APEC 5151: Applied Microeconomics: firm and household
Fall 2015 Midterm Exam-ANSWER KEY

1. (6 points) Suppose the demand function for coffee is $Q = 1 - p_c + 2p_t + 0.1Y$ where Q is the quantity of coffee demanded, p_c and p_t are the prices of coffee and tea per kilogram, respectively, and Y is the average consumer income. What are the own- and cross-price elasticities of demand for coffee at $p_c = 2$, $p_t = 1$ and $Q = 2$. Please interpret your findings.

- $\frac{\partial Q}{\partial p_c}\frac{p_c}{Q} = -1\frac{2}{2} = -1$, own price elasticity of coffee is unitary elastic.
  - A one-percent increase in coffee price decreases its quantity demanded by one percent.
- $\frac{\partial Q}{\partial p_t}\frac{p_t}{Q} = 2\frac{1}{2} = 1$, cross price elasticity of coffee is unitary elastic.
  - A one-percent increase in tea price increases quantity demanded of coffee by one percent.

2. (6 points) If the price of tea increases to 1.5 per kilogram, does the quantity of coffee demanded increase or decrease? By how much?

It would increase the quantity demanded of coffee because the cross-price elasticity is positive. If tea price increases from 1 to 1.5 dollars the change is fifty percent. Because cross-price elasticity is unitary elastic, for a fifty-percent increase in price of tea quantity of coffee demanded would increase by fifty percent.

3. (6 points) The slope of the Marshallian demand curve for a normal good is indeterminate. True or False? Please explain.

FALSE. For a normal good the income effect is positive. Therefore, the slope of the Marshallian demand curve is always negative. For example, an increase in the price of the good will always decrease its quantity demanded.

- $\epsilon^*_{x,p} = \underbrace{\epsilon^h_{x,p}|_{u=u^0}}_{-} - s_x \underbrace{\epsilon^M_{x,I}}_{+} < 0$

Use the following information to answer question 4-6. Suppose the uncompensated demand functions for x_1 and x_2 are given as: $x_1 = x_1^*(p_1, p_2, M)$ and $x_2 = x_2^*(p_1, p_2, M)$, where p_i is the price of good i, and M is income. Assume $\underline{x_1 \text{ is inferior}}$ and $\underline{x_2 \text{ is a normal good}}$.

4. (6 points) What is income effect? On a graph, explain the effect of an increase in income on x_1. Please clearly label your graph.

The income effect is the of a change in income on the quantity demanded of good or service. For an inferior good the income effect is negative. That is, an increase in income decreases the quantity demanded of the inferior good. Please see the graph that is provided in your lecture notes or the textbook.

5. (6 points) Suppose that price of x_1 is decreased, denote as p_1. On a graph, explain the effect of a decrease in p_1 on x_1. Please clearly label your graph.

For an inferior good, the total change in quantity demanded of x_1 will be positive due to a decrease in its price. However, the substitution and income effects are in opposite directions. The change due to the substitution effect is larger than the total change. Please see the graph that is provided in your lecture notes or the textbook.

6. (6 points) Suppose that x_1 and x_2 are gross substitutes. On a graph, explain the effect of a decrease in p_1 on x_2. Please clearly label your graph.

If the goods are gross substitutes a decrease in p_1 would decrease the quantity demanded of x_2. Please see the graph that is provided in your lecture notes or the textbook.

7. (6 points) Given a twice differentiable concave utility function $u(x_1, x_2)$, goods prices p_1, p_2, and income M write down the consumer's expenditure minimization problem. Provide the Lagrangian function.

- $\underset{x_1,x_2}{\text{Min}}\; p_1x_1 + p_2x_2$ s.t. $u^0 = u(x_1, x_2)$
- $\underset{x_1,x_2,\lambda}{\text{Min}}\; \text{L} = p_1x_1 + p_2x_2 + \lambda(u^0 - u(x_1, x_2)$

8. (6 points) Derive and interpret the first order conditions of the cost minimization problem in (8). What is the interpretation of Lagrange multiplier?

- $L_1 = \frac{\partial L}{\partial p_1} = p_1 - \lambda u_1 = 0,\ L_2 = \frac{\partial L}{\partial p_2} = p_2 - \lambda u_2 = 0,\ L_\lambda = \frac{\partial L}{\partial \lambda} = u^0 - u(x_1, x_2) = 0$

Solve that at the optimum $\frac{p_1}{p_2} = \frac{u_1}{u_2}$. The optimum condition states that the expenditure minimizing consumer choses goods at levels where the ratio of the prices of goods equals to the ratio of their marginal utilities. The lagrange multiplier of this problem can be interpreted as the inverse of the marginal utility of income.

