Suggested Solutions to Problem Set #6

Part A.

15.2 a. A monopolist maximizes $Q(a-bQ-c)$, yielding first-order condition $a-2bQ-c=0$ and the monopoly outcome

$$Q^m = \frac{a-c}{2b}$$

$$P^m = \frac{a+c}{2}$$

$$\Pi^m = \frac{(a-c)^2}{4b}.$$

b. Cournot firm 1 maximizes $q_1[a-b(q_1+q_2)-c]$, yielding first-order condition $a-2bq_1-bq_2-c=0$ and best-response function

$$q_1 = \frac{a-bq_2-c}{2b}.$$

Symmetrically for firm 2,

$$q_2 = \frac{a-bq_1-c}{2b}.$$

The Nash equilibrium outcome is

$$q_i^c = \frac{a-c}{3b}$$

$$P^c = \frac{a}{3}+\frac{2c}{3}$$

$$\pi_i^c = \frac{(a-c)^2}{9b}.$$

d. Cournot firm i maximizes $q_i[a-b(Q_{-i}+q_i)-c]$, yielding first-order condition $a-bQ_{-i}-2bq_i-c=0$. Once the first-order condition has been taken, we can apply the fact that firms are symmetric and so the equilibrium will be symmetric. Substituting $Q_{-i}^c=(n-1)q_i^c$ into the first-order condition and solving for q_i^c yields

$$q_i^c = \frac{a-c}{b(n+1)}.$$

Therefore,

$$Q^c = \frac{n}{n+1}\cdot\frac{a-c}{b}$$

$$P^c = \frac{a+nc}{n+1}$$

$$\pi_i^c = \frac{(a-c)^2}{b(n+1)^2}$$

$$\Pi^c = \frac{n(a-c)^2}{b(n+1)^2}.$$

15.3 a. Skipping preliminary calculations, firm 1's best-response function is

$$q_1 = \frac{1-q_2-c_1}{2}.$$

Firm 2's is

$$q_2 = \frac{1-q_1-c_2}{2}.$$

Solving simultaneously,

$$q_1^c = \frac{1-2c_1+c_2}{3}$$

$$q_2^c = \frac{1-2c_2+c_1}{3}.$$

Further,

$$Q^c = \frac{2-c_1-c_2}{3}$$

$$P^c = \frac{1+c_1+c_2}{3}$$

$$\pi_i^c = \frac{(1-2c_i+c_j)^2}{9}$$

$$\Pi^c = \pi_1^c + \pi_2^c$$

$$CS^c = \frac{(2-c_1-c_2)^2}{18}$$

$$W^c = \Pi^c + CS^c.$$

15.9 Herfindahl index of market concentration

a. Reprising the analysis from Problem 15.2, firm i's profit is $q_i(a-bq_i-bQ_{-i}-c)$ with associated first-order condition $a-2b-bQ_{-i}-c=0$. Imposing symmetry $[Q_{-i}^* = (n-1)q_i^*]$ and solving,

$$q_i^* = \frac{a-c}{(n+1)b}.$$

Further,

$$Q^* = \frac{n(a-c)}{(n+1)b}$$

$$P^* = \frac{a+nc}{n+1}$$

$$\Pi^* = n\pi_i^* = \frac{n}{b} \cdot \left(\frac{a-c}{n+1}\right)^2$$

$$CS^* = \frac{n^2}{2b} \cdot \left(\frac{a-c}{n+1}\right)^2$$

$$W^* = \frac{n(n+2)}{2b} \cdot \left(\frac{a-c}{n+1}\right)^2 .$$

Because firms are symmetric, $s_i = 1/n$, implying

$$H = n \cdot \left(\frac{1}{n}\right)^2 = \frac{1}{n}.$$

b. We can obtain a rough idea of the effect of merger by seeing how the variables in part a change with a reduction in n. Per-firm output, price, industry profit, and the Herfindahl index increase. Total output, consumer surplus, and welfare decrease.

c. Substituting $c_1 = c_2 = 1/4$ into the answers for 15.3, we have $q_i^* = 1/4$, $Q^* = 1/2$, $P^* = 1/2$, $\Pi^* = 1/8$, $CS^* = 1/8$, $W^* = 1/4$, $H = 1/2$.

d. Substituting $c_1 = 0$ and $c_2 = 1/4$ into the answers for 15.3, we have $q_1^* = 5/12$, $q_1^* = 2/12$, $Q^* = 7/12$, $P^* = 5/12$, $\Pi^* = 29/144$, $CS^* = 49/288$, $W^* = 107/288$, $H = 29/49$.

e. Comparing part a with part b suggests that increases in the Herfindahl index are associated with lower welfare. The opposite is evidenced in the comparison of parts c-d: welfare and the Herfindahl increase together. General conclusions are thus hard to reach.

