1x_1 + p_2x_2 = m$

$\mathcal{L} = U(x_1x_2) + \lambda(m - p_1x_1 - p_2x_2)$ [* can you pick λ? *]

FOC:

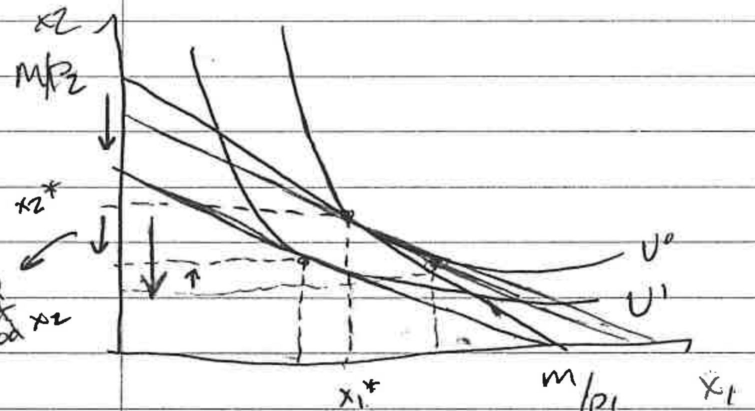
$\partial\mathcal{L}/\partial x_i$ = $\partial U/\partial x_i - \lambda p_i = 0$

$\partial\mathcal{L}/\partial\lambda = m - p_1x_1 - p_2x_2 = 0$ makes sure you're inside the budg. const.

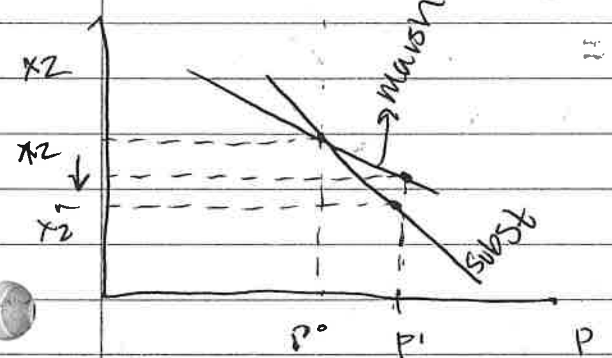
$$\left.\begin{aligned} MU_1 &= \lambda p_1 \\ MU_2 &= \lambda p_2 \end{aligned}\right\} \lambda = \frac{MU_1}{p_1} = \frac{MU_2}{p_2}$$

Want soln: $x_1(p_1, m)$ + $x_2(p_1, m)$

Microcont Sept 4 cont



hecksian dmd
m = supst effect.
where new slope meets
old U^0
m = income effect
inc ↓ dmd ↑ → inferior good



Law of dmd

own substitution effects are always neg. [monotic & strict concav]

$$\frac{\partial x_i(p,u)}{\partial p_i} < 0 \quad \forall i = 1, \dots n$$

Recall $x_i^h(p,u) = \frac{\partial E(p,u)}{\partial p_i}$ shep lem.

$$\frac{\partial^2 E(p,u)}{\partial p_i^2} < 0$$ by concavity

"Modern" law of demand:
a decrease in price of a normal good will cause quantity dmd to increase

subst + income effect move in ops. direction of price Δ

if ↓ΔP = ↓Δdmd; good is inferior + (large inc effect)

Δp: Ford Ranger , substitutes? lots
% income? large

$\downarrow$

Big Dmd Δ

$x_i^h(pu) = x_i^M(p E(pu)$

$x_i^h(pu = x_1(p, E(p,u))$ p shows up x2

$$\frac{\partial x_i^h(pu)}{\partial p_i} = \underbrace{\frac{\partial x_1(p,E(p,u)}{\partial p_1}}_{\text{direct effect of } \Delta p_1 \text{ on Dmd } x_1} + \underbrace{\frac{\partial x_1(p, E(pu))}{\partial m} \frac{\partial E(pu)}{\partial p_1}}_{\text{chain rule}}$$

$x^h(pu) \approx x(pm)$

$$\cancel{\frac{\partial x_i^h(pu)}{\partial p_i} = \frac{\partial x(p,E(pu)}{\partial p_i} + \frac{\partial x_1(p,E(pu)}{\partial m} \partial}$$

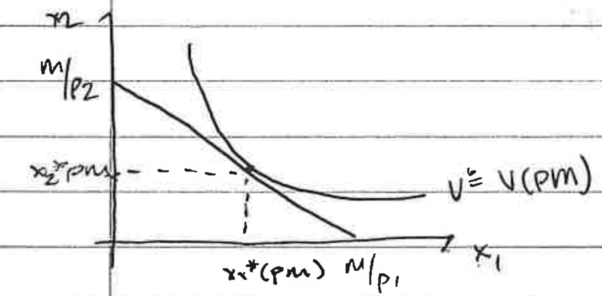
$$\frac{\partial x_i^h(pu)}{\partial p_i} - x(pm)\frac{\partial xi(pm)}{\partial m} = \frac{\partial xipm}{\partial pi} \quad x^h(pu) \approx x(pm)$$

$$\frac{\partial xi(pm)}{\partial pi} = \frac{\partial x_i^h(pu)}{\partial pi} - xi(pm)\frac{\partial xi(pm)}{\partial m}$$

Micro start

Sept 4

Duality in consumer theory → see page



Assume nowon:
Preferences are: complete
transitive
Strongly monotonic
Quasi concave → utility
↳ ~~[illegible]~~

① p, m are fixed
② want to get to utility level $V(p,m)$
↳ get $E(pU) = m = E(p, V(pm))$
↳ U becomes ~~objective~~ const & budg. is objective
↳ get same m as $x_1^*(pm)$ OR $x_2^*(pm)$
③ $V(p,m)$ is largest utility attainable @(p,m) we ~~as~~ also have $V(p, E(pU)) == U$

pt: $V(p, E(pU)) == U$
$E(p, V(pm)) == m$
to go between just invert

Solving expend min

Slutsky's equation (the goes both ways)

pt #2: Roys ID
$x_i h = \partial E(pU)/\partial p_i$

① $x_i(p,m) = x_i^h(p, V(pm))$
② $x_i^h(pU) = x_i(p, E(pU))$

$$\frac{\partial x_i(pm)}{\partial p_i} = \frac{\partial x_i^h(pU)}{\partial p_i} - x_i(pm)\frac{\partial x_i(pm)}{\partial m}$$

"total effect" "subst" effect "income" effect.

