Sinusoidal Steady State Analysis:-

Sinusoid:- Sinusoid is a signal that has the form of the Sine or Cosine function.

A Sinusoidal Current is a Current reverses at regular time intervals and has alternatively positive and negative Values. Circuits driven by Sinusoidal Current or Voltage Sources are Ac Circuits.

Consider the sinusoidal Voltage,

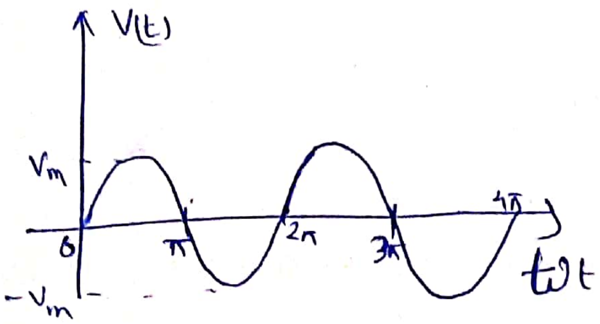
$$V(t) = V_m \sin \omega t \quad - (1)$$

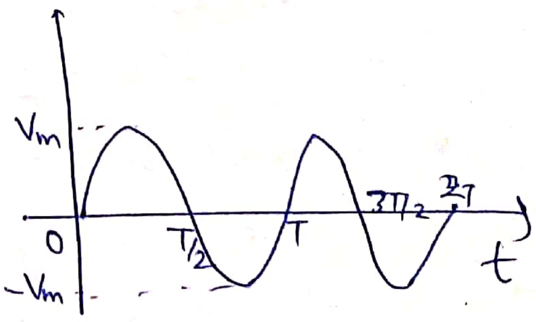
V_m = the amplitude of Signal

ω = Angular frequency in radian/sec

ωt = argument of sinusoid.

$$\boxed{T = 2\pi/\omega}$$





$$\boxed{f = 1/T}$$

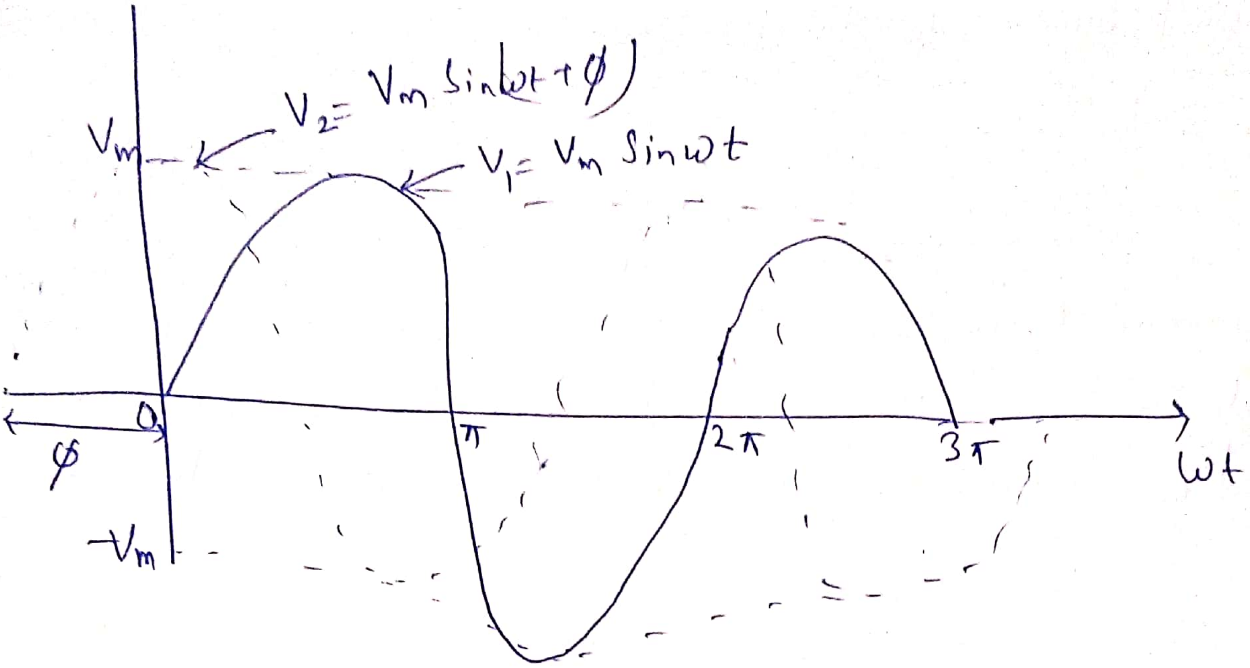
Let us now Consider a more general expression of Sinusoid,

$$V(t) = V_m \sin(\omega t + \phi)$$

$\omega t + \phi$ = Argument in degrees or radians

ϕ = Phase —— " —— " —— "

If $V_1(t) = V_m \sin \omega t$, $V_2(t) = V_m \sin(\omega t + \phi)$



$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Also,

$$\sin(\omega t \pm 180^\circ) = -\sin\omega t$$
$$\cos(\omega t \pm 180^\circ) = -\cos\omega t$$
$$\sin(\omega t \pm 90) = \pm\cos\omega t$$
$$\cos(\omega t \pm 90) = \mp\sin\omega t$$

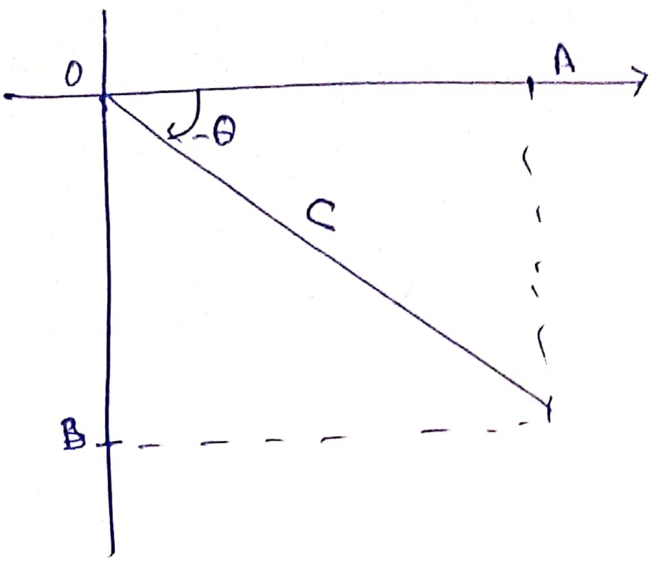
In order to compare two signals or find the resultant signals we need to ~~have~~ express both in Cosine or Sign forms with positive amplitudes.