B. To find the Nash equilibrium, we will analyze each of the 4 scenarios to calculate the firms' payoffs.

1. *The incumbent firm does not invest, the second firm does not enter:* In this case the incumbent firm makes monopoly profits in both periods. By setting its $MR = MC$, $74 - 18Q = 20$ find that $Q = 3$ and $P = 47$. Its profits in each period is then $\pi_1 = TR - TC = (47)(3) - 15 - 20(3)$, or $\pi_1 = 66$. Total profits in two periods $\pi_1 = 132$, with the second firm making zero profit the payoffs under this scenario are: $(\pi_1, \pi_2) = (132, 0)$.

2. *The incumbent firm does not invest, the second firm enters:* In this case the incumbent firm makes monopoly profits in the first period, \$66. In period 2, the market structure is Cournot duopoly. The residual demand each of firm 1 is: $p = (74 - 9q_2) - 9q_1$. By setting $RMR_1 = MC_1$, we find that the best response function of firm 1 is $(74 - 9q_2) - 18q_1 = 20$, or $q_1 = 3 - q_2/2$. Since the two firms are identical the best response function of firm 2 is $q_2 = 3 - q_1/2$. The Cournot-Nash equilibrium is the intersection of the two BR functions $q_1 = 3 - (3 - q_1/2)/2$, or $q_1 = 2$. Similarly, $q_2 = 3 - (2)/2 = 2$. Total output $Q = 2 + 2 = 4$, market price $p = 74 - 9(4) = 38$. Profit of each firm in period 2: $\pi = (38)(2) - 15 - 20(2) = 21$. The total profit of each firm under this scenario is then $(\pi_1, \pi_2) = (87, 21)$.

3. *The incumbent firm invests, the second firm does not enter:* In this case the incumbent firm makes monopoly profits in both periods. However, while its profits in the first period is equal to its monopoly profits minus R&D costs, \$66 - \$63.5 = \$2.5, in period 2 the firm enjoys extra profits due to its lower marginal cost, $MC_1 = 2$.

   In period 2, the incumbent firm sets its market quantity at a level where MR = MC. Then, the monopoly quantity is $74 - 18Q = 2$, or $Q = 4$. At this quantity, the market price is $P = 74 - 9(4) = 38$. Its profits are $\pi = (38)(4) - 15 - 2(4) = 129$. Thus, under this scenario the incumbent firm makes a total of 131.5, while the second firm makes zero. $(\pi_1, \pi_2) = (131.5, 0)$.

4. *The incumbent firm invests, the second firm enters:* Just as in (3) the incumbent firm makes \$2.5 in period 1. In period 2, the market is a Cournot duopoly. However, in this case the incumbent has a cost advantage over firm 2 with marginal cost, $MC_1 = 2$.

   To find the Cournot-Nash outcome we seek to find the best response functions of each firm. Note that, because neither the marginal revenue nor the marginal cost facing firm 2 has changed its best response function is the same:

   - $q_2 = 3 - 0.5q_1$

   On the other hand, Firm 1 has a new best response function because its MC is now lower. Setting $RMR_1 = MC_1$ and solving for q_1

