Imperfect Competition Models: Monopoly and Oligopoly

I. Introduction

A. Market power and strategic behavior
1. One area of economics where analysis of strategic behavior has always been of central importance is analysis of imperfect competition
2. When there are a small number of firms some firms will have a sizeable market share. Firms will realize that they can influence market price through their actions ("market power")
3. Cases:
   a. Monopoly (single seller); monopsony (single buyer)
   b. Oligopoly (multiple sellers); oligopsony (multiple buyers)
4. Firms will then behave strategically vis-à-vis consumers (monopoly) and vis-à-vis other firms and consumers (oligopoly)

B. Exploration of strategies
1. Start with simple cases – strategies over prices and quantities in static games
2. Price discrimination and non-linear pricing
3. Product differentiation
4. Later in the course – repeated play and collusion, strategic investment, entry and entry deterrence

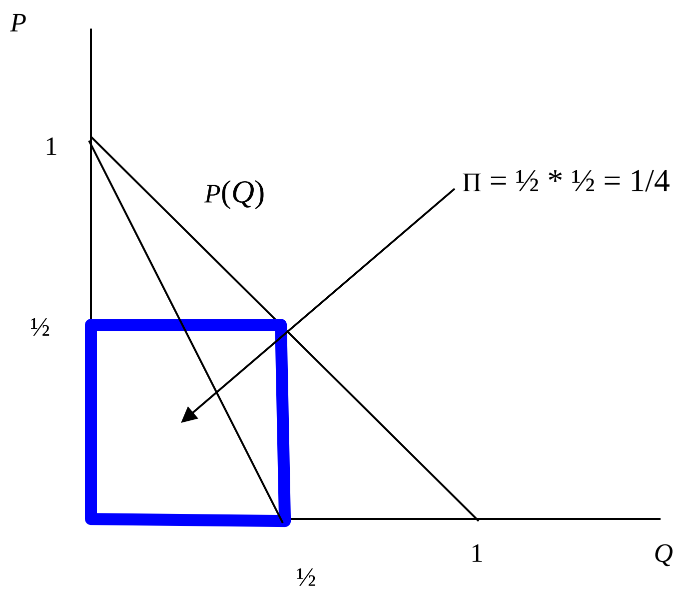
II. Monopoly

A. Standard analysis: monopolist chooses quantity to maximize profits
1. Inverse demand function: $P(Q)$, $P'(Q) < 0$
2. Cost function: $C(Q)$, $C'(Q) \geq 0$
3. Payoff (profit): $\pi(Q) = P(Q)Q - C(Q)$
4. First order necessary condition:
$$P(Q) + P'(Q)Q - C'(Q) = 0$$
$$MR = MC$$
5. Rewriting in terms of elasticity:
$$P(Q)\left[1 + \frac{P'(Q)Q}{P(Q)}\right] = C'(Q)$$
$$P(Q)\left[1 - \frac{1}{\varepsilon}\right] = C'(Q)$$
   where ε is the elasticity of demand.

   Note: in perfect competition, demand to the individual firm is perfectly elastic ($\varepsilon \to \infty$) so that $P(Q) = C'(Q)$
6. Second order condition (concave profit function):
$$2P'(Q) + P''(Q)Q - C''(Q) \leq 0$$
   We don't necessarily know that $P''(Q^*) \leq 0$ or that $C''(Q^*) \geq 0$. But we need the equation as a whole to be satisfied for the solution to be profit maximum.
7. Geometry of monopoly solution:
   a. Consider the simple case with $P(Q) = 1 - Q$; $C(Q) = 0$.
   b. Must price somewhere along the demand curve

c. Profit is rectangle: $QP(Q)$

d. Profit maximizing solution is to choose a square: Q = ½ and P = ½



C. Welfare analysis (partial equilibrium)

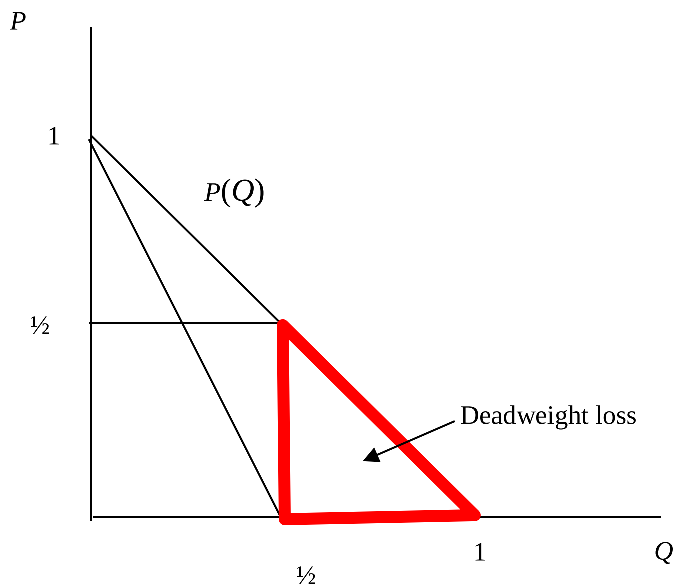
a. Total surplus = consumer surplus + producer surplus:
$W(Q) = U(Q) - P(Q)Q + P(Q)Q - C(Q) = U(Q) - C(Q)$

