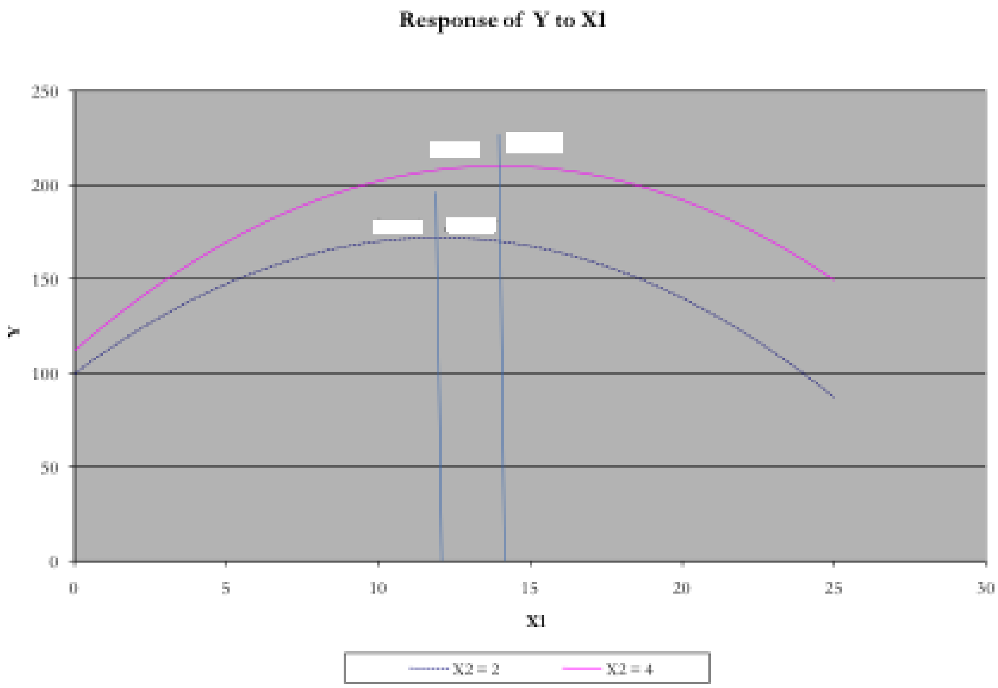
Solutions to Problem Set #3: Producer Theory

In this assignment we will mathematically and graphically investigate the properties of a single output, production function with two variable inputs. Assume that the following quadratic production function describes the technical relationship between output and the two inputs. Also, assume that $x_1, x_2 \in [0, 25]$.

$$y = 80 + 10x_1 + 12x_2 - .5x_1^2 - x_2^2 + x_1x_2$$

1. Graph both $y = f(x_1|x_2 = 2)$ and $y = f(x_1|x_2 = 4)$ on the same figure. (Use Excel to calculate the values of y for each integer value of $x_1 \in [0, 25]$, first fixing the value of x_2 at 2 then at 4). On each curve, identify the stages of production.

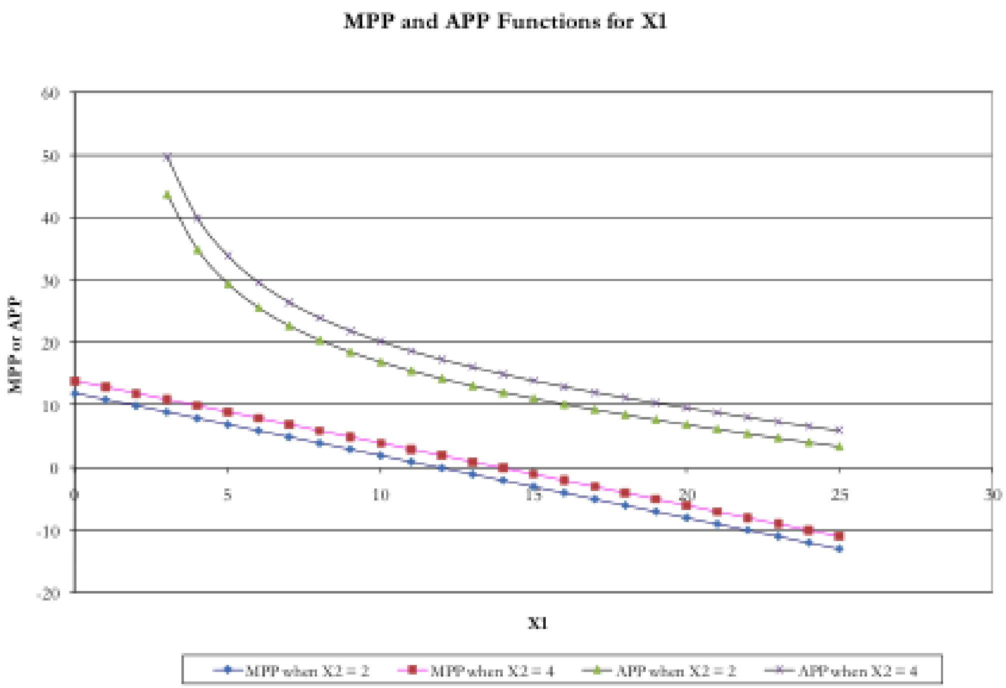


2. Discuss the similarities and differences between the two curves. Is output more or less responsive to x_1 at the higher level of x_2? Can you comment on the technical relationship between the inputs?

