Experiment No. 10

To study the behavior of Series-Parallel RLC ckt at Resonance. ↗ Circuit

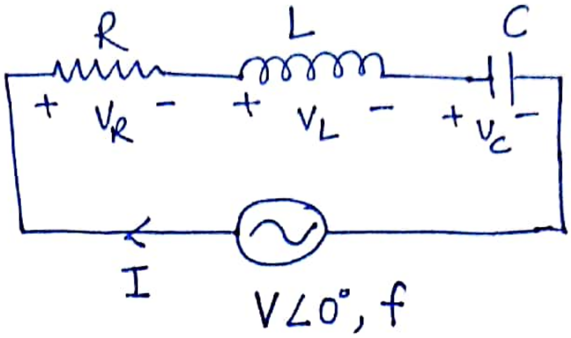
Series RLC Resonance Circuit

- Inductive Reactance,

  $X_L = 2\pi f L = \omega L$

- Capacitive Reactance,

  $X_C = \frac{1}{2\pi f C} = \frac{1}{\omega C}$



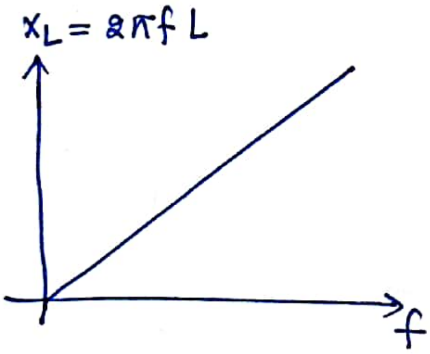
Note: when $X_L > X_C$, the circuit is inductive
when $X_C > X_L$, the circuit is capacitive

The total circuit impedance,

$$Z = R + jX_T$$

where,

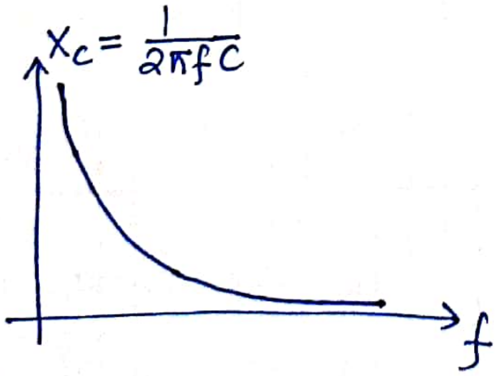
$$X_T = X_L - X_C \qquad X_L > X_C$$
$$= X_C - X_L \qquad X_C > X_L$$



