AFRE 802
Statistical Methods for Agricultural, Food, & Resource Economists

Linear models & estimation by least squares – Part 1 of 3
(WMS Ch. 11.1-11.3, Wooldridge pp. 22-37)

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GAME PLAN

- Collect Ch. 10 HW
- Hand out graded exercise (due Thursday)

- Linear models & estimation by least squares – Part 1 of 3
  - The simple linear regression model
    - Examples
    - Terminology
    - Assumptions about the error term (and concept of endogeneity)
    - Deriving estimates of the simple linear regression parameters: ordinary least squares (OLS)
    - Compute OLS estimates by hand & in Stata

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Simple Linear Regression

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The simple linear regression model: motivation

- Suppose *y* and *x* are two variables that represent some population
- **How does *y* change when *x* changes? What is the causal effect (*ceteris paribus* effect) of *x* on *y*?**
- *Examples?*

y	*x*
Corn yield	Fertilizer
Beef demand	Beef price
Wheat acreage	Wheat price
Hourly wage	Years of education
Community crime rate	# of police officers

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The simple linear regression model

$$y = \beta_0 + \beta_1 x + u$$

β_0 (intercept) and **β_1 (slope)** are the population **parameters to be estimated**

- u is the **error term** or **disturbance**
  - u for "**unobserved**"
  - Represents all factors other than x that affect y
  - Some use ε instead of u
- Terminology for y and x:

y	*x*
Dependent variable	Independent variable
Explained variable	Explanatory variable
Response variable	Control variable
Predicted variable	Predictor variable
Regressand	Regressor
	Covariate

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To use data to get unbiased estimates of β_0 *and* β_1, need to restrict the relationship b/w x and u

$$y = \beta_0 + \beta_1 x + u$$

1. $E(u) = 0$ (not restrictive if have an intercept, β_0)
2. *** $E(u|x) = E(u)$ (i.e. the average value of u does not depend on the value of x)

#1 & #2 → $\boxed{E(u|x) = E(u) = 0 \quad \text{(zero conditional mean)}}$

- If this holds, **x is "exogenous"**; but **if x is correlated with u, x is "endogenous"** (next class)

What does this assumption imply below?

- $yield = \beta_0 + \beta_1 fertilizer + u$, where u is unobserved land quality (*inter alia*)
- $wage = \beta_0 + \beta_1 educ + u$, where u is unobserved ability (*inter alia*)

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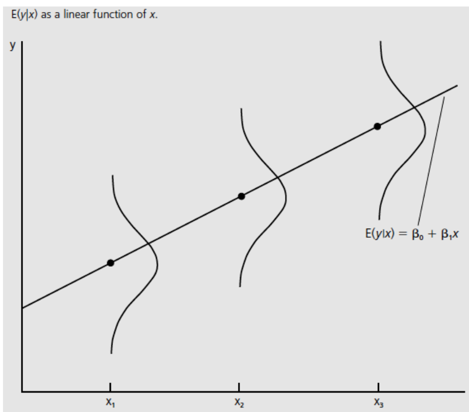
What is E(y|x) if we assume E(u|x)=0?

Hint: Apply the rules for conditional expectations.

$y = \beta_0 + \beta_1 x + u$

$$\boxed{E(y|x) = \beta_0 + \beta_1 x}$$



Source: Wooldridge (2003)