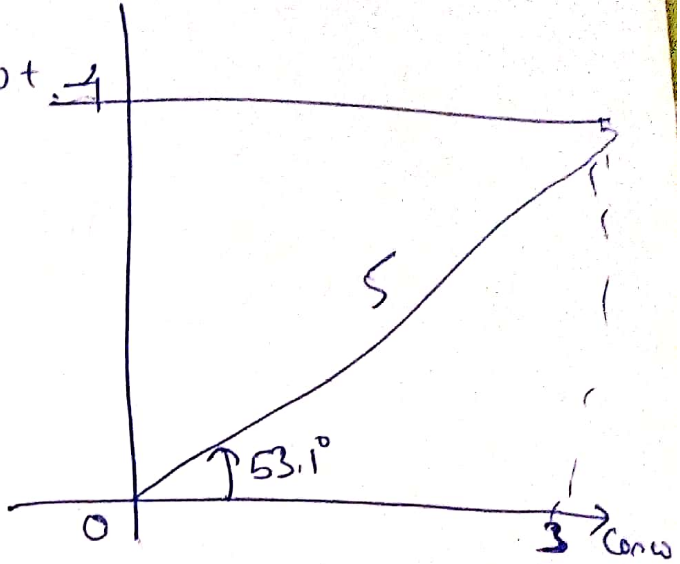
$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$
$$\Rightarrow \quad A\cos\omega t + B\cos(\omega t - \pi/2)$$
$$\Rightarrow \quad C\cos(\omega t - A/2)$$

Where $C = \sqrt{A^2 + B^2}$, $\theta = \tan^{-1}(B/A)$

If

$$3\cos\omega t - 4\sin\omega t = 5\cos(\omega t + 53.1^\circ)$$





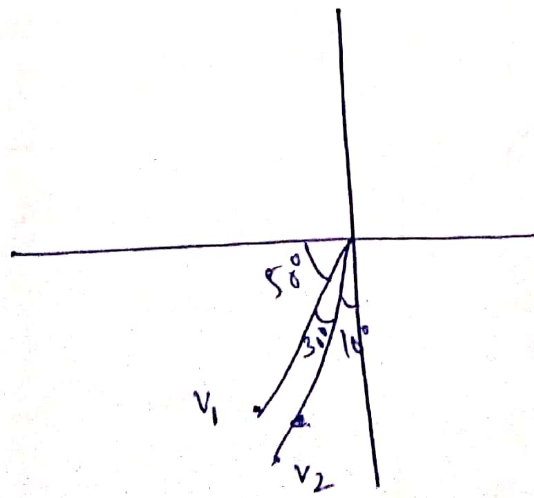
Prob. Calculate the phase angle b/w $V_1 = -10\cos(\omega t + 50°)$ and $V_2 = 12\sin(\omega t - 10°)$. State which sinusoid is leading.

Soln:-

$$V_1 = -10\cos(\omega t + 50°) = 10\cos(\omega t + 50° + 180°)$$
$$= 10\cos(230 + \omega t) \quad \text{— (1)}$$

$$V_2 = 12\sin(\omega t - 10°) = 12\cos\{90° - (\omega t - 10)\}$$
$$= 12\cos\{100 - \omega t)$$
$$= 12\cos(\omega t - 100) \quad \text{— (2)}$$

$$V_1 = 10\cos(\omega t + 230°), \quad V_2 = 12\cos(\omega t - 100)$$



Conωt

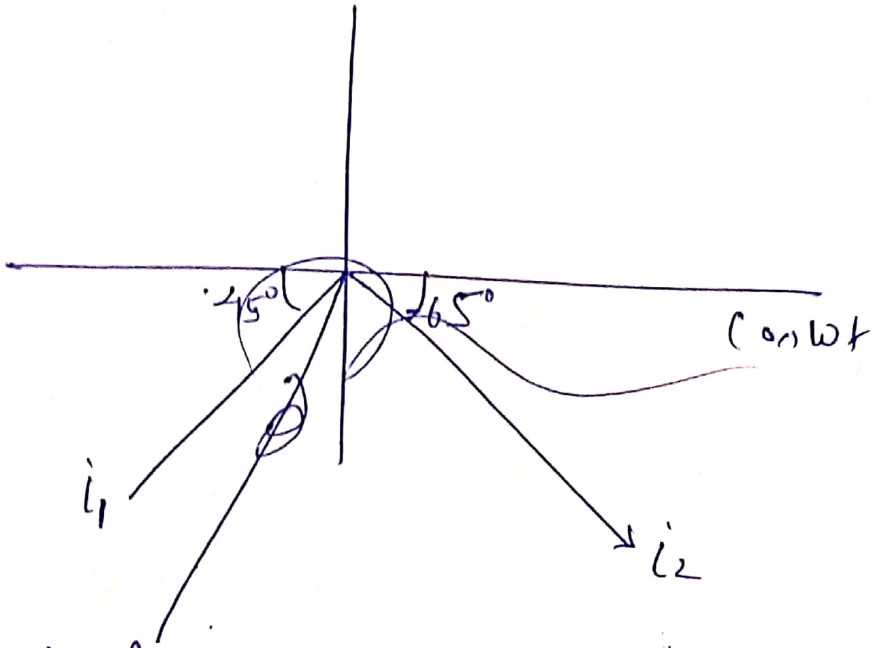
Hence V_2 leads by 30° with V_1.

Prob.

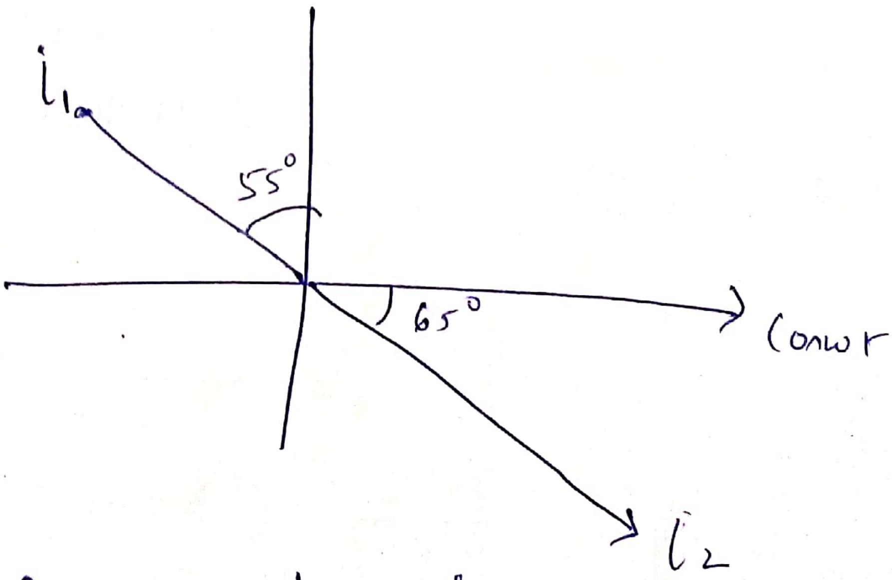
$i_1 = -4\sin(377t + 55^\circ)$, $i_2 = 5\cos(377t - 65^\circ)$

Soln:-

$i_1 = 4\cos(377t \overset{+145}{\cancel{+235}}^\circ)$, $i_2 = 5\cos(377t - 65^\circ)$



