16
DC Machines

Objectives : After completing this Chapter, you will be able to :

- State the importance of dc machines.
- Describe the basic construction of a dc machine.
- Describe the principle of working of a dc machine.
- Explain the process of commutation in dc machines.
- State the difference between lap winding and wave winding and state the number of parallel paths in these two types of windings.
- Derive the emf equation for a dc machine.
- State the difference between the dc generator and dc motor, in respect of the induced emf and the terminal voltage.
- State different ways of establishing magnetic field in dc machines.
- State the two effects of the armature reaction on the magnetic flux.
- State different types of losses occurring in a dc machine and the factors on which these losses depend.
- State three types of efficiency of a dc generator.
- Derive the condition for maximum efficiency.
- State and explain the open-circuit characteristic (OCC) and load characteristic of a dc generator.
- Explain how the voltage builds up in a self-excited generator.
- State the meaning of critical resistance and critical speed in relation to voltage build-up in a dc generator.
- Draw the load characteristic of a compound dc generator, and explain how it depends on the relative ampere-turns of shunt and series fields.
- Draw the equivalent circuit of a dc motor.
- Derive the expression for the torque developed in a dc motor.
- Draw the torque and speed characteristics of shunt, series, and compound motors.
- State the need of a starter in a dc motor, and explain the working of the three-point starter.

16.1 IMPORTANCE

The dc machines were the first electrical machines invented. An elementary dc motor drove an electric locomotive in Edinburgh in 1839, although it took another forty years before dc motors were commercially used. It is still the best motor to drive trains and cranes.

The dc machine can be used either as a motor or a generator. However, because semiconductor rectifiers can easily convert ac into dc, dc generators are not needed except for remote operations. Even in the automobiles, the dc generator has been replaced by the *alternator* plus diodes for rectification. Nevertheless, the generator operations must be discussed because motors operate as generators in braking and reversing.

Portable devices powered by batteries require dc motors, such as portable tape players, walkman, window-lifters, etc. Also, the dc motor is readily controlled in speed and torque and hence is useful for control systems. Examples are robots, elevators, machine tools, rolling mills, etc.

16.2 CONSTRUCTION OF A DC MACHINE

Figure 16.1 shows the basic structure of a dc machine (a motor or a generator). The machine has following important parts.

Stator Magnetic Structure

Figure 16.1*a* shows the magnetic structure of a four-pole dc machine. Its main components are described below.

(*i*) Yoke : It is the outermost cylindrical part which serves two purposes. First, it acts as a supporting frame for the machine, and secondly, it provides a path for the magnetic flux. It is made of cast iron, cast steel, or forged steel. Usually, small machines have cast-iron yokes.

(*ii*) Poles : The machine has salient poles. The *pole cores* are fixed inside the yoke, usually by bolts. The cross-section of the pole core is rectangular. By attaching a *pole shoe,* the end of the pole is made to have a cylindrical surface. The cross-sectional area of the pole shoe is considerably larger than that of the pole core to leave as little inter-pole space as practical. This is done to reduce the leakage flux. The poles are made of cast steel, or forged steel. Each pole carries a *field coil* (or exciting coil). Small machines usually use permanent magnets.

(*iii*) Field Coils : The field coils are wound on the pole cores and are supported by the poles shoes. All coils are identical and are connected in series such that on excitation by a dc source, alternate N and S poles are made. Thus, a machine always has even number of poles. The magnetic flux distribution approximates a square wave, as shown in Fig. 16.2 for the four-pole structure shown in Fig. 16.1. The flux is taken positive in the radially inward direction. Note that the yoke carries one-half of the pole flux Φ. Therefore, the cross-section of the yoke should be selected accordingly.

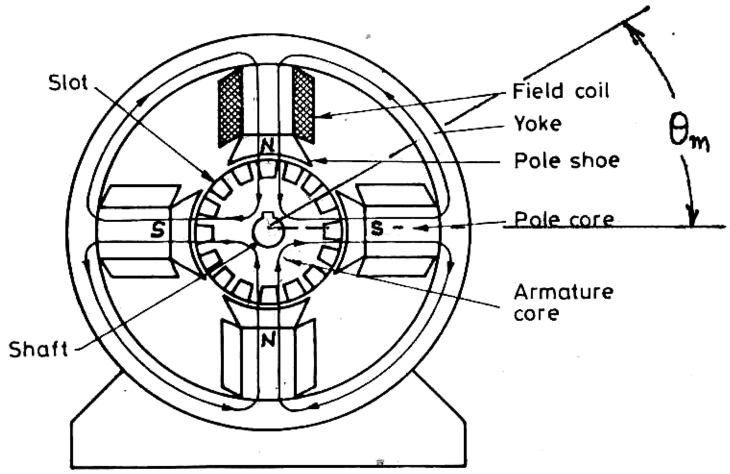


Fig. 16.1 Main parts of a dc machine.

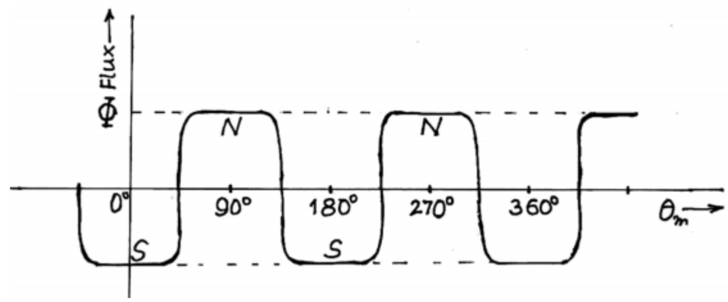


Fig. 16.2 Magnetic flux distribution for four-pole dc machine.

Rotor

The rotor is the inner cylindrical part having armature and commutator-brush arrangement. It is mounted on the shaft of the motor.

(*i*) Armature : The armature core consists of steel laminations, each about 0.4 – 0.6 mm thick, insulated from one another. The purpose of laminating the core is to reduce the eddy-current loss. Slots are stamped on the periphery of the laminations to accommodate the armature winding. The top of the slot have a groove in which a wedge can be fixed. After the winding *conductors* are put into the slots, the wedge is inserted. The wedge prevents the conductors from flying out due to the centrifugal force when the armature rotates. The axial length of the armature is the same as that of the poles on the yoke. The term *conductor* refers to the active portion of the winding, namely that part which cuts the flux when the rotor rotates, thereby generating an alternating emf.

(*ii*) Commutator : It consists of a large number of wedge-shaped copper segments or bars, assembled side by side to form a ring. The segments are insulated from one another by thin mica sheets. Each segment is connected to a coil-end of the armature winding, as shown schematically in Fig. 16.3. The radial lines represent the active lengths of the rotor conductors. The commutator is a part of the rotor and participates in its rotation.

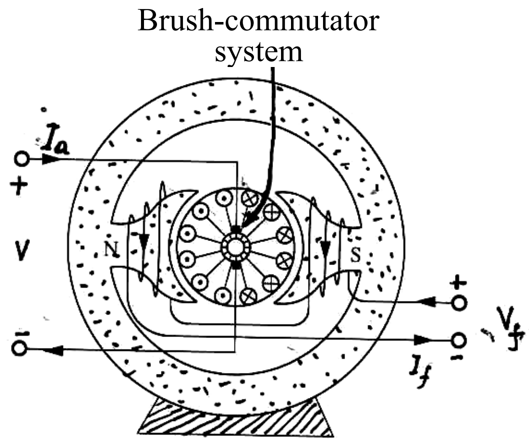


Fig. 16.3 A two-pole dc motor with a brush-commutator system.

(*iii*) Brushes : Two stationary brushes, made of carbon, are pressed against the commutator with the help of a spring fitted in a brush-gear. The brush-commutator system provides two related functions : (*i*) electrical connection is made with the moving rotor, and (*ii*) a steady or direct voltage is obtained from the alternating emf generated in the rotating conductors.

Process of Commutation

The width of a brush is made a little more than the width of a commutator segment and the mica insulation. Whenever, a brush spans two commutator segments, it short-circuits the two coils connected to these segments. On the two sides of the *magnetic neutral axis* (MNA), the conductors of the armature winding carry currents in opposite directions. The brushes are aligned along the MNA, so that they make contact with conductors which are moving midway between the poles and therefore have no emf induced in them. Thus, the reversal of current directions in the two short-circuited coils can take place with least sparking.

Commutation means the process of current collection by a brush, or the changes that take place in the coils during the period of short-circuit by a brush. The reversal of current in a coil during the commutation period sets up a self-induced emf in the coil undergoing commutation. This emf, called *reactance voltage*, opposes the reversal of current.

16.3 ARMATURE CURRENT AND FLUX

In the two-pole dc motor shown in Fig. 16.3, I_f is the *field current* (or *exciting current*) supplied to the stator field winding from the source V_f. Current I_a is the current supplied to the armature from the dc mains of voltage V. Because of the brush-commutator system, the currents in the conductors on the right side are into the paper and in the conductors on the left side are out of the paper.

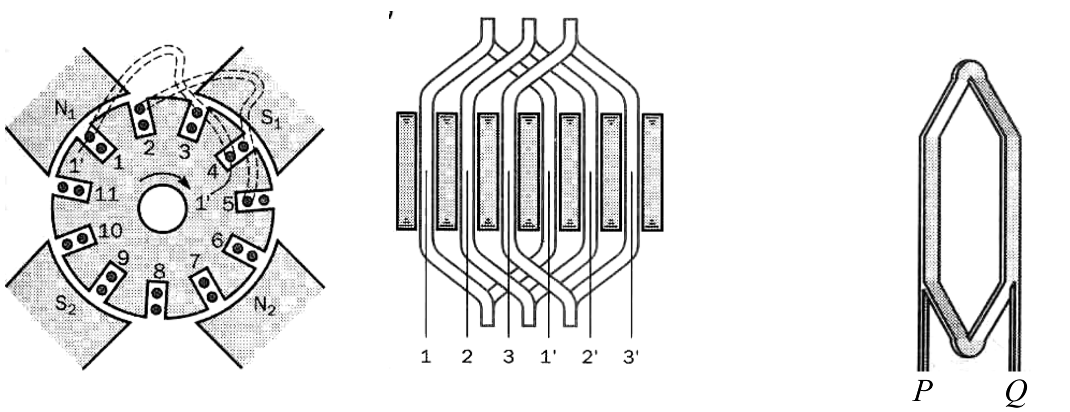
The currents in the armature conductors produce their own flux. According to right-hand thumb rule, the flux produced will be upwards. This is equivalent to making the bottom of the rotor a north pole and the top a south pole. These poles are attracted to their opposites on the stator. Thus, a counterclockwise torque is produced on the rotor.

16.4 ARMATURE WINDING

When the armature rotates, a small emf is induced in each conductor. Large emf can be obtained if a number of conductors are connected in series such that their emfs add up as we travel along the circuit. Thus, a conductor under *N*-pole has to be connected to a conductor under *S*-pole. To make all the coils identical, it is most convenient and practical to connect conductors housed in slots one pole-pitch[1] apart.

Double-Layer Armature Winding

Normally, the armature winding is arranged in double layer, as shown in Fig. 16.4*a* for a four-pole armature with 11 slots. First, a coil is wound in the correct shape and then it is assembled on the core. For making all the coils similar in shape, it is necessary that if side 1 of a coil occupies the outer half of a slot under N_1 pole, the other side 1' occupies the inner half of another slot in similar position under S_1 pole. This brings in a kink in the end connections so that the coils may overlap one another as they are assembled. Figure 16.4*b* shows three coils 1-1', 2-2' and 3-3', are arranged in the slots so that their end connections overlap one another.



(*a*) Arrangement of double-layer winding

(*b*) Arrangement of overlap of end connections.

(*c*) Ends of a coil.

Fig. 16.4 Double-layer armature winding.

Note that a coil may have a number of turns. The two ends of a coil are brought out to *P* and *Q*. As far as the connections to the commutator segments are concerned, the number of turns on each coil is of no consequence.

With 11 slots, it is impossible to make the distance between 1 and 1' exactly a pole pitch. In Fig. 16.4*a*, one side of coil 1-1' is shown in slot 1 and the other side is in slot 4. Thus, the *coil span* is $4 - 1 = 3$. In practice, the coil span must be a whole number and is approximately given as

[1] Pole-pitch is number of conductors per pole.

$$\text{Coil span} = \frac{\text{Total number of slots}}{\text{Total number of poles}} \qquad \text{(a whole number)}$$

In the example shown in Fig. 16.4, we had taken a very small number of slots (only 11) for the sake of simplicity. In actual machines, the number of slots per pole is 10 – 15.

Two Types of Winding

Once the coils are formed, they are to be connected in series through the commutator segments so that more emf is made available. The end of one coil is connected to the start of another coil. There are two ways of making such connections resulting in two types of windings described below.

(1) Lap Winding : As shown in Fig. 16.5*a*, finishing end of one coil is connected via the commutator segment to the starting of the adjacent coil under the same pole. This winding is called lap winding because the sides of the successive coils overlap each other. A lap winding has as many parallel paths between the positive and negative brushes as there are poles.

(2) Wave Winding : In wave winding, as shown in Fig. 16.5*b*, one side of a coil under one pole is connected to the other side of a coil which occupies approximately the same position under the next pole, through back connection. The second coil-side is then connected forward to another coil-side under the next pole. (In lap winding, the second coil-side is connected back through the commutator segment to a coil-side under the original pole.) A wave winding has only two paths in parallel, irrespective of the number of poles.

