Applied Microeconomics: Firm and Household

Lecture 7: Demand Relationships and Welfare Measures

Jason Kerwin

Department of Applied Economics
University of Minnesota

September 27, 2016

Outline

- An alternative derivation of the Slutsky equation
- Useful elasticity formulas:
  - Homogeneity
  - Engel aggregation
  - Cournot aggregation
- Welfare measures:
  - Consumer surplus (CS)
  - Equivalent compensation (EV)
  - Compensating variation (CV)

An alternative derivation of the Slutsky equation

The derivation of the Slutsky equation using the traditional approach is useful to know especially in dealing with nonstandard problems. Next, we discuss the modern approach to derivation of the Slutsky equation.

Recall that at the tangency of a budget line and an indifference curve $x^h = x^*$. Thus, we can write

- $x_1^h(p_1, p_2, u^0) = x_1^*(p_1, p_2, M^*(p_1, p_2, u^0))$

By taking the first derivative of both sides w.r.t p_1 we obtain

- $\frac{\partial x_1^h}{\partial p_1} = \frac{\partial x_1^*}{\partial p_1} + \frac{\partial x_1^*}{\partial M^*} \underbrace{\frac{\partial M^*}{\partial p_1}}_{=x_1^h = x_1^*}$
- $\frac{\partial x^h}{\partial p_1} = \frac{\partial x_1^*}{\partial p_1} + x_1^* \frac{\partial x_1^*}{\partial M^*}$

An alternative derivation of the Slutsky equation

Similarly, the Slutsky equation for cross-price effects is:

- $\frac{\partial x_1^h}{\partial p_2} = \frac{\partial x_1^*}{\partial p_2} + x_2^* \frac{\partial x_1^*}{\partial M^*}$

As a general expression,

- $\frac{\partial x_i^h}{\partial p_j} = \frac{\partial x_i^*}{\partial p_j} + x_j^* \frac{\partial x_i^*}{\partial M^*}$, for $i, j = 1, ..., n$.

The Slutsky equation in elasticity form

A useful way of expressing the Slutsky Equation is in its elasticity form. To get this formulation we multiply both sides of equation by $\frac{p_j}{x_i}$ and multiply the second term on the right-hand side by $\frac{M}{M} = 1$

- $\frac{\partial x_i^h}{\partial p_j}\frac{p_j}{x_i} = \frac{\partial x_i^*}{\partial p_j}\frac{p_j}{x_i} + x_j\frac{p_j}{x_i}^*\frac{\partial x_i^*}{\partial M^*}\frac{M}{M}$, for $i, j = 1, ..., n$.
- $\frac{\partial x_i^h}{\partial p_j}\frac{p_j}{x_i} = \frac{\partial x_i^*}{\partial p_j}\frac{p_j}{x_i} + \frac{x_j p_j}{M}^*\frac{\partial x_i^*}{\partial M^*}\frac{M}{x_i}$, for $i, j = 1, ..., n$.
- $\epsilon_{ij}^h = \epsilon_{ij}^* + s_j^*\epsilon_{iM}^*$, for $i, j = 1, ..., n$.

That is, the price elasticity of Hicksian demand can be expressed as a function of i) the price elasticity of Marshallian demand, ii) the share of consumer's budget spent on good j, and iii) the income elasticity of good i.

Useful Elasticity Formulas: homogeneity

The Marshallian demand, $x_1^*(p_1, p_2, M)$, is HOD 0 in prices and income, by Euler's theorem:

- $\frac{\partial x_1^*}{\partial p_1} p_1 + \frac{\partial x_1^*}{\partial p_2} p_2 + \frac{\partial x_1^*}{\partial M} M = 0$, divide both sides by x_1^*
- $\epsilon_{11}^* + \epsilon_{12}^* + \epsilon_{1M}^* = 0$
- $\epsilon_{ii}^* + \epsilon_{ij}^* + ... + \epsilon_{in}^* + \epsilon_{1M}^* = 0$, for the case of n goods.

That is, the sum of own-price, cross-price and the income elasticities of the Marshallian demand is equal to zero.

Useful Elasticity Formulas: homogeneity

The Hicksian demand $x_1^h(p_1, p_2, u^0)$ is HOD 0 in prices, by Euler's theorem

- $\frac{\partial x_1^h}{\partial p_1}p_1 + \frac{\partial x_1^h}{\partial p_2}p_2 = 0$, divide both sides by x_1^h
- $\epsilon_{11}^* + \epsilon_{12}^* = 0$
- $\epsilon_{ii}^h + \epsilon_{ij}^h + ... + \epsilon_{in}^h = 0$, for the case of n goods.

That is, the sum of the own- and cross-price elasticities of the Hicksian demand is equal to zero.

Useful Elasticity Formulas: Engel aggregation

Engel's law: As income increases the share of income spent on food decreases.

- Income elasticity of demand for food < 1

Engel's law implies that the income elasticity of all nonfood items must be > 1.

We can establish the formal relationship between income elasticities by differentiating the budget constraint, $p_1 x_1^* + p_2 x_2^* = M$, with respect to income.

- $p_1 \frac{\partial x_1^*}{\partial M} + p_2 \frac{\partial x_2^*}{M} \equiv 1$

Useful Elasticity Formulas: Engel aggregation

- $p_1 \frac{\partial x_1^*}{\partial M} + p_2 \frac{\partial x_2^*}{M} \equiv 1$

Multiplying the first term by $\frac{x_1^* M}{x_1^* M} = 1$ and the second term by $\frac{x_2^* M}{x_2^* M} = 1$ we obtain

- $\frac{p_1 x_1^*}{M} \frac{\partial x_1^*}{\partial M} \frac{M}{x_1^*} + \frac{p_2 x_2^*}{M} \frac{\partial x_2^*}{M} \frac{M}{x_2^*} \equiv 1$
- $s_1 \epsilon_{1M}^* + s_2 \epsilon_{2M}^* = 1$

For the *n*-good case

- $s_1 \epsilon_{1M}^* + s_2 \epsilon_{2M}^* + \ldots + s_n \epsilon_{nM}^* = 1$

That is, the weighted sum of the income elasticities of all goods is equal to 1, where the weights are the shares of income spent on each good.