Normal goods: Δp salt inelastic ; are there many substitutes for salt? (not really) small
do people spend alot of \$ on salt (not directly) small

Micro cont

Aug 30

Look @ income + subst effects → Modern theory of dmd
↳ means not predicated ~~of~~ on DMR

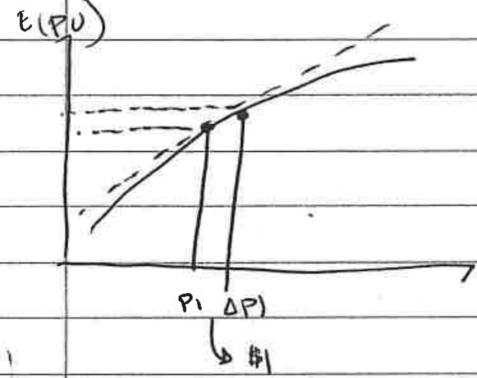
ϕ → Marginal ~~impact~~ cost of one unit of Utility

Properties of E(pu)

1. E(pu) is denominated in currency
   E=0 for lowest level of utility
2. cont. on $\mathbb{R}^n_{++} \times U \to \mathbb{R}^n_{++} \cdot \mathbb{R}$
3. Homogeneous of degree 1 in P; if p↑ by α, M also ↑ α
4. non decreasing in prices
5. increasing in U
6. concave in P
7. shepards lemma: $x_i^H(pu) = \frac{\partial E(pu)}{\partial p_i}$

$\frac{\partial E(pu)}{\partial p_i} = x^H(pu) \geq 0$ (shep lem) (lenc in p) 7 4

$\frac{\partial^2 E(pu)}{\partial p_i^2} = \frac{\partial x^H_i(pu)}{\partial p_i} < 0$ 6



what does this imply about Hicksian dmd f(·)
E is concave but always downward sloping

p_1

> Point: E is concave b/c subst.; if it was a straight line then its a 1:1 \$ per \$ compensation; largest compensation amt is the ΔP.

$X_1 = 10$
$P_1 = \$1$
$P_2 = \$2$

need about \$10

small Δ in p needs to give similar Δ in E

→ Ignores substitution. but P↑ subst away to x_2

$$\frac{dV}{dm} = \frac{dU}{dx_1}\frac{dx_1}{dm} + \frac{dU}{dx_2}\frac{dx_2}{dm} + \lambda - \lambda p_1 \frac{dx_1}{dm} - \lambda p_2 \frac{dx_2}{dm}$$

w income effect

$$dV/dw = \frac{dx_1}{dm}\Big(\underbrace{\frac{dU}{dx_1} - \lambda p_1}_{\to 0}\Big) + \frac{dx_2}{dm}\Big(\underbrace{\frac{dU}{dx_2} - \lambda p_2}_{\to 0}\Big) + \lambda$$

Roys ID =

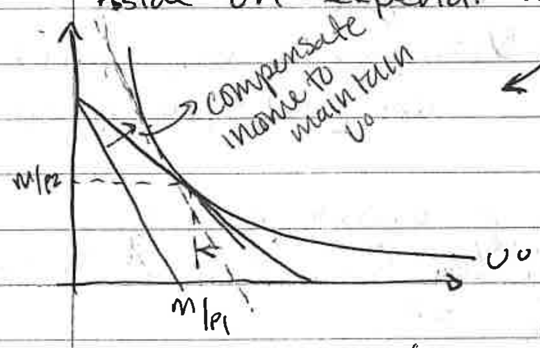
$$x_i^*(pm) = -\frac{dV/dp_i}{dV/dm} = -\frac{-\cancel{\lambda} x_1(p_1 p_2 m)}{\cancel{\lambda}}$$

$$x_i^*(pm) = -(-x_i^*(p_1 p_2 m))$$

$$x_i^*(pm) = x_i^*(p_1 p_2 m)$$ ☑ ☺

envelope theorum: if theres a Δ in a parameter that affects decision w/in an optimal value $f(\cdot)$ → only have to eval DIRECT EFFECTS

Aside on expend. min:



start w/ V_0 go to B

const

$$\underset{x_1 x_2 \phi}{\text{Min}}\ \mathcal{L} = p_1x_1 + p_2x_2 + \phi(U^0 \cancel{(x_1,x_2)} - U(x_1,x_2))$$

min expenditures st. getting certain level of U.

$\mathcal{L}_{x_1} = p_1 - \phi\, dU/dx_1 = 0$

foc $\mathcal{L}_{x_2} = p_2 - \phi\, dU/dx_2 = 0$

$\mathcal{L}_\phi = U^0 - U(x_1 x_2) = 0$

Argmin: $x^H(pU)$

How you subst to maintain utility → tells us about substitution not income

THIS IS COMENSATED DMD

MICRO START → Thursd.

Aug 30

Notes:

① there is a 2^{nd} deriv. test for quasi-convex

② Checking for convexity on quiz: should have been $\partial(\partial x_2/\partial x_1)/\partial x_1$ not 2nd total different.

③ Axis of budget constraint m/p not p/m

$$\max \mathcal{L} = U(x_1 x_2) + \lambda(m - P_1 x_1 - P_2 x_2)$$

FOC

$$\partial \mathcal{L}/\partial x_1 = \partial U/\partial x_1 - \lambda p_1 = 0$$
$$\partial \mathcal{L}/\partial x_2 = \partial U/\partial x_2 - \lambda p_2 = 0$$
$$\partial \mathcal{L}/\partial \lambda = m - p_1 x_1 - p_2 x_2 = 0$$

λ = marg. util of income

$MU_i = \lambda p_i$

Optimal val function

$V(pm)$ → indirect util

$E(pU)$ → expend fc.

ARGMAX = $x^*(pm)$ (marshallian)

→ create $U(x^*(pm)) = V(pm)$

Property #6: Roy's ID

$$x_i^*(pm) = \frac{-\partial V/\partial p_i}{\partial V/\partial m}$$

for Roy's ID

Aside: dfn of $V(p,m)$

$$V(pm) = U(x_1(p_1, p_2 m), x_2(p_1 p_2 m)) + \lambda(m - p_1 x_1(p_2 p_2 w) - p_2 x_2(p_1 p_2 w))$$

[envelope therom]

eval @ optimal

consider p_1: occurs 4 times!

$$\partial V/\partial p_1 = \partial U/\partial x_1 \, \partial x_1/\partial p_1 + \partial U/\partial x_2 \, \partial x_2/\partial p_1 - \lambda x(p_1, p_2 m)$$

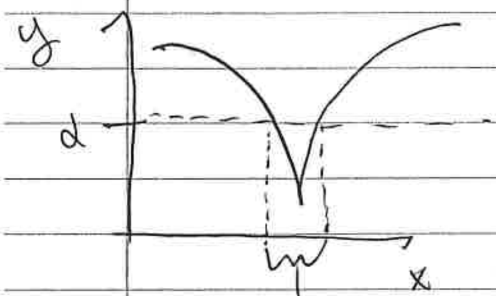
$$\partial V/\partial p_1 = \partial U/\partial x_1 \frac{\partial x_1}{\partial p_1} + \frac{\partial U}{\partial x_2}\frac{\partial x_2}{\partial p_1} - \lambda x_1(p_1 p_2 m) - \lambda p_1 \frac{\partial x_1}{\partial p_1} - \lambda p_2 \frac{\partial x_2}{\partial p_1}$$

$$= \partial x_1/\partial p_1 \underbrace{(\partial U/\partial x_1 - \lambda p_1)}_{=0 \text{ b/c FOC}} + \partial x_2/\partial p_1 \underbrace{(\partial U/\partial x_2 - \lambda p_2)}_{=0} - \lambda x_1(p_1 p_2 m)$$

$$\partial V/\partial p_1 = -\lambda x_1(p_1 p_2 m) < 0$$

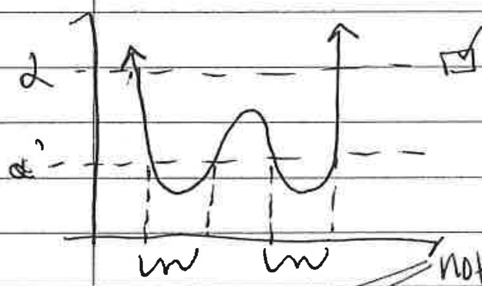
Micro cont (2)