   Both curves display the second and third stages of production. Output is more responsive at higher levels of x_2. This indicates that the two inputs are technically complementary. We can also verify the technical relationship between the inputs by showing that $\frac{\partial y^2}{\partial x_1 \partial x_2} = 1 > 0$.

3. Derive the average product and marginal product functions of x_1, (AP_1 and MP_1). On a new figure, graph AP_1 and MP_1 for $x_2 = 2$ and $x_2 = 4$. Identify the stages of production on each of these graphs.

- $AP_1 = \frac{80}{x_1} + 10 + 12\frac{x_2}{x_1} - .5x_1 - \frac{x_2^2}{x_1} + x_2$
- $MP_1 = 10 - x_1 + x_2$

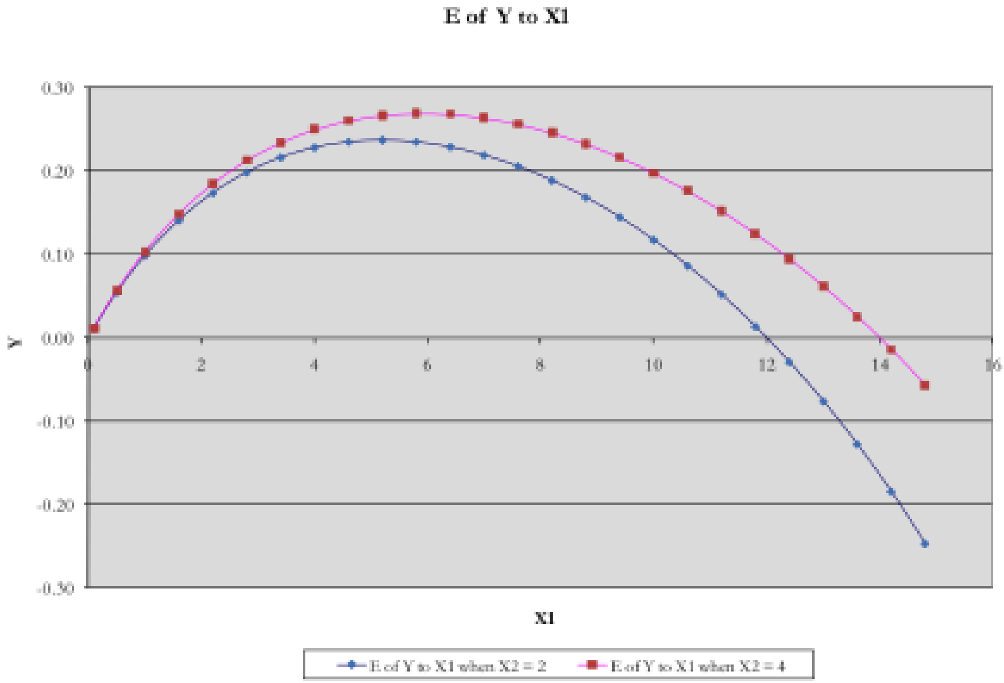


In both cases, MP is below AP for all x_1. Therefore, the functions do not display stage one of production. Stage two is delineated from stage three at the point where $MP_1 = 0$.

4. Derive the factor elasticity of x_1, (ϵ_1). On a new figure, graph ϵ_1 for $x_2 = 2$ and $x_2 = 4$. Interpret your finding. (Note: for a better display, draw this graph for $x_1 \in [0, 15]$.

- $\epsilon_{x_1} = \frac{MP_1}{AP_1} = \frac{x_1(10-x_1+x_2)}{80+10x_1+12x_2-.5x_1^2-x_2^2+x_1x_2}$

ϵ_{x_1} measures the percent change in output resulting from a one percent change in factor 1. See the graph below. The interpretation is similar to that of the graph in question 2. The graph shows that the percent change in output due to a one percent change in factor 1 is larger at higher levels of x_2. That is, output is more responsive to x_1 at higher levels of x_2.



5. Find the optimal levels of x_1, x_1^*, – the values that maximize output – when $x_2 = 2$ and $x_2 = 4$. Also, calculate the maximum output, y^*, for each case.
   - $MP_1 = 10 - x_1 + x_2$. The maximum output is where $MP_1 = 0$, so $x_1^* = 10 + x_2$. For $x_2 = 2$ and $x_2 = 4$, $x_1^* = 12$ and $x_1^* = 14$, respectively.
   - $y = 80 + 10x_1 + 12x_2 - .5x_1^2 - x_2^2 + x_1x_2$, so $y(x_2 = 2) = 80 + 10x_1 + 24 - .5x_1^2 - 4 + 2x_1 = 100 + 12x_1 - .5x_1^2$ and $y(x_2 = 4) = 80 + 10x_1 + 48 - .5x_1^2 - 16 + 4x_1 = 112 + 14x_1 - .5x_1^2$. Therefore $y^*(x_2 = 2) = 172$ and $y^*(x_2 = 4) = 210$.