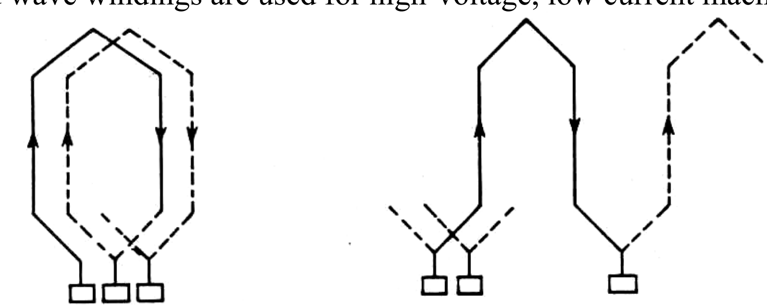
Thus, if a machine has P poles, the *number of parallel paths* in armature winding is

$$A = P \qquad \text{(for lap winding)}$$

and

$$A = 2 \qquad \text{(for wave winding)}$$

Hence, it may be said that, in general, lap windings are used for low-voltage, heavy-current machines, and wave windings are used for high-voltage, low current machines.



(*a*) Lap winding. (*b*) Wave winding.

Fig. 16.5 Types of armature windings.

Example 16.1 The armature of an eight-pole dc generator has 480 conductors. The magnetic flux and the speed of rotation are such that the average emf generated in each conductor is 2.1 V, and each conductor is capable of carrying a full-load current of 200 A.

Calculate the terminal voltage on no load, the output current on full load and the total power generated on full load, when the armature is (*a*) lap-wound, and (*b*) wave-wound.

Solution **:** (*a*) With the armature lap-wound, the number of parallel paths, $A = P = 8$. Therefore, the number of conductors per path is

$$\frac{Z}{A} = \frac{480}{8} = 60$$

Therefore, the terminal voltage on no load,

$$E = e \times \left(\frac{Z}{A}\right) = 2.1 \times 60 = \mathbf{126\ V}$$

The output current on full load,

$$I_L = \text{Full-load current per conductor} \times \text{ no. of parallel paths}$$

$$= 200 \times 8 = \mathbf{1600\ A}$$

The total power generated on full load,

$$P_o = I_L \times E = 1600 \times 126 = 201\ 600\ \text{W} = \mathbf{201.6\ kW}$$

(*b*) With the armature wave-wound, the number of parallel paths, $A = 2$. Therefore, the terminal voltage on no load,

$$E = e \times \left(\frac{Z}{A}\right) = 2.1 \times \frac{480}{2} = \mathbf{504\ V}$$

The output current on full load, $I_L = 200 \times 2 = \mathbf{400\ A}$

The total power generated on full load,

$$P_o = I_L \times E = 400 \times 504 = 201\ 600\ \text{W} = \mathbf{201.6\ kW}$$

Note that the total power generated by a given machine is the same whether the armature is lap-wound or wave-wound.

16.5 EMF EQUATION FOR A DC GENERATOR

Let there be *P* number of poles and let Φ be the magnetic flux per pole in the dc generator. Let *Z* be the total number of conductors and let *A* be the number of parallel paths on the armature winding. Let the rotational speed of the rotor be *N* rpm.

Consider one revolution of the rotor. As the rotor makes *N* revolutions in one minute, it makes *N*/60 revolutions in one second. In other words, the speed of rotation is *N*/60 rps. Therefore, the time (in seconds) taken in making one revolution is

$$\Delta t = \frac{1}{N/60} = \frac{60}{N}$$

As Φ is the magnetic flux per pole and there are *P* poles, the total flux traversed in one revolution by a conductor on the armature is *P*Φ. That is, for a single conductor the change in flux in one revolution is

$$\Delta\Phi = P\Phi$$

Therefore, the induced emf per conductor is given by Faraday's law as

$$e = \frac{\Delta\Phi}{\Delta t} = \frac{P\Phi}{60/N} = \frac{NP\Phi}{60}$$

...(16.1)

The conductors are connected to make coils, and the coils are connected to form parallel paths. The brushes collect the emf from all these identical parallel paths. The net emf E generated in the machine is same as the total emf in one parallel path. As the armature has Z number of conductors, and there are A number of parallel paths, the number of conductors per parallel paths is Z/A. Therefore, using Eq. 16.1 we can write the expression for the net emf E generated in the dc machine as

$$E = e\left(\frac{Z}{A}\right) = \frac{NP\Phi}{60}\left(\frac{Z}{A}\right)$$

or
$$E = \frac{\Phi ZNP}{60A}$$

...(16.2)

Remember, the number of parallel paths,

$A = P$ (for lap winding)

and $A = 2$ (for wave winding)

Example 16.2 A 4-pole, 1200-rpm dc generator has a lap-wound armature having 65 slots and 12 conductors per slot. If the flux per pole is 0.02 Wb, determine the emf induced in the armature.

Solution **:** The total number of conductors, $Z = 65 \times 12 = 780$

For lap winding, the number of parallel paths, $A = P = 4$.

Therefore, using Eq. 16.2, the total emf induced is given as

$$E = \frac{\Phi ZNP}{60A} = \frac{0.02 \times 780 \times 1200 \times 4}{60 \times 4} = \mathbf{312\ V}$$

Example 16.3 The induced emf in a dc machine while running at 500 rpm is 180 V. Assuming constant magnetic flux per pole, calculate the induced emf when the machine runs at 600 rpm.

Solution **:** The induced emf is given by Eq, 16.2 as

$$E = \frac{\Phi ZNP}{60A} = KN$$

where K is a constant for the machine. Therefore, we have

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{or} \quad E_2 = \frac{N_2}{N_1}E_1 = \frac{600}{500} \times 180 = \mathbf{216\ V}$$

Example 16.4 The induced emf in a dc generator running at 750 rpm is 220 V. Calculate (*a*) the speed at which the induced emf is 250 V (assume the flux to be constant), and (*b*) the required percentage increase in the field flux so that the induced emf is 250 V, while the speed is only 600 rpm.

Solution **:** **(***a***)** From Eq. 16.2, if the flux is constant, we have

$$E = KN$$

where K is a constant for the machine. Therefore, we have

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{or} \quad \mathrm{N}_2 = \frac{E_2}{E_1} N_1 = \frac{250}{220} \times 750 = \mathbf{852\ rpm}$$

(*b*) Here, neither the speed nor the flux remains constant. Therefore, from Eq. 16.2, we can write

$$E = K'\Phi N$$

where K' is a constant. Thus, we have

$$\frac{E_2}{E_1} = \frac{\Phi_2 N_2}{\Phi_1 N_1} \quad \text{or} \quad \frac{\Phi_2}{\Phi_1} = \frac{E_2}{E_1} \times \frac{N_1}{N_2} = \frac{250}{220} \times \frac{750}{600} = 1.42$$

Thus, the required percentage increase in flux is

$$(1.42 - 1.00) \times 100 = \mathbf{42\ \%}$$

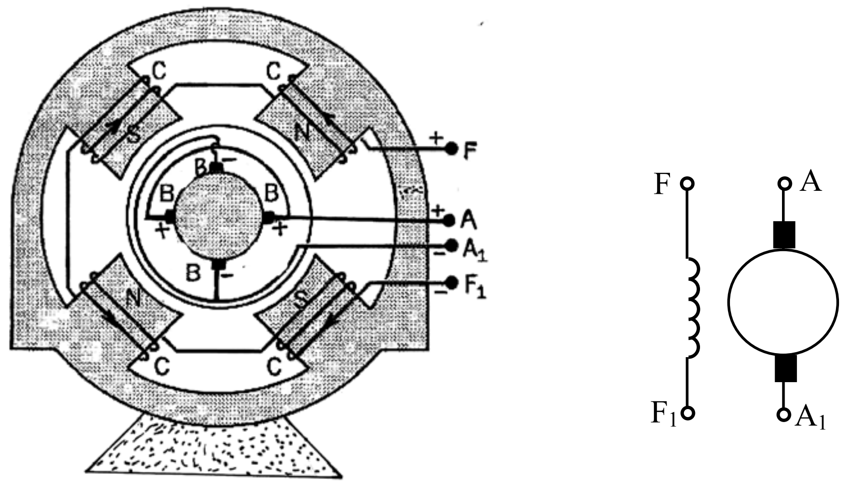
16.6 TYPES OF DC MACHINES

There are several ways of exciting the stator field winding of a dc machine. Each method of field connections gives different characteristics.

Consider a four-pole dc machine shown in Fig. 16.6*a*. The four brushes *B* make contact with the commutator. The brushes are situated in between the north and south poles. The positive brushes are connected to the positive terminal *A* and the negative brushes to the negative terminal A_1. The terminals A-A_1 are used to make connection to the armature winding. Note that the brushes are situated half-way between the north and south poles. This position enables them to make contact with conductors in which little or no emf is being generated. As a result, least sparking is produced when the contact of a brush changes over from one segment to the next during the rotation of the armature.

The four exciting or field coils *C* are connected in series and the ends are brought out to terminals *F* and F_1. The four coils are so connected as to produce *N* and *S* poles alternately. The arrowheads on the coils indicate the direction of the field current I_f when a dc supply is connected to the terminals F-F_1.

Symbolically, a dc machine is represented as shown in Fig. 16.6*b*. The circle represents the armature and the commutator. Only two brushes, placed diametrically opposite, are shown. The field winding is shown separately. Some machines are designed to have more than one field-winding.



(*a*) Armature and field connections. (*b*) Symbolic representation.

Fig. 16.6 A dc machine.

A DC Machine as Generator or Motor

There is no difference of construction between a dc generator and a dc motor. In fact, the only difference is that in a generator the generated emf is greater than the terminal voltage, whereas in a motor the generated emf is less than the terminal voltage.

Let us consider a dc machine *D* whose field winding is connected in shunt across the armature terminals, through a regulator resistor *R*, as shown in Fig. 16.7. Such machine is called *shunt-wound machine.* Let it be driven by an engine and be connected through a centre-zero ammeter *A* to a battery *B*. If we adjust the field regulator *R* such that the reading on *A* is zero, then the emf E_D generated in *D* is exactly equal to the emf E_B of the battery.

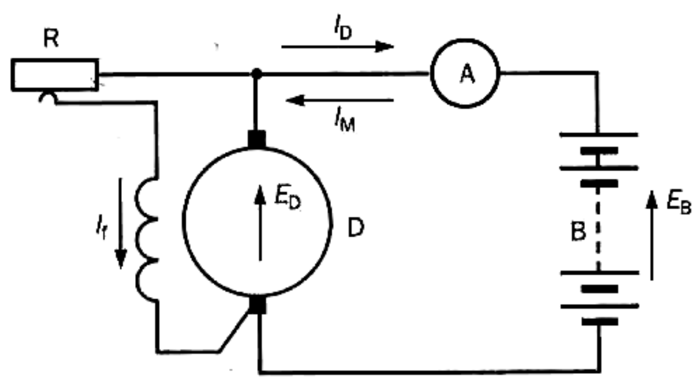


Fig. 16.7 Shunt-wound machine as generator or motor.

Next, let us reduce *R* to increase the field current I_f and hence the magnetic flux Φ. This results in an increased emf E_D generated in machine *D* (see Eq, 16.2). Now, since the emf E_D exceeds emf E_B, the excess emf is available to circulate a current I_D through the resistance R_a of the armature circuit, and the battery. Since the current I_D is in the same direction as the emf E_D, the machine D is working as a ***generator*** of electrical energy. Note that the battery B is getting charged and hence working as a load on the generator.

Next, suppose that we cut off the supply of oil to the engine driving machine D. The speed of the machine falls, the emf E_D decrease, current I_D gets reduced, until when $E_D = E_B$, there is no circulating current I_D. But E_D continues to decrease and becomes less than E_B. Therefore, the current I_M through the ammeter A flows in the reverse direction. The battery B is now supplying electrical energy to drive machine D as an electric ***motor***.

Note that the direction of field current I_f is the same whether the machine D works as a generator or as a motor. The relationship between the emf, the current and the terminal voltage can now be expressed when the machine D works as a generator or as a motor. Let E be the emf generated in the armature of the machine[2], V the terminal voltage, R_a the resistance of the armature circuit, and I_a the armature current.

As Generator : The current I_a flows in the same direction as the generated emf E, and the terminal voltage V is less than the emf E due to the armature-circuit voltage-drop. Thus, we have

$$V = E - I_a R_a \qquad \text{...(16.3)}$$

As Motor : The current I_a flows in the opposite direction to that of the generated emf E, and the terminal voltage V is more than the emf E due to the armature-circuit voltage-drop. Thus, we have

$$V = E + I_a R_a \qquad \text{...(16.4)}$$

Types of DC Generators

The type of dc machine depends on the way the magnetic flux is established in it. Though equally applicable to motors, let us describe different types of dc generators. There can be three ways of establishing magnetic flux in a dc generator :

(1) Using a permanent magnet, (called ***permanent magnet generators***).

(2) Using some external source to excite the field coils, (called ***separately excited generators***).

(3) Using the armature supply to excite the field coils, (called ***self-excited generators***).

In describing various relations for a dc generator, following notations for different quantities are used :

I_a = armature current

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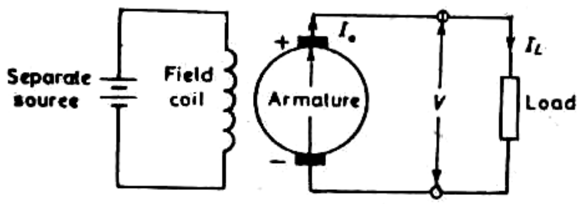
[2] Note that the emf in the armature is generated both in the generator as well as in the motor. In the generator, it is due to this emf that the current flows in the external electric circuit. In a motor, this emf opposes the applied voltage V, and hence it is called ***back emf***.

