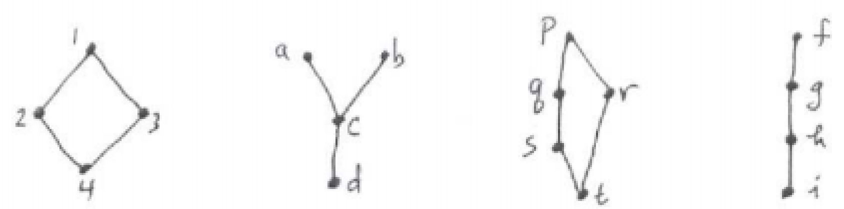
Tutorial 2
RELATIONS

Question 1 Let A = {a, b, c} and B = {p, q}. Find

(i) $A \times B$
(ii) $B \times A$
(iii) $A \times A$
(iv) $B \times B$

Question 2 Which of the following Hasse diagrams represent lattices?

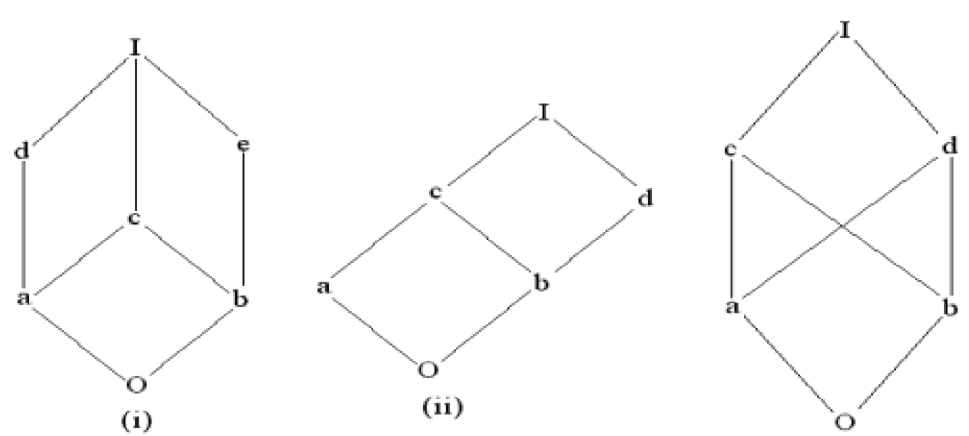


(I) (II) (III) (IV)

Question 3 Determine the domain and range of the following relations

(i) {(1, 2), (1, 4), (1, 6), (1, 8)}
(ii) $\{(x, y) : x \in N, y \in N \text{ and } x + y = 10\}$
(iii) $\{(x, y) : x \in N, x<5, y = 3\}$
(iv) $\{(x, y) : y = |x - 1|, x \in Z \text{ and } |x| \leq 3\}$

Question 4 Which of the partially ordered sets in figures (i), (ii) and (iii) are lattices? Justify your answer.



(i) (ii) (iii)

Question 5 **Let R be the relation on Z defined by a R b if and only if a – b is an even integer. Find (i) R, (ii) domain R, (iii) range of R.**

Question 6 Consider the relation on $A = \{a, b, c, d, e\}$.

$$R = \{(a, a), (a, c), (b, c), (c, d), (e, a), (d, b)\}.$$

Draw the corresponding digraph.

Question 7 **Let A = {1, 2, 3}, B = {2, 3, 4} and C = {4, 5}. Verify that**

(i) A × (B ∩ C) = (A × B) ∩ (A × C)

(ii) A × (B ∪ C) = (A × B) ∪ (A × C)

Question 8 **If R is the relation "less than" from A = {1, 2, 3, 4, 5} to B = {1, 4, 5}, write down the set of ordered pairs corresponding to R. Find the inverse relation to R.**

Question 9 For $W_2 = \mathbb{N} \times \mathbb{N}$. Define LEX_2 to be the relation such that, for $a = (a_1, a_2) \in W_2$ and $b = (b_1, b_2) \in W_2$, we say that aW_2b if either

(a) $a_1 < b_1$, or

(b) $a_1 = b_1$ and $a_2 \leq b_2$.

Is (W_2, LEX_2) a partial order?

Question 10 Let T be the set of all triangles in a plane with R a relation in T given by R = {(T1, T2) : T1 is congruent to T2}. Show that R is an equivalence relation.

Question 11 **If R is the relation in N x N defined by (a,b) R (c,d) if and only if a + d = b + c, show that R is an equivalence relation.**

Question 12 **Let A = {1, 2, 3, 4} and B = {x, y, z}. Let R be a relation from A to B defined by R = {(1, x), (1, z), (3, x), (4, y)}. Find the domain and range of R.**

Question 13 For $k \in \mathbb{N}$, let $D_k = \{a \in \mathbb{N} : a \mid k\}$ be the set of divisors of k. Define L_k to be the relation such that, for $a, b \in D_k$, we say that aL_kb if $a \mid b$. Is (D_{12}, L_{12}) a partial order? If so draw the Hasse diagram.

Question 14 For any positive integer n, let $I_n = \{x | 1 \le x \le n\}$. Let the relation "divides" be written as $a \mid b$ **iff a divides b** or b = ac for some integer c. Draw the Hasse diagram and determine whether $[I_{12}; \mid]$ is a lattice.

Question 15 Define R to be the relation such that, for $a, b \in \mathbb{Z}$, we say that aRb if $|a-1| \le |b-1|$. Is $(\mathbb{Z}, R)$ a partial order? If so draw the Hasse diagram.

Question 16 Let m be a positive integer with m>1. Determine whether or not the following relation is an equivalent relation.

$$R = \{(a, b) \mid a \equiv b \pmod m\}$$

Question 17 Draw the Hasse diagram for the lattice D_{18} consisting of the divisors of 18 with the partial order of divisibility.