AFRE 802
Statistical Methods for Agricultural, Food, & Resource Economists

Wrap-up of Ch. 1 and

Introduction to Probability – Part 1 of 2 (WMS Ch. 2.1-2.6)

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GAME PLAN

1. Review
2. Finish **Intro to Stats** (Ch. 1) - normal distribution & the empirical rule
3. Start **Probability** (Ch. 2)
   a. What is probability?
   b. Set notation & Venn diagrams
   c. Probabilistic models
   d. The sample point method for calculating the probability of an event
   e. Some tools for counting sample points

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REVIEW

- *What are the main objectives of statistics?*
- *What are some ways we can summarize data?*
- *Formulas for sample mean, variance, and standard deviation?*

Sample mean

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

Sample variance

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$$

Sample std. dev.

$$s = \sqrt{s^2}$$

- *Population analogues?*
  - μ (pop. mean), σ^2 (pop. variance), σ (pop. std. dev.)

REVIEW (cont'd) - *Any questions on example?*

Obs. #	Ha cultivated	$y_i - \bar{y}$	$(y_i - \bar{y})^2$
1	2	-0.27	0.0729
2	0	-2.27	5.1529
3	2	-0.27	0.0729
4	1.1	-1.17	1.3689
5	2.5	0.23	0.0529
6	0.5	-1.77	3.1329
7	5.5	3.23	10.4329
8	1.1	-1.17	1.3689
9	7	4.73	22.3729
10	1	-1.27	1.6129
N = 10	Sum = 22.7		Sum = 45.6410

Sample mean

$$\bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i = \frac{22.7}{10} = 2.27$$

Sample variance

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2 = \frac{45.6410}{9} = 5.0712$$

Sample std. dev.

$$s = \sqrt{s^2} = \sqrt{5.0712} = 2.2519$$

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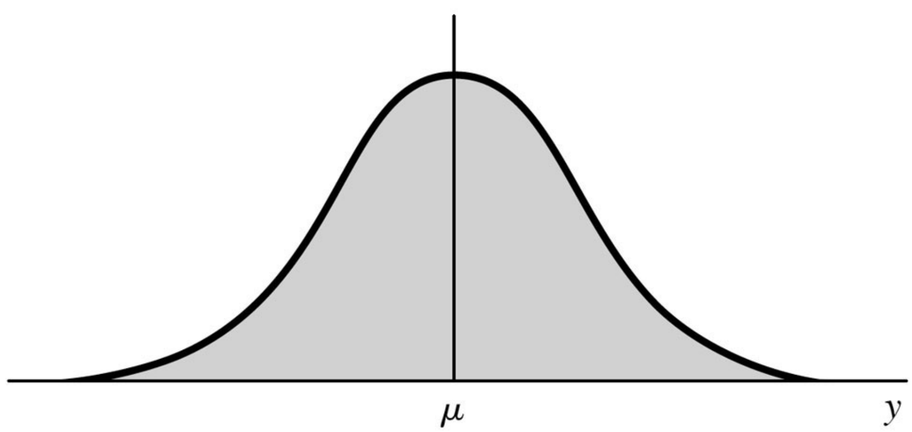
Additional example – handout

- For you to try at home
- Graded in-class exercises will be similar in spirit
- The key is to review material and examples from class

Normal distribution

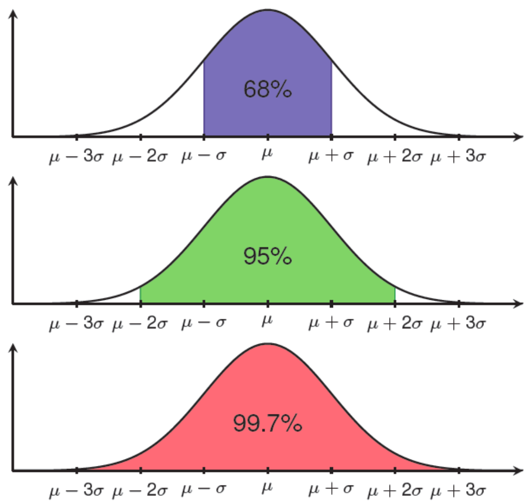
- Bell-shaped curve, symmetric



The "empirical rule"

For a distribution that is <u>approximately</u> normal:

•68% of obs. are w/in σ of μ
•95% of obs. are w/in 2σ of μ
•Almost all obs. (99.7%) are w/in 3σ of μ



What is the probability of being more than 2σ from the mean?