R_a = net resistance of the armature circuit
E = emf generated in the armature winding
I_{se} = current through series field coil
R_{se} = resistance of the series field coil
I_{sh} = current through shunt field coil
R_{sh} = resistance of the shunt field coil
I_L = current supplied to the load
V = terminal voltage across the load
R_L = load resistance

(1) Permanent Magnet Generators : These do not find many applications in the industry, because of their low efficiency. However, low-power, low-cost, small size machines do use permanent magnet.

(2) Separately Excited Generators : As shown in Fig. 16.8, the field coils are excited from a storage battery or from a separate dc source.



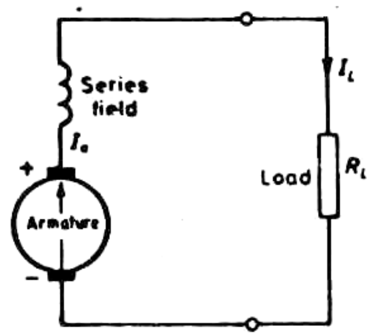
(*i*) $I_a = I_L = I$ (say)
(*ii*) $V = E - IR_a$
(*iii*) Power developed = EI
(*iv*) Power delivered = VI

Fig. 16.8 Separately excited dc generator.

(3) Self-Excited Generators : The field coils are excited by the dc voltage generated by the generator itself. Such generators are further subdivided into following three categories :

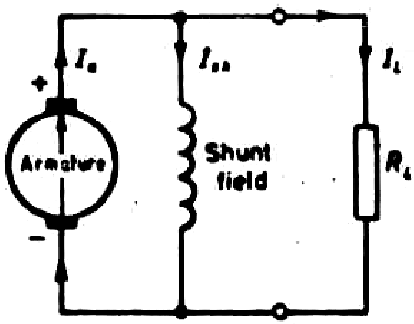
(***a***) ***Series-Wound Generators*** **:** The field coils are connected in series with the armature circuit (Fig. 16.9*a*).

(***b***) ***Shunt-Wound Generators*** **:** The field coils are connected across the armature circuit (Fig. 16.9*b*).



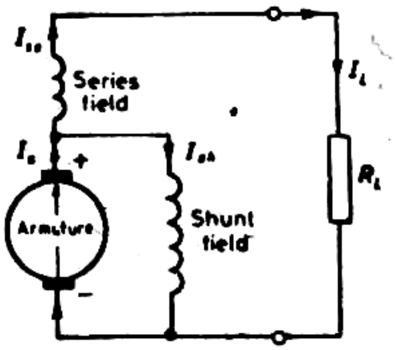
(i) $I_a = I_{se} = I_L = I$ (say)
(ii) $V = E - I\,(R_a + R_{se})$
(iii) Power developed $= EI$
(iv) Power delivered $= VI$

(*a*) Series wound.



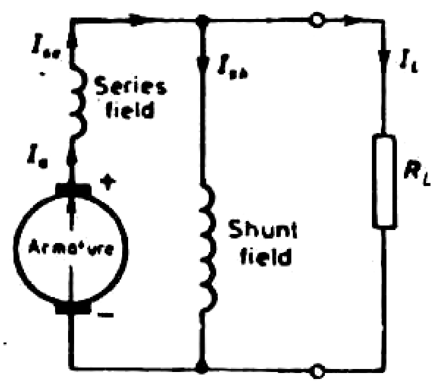
(i) $I_{sh} = V/R_{sh}$
(ii) $I_a = I_{sh} + I_L$
(iii) $V = E - I_aR_a$
(iv) Power developed $= EI_a$
(v) Power delivered $= VI_L$

(*b*) Shunt wound.



(i) $I_{se} = I_L$
(ii) $I_{sh} = (V + I_{se}R_{se})/R_{sh}$
(iii) $I_a = I_{sh} + I_L$
(iv) $V = E - I_aR_a - I_{se}R_{se}$
(v) Power developed $= EI_a$
(vi) Power delivered $= VI_L$

(*c*) Short-shunt compound wound.



(i) $I_a = I_{se} = I_{sh} + I_L$
(ii) $I_{sh} = V/R_{sh}$
(iii) $V = E - I_aR_a - I_aR_{se}$
$= E - I_a\,(R_a + R_{se})$
(iv) Power developed $= EI_a$
(v) Power delivered $= VI_L$

(*d*) Long-shunt compound wound.

Fig. 16.9 Self-excited dc generators.

(*c*) ***Compound-Wound Generators*** **:** There are two windings on each pole, one connected in series and the other in parallel with the armature circuit. The compound-wound generators may again be of two types :

(*i*) ***Short-Shunt*** in which the shunt field winding is connected in parallel with the armature (Fig. 16.9*c*).

(*ii*) ***Long-Shunt*** in which the shunt field winding is connected in parallel with both the armature and series winding (Fig. 16.9*d*).

The compound-wound generators can also be classified, from another point of view, in two classes, viz., *differential compound generators* and *cumulative compound generators* depending on the fact whether the series field opposes or supports the shunt field, respectively.

Example 16.5 A shunt-wound dc generator delivers 496 A at 440 V to a load. The resistance of the shunt filed coil is 110 Ω and that of the armature winding is 0.02 Ω. Calculate the emf induced in the armature.

Solution **:** The current through the shunt-field coil is given as

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{110} = 4 \text{ A}$$

$\therefore$ Armature current, $I_a = I_L + I_{sh} = 496 + 4 = 500 \text{ A}$.

Therefore, the generated emf is

$$E = V + I_a R_a = 440 + (500 \times 0.02) = \mathbf{450\ V}$$

Example 16.6 A 4-pole shunt generator with lap connected armature has armature and field resistances of 0.2 Ω and 50 Ω, respectively. It supplies power to 100 lamps, each of 60 W, 200 V. Calculate the total armature current, the current per path and the generated emf. Allow a brush drop of 1 V at each brush.

Solution **:** The current taken by each lamp, $I_l = \frac{P}{V} = \frac{60}{200} = 0.3\,\text{A}$

Since all the lamps are connected in parallel, the total load current is

$$I_L = 100 \times I_l = 100 \times 0.3 = 30 \text{ A}$$

Shunt field current, $I_{sh} = \frac{V}{R_{sh}} = \frac{200}{50} = 4 \text{ A}$

$\therefore$ Armature current, $I_a = I_{sh} + I_L = 30 + 4 = \mathbf{34\ A}$

For lap winding, the number of parallel paths, $A = P = 4$. Thus,

The current per path, $I_c = \frac{I_a}{A} = \frac{34}{4} = \mathbf{8.5\ A}$

Generated emf, $E = V + I_a R_a + \text{brush-drop} = 200 + 34 \times 0.2 + 2 \times 1 = \mathbf{208.8\ V}$

Example 16.7 A short-shunt compound-wound dc generator supplies a load current of 100 A at 250 V. The generator has following winding resistances :

Shunt field = 130 Ω, armature = 0.1 Ω, and series field = 0.1 Ω

Find the emf generated, if the brush drop is 1 V per brush.

Solution **:** Refer to Fig. 16.9*c*. The series-field current, $I_{se} = I_L = 100\,\text{A}$

The voltage drop across the series field, $V_{se} = I_{se}R_{se} = 100 \times 0.1 = 10\ \text{V}$

The voltage drop across the shunt field, $V_{sh} = V + V_{se} = 250 + 10 = 260\ \text{V}$

The shunt-field current, $I_{sh} = \dfrac{V_{sh}}{R_{sh}} = \dfrac{260}{130} = 2\,\text{A}$

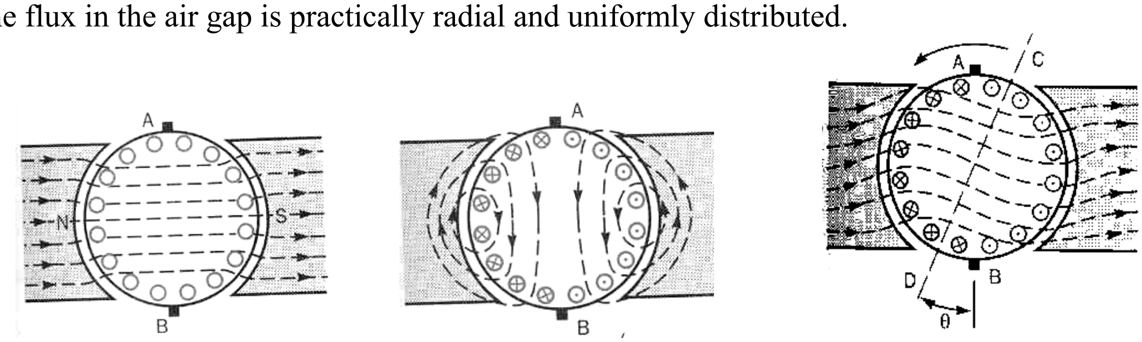
∴ The armature current, $I_a = I_L + I_{sh} = 100 + 2 = 102\ \text{A}$

The generated emf, $E = V + V_{se} + I_a R_a + \text{brush-drop}$

$= 250 + 10 + 102 \times 0.1 + 2 \times 1 = \mathbf{272.2\ V}$

16.7 ARMATURE REACTION

The effect of armature ampere-turns upon the value and distribution of the magnetic flux entering and leaving the armature core is called *armature reaction.* Let us, for simplicity, consider a two-pole dc machine, as shown in Fig. 16.10*a*. The brushes *A* and *B* are placed in the Geometric Neutral Plane (GNP). For the sake of clarity, we have omitted the slots on the armature and shown the conductors uniformly distributed. The figure shows the flux distribution due to the field current alone (i.e., when there is no armature current). Note that the flux in the air gap is practically radial and uniformly distributed.



(*a*) Flux due to field current alone.

(*b*) Flux due to armature current alone.

(*c*) Resultant flux due to field and armature currents.

Fig. 16.10 Flux distribution in a dc machine.

Now, suppose that the dc machine is to work as a motor rotating in counterclockwise direction. To produce a counterclockwise torque, the current is made to flow through the armature conductors in direction shown in fig. 16.10*b*. The figure also shows the flux distribution due to this current alone (assumed no flux due to the field winding). Note that at the centre of the armature and in the pole shoes, the direction of this flux is at right angles to that due to the field winding. For this reason, the flux due to the armature current is called ***cross flux***.

The pole tip which is first met by a point on the armature during its revolution is known as the *leading tip* and the other as *trailing tip*.

Figure 16.10*c* shows the resultant distribution of the flux due to the combination of the fluxes in Figs. 16.10*a* and *b*. We find that over the trailing halves of the pole faces the cross flux is in opposite direction to the main flux, thereby reducing the flux density. On the other hand, over the leading halves of the pole faces the cross flux is in the same direction as the main flux, thereby strengthening the flux density. However, if the teeth are strongly saturated under no load, the strengthening of the flux at the leading pole tips would not be as much as the weakening of the flux at the trailing pole tips. Therefore, the total flux would be somewhat reduced. Hence, the ***demagnetization effect*** is one of the consequences of the armature reaction.

Another important consequence of the armature reaction is to ***distort the flux distribution***. As shown in Fig. 16.10*c*, the Magnetic Neutral Plane (MNP) is shifted through an angle θ from *AB* to *CD*, in a direction opposite to rotation[3].

Thus, the armature reaction has two components, namely, the *demagnetizing component* and the *distorting component.* With the increase in the armature current (or load), both these components increase. At times, when the machine is working as a generator and if the 'short-circuit' or 'excessive-overload' condition occurs, the demagnetizing component may even reverse the polarity of the main poles.

Remedy : The adverse effect of armature reaction can be neutralized by shifting the brushes to the magnetic neutral plane and by increasing the air gap at pole tips.

16.8 LOSSES IN A DC MACHINE

Various losses occurring in a dc machine are as follows.

(1) Copper losses

Copper loss occurs in armature winding, in field winding and brush contacts.

(*i*) **Armature Copper Loss :** It is given as $I_a^2 R_a$. This loss amounts to about 30 to 40 % of the full-load losses.

(*ii*) **Field Copper Loss :** It is given as $I_{sh}^2 R_{sh}$ for shunt-wound machine and as $I_{se}^2 R_{se}$ for series wound machine. This loss amounts to about 20 to 30 % of the full-load losses. For shunt-wound machine, it remains practically constant; but for a series-wound machine, it increases with the load.

(*iii*) **Brush Contact Loss :** This loss occurs due to the resistance of the brush contact with the commutator. This is usually included in armature copper loss.

(2) Magnetic (or Iron) losses

Since the current in the armature winding is alternating at a frequency *f*, the flux produced is also alternating. Some of this flux also enters the pole cores. The magnetic loss, therefore,

[3] If the machine works as a generator, the magnetic neutral plane shifts by angle θ in the direction of rotation.

mainly occurs in the armature core. This loss amounts to about 20 to 30 % of the full-load losses. There can be two types of magnetic (or iron) losses :

(*i*) Hysteresis Loss $= B_{\max}^{1.6} f$

(*ii*) Eddy-current Loss $= B_{\max}^{2} f^{2}$

(3) Mechanical Losses

There are two types of mechanical losses.

(*i*) Air Friction (or Windage) Loss : It occurs due to rotation of the armature.

(*ii*) Bearing Friction Loss : It occurs at the ball-bearing fixed on the rotor.

Mechanical losses are about 10 to 20 % of the full-load losses. Mechanical losses taken together are also called ***stray losses.***

16.9 EFFICIENCY OF A DC GENERATOR

Following types of efficiencies can be defined for a dc generator.

