

Econometrics Qualifying Exam  
May 18, 2012

Answer 5 out of 6 questions

Show work fully and write neatly. Good luck.

1. An econometrician estimates the following equation for the price of beef:

$$P_t = \beta_0 + \beta_1 P_{t-1} + \beta_2 M_t + u_t$$

where  $u_t$  is a random shock and  $M_t$  is some policy variable, say money growth. The econometrician estimates that  $\beta_2$  is positive and statistically significant at the  $p=0.07$  level.

- a) What does this say about policy effectiveness?
- b) Could  $\beta_2$  be estimated as positive even if the policy is ineffective? Use a combination of statistical theory, mathematics and verbal explanation to fully justify your answer.
2. Discuss an example of a linear regression model where it is necessary to use an instrumental variable (IV) estimator. Provide the formula for this estimator and thoroughly discuss its asymptotic properties.
3. Consider the standard linear regression model  $y = \beta_0 + \beta_1 x + u$  with  $x$  being a single regressor and the model satisfying the Gauss-Markov assumptions. The usual OLS estimators  $\hat{\beta}_0$  and  $\hat{\beta}_1$  are unbiased for their respective population parameters. Let  $\tilde{\beta}_1$  be the estimator of  $\beta_1$  obtained by assuming the intercept is zero.
- a) Find  $E(\tilde{\beta}_1)$  in terms of  $x_i$ ,  $\beta_0$  and  $\beta_1$ , and identify all conditions required for  $\tilde{\beta}_1$  to be unbiased for  $\beta_1$ .
- b) Find the variance of  $\tilde{\beta}_1$  and compare it to  $\hat{\beta}_1$ .
- c) Discuss the trade-off one faces when choosing between  $\tilde{\beta}_1$  and  $\hat{\beta}_1$ . Explain when you would advise an analyst to use  $\tilde{\beta}_1$ , and when you would advise him to use  $\hat{\beta}_1$ .

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4. A particular heart disease has a prevalence of 1/1000 people. A test to detect this disease has a false positive rate of 5% (meaning 5% of healthy people incorrectly are tested as being ill.). Assume that the test diagnoses correctly every person who has the disease. What is the chance that a randomly selected person found to have a positive result actually has the disease?
5. For the model  $y = X\beta + \varepsilon$ , where  $X = (100 \times 5)$  and  $\varepsilon \sim N(0, \sigma^2)$ ,
- Show all the steps to efficient estimation of the model while imposing the restrictions:  $\beta_2 = \beta_3$  and  $\beta_4 + \beta_5 = 1$ .
  - Show all the steps to testing those joint restrictions.
  - Show all the steps to testing the first restriction ( $\beta_2 = \beta_3$ ) while imposing the second ( $\beta_4 + \beta_5 = 1$ ).
6. Suppose that you need to estimate a system of two simultaneous equations, one of which is just identified and the other being over-identified. Discuss and compare the properties of the instrumental variable, two-stage and three-stage least squares estimator for the parameters of each of those two equations.