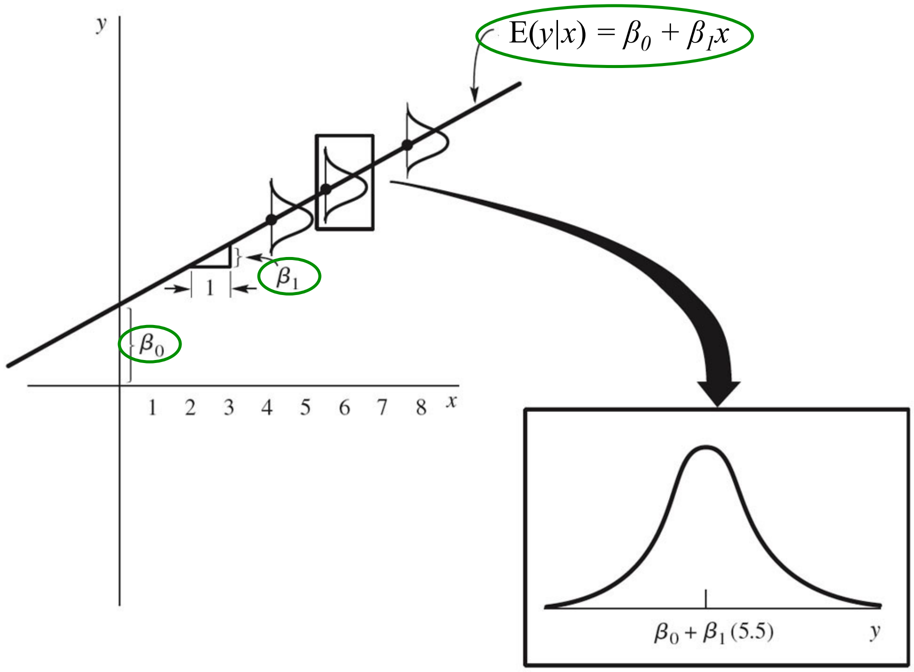
What is $\frac{\partial E(y|x)}{\partial x}$ *and how do we interpret this result?*

$$\frac{\partial E(y|x)}{\partial x} = \beta_1$$

Interpretation: β_1 is the expected change in y given a one unit increase in x, *ceteris paribus* (slope)

What is the interpretation of β_0*?*

Interpretation: β_0 is the expected value of y when $x = 0$ (intercept)



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Why is it called linear regression?

$$y = \beta_0 + \beta_1 x + u$$

- **Linear in parameters**, β_0 and β_1
- Does NOT limit us to linear relationships between x and y
- But rules out models that are nonlinear in parameters, e.g.:

$$y = \frac{1}{\beta_0 + \beta_1 x} + u$$

$$y = \Phi\left(\beta_0 + \beta_1 x\right) + u$$

$$y = \frac{\beta_0}{\beta_1} x + u$$

Estimating β_0 and β_1

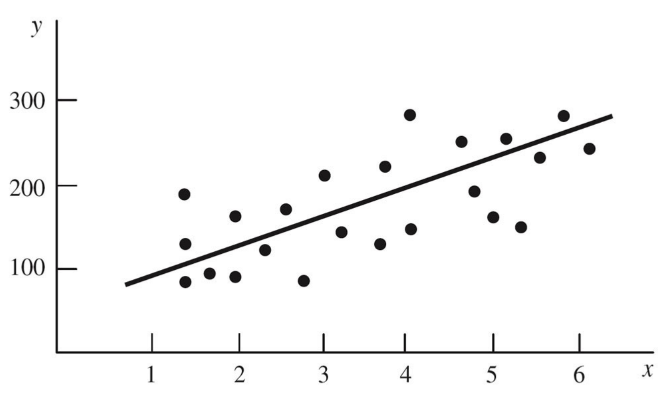
$$y = \beta_0 + \beta_1 x + u$$

- Suppose we have a random sample of size N from the population of interest. Then can write:

$y_i = \beta_0 + \beta_1 x_i + u_i, \quad i = 1, 2, 3, \ldots, N$

We don't know β_0 and β_1 but want to estimate them.

How could we use the data in our sample to estimate β_0 and β_1?



Recall 3 common methods of estimation

1. Method of moments
2. Maximum likelihood
3. Least squares

- All 3 of these approaches lead to the same estimators for β_0 and β_1 (under certain assumptions)
- We'll focus on the least squares approach
- See Wooldridge (2003: 27-29) for method of moments discussion and his panel data book for MLE discussion

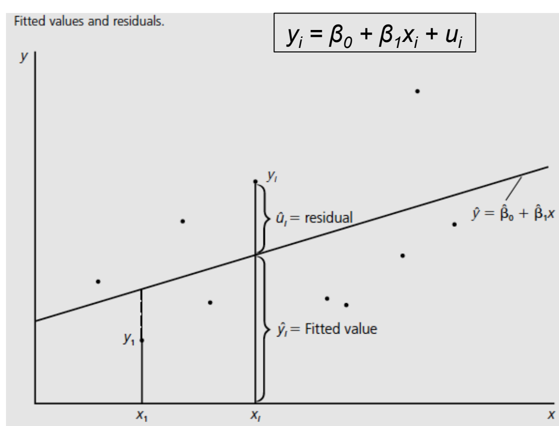
(Ordinary) least squares (OLS) approach

- *What was the gist of this approach?*
  - Choose estimator to minimize the sum of squared deviations b/w observed & estimated values
- "Fitted" values of y and residuals:

Fitted (estimated, predicted) values of y: $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$

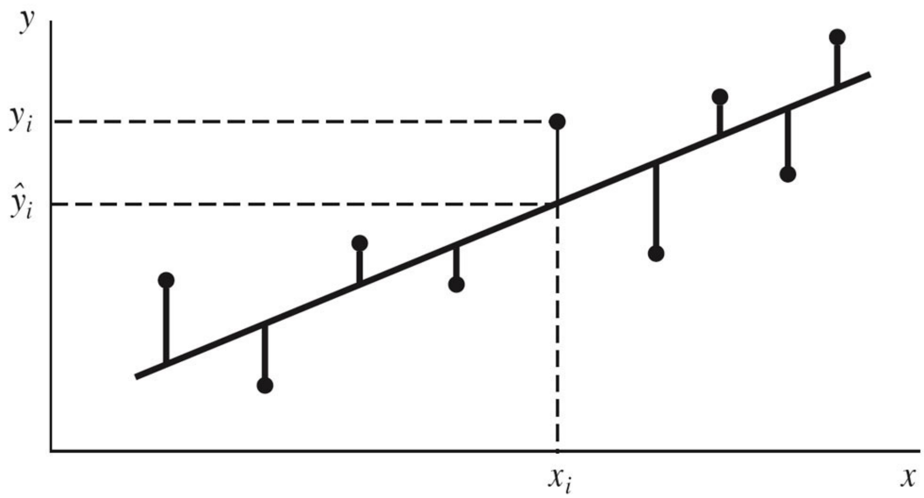
Residuals:
$$\hat{u}_i = y_i - \hat{y}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$$

OLS:
Choose $\hat{\beta}_0$ and $\hat{\beta}_1$ to minimize:
$$\sum_{i=1}^{N} \hat{u}_i^2 = \sum_{i=1}^{N} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i\right)^2$$



Source: Wooldridge (2003)

Where are the fitted values and residuals in this figure?



The OLS estimators for β_0 and β_1

$$\boxed{\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2}} \qquad \boxed{\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}}$$

Work through proof of formulas above. **This is a proof you should know.**

Another useful expression for $\hat{\beta}_1$:

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{N} x_i y_i - \frac{1}{N}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2}$$

Proof will be posted to D2L.

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Obtaining OLS estimates – example (by hand)

EXAMPLE 11.1

Use the method of least squares to fit a straight line to the $n = 5$ data points given in Table 11.1.

$$\boxed{\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^{N}(x_i - \overline{x})^2} = \frac{\sum_{i=1}^{N} x_i y_i - \frac{1}{N}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2}}$$

$$\boxed{\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}}$$

Table 11.1 Data for Example 11.1

x	y
−2	0
−1	0
0	1
1	1
2	3

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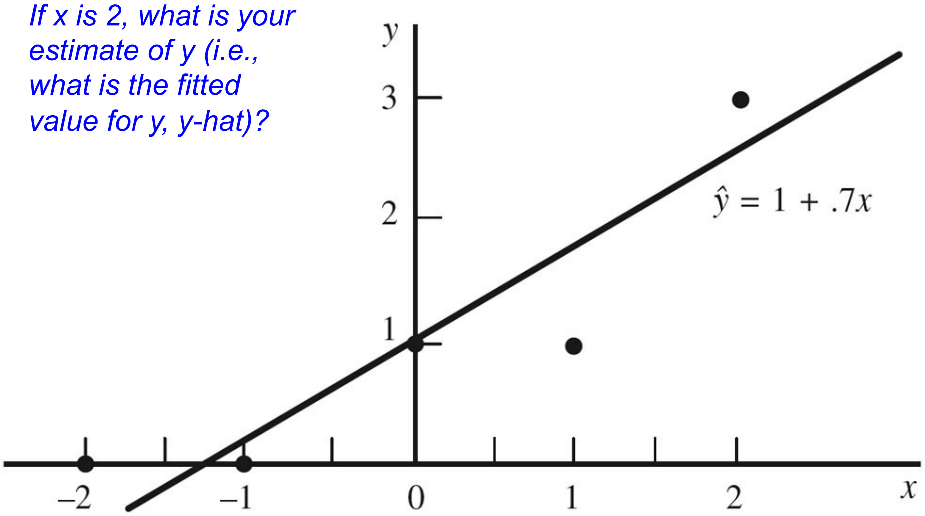
If x is 2, what is your estimate of y (i.e., what is the fitted value for y, y-hat)?