Aug 28 cont



~~not~~ quasi convex

quasi convex (this area all xs ~~less~~ that correspond to $y \leq \alpha$



not quasiconvex

V(pm) is quasiconvex in p+m

$\left[\begin{array}{l} V(\tilde{p}, \tilde{m}) \text{ max utility under } \tilde{p}\,\tilde{m} \\ V(\hat{p}, \hat{m}) \text{ max utility under } (\hat{p}, \hat{m}) \end{array} \right.$

take convex combo of prices + income (just 2 pts)

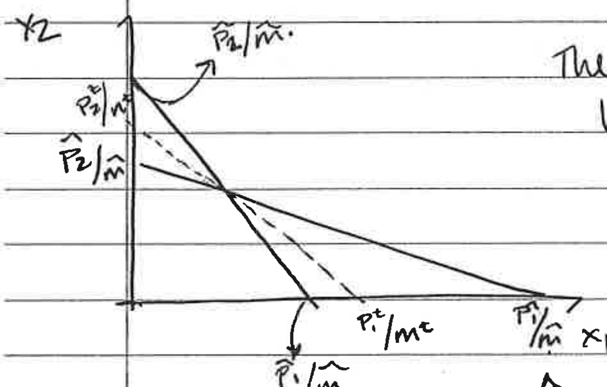
$\hat{p}\ \tilde{p}$ + $\hat{m}\ \tilde{m}$

$$\left[\begin{array}{ll} p_1^t = t\tilde{p}_1 + (1-t)\hat{p}_1 & m^t = tm + (1-t)\hat{m} \\ p_2^t = t\tilde{p}_2 + (1-t)\hat{p}_2 & \end{array} \right]$$

new P/m

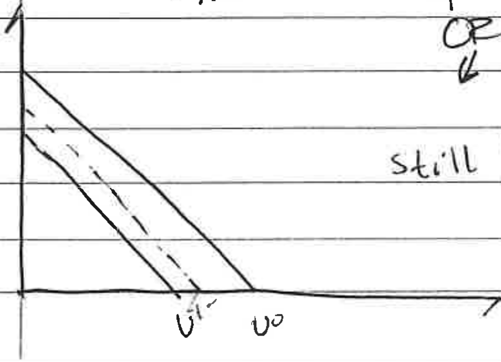
Quasiconvex says:

$$V(p^t m^t) \leq \max \left(V(\tilde{p}, \tilde{m}), V(\hat{p}\,\hat{m}) \right)$$



The convex combo is inbetween here → therefore U is bounded w/in Max. ; can be just as good but never better.

OR



still bounded w/in U° + U'

interpreting λ:

$\partial \mathcal{L}/\partial m = \lambda$ units of $\mathcal{L}$ at soln : util/\$

[λ is marginal utility of income]

OR MU of 1 (one) dollar

{The marginal "objective f(·)" for a 1 unit Δ in "constraint"}

Solve for $x^*(pm)$: Marshallian dmd

plug back in to $U f(\cdot)$

$U(x(pm)) = V(pm)$

$V(pm) \rightarrow$ properties

1. continuous in p+m dimensions
2. Homogeneous of degree 0 in prices + income
3. increasing in income
4. ~~decreasing~~ in prices (non increasing)
5. quasi convex in price + income
6. Roy's identity: $x_i^*(p,m) = \partial V(pm)/\partial p_i \,/\, \partial V(pm)/\partial m$

2. $V \Delta = 0$ of αp & αm

let $U = x_1 x_2^2 \Rightarrow V(pm) = 4/27 \; m^3/p_1 p_2^2$

~~add~~ multip by α for both w + p nothing Δ

3. $V\uparrow$ w/ $\uparrow w$ $\partial V/\partial w > 0$
4. $V\downarrow$ w $\uparrow p$ $\partial V/\partial p_i \leq 0$ (worse off or unΔ)
5. quasiconvex w/ univariate function

$y = f(x)$

dfn: a function $f(x)$ defined on a convex subset $S \in \mathbb{R}^n$ is quasi convex if $\forall$ real # (α) the set C_α^- is dfn as

$C_\alpha^- \equiv \{x \in S : f(x) \leq \alpha\}$ is a convex set.

Micro

Aug 28 Tuesday

Quiz1 Question

consider the utility fcs : $U = x_1^{-1/2} x_2^{1/2}$

evaluate whether this preference relation satisfies some version of non satiation + convexity

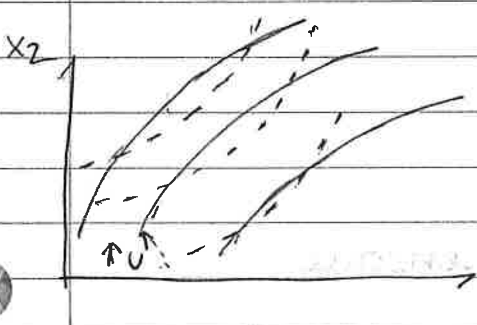
1st: FOC for MU → how consumer feels about quant of goods

FOC

$\mathcal{L}x_1 = -\frac{1}{2}x^{-3/2}x_2^{1/2} < 0$ → "a bad"

$\mathcal{L}x_2 = \frac{1}{2}x^{-1/2}x^{-3/2} > 0$

local ~~monotonic~~ nonsatiation only ↑ w/ ↑ x_2 or ↓ x_1



$$\frac{dx_2}{dx_1} = -\frac{MU_1}{MU_2} \rightarrow -\frac{-\frac{1}{2}x_1^{-3/2}x_2^{1/2}}{\frac{1}{2}x_1^{-1/2}x_2^{-1/2}}$$

not convex → concave

actual lines

$$= x_2 x_1^{-1}$$

$$= \frac{x_2}{x_1} > 0$$

$$\frac{d^2x_2}{dx_1^2} = -\frac{x_2}{x_1^2} < 0$$

pos slope but mono neg

Preferences + Budget constraints

↳ optimization $V(pm)$, properties of $V(pm)$ DUALITY

optimization problem: Max $U(x)$ st $M \geq p'x$

$$\underset{x \in X^*}{\mathcal{L}} = U(x) + \lambda(m - p'x)$$

$$d\mathcal{L}/dx_i = dU/dx_i + \lambda p_i = 0$$

$$\mathcal{L}_\lambda = m - p'x = 0$$

impose consumer spend all of budg. const → also assume entr. soln. ∀x

Also assuming xs are "goods"

$\frac{1}{p} = p^{-1}$

how does x_1^* Δ w/ Δm

$\frac{dx_1^*}{dm} = \frac{1}{3P} > 0$

$\frac{dx_1^*}{dp_1} = -\frac{1}{3}\frac{m}{P_1^2} < 0$ Downward sloping dmd curve

ex: optimal val function

$V(P,m) = U(x_1(P,m), x_2(P,m)^2)$

$V(P,m) = \frac{1}{3}\frac{m}{P_1}\left(\frac{2}{3}\frac{m}{P_2}\right)^2$

$= \frac{1}{3}\frac{m}{P_1}\frac{4}{9}\frac{m^2}{P_2^2}$

$V(p,m) = \frac{4}{27} m^3/P_1P_2^2$ [max utility @ given p + m]

