APEC 5151: Applied Microeconomics: Firm and Household
Fall 2016 Midterm Exam
ANSWER KEY

NAME:

Do not open the test until you are instructed to do so. Everyone will start the test at the same time.

There are 100 possible points on this test, and 15 total questions. You will have 75 minutes. If you answer a question every four minutes (or 1.67 points per minute) you will have 15 minutes at the end to check your work. The two bonus points at the end will be added to your score but cannot raise it above 100.

You are allowed to use a simple (non-graphing) calculator during the test, but you should not need a calculator to answer any of the questions. You can leave any fractions in unsimplified form.

If you do not know how to answer a question, write down what you do know, or skip it and come back to it later.

Be sure to answer all parts of each question, as several require multiple answers. Please show all steps of your work. If you use a graph in answering a question, please make sure it is labeled appropriately.

1. (6 points) The slopes of the Marshallian and Hicksian demand must always have the same sign. True or False? Explain your answer.

False. The Marshallian demand includes both income and substitution effects and the Hicksian demand contains only substitution effects. The Marshallian demand can slope upward if the income effect is negative (inferior goods) and specifically if it is larger than the substitution effect. This is true for a Giffen good.

For questions 2 and 3, let $u(x_1, x_2)$ be a twice-differentiable utility function.

2. (6 points) Explain in words what the *mathematical* meaning of $u_1(x_1, x_2)$ is. Also, explain in words what the *economic* meaning of $u_1(x_1, x_2)$ is.

Mathematical: it is the partial derivative of the utility function with respect to good 1. Economic: it is the change in the consumer's level of utility when the amount of good 1 consumed increases by 1 unit - the *marginal utility* of good 1.

3. (6 points) Explain in words the *economic* meaning of the statement $u_{12}(x_1, x_2) > 0$. Give a realistic example of two goods for which the statement will be true. This statement means that

the marginal utility of good 1 increases when the agent consumes more of good 2: consuming more of good 2 increases the payoff to consuming an additional unit of good 1. The two goods are complements. Answers will vary. This could hold for peanut butter and bread, for example.

4. (7 points) A consumer's unconstrained optimization problem is

$$\max_{x} u(x;t) = ln(x) - 2x + 1/e^{tx} + 5$$

. The choice variable is x and t is a parameter. Assume u is a concave function of x, and let the optimized value of x be $x^*(t)$. Find an expression for the comparative static of x^* with respect to t, $\partial x^*(t)/\partial t$. In what direction does x^* move if t increases?

$$\begin{aligned} u' = 1/x - 2 - te^{-tx} &= 0 \\ 1/x - 2 &= te^{-tx} \\ \partial U'/\partial t &= t^2 e^{-tx} \\ \partial U'/\partial x &= 1/x^2 + t^2 e^{-tx} \\ \partial x/\partial t = -(\partial U'/\partial t)/(\partial U'/dx) &= -[t^2 e^{-tx}]/[1/x^2 + t^2 e^{-tx}] \end{aligned}$$

The denominator is negative. The numerator is a positive number with a leading minus sign so is negative. The whole expression is hence positive. The comparative static is positive so x^* will increase when t rises.

For problems 5 through 9, use the following assumptions:

Let a firm's production function be $y = 10x_1^{0.75}x_2^{0.75}$. The price of input 1 is w_1 and the price of input 2 is w_2. The price of output is p.

5. (6 points) Set up the firm's profit maximization problem.

$$
\begin{aligned}
max_{x_1,x_2}\pi &= p[10x_1^{0.75}x_2^{0.75}] - w_1x_1 - w_2x_2 \\
\partial\pi/\partial x_1 &= p7.5x_1^{-0.25}x_2^{0.75} - w_1 = 0 \\
\partial\pi/\partial x_2 &= p7.5x_1^{0.75}x_2^{-0.25} - w_2 = 0
\end{aligned}
$$

6. (7 points) Solve for the profit-maximizing level of output y (the supply function).

$$
\begin{aligned}
p7.5x_1^{-0.25}x_2^{0.75} &= w_1 \\
p7.5x_1^{0.75}x_2^{-0.25} &= w_2 \\
\frac{p7.5x_1^{-0.25}x_2^{0.75}}{p7.5x_1^{0.75}x_2^{-0.25}} &= \frac{w_1}{w_2} \\
\frac{x_1^{-0.25}x_2^{0.75}}{x_1^{0.75}x_2^{-0.25}} &= \frac{w_1}{w_2} \\
\frac{x_2}{x_1} &= \frac{w_1}{w_2} \\
x_2 &= x_1\frac{w_1}{w_2} \\
max_{x_1}\pi = p[10x_1^{0.75}(x_1\frac{w_1}{w_2})^{0.75}] &- w_1x_1 - w_2(x_1\frac{w_1}{w_2}) \\
max_{x_1}\pi = p[10x_1^{1.5}(\frac{w_1}{w_2})^{0.75}] &- 2w_1x_1 \\
\partial\pi/\partial x_1 = 15p(\frac{w_1}{w_2})^{0.75}x_1^{0.5} &- 2w_1 = 0
\end{aligned}
$$

