# Solution - Tutorial - 6

**Soln. 1 :-** $V = 3\cos 4t + 4\sin 4t$

$$V = \sqrt{(3)^2+(4)^2}\,\cos\{4t - \tan^{-1}(4/3)\} = 5\cos(4t - 53.13^\circ)$$

$\because$ If $X = A\cos\theta + B\cos\theta$

$$X = C\cos(\theta - \tan^{-1}(B/A))$$

where $C = \sqrt{A^2+B^2}$

**Soln. 2 :-**

$$i(t) = 2\cos(6t+120^\circ) + 4\sin(6t-60^\circ)$$

$$= \cancel{2\sin(90-(6t-120^\circ))}$$

$$= 2\cos(6t+120^\circ) + 4\cos(6t-60^\circ-90^\circ)$$

$$= 2\angle 120^\circ + 4\angle -150^\circ$$

$$= 4.472\angle -176.56$$

$$= -4.464 - 0.267j$$

$$= 4.464\cos(6t - 176.56^\circ)$$

$$= 4.464\sin(6t - 176.56^\circ + 90^\circ)$$

$$= 4.464\sin(6t - 86.56^\circ)$$

**Soln. 3 :-**

$$V = 3\cos 3t$$

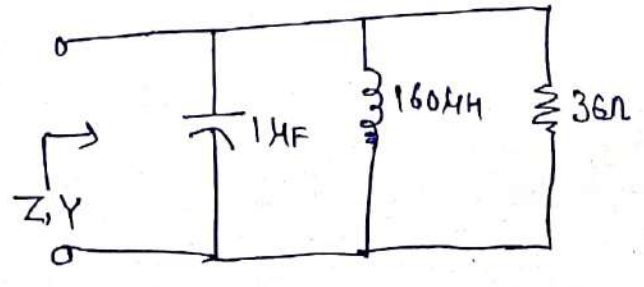
$$i = -2\sin(3t+100^\circ)$$

$$i = 2\cos(3t+100+90^\circ)$$

$$i = 2\cos(3t+190^\circ)$$

Hence Current leads with Voltage by 90°.

**Soln. 4 :-**

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$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$Y = Y_1 + Y_2 + Y_3$$

$$= j\omega C + \frac{1}{j\omega L} + \frac{1}{R}$$

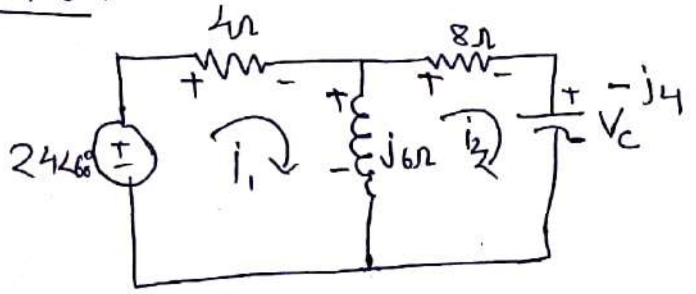
$$= j \times 2\pi \times 10^4 \times 10^{-6} + \frac{1}{j \times 2\pi \times 10^4 \times 160 \times 10^{-6}} + \frac{1}{36}$$

$$= 0.062j - j0.099 + \frac{1}{36}$$

$$\boxed{Y = -0.037j + 0.027}$$

$$\boxed{Z = 12.87 + 17.63j}$$

Soln. 5:-




Applying KVL in Mesh 1

$$24\angle 60^\circ - 4i_1 - j6(i_1 - i_2) = 0$$

$$\Rightarrow (4+j6)i_1 - j6i_2 = 24\angle 60^\circ \quad \text{--- (1)}$$

Applying KVL in Mesh 2

$$-8i_2 + j4i_2 + j6(i_1 - i_2) = 0$$

$$\Rightarrow j6i_1 - (8+j2)i_2 = 0 \quad \text{--- (2)}$$

$$\begin{bmatrix} 4+6j & -j6 \\ j6 & -(8+j2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 24\angle 60^\circ \\ 0 \end{bmatrix}$$