Are such values relatively common or rare?

We'll use this a lot when we get to hypothesis testing later in the course

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The "empirical rule"

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•68% of obs. are w/in σ of μ

•95% of obs. are w/in 2σ of μ

•Almost all obs. (99.7%) are w/in 3σ of μ

<u>EXAMPLE</u>

Scores on the quantitative portion of a recent GRE were approximately normally distributed and averaged 151 with a standard deviation of 9.

a. Approximately what percentage of test-takers got scores between 142 and 160?
b. An elite grad school only considers applicants with quantitative GRE scores in the top 2.5%. What minimum score would this be?

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PROBABILITY
(WMS Chapter 2)

What is probability?

- "One's belief in the occurrence of a future event" (WMS)
- The likelihood of something happening
- A measure of the likelihood that an event will occur

When does the concept of probability come up in your work?

→ Probability theory is the foundation of statistical inference

→ We'll study how to measure it & how it helps us draw inferences

→ Focus on **random** or "**stochastic**" events – i.e., events that cannot be predicted with certainty but whose relative frequency is stable in large # of trials. *Examples?*

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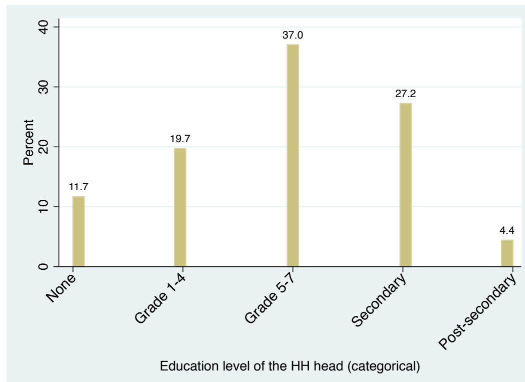
Probability and relative frequencies

- Can use relative frequencies to get a sense of probabilities

EX) Based on these data, what is the likelihood that a randomly selected smallholder farm HH head has some formal education?



Education levels of the household heads of Zambian smallholder farms

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Set notation

- Let *A, B, C, ...* denote **sets of points**
- Let $A = \{a_1, a_2, a_3\}$ denote that a_1, a_2, and a_3 are **elements of set** *A*
- Let *S* be the "universal set" (the set of all elements under consideration)
- Let Ø denote a "null set" or "empty set" (contains no points)
- Use Venn diagrams and set notation to describe the relationships between sets
- **Exercise**: Break into pairs and go through slides 12-18 and do examples

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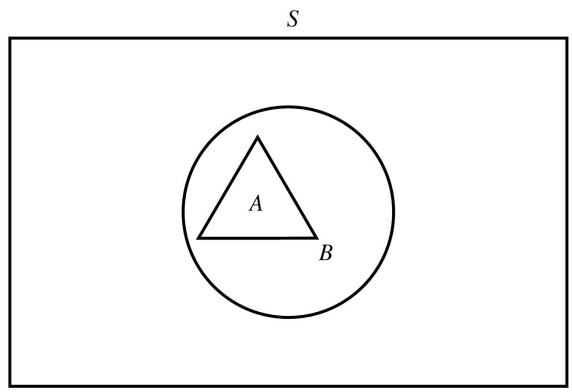
Set notation (cont'd)

- *A* is a **subset** of *B* (all points in *A* are in *B*): $A \subset B$
- EX) *A*={2, 4, 6}, *B*={0, 2, 4, 6, 7, 10}, so all points in *A* are also in *B*



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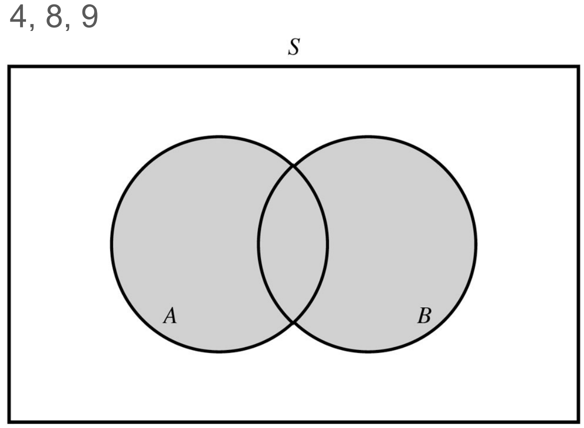
Set notation (cont'd)

