Description and Analysis of Systems

Linearity and LTI Systems

- If a system is both homogeneous and additive it is *linear*.
- If a system is both linear and time-invariant it is called an *LTI* system
- Some systems which are non-linear can be accurately approximated for analytical purposes by a linear system for small excitations

Example -

$$y(t) = tx(t)$$

$$y_1(t) = tax_1(t)$$

$$y_2(t) = tbx_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$y_3(t) = tx_3(t) = t[ax_1(t) + bx_2(t)]$$

$$y_3(t) = y_1(t) + y_2(t) \rightarrow \text{Linear System}$$

Example -

$$y[n] = \mathrm{Re}\{x[n]\}$$

$$x_1[n] = r[n] + js[n]$$

$$y_1[n] = r_1[n]$$

$$x_2[n] = jx_1[n] = jr[n] - s[n]$$

$$y_2[n] = \mathrm{Re}\{x_2[n]\} = -s[n] \neq jy_1[n]$$

System violates homogeinity property.

Hence, it is non linear system.

PROPERTIES OF LINEAR SYSTEMS:

- **Superposition:**

If $\qquad x_k[n] \rightarrow y_k[n]$

Then $\qquad \sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$

- **For linear systems, zero input zero output**

"Proof" $\qquad 0 = 0 \cdot x[n] \rightarrow 0 \cdot y[n] = 0$

Properties of Linear Systems

- A linear system is causal if and only if it satisfies the condition of <u>initial rest</u>:

$$x(t) = 0 \text{ for } t \leq t_0 \rightarrow y(t) = 0 \text{ for } t \leq t_0 \ (*).$$

System Interconnections

Series or cascade Interconnection

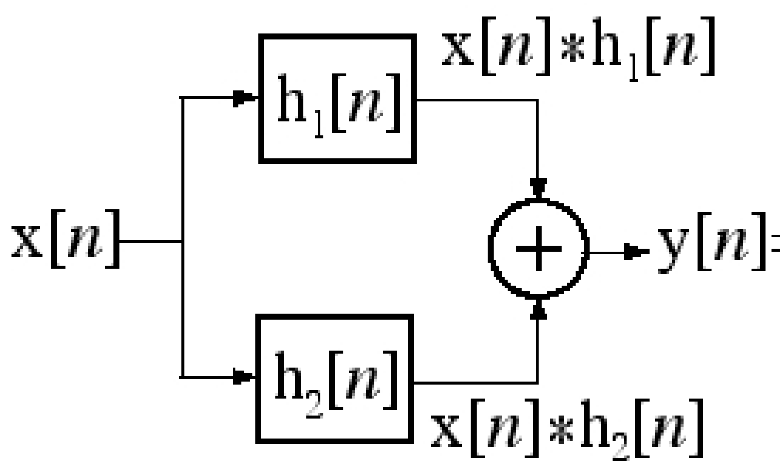
If the response of one system is the excitation of another system the two systems are said to be *series or cascade* connected.

$$\mathrm{x}[n] \longrightarrow \boxed{\mathrm{h}_1[n]} \longrightarrow \mathrm{x}[n]*\mathrm{h}_1[n] \longrightarrow \boxed{\mathrm{h}_2[n]} \longrightarrow \mathrm{y}[n]=\{\mathrm{x}[n]*\mathrm{h}_1[n]\}*\mathrm{h}_2[n]$$

System Interconnections

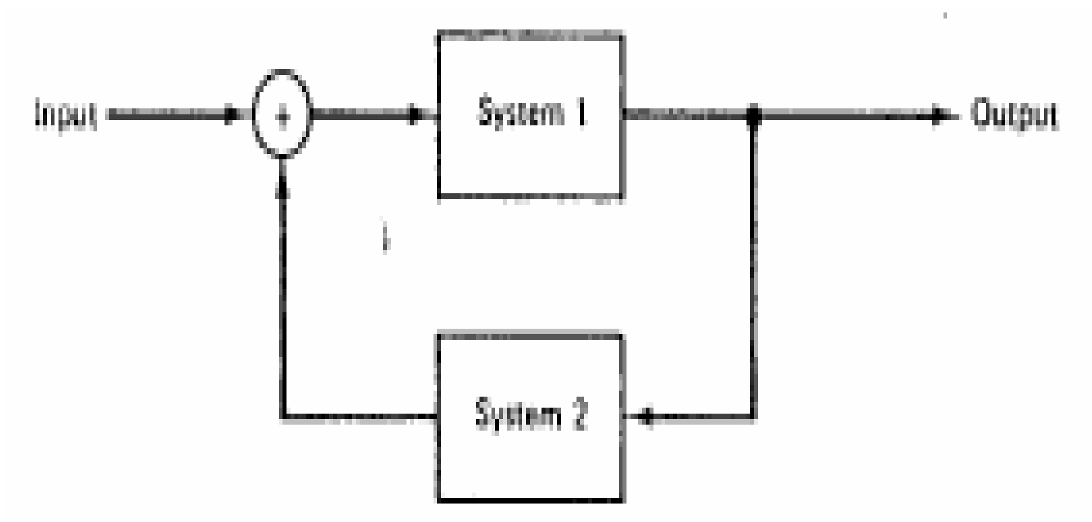
Parallel Interconnection

If two systems are excited by the same signal and their responses are added they are said to be *parallel* connected.