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$$\Delta = \begin{bmatrix} A & B \\ B & D \end{bmatrix} ; \quad \Delta_1 = \begin{bmatrix} A & B \\ E & F \end{bmatrix} \quad \Delta_2 \begin{bmatrix} E & A \\ F & B \end{bmatrix}$$

From Cramer's Rule,

$$i_1 = \Delta_1/\Delta \quad \& \quad i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} E & B \\ F & D \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} A & E \\ C & F \end{bmatrix}$$

$$i_1 = \Delta_1/\Delta = \frac{DE - BF}{AD - BC}$$

$$i_2 = \Delta_2/\Delta = \frac{AF - EC}{AD - BC}$$

Where, $A = 4 + 6j$, $B = -6j$, $C = 6j$, $D = -(8 + j2)$

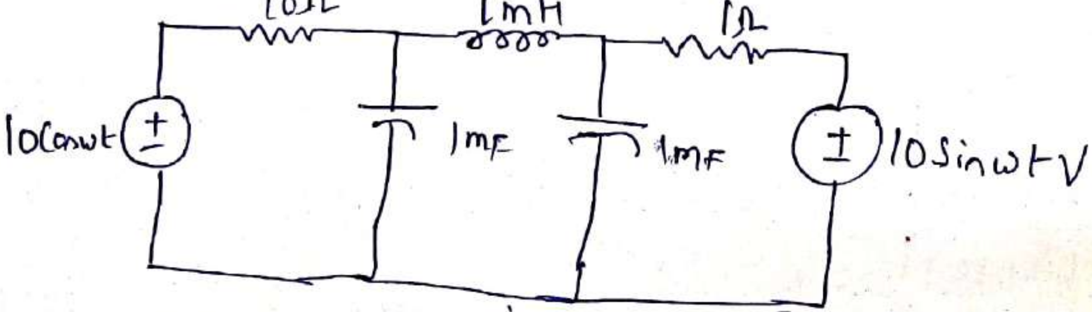
$E = 24\angle 60^\circ$, $F = 0$

Substituting in above Eqn, We have,

$$i_1 = 2.184 + j1.213$$

$$i_2 = -0.4706 + 1.756j$$

Soln. 6 :-




$$X_L = j\omega L = j \times 10^3 \times 10^{-3} = j\,\Omega$$

$$X_C = \frac{-J}{10^3 \times 10^{-3}} = -j\Omega$$

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Applying KVL in Loop 1,

$$10\angle 0^\circ = 10 i_1 - j(i_1 - i)$$

$$10\angle 0^\circ = (10 - j) i_1 - j i \quad -(1)$$

Applying KVL in Loop 2,

$$-ji + j(i - i_2) - j(i_1 - i) = 0$$

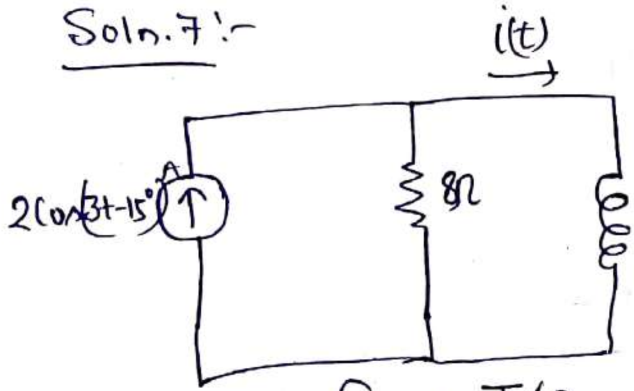
$$\Rightarrow \quad ji - ji_1 - ji_2 = 0 \quad -(2)$$

Applying KVL in Loop 3,

$$-i_2 - 10\angle -90^\circ - j(i - i_2) = 0$$

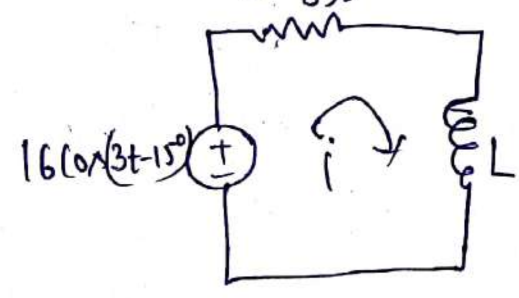
$$\Rightarrow \quad -ji + i_2(-1 + j) = 10\angle 90^\circ \quad -(3)$$

