Econ 8010 HW3

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1. The Acme Widget Corporation produces a single output (widgets) $q \in \mathbb{R}_+$ using inputs $z \in \mathbb{R}^3_+$. Its production set is given by

$$\{(q, z) : 1 - q^{\frac{1}{\alpha}}(\beta z_1^\rho + \gamma z_2^\rho + \delta z_3^\rho)^{\frac{1}{\rho}} \leq 0\}$$

for $\alpha < 0$, $\rho < 1$, and $\beta, \gamma, \delta > 0$.

   (a) Find Acme Widget Corporation's production function.

   (b) Can we assume without loss of generality that $\alpha = -1$? That $\alpha = -\rho$? Why or why not?

   (c) Can we assume without loss of generality that $\beta + \gamma + \delta = 1$? Why or why not?

   (d) A production function f is said to have

   - **increasing returns to scale** if $f(\alpha z) > \alpha f(z)$ for all $z \in \mathbb{R}^L$ and $\alpha > 1$
   - **decreasing returns to scale** if $f(\alpha z) < \alpha f(z)$ for all $z \in \mathbb{R}^L$ and $\alpha > 1$
   - **constant returns to scale** if it is homogeneous of degree one.

   Under what circumstances does Acme Widget Corporation's production function have increasing returns to scale? Constant returns to scale?

(e) Find Acme's cost function $c(w, q)$ and conditional factor demands $z(w, q)$. If its production function has increasing returns to scale, what does that tell you about $\frac{\partial^2}{\partial q^2} c(w, q)$?

(f) Find Acme's output supply correspondence $q(w, p)$.

2. A consumer has Cobb-Douglas utility

$$u(x_1, x_2) = \frac{3}{4} \log x_1 + \frac{1}{4} \log x_2$$

Suppose that p_1 increases from 1 to 2.

(a) What other information do you need to calculate the EV and CV of this price change? Why?

(b) Suppose that $p_2 = 2$ and $w = 12$. Calculate the EV and CV of this price change.