At $f = 0$, $X_L = 2\pi f L = 0$

At $f = \infty$, $X_L = 2\pi f L = \infty$

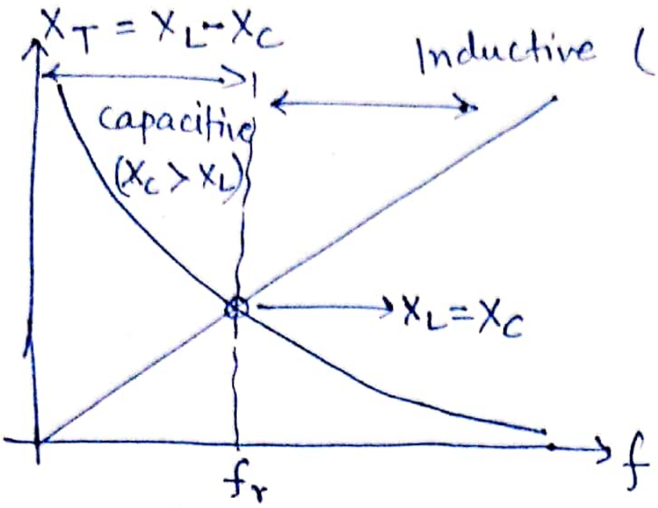
$\{X_L \propto f\}$



At $f = 0$, $X_C = \frac{1}{2\pi f C} = \infty$

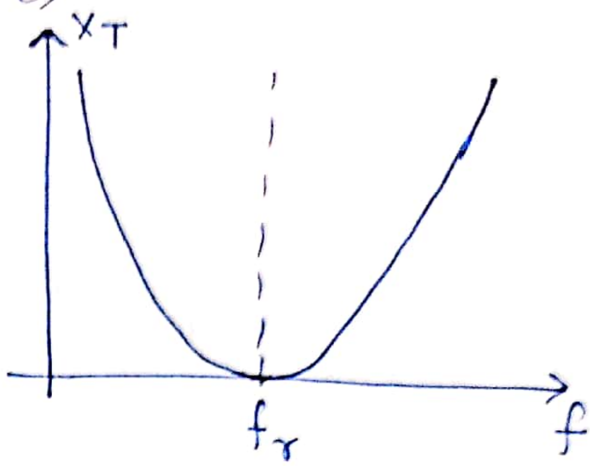
At $f = \infty$, $X_C = 0$

$[X_C \propto f^{-1}]$



(series Resonance)

$\Rightarrow$



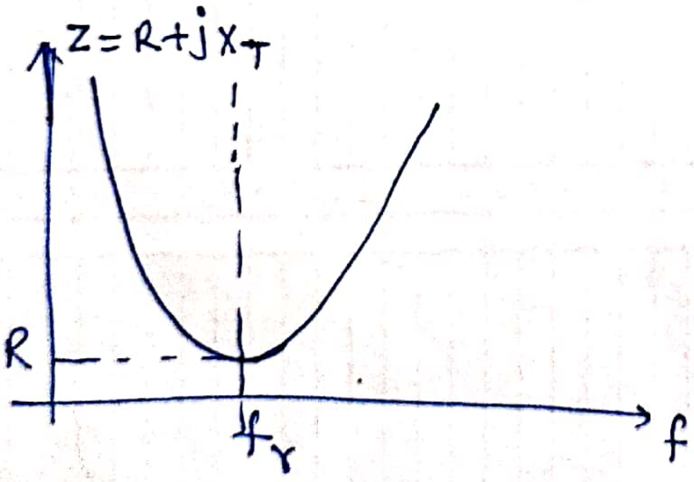
<u>At $f = f_r$</u> $\Rightarrow X_L = X_C$ $\Rightarrow Z = R + j(X_L - X_C)$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$\boxed{Z = R}$ at $f = f_r$

$$f_r = \frac{1}{2\pi\sqrt{LC}} \quad (\text{Hz})$$

$$\omega_r = \frac{1}{\sqrt{LC}} \quad (\text{rad})$$



$Z = R + j(X_L - X_C)$ ohm
↳ complex quantity

$Z = |Z| \angle \phi$ → phase
↳ magnitude

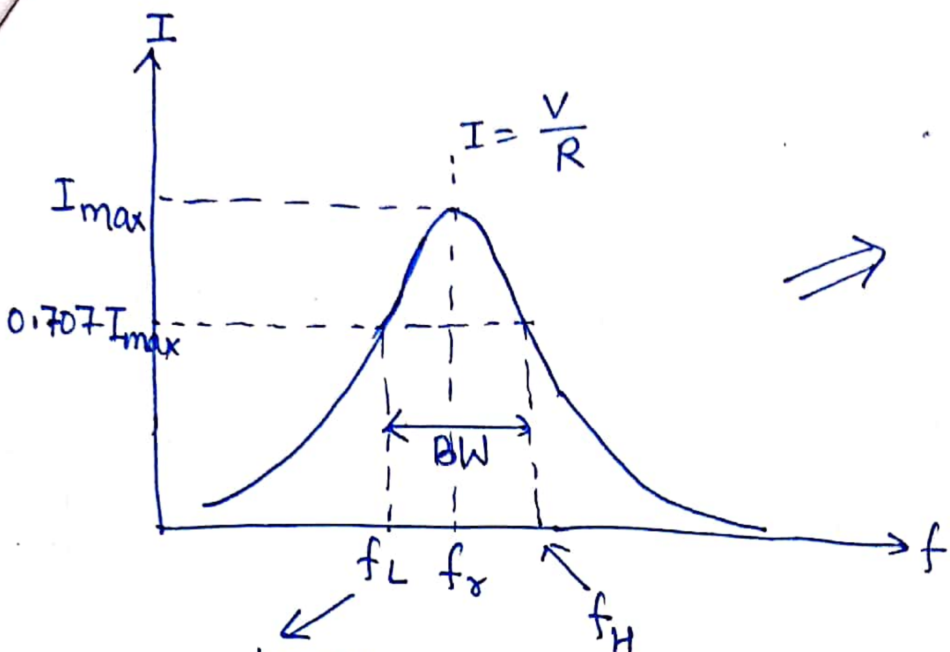
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