Soln. 7 :-




Given that $i(t) = B\cos(3t - 51.87^\circ)$

Applying Source T/F,




The Current through the inductor is,

$$i(t) = B\cos(3t - 51.87^\circ)$$

$$\Rightarrow \frac{V}{R + j\omega L} = B\cos(3t - 51.87^\circ)$$

$$\Rightarrow \frac{16\cos(3t - 15^\circ)}{8 + j\omega L} = B\cos(3t - 51.87^\circ)$$

$$\Rightarrow \frac{16}{8 + j3L}\angle -15^\circ = B\angle -51.87^\circ \quad -(1)$$

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Comparing Angle or Phase term of both sides.

$$-51.87^\circ = -15^\circ - \tan^{-1}(\beta L/8)$$

$$\Rightarrow \quad \cancel{+}36.87^\circ = \cancel{+}\tan^{-1}(\beta L/8)$$

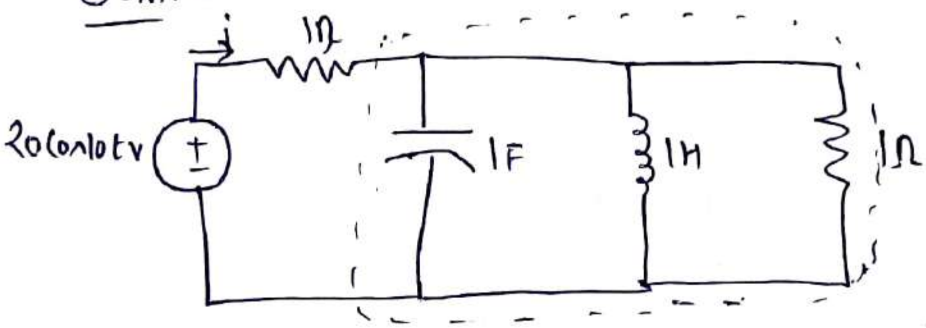
$$\Rightarrow \quad \boxed{L = 2H}$$

Comparing Magnitude terms,

$$B = \frac{16}{\sqrt{64+9L^2}}$$

$$\Rightarrow \quad \boxed{B = \frac{16}{16} = 1.6}$$

Soln. 8 :-




$$\frac{1}{Z_1} = \frac{1}{R_1} + \frac{1}{X_L} + \frac{1}{X_C}$$

$$\Rightarrow \quad Y_1 = 1 + \frac{1}{j10\times 1} + j\times 10\times 1$$

$$= 1 + \frac{1}{j10} + j10$$

$$= \frac{j10 + 1 - 100}{j10}$$

$$Z_1 = \frac{j10}{j10-99}$$

$$Z_{eq} = 1 + \frac{j10}{j10-99} = \frac{j20-99}{j10-99}$$

$$i = \frac{20\cos 10t}{\left(\frac{j20-99}{j10-99}\right)} = \frac{20\angle 0}{1.01 - 0.99i} = 19.703\angle 5.653^\circ$$

$$= 19.703\cos(10t + 5.653^\circ)$$

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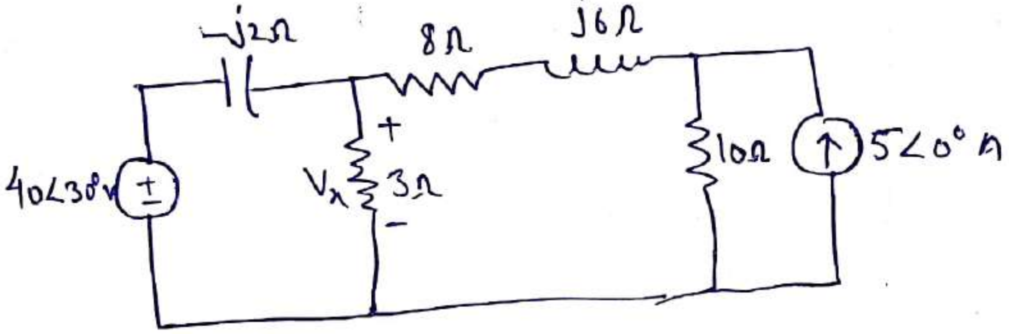
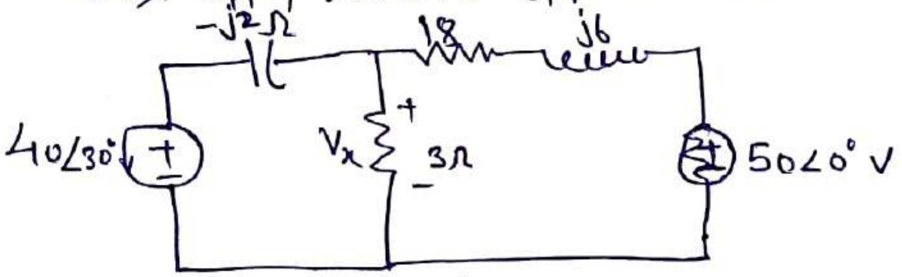
Soln. 9 :-