b. Welfare maximization:
$$W'(Q) = U'(Q) - C'(Q) = 0$$
$$\Rightarrow P(Q) = C'(Q)$$

c. Monopoly : $P(Q) + P'(Q)Q - C'(Q) = 0;\ P(Q) > C'(Q)$

d. Monopolist sells too little – does so in order to raise price to earn positive profits.

e. Deadweight loss $\int_{Q_M}^{Q^*} [P(Q) - C'(Q)]dQ$

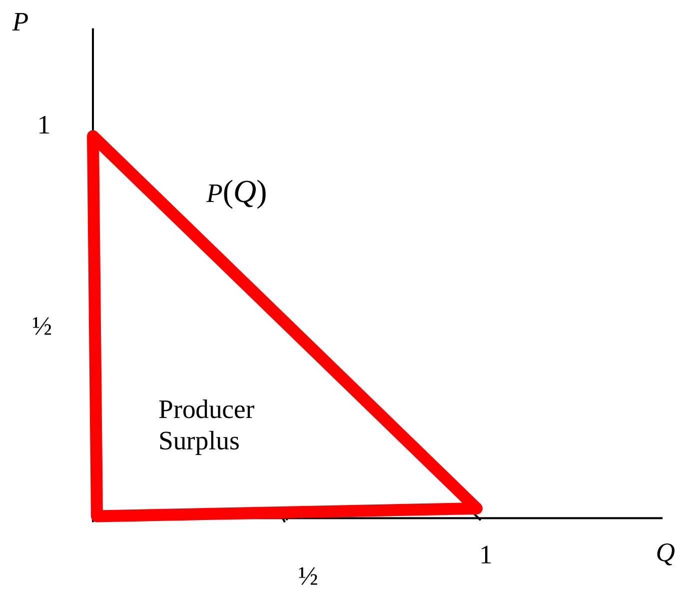


f. In the case where $P(Q)$ = 1- Q, $C(Q)$ = 0, deadweight loss is

$$\int_{Q_M}^{Q^*}[P(Q)-C'(Q)]dQ = \int_{1/2}^{1}[1-Q]dQ = [Q-Q^2/2]|_{1/2}^{1} = 1-1/2-1/2+1/8 = 1/8$$

g. In the standard monopoly solution: gains from trade are left unrealized – monopolist can produce additional units of the good for less cost than what consumers are willing to pay. In some sense, the monopolist isn't doing a very good job of maximizing profits.

D. Price discrimination and non-linear pricing strategies
   1. The monopolist can capture more profit by being more sophisticated about pricing
   2. Perfect price discrimination
      a. Suppose that each consumer, i = 1, 2, …N, demands one unit of a good, with valuation of v_i
      b. Suppose that the monopolist has complete information – knows every individual's v_i
      c. Monopolist can extract ALL surplus by setting an individual price for each consumer $p_i = v_i$
      d. Result is efficient: maximize total surplus (but all surplus is producer surplus)
      e. Result relies on monopolist knowing everything about consumer demand. So is the inefficiency of monopoly really a problem of market power or one of information? Could argue that it is an information problem.



3. Perfect discrimination part 2: Two-part tariffs
   a. Suppose consumer has typical downward sloping demand for a product *P*(*Q*)
   b. Monopolist charges a two-part tariff: $T(Q) = A + pQ$, where *A* is a fixed fee ("gate fee") and *p* is a price per unit
   c. By charging a non-linear two-part tariff, the monopolist can again extract all of the rent
   d. Monopolists problem:
   $$Max\ \ A + pQ - C(Q)$$
   $$s.t.\ \ A + pQ \leq \int_0^q P(x)dx$$
   e. The constraint is an example of an "individual rationality" constraint – the consumer will not purchase the good at all if they get negative utility from doing so.
   f. Suppose we make the constraint binding, then the problem is
   $$Max \int_0^Q P(x)dx - C(Q)$$
   $$\Rightarrow P(Q^*) - C'(Q^*) = 0$$
   where Q^* is the quantity at which price equals marginal cost.
   g. Set the efficient price: price equals marginal cost (maximize surplus)
   h. Then set *A* to make the constraint binding (have all of the surplus go to the monopolist): $A = \int_0^{Q^*} P(x)dx - P(Q^*)Q^*$

i. Total revenue earned by the monopolist:

$$A + pQ = \int_0^Q P(x)dx - pQ + pQ = \int_0^Q P(x)dx$$

j. Total profit: $\int_0^{Q^*} P(x)dx - C(Q^*)$

4. Multi-market price discrimination
   a. Question: Why do airlines charge different fares between city pairs depending on whether you stay over a Saturday night? There is no cost difference to them
   b. Suppose the monopolist produces a single product and sells it to different markets. Let market demand in market *i* be given by $q_i = D_i(P_i)$
   c. Profit maximizing problem for the multi-market monopolist:

$$Max \sum_{i=1}^{N} [P_i D_i(P_i)] - C\left(\sum_{i=1}^{N} D_i(P_i) \right)$$

$$\text{First order condition}: P_i D_i'(P_i) + D_i(P_i) - C'(.)D_i'(P_i) = 0 \ \ \textit{for all } i$$

$$\Rightarrow P_i - C'(.) = -\frac{D_i(P_i)}{D_i'(P_i)}$$

$$\Rightarrow \frac{P_i - C'(.)}{P_i} = -\frac{D_i(P_i)}{D_i'(P_i)P_i} = \frac{1}{\varepsilon_i}$$

   where ε_i is the elasticity of demand in market *i*.
   d. Profit maximizing solution is to charge the highest prices to markets with the most inelastic demand.
   e. Answer to question about Saturday stay: tourists have more elastic demand and they tend to stay over the weekend. Business travelers have inelastic demand and don't stay over the weekend

III. Oligopoly

A. Issues
1. Monopoly and perfect competition are polar opposite cases – but both are relatively simple
2. Oligopoly – must consider strategic behavior of firms
3. No single model of oligopoly: Do firms set prices (Bertrand)? Set quantities (Cournot)?
4. Are products of different firms identical (homogeneous products) or imperfect substitutes (heterogeneous products)
5. Rich area of study with many possibilities

B. Quantity setting: Cournot Oligopoly (Cournot 1838)
1. There are *N* firms indexed by *i*, *i* = 1,2, …, *N*
2. Each firm simultaneously chooses quantity, q_i

3. The outputs of all firms are perfect substitutes (homogeneous products). Total output: $Q = \sum_{i=1}^{N} q_i$
4. Payoff (profit) function:
   a. Market inverse demand function: $P(Q)$, with $P'(Q) < 0$
   b. Cost function for firm i: $C_i(q_i)$, $C'(.) > 0$, $C''(.) \geq 0$
   c. Profit: $\Pi_i(q_i, Q) = P(Q)q_i - C_i(q_i)$
5. Solving for Cournot (Nash) equilibrium:
   a. First order condition: $\frac{\partial \Pi_i(q_i, Q)}{\partial q_i} = P(Q) + P'(Q)q_i - C_i'(q_i) = 0$
   b. The first order condition is an implicit function that gives firm i's best response function $q_i^*(q_{\sim i})$. This is also called a reaction function.
   c. Cournot (Nash equilibrium): each firm plays a best response given others firms' strategies. $\Pi_i(q_i^*, q_{\sim i}^*) \geq \Pi_i(q_i, q_{\sim i}^*)$ for all $q_i \geq 0$, for all i
6. Oligopoly with linear demand, constant marginal cost
   a. Players: $i = 1, 2, \ldots, N$
   b. Strategies: $q_i \geq 0$
   c. Payoffs: $\Pi_i(q_i, Q_{\sim i}) = (a - q_i - Q_{\sim i})\, q_i - c\, q_i$
   d. Best response function: maximize payoffs for a given strategy of the rival player. First order condition (maximize objective with respect to choice holding other player's strategy constant):
      $\frac{\partial \pi_i(q_i, Q_{\sim i})}{\partial q_i} = a - 2q_i - Q_{\sim i} - c = 0$
      or $q_i(Q_{\sim i}) = (a - c - Q_{\sim i})/2$
   e. Nash equilibrium: must satisfy best response functions (N linear equations in N unknowns): $a - 2q_i - Q_{\sim i} - c = 0$ for all i
   f. Sum over the N firms:
$$\sum_{i=1}^{N}[a - 2q_i - Q_{\sim i} - c] = 0$$
$$\sum_{i=1}^{N}[a - q_i - Q - c] = 0$$
$$Na - \sum_{i=1}^{N} q_i + NQ - Nc = 0$$
$$Q = \frac{N(a-c)}{N+1}$$
   g. Use this to solve for q_i:
$$a - q_i - Q - c = 0$$
$$a - q_i - \frac{N(a-c)}{N+1} - c = 0$$

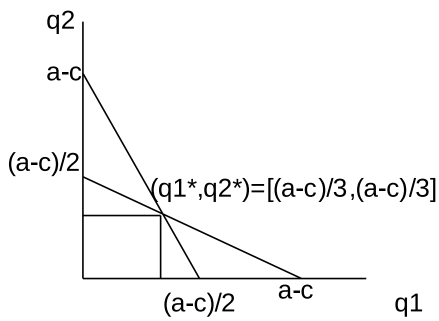
$$q_i = \frac{(a-c)}{N+1}$$

$q_1\,(q_2) = (a - c - q_2)/2$

$q_2\,(q_1) = (a - c - q_1)/2$

h. Case of duopoly (N=2):