Hence i_2 leads, ~~35+45 = 80°~~



i_1 leads by i_2 210°

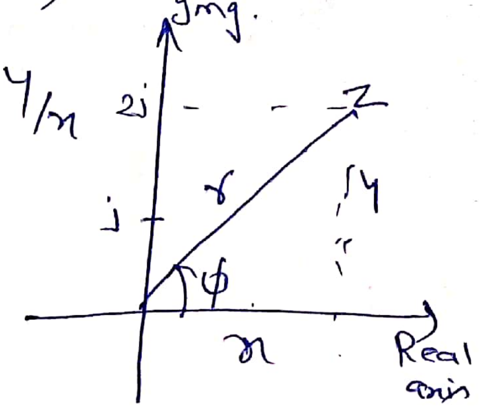
Phasors:- A phasor is a Complex number that represents the amplitude and phase of a sinusoid.

$$Z = x + iy \quad \text{(Rectangular form)}$$

$$Z = r \angle \phi \quad \text{(Polar form)}$$

$$Z = r e^{j\phi} \quad \text{(Exponential form)}$$

$$r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1} y/x$$



$$x = r\cos\phi, \quad y = r\sin\phi$$

$$Z = x + iy = r\cos\phi + i\, r\sin\phi$$

Addition:- $\cancel{x+iy}$ $Z_1 + Z_2 = (x_1 + x_2) + i(y_1 + y_2)$

Subtraction:- $Z_1 - Z_2 = (x_1 - x_2) + i(y_1 - y_2)$

Multiplication:- $Z_1 Z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

Division:- $\frac{Z_1}{Z_2} = r_1/r_2 \angle \phi_1 - \phi_2$

Reciprocal: $\frac{1}{Z} = \frac{1}{r} \angle -\phi$

Square root:- $\sqrt{Z} = \sqrt{r}\, e^{j\phi/2} = \sqrt{r} \angle \phi/2$

Complex Conjugate:- $Z^* = x - jy = r \angle -\phi$

Time domain Representation	Phasor domain Representation
$V_m \cos(\omega t + \phi)$	$V_m \angle +\phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - \pi/2$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

Prob. Using the phasor approach, determine the current $i(t)$ in a circuit described by the integrodifferential equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t + 75^\circ)$$

Soln:-

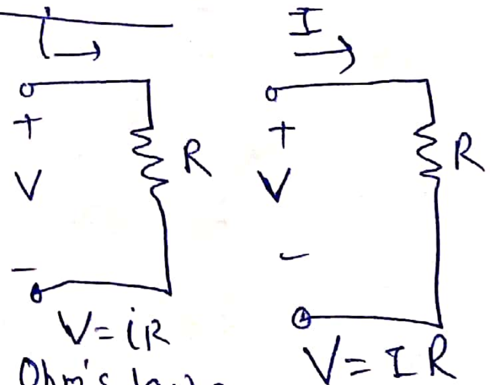
$$4I + \frac{8I}{j\omega} - 3j\omega I = 50\angle 75^\circ$$

$$(4 - j4 - 6j)\,I = 50\angle 75^\circ \quad (\because \omega = 2)$$

$$\Rightarrow \boxed{I = \frac{50\angle 75^\circ}{4 - j10} = 4.642\angle 143.2^\circ \text{ A}}$$

Phasor Relationships for Circuit elements :-

Resistor :-



If the current through the resistor R is -

$$i = I_m \cos(\omega t + \phi).$$

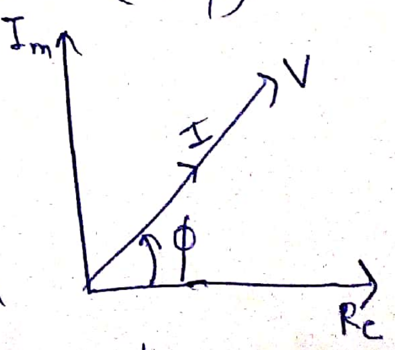
the voltage across it is given by Ohm's law as

$$V = iR \Rightarrow V = R I_m \cos(\omega t + \phi)$$

the phasor form of this voltage is -

$$V = R I_m \angle \phi$$

$$\boxed{V = RI}$$



Showing that the voltage-current relation for the resistor in the phasor domain continues to be Ohm's law as in time domain. Note that Voltage & Currents are in phase.

Inductor :- For the inductor L, assume the current through it is $i = I_m \cos(\omega t + \phi)$. The voltage across the inductor is -

$$V = L\frac{di}{dt} = L\frac{d}{dt} I_m \cos(\omega t + \phi)$$

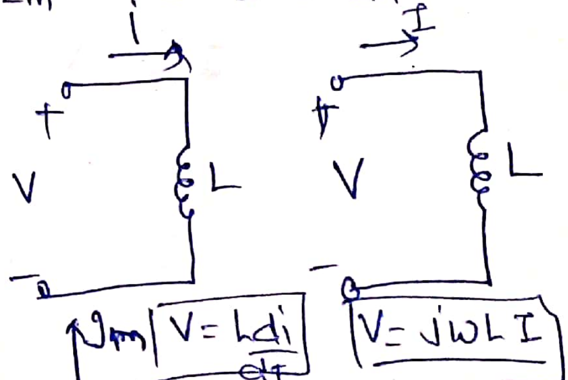
$$V = -\omega L I_m \sin(\omega t + \phi)$$

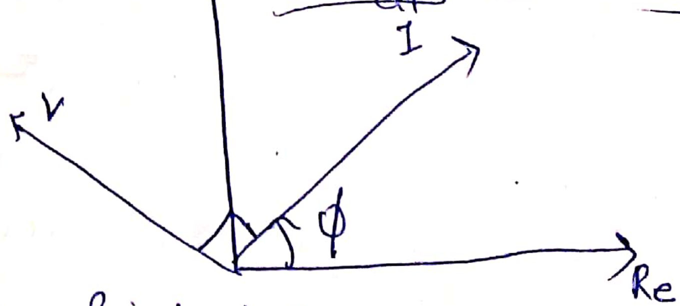
$$V = \omega L I_m \cos(\omega t + \phi + 90^\circ)$$

$$V = \omega L I_m e^{j(\phi + 90^\circ)} = \omega L I_m e^{j\phi} e^{j90} = \omega L I_m \angle \phi + 90^\circ$$

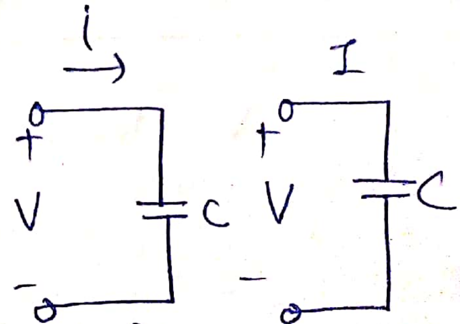
Also,

$$\boxed{V = j\omega L I}$$





Capacitor :- It shows that Voltage & current are 90° out of phase, in case of inductor.