Nodal analysis is the best way to use on this problem. To do this apply source t/f to make the problem easier.



Applying Nodal Analysis into Node,

$$\frac{V_x - 40\angle 30^\circ}{-j2\Omega} + \frac{V_x}{3} + \frac{V_x - 50\angle 0^\circ}{18+j6} = 0$$

$$\Rightarrow \frac{V_x}{-j2} + \frac{V_x}{3} + \frac{V_x}{18+j6} = \frac{40\angle 30^\circ}{-j2} + \frac{50\angle 0^\circ}{18+j6}$$

$$\Rightarrow \frac{(54+j18)V_x + (12-36j)V_x - j6V_x}{-6j(18+j6)} = 20\angle 120^\circ + 2.635\angle -18.43^\circ$$

$$\Rightarrow V_x(66-24j) = 2062.008\angle 4289$$

$$\boxed{V_x = 29.36\angle 62.87^\circ \text{ volt}}$$

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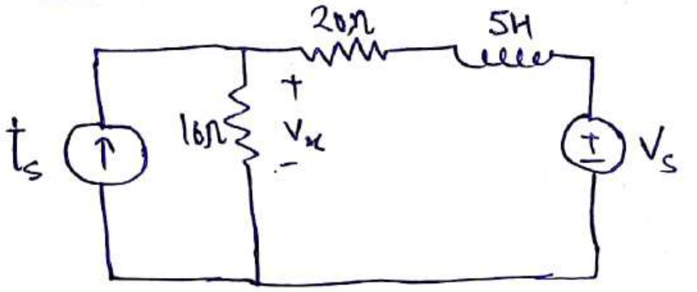
Soln. 10 :-



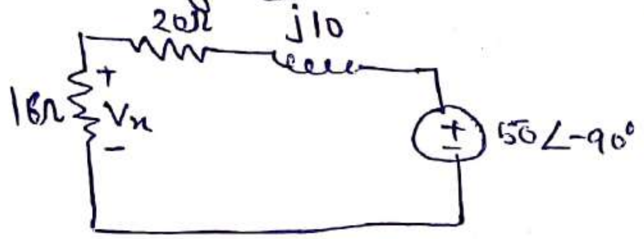

Given that -

$V_s = 50 \sin 2t = 50 \angle -90^\circ$

$i_s = 12 \cos(6t + 10^\circ) A$

$i_s = 12 \angle 10^\circ$

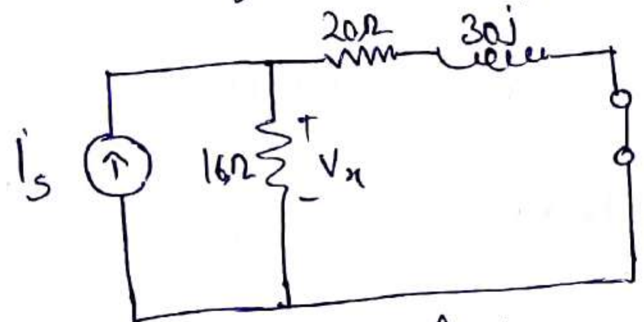
Considering $V_s = 50 \angle -90^\circ$ voltage source, & $\omega = 2$