Indirect utility function

1. cont on $\mathbb{R}^n_+ \times \mathbb{R}$
2. homogenous of degree 0 (p + w)
3. incr in w
4. decr in p
5. quasiconvex in P + w
6. Roys ID = $x_i(p,m) = -\frac{dV}{dp_i} \Big/ \frac{dV}{dm}$

→ application of the envelope therom (to be shown later)

~~AAP...~~

$x = 90/1+8 = 90/9 = 10$

$x = 10$

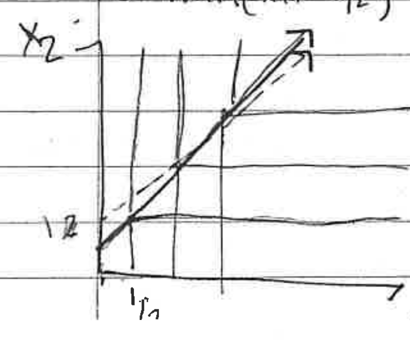
$x_2 = 10$ $x(4) = 40$

$x_1 = 20$ $x(1) = 20$

[clean up later] DISREGARD

end class

$U = \min(x_1, x_2/2)$



$2x_2 : x_1$

use budget constraint

$2x_1 = x_2$

$\cancel{m = P_1 2x + P_2 x}$

$m = x(P_1 2 + P_2)$

$\cancel{m = p_1 2x + P_2 x_2}$

MICRO cont.

Aug 23 cont

Example 1: $U = x_1 x_2^2$ $\quad \mathbb{R}^n_+ \Rightarrow \mathbb{R}$ $\quad x^0 \succsim x^1$

Derive formula for indif. curve:

$$\left.\begin{aligned} U = 100 = x_1 x_2^2 \\ x_2 = \sqrt{100/x_1} = \frac{10}{\sqrt{x_1}} \text{ [indif curve.]} \end{aligned}\right\}$$

$$\max_{x\lambda} U = x_1 x_2^2 \quad \text{st} \quad p_1x_1 + p_2x_2 = m$$

$$\mathcal{L} \; x_1 x_2^2 + \lambda(m - p_1x_1 - p_2x_2)$$

FOC:

$$\mathcal{L}_1 = x_2^2 - \lambda p_1 = 0 \qquad \frac{x_2^2}{p_1} = \frac{2x_1x_2}{p_2}$$

$$\mathcal{L}_2 = 2x_1x_2 - \lambda p_2 = 0$$

$$x_2^2 p_2 = 2x_1x_2p_1 \qquad p_1\left(\frac{x_2p_2}{2p_1}\right) + p_2x_2 = m$$

$$x_2p_2 = 2x_1p_1 \qquad 3x_2p_2/2 = m$$

$$\frac{x_2p_2}{2p_1} = \boxed{x_1} \qquad 3x_2p_2 = 2m$$

$$\frac{\left(2m/3p_2\right)p_2}{2p_1} = x_1 \qquad \boxed{x_2} = 2m/3p_2$$

$$\frac{2/3\,m}{2p_1} = x_1$$

$$m/3p_1 = x$$

soln

$$x_1^* = m/3p_1$$

$$x_2^* = 2m/3p_2$$

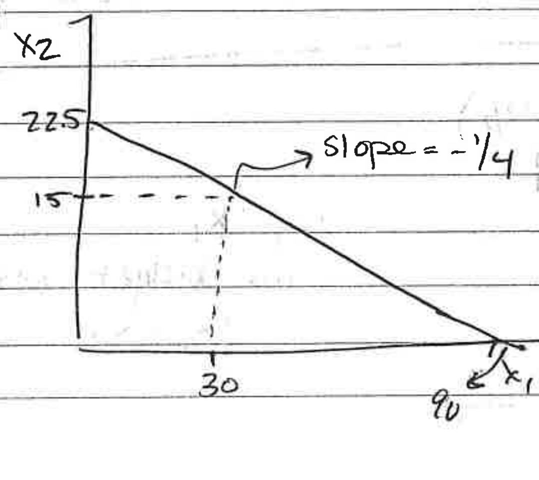
Note

$$\boxed{U = x_1^{\alpha} x_2^{\beta} \rightarrow x_1^* = \frac{\alpha}{\alpha+\beta}\frac{m}{p_1} \qquad x_2^* = \beta/_{\alpha+\beta}\; m/p_2}$$

$m = 90$

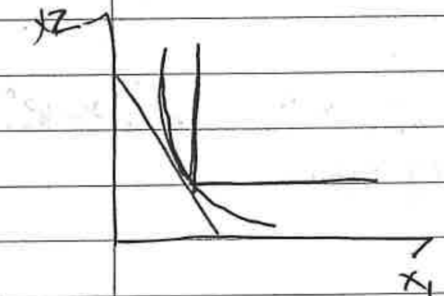
$p_1 = 1$

$p_2 = 4$



$$x_1^* = \frac{1}{3}\,\frac{90}{1} = 30$$

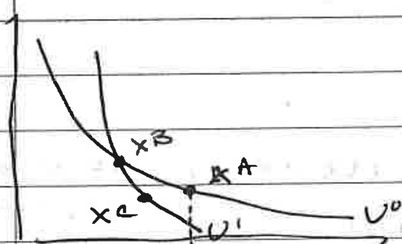
$$x_2^* = \frac{2}{3}\,\frac{90}{4}\; 15$$



perfect substitutes
perfect complements.
standard (emperfect substitutes)

[Why MRS is emportant; discusses how one will give up x_1 for more x_2 @ diffenent areas of endif curve]

[Proove endif curves don't cross] Boo!
↳ don't have to be parallel but can't cross
proof by contradiction → suppose endif curves cross.



$x^B \sim x^A$ dfn endif curve
$x^C \sim x^B$ dfn endif curve
$\therefore x^A \sim x^C$ transitive
violates non satiaton; $x^A >> x^C$

[Now to solve the consumer problem]
Constrained optimizaton:

$\underset{x_i, \forall i\, 1-n}{\text{Max}}\ U(x)$ subject to $xP' = m$

to stay in feasable set → $m = P'x$; $\lambda(0) = 0$

$\underset{x\lambda}{\mathcal{L}} = U(x) + \lambda(m - P'x)$

FOC
$\partial\mathcal{L}/\partial x_i = \partial U/\partial x_i - \lambda P_i = 0$
$\partial\mathcal{L}/\partial\lambda = m - P'x = 0$
} necessicary

* solve for lambda; get $\frac{MU_i}{P_i}$

START

Micro Aug 23 Thursday

Review:

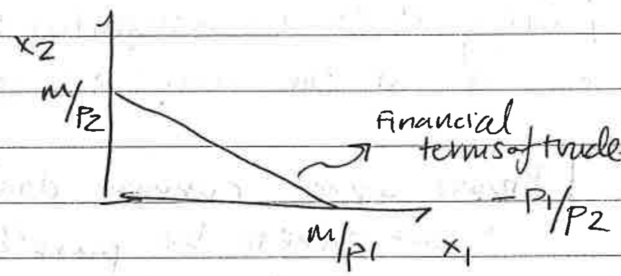
~~Comod space~~: comodity space: N-domentional $x^A = \{x_1^A, x_2^A \ldots x_n^A\}$ $x \in \mathbb{R}_+^n$