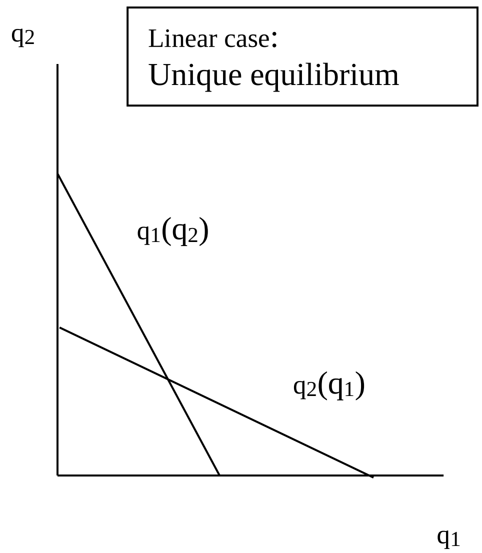
$q_1^* = q_2^* = (a - c)/3$

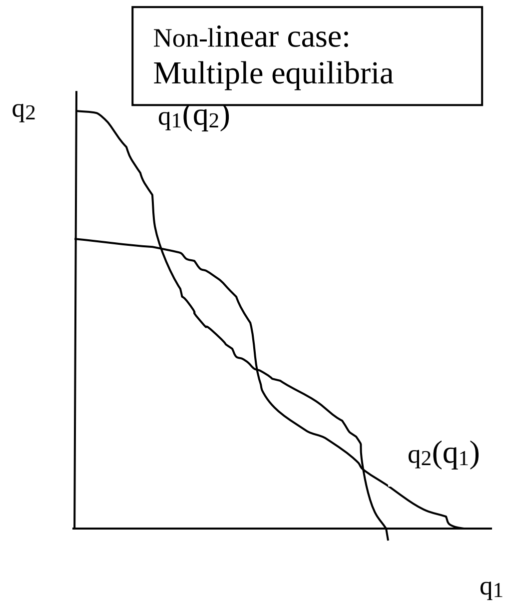


i. Note: Nash equilibrium is convincing once you are at the equilibrium. Once at equilibrium there are no profitable deviations (i.e., no player has an incentive to change strategy). But how do players get to this equilibrium in the first place? Adjustment to other players' strategy? But the game is static. A convincing answer of how players arrive at equilibrium depends on analysis of dynamics of non-equilibrium play or analysis of learning in games.

j. Can think about how firms might reason their way to equilibrium ("false dynamics"). Suppose one firm starts as a monopolist (produces (a-c)/2. The other firm starts to produce, its best response is (a-c)/4. But then the first firm will produce a bit less, but then the second firm will produce a bit more…until the firms converge to the Cournot-Nash equilibrium where each firm produces (a-c)/3.

7. Does Cournot equilibrium exist and is it unique?
   a. Existence: can use existence proof for Nash equilibrium
   b. Uniqueness: not necessarily unique. Do reaction functions cross only once?



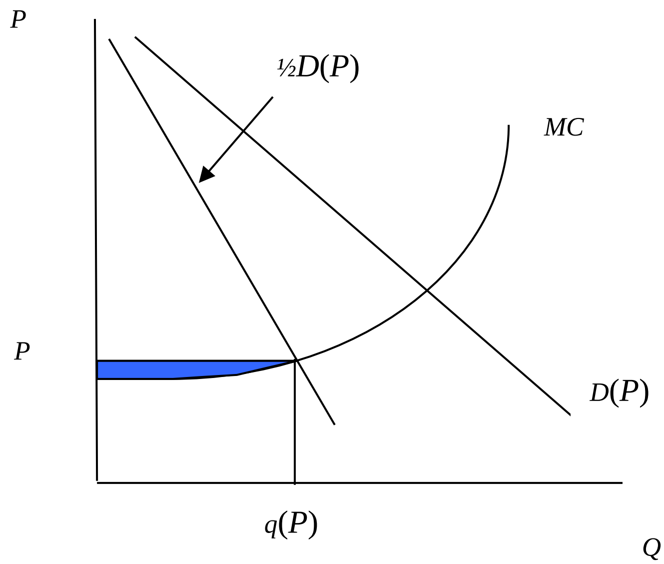


c. Sufficient condition for uniqueness: reaction function (best response function) for firm 1 is steeper than the reaction function for firm 2.
d. This will be true in the linear case as the absolute value of the slope of $q_1^*(q_2) = 2$, the absolute value of the slope of $q_2^*(q_1) = 1/2$
e. See Tirole p. 224- 226 for sufficient conditions for existence and uniqueness of Nash equilibrium in a Cournot game.