$$7.5p\frac{w_1^{-0.25}}{w_2^{0.75}}x_1^{0.5} = 1$$
$$x_1^{0.5} = \frac{w_1^{0.25}w_2^{0.75}}{7.5p}$$
$$x_1 = [\frac{w_1^{0.25}w_2^{0.75}}{7.5p}]^2$$
$$x_1 = [\frac{w_1^{0.5}w_2^{1.5}}{56.25p^2}]$$
$$x_2 = [\frac{w_1^{0.5}w_2^{1.5}}{56.25p^2}]\frac{w_1}{w_2}$$
$$x_2 = [\frac{w_1^{1.5}w_2^{0.5}}{56.25p^2}]$$
$$y = 10x_1^{0.75}x_2^{0.75} = 10[\frac{w_1^{0.5}w_2^{1.5}}{56.25p^2}]^{0.75}[\frac{w_1^{1.5}w_2^{0.5}}{56.25p^2}]^{0.75}$$
$$y = 10[\frac{w_1^2w_2^2}{7.5^4p^4}]^{0.75} = 10\frac{w_1^{1.5}w_2^{1.5}}{7.5^3p^3}$$

7. (7 points) Verify that Hotelling's Lemma holds for the supply function.

$$\pi(p, w_1, w_2) = p[10(\frac{w_1^{0.5}w_2^{1.5}}{56.25p^2})^{1.5}(\frac{w_1}{w_2})^{0.75}] - 2w_1[\frac{w_1^{0.5}w_2^{1.5}}{56.25p^2}]$$
$$= [10\frac{(w_1^{0.5}w_2^{1.5})^{1.5}}{56.25^{1.5}p^2}(\frac{w_1}{w_2})^{0.75}] - 2w_1[\frac{w_1^{0.5}w_2^{1.5}}{56.25p^2}]$$
$$y = \partial\pi/\partial p$$
$$= -2[10\frac{(w_1^{0.5}w_2^{1.5})^{1.5}}{56.25^{1.5}p^3}(\frac{w_1}{w_2})^{0.75}] + 4w_1[\frac{w_1^{0.5}w_2^{1.5}}{56.25p^3}]$$
$$= \frac{1}{p^3}[-20\frac{(w_1^{0.5}w_2^{1.5})^{1.5}}{56.25^{1.5}}(\frac{w_1}{w_2})^{0.75} + 4w_1\frac{w_1^{0.5}w_2^{1.5}}{56.25}]$$
$$= \frac{1}{p^3}[-20\frac{(w_1^{0.75}w_2^{2.25})}{56.25^{1.5}}(\frac{w_1}{w_2})^{0.75} + 4\frac{w_1^{1.5}w_2^{1.5}}{56.25}]$$
$$= \frac{1}{p^3}[-20\frac{w_1^{1.5}w_2^{1.5}}{56.25^{1.5}} + 4\frac{w_1^{1.5}w_2^{1.5}}{56.25}]$$
$$= \frac{w_1^{1.5}w_2^{1.5}}{p^3}[\frac{-20}{56.25^{1.5}} + \frac{4}{56.25}]$$
$$= \frac{w_1^{1.5}w_2^{1.5}}{p^3}[\frac{-20}{7.5^3} + \frac{4}{7.5^2}]$$
$$= \frac{w_1^{1.5}w_2^{1.5}}{p^3}[\frac{-20}{7.5^3} + \frac{30}{7.5^3}]$$
$$= \frac{w_1^{1.5}w_2^{1.5}}{p^3}[\frac{10}{7.5^3}]$$

Hence Hotelling's Lemma holds for this supply function.

8. (6 points) Does the firm's production function have diminishing, constant, or increasing returns to scale? The exponents add to 1.5 which is greater than 1. Increasing returns to scale.
9. (6 points) Compute the firm's marginal rate of technical substitution between inputs. Explain in words what the RTS means.