Budget set (constraints) is a subset of commodity space feasable for given P, m

$m = P'x = p_1x_1 + p_2x_2 + \ldots p_nx_n$

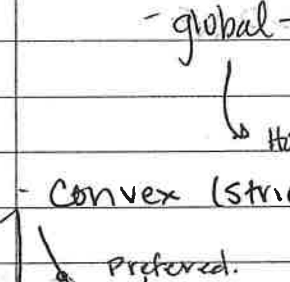
2 Space: solve for ~~x_2~~ 2 space: $m = p_1x_1 + p_2x_2$

solve for $x_2 = m/p_2 - p_1/p_2 x_1$



Consumer preferences: $\succ$

- Completeness → $\succeq$ $\forall x \in \mathbb{R}_+^n$ (can Rank all x)
- Transitivity
- non satiation (local or global)
  - local → @ least 1 directon to go in where we are better off (at least 1 MU (+))
  - global → ~~preference~~ for 2 bundles identical except for 1 comodity where X_B has more of x_n; $\cancel{A}$ $X_B \succ X_A$
    - Holds for all commodities; all x in X are desireable
- Convex (strict or weak): convexity of preferences:



Utility functions (YAY)

$U: \mathbb{R}_+^n \to \mathbb{R}$ (a rank) st. $\forall$ $x^0 \in x' \in \mathbb{R}_+^n$ $x^0 \succeq x' \iff U(x^0) > U(x')$

- ordinal operator (Rank not intensity)
- we can take any monotonic Δ of U; $V = f(U)$ + can still preserve Ranking
- the level sets of this function: indifference curve!

COM

descrete choice model

↳ Train chapt 2

descrete choice → Y/No buy
1, 2, 3 options
etc

Char
- → mutually exclusive options
- → exaustive (model all sets that could be chosen)
- → finite # choices

RUM: Random utility model
↳ econometrics

assume linear util. fc.)

$U_a = \alpha_a + B_p P_a + B_q q_a + \gamma m + \nu_a$ ① → Random error

$U_B = \alpha_B + B_p P_B + B_q q_B + \gamma m + \nu_B$ ②

$U_C = \underbrace{\alpha_C + B_p P_C + B_q q_C + \gamma m}_{\text{determanistic}} + \nu_C$ ③

determanistic → B_q are all same

B_p are all same

prob(select A) = prob. $U_A > U_b$ & $U_a > U_c$
① > ⑧

Assume $F(\varepsilon)$ = max like est.

$\ln(\alpha, B_q, \gamma)$

α: Brand fix E

B_q: marg-U of qual

γ: marg-U inc

no units →

$MRS = \frac{MU_1}{MU_2}$

$\boxed{MWTP \rightarrow -\frac{B_i}{\gamma}}$

using for constrained optimiz?

extra row def

Bordered Hessian

$$\left[\begin{array}{cc|c} U_{11} & U_{12} & -P_1 \\ U_{21} & U_{22} & -P_2 \\ \hline -P_1 & -P_2 & 0 \end{array}\right] = |H_B|$$

derives wrt λ

pos is neg semidef [determinate as a whole]

$(-)(+)(-)$ is pos semi

~~max~~ local v. global: neg def @least local

unit ef > other than anyother or $\forall$ p,m

↳ neg def

Applications

dmd modeling → Dmd systems

$x_1 = \alpha_1 + B_1 P_1 + B_{12} P_2 + \gamma_1 m$ Zhen et al. AJAE 2020

$x_2 = \alpha_2 + B_2 P_2 + B_{21} P_1 + \gamma_2 m$

doing w/ ~10 goods → impose bul. budget

Slutsky matrix → ~~hessian of E(pu)~~ : symmet + neg semidef.

also Cournot + Engel

$$|E(pu)| = \frac{\partial^2 E(pu)}{\partial p^2} = \frac{\partial x^h(pu)}{\partial p}$$

(concave) neg def. slopes/cross price effect of hix dmd

$$\frac{\partial x_i^h(pu)}{\partial P_j} = \frac{\partial x_i(pm)}{\partial P_j} + x_j(pm)\frac{\partial x_i}{\partial m}$$

TE slope of marsh dmd

dmd x_j

II good i

Micro start

~~ttw~~ sept 18

over view + Review

max utility: max v st $m \geq p'x$

$\mathcal{L}$ to solve ; λ - marginal v of encome

↓

arg max: $x(pm)$ marshldnd

[see duality ws]

arg max vs Argmen: constrained differently

$x^*(pm) = x^h(p, v(pm))$

if $\bar{v}$ is sameas $v(pm)$ use same p', m'

Key pt perameter space is the same

Right

~~$x^*(pE(pv) = x^h(p, v(pm)$~~ Not Right

Comparative statics → compare equilibria

$m = \sum_i^n p_i x_i (pm)$ → HAS TO HOLD "Balance budget identity

derive w/RT m → engel Aggrigation → Relation b/w dmd + \$

derive WRT P → cournot agg. →

SOC eval curv. of multi variate $f(\cdot)$

↳ Hessian ; nxn matrix of 2nd derivitive

Look @ leading principal minors

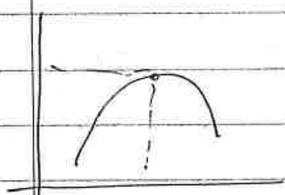
$$H = \begin{bmatrix} f_{11} & f_{12} & f_{13} & f_{41} \\ f_{21} & f_{22} & & \\ f_{31} & & & \\ f_{41} & & & f_{44} \end{bmatrix}$$

get n determinates

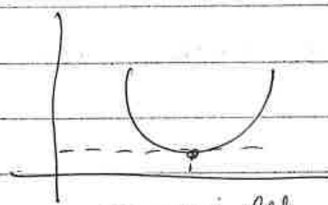
pos semi def → Postive (all

neg semi def → start neg + Switch sign

Multivariate world: Hessian ~~matrix~~ matrix: [n×n]



neg (semi) def Hessian[]



pos semi def Hess[]

* Note this is for local + global → need to do grid search to check whole domain

$y''(x) = 0$ or $> + <$ to 0 around pt (undet Hessian) saddle pt

eval of Hessian:

$$\begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \rightarrow & \partial f^2 / \partial x_1 \partial x_2 \\ \downarrow & \searrow & \downarrow \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \rightarrow & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

X^*

→ for local

if holds for all x^* then global

Leading 5×5

$H_1 = f_{11}$

$H_2 = f_{11} f_{22} - f_{12} f_{21}$

$H_3 = f_{11}(f_{22} f_{33} - f f_{23} f_{32}) - f_{12}(f_{21} f_{33} -$

$$\begin{vmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{vmatrix}$$

$[f_{11} f_{22} f_{33} + f_{13} f_{21} f_{32} + f_{31} f_{12} f_{23}]$
$- [f_{31} f_{22} f_{12} + f_{21} f_{12} f_{3} + f_{11} f_{32} f_{23}$

OR DETERMANITES

Neg def: $(-1)^i |H_i| > 0 \quad \forall i = 1 \ldots N$ alt signs...