- **Union** of *A* and *B* (all points in *A* or *B* or both): *A* U *B*
- EX) *A*={1, 8, 9}, *B*={0, 2, 4, 8}. *What is A U B*?
  - 0, 1, 2, 4, 8, 9



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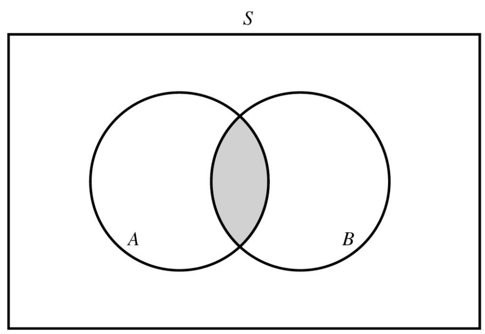
Set notation (cont'd)

- **Intersection** of *A* and *B* (all points in both *A* and *B*):
  $A \cap B$
- EX) A={1, 8, 9}, B={0, 2, 4, 8}. *What is* $A \cap B$?
  - 8



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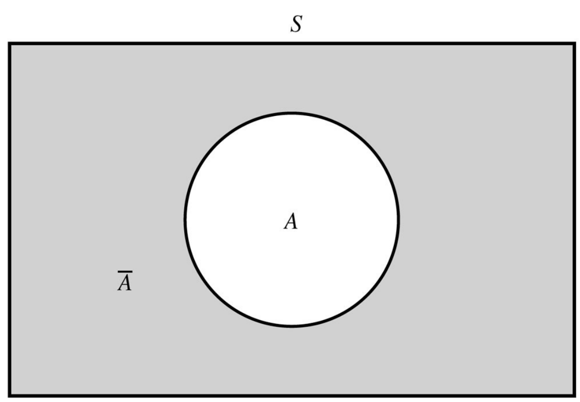
Set notation (cont'd)

- **Complement** of *A* (all points not in *A*): $\bar{A}$
- EX) S={0, 1, 2, 4, 8, 9}, A={1, 8, 9}. *What is* $\bar{A}$?
  - 0, 2, 4

Note:

$A \cup \bar{A} = S$

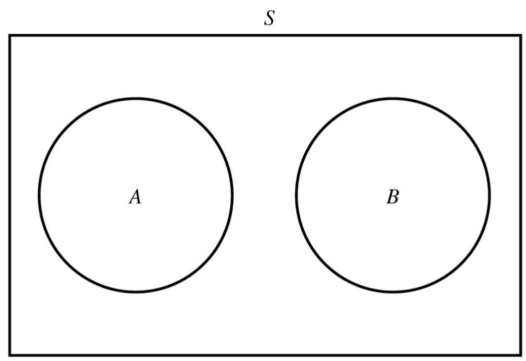


Set notation (cont'd)

- *A* and *B* are **mutually exclusive** (or **disjoint**) if no points are in both *A* and *B*, i.e.: $A \cap B = \varnothing$
  EX) *A*={1, 8, 9}, *B*={0, 2, 4}.

Also note:

$A \cap \bar{A} = \varnothing$



Set notation (cont'd)

- **Distributive laws:**

$$(i)\ A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(ii)\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- *EX) Suppose S={1, 2, 3, 4, 5, 6}, A={1, 2}, B={1, 3}, and C={2, 4, 6}. Show that (i) and (ii) hold.*

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Set notation (cont'd)

- **DeMorgan's laws**

$$(i)\ \overline{(A \cap B)} = \overline{A} \cup \overline{B}$$

$$(ii)\ \overline{(A \cup B)} = \overline{A} \cap \overline{B}$$

- *EX) Suppose S={1, 2, 3, 4, 5, 6}, A={1, 2}, B={1, 3}, and C={2, 4, 6}. Show that (i) and (ii) hold.*

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Experiments and sample spaces

- **Experiment**
  - WMS: "the process by which an observation is made"
  - A process that is repeated under (nearly) identical conditions and terminates with an outcome ("events")
- **Random experiment**: experiment whose outcome cannot be predicted with certainty, but every possible outcome can be described prior to the experiment
- **Sample space** (call is ***S***): the set of all possible outcomes from a random experiment