Capacitor :- $V = V_m \cos(\omega t + \phi)$

$$i = C\frac{dv}{dt} = -V_m \omega C \sin(\omega t + \phi)$$

$$i = V_m \omega C \cos(\omega t + \phi + 90^\circ)$$

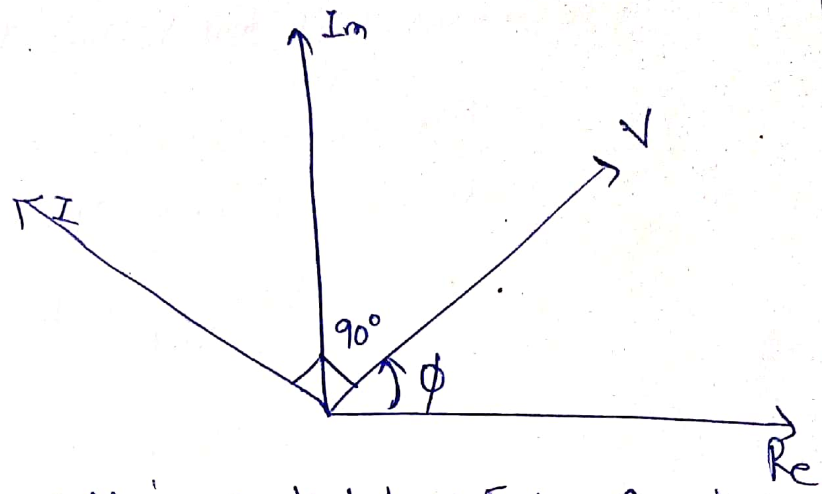
$$i = \omega C V_m \angle \phi + 90^\circ$$

$$\boxed{i = V_m \omega C e^{j\phi} e^{j90} = V_m j\omega C}$$

$V = \frac{1}{C}\int i\,dt \rightarrow i = C\frac{dv}{dt}$

$V = \frac{1}{j\omega C} I$

$I = j\omega C V$



Ex:- If Voltage $V = 10\cos(100t + 30°)$ is applied to a 50 μF Capacitor, Calculate the Current through the inductor.

Soln:-

$I = j\omega C V$

$\cancel{I =} \; V = \cancel{j\omega L I}$

$\Rightarrow I = \cancel{\dfrac{V}{j\omega L}} = \cancel{10\angle}$

$\Rightarrow I = j100 \times 50 \times 10^{-6} \times 10\angle 30°$

$= j(5 \times 10^{-3}) \times 10\angle 30°$

$= 5 \times 10^{-3} \angle 90° \times 10\angle 30°$

$I = 50 \times 10^{-3} \angle 120°$

$$\boxed{i = 50\cos(\omega t + 120°)\ \text{mA}}$$

Impedance and admittance:-

$V = RI, \quad V = j\omega L I, \quad V = \dfrac{I}{j\omega C}$

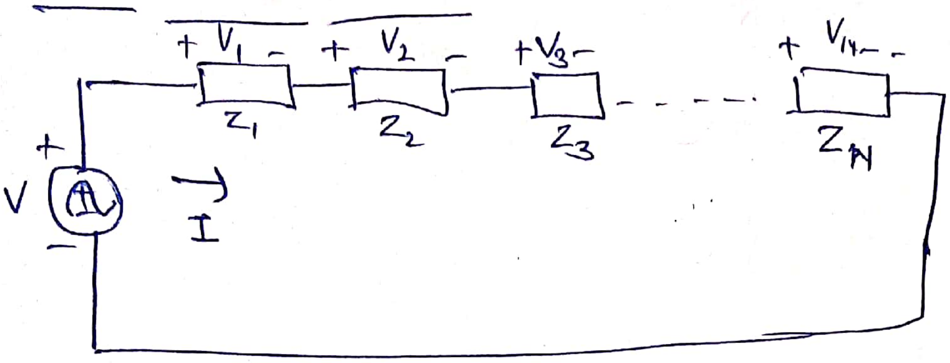
$\Downarrow$

$\dfrac{V}{I} = R, \quad \dfrac{V}{I} = j\omega L, \quad \dfrac{V}{I} = \dfrac{1}{j\omega C}$

$Z = V/I$ or $V = ZI$

The Impedance Z of a Circuit is the ratio of the Phasor Voltage V to the Phasor Current I.

Impedance Combinations :-

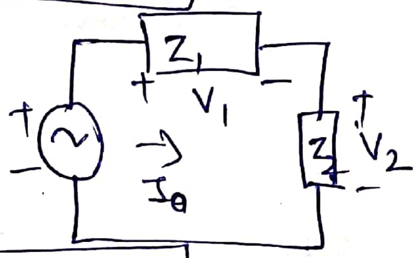


$$Z_{eq} = \frac{V}{I} = Z_1 + Z_2 + Z_3 + \dots + Z_N$$

or

$$\boxed{Z_{eq} = Z_1 + Z_2 + Z_3 + \dots + Z_N}$$

Voltage Divider :-

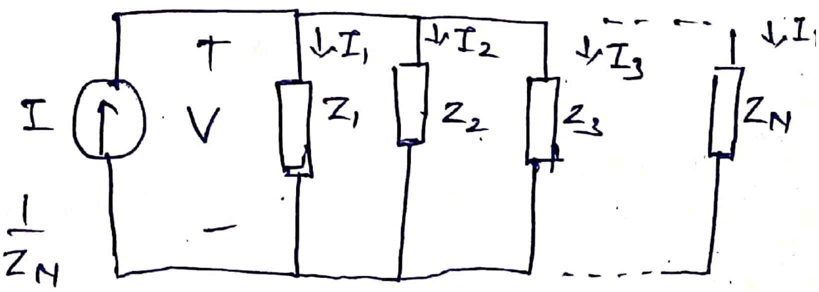


$$I = \frac{V}{Z_1 + Z_2}$$

Since, $V_1 = Z_1 I$, $V_2 = Z_2 I$

$$\boxed{V_1 = \frac{Z_1 V}{Z_1 + Z_2}} \qquad \boxed{V_2 = \frac{Z_2 V}{Z_1 + Z_2}}$$

Current Divider :-

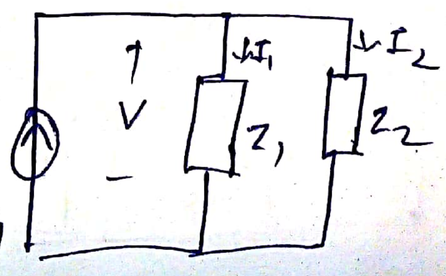


$$I = I_1 + I_2 + I_3 + \dots + I_N$$

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots + \frac{1}{Z_N}$$

$$\boxed{Y_{eq} = Y_1 + Y_2 + Y_3 + \dots + Y_N}$$

$$Z_{eq} = \frac{1}{Y_{eq}} =$$



$$\boxed{I_1 = \frac{Z_2 I}{Z_1 + Z_2}} \qquad \boxed{I_2 = \frac{Z_1 I}{Z_1 + Z_2}}$$

Element	Impedence	Admittance
Resistance(R)	$Z = R$	$Y = 1/R$
Inductance(L)	$Z = j\omega L$	$Y = 1/j\omega L$
Capacitance (C)	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

Equivalent ckts at high frequencies and dc :-

L

DC $Z = j\omega L = 0$, S.C

high frequency $Z = j\omega L = \infty$, O.C

DC $Z = \frac{1}{j\omega C} = \infty$, O.C

High frequency $Z = \frac{1}{j\omega C} = 0$, S.C.

$$Z = R + jX = |Z| \angle \theta$$

$$|Z| = \sqrt{R^2 + X^2}, \quad \theta = \tan^{-1}(X/R)$$

The admittance Y is the reciprocal of Impedance, measured in siemens.

$$Y = 1/Z = \frac{1}{R + jX}$$

$$Y = G + jB = \frac{1}{R + jX}$$

$$G + jB = \frac{R - jX}{R^2 + X^2}$$

$$\Rightarrow \boxed{G = \frac{R}{R^2 + X^2}}, \quad \boxed{B = \frac{-X}{R^2 + X^2}}$$

G, is the Conductance and B is the Susceptance.