Pos def: $H_i > 0 \quad \forall i = 1 \ldots N$

micro cont

Sept 13

suppose $x_1 > 0$ $x_2 = 0$

$12 x_1^{-1/2} = \lambda p$ $\quad x_2 = 0$

$1 < \lambda$ $\quad$ good $25 \cdot 2 = 50$ $\quad U = 24(25)^{1/2} = \boxed{120}$

$x = \frac{m}{p} = 25$

Suppose $x_1 = 0$ $x_2 > 0$

$1 = \lambda p_1$ $\quad \boxed{U = 50}$

$\lambda = 1$

$x_2 = 50$

$$\boxed{\begin{array}{l} x_1 > 0 \quad x_2 = 0 \quad \text{optimal} \\ x_1 = 25 \quad x_2 = 0 \end{array}}$$

Max $U(\cdot)$ st linear constraint

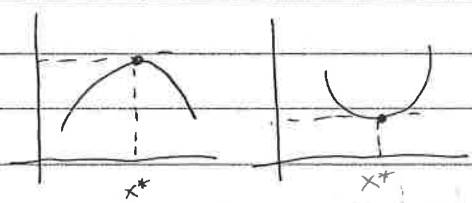
necess foc:

$U_i \leq 0$ allows cornersoln $\quad$ NOT SUFFICIENT

$U_i = 0$ exterior soln

suffic: SOC

Univ:



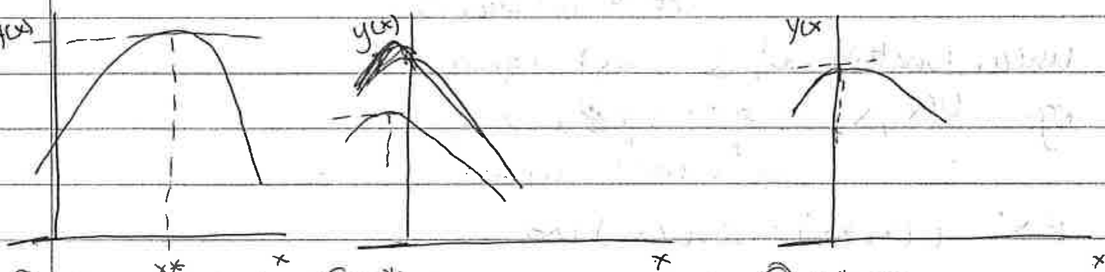
What happening around function @ extreampt

• = critical pt → 0 slope

$\left\{ \begin{bmatrix} y''(x) < 0 \\ \text{concave} = \text{max} \end{bmatrix}, \begin{bmatrix} y''(x) > 0 \\ \text{convex} = \text{min} \end{bmatrix} \right\}$

$$\max_{x_1 x_2 \lambda \geq 0} \mathcal{L} = u(x_1 x_2) + \lambda(m - p_1x_1 - p_2x_2)$$

univariate case:



x no longer restricted to >0

① $x > 0$ $\quad y'x = 0$

② $x^* = 0$ $\quad y'(x) < 0$

③ $x^* = 0$ $\quad y'(x) = 0$

need new foc:

$\mathcal{L}_1 = \partial U/\partial x_1 - \lambda p_1 \leq 0$

$\mathcal{L}_2 = \partial U/\partial x_2 - \lambda p_2 \leq 0$

$\mathcal{L}_\lambda = m - p_1x_1 - p_2x_2 \geq 0$

$y'(x) \leq 0$

comp slack conditons:

$\mathcal{L}_1 \cdot x_1 = 0$

$\mathcal{L}_2 \cdot x_2 = 0$ $\quad$ either or both are 0

$\mathcal{L}_\lambda \cdot \lambda = 0$

argmax $x \geq 0$

KT example

$U(x_1 x_2) = 24x_1^{1/2} + x_2$ $\qquad \mathcal{L} = 24x^{1/2} + x_2 + \lambda(m - p_1x_1 - p_2x_2)$

kuhn tucker depends on $m + p$

$\mathcal{L}_1 = 12x^{-1/2} - \lambda p_1 \leq 0$	$x_1 \geq 0$	$x_1[12x^{-1/2} - \lambda p_1] = 0$
$\mathcal{L}_2 = 1 - \lambda p_2 \leq 0$	$x_2 \geq 0$	$x_2[1 - \lambda p_2] \leq 0$
$\mathcal{L}_\lambda = m - p_1x_1 - p_2x_2 \geq 0$	$\lambda \geq 0$	$\lambda[m - p_1x_1 - p_2x_2] =$

Let $P_1 = 2$ $\quad P_2 = 1$ $\quad m = 50$

$12x_1^{-1/2} = \lambda p_1 = 2\lambda$

$1 = \lambda = 6x_1^{-1/2}$

$\cancel{x_1 > 0}$ ①

$\boxed{x_1 = 36}$ $\rightarrow$ Bust budg Req $52 > 50$

Mecro start

Sept 13

Quasi linear preferences: utility fc.) has 2 key attributes

① additively seperable in 2 or more goods

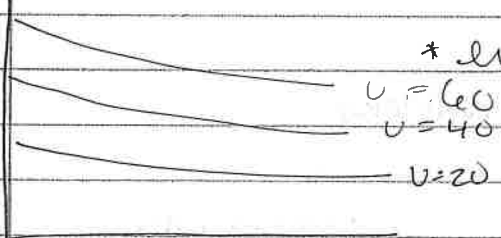
$u(x_1, x_2) = g(x_1) + h(x_2)$ where $g(\cdot)$ & $h(\cdot)$ are some functions

② its linear function of @ least 1 good

eg: $U(x_1, x_2) = \underbrace{g(x_1)}_{\text{non linear}} + \underbrace{\alpha x_2}_{\text{linear}}$

Ex: $U(x_1, x_2) = \ln(x_1) + x_2$

- many utilties are indep of 1 another
- MV for x_2 is const



* ind if curves → vert. ~~transitions~~ translations

set $V = 20 \Rightarrow x_2 = \frac{20 - \ln x_1}{\alpha}$

Quasi linear used for: tractibility → allows corner sol'n.

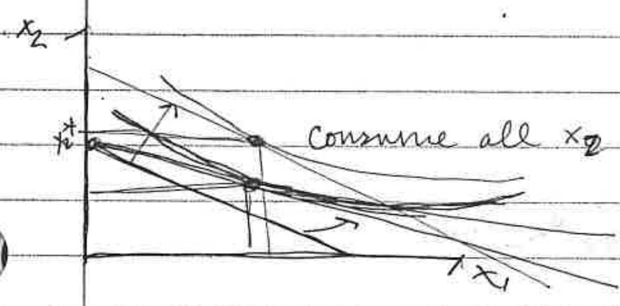
Kuhn-Tucker conditions:

3 cases: $x_1, x_2 > 0$, $x_1 = 0\ x_2 > 0$, $x_2 = 0\ x_1 > 0$

$\max_{x_1, x_2\ \lambda \geq 0} \mathcal{L} = U(x_1, x_2) + \lambda(m - p_1 x_1 - p_2 x_2)$

Aside

a consumer may not purchase a good if p is too high OR AKA income is too low



consume all x_2 + no x_1

if $p_1 \downarrow m$

price ↓ enough to cause interior sol'n

if inc ↑ m

1) total effect

micro cont

sept 11

$$\frac{-x_j(pm)\,p_j}{m} = \sum_{i=1}^{n} \cancel{p_i} \; s_i \, \varepsilon_{ij}$$

if $p_j \uparrow$ take net equal to

Asside $\varepsilon_{ij} = \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i}$