(1) Mechanical Efficiency, $\eta_m = \dfrac{\text{Total watts generated in armature}}{\text{Mechanical power supplied at the input}}$

$$= \frac{EI}{\text{hp} \times 746} \qquad ...(16.5)$$

(2) Electrical Efficiency, $\eta_e = \dfrac{\text{Total watts available to the load}}{\text{Total watts generated}}$

$$= \frac{VI}{EI} \qquad ...(16.6)$$

(3) **Commercial or Overall Efficiency,** $\eta_c = \dfrac{\text{Total watts available to the load}}{\text{Mechanical power supplied}}$

$$= \frac{VI}{\text{hp} \times 746} \qquad ...(16.7)$$

It is obvious that $\eta_c = \eta_m \times \eta_e$.

Condition for Maximum Efficiency

Due to the losses occurring in the generator, its efficiency is not cent percent. The total losses P_t can be divided in to two categories : (*i*) constant losses, P_c, and (*ii*) variable losses, P_v. The copper loss in armature winding (i.e., $I_a^2 R_a$) is the only loss that varies with the load current. Other losses remain almost constant. Now,

$$\text{Efficiency, } \eta = \frac{\text{Output}}{\text{Output + Total losses}} = \frac{VI}{VI + (P_v + P_c)} = \frac{VI}{VI + I_a^2 R_a + P_c}$$

$$= \frac{VI}{VI + I^2 R_a + P_c} \qquad \text{(Since, } I_a \approx I\text{)}$$

$$= \frac{1}{1 + \left(\frac{IR_a}{V} + \frac{P_c}{VI} \right)}$$

For efficiency to be maximum, the denominator of the above expression should be minimum, for which we must have

$$\frac{d}{dI}\left\{ 1 + \left(\frac{IR_a}{V} + \frac{P_c}{VI} \right) \right\} = 0 \qquad \text{or} \qquad \frac{R_a}{V} - \frac{P_c}{VI^2} = 0$$

or $$I^2 R_a = P_c$$

...(16.8)

This shows that maximum efficiency is obtained when the variable loss equals constant loss. Thus, the load current corresponding to the maximum efficiency is given by

$$I^2 = P_c / R_a \qquad \text{or} \qquad I = \sqrt{P_c / R_a}$$

...(16.9)

Example 16.8 A shunt generator gives full-load output of 30 kW at a terminal voltage of 200 V. The armature and shunt-field resistances are 0.05 Ω and 50 Ω, respectively. The iron and friction losses are 1000 W. Calculate (*i*) the emf generated, (*ii*) the copper losses, and (*iii*) the efficiency.

Solution **:** (*i*) $I_L = \frac{30\,\text{kW}}{200\,\text{V}} = 150\ \text{A}; \ I_{sh} = \frac{200\ \text{V}}{50\ \Omega} = 4\ \text{A}; \ I_a = I_L + I_{sh} = 150 + 4 = 154\ \text{A}$

The emf generated, $E = V + I_a R_a = 200 + 154 \times 0.05 = \mathbf{207.7\ V}$

(*ii*) The copper losses $= I_{sh}^2 R_{sh} + I_a^2 R_a = 4^2 \times 50 + 154^2 \times 0.05 = \mathbf{1985.8\ W}$

(*iii*) The efficiency, $\eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{30\,000}{30\,000 + (1000 + 1985.8)} = 0.9095\ \text{pu} = \mathbf{90.95\%}$

Example 16.9 A dc shunt generator, with shunt-field resistance of 52.5 Ω, supplies full-load current of 195 A at 210 V. Its full-load efficiency is 90 % and it has stray losses of 710 W. Determine its armature resistance and the load current corresponding to maximum efficiency.

Solution **:** The output power of the generator, $P_o = V_o \times I_L = 210 \times 195 = 40.95\ \text{kW}$

$\therefore$ The input power, $P_{\text{in}} = \frac{P_o}{\eta} = \frac{40.95\ \text{kW}}{0.90} = 45.5\ \text{kW}$

$\therefore$ Total losses $= P_{\text{in}} - P_o = 45.5 - 40.95 = 4.55\ \text{kW}$

Shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{210\,\text{V}}{52.5\,\Omega} = 4\,\text{A}$

$\therefore$ Armature current, $I_a = I_L + I_{sh} = 195 + 4 = 199$ A

Shunt-field copper loss $= I_{sh}^2 R_{sh} = 4^2 \times 52.5 = 840$ W

$\therefore$ Constant losses $= 840 + 710 = 1550$ W

Thus, the armature copper loss, $I_a^2 R_a = 4550 - 1550 = 3000$ W

Hence, the armature resistance, $R_a = \dfrac{\text{Aramture copper loss}}{I_a^2} = \dfrac{3000}{199^2} = \mathbf{0.0757\,\Omega}$

For maximum efficiency, we must have

Variable losses = Constant losses

or $$I_a^2 R_a = P_c$$

$$\therefore \qquad I_a = \sqrt{\frac{P_c}{R_a}} = \sqrt{\frac{1550}{0.0757}} = \mathbf{143.1\ A}$$

16.10 CHARACTERISTICS OF DC GENERATORS

There are following three important characteristics of a dc generator.

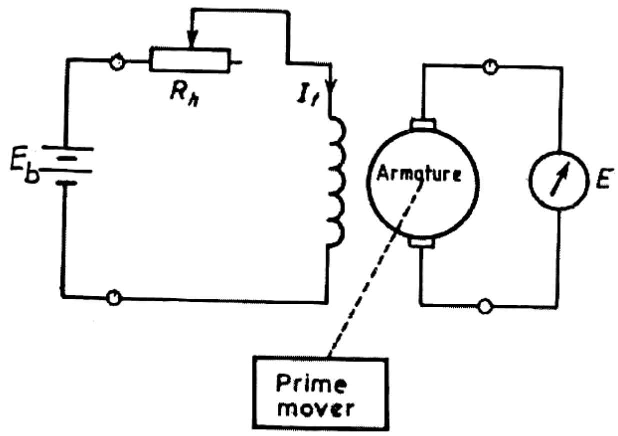
1. **Open-Circuit, Magnetization, or No-Load Characteristic :** It provides the relationship between the no-load emf E generated in the armature and the field (or exciting) current I_f.
2. **Load (or External) Characteristic :** It shows the relationship between the terminal voltage V and the load current I_L. It is also called the ***performance characteristic*** or ***voltage regulation curve***.
3. **Internal Characteristic :** It gives the relationship between the emf E generated in the armature (after considering the demagnetizing effect of armature reaction) and the armature current I_a.

The first two characteristics, which we shall be discussing, are more important to know the performance of the generator.

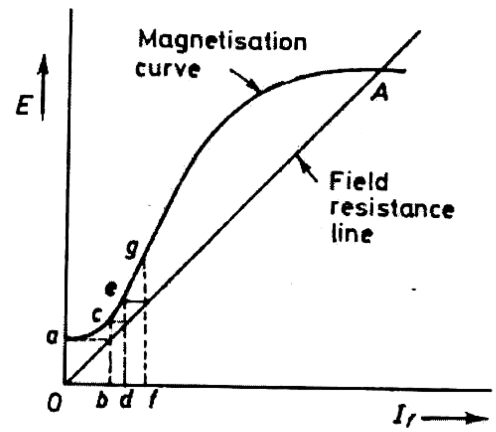
Open-Circuit Characteristic (OCC)

To understand how the self-excitation process takes place, we must know the magnetization curve of the machine. This curve is sometimes called the *saturation curve*. Strictly speaking, the magnetization curve represents a plot of magnetic flux (in the air gap) versus field winding mmf. However, if the speed N is fixed, the magnetization curve represents a plot of the open-circuit induced emf E (in the armature) as a function of field-winding current I_f. This is why this curve is called *open-circuit characteristic* (*OCC*) of the machine.

For plotting the OCC of a self-excited generator, the generator is separately excited by a battery of E_b. The generator is driven by a motor or any other prime-mover at a fixed speed and its armature terminals are left open. A voltmeter (of high resistance) is used for measuring the induced emf E, as shown in Fig. 16.11*a*.



(*a*) The circuit arrangement.



(*b*) The characteristic curve.

Fig. 16.11 Open-circuit characteristic (OCC) of a dc generator.

Figure 16.11*b* shows a typical magnetization curve or OCC of a generator, for a constant speed of rotation of the armature. Note that the emf E is not necessarily zero for $I_f = 0$. It happens because the machine has been previously used and some *residual magnetism* is left. If that were not the case, the magnetization curve would start from the origin. As the exciting current I_f is increased (by decreasing the rheostat R_h in the field circuit), the flux per pole increase and consequently the induced emf E increases.

The magnetic path in a dc generator consists of partly the air gap and partly iron (the pole shoes and the armature core). For low flux density, the iron has high permeability and therefore offers negligible reluctance. Hence, the total reluctance of the magnetic path is almost that of the air gap. Consequently, the flux (and hence the induced emf E) varies *linearly* with the exciting current I_f. The OCC curve is a straight line. However, for high flux densities, the permeability of iron reduces due to magnetic saturation and hence its reluctance is no longer negligible. A stage is reached when the flux does not proportionately increase with increase in the current I_f. The curve starts levelling off.

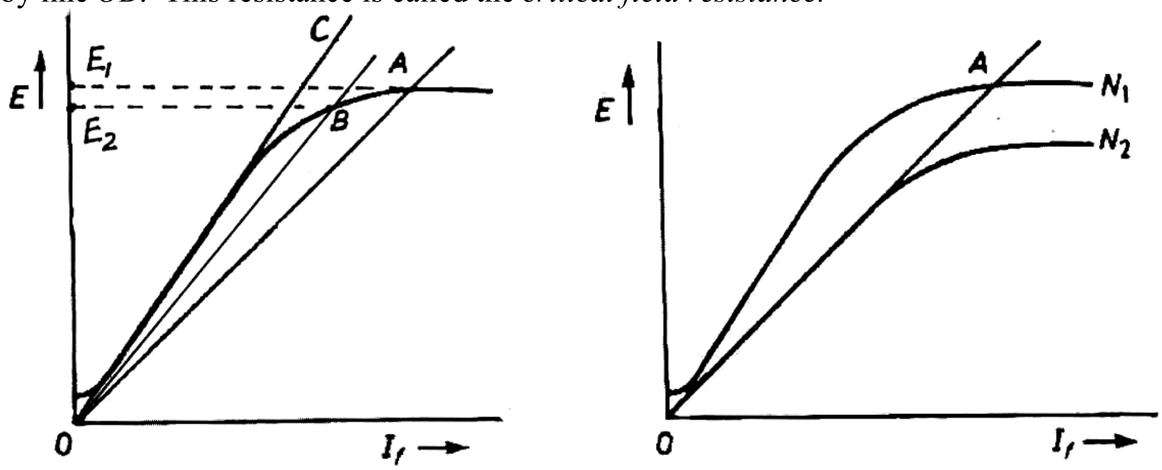
The Field Resistance Line : In Fig. 16.11*b*, the straight line *OA* represents the *field resistance line*. It is a plot of the current caused by the voltage E_b applied to the field circuit. Since, we are drawing the OCC of a self-excited generator, when the generator is actually put to use, the voltage E_b would be the same as the armature voltage E. The slope of this line is E/I_f is a constant and is equal to the total resistance R_F of the field circuit. Note that the resistance R_F represents the sum of the field winding resistance R_f and the active portion of the rheostat resistance R_h.

Building Up of Voltage : Let us now examine how voltage is built up in the self-excited generator. Assume that the generator has been used previously and hence has some residual magnetism left at its poles. If the machine is running at constant speed, a small emf *Oa* is induced in the armature due to the residual magnetism, even if the field current I_f is zero in the beginning. The small emf *Oa* causes a feeble current *Ob* in the field winding, as given by the field resistance line *OA*. This field current produces more flux and a larger emf *bc* is induced. This increased emf causes an even larger field current *Od.* This produces more emf *de*, which in turn causes more field current *Of*, and so on. This process of voltage build up continues until the induced emf is just enough to produce a field current to sustain it. This corresponds to the point *A*, the point of intersection of the OCC curve and the field resistance line.

Note that for the voltage to build up, following three conditions must be satisfied :

(*i*) There must be residual magnetism.
(*ii*) The field winding mmf must act to aid this residual flux.
(*iii*) The field resistance line must intersect the OCC curve at some point.

Critical Field Resistance : Let us consider the third point given above, in some detail. Corresponding to field resistance line *OA*, the emf induced is E_1. For a larger value of the field resistance, the slope of the line increases (line *OB* in Fig. 16.12*a*). This line cuts the OCC curve at a lower voltage E_2. Hence, the larger the field resistance, the smaller is the emf generated. Now suppose that the field resistance is increased to a value corresponding to line *OC*, which just touches the initial straight part of the OCC curve. When the generator is run, the final emf induced will be low, as the voltage build-up process cannot start. Thus, we conclude that voltage build-up takes place only if the field resistance is *less than* that given by line *OB*. This resistance is called the *critical field resistance.*



(*a*) Defining critical field resistance. (*b*) Defining critical speed.

Fig. 16.12 The OCC curves for a dc generator.