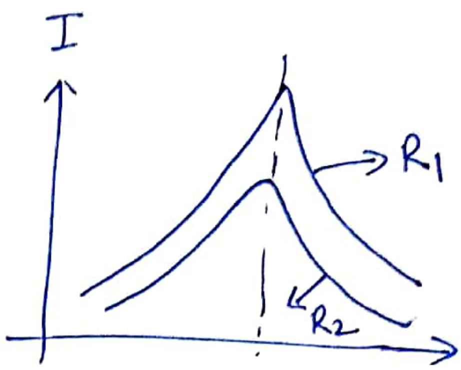
Now, total current,

$$I = \frac{V\angle 0^\circ}{Z} = \frac{V\angle 0^\circ}{|Z|\angle\phi}$$

$$I = \frac{V}{|Z|} \angle -\phi$$



$\Rightarrow$



$(R_1 < R_2)$

$\Rightarrow$ Current is maximum if $X_L = X_C$ i.e. at resonance.

Bandwidth, $BW = f_H - f_L$

where, $f_L = \frac{1}{2\pi}\left[-\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]$

$$f_H = \frac{1}{2\pi}\left[\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}\right]$$

Also, $BW = \frac{f_r}{Q}$

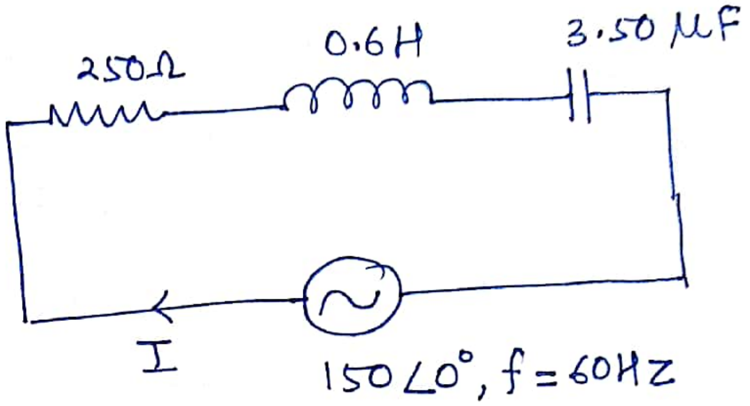
where, $Q = \frac{\omega_r L}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

so, $BW = \frac{f_r}{Q} = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{1}{R}\sqrt{\frac{L}{C}}} = \frac{R}{L}$ (rads) $= \frac{R}{2\pi L}$ (Hz)

$$\boxed{BW = \frac{f_r}{Q} = f_H - f_L = \frac{R}{2\pi L} \text{ Hz}}$$

$V_R = IR = IR$
$V_L = I X_L \angle 90^\circ = I \cdot jX_L$
$V_C = I X_C \angle -90^\circ = I \cdot (-jX_C)$

Q1. A series RLC circuit is shown in fig. Find (a) the total impedance of the circuit (b) the maximum current in the circuit, (c) the phase angle, (d) the resonant frequency, (e) Quality factor, Q, (f) Bandwidth, and (g) voltage across R, L, and C.