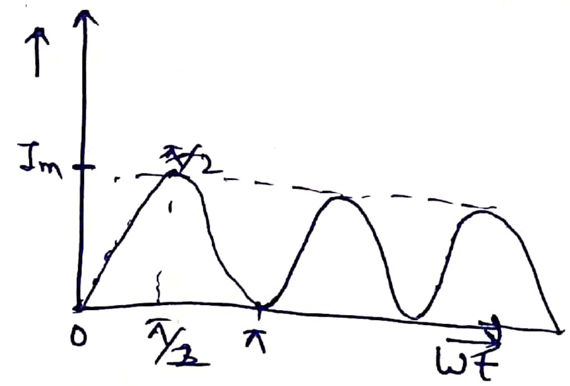
Average of a Sine Wave :- (Average Value)

* The average of a Sine Wave over a full Cycle is zero but average Value of half Cycle exists.

$$I_{av} = \frac{1}{T}\int_0^T i\, d(\omega t) = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t\, d(\omega t)$$

$$= \frac{-I_m}{\pi}\left[\cos\omega t\right]_0^{\pi} \Rightarrow \frac{-I_m}{\pi}\left[\cos\pi - \cos 0\right]$$

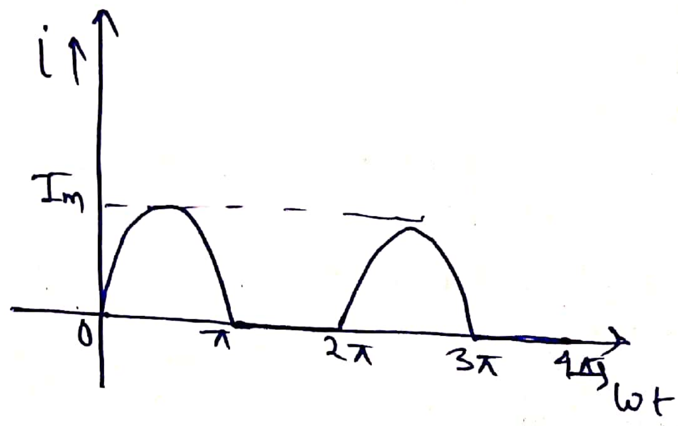
$$\boxed{I_{av} = \frac{2I_m}{\pi} = 0.637 I_m}$$



Output of half Wave Rectifier :-

$$I_{av} = \frac{1}{T}\int_0^T i\, d(\omega t)$$

Now, $T = 2\pi$, but i does not exists from π to 2π.



$$I_{av} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin\omega t\, d(\omega t)$$

$$= \frac{-I_m}{2\pi}\left[\cos\omega t\right]_0^{\pi}$$

$$\boxed{I_m = I_m/\pi = 0.318\pi}$$

This implies that, average Value of ~~full~~ half Wave rectifier is $\frac{1}{2}$ that of full Wave ~~rectified~~ rectifier.

Effective or RMS Value:-

It is equivalent to dc current, which when flows through the given circuit, produces same amount of heating.

$$P = i^2 R = R I_m^2 \sin^2 \omega t$$

$$P = R I_m^2 \left(\frac{1 - \cos 2\omega t}{2} \right)$$

$$\boxed{P_{ave} = \frac{R I_m^2}{2}}$$

Compare the above ac Average power and Dc power,

$$I^2 \cancel{R} = \frac{\cancel{R} I_m^2}{2}$$

$$\Rightarrow \boxed{I_{eff} = I_m/\sqrt{2}} \Rightarrow \boxed{I_{rms} = I_m/\sqrt{2}}$$

Let us again examine the procedure of finding the I_{eff}.

$$P_{av} = \frac{1}{T} \int_0^T i^2 R\, dt$$

$$= \left[\frac{1}{T} \int_0^T i^2 dt \right] R$$

This power, we equated to $I_{eff}^2 R$.

$$I_{eff}^2 \cancel{R} = \left[\frac{1}{T} \int_0^T i^2 dt \right] \cancel{R}$$

$$\Rightarrow \boxed{I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt}}$$

* Note that the rms value is always greater than the average value, except for a rectangular wave.

* For Full Wave Rectifier, I_{rms} is the same as that for A.C. as derived above.

For Half Wave Rectifier, Irms can be determined as,

$$\boxed{I_{rms} = I_m/2}$$

Here's How,

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, d(\omega t)}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\omega t\, d(\omega t)}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\omega t}{2}\, d(\omega t)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left\{\left(\pi - \frac{\sin 2\pi}{2}\right) - (0-0)\right\}}$$

$$= \sqrt{\frac{I_m^2}{4\cancel{\pi}} \times \cancel{\pi}}$$

$$\boxed{I_{eff} = I_{rms} = \frac{I_m}{2}}$$

Form Factor:- It is the ratio of rms value to the rms value to the average value of an alternating quantity.

$$K_f = \frac{I_{rms}}{I_{av}} = 1.11 \text{ for Sinusoidal.}$$

Peak factor:- It is the ratio of maximum to rms value.

$$K_p = I_m / I_{rms} = 1.414 \text{ for Sinusoidal}$$

Instantaneous and Average Power :-

The Instantaneous Power $p(t)$ absorbed by an element is the Product of instantaneous Voltage $v(t)$ and instantaneous Current $i(t)$.

$$\boxed{p(t) = v(t)\, i(t)} \quad -(1)$$

The instantaneous power is the power at any instant of time.

Let the Voltage and Current at the terminals of the Circuit be-

$$v(t) = V_m \cos(\omega t + \theta_v) \quad -(2)$$

$$i(t) = I_m \cos(\omega t + \theta_i) \quad -(3)$$

Substituting (2) & (3) in (1),

$$p(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{V_m I_m}{2}\left[\cos(2\omega t + \theta_v + \theta_i) + \cos(\theta_v - \theta_i)\right]$$

$$p(t) = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) \quad -(4)$$

Thus the average Power is given by,

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{T}\times\frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)\int_0^T dt + \frac{1}{2} V_m I_m \times \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt \quad -(5)$$

The first Integrand is Constant, and the average of a Constant is the Same Constant. The Second integrand is a Sinusoid. We know that the average of a Sinusoid over its period is zero.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \quad -(6)$$

A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

Ex:- $V(t) = 120\cos(377t+45^\circ)$ V and $i(t) = 10\cos(377t-10^\circ)$ A

Soln:- find the instantaneous power and the average power absorbed by the Passive linear N/W.

