# MARKETS & MARKET POWER

**APEC 8002, Fall 2015** **Instructor: Terry Hurley**

## INTRODUCTION

Now that we have characterized both consumer and producer behavior it is time to put them together in a market in order to reinforce a range of important results regarding prices and quantities, most of which should be familiar. Our analysis will focus on one or a couple of markets at a time, so it is referred to as partial equilibrium analysis. The alternative to partial equilibrium analysis is general equilibrium analysis, which considers all markets at once and will be taken up more in APEC 8004. While partial equilibrium analysis is more tractable in terms of results, it is important to remember that it can miss indirect effects that can only be captured in general equilibrium analysis. Still, many economists have and do find the partial equilibrium framework quite useful for their research.

We will begin our analysis with the simplest model that assumes perfectly competitive markets. The basic idea behind a perfectly competitive market is that there are enough consumers and producers such that no one producer or consumer can influence price on their own. We will then explore what happens when we relax this assumption to allow for market power in the form of monopoly or monopsony. With a monopoly, you have a single seller that realizes how much it chooses to sell can affect the price it receives. With a monopsony, you have a single buyer that realizes how much it buys can affect the price it pays. After exploring these two extreme benchmarks for characterizing how resources are allocated, we will turn to a variety of more intermediate cases: oligopoly markets, rent seeking, and bilateral bargaining. These more intermediate cases are best described and analyzed in the context of games of strategy, which is what we will spend our time on in APEC 8003.

## PERFECT COMPETITION

Suppose we have $i = 1,\ldots, I$ consumers. In APEC 8001, you established the conditions under which we can derive individual Marshallian demands that depend on the prices of commodities and individual wealth: $\mathbf{x}^i(\mathbf{p}, w_i)$. We can aggregate these individual demands by simply summing: $\mathbf{x}(\mathbf{p},\mathbf{w}) = \sum_{i=1}^{I}\mathbf{x}^i(\mathbf{p},w_i)$ where $\mathbf{w} = (w_1, \ldots, w_I)$. In general, the aggregate demand will depend on individual tastes, prices and the distribution of income; though there are restrictive circumstances when it is possible to write aggregate demand as a function of aggregate income.

Suppose we also have $j = 1,\ldots, J$ producers. We have established the conditions under which we can derive individual supply functions that depend on technology, commodity prices, and other factors: $\mathbf{y}^j(\mathbf{p})$. We can also aggregate these individual supplies by simply summing: $\mathbf{y}(\mathbf{p}) = \sum_{j=1}^{J} \mathbf{y}^j(\mathbf{p})$.

In a partial equilibrium analysis, we are concerned with a single market, and price and quantity determination in this market. We will pick the market for commodity $l$. Note that at this level of generality, some producers may actually be consumers of the commodity. To keep things more transparent, we will stick everything but the price of the commodity of interest into a vector $\boldsymbol{\alpha}$ and get rid of the $l$ subscript, so we can write aggregate demand as $x(p, \boldsymbol{\alpha})$ and aggregate supply as $y(p, \boldsymbol{\alpha})$. A partial equilibrium is defined by the conditions of no excess supply and no excess demand, which means the equilibrium price $p^*$ solves $x(p^*, \boldsymbol{\alpha}) = y(p^*, \boldsymbol{\alpha})$ and the equilibrium quantity is $q^* = x(p^*, \boldsymbol{\alpha}) = y(p^*, \boldsymbol{\alpha})$. Now we can sit around and ponder how individuals in a market ultimately come to $p^*$ and $q^*$, but the fact that a market will gravitate to this equilibrium is now fairly well established in the experimental literature under a broad range of institutional rules thanks to the contributions of Vernon Smith and those who followed. There also has been lots of theorizing around this point that we will not go into, but MWG and Varian do if you are interested.