Solution:

$$X_L = 2\pi f L = 2\pi \times 60 \times 0.6\ \Omega = 226.2\ \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 60 \times 3.5 \times 10^{-6}}\ \Omega = 757.88\ \Omega$$

(a) $$Z = R + j(X_L - X_C) = 250 + j(226.2 - 757.88)$$

$X_C > X_L$ ⇒ the circuit is capacitive in nature

$$Z = 250 - j\,531.68 = 587.52 \angle -64.82^\circ$$

(b) max^m current flow in the circuit at resonance

$$I_{max} = \frac{150}{250} = \frac{V}{R} = \cancel{0.106}\ 0.6A$$

(c) phase angle, $\phi = -64.82^\circ$

(d) ⇒ $$f_r = \frac{1}{2\pi\sqrt{LC}} = 109.83\ Hz$$

(e) $Q = \frac{X_L}{R} = \frac{226.2}{250} = 0.9048$

(f) $BW = \frac{f_r}{Q} = \frac{109.83}{0.9048} = 121.38$ Hz

(g) Total current in the circuit

$$I = \frac{V}{Z} = \frac{150 \angle 0^\circ}{587.52 \angle -64.82^\circ} = 0.255 \angle 64.82^\circ$$

i.e current leads voltage by 64.82°.

$$V_R = IR = 0.255 \times 250 \angle 64.82^\circ$$
$$= 63.75 \angle 64.82^\circ \text{ volt}$$

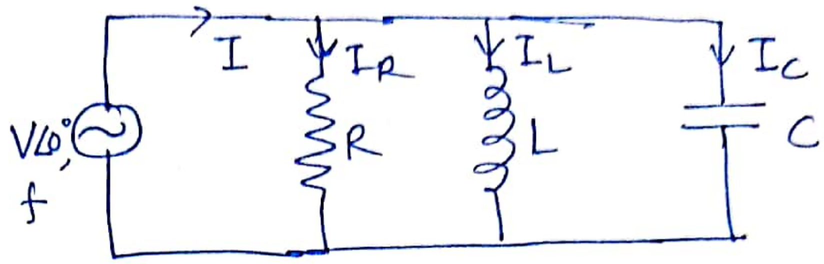
$$V_L = I \cdot jX_L = IX_L \angle 90^\circ = 0.255 \times 226.2 \angle 64.82^\circ + 90^\circ$$
$$= 57.681 \angle 154.82^\circ \text{ volt}$$

$$V_c = I \cdot (-jX_c) = IX_c \angle -90^\circ$$

$$= 0.255 \angle 64.82^\circ \times 757.88 \angle -90^\circ$$

$$V_c = 193.2594 \angle -25.18^\circ \text{ volt}$$

Parallel RLC Resonance Circuit



Conductance, $G = \frac{1}{R}$

Inductive susceptance, $B_L = \frac{1}{X_L} = \frac{1}{2\pi f L}$

capacitive Susceptance, $B_C = \frac{1}{X_C} = 2\pi f C$

Admittance, $Y = \frac{1}{R} + j(B_C - B_L) = G + jB_T$

$$= G + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$Y = |Y| \angle \phi$$

where, $|Y| = \sqrt{G^2 + (B_C - B_L)^2} = \sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$

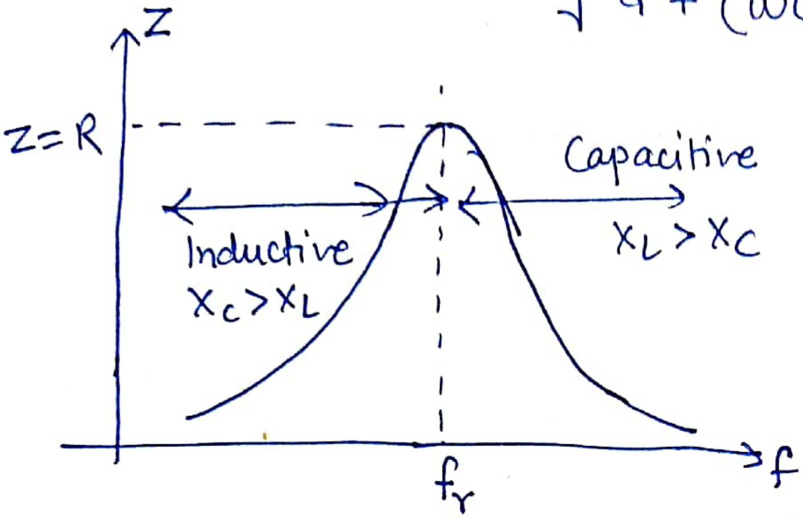
$$\phi = \tan^{-1}\left(\frac{B_C - B_L}{G}\right)$$