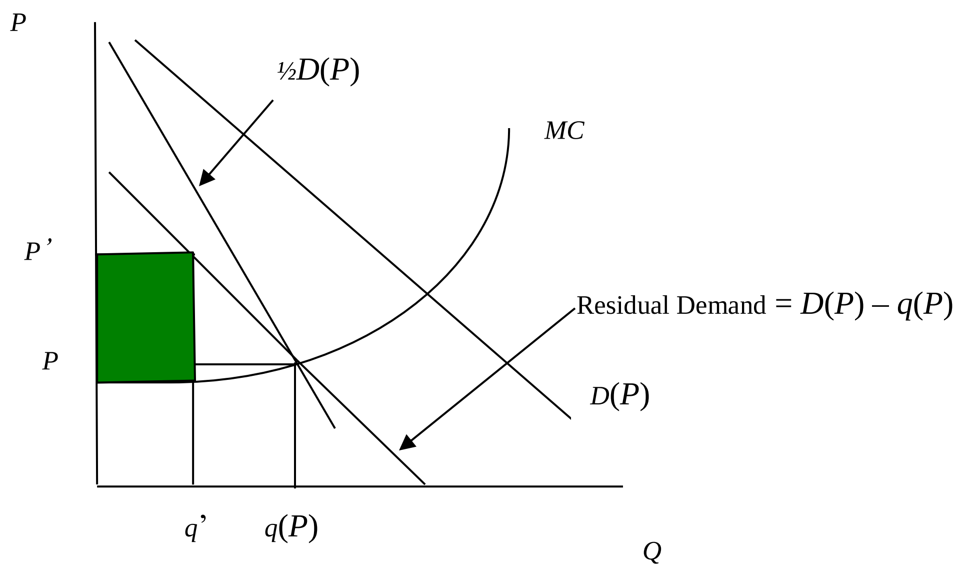
C. Bertrand equilibrium (Bertrand 1883)
1. There are N firms indexed by i, $i = 1,2, \ldots, N$
2. Each firm simultaneously chooses price, p_i
3. Each firm has constant marginal costs of production of c. Cost function: $C_i(q_i) = cq_i$.
4. $Q = D(p)$ is the market demand curve
5. The outputs of all firms are perfect substitutes (homogeneous products)
6. Define $p_j^{\min}$ as the minimum price of all $j \neq i$
7. Payoff (profit) function:
   a. Since products are perfect substitutes, consumers will choose on the basis of price alone. Lowest price firm will get all demand.
   b. $$\Pi_i(p_i, p_{\sim i}) = \begin{cases} (p_i - c)D(p_i) \text{ for } p_i < p_j^{\min} \\ (p_i - c)D(p_i)S_i \text{ for } p_i = p_j^{\min}, 0 \le S_i \le 1 \\ 0 \text{ for } p_i > p_j^{\min} \end{cases}$$
   where S_i is the market share for firm i when it ties with some other firm for lowest price
   c. Note that there is a discontinuity at $p_i = p_j$ (for lowest price among other firms)
8. Bertrand equilibrium

a. $p_i = p_j = c$ for at least two firms i and j and all firms $k = 1, 2, \ldots, N$, set price $p_k \geq c$
b. Proof:
   i. Suppose $p_j^{\min} > c$, then the best response of firm i is to set $p_i = p_j^{\min} - \varepsilon$ where ε is a small positive number
   ii. But then all other firms earn zero profit. The best response of some firm $j \neq i$ is to set p_j just below p_i.
   iii. But then firm i should set p_i just below p_j, and so on…
   iv. Firms will undercut for any price above c
   v. No firm has an incentive to set price below c
   vi. Bertrand (Nash) equilibrium: $p_i = p_j = c$ for at least two firms i and j and all firms $k = 1, 2, \ldots, N$, set price $p_k \geq c$
c. The Bertrand equilibrium is the same as the perfectly competitive outcome.
d. Bertrand paradox: even two firms is enough to get the competitive outcome

9. Solutions to the Bertrand paradox: why price competition might not lead to competitive equilibrium outcome
   a. Increasing marginal cost (see below)
   b. Product differentiation (see below)
   c. Repeated play (will talk about this with dynamic games)
10. Price competition with increasing marginal costs
   a. Price equal to marginal cost is no longer the Nash equilibrium solution
   b. Example: consider a duopoly where each firm has identical costs, and where marginal costs rise with production. Suppose firms start by setting $p_i = p_j = P = C'(D(P)/2)$. Define $q(P) = \frac{1}{2}D(P)$.