$$V_x = \frac{50 \angle -90^\circ}{36 + j10} \times 16$$

$$\boxed{V_{x_1} = 21.411 \angle -105.524}$$

Considering $i_s = 12 \angle 10^\circ$, & $\omega = 6$,




Applying Nodal Analysis,

$$12 \angle 10^\circ = \frac{V_x}{16} + \frac{V_x}{20 + 30j}$$

$$\Rightarrow 6922.65 \angle 66.31^\circ = (20 + 30j) V_x + 16 V_x$$

$$\Rightarrow 6922.65 \angle 66.31^\circ = (36 + 30j) V_x$$

$$\Rightarrow \boxed{V_{x_2} = 147.72 \angle 26.504}$$

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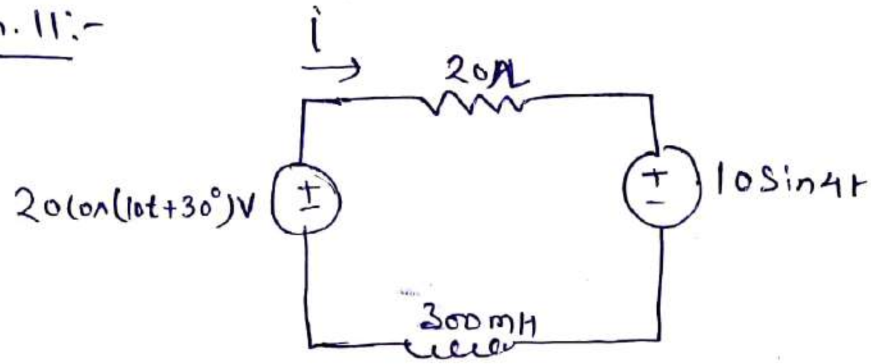
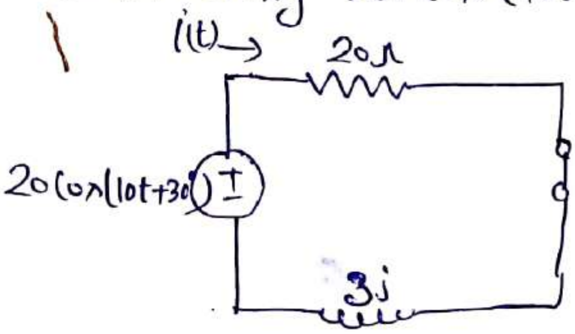
$$V_x = V_{x_1} + V_{x_2}$$

$$\boxed{V_x = 21.411\cos(2t - 105.524^\circ) + 147.72\cos(6t + 26.504^\circ)}$$

Soln. 11:-




Considering $20\cos(10t+30^\circ)$V Voltage Source, & $\omega = 10$

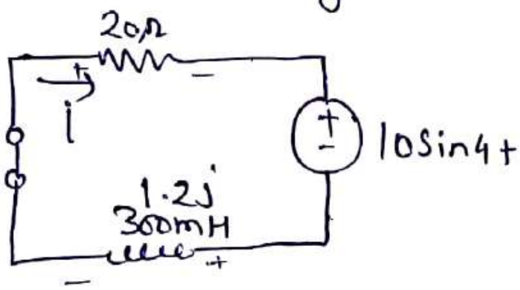



$$i(t) = \frac{20\angle 30^\circ}{20 + 3j}$$

$$= 0.989\angle 21.47^\circ$$

$$i_1(t) = 0.989\cos(10t + 21.47^\circ)$$

Considering $10\sin 4t$ V Voltage Source & $\omega = 4$,




$$i(t) = -\frac{10\sin 4t}{20 + 1.2j} = -\frac{10\angle 0^\circ}{20 + 1.2j} = -0.499\angle -3.43^\circ$$

$$i_2(t) = -0.499\sin(4t - 3.43^\circ)$$

$$= 0.499\sin(4t - 3.43^\circ + 180^\circ)$$

$$= 0.499\sin(4t + 176.57^\circ)$$

therefore,

$$\boxed{i(t) = i_1(t) + i_2(t) = 0.989\cos(10t + 21.47^\circ) + 0.499\sin(4t + 176.57^\circ)\ \text{A}}$$

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