ApEc 8001
Applied Microeconomic Analysis: Demand Theory

Lecture 13: General Equilibrium: The Edgeworth Box (MWG, Ch. 15, pp.515-525)

I. Introduction

In all lectures thus far, prices have been assumed to be fixed; the focus has been on how consumers choose their consumption bundles given a fixed set of prices. **General equilibrium theory** considers the **simultaneous determination of both prices and consumption bundles**. One way to express this is that both prices and quantities (of consumption bundles) are "endogenous". In fact, each consumer's wealth will also be endogenous.

The only "exogenous" (fixed) components of the economy are the number of consumers, their preferences, the total endowment of each type of good, and the initial allocation of goods to each consumer, which are the "wealth" or "endowment" of each consumer.

Today we will focus on the case of only two consumers and only two commodities. In Apec 8004 you will have models that allow for large numbers of both consumers and commodities.

II. Pure Exchange: The Edgeworth Box

A **pure exchange economy** is an economy that consists of consumers who trade goods, but there is **no production process**. Each "agent" (consumer) has an initial stock ("endowment") of commodities, and given the prevailing prices each agent trades with other agents so as to maximize his or her utility.

The **simplest economy** has only **two consumers** and **two goods**. Both consumers are assumed to **act as "price takers"** (they assume that they cannot do anything that can change the prices). This is somewhat unreasonable for the case of only two consumers, but it is much more reasonable for the more general case of many consumers, so to get started we will impose this assumption.

More specifically, assume that there are two consumers, $i = 1, 2$, and two commodities, $\ell = 1, 2$. Consumer i's consumption vector is denoted by $x_i = (x_{1i}, x_{2i})$. Assume that each consumer's consumption set $X \in \mathbb{R}^2_+$, and that he or she has a preference relation $\succsim_i$ over the consumption bundles in this set.

Each consumer i has an initial endowment of good ℓ, denoted by $\omega_{\ell i}$. The **endowment vector** of consumer i is denoted by $\omega_i = (\omega_{1i}, \omega_{2i})$. The **total endowment** of good ℓ in this economy is denoted by $\overline{\omega}_\ell = \omega_{\ell 1} + \omega_{\ell 2} > 0$.

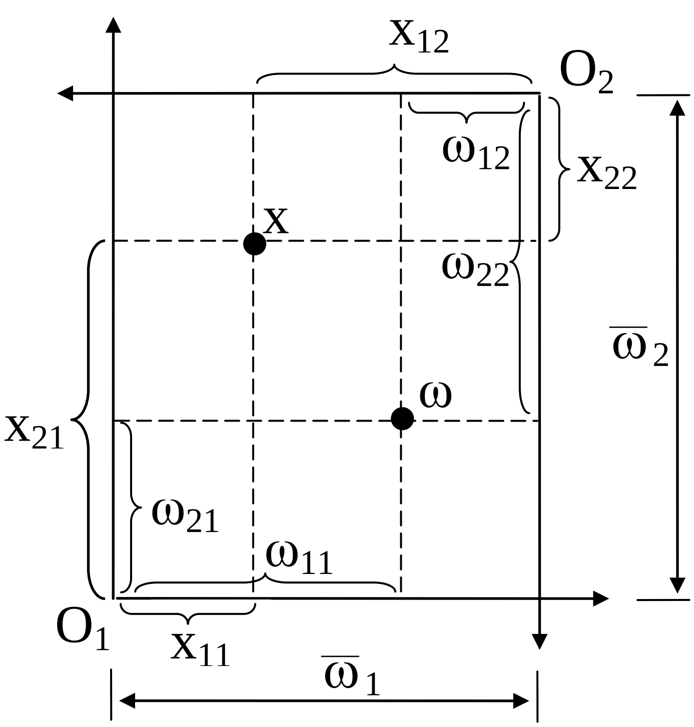
An **allocation** $x \in \mathbb{R}^4_+$ is an assignment of a nonnegative **consumption vector to each consumer**: $x = (x_1, x_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$. An allocation is **feasible if**:

$$x_{\ell 1} + x_{\ell 2} \leq \overline{\omega}_\ell \quad \text{for } \ell = 1, 2$$

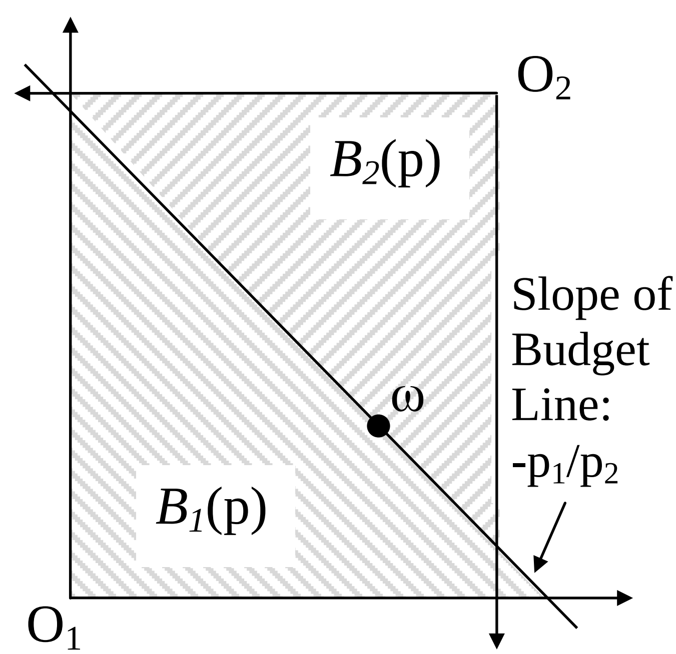
Note that the expression ≤ implicitly assumes that total consumption in the economy could be less than the total endowment; that is any unwanted amount can be ignored ("free disposal"). If the above expression is an equality, then we are limiting the feasible allocations to those that are "non-wasteful".