similar to series RLC Resonance. circuit —

Resonance occurs when $X_L = X_C$ at $f = f_r$

Again, $\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} = \frac{1}{\sqrt{LC}} \text{ (rad)}}$

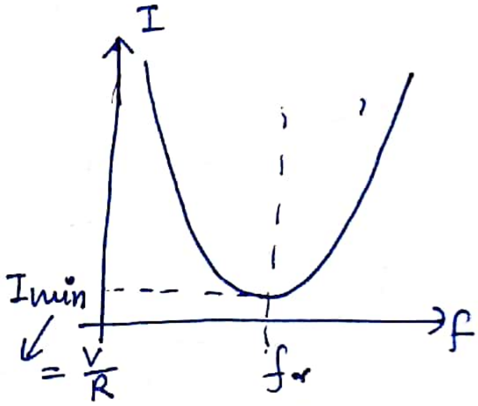
Impedance, $Z = \frac{1}{Y} = \frac{1}{\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}} \angle\phi$



(Parallel Resonance)

The current, $I = \frac{V\angle 0^\circ}{Z} = V\angle 0^\circ \cdot Y$

$I = V\sqrt{G^2 + \left(\omega C - \frac{1}{\omega L}\right)^2} \angle\phi$



The current in the circuit is minimum, if $\omega C = \frac{1}{\omega L}$

$I_R = \frac{V}{R}$

$I_L = \frac{V}{jX_L} = \frac{V}{2\pi f L} \angle -90^\circ$

$I_C = \frac{V}{-jX_C} = V \cdot 2\pi f C \angle +90^\circ$

$\boxed{I = I_R + I_L + I_C}$

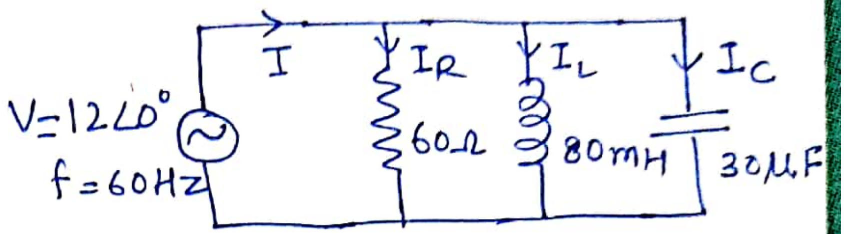
$I =$ vector sum of (I_R, I_L, I_C)

$I = \sqrt{I_R^2 + (I_L - I_C)^2}$

Quality factor, $Q = \frac{R}{X_L} = \frac{R}{X_C} = R\sqrt{\frac{C}{L}}$

$BW = f_H - f_L = \frac{f_r}{Q} = \frac{\frac{1}{2\pi\sqrt{LC}}}{R\sqrt{\frac{C}{L}}} = \frac{1}{2\pi\sqrt{LC}} \cdot \frac{\sqrt{L}}{R\sqrt{C}} = \frac{1}{2\pi RC}$

Q2. Calculate the impedance in the circuit shown below as well as the current flowing in each element.