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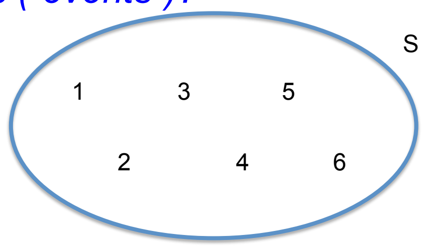
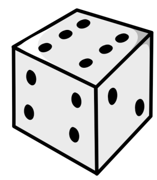
Sample space - example

- *If you roll a fair die once, what is the sample space (S), i.e., what are all of the possible outcomes ("events")?*



- **Simple vs. compound events**
  - Simple: can't be decomposed further
  - Compound: can be decomposed into simple events. *Example in die roll experiment above?*



Probabilistic model for an experiment with a discrete sample space

- Each simple event associated with an experiment corresponds to a "**sample point**"
  - Compound event = collection of sample points
- **Discrete sample space**: finite or countable number of distinct sample points
- Once we identify the sample space, we can assign a probability to each sample point
  - E.g., Rolling a fair die – probability of each sample point is 1/6
- *What are some characteristics of these probabilities (e.g., range, sum)?*

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Let *S* be the sample space of an experiment,

A denote events in (outcomes of) the experiment, and

P(A) be the probability of event *A*.

When assigning probabilities, 3 conditions must hold:

1. $0 \leq P(A) \leq 1$
2. $P(S) = 1$ (sum of probabilities of all simple events in $S = 1$)
3. The probability of the union of mutually exclusive events is the sum of the probabilities of those events. i.e.,

If $A_i \cap A_j = \varnothing$ for all $i \neq j$, then

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots A_k) = P(A_1) + P(A_2) + P(A_3) + \ldots + P(A_k) = \sum_{i=1}^{k} P(A_i)$$

- *Do these conditions hold for the probabilities we assigned to our fair die tossing experiment?*
- *What is the probability of getting a 2 or a 5?*

Two methods for calculating the probability of an event

1. The **sample point method** (today)

2. The **event-composition method** (Thursday)

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The sample-point method for calculating the probability of an event

Essentially what we just did intuitively with the die-rolling experiment. More formally:

1. Define the experiment
2. Define the sample space (S) by identifying all of the possible <u>simple</u> events / outcomes (call these E_i)
3. Assign a probability to each simple event. Be sure these probabilities satisfy:
$$0 \leq P(E_i) \leq 1 \quad \text{and} \quad \sum_i P(E_i) = 1$$
4. Define the event of interest (call it A) and decompose it into its component simple events
5. Find $P(A)$ by summing the probabilities of these simple events

Probability of an event when sample points are all equally probable

- *EX) If have 100 equally probable sample points, and compound event A contains 10 of those sample points, what is P(A)?*

- If a sample space contains N sample points that <u>can occur with equal probability</u>, and compound event A contains n_A of those sample points, then

$$P(A) = \frac{n_A}{N} = \frac{\text{number of ways } A \text{ can occur}}{\text{total number of outcomes}}$$

The sample point method: pros & cons

- **Pros**:
  1. Simple / intuitive
  2. Can always be applied when there is a finite or countable set of sample points
- **Cons**:
  1. Not foolproof: we might forget some sample points
  2. Can be tedious / time-consuming if a lot of events

→ Can use results from "**combinatorial analysis**" to more systematically count sample points.

4 tools from combinatorial analysis to help us count sample points

1. The "mn" rule
2. Permutations
3. Combinations
4. Partitioning *n* objects into *k* non-overlapping groups

Tools for counting sample points

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1. The "mn" rule

- Suppose there are 2 groups:
  - One contains m elements ($a_1, a_2, \ldots, a_m$)
  - One contains n elements ($b_1, b_2, \ldots, b_n$)
- If we pick 1 element from each group, how many distinct pairs can we form?
  - **mn**
  - *EX) If roll a die and toss a coin, how many distinct pairs are possible (e.g., (1, H), (5, T), etc.)?*
- More generally, if there are p groups, and the 1st has n_1 elements, the 2nd has n_2 elements,…, and the p^{th} has n_p elements, then we can form

$$n_1 \, X \, n_2 \, X \ldots X \, n_p$$

distinct sets containing one element from each group

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Example of the "mn" rule

- *Suppose a license plate has 3 letters followed by 3 numbers. How many distinct license plates are possible? (Letters & numbers can be repeated.)*