9. (6 points) Theoretically, Compensating Variation (CV) or Equivalent Variation (EV) provide the "true" measure of the change in welfare due to a price change. However, in most of the empirical studies of demand Consumer Surplus (CS) is commonly used to measure welfare changes. Please discuss why CS provides a good and convenient measure of welfare.

CV is a function of the utility level before the change, and EV is function of the utility level after the change. These measures represent the area between their corresponding Hicksian demand curve and price. The problem is that utility level is not observable. In empirical analysis we usually estimate Marshallian demand curves because it is a function of observable variables such as prices and income. Thus, CS is a convenient measure because it is the area between the Marshallian demand curve and price. Also, CS is a good measure because (for certain functional forms) it provides a value between EV and CV measures.

Given below is a two input production function. Use production function 2 to answer questions 10-12.

$y = 5 + 3x_1 + 2x_2 - 0.1x_1^2 - 0.1x_2^2 - 0.1x_1x_2$

10. (6 points) Is either input in production function 2 essential? Can you characterize the factor interdependency?

Neither of the inputs is essential. Because when $x_1 = 0$ $y > 0$, also when $x_2 = 0$ $y > 0$. The factors are technically competitive because an increase in quantity of factor 2 decreases the prodcutivity of factor 1, and vice versa, $\frac{\partial y^2}{\partial x_1 \partial x_2} = -0.1 < 0$.

11. (6 points) What is the rate of technical substitution between inputs, RTS_{12}? In words, what does RTS represent?

$\frac{\partial y}{\partial x_1} = 3 - 0.2x_1 - 0.1x_2$, $\frac{\partial y}{\partial x_1} = 2 - 0.2x_2 - 0.1x_1$.

- $RTS = \frac{3-0.2x_1-0.1x_2}{2-0.2x_2-0.1x_1}$.
- RTS shows the trade off between factors of production. It represents the rate at which x_2 needs to be decreased in order to achieve the same level output if x_1 is increased.

12. (6 points) What do the ridgelines represent? Derive the ridgeline equations of this function?

Ridgelines bound the economically rational region (the range of inputs) on an isoquant map.

- $RTS = 0 \rightarrow 3 - 0.2x_1 - 0.1x_2 = 0 \rightarrow x_2 = \frac{3-0.2x_1}{0.1}$.
- $RTS = \infty \rightarrow 2 - 0.2x_2 - 0.1x_1 = 0 \rightarrow x_2 = \frac{2-0.1x_1}{0.2}$

13. (7 points) In words, explain why a profit maximizing firm never operates in the 1^{st} production stage for given input and output prices.

The 1^{st} production is characterized by increasing returns to scale. When there is increasing returns to scale, at constant prices of inputs and outputs increasing production will always increase profits. Because increasing all inputs by the same amount will generate proportionally more output. Therefore, the firm will keep increasing its production and until it exhausts the scale economies.

14. (7 points) Consider the following factor demand function $x(p, w) = p/w^2$, where p is output w is input price. Is this a legitimate factor demand function for a profit maximizing firm?

No. Because it is not homogeneous of degree zero. $x(tp, tw) = \frac{p}{tw^2} \neq p/w^2$.

15. (7 points) If a price-taking firm's production function is given by $y = x^{0.5}$. Show that its optimal supply function is $y(p, w) = p/2w$ where p is output w is input price.

- $\underset{x}{\text{Max}}\ \Pi = py(x) - wx$
- $\frac{\partial \Pi}{\partial x} = 0.5px^{-0.5} - w = 0 \rightarrow x = \frac{p^2}{4w^2}$
- $y = x^{0.5} \rightarrow y = \frac{p}{2w}$

16. (7 points) A profit maximizing firm with production function $y = f(x_1, x_2, x_3)$ will always increase the quantity demanded demand for factors $\mathbf{x}$ due to an increase in output price. True or False? Please explain.

FALSE. It depends on the technical relationships between the inputs. A firm will increase its production due to an increase in the price of output. But it is possible that the firm can increase production by increasing only a subset of the inputs. That is, some inputs may not increase or may even decrease while production is increased due to an increase in output price.

Bonus (2 points) Who won the Nobel Prize in Economic Sciences this year? What is one of his/hers important contributions that we discussed in class?

Angus Deaton. Almost Ideal Demand System.