<u>solution</u>:

$$X_L = 2\pi f L = 2\pi \times 60 \times 80 \times 10^{-3} = 30.16\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 60 \times 30 \times 10^{-6}} = 88.42\,\Omega$$

$$Z = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{60}\right)^2 + \left(\frac{1}{30.16} - \frac{1}{88.42}\right)^2}}$$

$$Z = 36.392\,\Omega$$ ✓

or,

$$G = \frac{1}{R} = 0.0166$$

$$B_L = \frac{1}{X_L} = 0.033$$

$$B_C = \frac{1}{X_C} = 0.0113$$

$$Y = G + j(B_C - B_L)$$

$$= 0.0166 + j(0.0113 - 0.033)$$

$$Y = 0.0166 - j0.0217$$ ✓

$$= \cancel{0.1674 \angle -7.44^\circ}$$

$$= 0.0273 \angle -52.58^\circ$$

$$Z = \frac{1}{Y} = \frac{1}{0.0273 \angle -52.58^\circ}$$

$$= 36.63 \angle 52.58^\circ$$ ✓

Current, $I = \frac{V}{Z} = \frac{12\angle 0^\circ}{36.63\angle 52.58^\circ}$

$I = 0.327\angle -52.58^\circ$ → that mean current lags voltage by 52.58°

$I_R = \frac{V}{R} = \frac{12}{60} = 0.2A$

$I_L = V/jX_L = \frac{12\angle 0^\circ}{X_L\angle 90^\circ} = \frac{12}{30.16}\angle -90^\circ$

$= 0.398\angle -90^\circ$

$I_C = \frac{V}{-jX_C} = \frac{12\angle 0^\circ}{X_C\angle -90^\circ} = \frac{12}{88.42}\angle +90^\circ$

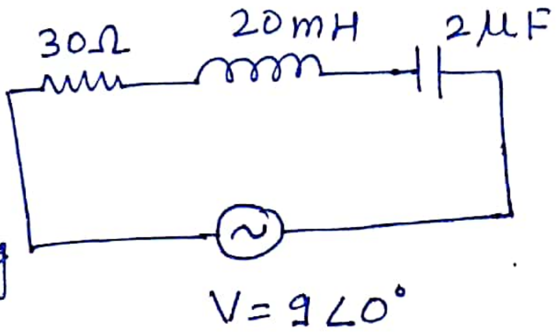
$= 0.136\angle 90^\circ$

<u>Also</u> $I = \sqrt{I_R^2 + (I_L - I_C)^2} = \sqrt{0.2^2 + (0.398 - 0.136)^2}$

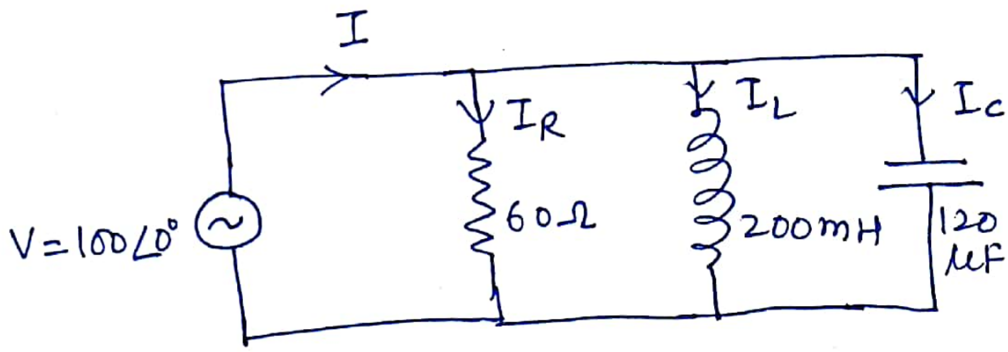
$= \underline{0.329\ A}$

Experiment No. 10 (Assignment)

Q1. Calculate the resonant frequency, the current at resonant, the voltage across the inductor and capacitor at resonance,
the quality factor
and the bandwidth
of the circuit.
Also sketch the corresponding current waveform for all frequencies,



Q2.



Calculate the resonant frequency, the quality factor and the bandwidth of the circuit, the circuit current at resonance.