$$n_1 \, X \, n_2 \, X \ldots X \, n_p$$

$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17{,}576{,}000$

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2. Permutations

- **Permutation** = an ordered arrangement of objects
- The number of ways of ordering *n* distinct objects taken *r* at a time (or the number of ways of filling *r* distinct positions drawing from *n* distinct objects without replacement) is:

$$P_r^n = \frac{n!}{(n-r)!} \quad where\ n! = n(n-1)(n-2)\cdots 1$$

- *EX) The the GSO president, VP, and secretary/treasurer can be selected from among 60 grad students. Each person can hold only 1 office. How many sample points are there in this experiment?*

$$P_3^{60} = \frac{60!}{(60-3)!} = \frac{60!}{57!} = 60 \cdot 59 \cdot 58 = 205{,}320$$

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3. Combinations

- **Combination**= a selection of a set of objects without regard to the order
  - Contrast to permutations where order matters
- The number of unordered subsets of size *r* chosen from *n* available objects (without replacement) is:

$$\binom{n}{r} = C_r^n = \frac{n!}{r!(n-r)!} = \frac{P_r^n}{r!}$$

- *EX) 3 grad students are selected from among the 60 to serve on an ad hoc committee. How many sample points are there in this experiment? Why is this fewer than in the previous example?*

$$\binom{60}{3} = C_3^{60} = \frac{60!}{3!(60-3)!} = \frac{60!}{3!57!} = \frac{60 \cdot 59 \cdot 58}{3 \cdot 2 \cdot 1} = 34{,}220$$

Examples of permutations or combinations you found?

4. # of ways of partitioning *n* distinct objects into *k* non-overlapping groups

- Let the groups contain $n_1, n_2, \ldots, n_k$ objects such that: $$\sum_{i=1}^{k} n_i = n$$
- Then the number of ways of partitioning the *n* distinct objects into *k* distinct groups is:

$$\binom{n}{n_1\ n_2\ \cdots\ n_k} = \frac{n!}{n_1!n_2!\cdots n_k!}$$

- *EX) The full GSO has 14 positions. These 14 people are divided into two 5-member and one 4-member subcommittees. How many sample points are there in this experiment?*

$$\frac{n!}{n_1!n_2!\cdots n_k!} = \frac{14!}{5!5!4!} = 252{,}252$$

The sample point method: example

If you randomly selected 2 days (without replacement) from a given 7-day week this semester, what is the probability that you would have AFRE 801 or 802 lecture both days? (Ignore weeks w/ holidays and the first week of class.)

1. List the sample space and assign probabilities to each sample point.
   - {MTu}, {MW}, {MTh}, {MF}, {MSa}, {MSu}
     {TuW}, {TuTh}, {TuF}, {TuSa}, {TuSu}, {WTh}, {WF}, {WSa}, {WSu},
     {ThF}, {ThSa}, {ThSu}, {FSa}, {FSu}, {SaSu} = 21
     so prob of each is 1/21
   - Can check with combination: ${}_7C_2$=7!/(2!5!)=(7*6)/2=21
2. Event A is that 2 of the days picked are Mon-Thurs. Count the number of sample points that correspond to event A.
   - 6 sample points or use combination: ${}_4C_2$=4!/(2!2!)=(4*3)/2=6
3. Calculate the probability of event A.
   - 6/21=3/7

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Tools for counting sample points

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Summary

- An essential step in the sample point method of calculating the probability of an event is to identify the total number of sample points in the sample space and the number of sample points that correspond to the event of interest
- When there is a small number of sample points, we can count the sample points by hand
- But when there are many sample points, tools from combinatorial analysis (the *mn* rule, permutations, combinations, etc.) can help us do this more efficiently and accurately

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Homework:

- Finish Ch. 1 HW → due Thurs. 9/7 (beginning of class)
- WMS Ch. 2 (part 1 of 2)
  - Set notation: 2.1, 2.2, 2.3
  - Probabilistic model w/ discrete sample space: 2.19
  - Sample point method: 2.25, 2.33
  - Tools for counting sample points: 2.36, 2.41, 2.43, 2.44, 2.51, 2.57

Next class:

- Wrap-up of probability including Bayes' Rule

Reading for next class:

- WMS Ch. 2: 2.7 through 2.13

Application to look into for next class:

- Find an example of how Bayes' Rule is used in your field