Point: $\frac{-x_j(pm)\,p_j}{m} = \sum_{i=1}^{n} s_i \, \varepsilon_{ij}$

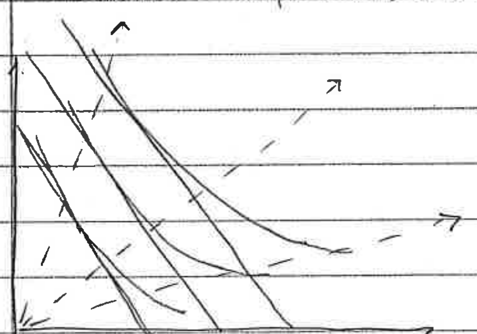
$\uparrow p_j \Rightarrow \downarrow$ purchasing power is proportional to x_j & m

Preference function → common forms

Theoretical forms are often symplystic (b/c tractable) and homothetic preference funcions - a preference $f(\cdot)$ is homothetic if its a monotonic transformaton of a linear homogenous $f(\cdot)$ (HOD 1) → "multiplicitive scaling behavior"

ex: $U = x_1^{1/2} x_2^{1/2}$ is linear homog → Σ expo $= 1$

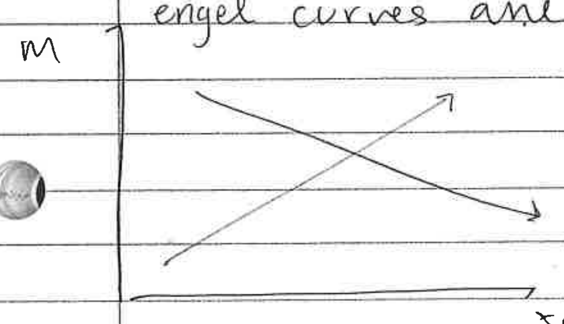
homothetic functions have level sets that are radial expansions from orgin



any cobb douglas: $x_1^{a} x_2^{b}$

perfect comp, perfect subs

engel curves are straight lines → a homothetic $f(\cdot)$ has linear engel curve



$$\frac{dx_1}{dm} = k$$

Balanced budget : ~~m~~ $m = \sum_i p_i x_i(pm)$ "adding up principal"

↳ ~~...~~ spend on budget line

2 properties from this

price aggrigator - cornot

income aggriy - engel

engel aggrigation (curves show relation b/w inc + dmd)

$1 = \sum_{i=1}^{n} p_i \frac{dx_i(pm)}{dm}$ why a \$ affects dmd via income

20¢ x_1 + 80¢ x_2 but should = \$1

how ever much \$ you get

a/t written in terms of elastc.

$dm/dm = \sum_i p \cdot \frac{\partial x_i}{\partial m} \Rightarrow 1 = \sum_i p_i \, dx_i/dm \Rightarrow 1 = \sum_i p \frac{dx_i}{dm} \frac{x_i m}{x_i m}$

$\Rightarrow 1 = \sum_i \frac{p_i x_i}{m} \frac{dx_i}{dm} \frac{m}{x_i} \Rightarrow 1 = \sum s_i \eta_i$

→ share of expend on i

→ inc elast

→ pos for normal good

neg for inferior good

POINT

[share wted sum of income elast = 1] engel aggrigation

Cournot aggrg: what happens to demand system w/ single price Δ

consider Δp_j

$m = \sum_i p_i x_i (pm)$

$\frac{dm}{dp_j} = 0 = \underbrace{\sum_{i \neq j}^{n} p_i \frac{dx_i(p_i m)}{dp_j}}_{\text{induced price}\Delta} + \underbrace{x_j(pm) + p_j \frac{dx_j}{dp_j}}_{\text{direct effect}}$ } → price effect on good j

scalars: deriv = 0

cross price effect → shift in dmd curve

"shifts in other dmd curves

$-x_j(pm) = \sum_{i=1}^{n} p_i \frac{dx_i(pu)}{dp_j}$ multip + divide both sides by p_j/m

share expend on good j

$\frac{-x_j(pm) p_j}{m} = \sum_{i=1}^{n} p_i \frac{dx_i(pm}{dp_j} \frac{p_j}{m}$ multip + divide by ~~x_j~~ $\frac{x_i}{x_i}$

micro start

sept 11

Duality in consumer theory

slutsky: $x(pm) = x^h(p, v(p,m))$
$x^h(pu) = x(p, E(pu))$] via inversion

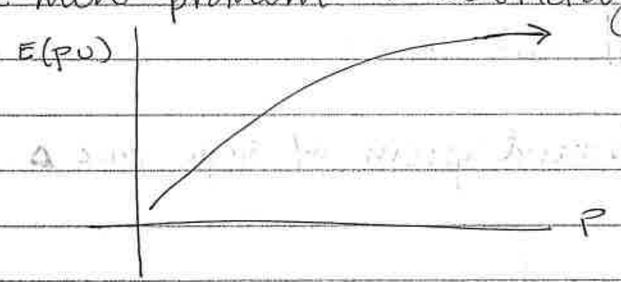
slutsky matrix → neg semidef and symmetric

review $\left[\frac{dx_i}{dp_j}\right]^{(n \times n)}$ $\quad \underset{\text{(subst matrix)}}{\underset{n \times n}{\sigma(pu)}} = \begin{bmatrix} \partial x_1^h/\partial p_1 & \rightarrow & \frac{\partial x_1^h}{\partial p_n} \\ \partial x_2^h/\partial p_1 & & \downarrow \\ \downarrow & & \\ \frac{\partial x_n^h}{\partial p_1} & \rightarrow & \frac{\partial x_n^h}{\partial p_n} \end{bmatrix}$

Aside:
hicksian world
price Δ = subst effect
no income effect

also = to → $\begin{bmatrix} \partial^2 E/\partial p_1^2 & \longrightarrow & \partial^2 E/\partial p_n \partial p_1 \\ \downarrow & \searrow & \downarrow \\ \partial^2 E/\partial p_1 \partial p_n & \rightarrow & \partial^2 E/\partial p_n^2 \end{bmatrix}$ → neg semi def.
symmetric → 2nd derivitives
↳ via youngs theroum

E min problem → strictly concave in P
ie



derive subst matrix so as to learn about slutsky matrix

$$\underset{[SE]}{\frac{\partial x_i^h}{\partial p_j}} = \underset{[TE]}{\frac{\partial x_i}{\partial p_j}} + \underbrace{x_j \cdot \frac{\partial x_i}{\partial m}}_{[IE]} \quad \text{(slutsky eq)}$$

*note i=j is diagonal else other sides.

slutsky matrix: $\begin{bmatrix} \frac{\partial x_1}{\partial p_1} + x_1 \frac{\partial x_1}{\partial m} & \longrightarrow & \frac{\partial x_1}{\partial p_n} + x_n \frac{\partial x_1}{\partial m} \\ \downarrow & \searrow & \downarrow \\ \frac{\partial x_n}{\partial p_1} + x_1 \frac{\partial x_n}{\partial m} & \longrightarrow & \frac{\partial x_n}{\partial p_n} + x_n \frac{\partial x_n}{\partial m} \end{bmatrix}$ neg semi def
symmetric

Bummer about slutsky matrix: is symmetric
+ neg def
meaning: Restricts perameters
(or semi def for cornersol

Slutsky:

$$\begin{bmatrix} B_f + x_f \gamma_{fm} & B_{fs} + x_s \gamma_{fm} \\ B_{sf} + x_f \gamma_{sm} & B_s + x_s \gamma_{sm} \end{bmatrix}$$

$B_{fs} + x_s \gamma_{fm} = B_{sf} + x_f \gamma_{sm}$

$\underbrace{B_{fs} = B_{sf}}_{\text{hicksian symmetry}}$ $\quad \underbrace{\gamma_{fm} = \gamma_{sm} = 0}_{\text{no inc effect}} \rightarrow x_i^h = x_i$ (easy way to make symmetry hold)

Not really satisfying to be in hicksian world

Bummer.