Soln:- $P(t) = V(t)\,i(t) = 1200\cos(377t+45^\circ)\cos(377t-10^\circ)$

$$\cos A\cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$\Rightarrow \quad P = 600[\cos(754t+35^\circ)+\cos 55^\circ]$$

$$\boxed{P = 344.2 + 600\cos(754t+35^\circ)}$$

The average power is,

$$\boxed{P_{ave} = 344.2\,W}$$

Ex:- Calculate the instantaneous power and average power absorbed by the passive linear N/W.

$V(t) = 330\cos(10t+20^\circ)$ V & $i(t) = 33\sin(10t+60^\circ)$ A

Soln:- $P(t) = V(t)\,i(t) = \frac{330\times33}{2}[\cos(20t+80^\circ)+\cos(-40^\circ)]$

$$= \cancel{4795}\,5.445\,[\cos 40 + 5.445\cos(20t+80^\circ)$$

$$P(t) = 3.5 + 5.445\cos(20t+80^\circ)\ kW$$

$$P_{ave} = 3.5\,kW$$

Ex:- $Z = 30 - j70\,\Omega$, $V = 120\angle 0^\circ$

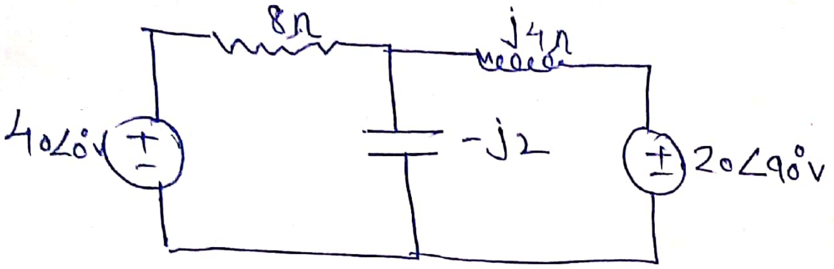
$$I = V/Z = \frac{120\angle 0^\circ}{30-j70} = \frac{120\angle 0^\circ}{76.16\angle -66.8^\circ} = 1.576\angle 66.8^\circ\,A$$

$$P_{ave} = \frac{1}{2}V_m I_m\cos(\theta_v - \theta_i)$$

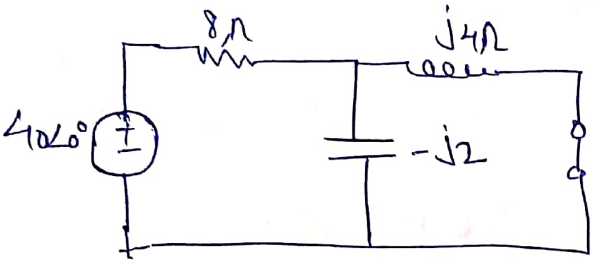
$$= \frac{1}{2}\times 120\times 1.576\cos(0-66.8^\circ)$$

$$\boxed{P_{ave} = 37.24\,W}$$

Calculate the average Power absorbed by each of the five elements in the Circuit.



Soln:- Considering 40∠0° Voltage Source



$$Z_{in} = (-j2 \,||\, j4) + 8\Omega$$

$$= \frac{-\cancel{j}2 \times j\cancel{4}}{\cancel{j2}} + 8\Omega$$

$$Z_{in} = 8 - j4$$

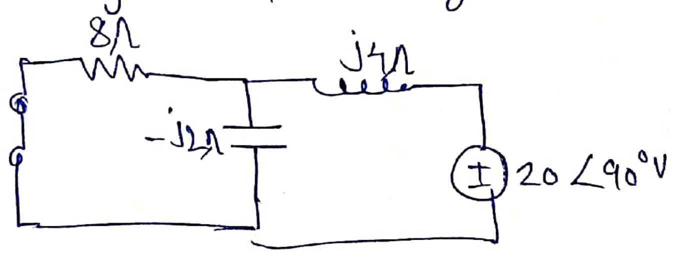
$$i_1 = \frac{V}{Z_{in}} = \frac{40\angle 0^\circ}{8-j4} = 4+2j = 4.472\angle 26.56^\circ$$

$$P_{ave} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 4.472 \times 40 \, (0 - 26.56^\circ)$$

$$P_{ave} = -80W$$

Considering 20∠90° Voltage Source,



$$Z_{in} = \frac{-j2 \times 8}{8 - j2} + j4$$

$$Z_{in} = \frac{-16j}{8-j2} + j4 = 0.4705 + 2.11j = 2.169\angle 77.47^\circ$$

$$i_2 = \frac{20\angle 90^\circ}{2.169\angle 77.47} = 9.22\angle 12.53$$

$$P_{ave} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \frac{1}{2} \times 20 \times 9.22 \cos(90 - 12.53)$$

$$P_{ave} = \cancel{92.2} \; -20\,W$$

Current through Resistor:-

$$P_{ave} = \frac{1}{2} |I|^2 R$$

$$= \frac{1}{2} \times (\cancel{4.47}\, i_1 - i_2)^2 \times R$$

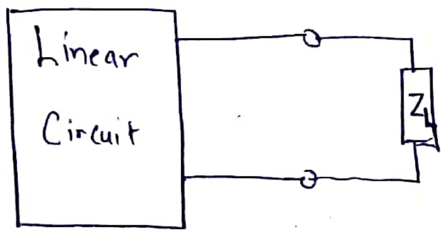
$$= \frac{1}{2} \times (4.472\angle 26.56 - 9.22\angle 12.53)^2 \times 8$$

$$= \frac{1}{\cancel{2}} \times (5.00\angle -180°)^2 \times \cancel{8}\; 4$$

$$\boxed{P_{ave} = 100\,W}$$

Maximum Average power transfer:-

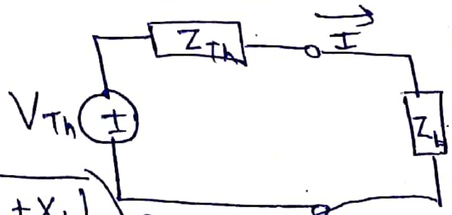
The Thevenin's Impedance Z_{Th} and the load impedance Z_L are,



$$Z_{Th} = R_{Th} + jX_{Th} \quad (1)$$

$$Z_L = R_L + jX_L \quad (2)$$

The Current through the load is,



$$I = \frac{V_{Th}}{Z_{Th} + Z_L} = \frac{V_{Th}}{(R_{Th} + R_L) + j(X_{Th} + X_L)} \quad (3)$$

The average Power delivered to the Load is,

$$P_{ave} = \frac{1}{2} |I|^2 R_L$$

$$P_{ave} = \frac{V_{Th}^2 R_L / 2}{\left[\sqrt{(R_{Th} + R_L)^2 + (X_L + X_{Th})^2}\right]^2} = \frac{V_{Th}^2 R_L / 2}{(R_{Th} + R_L)^2 + (X_L + X_{Th})^2}$$

Our objective is to adjust the load ~~impedance~~ Parameters R_L and X_L so that P is maximum. To do this we set $dP/\partial R_L$ and $\partial P/\partial X_L$ equal to zero.