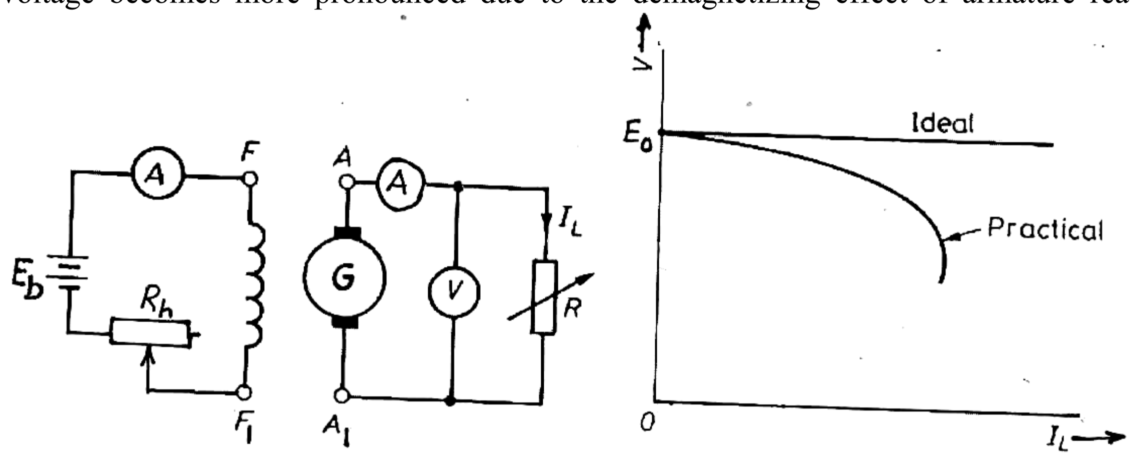
Critical Speed : We know that the emf induced in a dc generator is directly proportional to the speed *N*. Therefore, a generator has different OCC curves for different speeds. Figure 16.12*b* shows two OCC curves---one for speed N_1 and the other for a lower speed N_2. It is evident that if the field resistance corresponds to line *OA*, the voltage builds up if the generator runs at speed N_1. However, if the generator runs at speed N_2, the same line *OA* becomes tangential to the initial part of the OCC curve for N_2. This means that the generator will fail to build up voltage.

The line OA gives the critical field resistance for speed N_2. Or, in other words, we can say that the speed N_2 is the *critical speed* for the field resistance given by OA. Thus, for a given value of field resistance, the lowest speed at which the generator can just build up the voltage is called the *critical speed.*

Load (or External) Characteristics of DC Generators

These characteristics depict the variation of the terminal voltage V with the load current I_L, when the speed and the exciting current are kept constant. Load characteristics of different type of generators are described below.

(1) Separately Excited Generator : This can be experimentally determined by using the circuit of Fig. 16.13*a*. The generator is driven at constant speed N and the field current I_f is kept constant at a value that gives an emf E_o with no load connected. The load is then gradually increased by connecting more lamps in parallel. For each load the terminal voltage V and load current I_L are measured and plotted. Figure 16.13*b* shows *ideal* and *practical* characteristics. Ideally, we would like that the terminal voltage V remains constant with the variation of load current I_L. But in practice, because of the drop in the armature circuit, the terminal voltage drops as the load is increased. For larger load currents, the drop in terminal voltage becomes more pronounced due to the demagnetizing effect of armature reaction.



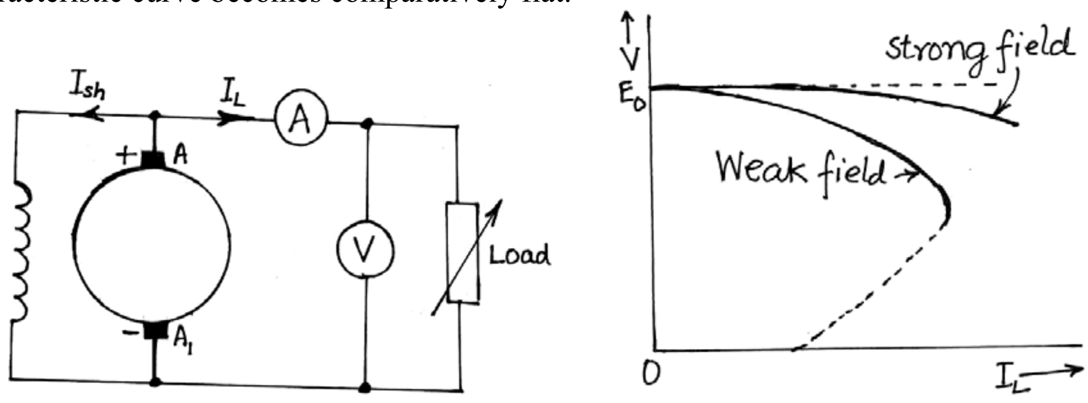
(*a*) The circuit arrangement. (*b*) The load characteristic.

Fig. 16.13 Separately excited dc generator.

(2) Shunt Generator : The circuit arrangement is shown in Fig. 16.14*a*, and the load characteristic is shown in Fig. 16.14*b*. The terminal voltage is maximum at no load. As *the load is increased, the terminal voltage gradually decreases.* Within the normal limits of the load, the terminal voltage falls by about 5 %. If an attempt is made to increase the load beyond the rated value, the fall in voltage becomes very rapid. Sometimes the fall becomes so fast that the characteristic curve turns backward. The dotted part of the curve indicates the ***unstable*** region of operation of the generator. There are two reasons why the voltage falls on increasing the load :

(*i*) Due to the armature resistance voltage drop, and
(*ii*) Due to the demagnetizing effect of the armature reaction.

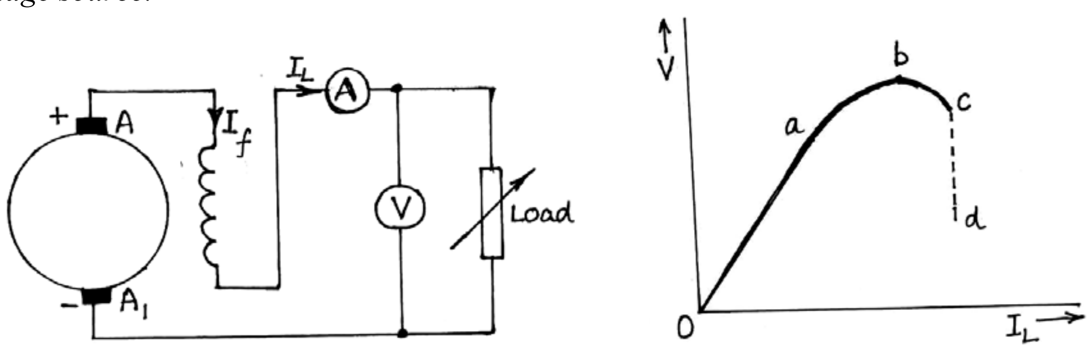
As shown in Fg.16.14*b*, if the shunt generator is designed with a strong field, the load characteristic curve becomes comparatively flat.



(*a*) The circuit arrangement. (*b*) The load characteristic.

Fig. 16.14 Shunt dc generator.

(3) Series Generator : The circuit connection is shown in Fig. 16.15*a*, and the load characteristic is shown in Fig. 16.15*b*. Here, the field current I_f is the same as the load current I_L. Therefore, at no load ($I_L = 0$), the field current and hence the flux is zero. As a result, the emf E induced in the armature too is zero. Up to a point *a*, *the terminal voltage V increases proportionately to the load current* I_L. This property makes a series generator suitable to work as a *booster*, which boosts up the supply voltage. From point *a* to point *b*, the increase in terminal voltage with load current is much less due to the magnetic saturation. Beyond point *b*, the terminal voltage starts falling due to the demagnetizing effect of the armature reaction. After point *c*, the voltage *falls steeply* as the armature reaction becomes prominent. In this region, the series generator may be used as a *constant current but variable voltage source.*

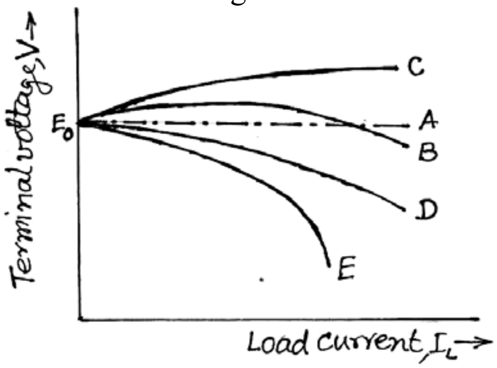


(*a*) The circuit arrangement. (*b*) The load characteristic.

Fig. 16.15 Series dc generator.

(4) **Compound-Wound Generators :** *Ideally*, we would like to have load characteristic of a generator as shown by the horizontal straight line *A*, in Fig. 16.16. This is possible neither from a shunt generator (Fig. 16.14*b*) nor from a series generator (Fig. 16.15*b*). However, a compound generator, either short-shunt or long-shunt as shown in Figs. 16.9*c* and *d*, respectively, can be designed to achieve a characteristic very near to ideal. It utilizes

opposing effects of both (*i*) the falling characteristic of a shunt generator, and (*ii*) the rising characteristic of a series generator.



A : Ideal

B : Level-compounded

C : Over-compounded

D : Under-compounded

E : Differential-compounded

Fig. 16.16 Load characteristics of compound dc generators.

Case* I :** We can have a combination of shunt and series excitations in such a way that the resultant terminal voltage varies very little over a range of load current (curve *B* in Fig. 16.16). The generator is then said to be ***flat or ***leve- compounded***. The terminal voltage *V* remains almost constant between the no-load and full-load.

Case* II :** In case the series field supports the shunt field (i.e., if the generator is *cumulative compounded*), and the series ampere turns are more than the shunt ampere turns, the terminal voltage *V* can be made to rise with load current (curve *C* in Fig. 16.16). Such a generator, known as ***over-compounded, can be used for supplying power over long distances. Whenever the load increases, the terminal voltage falls due to large voltage drops in transmission lines. The terminal voltage at load end can easily be re-adjusted by the over compounded generator.

Case* III :** In case the series ampere turns are less than the shunt ampere turns, the terminal voltage *V* falls as the load increases (curve *D* in Fig. 16.16). Such generators are said to be ***under-compounded. Under-compounding is useful where a short might occur, e.g., in an arc welding machine.

Case* IV :** If the series ampere turns oppose the shunt ampere turns, the generator is said to be ***reverse or ***differential-compounded***. For such generators, the terminal voltage *V* falls very rapidly as the load current increases (curve *E* in Fig. 16.16).

Example 16.11 A dc shunt generator is to be converted into a level-compounded generator by adding a series field winding. From a test on the machine with shunt excitation only, it is found that the shunt current is 4 A to give 440 V on no load and 6 A to give the same voltage when the machine is supplying its full load of 100 A. The shunt winding has 1500 turns per pole. Find the number of series turns required per pole.

***Solution* :** Ampere turns per pole required on no load $= 4 \times 1500 = 6000$ At

Ampere turns per pole required on full load $= 6 \times 1500 = 9000$ At

Hence, ampere turns per pole to be provided by the series winding

$$= 9000 - 6000 = 3000 \text{ At}$$

Since, the full-load current is 100 A, the number of turns per pole needed in the series winding, $\tau_{se} = \dfrac{3000}{100} = \mathbf{30}$

16.11 DC MOTORS

In construction, a dc motor is no different from a dc generator. As in case of dc generators, there are three types of dc motors : (*i*) shunt, (*ii*) series, and (*iii*) compound. Unlike the series generators, the dc series motors find wide applications, especially for traction type of loads.

When the motor terminals are connected to dc mains supply, a current flows in the field winding as well as in the armature winding. In a shunt motor, the two currents have different values. But in a series motor, the two currents are the same.

Equivalent Circuit of a DC Motor

Like a dc generator, a dc motor too has induced emf E in the armature, given by the same equation (Eq. 16.4):

$$E = \frac{\Phi Z N P}{60 A} \qquad \text{...(16.10)}$$

However, this induced emf opposes the supply voltage V and hence it is treated as *counter* or *back emf*. The equivalent circuit of a dc shunt motor is depicted in Fig. 16.17. Note that the terminal voltage V must be equal to the sum of induced emf E and voltage drop in the armature. Similarly, the line current I_L is equal to the sum of the armature current I_a and field current I_f. That is,

$$V = E + I_a R_a \qquad \text{...(16.11)}$$

and

$$I_L = I_a + I_f \qquad \text{...(16.12)}$$

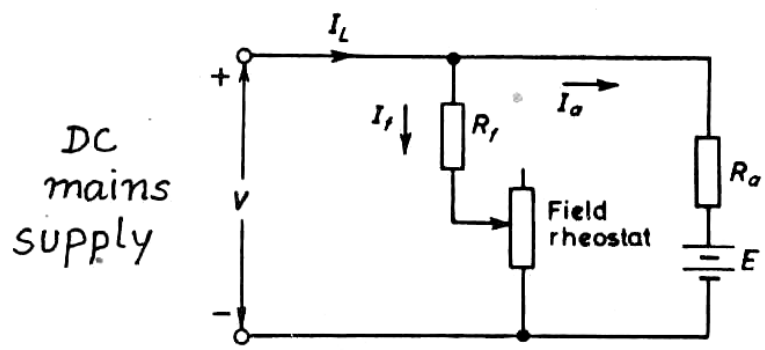


Fig. 16.17 Equivalent circuit of a dc shunt motor.

Speed Regulation of a DC Motor

For a given machine, A, Z and P are fixed, so that the expression for induced emf E (Eq. 16.10) can be written as

$$E = kN\Phi$$

where $$k = \frac{ZP}{60A} \qquad \text{(a constant)}$$

Substituting for E in Eq. 16.11, we get

$$V = kN\Phi + I_a R_a \qquad \Rightarrow \qquad N = \frac{V - I_a R_a}{k\Phi} \qquad ...(16.13)$$

The value of the voltage drop $I_a R_a$ is usually less than 5 % of the terminal voltage V, so that the above equation can be written as

$$N \approx \frac{V}{k\Phi} \qquad \text{or} \qquad N \propto \frac{V}{\Phi} \qquad ...(16.14)$$

It means that the speed of a dc motor is approximately proportional to the applied voltage V and inversely proportional to the flux Φ. All methods of controlling the speed involve the use of either or both of these relationships.