Next time:

$m = \Sigma p_i x_i (p, m)$ walrus law

get:

cournot aggrigation

engel aggrigation

MICRO cont

sept 6

using Hessian:

$f(x_1, x_2) = y$

$\nabla f = \begin{matrix} df/dx_1 \\ df/dx_2 \end{matrix}$ how range moves in x_1 + x_2 directions

Hessian: $\begin{bmatrix} d^2f/dx_1^2 & d^2f/dx_2 dx_1 \\ d^2f/dx_1 dx_2 & d^2f/dx_2^2 \end{bmatrix}$

principal minors!

1st: $d^2f/dx_1^2 \rightarrow$ negitive

2nd: $\hookrightarrow (d^2f/dx_1^2 \cdot d^2f/dx_2^2) - (df/dx_2dx_1 \cdot df/dx_1x_2)$

$\rightarrow$ positive

1st (−), 2nd (+) } Strict concav $\rightarrow$ neg def

$\hookrightarrow$ expen function

1st (−) or 0, 2nd (+) or 0 } semi def $\rightarrow$ convex (not strict) $\rightarrow$ expend function

allows corner sol'n

(Behavior of unafordable things)

$$\underbrace{\frac{dx_i h(pu)}{dp_i}}_{SE} = \underbrace{\frac{dx(pm)}{dP_i}}_{TE} + \underbrace{x_i(pm)\frac{dx_i(pm)}{dm}}_{IE}$$

② $x_i(pu) = x_i(p, E(pu))$

$$|E(pu)| = \begin{bmatrix} dx_i h/dp_i & \cdots & dx_i h/dp_n \\ \frac{dx_n h}{dp_i} & \cdots & \frac{dx_h^n}{dp_n} \end{bmatrix} = \text{Slutsky Matrix} \begin{bmatrix} \frac{dx_i(pm)}{dp_i} + x_i(pm)\frac{dx_i(pm)}{dm} & \rightarrow \\ \frac{dx_2(pm)}{dp_1} + x_1(pm)\frac{dx_2(pm)}{dm} \\ \frac{dx_n(pm)}{dp_1} + x_1(pm)\frac{dx_n(pm)}{dm} & \rightarrow \frac{dx_n(pm)}{dp_n} + x_n\frac{dx_n}{dm} \end{bmatrix}$$

neg def

(or semidef for corner soln)

ex

2 good system: food + shelter

$x_f = \alpha_f + \beta_f p_f + \beta_{fs} P_s + \gamma_{fm} M$

$$XS = \underbrace{\alpha_s}_{inte} + \underbrace{\beta_{sf} P_s + P_{ss}}_{price\ effect} + \underbrace{\gamma_{sm} M}_{income\ effect}$$

strictly

SE cont: if $U(\cdot)$ is quasi concave $\rightarrow E(pu)$ is strictly concave

duality relation [$\frac{\partial^2 E(p,w)}{\partial p_i^2} \leq 0 \Rightarrow \frac{\partial x_i^h(pu)}{\partial p_i} < 0$

[Subst effects always obey law of dmd]

income effect: relationship b/w price and quantity, allowing it to Δ depends on the nature of the good in question

(A) normal good: IE is opps. price Δ

(B) inferior good: IE is same sign as price Δ

$\frac{\partial E(pu)}{\partial p_i} = x_i^h(pu)$ by shep. lem

$\frac{\partial^2 E(pu)}{\partial p_i^2} = \frac{\partial x_i^h(pu)}{\partial p_i} < 0$ by concavity

Theroum: Symmetric substitution

$\frac{\partial^2 E(pu)}{\partial p_i \partial p_j} = \frac{\partial^2 E(pu)}{\partial p_j \partial p_i}$ Youngs Theroum

$\frac{\partial x_i^h(pu)}{\partial p_j} = \frac{\partial x_j^h(pu)}{\partial p_i}$

Substitution matrix

~~SM~~ $\sigma(pu)$ = $\begin{pmatrix} \frac{\partial x_1^h}{\partial p_1} & \rightarrow & \frac{\partial x_1^h}{\partial p_n} \\ \vdots & \searrow & \downarrow \\ \frac{\partial x_n^h}{\partial p_1} & \rightarrow & \frac{\partial x_n^h}{\partial p_n} \end{pmatrix}$ second deriv of E ~~marsh~~

Collection of all cross partials

neg semidef if $E(pu)$ is concave

neg def if $E(pu)$ is strictly concave

Hessian! woo

gradiant: $n \times 1$ vector of 1st deriv. FOC = 0 (necess for extream pts)

Hessian: $n \times n$ ~~vec~~ matrix of 2nd deriv

SOC $f''(x) < 0 \rightarrow$ max strictly concave

MICRO START

sept 6

U Max Emin

$x(pm) \leftrightarrow x^h(pU)$

$V(pm) \leftrightarrow E(pU)$

Dmd: P & Q are inversely related for normal goods; if P & Q are positively related the good is inferior

Slutsky:

① $x_i(p,m) \equiv x_i^h(p, V(pm))$

② $x_i^h(pU) \equiv x_i(p, E(pU))$

Relation #1

$$\underbrace{\frac{dx_i(pm)}{dp_i}}_{\text{Total effect}} = \underbrace{\frac{dx_i^h(p, \overset{U}{\overline{V(pU)}})}{\partial p_i}}_{\text{subst effect}} + \underbrace{\frac{dx_i^h(p,V(pU))}{dV} \frac{dV(pU)}{dp_i}}$$

$$+ \frac{dx_i^h(p,V(pm))}{dV(pm)} \left(-x_i(pm)\frac{dV(pm)}{dm}\right)$$

envelope theorem:

Roys ID $x_i(pm) = \frac{dV/dp_i}{dV/dm}$

$$= \frac{dx_i^h}{dp_i} + (-x_i(pm)) \cdot \frac{dx_i^h(pU)}{dV(pm)} \cdot \frac{dV(pm)}{dm}$$

$$\underbrace{\frac{\partial x_i(pm)}{\partial p_i}}_{\text{total}} = \underbrace{\frac{\partial x_i^h}{\partial p_i}}_{\text{subst.}} \underbrace{- x_i(pm)\frac{\partial x_i(pm)}{\partial m}}_{\text{inc effect}}$$

SE: if preferences are strictly convex; ↑p_i w/ p_j + U const will ↓ x_i