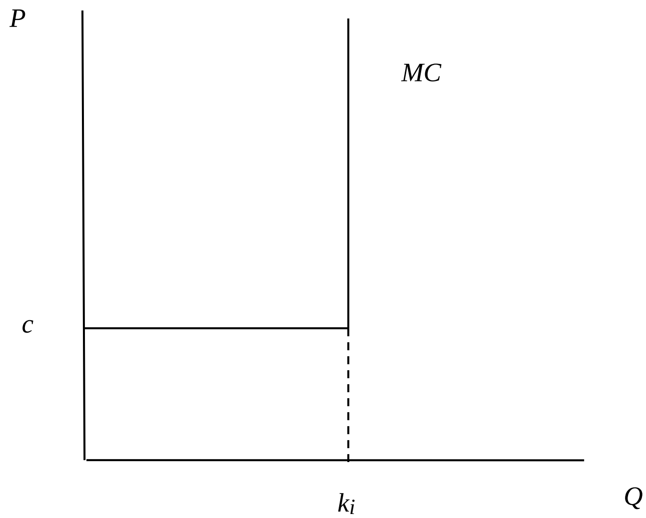


c. Note: following this strategy gives each firm profit equal to the blue shaded area.
d. Now suppose that one of the firms raises its price. The lower price firm will not choose to serve all demand because doing so would cause it to lose money so suppose it continues to sell $q(P)$. Subtracting this amount of demand from market demand yields residual demand: $D(P) - q(P)$
e. Now the higher price firm earns profits equal to the area shaded in green by serving the residual demand curve customers. This profit is higher than what it earned before, so the original strategy is not a Nash equilibrium strategy.



f. With price competition and rising marginal costs there is no pure strategy Nash equilibrium – only mixed strategy Nash equilibrium

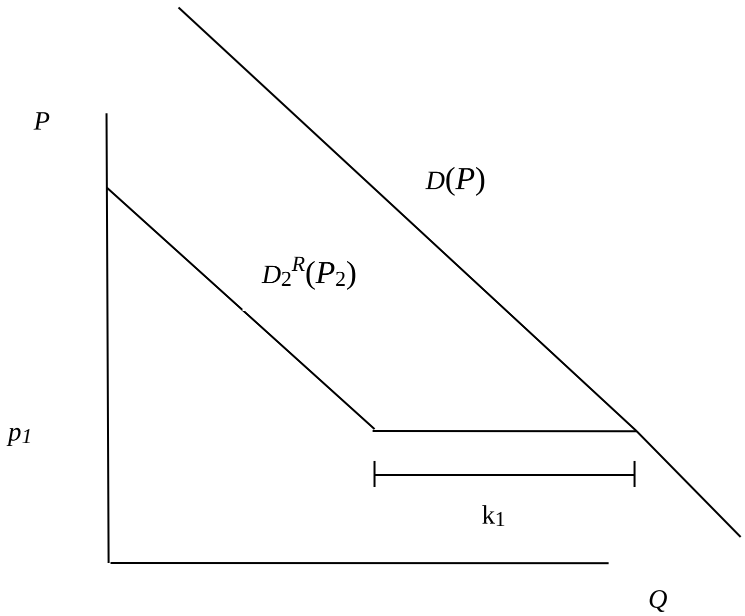
8. Capacity constraints: Edgeworth variant of the Bertrand model
   a. Restrict attention to a duopoly game
   b. Suppose both firms have constant marginal cost (c) up to a capacity constraint, k_i, $i = 1, 2$.



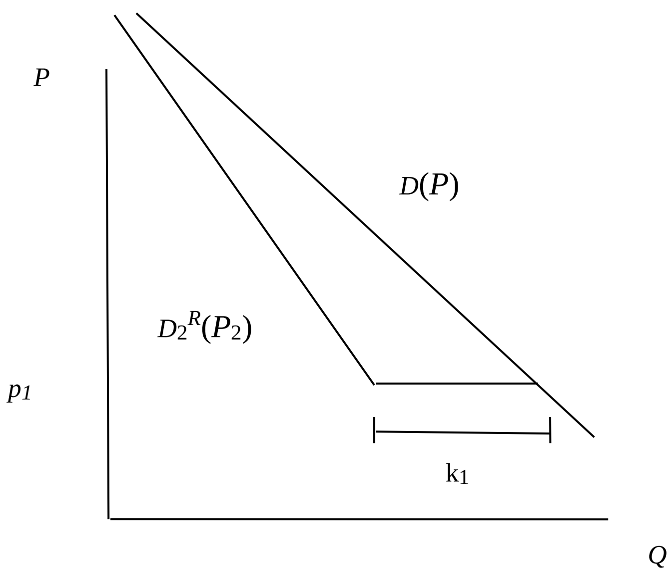
c. Rationing rules: if rival firms are producing at capacity and cannot satisfy all demand, there will be residual demand. Firm with the higher price can still serve some customers. Can have variety of rationing rules, which determine which consumers go to which firms: efficient rationing rule, proportional rationing rule…

d. Efficient rationing rule: suppose $p_1 < p_2$. Residual demand for firm 2 is defined as $D_2^R(p_2) = \begin{cases} D_2(p_2) - k_1 \text{ for } D_2(p_2) \geq k_1 \\ 0 \text{ otherwise} \end{cases}$.