Calculations using alternative formula

Table 11.2 Calculations for finding the coefficients

x_i	y_i	$x_i y_i$	x_i^2
-2	0	0	4
-1	0	0	1
0	1	0	0
1	1	1	1
2	3	6	4
$\sum_{i=1}^{n} x_i = 0$	$\sum_{i=1}^{n} y_i = 5$	$\sum_{i=1}^{n} x_i y_i = 7$	$\sum_{i=1}^{n} x_i^2 = 10$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{N}(x_i - \bar{x})^2} = \frac{\sum_{i=1}^{N} x_i y_i - \frac{1}{N}\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sum_{i=1}^{N} x_i^2 - \frac{1}{N}\left(\sum_{i=1}^{N} x_i\right)^2} = \frac{7 - \frac{1}{5}(0)(5)}{10 - \frac{1}{5}(0)^2} = 0.7$$

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Basic Stata Commands

- **regress** *y x* Linear regression of *y* on *x*
  - EX) regress wage educ
- **predict** *newvar1*, **xb** Compute fitted values
  - EX) predict wagehat, xb (I just made up the name wagehat)
- **predict** *newvar2*, **resid** Compute residuals
  - EX) predict uhat, resid (I just made up the name uhat)

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Obtaining OLS estimates – example (Stata)

Wooldridge (2003) Example 2.4: Wage and education
Use Stata to run the simple linear regression of wage (*y*) on educ (*x*).

$$\boxed{wage_i = \beta_0 + \beta_1 educ_i + u_i}$$

Command: regress wage educ (or: reg wage educ)

What are β_0 and β_1 below?

```
reg wage educ

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  1,   524) =  103.36
       Model |  1179.73204     1  1179.73204           Prob > F      =  0.0000
    Residual |  5980.68225   524  11.4135158           R-squared     =  0.1648
-------------+------------------------------           Adj R-squared =  0.1632
       Total |  7160.41429   525  13.6388844           Root MSE      =  3.3784

------------------------------------------------------------------------------
        wage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        educ |   .5413593    .053248    10.17   0.000     .4367534    .6459651
       _cons |  -.9048516   .6849678    -1.32   0.187    -2.250472    .4407687
------------------------------------------------------------------------------
```

$\boxed{\hat{\beta}_1}$ $\boxed{\hat{\beta}_0}$

Homework:

- WMS Ch. 11:
  - 11.1, 11.3 (do calculations by hand)
  - 11.4 and 11.5 (for these two problems, do calculations using Excel and the formulas on slide 17, and in Stata using the "regress" command; the data are on D2L in the Stata folder)
- Try to complete all Ch. 11 HW before last day of class (Dec. 8) so that we can go over it then. (You won't turn in Ch. 11.)

Next class:

- Linear regression part 2 of 3
  - Properties of OLS estimators

Reading for next class:

- WMS Ch. 11: section 11.4
- Wooldridge *Introductory Econometrics* (2003): pp. 38-60, 101-102

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Aside: NPR "Hidden Brain" example of a natural experiment, and when it might be reasonable to assume E(u|x)=E(u)

- Listen for the following:
  - *What is the dependent variable?*
  - *What is the main explanatory variable of interest?*
  - *Why might it be reasonable to assume E(u|x)=E(u) here?*
  - *What is a natural experiment?*
- Dependent variable: cognitive function of elderly
- Main explanatory variable: wealth
- E(u|x)=E(u) might be reasonable – Congress computational mistake – people in one cohort got higher benefits that next cohort (level of benefits shouldn't be correlated with unobservables)

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Aside: Natural experiments

A natural experiment occurs when some exogenous event—often a change in government policy—changes the environment in which individuals, families, firms, or cities operate. A natural experiment always has a control group, which is not affected by the policy change, and a treatment group, which is thought to be affected by the policy change. Unlike with a true experiment, where treatment and control groups are randomly and explicitly chosen, the control and treatment groups in natural experiments arise from the particular policy change. (Wooldridge, 2003: 417)

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