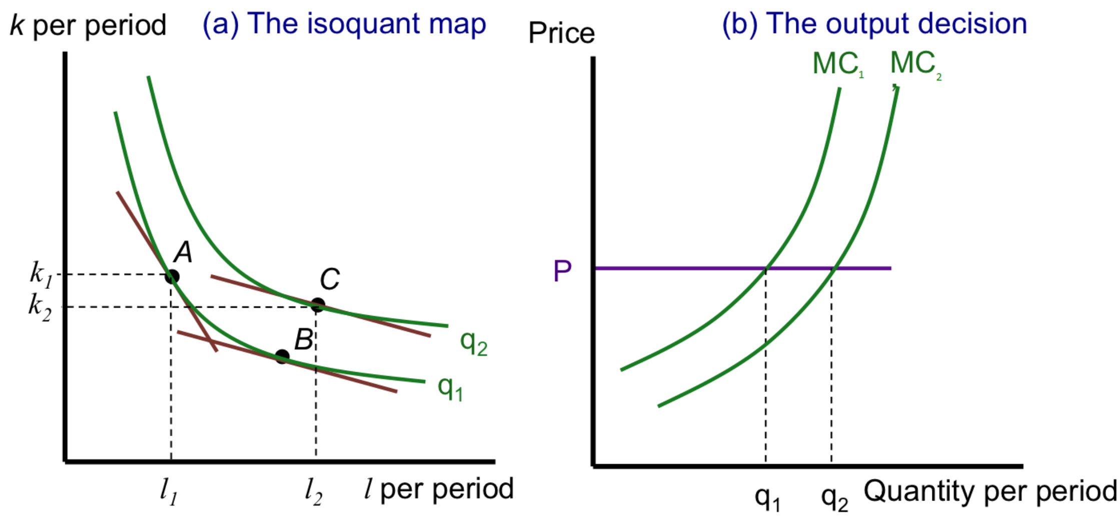
$$\begin{aligned} MRTS_{12} &= MP_1/MP_2 \\ &= [p7.5x_1^{-0.25}x_2^{0.75}]/[p7.5x_1^{0.75}x_2^{-0.25}] \\ &= [x_1^{-0.25}x_2^{0.75}]/[x_1^{0.75}x_2^{-0.25}] \\ &= [x_2/x_1] \end{aligned}$$

The MRTS is the number of units of input 2 the firm needs to add if it reduces its level of input 1 by one unit and holds output constant.
10. (6 points) Explain in words why a profit-maximizing firm never chooses to produce in stage 3 of production.

In stage 3 of production, additional inputs lead to decreases in output. Inputs cost money, so you would be better off using less of them - you would produce more output for a lower price, raising overall profits.

11. (6 points) Suppose a firm has a Cobb-Douglas production function with two inputs, so the inputs are neither perfect complements nor perfect substitutes but somewhere in between. Draw a graph showing how the firm's optimal levels of each input change when the price of one input increases. Be sure to show both the output and substitution effects.

Figure should look like the left panel of the graph below, but with the order of the changes flipped - we start on a higher isoquant, pivot the isocost line along that isoquant, and then shift inward to a lower isoquant.



For problems 12 through 15, use the following assumptions:

Consumers have a strictly concave utility function, $u(S, B; H)$. S is the quantity of SPAM consumed (in cans), B is the number of burgers consumed, and H is the consumer's degree of exposure to Hawaiian culture (a parameter in the model). Let I be total income, p_S be the price of a can of SPAM and p_B be the price of a burger.

12. (6 points) Set up the consumer's constrained utility maximization problem and write down the first-order conditions.

$$\begin{aligned} L &= u(S, B; H) + \lambda(I - p_S S - p_B B) \\ L_S &= u_S(S, B; H) + p_S = 0 \\ L_B &= u_B(S, B; H) + p_B = 0 \\ L_\lambda &= I - p_S S - p_B B = 0 \end{aligned}$$

13. (6 points) Write the demand functions for SPAM and burgers in implicit form.

$$\begin{aligned} S^* &= S^*(p_B, p_S, I; H) \\ B^* &= B^*(p_B, p_S, I; H) \end{aligned}$$

14. (6 points) SPAM is an important part of Hawaiian culture. People who are more exposed to Hawaiian culture derive more utility from consuming a can of SPAM. Write down an inequality that represents this fact in terms of the economic model you set up above.

$$u_{SH} > 0$$

15. (7 points) How would you expect increases in H to affect the consumption of burgers, *ceteris paribus*? Justify your answer using the results from question 14.

Consumption of burgers should decrease. Consumers pick the optimal mix of burgers and SPAM by setting MU per dollar equal across the two goods:$MU_S/p_S = MU_B/p_B$ A rise in H will increase MU for SPAM, so consumers will rebalance the equation by buying more spam (pushing down the left-hand side) and fewer burgers (pushing up the right-hand side).

Bonus (2 points) Explain why all the historical increases in the US minimum wage have been short-run, rather than long-run, increases.

Minimum wage increases have been set in nominal terms. Rea minimum wages hence rise, then are eroded away over time, generating a "sawtooth" pattern.