Non-wasteful allocations can be shown in an **Edgeworth box**:

Edgeworth Box



Budget Sets



In these diagrams, O_1 is the origin for the first consumer and O_2 is the origin for the second consumer. The **first diagram** shows the initial allocations for both, denoted by ω, and the final consumption point for both, denoted by x. The first **point (ω)** indicates the **endowment** of each good **for each consumer**. **After trading** ("exchange") with each other, the **consumers end up at** the **point x**, which shows the consumption of each good for each consumer. For both diagrams, the length (horizontal distance) of the box is the total endowment of good 1 for the economy, that is $\overline{\omega}_1$, and the height (vertical dist.) of the box is the total endowment of good 2 for the economy, that is $\overline{\omega}_2$.

In this "economy", the **wealth** of the two consumers is no longer given; rather it **depends on endowments and prices**:

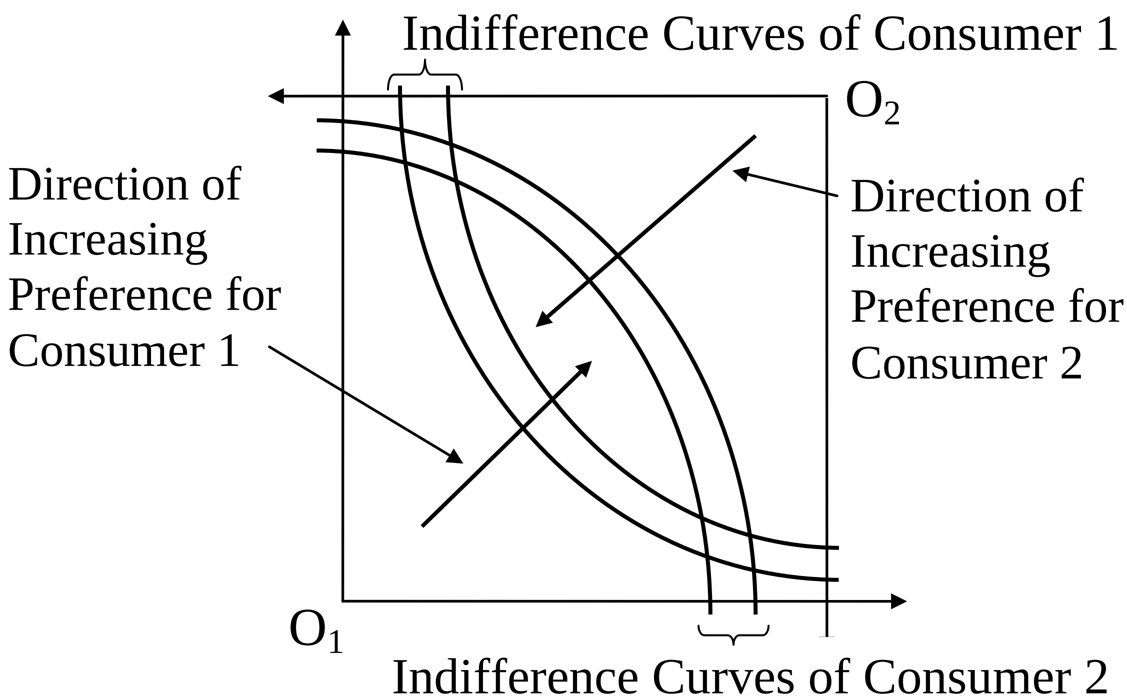
$$\text{wealth of consumer } i = p\cdot\omega_i = p_1\omega_{1i} + p_2\omega_{2i}$$

Thus the budget set of each consumer depends on prices and the endowment of commodities (ω). Here is the budget set for consumer i:

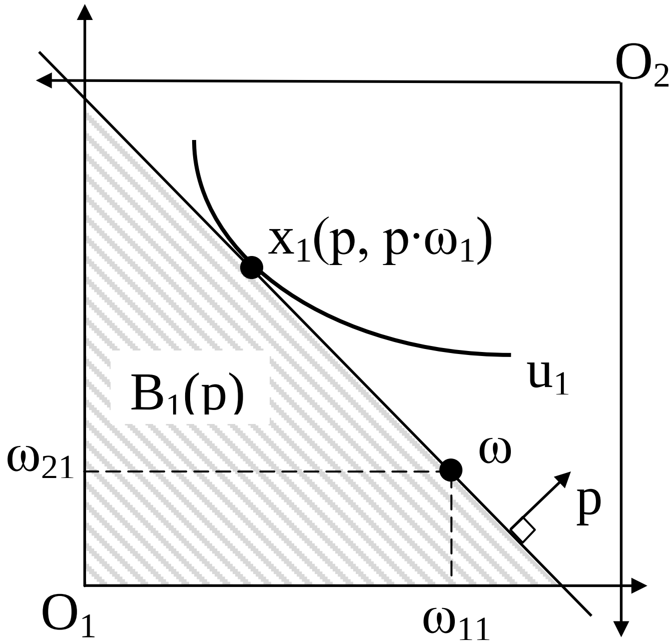
$$B_i(p) = \{x_i \in \mathbb{R}^2_+ : p\cdot x_i \leq p\cdot\omega_i\}$$

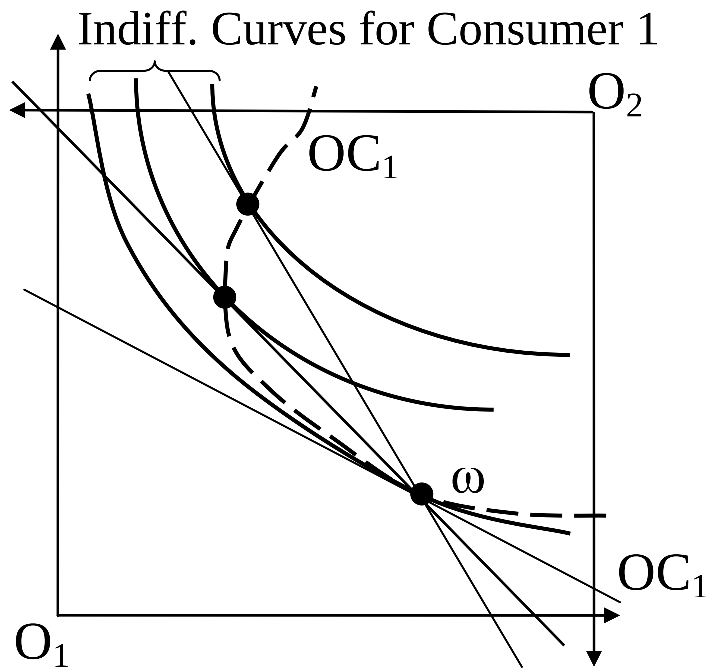
The **second figure** on the previous page shows the **budget sets for the two consumers**. The budget line, which has a slope of $-p_1/p_2$ is the set of all possible allocations that are consistent which both budget sets.

So what is the process by which the two consumers move from point ω to point x? To see how this works, **add each consumer's preferences to the Edgeworth box**:



Given these preferences, and a set of relative prices p_1/p_2, **how is the equilibrium** level of consumption for both consumers **achieved?** The following two diagrams explain the process for the first consumer:

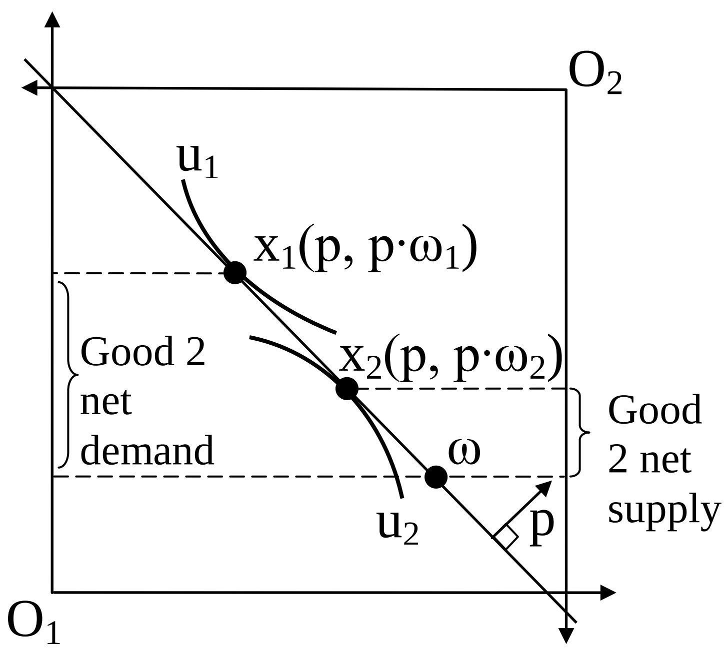




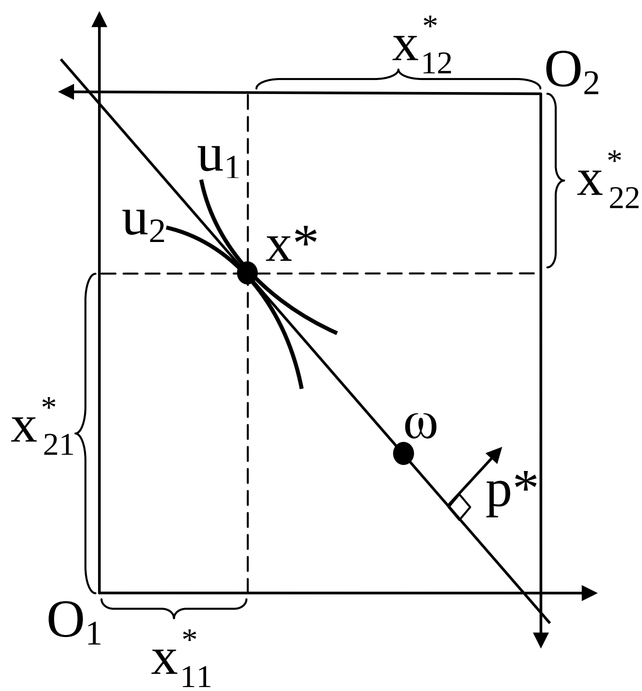
The **diagram at the left** shows **Consumer 1's budget set** B_1(p), which is determined by his or her endowment, ω, and prices, p. We assume that preferences $\succsim$ are strictly convex, continuous & strongly monotone. For those prices the **highest utility curve Consumer 1 can reach is point $\mathbf{x_1}$**. That is, given prices p consumer 1 wants to trade some of good 1 in return for good 2, to move from point ω to x_1.

The **diagram at the right** shows **three different points** which are analogous to point x_1 in the diagram at the left, **for three different sets of prices**. More generally, for each possible price vector (each possible slope of the price line) there is an "offer" for which Consumer 1 is will to trade good 1 or good 2 for the other good. **Connecting** all of **these points** produces the **offer curve**, denoted by OC_1.

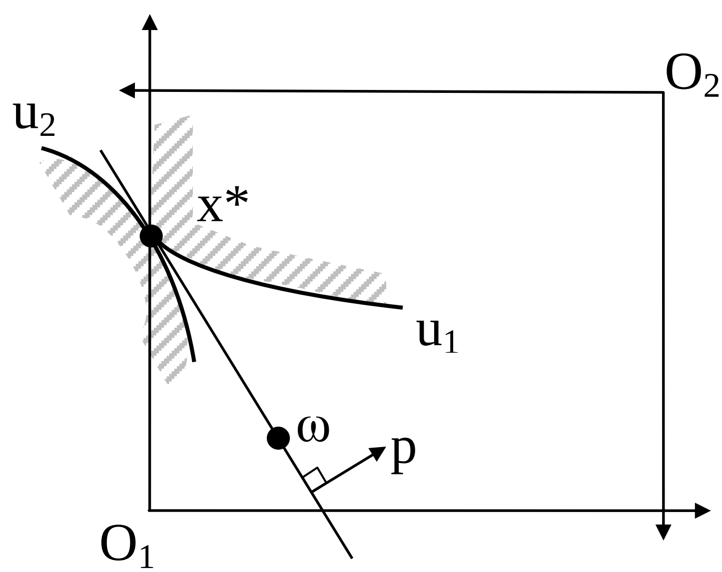
Excess demand for good 2



Walrasian Equilibrium



Walrasian Equilibrium "at the boundary"



Next, let's bring in the second consumer. The **figure on the upper left** shows, for some price vector p, the chosen **consumption bundles** of consumers 1 and 2, which are labeled x_1 and x_2, respectively. Note that they **do not** "**match**". At this price, Consumer 2 would like to supply a relatively small amount of good 2 but consumer 1 would like to purchase a much larger amount of good 2, so **supply does not equal demand** at these (relative) prices.

In contrast, in the **diagram at the upper right supply equals demand** for both goods. This leads to the following definition:

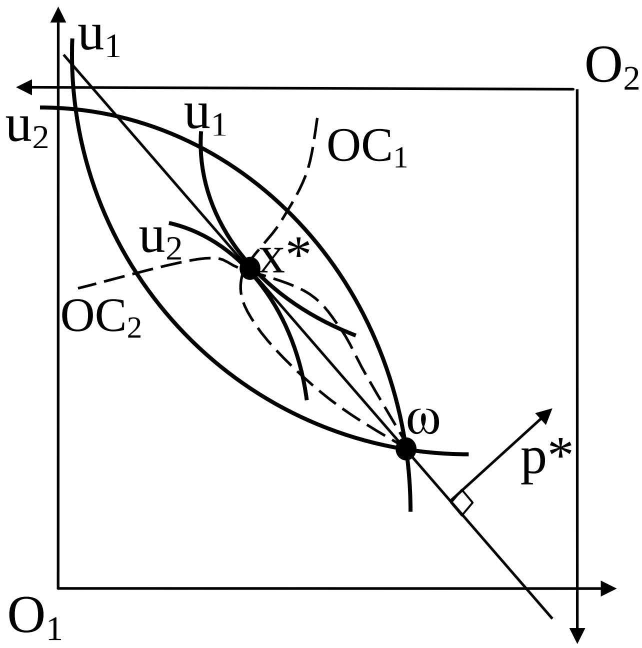
Definition: A **Walrasian** (or "competitive") **equilibrium** for an Edgeworth box economy is a price vector p* and an allocation $x^* = (x_1^*, x_2^*)$ in the Edgeworth box such that, for i = 1, 2:

$$x_i^* \succsim_i x_i' \text{ for all } x_i' \in B_i(p^*)$$

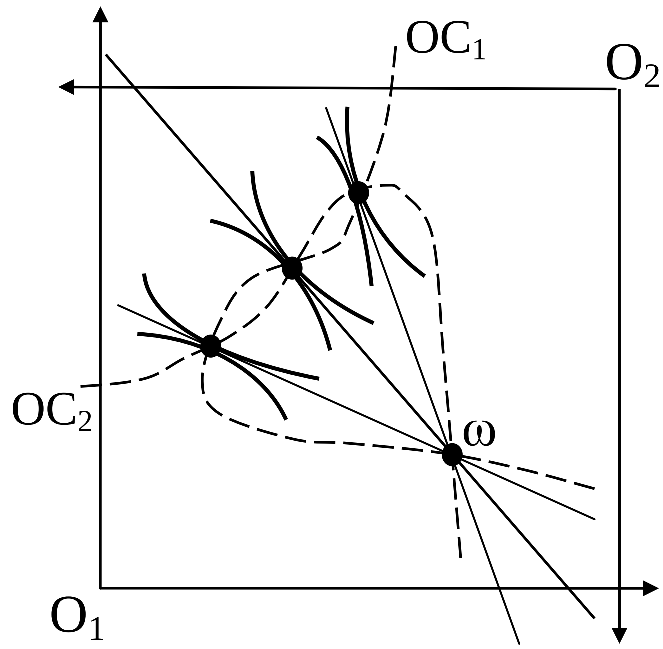
Question: Is the third figure on p.7 a Walrasian equilibrium according to this definition?

Perhaps the best way to see how a Walrasian equilibrium is determined is to plot out the offer curves for both consumers. This is done in the following diagrams:

Offer curves intersect at the Walrasian equilibrium



Offer curves with multiple equilibria



In the figure at the left, the offer curve of Consumer 1, the dashed line labeled OC_1, indicates what Consumer 1 is willing to trade for given various (relative) prices p. A similar offer curve is shown for Consumer 2, the dashed line labeled OC_2. Only at the point x*, which corresponds to the (relative) price p*, does the supply equal demand for both goods. At that price, the value of x_i for both consumers is the optimal amount given the budget constraints they face.

The figure at the right shows that it is possible that there is more than one price for which there is a Walrasian equilibrium. Further assumptions are needed to determine which equilibrium will be the one “chosen”.

III. Some Examples

In this section we examine two examples for specific functional forms of the two consumers' utility functions.

Example 1. Assume that each consumer has the following Cobb-Douglas utility function:

$$u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} x_{2i}^{1-\alpha}, \quad \text{for } i = 1, 2$$

We saw in Lecture 5 that the corresponding demands are:

$$x_1 = \frac{\alpha w}{p_1}, \quad x_2 = \frac{(1-\alpha)w}{p_2}$$

For each consumer, wealth (w) is not a given amount of money but an endowment of each of the two goods. Assume that Consumer 1 has an endowment $\omega_1 = (1, 2)$ and Consumer 2 has an endowment $\omega_2 = (2, 1)$. Then **the demands**, which **map out the offer curves**, are:

$$OC_1(p) = \left(\frac{\alpha(p_1 + 2p_2)}{p_1}, \frac{(1-\alpha)(p_1 + 2p_2)}{p_2} \right)$$

$$OC_2(p) = \left(\frac{\alpha(2p_1 + p_2)}{p_1}, \frac{(1-\alpha)(2p_1 + p_2)}{p_2} \right)$$

One way to determine the equilibrium (relative) prices is to recall that total demand for each good must equal 3.

Applying this to the first good implies:

$$\frac{\alpha(p_1^* + 2p_2^*)}{p_1^*} + \frac{\alpha(2p_1^* + p_2^*)}{p_1^*} = 3$$

This expression simplifies to $\alpha + 2\alpha(p_2^*/p_1^*) + 2\alpha + \alpha(p_2^*/p_1^*) = 3\alpha + 3\alpha(p_2^*/p_1^*) = 3$, which implies:

$$\frac{p_1^*}{p_2^*} = \frac{\alpha}{1-\alpha}$$

These prices also equate total demand with the endowment (also = 3) for good 2. (You should check this on your own.) This is a **general feature of an economy** with two goods. The **prices that clear the market for one** of the goods **will also clear the market for the other** good.

Example 2. Next, consider the following utility functions for the two consumers:

$$u_1(x_{11}, x_{21}) = x_{11} - (1/8)x_{21}^{-8}$$

$$u_2(x_{12}, x_{22}) = -(1/8)x_{12}^{-8} + x_{22}$$

These utility functions are **quasilinear**; for Consumer 1 the numeraire commodity is good 1, and for Consumer 2 the numeraire commodity is good 2.