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$$\frac{\partial P}{\partial X_L} = \frac{-|V_{Th}|^2 R_L (X_{Th}+X_L)}{\left[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2\right]^2} \quad -(5)$$

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2\left[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2-2R_L(R_{Th}+R_L)\right]}{2\left[(R_{Th}+R_L)^2+(X_{Th}+X_L)^2\right]^2} \quad -(6)$$

Setting $\partial P/\partial X_L$ to zero given,

$$X_L = -X_{Th} \quad -(7)$$

and setting $\partial P/\partial R_L$ to zero results in,

$$R_L = \sqrt{R_{Th}^2+(X_{Th}+X_L)^2} \quad -(8)$$

Combining (7) & (8), $R_L = R_{Th}$ & $X_L = -X_{Th}$

$$Z_L = R_{Th} - jX_{Th} = Z_{Th}^* \quad -(9)$$

For maximum average power transfer, the load impedance Z_L must be equal to the complex conjugate of the thevenin's impedance Z_{Th}.

Substituting (9) into (4),

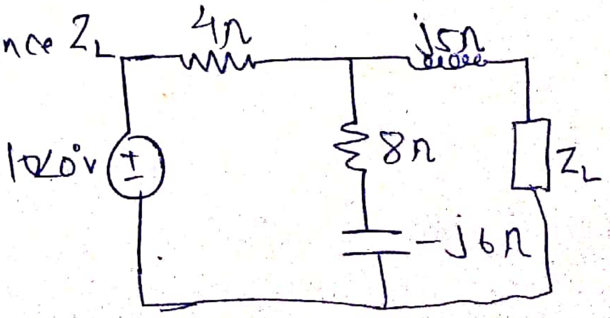
$$\boxed{P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}}$$

If the load is purely real i.e, $X_L = 0$, from (8)

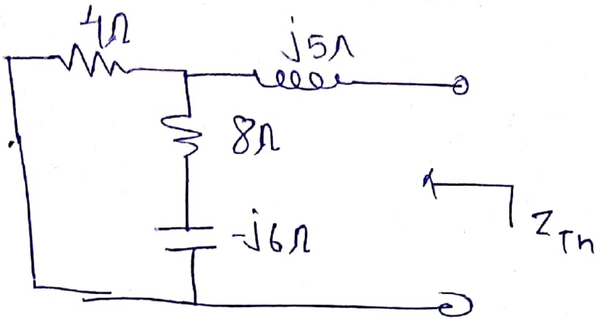
$$R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = |Z_{Th}|$$

For a purely resistive load; the load impedance is equal to the magnitude of the Thevenin's impedance.

Prob. :- Determine the load impedance Z_L that maximizes power drawn from circuit. What is the maximum average power.



Finding Z_{Th}:-



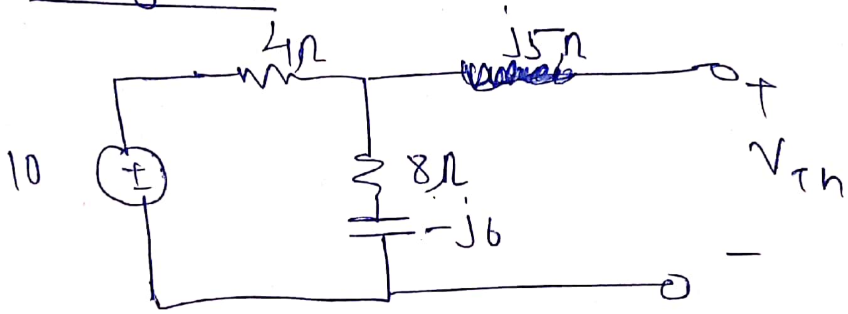
$$Z_{Th} = (4\Omega \parallel 8-j6) + j5$$

$$= \frac{4(8-j6)}{12-j6} + j5$$

$$= 5.34\angle 56.70$$

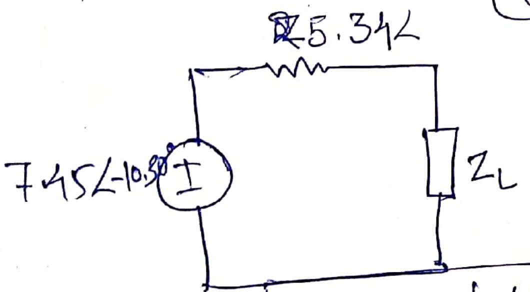
$$Z_{Th} = 2.933 + 4.466i$$

Finding V_{Th}:-



$$\Rightarrow \quad i = \frac{10}{4+8-j6} = \frac{10}{12-j6}$$

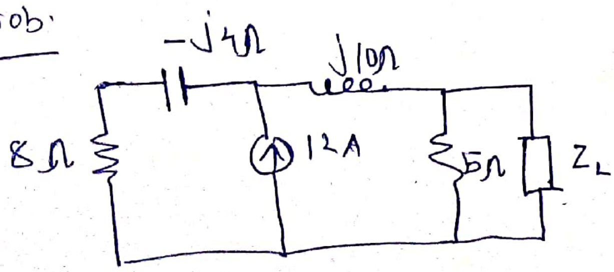
$$V_{Th} = \frac{(8-j6) \times 10}{(12-j6)} = \cancel{0.44-0.88j} = 7.45\angle -10.30$$



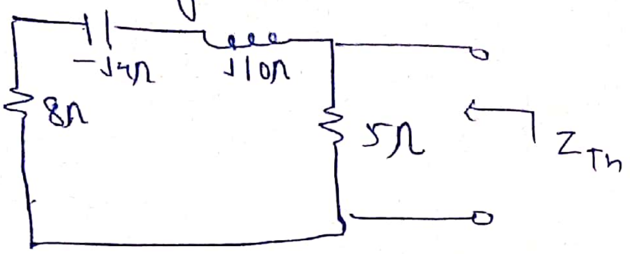
$$Z_L = Z_{Th}^{*} = 2.933 - 4.466i\ \Omega$$

$$\boxed{P_{max} = \frac{|V_{max}|^2}{8R_{Th}} = \frac{(7.45\angle -10.30)^2}{8 \times 2.933} = 2.368\,W}$$

Prob.

