## Department of Agricultural & Applied Economics Microeconomics Qualifying Exam

May 27, 2011 10:00 a.m. - 3:00 p.m.

- 1. The theory of consumer behavior gives us several theorems. Such as:
  - a. Slutsky equation.
  - b. Homogeneity condition.
  - c. Symmetry condition.
  - d. Engel aggregation condition for income elasticities.

For each theorem, you should complete the following tasks:

(i) Specify algebraically what each theorem states.

(ii) Explain in words what you think the algebra says.

- (iii) Discuss explicitly how important and what useful role each theorem plays in empirical analysis conducted by the applied economists when actually estimating demand functions.
- 2. Consider a price-taking firm that produces an output q using inputs  $z_1$  and  $z_2$  according to the production function  $f(z_1, z_2) = \sqrt{\min\{\alpha_1 z_1, \alpha_2 z_2\}}$  where  $\alpha_i > 0, i = 1, 2$ . Let p be the output price and  $w_1, w_2$  denote input prices.
  - a. Derive the cost function of this firm.
  - b. Derive the profit function of this firm.
  - c. Derive the firm's supply function q(p). What is the sufficient condition for  $q^*$  to be a maximum?
  - d. Determine whether the technology of this firm displays increasing, constant, or decreasing returns to scale.
  - e. Suppose that initially  $\alpha_1=\alpha_2=1$  and a new technology becomes available such that  $\alpha_1 = 2$ . What is the maximum amount that the producer is willing to pay for access to this technology?
- 3. Consider a pure exchange economy consisting of two-consumers (denoted A and B) and two goods (denoted  $x_1$  and  $x_2$ ). Preferences and initial endowments for each consumer are given by

$$U^A(x_1^A, x_2^A) = (x_1^A x_2^A)^2$$

$$(e_1^A, e_2^A) = (4,4)$$

$$U^{B}(x_{1}^{B}, x_{2}^{B}) = \ln(x_{1}^{B}) + 2\ln(x_{2}^{B})$$
  $(e_{1}^{B}, e_{2}^{B}) = (1.6)$ 

$$(e_1^B, e_2^B) = (1,6)$$

- a. Solve for the set of Pareto-efficient allocations in this economy.
- b. Solve for the Walrasian equilibrium in this economy.
- c. Solve for the commodity allocations that would maximize social welfare under a Nietzschean social welfare function  $W = max(U^A, U^B)$ .
- d. Graph an Edgeworth box for this economy. Label indifference curves for each agent, initial endowments, the Walrasian equilibrium, the contract curve, and the core.

4. Assume two competing firms selling a homogeneous product. The market price, P, is determined by (inverse) market demand:

$$P = a - bQ$$
, if  $a > bQ$ ,  $P = 0$  otherwise,

where  $Q = (q_1 + q_2)$  is total output. The cost function for each firm is represented by:

$$C_i = c^0 q_i + d$$
,  $i = 1, 2$ .

If we let a = 14, b = 1,  $c^0 = 2$ , and d = 5, answer the following questions (show all work):

- a. What are the Cournot reaction functions?
- b. What are the Cournot equilibrium outputs, price(s), firm profits, consumer surplus, and deadweight loss?
- c. If firm one is a leader and firm two a follower, what are the Stackelberg equilibrium outputs, price(s), firm profits, consumer surplus, and deadweight loss?
- d. If the two firms collude, what is the equilibrium total output, price, and total profit?
- 5. Assume that in an economy there is one private good, Q (that is rival and exclusive in consumption), and one public good, y (that is non-rival and non-exclusive in consumption).
  - a. Define what is meant by a rival, exclusive private good and give an example.
  - b. Define what is meant by a non-rival, non-exclusive public good and give an example.
  - c. Suppose the production possibilities curve for this economy is given by:

 $y^2 + Q^2 = 320,000$  and the economy has 100 households with identical preferences for

the public and private goods given by:  $U = x_j y$ , j = 1, 2, ..., 100, where  $Q = \sum_{j=1}^{100} x_j$ .

Determine the optimal levels of Q and y.