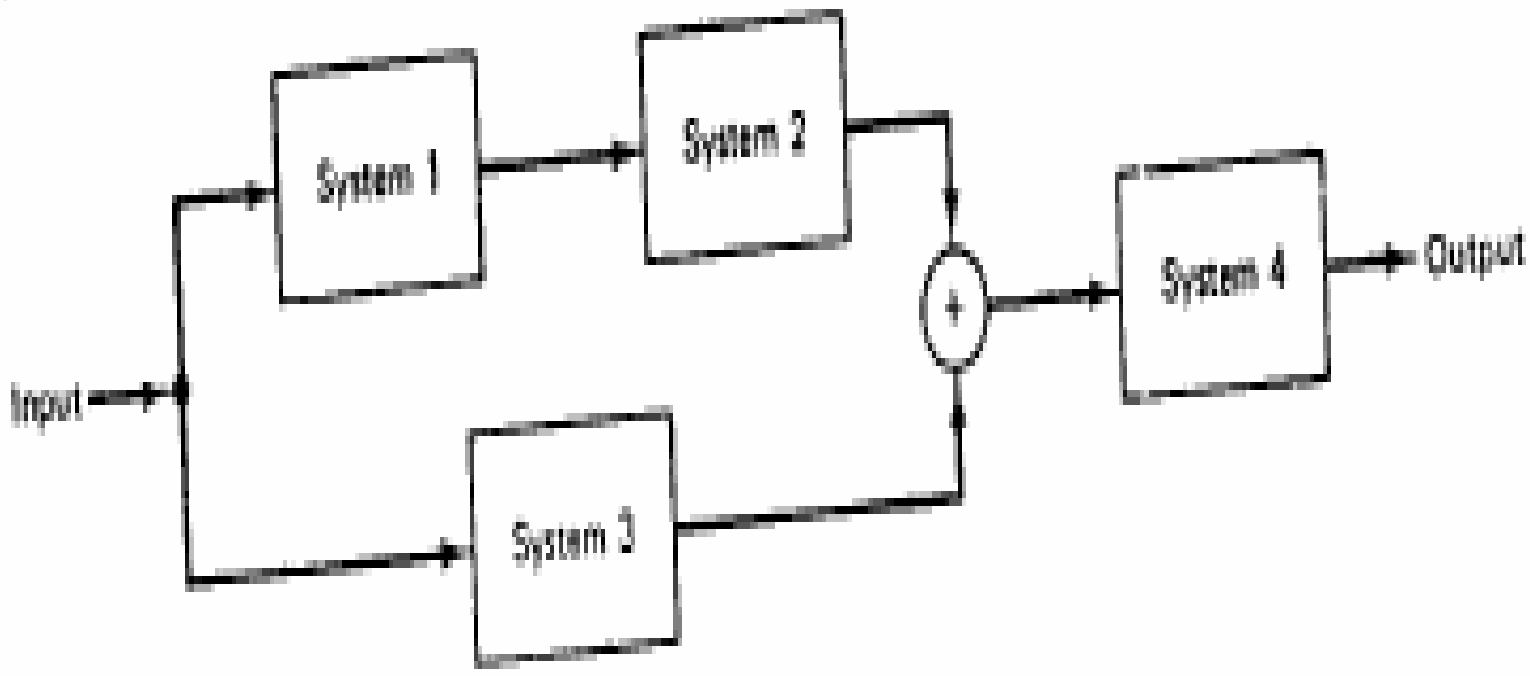
System Interconnections

Feedback Interconnection



System Interconnections

Series/Parallel Interconnection



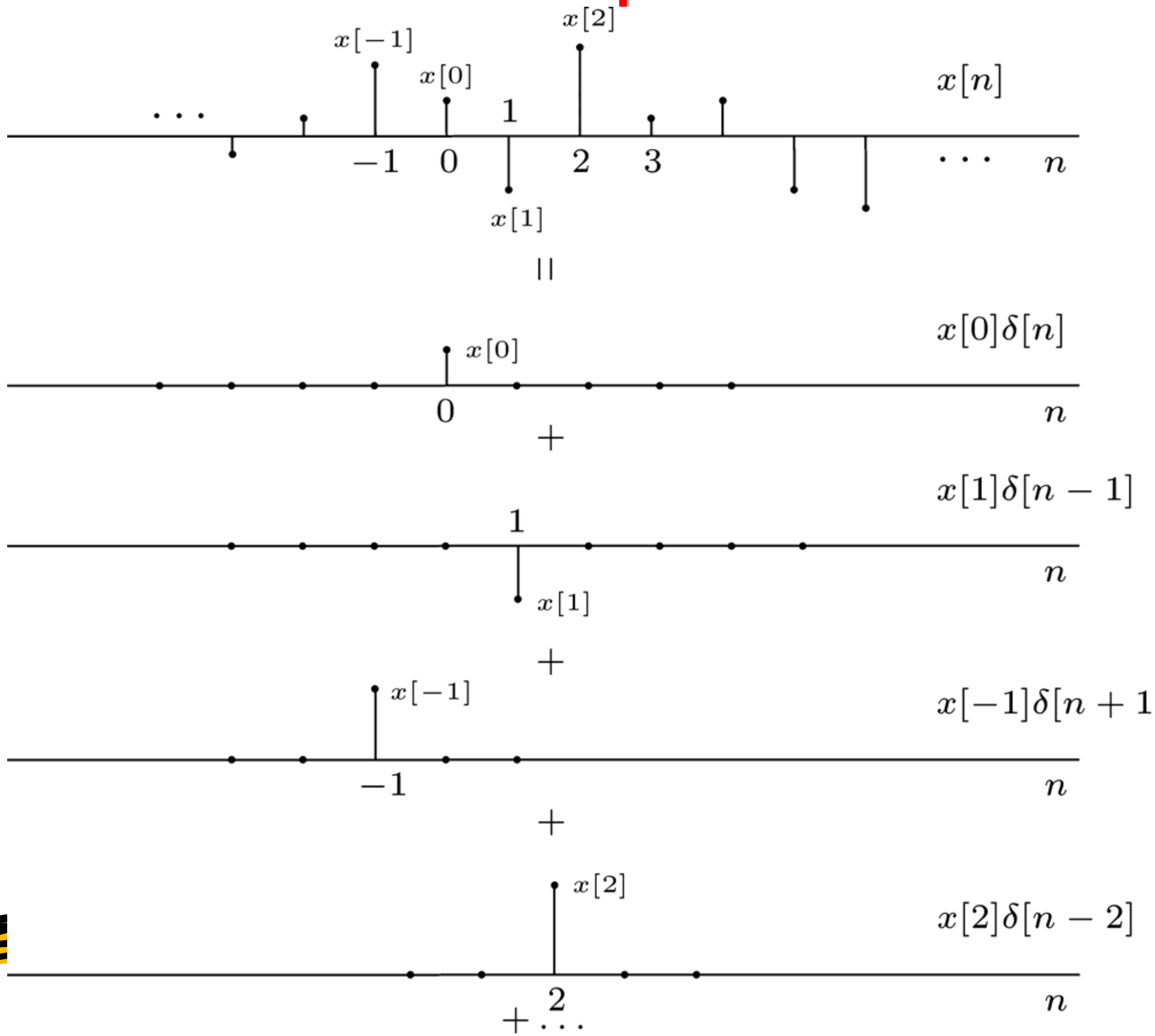
Exploiting Superposition and Time-Invariance:

$$x[n] = \sum_k a_k x_k[n] \xrightarrow{Linear System} y[n] = \sum_k a_k y_k[n]$$

For LTI Systems (CT or DT) there are two natural choices for building blocks:

DT	**Shifted unit samples**
CT	**Shifted unit impulses**

Representation of DT Signals Using Unit Samples



That is ...

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \cdots$$

$$\Downarrow$$

$$x[n] = \sum_{k=-\infty}^{\infty} \underbrace{x[k]}_{\text{Coefficients}} \, \underbrace{\delta[n-k]}_{\text{Basic Signals}}$$

Therefore all discrete time signals could be written in terms of shifted unit samples.

$$x[n] \longrightarrow \boxed{\text{DT System}} \longrightarrow y[n]$$

- Suppose the system is linear, and define $h_k[n]$ as the response to $\delta[n - k]$:

$$\delta[n-k] \rightarrow h_k[n]$$

From superposition: $$\Downarrow$$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow y[n] = \sum_{k=-\infty}^{\infty} x[k]h_k[n]$$

$$x[n] \longrightarrow \boxed{\text{DT System}} \longrightarrow y[n]$$

- Now suppose the system is **LTI**, and and define the *unit sample response h[n]:*

$$\delta[n] \rightarrow h[n]$$

$$\Downarrow$$

From TI:

$$\delta[n-k] \rightarrow h[n-k]$$

From LTI:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k] \rightarrow y[n] = \underbrace{\sum_{k=-\infty}^{\infty} x[k]h[n-k]}_{\text{Convolution Sum}}$$