

# Digital Electronics

∴ no. of bits

$$\therefore \alpha^3 = 8 \quad (8 \rightarrow \text{Octal})$$

∴ 3 bits in octal

$$2^4 = 16 \quad (16 \rightarrow \text{Hex.})$$

4 bits in Hex.

## Signed integer representation

- ① Signed matrix/Magnitude
- ② Diminished-Radix Complement
- ③ Radix Complement

### ① Signed Matrix/Magnitude

Most Significant bit (MSB) used for sign of the no.

In Binary

(-) for -

(+) for 0

$$-5 = (\underline{1}0000101)_2 = (85)_{16}$$

$$+5 = (\underline{0}0000101)_2 = (05)_{16}$$

$$+0 = (00000000)_2 = (00)_{16}$$

$$-0 = (10000000)_2 = (80)_{16}$$

Total nos represented by signed

$$-(\alpha^{n-1}-1) \text{ to } (\alpha^{n-1}-1)$$

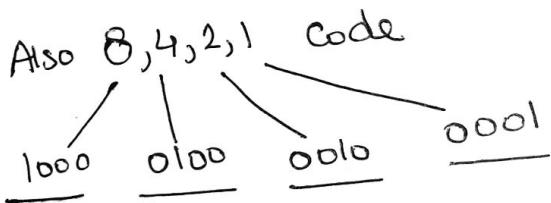
One's Complement Method (Only do the complement)  
Two's Complement Method (Complement & add 1)

Maggie (Mano) - Book

### Codes

- ① BCD → Binary coded decimal  
② Excess-3  
③ Gray  
④ Alphanumeric

### Natural BCD Codes



### Excess-3 Code

2 3  
0010 0011  
Add ③ to both digits

$$\begin{array}{r} 0010 \quad 0011 \\ 0011 \quad 0011 \\ \hline 0100 \quad 0110 \end{array}$$

↓  
56

### Gray-Code

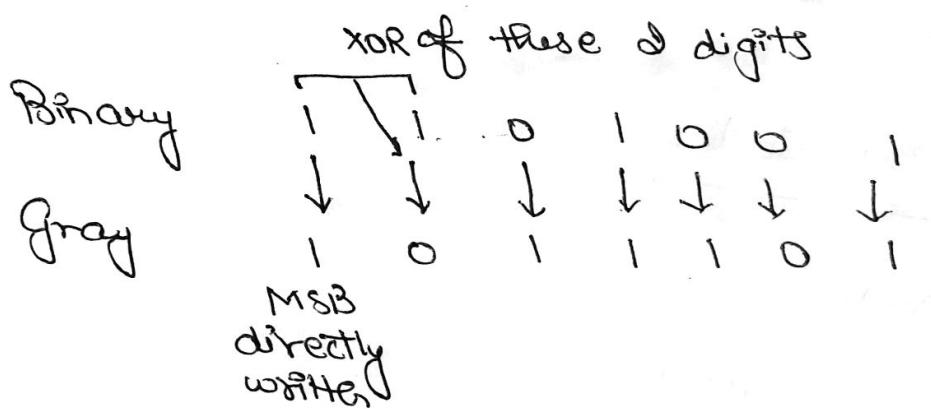
→ Not a weighted code  
→ Reflected code

Only 0 & 1 have values 0, 1

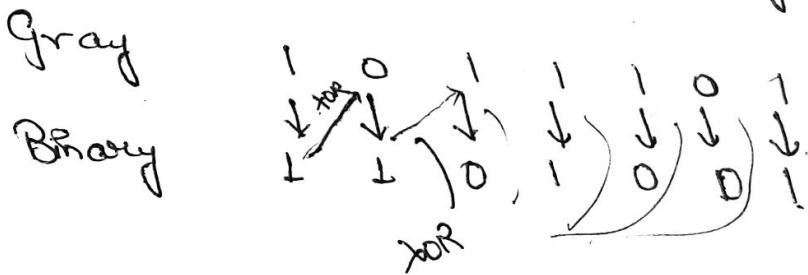
0	-	0 0 0
1	-	0 0 1
2	-	0 1 1
3	-	0 1 0
4	-	1 1 0
5	-	1 1 1
6	-	1 0 1
7	-	1 0 0

Reflection from this

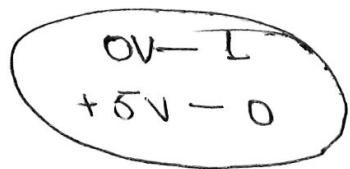
### Conversion of Binary to Gray Code



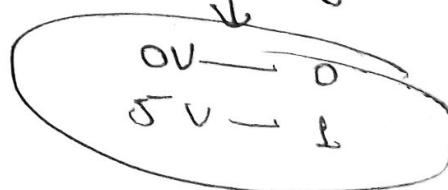
Similarly for Gray  $\rightarrow$  Binary

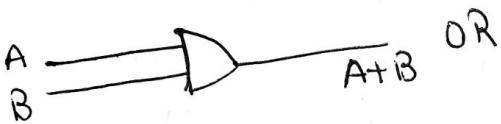
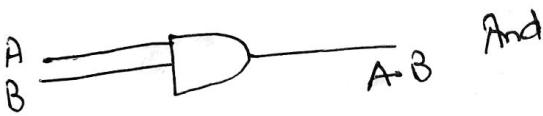
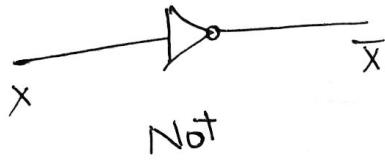
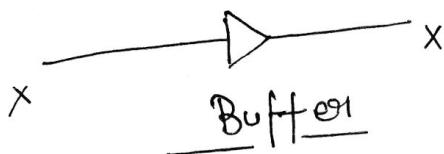