Useful Elasticity Formulas: Cournot aggregation

Finally, as Cournot once did, we would like to know how a change in a single price might affect the demand for all goods.

To this end, we differentiate the budget constraint, $p_1 x_1^* + p_2 x_2^* = M$, with respect to the price of good 1, p_1, to obtain

- $x_1^* + p_1 \frac{\partial x_1^*}{\partial p_1} + p_2 \frac{\partial x_2^*}{\partial p_1} \equiv 0$

Useful Elasticity Formulas: Cournot aggregation

- $x_1^* + p_1 \frac{\partial x_1^*}{\partial p_1} + p_2 \frac{\partial x_2^*}{\partial p_1} \equiv 0$

Multiplying all terms by $\frac{p_1}{M}$, the second term by $\frac{x_1}{x_1} = 1$ and the third term by $\frac{x_2}{x_2} = 1$ we obtain

- $\frac{p_1 x_1^*}{M} \frac{\partial x_1^*}{\partial p_1} \frac{p_1}{x_1^*} + \frac{p_2 x_2^*}{M} \frac{\partial x_2^*}{\partial p_1} \frac{p_1}{x_2^*} \equiv -\frac{p_1 x_1^*}{M}$
- $s_1 \epsilon_{11}^* + s_2 \epsilon_{21}^* = -s_1$

For the *n* good case

- $s_1 \epsilon_{1j}^* + s_2 \epsilon_{2j}^* + ... + s_n \epsilon_{nj}^* = -s_j$

The equation states that the weighted sum of the elasticities of all goods with respect to price of an arbitrary good *j* equals to the negative of the budget spent on good *j*, where the weights are the budget spent on each good.

Changes in Utility and Welfare Measures

Thus far we have analyzed change in demand due to a change in prices/income. Next, we seek to evaluate change in welfare due to a change in price. To motivate our discussion we first summarize the utility maximization problem:

- $\underset{x}{Max}\ \ U(x)\ \ \text{s.t. } px = M$
- optimal demand is given by $x^*(p, M)$
- indirect (maximized) utility is given by $u(p, M) = u(x^*(p, M))$

Denote the initial set of prices and income as (p^0, M^0), and the set of prices and income after a price change as $(p^{'}, M^{'})$. We can compute the change in utility using the indirect utility function.

- $\Delta u = u(p^{'}, M^{'}) - u(p^0, M^0)$

Question: Is this measure at all useful?

Equivalent and Compensating Variation

We seek to measure relative costs of reaching a given standard of living under two different situations:

Definitions:

- CV: The dollar amount which, when taken away from the consumer after the price/income changes, leaves her with the initial utility level
  - Implicitly defined as: $u(p^{'}, M^{'} - CV) = u(p^0, M^0)$
  - $CV > 0$ is maximum amount willing to pay for the change to occur
  - $CV < 0$ is minimum compensation required for the change
- EV: The dollar amount which, when paid to the consumer at initial prices and income, gives her the same level of utility they would have enjoyed at the new prices/income
  - Implicitly defined as: $u(p^{'}, M^{'}) = u(p^0, M^0 + EV)$

Equivalent and Compensating Variation

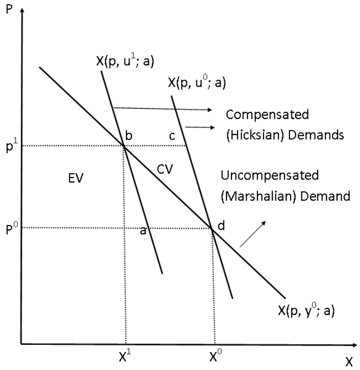


Figure: Welfare measures in case of a price hike

- $EV = P^0 P^1 ba$
- $CV = P^0 P^1 cd$
- $CS = P^0 P^1 bd$

In case of a price increase:

- $|CV| > |CS| > |EV|$

In case of zero income effect

- $|CV| = |CS| = |EV|$

Note that, in case of a price decrease the rankings reverse

- $|EV| > |CS| > |CV|$

Measuring Consumer Surplus, CS

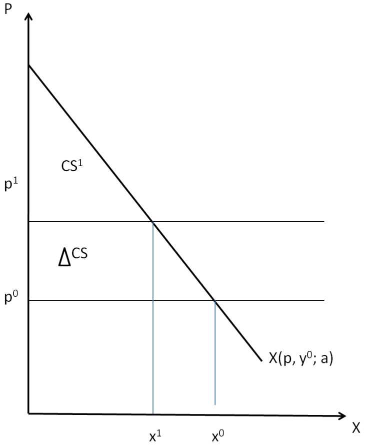
Should we measure welfare changes using EV or CV?

Consumer Surplus measure provides a convenient compromise between these two measures

Measuring CS is convenient because most empirical work on demand estimates Marshallian demand curves.

- Consumer surplus is measured by the area below the Marshallian demand curve and above price
- Shows what an individual would pay for the right to make voluntary transactions at this price (willingness to pay)
- Changes in consumer surplus measure the welfare effects of price changes

Measuring Consumer Surplus, CS



In this example:

- $CS^0 = \Delta CS + CS^1$
- A price increase reduces the CS by ΔCS