   - $74 - 9q_2 - 18q_1 = 2$

- $q_1 = 4 - 0.5q_2$

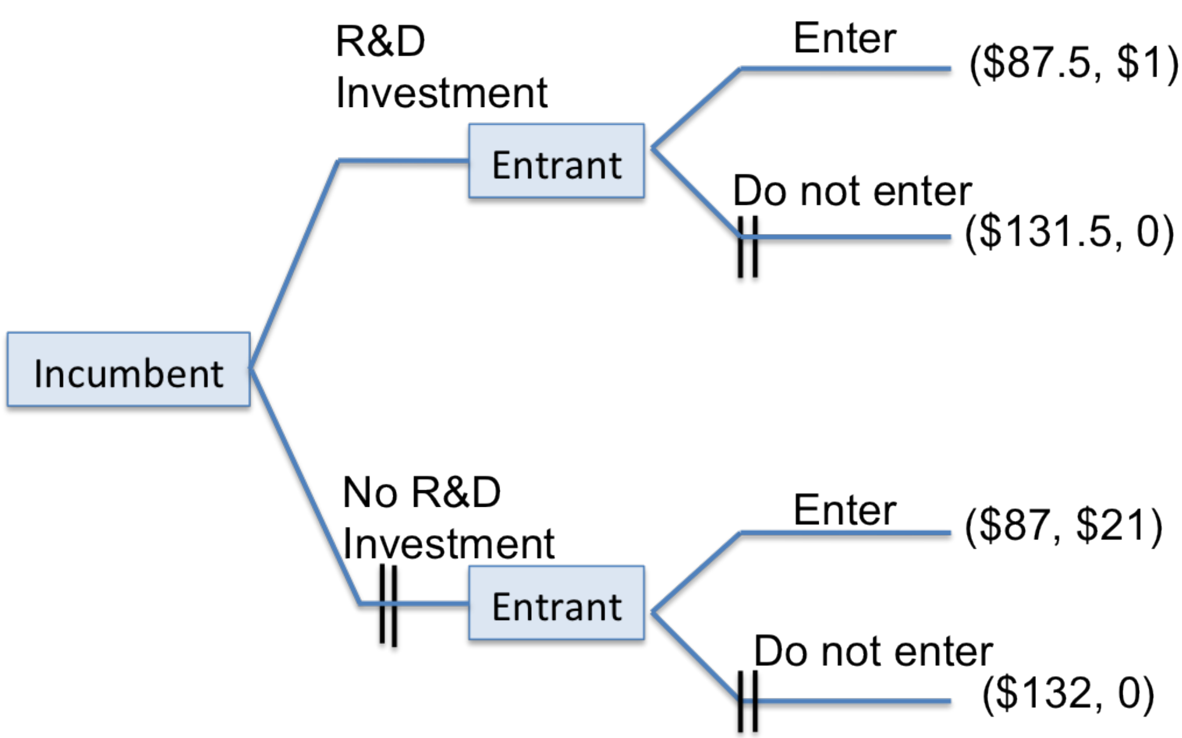
Thus, Cournot-Nash equilibrium is

- $q_1 = 4 - 0.5(3 - 0.5q_1)$
- $q_1 = 2.5 + 0.25q_1$
- $q_1 = 10/3$

Thus, $q_2 = 3 - 0.5(10/3) = 4/3$. In period 2, the Cournot-Nash equilibrium is $(q_1^c, q_2^c) = (10/3, 4/3)$. Total output is $10/3+4/3 = 14/3$. The market price is $P = 74-9(14/3) = 32$. The profits of each firm are: $\pi = (32)(10/3) - 15 - 2(10/3) = 85$ and $\pi = (32)(4/3) - 15 - 20(4/3) = 1$. At the end of period 2, the incumbent makes a total of \$87.5, while the second firm makes \$1. $(\pi_1, \pi_2) = (87.5, 1)$.

Since firm 2 is better off entering the market regardless whether incumbent firm invests or not, and the incumbent firm earns higher firms by investing if firm 2 is going to enter, the incumbent firm is better off investing.

2. We can summarize the results of the R&D game in an extensive form. The Nash equilibrium of this game is (invest, enter) = (87.5, 1).



C.1 The players' strategies are

- player 1= {A, B}, player 2= {CE, CF, DE, DF}, player 3= {GI, GJ, HI, HJ}

2. There are three subgames. Subgame 1: the whole game tree, which starts from node 1 and includes everything thereafter. Subgame 2: the second subgame starts at node 2 at the end of branch A and includes everything thereafter. Subgame 3: the third subgame starts at node 2 at the the end of branch B and includes everything thereafter.

3. We can find the subgame perfect Nash equilibrium by using backward induction. That is, we find the solutions to each subgame at every step. Subgame 2 is a simultaneous move game between players 2 and 3. There is a dominant strategy solution to subgame 2, which is (C, H). Subgame 3 is also a simultaneous move game between players 2 and 3. There is a dominant strategy solution to subgame 3, which is (E, J). The unique subgame perfect Nash equilibrium of this game is (B, CE, HJ), where B is the strategy for player 1, CE is the strategy for player 2, and HJ is the strategy for player 3. This leads to an equilibrium path of play of BEJ and equilibrium payoffs (3, 7, 2).