Often times, we are interested in how an exogenous change in factors underlying a market like the distribution of income will influence the equilibrium price and quantity. To answer this question, we can appeal to the implicit function theorem:

**PC1** $$\frac{dp^*}{d\alpha_h} = -\frac{\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial \alpha_h} - \frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial \alpha_h}}{\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial p} - \frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial p}}$$

where $\alpha_h$ might be the price of another commodity, aggregate income or some other factor related to individual utility or production possibilities. Note that $y$ in our context is a net output because consumer theory assumes $x \geq 0$. Therefore, we know $\frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial p} \geq 0$. The sign of $\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial p}$ is complicated by wealth effects, but if the commodity is normal in the aggregate, it will be negative. Therefore, the sign of $\frac{dp^*}{d\alpha_h}$ for a normal good depends on the numerator. The first term in the numerator is what we tell principle students is a shift in demand due to a change in $\alpha_h$, while the second term is a shift in supply. Note that if $\alpha_h$ relates only to production, then $\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial \alpha_h} = 0$. Alternatively, if it relates to individual wealth or preferences, then $\frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial \alpha_h} = 0$. To figure out how the equilibrium quantity will change, we can differentiate $q^* = x(p^*, \boldsymbol{\alpha})$ with respect to $q^*$ and $\alpha_h$:

**PC2**

$$\frac{dq^*}{d\alpha_h} = \frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial \alpha_h} + \frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial p}\frac{dp^*}{d\alpha_h}$$

$$= \frac{\frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial \alpha_h}\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial p} - \frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial p}\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial \alpha_h}}{\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial p} - \frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial p}}$$

$$= \frac{\varepsilon_x \frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial \alpha_h} + \varepsilon_y \frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial \alpha_h}}{\varepsilon_x + \varepsilon_y}$$

where $\varepsilon_x = \left|\frac{\partial x(p^*,\boldsymbol{\alpha})}{\partial p}\frac{p}{x(p^*,\boldsymbol{\alpha})}\right|$ and $\varepsilon_y = \frac{\partial y(p^*,\boldsymbol{\alpha})}{\partial p}\frac{p}{y(p^*,\boldsymbol{\alpha})}$ are the own-price elasticities of demand and supply.

We could have also differentiated $q^* = y(p^*, \boldsymbol{\alpha})$ with respect to $q^*$ and $\alpha_h$, which would have yielded the exact same result. The sign of equation PC2 will be opposite the sign of the numerator. Intuitively, the first term in the numerator reflects a shift in the supply function and movement on the demand function, while the second term captures a shift in the demand function and movement along the supply function. Again, if the commodity is normal, we know the direction of the movements along the demand and supply, so it is the shift in demand and supply that confound the result.

Equations PC1 and PC2 give us the foundation for predicting changes in price and quantity based on exogenous market shocks that we all learned in our first economics class. These results are summarized in the table below.

<table>
<tr><td></td><td></td><td colspan="2">$\frac{\partial y(p^*,\alpha)}{\partial \alpha_h}$</td></tr>
<tr><td></td><td></td><td>> 0<br>(Supply Shift Out)</td><td>< 0<br>(Supply Shift In)</td></tr>
<tr><td rowspan="2">$\frac{\partial x(p^*,\alpha)}{\partial \alpha_h}$</td><td>> 0<br>(Demand Shift Out)</td><td>$\frac{dp^*}{d\alpha_h} = ?$ and $\frac{dq^*}{d\alpha_h} > 0$</td><td>$\frac{dp^*}{d\alpha_h} > 0$ and $\frac{dq^*}{d\alpha_h} = ?$</td></tr>
<tr><td>< 0<br>(Demand Shift In)</td><td>$\frac{dp^*}{d\alpha_h} < 0$ and $\frac{dq^*}{d\alpha_h} = ?$</td><td>$\frac{dp^*}{d\alpha_h} = ?$ and $\frac{dq^*}{d\alpha_h} < 0$</td></tr>
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It is also instructive to use this framework to take a closer look at the implications of a tax on the equilibrium price and quantity. Fundamentally, a tax drives a wedge between the price consumers pay and producers receive such that consumers pay more than producers receive. If $p^c$ is the price paid by consumers, $p^p$ is the price received by producers, and $t$ is a unit tax, we will have $p^c = p^p + t$. Now when we solve for a market equilibrium, we will need to solve for two prices, not just one, which can be accomplished by solving for the $p^{c*}$ and $p^{p*}$ that satisfy $p^{c*} = p^{p*} + t$ and $x(p^{c*}, \alpha) = y(p^{p*}, \alpha)$. Substitution yields $x(p^{p*} + t, \alpha) = y(p^{p*}, \alpha)$, such that the implicit function theorem then implies