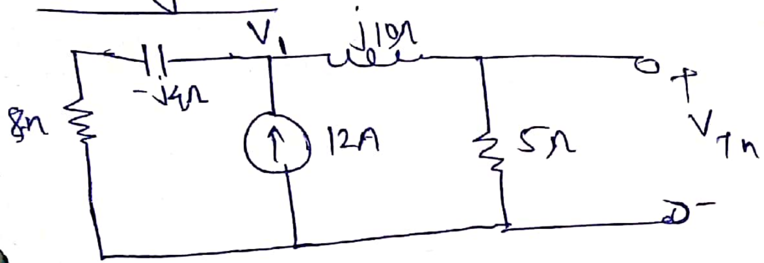


Soln:- Finding Z_{Th}



$$Z_{Th} = (8 - j4 + j10) \parallel 5$$
$$= \frac{(8 + j6) \times 5}{13 + j6}$$
$$= 3.49 \angle 12.09$$
$$= 3.414 + 0.731j$$

Finding V_{Th}:-



$$12 = \frac{V_1}{8 - j4} + \frac{V_1 - V_{Th}}{j10}$$

$$\Rightarrow 12 = V_1(0.1 - 0.05j) + 0.1j\, V_{Th} \quad - (1)$$

Also,
$$\frac{V_{Th}}{5} + \frac{V_{Th} - V_1}{j10} = 0$$

$$\Rightarrow (0.2 - 0.1j)V_{Th} + 0.1j\, V_1 = 0 \quad - (2)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ V_{Th} \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} E & B \\ F & D \end{bmatrix} = DE - BF$$

$$\Delta_2 = \begin{bmatrix} A & E \\ C & F \end{bmatrix} = AF - CE$$

$$\Delta = AD - BC$$

$$V_1 = \Delta_1/\Delta = \frac{84.70 + 98.8}{83.81 \angle 12.09} = \cancel{130.158} \angle 49.39$$

$$V_{Th} = \Delta_2/\Delta = \cancel{58.20 \angle 14.0} \quad 37.48 \angle -51.$$

$$P_{max} = \frac{|V_{Th}|^2}{8 \times R_{Th}} = \frac{(\cancel{58.20})^2}{8 \times 3.414} = \frac{(37.48)^2}{8 \times 3.414}$$
$$= 51.43\,W$$

Apparent Power & Power factor:-

The apparent power is the product of the rms values of voltage and current.

$$\boxed{S = V_{rms} I_{rms}}$$

Average power

$$P_{ave} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\text{Power factor } pf = P_{ave}/S = \cos(\theta_v - \theta_i)$$

Ex:- Obtain the power factor and the apparent power of a load whose impedance is $Z = 60 + j40\,\Omega$, when the applied voltage is $V(t) = 320\cos(377t + 10^\circ)$ V

Soln:-

$$i(t) = \frac{V(t)}{Z} = \frac{320\angle 10^\circ}{60 + j40} = 4.43(-23.69^\circ)$$

$$\text{Apparent power} = V_{rms} I_{rms}$$

$$= \frac{1}{2} \times 320 \times 4.437 = 709.92 \text{ VA}$$

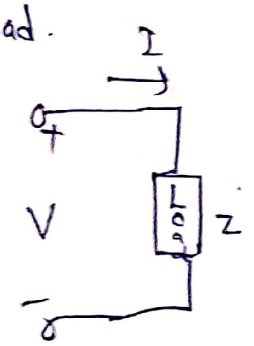
$$\text{Power factor} = \cos(\theta_v - \theta_i) = \cos(10^\circ + 23.69^\circ)$$

$$= 0.8320 \text{ (lagging)}$$

Complex power:-

Complex Power :- Complex Power is used to find the total effect of Parallel loads. Complex power is important in power analysis because It Contains all the information Pertaining to the power absorbed by a given load.

Consider the ac load in fig. providing Phasor form, $V = V_m \angle \theta_v$ and $I = I_m \angle \theta_i$ of Voltage $v(t)$ and Current $i(t)$, the Complex power S absorbed by the ac load is the Product of the Voltage and the Complex Conjugate of the Current, or



$$S = \frac{1}{2} V I^* \quad -(1)$$

$$\Rightarrow \quad S = V_{rms} \times I_{rms}^* \quad -(2)$$

where
$$V_{rms} = \frac{V_m}{\sqrt{2}} \angle \theta_v \quad -(3)$$

&
$$I_{rms} = \frac{I_m}{\sqrt{2}} \angle \theta_i \quad -(4)$$

$$\Rightarrow \quad I_{rms}^* = \frac{I_m}{\sqrt{2}} \angle -\theta_i \quad -(5)$$

from Eqn (2), (3) & (5) we have,

$$S = \frac{V_m I_m}{2} \angle \theta_v - \theta_i \quad -(6)$$

$$S = V_{rms} I_{rms} \left[\cos(\theta_v - \theta_i) + j \sin(\theta_v - \theta_i) \right]$$

The Complex Power may also be expressed in terms of load impedance Z. It Can be Written as,

$$Z = \frac{V}{I} = \frac{V_{rms}}{I_{rms}} = \frac{V_{rms}}{I_{rms}} \angle \theta_v - \theta_i \quad -(7)$$

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Thus, $V_{rms} = Z I_{rms}$

Hence,

$$S = I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*} = V_{rms} I_{rms}^* \quad (8)$$

Since, $Z = R + jX$, Thus,

$$S = I_{rms}^2 (R + jX) = P + jQ \quad (9)$$

Where,

$$P = Re(S) = I_{rms}^2 R \quad (10)$$

$$\& \quad Q = Im(S) = I_{rms}^2 X \quad (11)$$

Where P is the Real Power & Q is the Reactive Power. Comparing Eqn. (9) & (6), We Can Write,

$$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) \quad (12)$$

$$\& \quad Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i) \quad (13)$$

Real Power P Can be expressed in Watts, whereas Q Can be expressed by VAR (Volt Ampere Reactive) to distinguish from Real Power.

1. $Q = 0$, for resistive loads (Unity pf)
2. $Q < 0$, for Capacitive loads (leading p.f) ⇒ Current leads
3. $Q > 0$, for Inductive loads (lagging pf) ⇒ Voltage leads or Current lags.

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Prob. The Voltage across a load is $V(t) = 60\cos(\omega t - 10^\circ)$ V and the Current through the element in the direction of the Voltage drop is $i(t) = 1.5\cos(\omega t + 50^\circ)$ A. Find -

(a) The Complex and Apparent powers (b) The real and reactive Powers, (c) the power factor and the load impedance.

Soln: (a) In the given Problem, Current leads by 60° from the Voltage, therefore phase difference $\theta_v - \theta_i = -60^\circ$

and $V_m = 60$, $I_m = 1.5$

therefore, $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{60}{\sqrt{2}}$

& $I_{rms} = \frac{1.5}{\sqrt{2}}$

The Complex power is given by,

$$S = V_{rms} I_{rms}[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

$$= \frac{60 \times 1.5}{2}[\cos(-60^\circ) + j\sin(-60^\circ)]$$

$$\boxed{S = 22.5 - j38.97}$$

(b) Apparent power $P = V_{rms} I_{rms} = \frac{60 \times 1.5}{2} = 45$ VA

(b) Real power $P = 22.5$ VA

Reactive power depends on, the power factor, whether is leading or lagging. in this Problem, PF is leading therefore Q will be -ve.

$$Q = -V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

$$\boxed{Q = -38.97 \text{ VAR}}$$

(c) Power factor = $\cos \angle \theta_v - \theta_i$

$= \cos(-60^\circ) = \frac{1}{2}$ (leading)

$$\boxed{Z = \frac{V}{I} = \frac{60\angle -10^\circ}{1.5\angle 50^\circ} = 40\angle -60^\circ\ \Omega}$$