When a motor is mechanically loaded, its speed decreases. If N_0 represents the no-load speed and N_f the full-load speed, the *percentage speed regulation* is defined as

$$\%\ \text{speed regulation} = \frac{N_0 - N_f}{N_f} \times 100\ \% \qquad ...(16.15)$$

Example 16.10 A 250-V dc shunt motor takes 41 A current while running at full load. The resistances of motor armature and of field windings are 0.1 Ω and 250 Ω, respectively. Determine the back emf generated in the motor.

***Solution* :** The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{250} = 1\text{ A}$

Therefore, the armature current, $I_a = I_L - I_{sh} = 41 - 1 = 40\text{ A}$

$\therefore$ Back emf, $E = V - I_a R_a = 250 - 40 \times 0.1 = \mathbf{246\ V}$

Example 16.11 A 4-pole, 440-V dc motor takes an armature current of 50 A. The resistance of the armature circuit is 0.28 Ω. The armature winding is wave-connected with 888 conductors and the useful flux per pole is 23 mWb. Calculate the speed of the motor.

***Solution* :** From Eq. 16.11, the generated emf is given as

$$E = V - I_a R_a = 440 - 50 \times 0.28 = 426\text{ V}$$

Using Eq. 16.10, we get the speed of the motor as

$$N = \frac{60AE}{\Phi ZP} = \frac{60 \times 2 \times 426}{0.023 \times 888 \times 4} \approx \mathbf{626\ rpm}$$

Example 16.12 A dc motor runs at 900 rpm from a 460-V supply. Calculate the approximate speed when the machine is connected across a 200-V supply. Assume the new flux to be 0.7 times the original flux.

Solution **:** If Φ is the original flux, then from Eq. 16.14 we have

$$900 = \frac{460}{k\Phi} \qquad \text{or} \qquad k\Phi = 0.511$$

When the supply voltage changes to 200 V, the new speed is given as

$$N' = \frac{V'}{k\Phi'} = \frac{V'}{k(0.7\Phi)} = \frac{V'}{0.7k\Phi} = \frac{200}{0.7\times 0.511} \cong \textbf{559 rpm}$$

16.12 TORQUE DEVELOPED BY A DC MOTOR

If we multiply each term of Eq. 16.11 by I_a, namely, the total armature current, we get

$$VI_a = EI_a + I_a^2 R_a$$

Here, VI_a represents the total electric power supplied to the armature, and $I_a^2 R_a$ represents the loss due to the armature resistance. The difference between these two quantities, namely EI_a, represents the electrical power that is converted to mechanical power by the armature. If τ_d is the torque, in newton-metres, exerted on the armature to develop the mechanical power ($= EI_a$), and N is the speed of rotation in rpm, then we have

$$\text{Mechanical power developed, } P_m = \frac{2\pi\tau_d N}{60} \text{ watts}$$

Hence, we have

$$\frac{2\pi\tau_d N}{60} = EI_a$$

$$= \frac{\Phi ZNP}{60A}\times I_a \qquad \text{(replacing } E \text{ by the expression of Eq. 16.10)}$$

Thus, the torque developed by the armature is given as

$$\tau_d = \frac{\Phi Z}{2\pi}\left(\frac{P}{A}\right) I_a \qquad \text{...(16.16)}$$

Since, for a given machine, Z, P and A are fixed, we can write

$$\tau_d \propto I_a \times \Phi \qquad \text{...(16.17)}$$

It means that *the torque developed in a given dc motor is proportional to the product of the armature current and the flux per pole*.

Note that all of the mechanical power developed, namely EI_a, by the armature is not available externally. Some of it is absorbed as friction loss at the bearing and at the brushes and some is wasted as hysteresis loss and in circulating eddy currents in the core. The useful torque available at the shaft, namely τ_{sh}, is less than the torque developed τ_d, because of these losses.

Example 16.13 A 6-pole, dc motor takes an armature current of 110 A at 480 V. The resistance of the armature circuit is 0.2 Ω, and flux per pole is 50 mWb. The armature has 864 lap-connected conductors. Calculate (*a*) the speed, and (*b*) the gross torque developed by the armature.

***Solution* :** (*a*) The generated emf, $E = V - I_a R_a = 480 - 110 \times 0.2 = 458\ \text{V}$

Using Eq. 16.10, we have

$$E = \frac{\Phi ZNP}{60A} \qquad \text{or} \qquad N = \frac{60AE}{\Phi ZP} = \frac{60 \times 6 \times 458}{0.05 \times 864 \times 6} \approx \mathbf{636\ rpm}$$

(*b*) Torque developed by the armature,

$$\tau_d = \frac{\Phi Z}{2\pi}\left(\frac{P}{A}\right) I_a = \frac{0.05 \times 864}{2\pi} \times \left(\frac{6}{6}\right) \times 110 \approx \mathbf{756\ Nm}$$

Example 16.14 A dc generator runs at 900 rpm when a torque of 2 kNm is applied by a prime mover. If the core, friction and windage losses in the machine are 8 kW, calculate the power generated in the armature winding.

***Solution* :** The power required to drive the generator,

$$P_{\text{in}} = \frac{2\pi\tau N}{60} = \frac{2\pi \times 2000 \times 900}{60} = 188\,562\ \text{W} = 188.6\ \text{kW}$$

Therefore, the power generated in the armature,

$$P_d = P_{in} - P_{\text{losses}} = 188.6 - 8 = \mathbf{180.6\ kW}$$

16.13 TORQUE AND SPEED CHARACTERISTICS OF A DC MOTOR

When no load is connected to the shaft of a dc motor, it develops only that much torque which overcomes the rotational (frictional) losses and the iron losses. How does the motor react to the application of a shaft load ? To answer this question, we require knowing the performance characteristics of the motor.

Speed Characteristics of DC Motors

The speed characteristic of a motor represents the variation of speed with the input current. Its shape can be easily derived from expression of Eq. 16.13, namely

$$N = \frac{V - I_a R_a}{k\Phi}$$

(1) Shunt Motor : The field winding of a shunt motor consists of many turns of thin wire and is connected in parallel with the armature. The flux Φ, therefore, remains constant. Since the drop $I_a R_a$ at full load rarely exceeds 5 % of *V*, the speed *N* is almost constant. Its speed characteristic may be represented by curve *A* in Fig. 16.18*a*. Thus, *a dc shunt motor is a **constant speed** motor.* In actual practice, the drop in speed with current is even less than that shown in figure. This is because as the armature current increases, the armature reaction tends to slightly reduce the main flux Φ. This reduction in flux causes an increase in speed that partially compensates the drop due to $I_a R_a$.

(2) Series Motor : In a series motor, the field winding is made of a few turns of thick wire and is connected in series with the armature. If R_{se} represents the resistance of this winding, the back emf is given as

$$E = V - I_a(R_a + R_{se})$$

Therefore, Eq. 16.13 modifies to

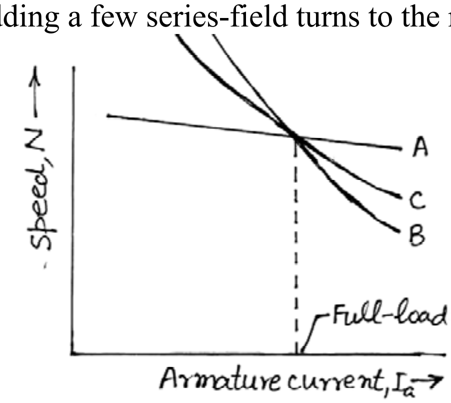
$$N = \frac{V - I_a(R_a + R_{se})}{k\Phi}$$

The flux Φ increases first in direct proportion to the armature current I_a and then less rapidly due to the magnetic saturation. Hence, the speed is roughly inversely proportional to the current, as indicated by the curve B in Fig. 16.18*a*. Thus, *a dc series machine is a **variable speed** motor.*

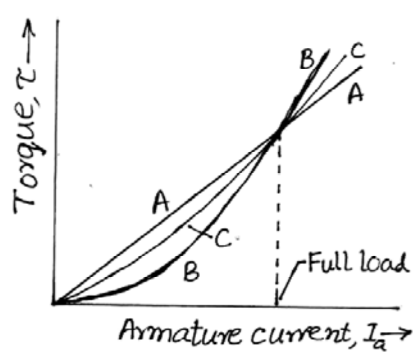
Note that if a dc motor is started with no load, the current (and hence the flux) is very low, and the speed may become dangerously high. It may fly to pieces due to such high speed. For the same reason, a series motor should never be used when there is a risk of the load becoming very low. For instance, the load should never be belt-connected, as it has the risk of breaking or slipping. The load to a dc series motor is either directly connected or geared to the shaft.

(3) Compound Motor : A compound motor has both a shunt winding and a series winding. The flux due to shunt field remains fixed, but that due to the series field increases with the current. Therefore, the total flux increases with the current, but not as rapidly as in a series motor. Hence, the speed characteristic (curve C in Fig. 16.18*a*) is in between those of the shunt and series motors. Depending upon the ratio of the shunt and series ampere-turns, any desired characteristic can be obtained.

In many cases, enough shunt field is provided to guarantee a safe no-load speed. Such motors are called *stabilized series motors*. Large shunt motors operating at high speeds face large fluctuations in speed due the line-voltage fluctuations. This problem can be reduced by adding a few series-field turns to the machine.



(*a*) Speed characteristic.



(*b*) Torque characteristic.

Fig. 16.18 Performance characteristics of dc motors.

Torque Characteristics of DC Motors

The torque characteristic of a motor represents the variation of the developed torque τ_d with the input current. Its shape can be easily derived from expression of Eq. 16.17, namely

$$\tau_d \propto I_a \times \Phi \qquad \text{or} \qquad \tau_d = k_t I_a \times \Phi$$

where, k_t is a constant for a machine.

(1) Shunt Motor : Since the flux Φ in a shunt motor is practically independent of the armature current, $\tau_d \propto I_a$, and hence the torque characteristic is represented by the straight line *A* in Fig. 16.18*b*.

(2) Series Motor : In a series motor, the flux Φ is approximately proportional to the armature current up to full load, so that $\tau_d \propto I_a^2$. Above full-load, magnetic saturation becomes more prominent and the torque does not increase so rapidly. The torque characteristic is represented by curve *B* in Fig. 16.18*b*.

(3) Compound Motor : The torque characteristic of a compound motor is in between those of the shunt and series motors, and is represented by curve *C* in Fig. 16.18*b*. The exact shape of curve *C* depends upon the relative value of the shunt and series ampere-turns at full-load.

From Fig. 16.18*b*, it is evident that for a given current below the full-load value, the shunt motor exerts the largest torque. But for a current above the full-load value, the series motor exerts the largest torque.

The maximum permissible current at starting is usually about 1.5 times the full-load value. Therefore, where a large starting torque is required such as for hoists, cranes, electric trains, etc., the series motor is the most suitable choice.

Example 16.17 A series motor runs at 600 rpm when taking a current of 110 A from a 230 V supply. The useful flux per pole for 110 A is 24 mWb and that for 50 A is 16 mWb. The The armature resistance and series-field resistance are 0.12 Ω and 0.03 Ω. Calculate the speed when the current has fallen to 50 A.

***Solution* :** The emf generated when the current is 110 A,

$$E_1 = V - I_a(R_a + R_{se}) = 230 - 110 \times (0.12 + 0.03) = 213.5 \text{ V}$$

But for a given machine, the emf is given as

$$E_1 = \frac{\Phi ZNP}{60A} = k\Phi_1 N_1$$

Hence, with 110 A, we have

$$213.5 = k \times 0.024 \times 600 \qquad \text{or} \qquad k = 14.83.$$

The emf generated when the current is 50 A,

$$E_2 = V - I_a(R_a + R_{se}) = 230 - 50 \times (0.12 + 0.03) = 222.5 \text{ V}$$

Hence, with 50 A, we have

$$222.5 = k \times \Phi_2 \times N_2$$

Therefore, new speed is

$$N_2 = \frac{222.5}{k \times \Phi_2} = \frac{222.5}{14.83 \times 0.016} \cong \mathbf{938\,rpm}$$

Example 16.18 A dc shunt motor has an armature resistance of 0.2 Ω. Calculate the current drawn by the machine when connected to 250-V dc supply, in each of the following cases :

(*a*) When the machine is at rest.

(*b*) When the machine is generating an emf of 200 V and is connected to the supply with correct polarities.

(*c*) When the machine is generating an emf of 250 V and is connected to the supply with correct polarities.

(*d*) When the machine is generating an emf of 250 V and is connected to the supply with reversed polarities.

***Solution* :** (*a*) When machine is at rest, $E = 0$.

$$\therefore \quad I_a = \frac{V - E}{R_a} = \frac{250 - 0}{0.2} = \mathbf{1250\ A}$$

(*b*) When $E = 200$ V.

$$\therefore \quad I_a = \frac{V - E}{R_a} = \frac{250 - 200}{0.2} = \mathbf{250\,A}$$

(*c*) When $E = 250$ V.

$$\therefore \quad I_a = \frac{V - E}{R_a} = \frac{250 - 250}{0.2} = \mathbf{0}$$

It means the machine is not consuming any power from the supply. The prime mover supplies power to meet the frictional and other losses in the machine.

(*d*) When E = -250 V (the negative sign is taken because the machine is connected to the supply with reversed polarities).

$$\therefore \quad I_a = \frac{V - E}{R_a} = \frac{250 - (-250)}{0.2} = \mathbf{2500\ A}$$

Example 16.19 A 6-pole, lap-connected dc series motor, with 864 conductors, takes a current of 110 A at 480 V. The armature resistance and the series-field resistance are 0.18 Ω and 0.02 Ω, respectively. The flux per pole is 50 mWb. Calculate (*a*) the speed, and (*b*) the gross torque developed by the armature.