PC3 $$\frac{dp^{p^*}}{dt} = -\frac{\frac{\partial x(p^{p^*}+t,\alpha)}{\partial p}}{\frac{\partial x(p^{p^*}+t,\alpha)}{\partial p} - \frac{\partial y(p^{p^*},\alpha)}{\partial p}} = -\frac{\varepsilon_x p^{p^*}}{\varepsilon_x p^{p^*} + \varepsilon_y (p^{p^*}+t)} < 0$$

for a normal commodity. The change in the equilibrium price paid by consumers is

PC4 $$\frac{dp^{c^*}}{dt} = \frac{dp^{p^*}}{dt} + 1 = -\frac{\frac{\partial y(p^{p^*},\alpha)}{\partial p}}{\frac{\partial x(p^{p^*}+t,\alpha)}{\partial p} - \frac{\partial y(p^{p^*},\alpha)}{\partial p}} = \frac{\varepsilon_y (p^{p^*}+t)}{\varepsilon_x p^{p^*} + \varepsilon_y (p^{p^*}+t)} > 0$$

Finally, with $q^* = x(p^{c*}, \alpha)$, we know

PC5 $\frac{dq^*}{dt} = \frac{\partial x(p^{c^*}, \boldsymbol{\alpha})}{\partial p} \frac{dp^{c^*}}{dt} < 0.$

What all this tells us is exactly what we learned as undergraduates. A tax decreases the price received by producers, increases the price paid by consumers, and decreases the equilibrium quantity. By how much depends on the own-price elasticities and the size of the tax wedge.

## MONOPOLY & MONOPSONY

When a producer has a monopoly, it can choose how much to sell and to whom — at least to the extent that it can discriminate among buyers. What we now want to explore is how all this affects prices and quantities.

The simplest case of monopoly is the case where the producer cannot discriminate among buyers, which means it has no reason to offer a different price to different buyers. Therefore, all the producer knows is that the demand for its product is $x(p)$ where I have now also gotten rid of $\alpha$ to reduce notational clutter. The problem facing the producer is to figure out how much to produce and what to charge consumers. Assuming the producer maximizes profit given the cost function $c(q)$, this problem can be written as

M1 $\max_{q,p} pq - c(q)$ subject to $q = x(p)$.

If everything is nicely differentiable, the Lagrangian and first order conditions for an interior optimum are

M2 $L = pq - c(q) + \gamma\big(q - x(p)\big),$

M3 $\frac{\partial L}{\partial p} = q^m - \gamma^m \frac{\partial x(p^m)}{\partial p} = 0,$

M4 $\frac{\partial L}{\partial q} = -\frac{\partial c(q^m)}{\partial q} + \gamma^m = 0$, and

M5 $\frac{\partial L}{\partial \gamma} = q^m - x(p^m) = 0$

where the superscript *m* is used to note that it is the monopolist's optimum choices. Combining equation M3 – M5 yields

M6 $$p^m\left(1-\frac{1}{\varepsilon}\right)=\frac{\partial c(q^m)}{\partial q}$$

where $\varepsilon=\left|\frac{p}{q}\frac{\partial x(p)}{\partial p}\right|$ is the price elasticity of demand, which is assumed to be negative for the typical downward sloping demand. The left-hand side of equation M6 is the marginal revenue, while the right-hand side is the marginal cost. Therefore, if demand is in fact downward sloping, we see from equation M6 that the monopolist will choose a price and output such that the price exceeds the marginal cost of production: $p^m>\frac{\partial c(q^m)}{\partial q}$. Remember that if the producer was a price taker, it would maximize profit where price equals marginal cost: $p=\frac{\partial c(q^*)}{\partial q}$. Also note that to ensure $q^*$ is a maximum, $\frac{\partial^2 c(q^*)}{\partial q^2}>0$ implying $q^*>q^m$. This is the classical monopoly result, but what does it really mean? What it means is that if a producer is the only one in the market, it will produce less if it behaves as a monopolist instead of as a price taking perfect competitor. This does not however mean that the producer would necessarily produce less as a monopolist with the market to itself, than as a perfect competitor in a market that is shared with others. The other important result to take from equation M6 is that the wedge between the price and marginal cost shrinks the more elastic the demand and that a monopolist will never produce where demand is inelastic ($\varepsilon<1$) because it would imply $1-\frac{1}{\varepsilon}<0$.