   Efficient rationing assumes that consumers queue in order of willingness-to-pay (so consumers with the highest willingness-to-pay buy first from the low price firm)



e. Proportional rationing rule: $D_2^R(p_2) = D_2(p_2)\left(\frac{D(p_1) - k_1}{D(p_1)}\right)$. Random assignment of consumers to each firm. Inefficient rule as some consumers with lower willingness-to-pay will be served compared to some consumers who are not served.



f. Which rationing rule holds depends upon context (queue or random assignment)
g. Analysis of capacity constrained price setting game
   i. For simplicity, assume $D(P) = 1 - P$, $c = 0$

ii. Small capacity case: $k_1 = k_2 < 1/3$ (Cournot outcome with these assumptions is (1/3, 1/3)). Equilibrium $p_1 = p_2 = p = 1 - k_1 - k_2$. Profit: $\Pi_i = (1 - k_1 - k_2)k_i$. Setting a lower price doesn't change sales (capacity constrained) so it just lowers profit. Setting a higher price would reduce sales but gain a higher price – but we know from Cournot analysis that best response when $q < 1/3$ for both firms is to raise output (lower price).

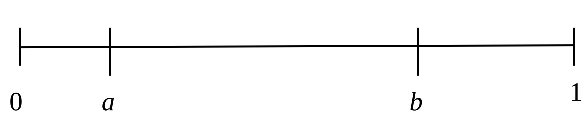
iii. Large capacity case: $1/3 < k_1 = k_2 < 1$. There does not exist a pure strategy Nash equilibrium.

iv. Example of the "Edgeworth cycle". Suppose that $k_1 = k_2 = ½$.
- Suppose we start with $p_1 = p_2 = 0$.
- If the other firm charges 0 it will sell its capacity of ½, residual demand (assuming efficient rationing) is
  $D_i^R(p_2) = 1 - (1/2) - p_2 = 1/2 - p_2$
- Firm i is a monopolist on the residual demand curve. The monopoly solution is $p_i = ¼$, $q_i = ¼$ Profit is $1/16 > 0$
- But if firm i sets $p_i = ¼$, then firm j should set $p_j = ¼ - \varepsilon$, but then firm i should undercut firm j…
- As price goes down to $p = 1/8$, so that profit falls to ½ * 1/8 = 1/16, then it is better to raise price up to $p_i = ¼$ and be the monopolist on the residual demand curve
- Edgeworth cycles – would expect price to cycle between ¼ and 1/8
- There is no pure strategy Nash equilibrium

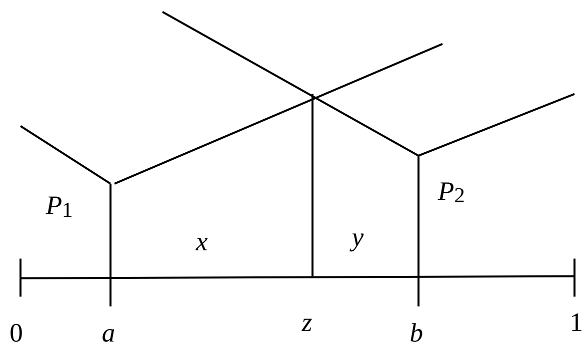
h. Kreps-Scheinkman (Bell Journal 1983): *Quantity precommitment and Bertrand competition yield Cournot outcomes*. Two stage game – first stage firms choose capacity, second stage – firms choose prices. Equilibrium for this model is to choose Cournot capacity and price so that produce at capacity.

D. Product differentiation: Hotelling Model of Spatial Competition

1. Introduction
   a. Most consumer products are not homogeneous but have characteristics that vary from brand to brand
   b. Example: cell phones, cars, computers, airlines…
   c. Brand loyalty – buy one brand even if that brand has a higher price if it has the right characteristics
2. Model setup
   a. Consumers are uniformly spread along a line of unit length. (Think of location as representing a measure of a product characteristic.) The density of consumers is one per unit length
   b. Two firms are located on the line one at a, the other at b.



c. Firms choose prices simultaneously: p_1, p_2
d. For simplicity, assume that production costs are 0 for both firms
e. Consumers like to consume a product that is close to them in space (matches more closely with their preferences on characteristics). They pay a cost from consuming a product that is not their ideal product. Let d be the distance from the consumer to the firm and let c be the cost to travel per unit distance (a measure of brand loyalty or preference for characteristics).
f. Total price for the consumer pays is price plus travel cost: $p + cd$
g. Consumers buy from the firm with the lowest overall price. Consumers to the left of z buy from firm 1 and to the right of z from firm 2.
h. Define the distance from a to z as x and the distance from z to b as y.