Solution* : (a*) The generated emf is given as

$$E = V - I_a(R_a + R_{se}) = 480 - 110 \times (0.18 + 0.02) = 458 \text{ V}$$

$$\therefore \qquad N = \frac{60AE}{\Phi ZP} = \frac{60 \times 6 \times 458}{0.05 \times 864 \times 6} = \mathbf{636\ rpm}$$

(*b*) Since, $\frac{2\pi\tau_d N}{60} = EI_a$, we have

$$\tau_d = \frac{60EI_a}{2\pi N} = \frac{60 \times 458 \times 110}{2 \times \pi \times 636} = \mathbf{756\ Mm}$$

Example 16.20 A 220-V, shunt motor, running at 700 rpm, has an armature resistance of 0.45 Ω and takes armature current of 22 A. What resistance should be placed in series with the armature to reduce the speed to 450 rpm ?

Solution : For speed, $N_1 = 700$ rpm :

$$E_1 = V - I_a R_a = 220 - 22 \times 0.45 = 210.1 \text{ V}$$

In a shunt motor, the flux remains constant. Hence,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} \qquad \text{or} \qquad \frac{E_2}{210.1} = \frac{450}{700} \qquad \Rightarrow \qquad E_2 = 135.06 \text{ V}$$

If R is the additional resistance placed in series with the armature, we have

$$E_2 = V - I_a(R_a + R)$$

$$\text{or} \qquad 135.06 = 220 - 22 \times (0.45 + R)$$

On solving the above, we get

$$R = \mathbf{3.411\ \Omega}$$

Example 16.21 A 230-V, dc series motor has an armature circuit resistance of 0.2 Ω and series field resistance of 0.1 Ω. At rated voltage, the motor draws a line current of 40 A and runs at a speed of 1000 rpm. Find the speed of the motor for a line current of 20 A at 230 V. Assume that the flux at 20 A line current is 60 % of the flux at 40 A line current.

Solution : The back emf when the line current is 40 A is given as

$$E_1 = V - I_a(R_a + R_{se}) = 230 - 40 \times (0.2 + 0.1) = 218 \text{ V}$$

The back emf when the line current is 20 A is given as

$$E_2 = V - I_a(R_a + R_{se}) = 230 - 20 \times (0.2 + 0.1) = 224 \text{ V}$$

The flux, $\Phi_2 = 0.6\Phi_1$. Here, in the two cases, only the flux and speed changes. Hence,

$$E = k\Phi N$$

$$\therefore \qquad \frac{E_2}{E_1} = \frac{N_2\Phi_2}{N_1\Phi_1} = \frac{N_2 \times 0.6\Phi_1}{N_1\Phi_1} = \frac{0.6N_2}{N_1}$$

$$N_2 = \frac{E_2 N_1}{E_1 \times 0.6} = \frac{224 \times 1000}{218 \times 0.6} \cong \mathbf{1713\ rpm}$$

16.14 STARTING OF DC MOTORS

When a dc motor is at rest, there is no back emf generated. Hence, if the motor is directly connected to the supply mains, a heavy current flows through the armature. This may result in a damage to the armature. It therefore becomes necessary to include a high resistance in series with the armature at the start. Once the motor picks up speed, the back emf is generated and then the series resistance can be gradually cut out. Ultimately, when the motor attains its normal (rated) speed, the entire resistance may be disconnected from the circuit. The device that provides this facility is called a ***starter***.

Only small dc motors (say, up to 2-3 watts) can be started by directly connecting to the supply mains, because of the following reasons :

(*i*) The resistance and inductance of the armature winding are generally quite high to limit the rush of the current.

(*ii*) Because of the low inertia of the rotor, the motor picks up speed quickly. Hence, the high current does not last long to cause any damage.

For heavy duty motors, we have to use either a *three-point starter* or a *four-point starter*. Here, we discuss only the three point starter

Three-Point Starter

It consists of a series starting resistance divided into several sections, each connected to a brass stud. As shown in Fig. 16.19, the starter is also provided with *no-volt release* and *overload release* facilities.

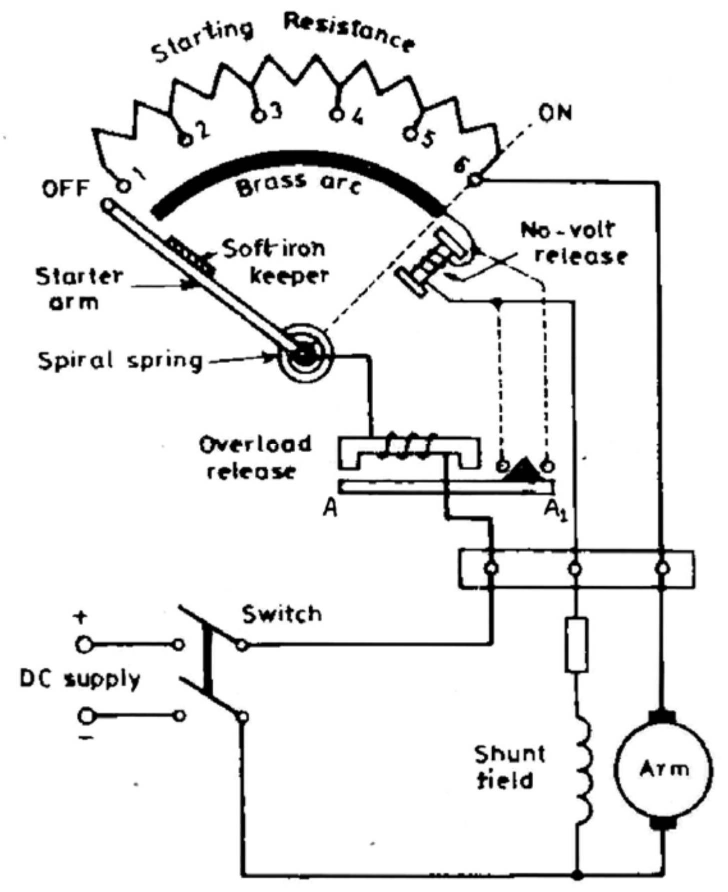


Fig. 16.19 Three-point starter for a dc machine.

Initially, the starter arm is at OFF position towards the left. After switching ON the dc supply, the starter arm is moved towards right. When connected to stud 1, the field circuit is directly connected to the supply through the brass arc. At the same time, the entire resistance is inserted in the armature circuit. Some current flows in the armature, developing a torque. The motor starts running, and an emf is generated. As the motor picks up speed, the starter arm is slowly moved towards right across studs 2, 3 …, etc. cutting out parts of the series resistance. Finally, when the arm is brought to ON position the motor attains the rated speed.

No-Volt Release : When brought to ON position, the starter arm is held by the no-volt release magnet against the pull of the spiral spring. The magnet is energized by the field current. Whenever the pull of this magnet weakens or becomes zero, the arm is released to go back to OFF position, automatically.

If this arrangement were not provided, the starter arm would remain in OFF position even when the supply goes off. When the supply is restored, the armature gets directly connected to the supply. This may cause heavy damage.

Another reason for putting no-voltage release is this. Suppose that due to some reason the field circuit becomes open. The flux and hence the back emf reduces to zero. The motor therefore starts drawing dangerously high current. This situation is avoided, as the no-volt release magnet de-energizes to release the arm to go back to OFF position, whenever the field current becomes zero.

Overload Release : This is provided to protect the motor against flow of excessive current due to overloads. The coil is connected in series with the motor. It carries full-load current. Whenever the machine is overloaded, it draws heavy current. This current flows through the coil of the electromagnet which then pulls the armature $A\ A_1$ upwards. This action short-circuits the no-volt release coil. The coil de-energizes and the starter arm is released to go back to OFF position.

ADDITIONAL SOLVED EXAMPLES

Example 16.22 A four-pole, lap-wound, dc shunt generator has a useful flux per pole of 0.08 Wb. The armature winding consists of 260 turns, each of resistance 0.006 Ω. Determine the terminal voltage of the generator when it is running at 1000 rpm and supplying a load current of 55 A.

***Solution* :** Total number of conductors, $Z = 260 \times 2 = 520$

For lap-wound, number of parallel paths, $A = P = 4$

Therefore, the emf generated,

$$E = \frac{\Phi ZNP}{60A} = \frac{0.08 \times 520 \times 1000 \times 4}{60 \times 4} = 693.34 \text{ V}$$

The total resistance of the winding, $R_w = 260 \times 0.006 = 1.56\ \Omega$

There are four parallel paths, and the resistance of one path, $R_1 = \dfrac{R_w}{4} = \dfrac{1.56}{4} = 0.39\ \Omega$

Thus, the net armature resistance, $R_a = \dfrac{R_1}{4} = \dfrac{0.39}{4} = 0.0975\ \Omega$.

Hence, the terminal voltage, $V = E - I_a R_a = 693.34 - 55 \times 0.0975 = \mathbf{687.98\,V}$

Example 16.23 An eight-pole, dc shunt generator has 778 wave-connected conductors on its armature. While running at 500 rpm, it supplies power to a load of 12.5 Ω at 250 V. The armature and the shunt-field resistances are 0.24 Ω and 250 Ω, respectively. Determine the armature current, the emf induced, and the flux per pole.

***Solution* :** The load current, $I_L = \dfrac{V}{R_L} = \dfrac{250}{12.5} = 20\ \text{A}$

The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{250} = 1\ \text{A}$

∴ Armature current, $I_a = 20 + 1 = \mathbf{21\ A}$

The emf induced, $E = V + I_a R_a = 250 + 21 \times 0.24 = \mathbf{255.04\ V}$

Since, $$E = \frac{\Phi Z N P}{60 A}$$

Therefore, $$\Phi = \frac{60AE}{ZNP} = \frac{60 \times 2 \times 255.04}{778 \times 500 \times 8} = 0.00983\ \text{Wb} = \mathbf{9.83\ mWb}$$

Example 16.24 Estimate the percentage reduction in speed of a dynamo working with constant excitation on 500-V bus bars to decrease its load from 500 kW to 250 kW. The resistance between the terminals is 0.015 Ω. Neglect the armature reaction.

***Solution* :** For the *first* case, the armature current, $I_a = \dfrac{P_o}{V} = \dfrac{500 \times 1000}{500} = 1000\ \text{A}$

Therefore, the induced emf is given as
$$E_1 = V + I_a R_a = 500 + 1000 \times 0.015 = 515\ \text{V}$$

In the *second* case, the armature current, $I_a = \dfrac{P_o}{V} = \dfrac{250 \times 1000}{500} = 500\ \text{A}$

Therefore, the induced emf is given as
$$E_2 = V + I_a R_a = 500 + 500 \times 0.015 = 507.5\ \text{V}$$

Since the excitation remains constant in the two cases, we have
$$E = \frac{\Phi Z N P}{60 A} \qquad \text{or} \qquad N = KE, \qquad \text{where } K \text{ is a constant.}$$
Hence, the fractional reduction in speed is given as
$$\frac{N_1 - N_2}{N_1} = \frac{K(E_1 - E_2)}{KE_1} = \frac{515 - 507.5}{515} = 0.01456\,\text{pu} = \mathbf{1.456\ \%}$$

Example 16.25 A dc series generator has external characteristic given by a straight line through zero to 50 V at 200 A. It is connected as a booster between a station bus-bar and a feeder of 0.3 Ω resistance. Calculate the voltage between the far end of the feeder and the bus-bar at a current of (*a*) 160 A, and (*b*) 50 A.

***Solution* :** The electrical energy is transmitted from station bus-bar to the feeder through a transmission line or an underground cable. A dc generator can work as a booster to compensate for the voltage drop in the transmission lines.

(*a*) *For* $I_L = 160$ A :

Voltage drop in the transmission lines, $V_t = I_L R_t = 160 \times 0.3 = 48$ V

From the external characteristic of the dc generator, the boost-in voltage supplied at 160 A is $V_b = (50/200) \times 160 = 40$ V. Hence, the voltage difference between the far end of the feeder and bus-bar is

$$V_d = V_t - V_b = 48 - 40 = \mathbf{8\ V}$$

(*b*) *For* $I_L = 50$ A :

Voltage drop in the transmission lines, $V_t = I_L R_t = 50 \times 0.3 = 15$ V

The boost-in voltage supplied by the booster at 50 A is $V_b = (50/200) \times 50 = 12.5$ V. Hence, the voltage difference between the far end of the feeder and bus-bar is

$$V_d = V_t - V_b = 15 - 12.5 = \mathbf{2.5\ V}$$

Example 16.26 A dc long-shunt compound generator delivers a load current of 50 A at 500 V, and has armature, series-field and shunt-field resistances of 0.05 Ω, 0.03 Ω and 250 Ω, respectively. Calculate the generated emf and the armature current. Allow 1.0 V per brush for contact drop.

***Solution* :** The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{500}{250} = 2$ A

$\therefore$ Armature current, $I_a = I_L + I_{sh} = 50 + 2 = \mathbf{52\ A}$

The generated emf is given as

$$E = V + I_a(R_a + R_{se}) + \text{brush drop} = 500 + 52(0.05 + 0.03) + 2 = \mathbf{506.16\ V}$$

Example 16.27 An 8-pole, dc generator has 500 conductors on its armature, and is designed to have 0.02 Wb of magnetic flux per pole crossing the air gap with normal excitation.

(*a*) What voltage will be generated at a speed of 1800 rpm, if the armature is (*i*) wave-wound, (*ii*) lap-wound ?

(*b*) If the allowable current is 5 A per path, what will be the kW generated by the machine in each case ?