Our simple monopoly assumed that the producer could not discriminate across consumers, but what if it could. For example, what if it was possible for the producer to charge each consumer a different price? Its decision problem could then be written as

M7 $$\max_{q,p^1,\ldots,p^I}\sum_{i=1}^{I}p^i x^i(p^i)-c(q)\text{ subject to }q=\sum_{i=1}^{I}x^i(p^i),$$

which for an interior solution would require

M8 $${p^i}^m\left(1-\frac{1}{\varepsilon^i}\right)=\frac{\partial c(q^m)}{\partial q} \text{ for } i = 1,\ldots, I$$

where $\varepsilon^i=\left|\frac{p^i}{x^i(p_l{}^i)}\frac{\partial x^i(p^i)}{\partial p^i}\right|$ for the typical downward sloping demand. Again, the left-hand side of equation M8 is the marginal revenue, while the right-hand side is the marginal cost. Equation M8 implies

M9 $${p^i}^m\left(1-\frac{1}{\varepsilon^i}\right)={p^j}^m\left(1-\frac{1}{\varepsilon^j}\right) \text{ for all } i, j = 1,\ldots, I,$$

which says the producer will distribute its output across consumers to equate the marginal revenues. Equation M9 also implies that it is optimal for ${p^i}^m>{p^j}^m$ when $\varepsilon^j>\varepsilon^i$. That is, the producer will set higher prices for consumers with more inelastic demands. Since $p^i=p^j$ for all *i*, *j* = 1,…,*I* is always an option for the producer in M7, its profit must be greater than when it does not price discriminate as in M1 or when it is a perfect competitor.

Price discrimination taken to the extreme is referred to as perfect price discrimination. With perfect price discrimination, the producer charges exactly what the consumer is willing to pay for each item that is sold. This implies that demand equals marginal revenue and that the producer will choose an output where the marginal cost equals demand. Therefore, it will choose the same output it would produce if it were to behave competitively. Though remember that the prices it charges will be quite different.

The results we have developed for monopoly have analogues in terms of monopsony. With a monopsony, there is a single buyer but many sellers, so the monopsonist has the opportunity to choose how much to pay in addition to how much to buy. To illustrate the similarities and differences between the monopsony and monopoly, we will consider an example where the buyer cannot discriminate between sellers. This example, will also serve to further motivate the utility of revenue functions.

Suppose the monopsonist has the revenue function $R(\mathbf{z})$ where $\mathbf{z}$ is a vector of inputs and the prices of its output are suppressed to avoid clutter. Let $r_n(z_n) > 0$ be the inverse supply for input $n = 1,\ldots, N$. Assume the monopsonist's objective is to maximize profit:

M10 $\max_{z_1,\ldots,z_N} R(\mathbf{z}) - \sum_{n=1}^{N} r_n\,(z_n)z_n.$

First order conditions for equation M10 are

M11 $\frac{\partial R(\mathbf{z}^m)}{\partial z_n} - \frac{\partial r_n({z_n}^m)}{\partial z_n}{z_n}^m - r_n({z_n}^m) = 0$ for $n = 1,\ldots, N$,

which can be rewritten as

M12 $\frac{\partial R(\mathbf{z}^m)}{\partial z_n} = r_n({z_n}^m)\left(1 + \frac{1}{\varepsilon_n}\right)$ for $n = 1,\ldots, N$

where the superscript *m* indicates the monopsonist's optimum and $\varepsilon_n = \frac{r_n({z_n}^m)}{{z_n}^m \frac{\partial r_n({z_n}^m)}{\partial z_n}}$ is the elasticity of supply for commodity *n*. Equation M12 looks much like equation M6. The left-hand side of equation M12 is the marginal revenue, while the right-hand side is the marginal cost. Typically we expect $\varepsilon_n > 0$ such that equation M12 implies $\frac{\partial R(z^m)}{\partial z_n} > r_n({z_n}^m)$. A monopsonist sets marginal revenue in excess of the price, while under perfect competition marginal revenue is set equal to price: $\frac{\partial R(\mathbf{z}^*)}{\partial z_n} = r_n({z_n}^*)$. While it would be nice to say that this implied ${z_n}^* > {z_n}^m$, we have to be careful because there are more moving parts here. In the monopoly problem, there was only one output and only one way for the producer to respond. In this monopsony case here, there are lots of inputs, which mean there are a lot of ways for a producer to respond so the analysis is not nearly as straightforward. Of course, if we assumed that a monopolist produced multiple outputs, then the results would also not be nearly as straight forward.