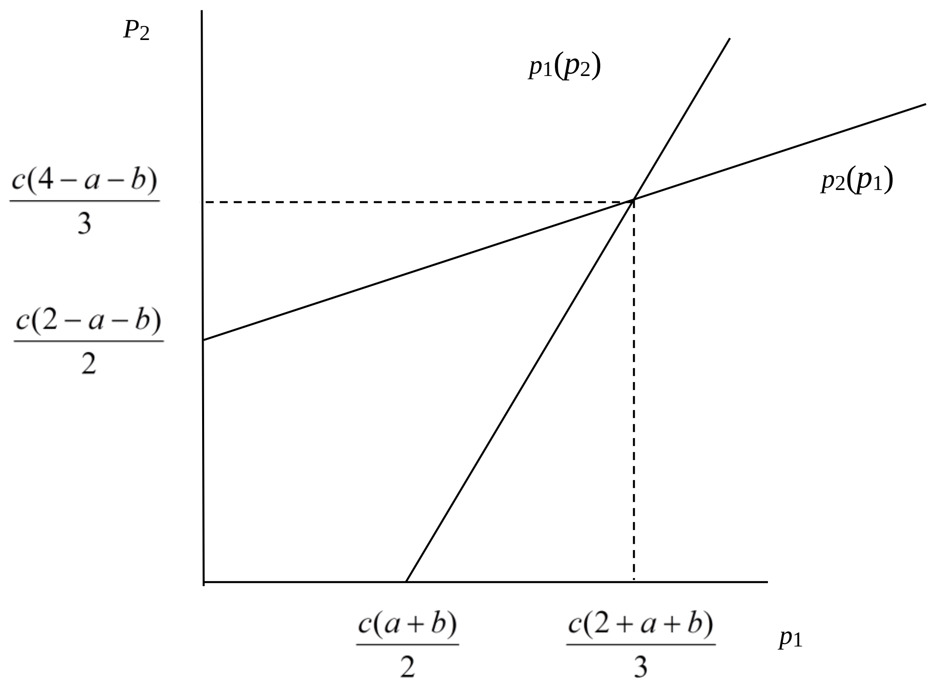


3. Analysis of Nash Equilibrium (assuming an interior solution)
a. Solving for market division point (z): at z, the price to consumer is equal so that we have $p_1 + cx = p_2 + cy$.
b. Note that $y = b - a - x$.
c. Substituting for y: $p_1 + cx = p_2 + c[b - a - x]$.
d. Then solving for x: $x = \dfrac{p_2 - p_1 + c(b-a)}{2c}$
e. The profit function for firm 1 is:
$$\Pi_1(p_1, p_2) = p_1(a + x) = p_1[a + \frac{p_2 - p_1 + c(b-a)}{2c}]$$
f. Take the derivative with respect to p_1 and simplify to find the best response function for firm 1: $p_1(p_2) = \dfrac{c(b+a) + p_2}{2}$
g. For firm 2, we want to follow a similar procedure. First note that $x = b - a - y$
h. Substituting for x: $p_1 + c[b - a - y] = p_2 + cy$.
i. Then solving for y: $y = \dfrac{p_1 - p_2 + c(b-a)}{2c}$
j. The profit function for firm 2 is:
$$\Pi_2(p_1, p_2) = p_2(y + 1 - b) = p_2[\frac{p_1 - p_2 + c(b-a)}{2c} + 1 - b]$$

k. Take the derivative with respect to p_2 and simplify to find the best response function for firm 2: $p_2(p_1) = \dfrac{c(2-b-a)+p_1}{2}$

l. Nash equilibrium: find the place where the best response functions cross.
$p_1{*} = \dfrac{c(2+a+b)}{3}$; $p_2{*} = \dfrac{c(4-a-b)}{3}$

m. Note that when $c = 0$, we are back to the homogeneous Bertrand model and $p_1 = p_2 = 0$.



4. The analysis above assumes that there is an interior solution (i.e., that firms split the market) but it is possible for one firm to undercut the market and take over the entire market.
    a. Firm 1 would take over the entire market if $p_1 + c(x+y) < p_2$
    $p_1 < p_2 - c(1 - a - b)$
    b. Profit for firm 1 when it takes over the entire market is simply p_1
    c. In order for the Nash equilibrium solution solved for above to be a Nash equilibrium one must check to see that it is not profitable to lower price and undercut. The logic is straightforward but the algebra is messy.
    d. One can show (with a bit of messy algebra) that there is interior Nash equilibrium as long as the two firms are not located too close together (i.e., if there is sufficient product differentiation). However, if the firms are quite close together then there is no pure strategy Nash equilibrium.

E. Summary
   1. Simple models of strategic competition among firms
   2. Cournot equilibrium (quantity competition) and Bertrand (price competition) are both examples of Nash equilibrium – best response given other firms' strategies
   3. Do firms choose prices or quantities in reality? More common to think of firms choosing price, but reality may be more complex (airlines – complicated pricing strategy depending on how fast the plane is filling up)
   4. We will return to oligopoly with more complex strategies when we cover dynamic games and games with incomplete information