***Solution* :** (*a*) (*i*) For wave-wound, $A = 2$.

$$\therefore \quad E = \frac{\Phi Z N P}{60A} = \frac{0.02 \times 500 \times 1800 \times 8}{60 \times 2} = \mathbf{1200\ V}$$

(*ii*) For lap-wound, $A = P = 8$.

$$\therefore \qquad E = \frac{\Phi ZNP}{60A} = \frac{0.02 \times 500 \times 1800 \times 8}{60 \times 8} = \mathbf{300\ V}$$

(*b*) (*i*) For wave-wound, A = 2. The current in armature path,

$$I_a = A \times 5 = 2 \times 5 = 10\ \text{A}$$

$$\therefore \quad \text{Power generated, } P_d = EI_a = 1200 \times 10 = 12\,000\ \text{W} = \mathbf{12\ kW}$$

(*ii*) For lap-wound, A = 8. The current in armature path,

$$I_a = A \times 5 = 8 \times 5 = 40\ \text{A}$$

$$\therefore \quad \text{Power generated, } P_d = EI_a = 300 \times 40 = 12\,000\ \text{W} = \mathbf{12\ kW}$$

Example 16.28 A shunt dc generator delivers 195 A at the terminal voltage of 250 V. The armature resistance and shunt-field resistance are 0.02 Ω and 50 Ω, respectively. The iron and friction losses are 950 W. Find (*a*) the emf generated, (*b*) the copper losses, (*c*) the output of the prime mover, and (*d*) the commercial, mechanical, and electrical efficiencies.

Solution : (*a*) $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{50} = 5\ \text{A}.$

$$\therefore \quad I_a = I_L + I_{sh} = 195 + 5 = 200\ \text{A}$$

The generated emf, $E = V + I_a R_a = 250 + 200 \times 0.02 = \mathbf{254\ V}$

(*b*) The armature copper loss $= I_a^2 R_a = 200^2 \times 0.02 = 800\ \text{W}$

The shunt-field copper loss $= V \cdot I_{sh} = 250 \times 5 = 1250\ \text{W}$

$\therefore$ The total copper losses $= 800 + 1250 = \mathbf{2050\ W}$

(*c*) The output power supplied to the load, $P_o = VI_L = 250 \times 195 = 48\,750\ \text{W}$

Total losses = Total copper losses + Stray losses = 2050 + 950 = 3000 W

$\therefore$ The output of the prime mover = The input to the generator, P_{in}

= The output + Total losses

= 48 750 + 3000 = 51 750 W = **51.75 kW**

(*d*) The electrical power produced in the armature is given as

$$P_e = P_{\text{in}} - \text{Stray losses} = 51750 - 950 = 50\,800\ \text{W}$$

$$\text{Mechanical Efficiency, } \eta_m = \frac{\text{Total watts generated in the armature}}{\text{Mechanical power supplied at the input}}$$

$$= \frac{50\,800}{51\,750} = 0.982\,\text{pu} = \mathbf{98.2\,\%}$$

$$\text{Electrical Efficiency, } \eta_e = \frac{\text{Total watts available to the load}}{\text{Total watts generated in the armature}}$$

$$= \frac{48\,750}{50\,800} = 0.959\ \text{pu} = \mathbf{95.9\ \%}$$

Commercial or Overall Efficiency, $\eta_c = \dfrac{\text{Total watts available to the load}}{\text{Mechanical power supplied}}$

$$= \frac{48\,750}{51\,750} = 0.942 \text{ pu} = \mathbf{94.2\ \%}$$

Example 16.29 A 4-pole, dc shunt motor working on 250 V takes a current of 2 A when running light (i.e., at no load) at 1000 rpm. Armature resistance and shunt-field resistances are 0.2 Ω and 250 Ω, respectively. (*a*) How much back emf is generated ? (*b*)What will be its back emf, speed and percentage speed drop if the motor takes 51 A at a certain load ?

Solution : (*a*) The shunt-field current, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{250} = 1 \text{ A}$

Therefore, armature current, $I_{a1} = I_{L1} - I_{sh} = 2 - 1 = 1 \text{ A}$

∴ Back emf, $E_1 = V - I_{a1}R_a = 250 - 1 \times 0.2 = \mathbf{249.8\ V}$

(*b*) The armature current, $I_{a2} = I_{L2} - I_{sh} = 51 - 1 = 50 \text{ A}$

∴ Back emf, $E_2 = V - I_{a2}R_a = 250 - 50 \times 0.2 = \mathbf{240\ V}$

As the motor is shunt wound, the flux remains constant. The emf generated is directly proportional to the speed. Hence, the new speed is given as

$$N_2 = N_1 \times \frac{E_2}{E_1} = 1000 \times \frac{240}{249.8} = \mathbf{961\ rpm}$$

$$\therefore \quad \% \text{ speed drop} = \frac{N_1 - N_2}{N_1} \times 100 = \frac{1000 - 961}{1000} \times 100 = \mathbf{3.9\ \%}$$

Example 16.30 Calculate the value of the starting resistance for the following shunt motor :
Output = 14 920 W; Supply = 240 V; Armature resistance = 0.25 Ω;
Efficiency at full load = 86 %
The starting current is to be limited to 1.5 times full-load current. Ignore the current in shunt winding.

Solution : The input power, $P_{\text{in}} = \dfrac{P_o}{\eta} = \dfrac{14\,920}{0.86} = 17\,349 \text{ W}$

$$\therefore \quad \text{Full-load line current, } I_L = \frac{P_{\text{in}}}{V} = \frac{17\,349}{240} = 72.3 \text{ A}$$

The permitted starting current, $I_{st} = 1.5 I_L = 1.5 \times 72.3 = 108.45 \text{ A}$

Required total resistance in the armature circuit at the starting is given as

$$R_t = \frac{V}{I_{st}} = \frac{240}{108.45} = 2.213\ \Omega$$

Therefore, extra resistance required at starting is

$$R_{st} = R_t - R_a = 2.213 - 0.25 = \mathbf{1.963\ \Omega}$$

Example 16.31 A 500-V, dc shunt motor takes 4 A on no load and runs at 1000 rpm. The armature resistance (including that of the brushes) is 0.2 Ω, and the field current is 1 A. On loading, if the motor takes a current of 100 A, determine its speed and estimate the efficiency at which it is working.

***Solution* :** At no load, $I_{a1} = I_{L1} - I_{sh} = 4 - 1 = 3\ \text{A}$

$$\text{Back emf}, E_1 = V - I_{a1}R_a = 500 - 3 \times 0.2 = 499.4\ \text{V}$$

Under loaded condition, $I_{L2} = 100\ \text{A}$; and $I_{a2} = I_{L2} - I_{sh} = 100 - 1 = 99\ \text{A}$

$$\therefore \quad \text{Back emf}, E_2 = V - I_{a2}R_a = 500 - 99 \times 0.2 = 480.2\ \text{V}$$

For a shunt motor, the flux remains constant and hence $E \propto N = kN$. Therefore,

$$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{480.2}{499.4} \times 1000 \simeq \mathbf{962\ rpm}$$

At no load, the power taken by the motor mainly meets the constant losses (iron and frictional losses). Hence,

$$\text{Constant losses}, P_c = VI_{L1} = 500 \times 4 = 2000\ \text{W}$$

On loading, the copper loss in shunt field winding is negligible compared to the copper loss in armature winding. Thus,

$$\text{The variable losses}, P_v = I_{a2}^2 R_a = 99^2 \times 0.2 = 1960\ \text{W}$$

The total input power, $P_{\text{in}} = VI_{L2} = 500 \times 100 = 50\,000\ \text{W}$

$$\therefore \quad \text{Efficiency}, \eta = \frac{P_{\text{in}} - (P_v + P_c)}{P_{\text{in}}} = \frac{50\,000 - (2000 + 1960)}{50\,000} = 0.92\ \text{pu} = \mathbf{92\ \%}$$

Example 16.32 A dc shunt generator running at 500 rpm delivers 50 kW at 250 V. It has an armature resistance of 0.02 Ω and a field-winding resistance of 50 Ω. Calculate the speed of the machine running as a shunt motor and taking a power of 50 kW at 250 V.

***Solution* :** The field current in both cases, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{250}{50} = 5\ \text{A}$

When working as a generator, the machine supplies a load of 50 kW at 250 V. Therefore, the load current,

$$I_L = \frac{50\,\text{kW}}{250\ \text{V}} = 200\ \text{A}; \quad \text{and} \quad I_{a1} = I_L + I_{sh} = 200 + 5 = 205\ \text{A}$$

Hence, the induced emf, $E_{a1} = V + I_{a1}R_a = 250 + 205 \times 0.02 = 254.1\ \text{V}$

When working as a motor, the machine takes a power of 50 kW at 250 V. The line current I_L is still 200 A. Out of this current, 5 A goes to the shunt field winding. Therefore, the armature current,

$$I_{a2} = I_L - I_{sh} = 200 - 5 = 195\ \text{A}$$

Hence, the induced emf, $E_{a2} = V - I_{a2}R_a = 250 - 195 \times 0.02 = 246.1\ \text{V}$

As the field current and hence the flux per pole is the same in the two cases, we should have

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \qquad \text{or} \qquad N_2 = \frac{E_2}{E_1} \times N_1 = \frac{246.1}{254.1} \times 500 = \mathbf{484\ rpm}$$

Example 16.33 A series motor takes 20 A at 400 V and runs at 250 rpm. The armature and field resistances are 0.6 Ω and 0.4 Ω, respectively. Find the applied voltage and the current to run the motor at 350 rpm, if the torque required varies as the square of the speed.

Solution **:** In a series motor, $\tau \propto I_a^2$. But here it is given that $\tau \propto N^2$. Hence, we conclude that

$$I_a \propto N = kN$$

$$\therefore \quad \frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1} \qquad \text{or} \qquad I_{a2} = \frac{N_2}{N_1} \times I_{a1} = \frac{350}{250} \times 20 = 28\,\text{A}$$

The back emfs generated in the two cases are

$$E_1 = V_1 - I_{a1}(R_a + R_{se}) = 400 - 20 \times (0.6 + 0.4) = 380\ \text{V}$$

and
$$E_2 = V_2 - I_{a2}(R_a + R_{se}) = V_2 - 28 \times (0.6 + 0.4) = (V_2 - 28)\ \text{V}$$

In a series motor, as the flux $\Phi \propto I_a$, the back emf generated is given as

$$E = \frac{\Phi ZNP}{60A} \propto \Phi N \propto I_a N$$

Therefore, we must have

$$\frac{E_2}{E_1} = \frac{I_{a2} N_2}{I_{a1} N_1}$$

or
$$\frac{V_2 - 28}{380} = \frac{28 \times 350}{20 \times 250} \quad \Rightarrow \quad V_2 = \mathbf{772.8\ V}$$

SUMMARY

1. There are two types of armature-windings in a dc machine : (*i*) Lap winding ($A = P$), and (*ii*) Wave-winding ($A = 2$); A is the number of parallel paths.

2. The emf generated in a dc machine, $E = \dfrac{\Phi ZNP}{60A}$.

3. For a ***generator***, $V = E - I_a R_a$; and for a ***motor,*** $V = E + I_a R_a$

4. Types of dc machines : (**1**) *Permanent magnet*, (**2**) *Separately excited*, and (**3**) *Self-excited* : (*a*) Series, (*b*) Shunt, and (*c*) Compound : (*i*) Short-shunt, and (*ii*) Long-shunt.

5. The ***armature reaction*** is the effect of armature ampere-turns upon the value and distribution of the magnetic flux entering and leaving the armature core. The flux due to armature reaction is called ***cross flux.*** It weakens and distorts the main flux.

6. Different losses in dc machines : (**1**) *Copper losses* : (*i*) Armature copper loss, (*ii*) Field copper loss, and (*iii*) Brush contact loss; (**2**) *Magnetic or iron losses* : (*i*) Hysteresis loss, and (*ii*) Eddy-current loss; (**3**) *Mechanical losses* : (*i*) Air friction (or windage) loss, and (*ii*) Bearing friction loss.

7. The condition for maximum efficiency of a dc generator is that the *constant losses* (*iron losses*) *must be equal to the variable losses* (*copper losses*).

8. Important characteristics of a dc generator : (**1**) Open-circuit, Magnetic, or No-load Characteristic, (**2**) Load, or External Characteristic, (**3**) Internal Characteristic.

9. ***Critical field resistance*** for a dc generator running at a given speed is the minimum value of field resistance for which the voltage build-up is possible.

10. ***Critical speed*** for a dc generator for given field resistance is the minimum speed for which the voltage build-up is possible.

11. Due to the armature voltage drop and the demagnetization effect of the armature reaction, the terminal voltage of a dc shunt generator slightly decreases as the load is increased.

12. In a dc series generator, the terminal voltage increases proportionately to the load current.

13. Ideally, we prefer a generator whose terminal voltage remains constant even on increasing the load. This can be achieved in a *level-compounded generator.* Depending upon whether the series ampere-turns are less than or more than the shunt ampere-turns, the generator may be *under-compounded* or *ove- compounded*.

14. The torque developed in a dc motor, $\tau_d = \frac{\Phi Z}{2\pi}\left(\frac{P}{A}\right)I_a$, or $\tau_d \propto I_a \times \Phi$

15. A dc shunt motor is almost a constant speed motor, and the torque, $\tau_d \propto I_a$.

16. A dc series motor is a variable speed motor, and the torque, $\tau_d \propto I_a^2$. For loads requiring large starting torque, the series motor is the best choice. For example, hoists, cranes, electric trains, etc.

17. If a dc motor is directly connected to the supply, a heavy damaging current flows through the armature. To avoid this, we use a *starter*.