Let the endowments for the two consumers be $\omega_1 = (2, r)$ and $\omega_2 = (r, 2)$, where r is chosen so that equilibrium prices are round numbers ($r = 2^{8/9} - 2^{1/9}$). With a little work, you can show that the two consumers have the following offer curves:

$$OC_1(p_1, p_2) = \left(2 + r\left(\frac{p_2}{p_1}\right) - \left(\frac{p_2}{p_1}\right)^{8/9},\ \left(\frac{p_2}{p_1}\right)^{-1/9} \right) >> 0$$

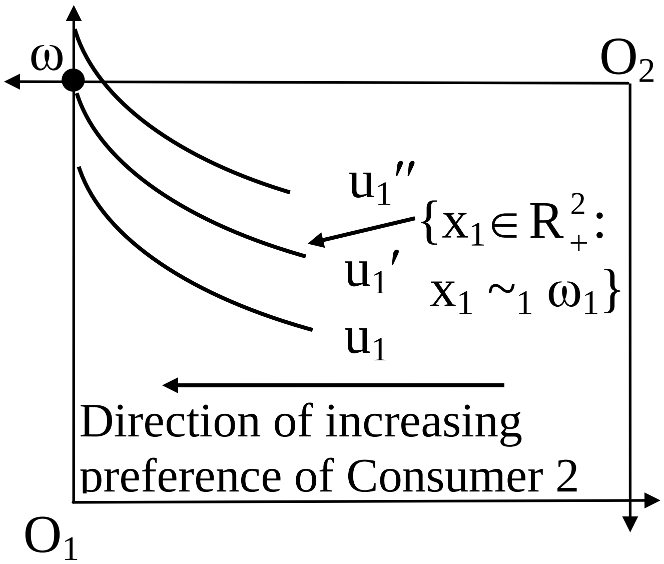
$$OC_2(p_1, p_2) = \left(\left(\frac{p_1}{p_2}\right)^{-1/9},\ 2 + r\left(\frac{p_1}{p_2}\right) - \left(\frac{p_1}{p_2}\right)^{8/9} \right) >> 0$$

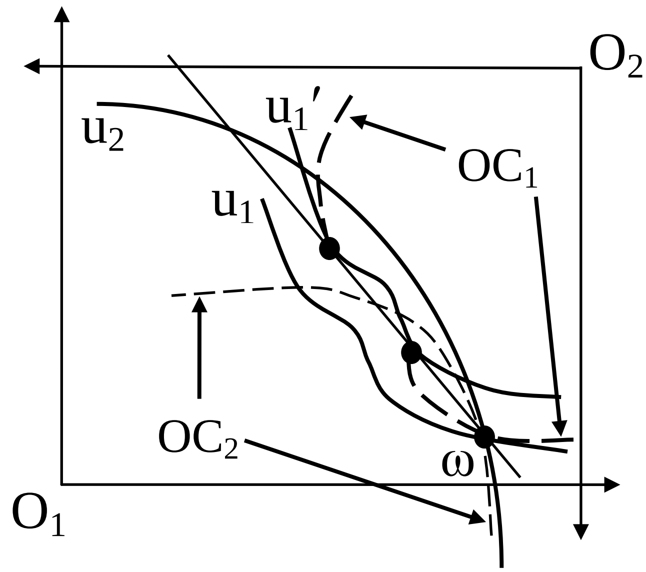
To obtain the equilibrium prices, equate the total demand for the second good with the total supply:

$$\left(\frac{p_2}{p_1}\right)^{-1/9} + 2 + r\left(\frac{p_1}{p_2}\right) - \left(\frac{p_1}{p_2}\right)^{8/9} = 2 + r$$

You should be able to show that **the following values of p_1/p_2 are solutions for this equation**: 2, 1 and 1/2. Thus in this case there are multiple Walrasian equilibria.

Finally, it is **possible that there is no Walrasian equilibrium**. Two examples are on the next page:





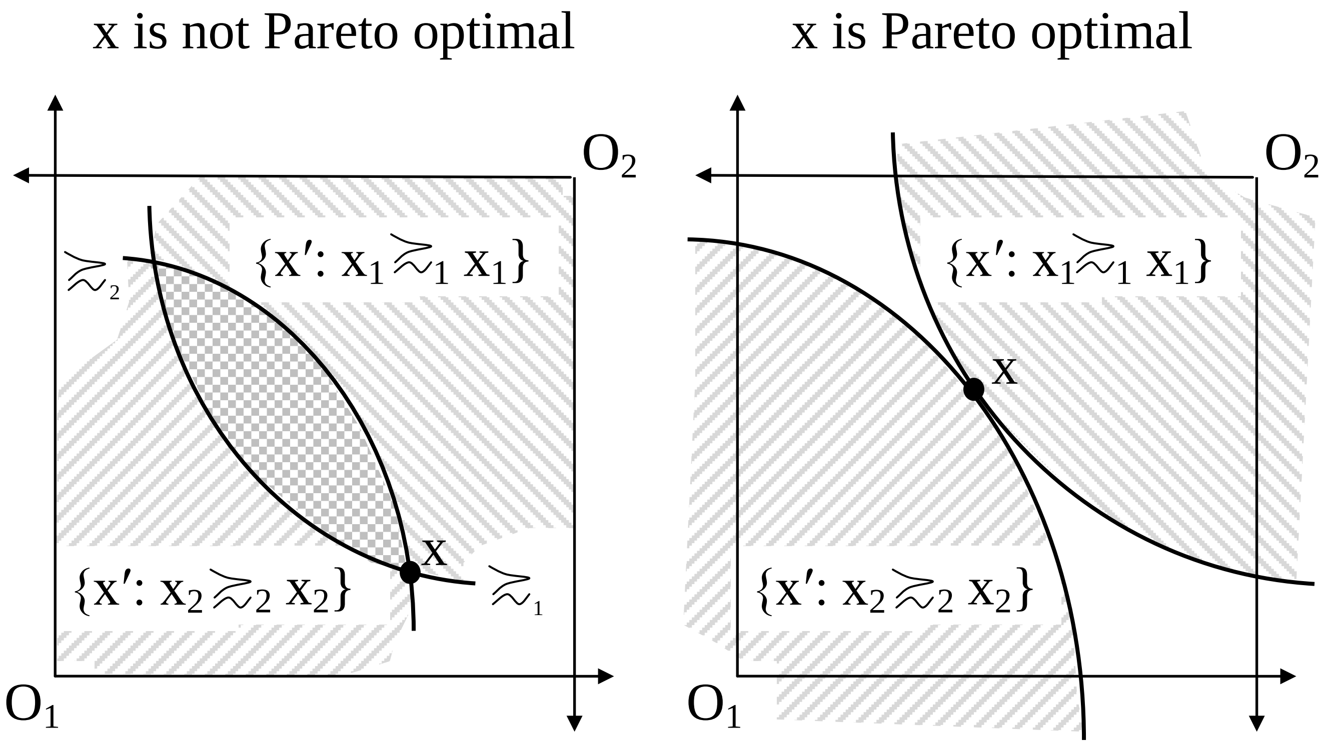
On the left, Consumer 2 wants to consume only good 1 for any relative prices, while Consumer 1 wants to trade good 2 to get some good 1 at any prices. On the right, Consumer 1's preferences are non-convex, so the offer curve has a gap. Apec 8004 explains the restrictions on preferences necessary to ensure that an equilibrium exists.

IV. Welfare Properties of Walrasian Equilibria

Edgeworth boxes are useful for explaining several welfare properties of Walrasian equilibria. **Pareto optimality** (also called **Pareto efficiency**) is the most fundamental concept of welfare economics. The following definition is for the two-consumer pure exchange economy:

Definition: An allocation x in the Edgeworth box is **Pareto optimal** if there is no other allocation x′ in that box with $x_i' \succsim_i x_i$ for i = 1, 2 and $x_i' \succ_i$ for at least one i.

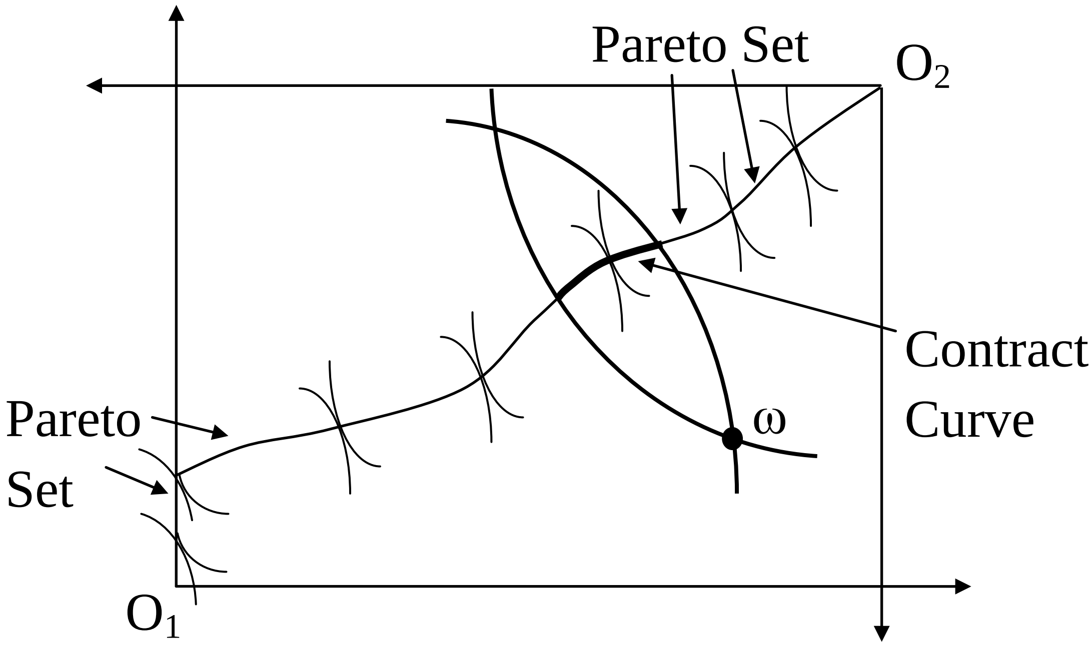
Consider the following diagrams:



In the diagram at the left, x is not Pareto optimal because all the darkly shaded points inside the “elipse” are points for which one or both of the consumers are better off relative to the point x. In contrast, in the diagram on the right x is Pareto optimal because there are no other points for which one (or both) consumer can be made better off while the other is not made worse off.

Note that for **Pareto optimal points in the interior of the Edgeworth box the indifference curves of the two consumers are tangent to each other**. See p.7 for a “boundary solution” that is Pareto optimal but the indifference curves are not tangent to each other.

Next, consider the following diagram:

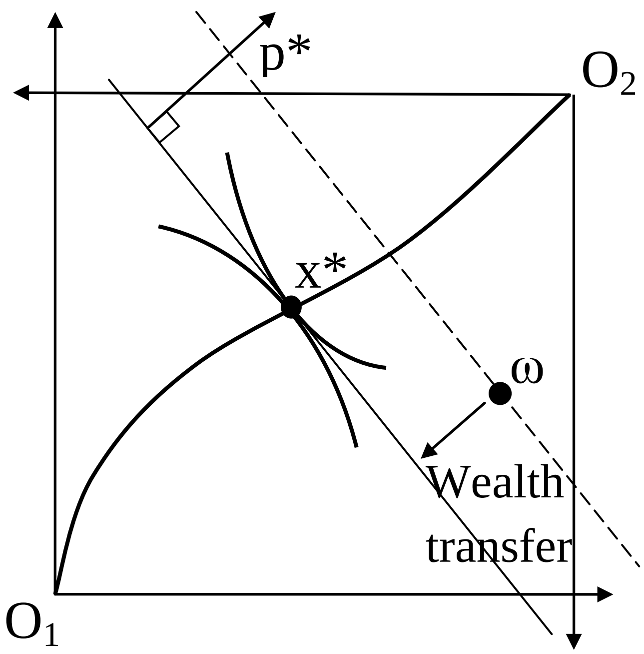


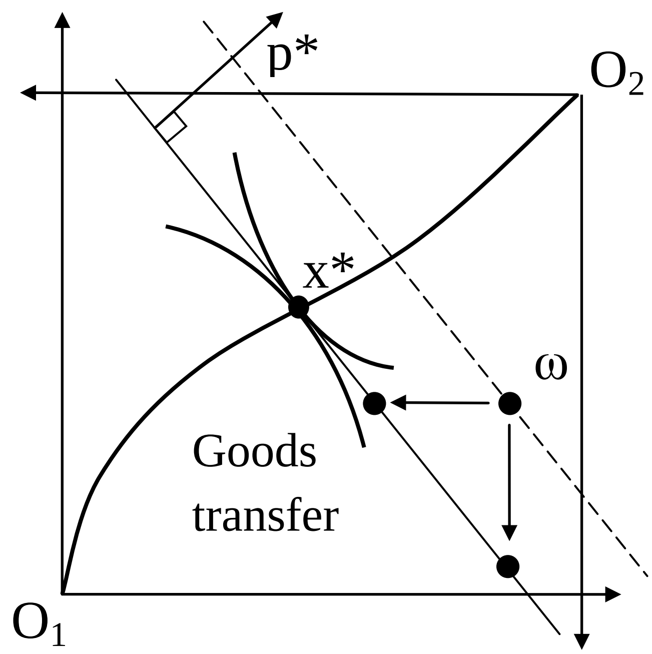
In this diagram, the set of all Pareto optimal allocations is called the **Pareto set**. For the initial endowment ω, the diagram also shows the **contract curve**: the **section of the Pareto set** where **each consumer is at least as well off as** he or she is **at the endowment point** (ω).

A **very important** conclusion is that **any Walrasian equilibrium allocation x* must lie on the Pareto set**. Look at the 2 Walrasian equilibria on p.7. The budget lines separate the two "at least as good as" sets. The only common point for the two sets is x*. Thus for any Walrasian equilibrium x* there is no feasible alternative allocation that can make one person or both people better off. Thus **all Walrasian equilibria are Pareto optimal**.

That Walrasian allocations yield Pareto optimal allocations is the **first fundamental theorem of welfare economics**.

A final point: it is often possible to reach any Pareto optimal allocation by reallocating wealth or initial endowments:





This expresses the **second fundamental theorem of welfare economics**. This leads to an important definition:

Definition: An allocation x* in the Edgeworth box is supportable as an **equilibrium with transfers** if there is a set of prices p* and wealth transfers T_1 and T_2 satisfying $T_1 + T_2 = 0$, such that for each consumer i we have:

$$x_i^* \succsim_i x_i' \quad \text{for all } x_i' \in \mathbb{R}^2_+ \quad \text{such that } p^* \cdot x_i' \leq p^* \cdot \omega_i + T_i$$

This is possible only if the preferences of both consumers are continuous, convex and strongly monotone.