Negative logic used

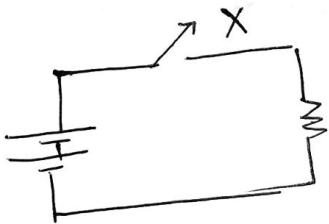


not true logic





Electrical Switches as logical Networks



$x=1 \rightarrow$  switch closed  
 $x=0 \rightarrow$  switch open

XOR

0 0	1
0 1	0
1 0	0
1 1	1

Minterms & Maxterms

1  
complement  
 $\circledcirc$

complement  
is 1

x	y	z	M <sub>0</sub>
0	0	0	$x'y'z'$
0	0	1	$x'y'z$
0	1	0	$x'yz'$
0	1	1	$x'yz$
1	0	0	$xy'z'$
1	0	1	$xy'z$
1	1	0	$xyz'$
1	1	1	$xyz$

	Max <sup>o</sup>	M <sub>0</sub>
$m_0$	$x+y+z$	M <sub>0</sub>
$m_1$	$x+y+z'$	M <sub>1</sub>
$m_2$	$x+y'+z$	M <sub>2</sub>
$m_3$	$x+y'+z'$	M <sub>3</sub>
$m_4$	$x'+y+z$	M <sub>4</sub>
$m_5$	$x'+y+z'$	M <sub>5</sub>
$m_6$	$x'+y'+z$	M <sub>6</sub>
$m_7$	$x'+y'+z'$	M <sub>7</sub>

Minterms are complements of maxterms

Eg:

$$\begin{aligned}x'yz &= m_3 \\m_3' &= (x'yz)' \\&= (x+y'+z') \\&= M_3\end{aligned}$$

a)

x	y	z	F <sub>1</sub>	F <sub>2</sub>	F <sub>3</sub>
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	0	1	0

$$F_1 = m_6$$

$$F_2 = (m_1, m_2, m_5, m_6, m_7)$$

$$F_2 = \sum m(1, 4, 5, 6, 7)$$

$$F_2 = x'y'z + xy'z' + x'y'z' + x'y'z + xy'z$$

This was sum of Products (take 1)

Now Product of Sum  
(take 0)

$$F_1 = \prod M(0, 1, 2, 3, 4, 5, 7)$$

$$F_2 = \prod M(0, 2, 3)$$

~~$$F_2 = x'y + xy' + xy^2$$~~

$$F_2 = (x+y+2) \cdot (x+y'+2) \cdot (x+y^2)$$

$(R-1)$ 's Complement

$R \rightarrow \text{Base}$

for  $n$ -veno.

$\bar{N} = \underbrace{(R^n - R^{-n})}_{\substack{\text{no. of digits} \\ \text{before decimal}}} - \underbrace{N}_{\substack{\text{no. of digits} \\ \text{after decimal}}} - \text{Number}$

Compo

$R$ 's Complement

for  $n$ -ve no.

$\bar{N}^* = R^n - N$

using 10's Complement ( $72532 - 3250$ )

$$M = 72532 \quad N = 03250$$

$$\therefore 10^5 \text{ Comp. of } N = 10^5 - 03250 \\ = 96750$$

$\underbrace{\phantom{00000}}_{5 \text{ dig.}}$

Now  $M+N = 1 \underline{69282}_{5 \text{ dig.}}$

discard

3250 - 72532

$$M = 03250$$

$$N = 72532$$

$$\text{10's Compl. of } N = 10^5 - 72532 \\ = 27468$$

$$\begin{array}{r} 100000 \\ 72532 \\ \hline \end{array}$$

$$\therefore M + N = 30718 \text{ (No Carry)}$$

∴ Taking comp. of 30718

$$10^5 - 30718$$

∴ Put -ve in front of Comp.

(R-1)'s Compl. use

$$\begin{array}{r} 72532 - 3250 \\ \hline \end{array}$$

$$M = 72532 \quad N = 03250$$

$$(R-1)'s = 9's \text{ Compl.} = 10^5 - 10 - 03250 \\ = 967\cancel{5}49$$

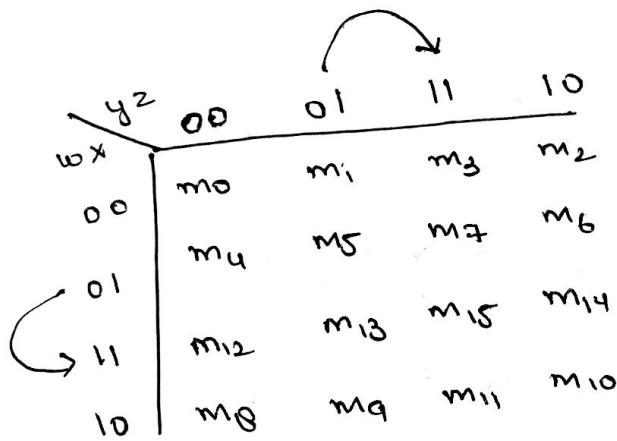
$$M + N = \begin{array}{r} 72532 \\ 03250 \\ \hline 105782 \end{array}$$

Add 1

$$\begin{array}{r} 011010 \\ 111110 \\ \hline \end{array}$$

$$\begin{array}{r} 011010 \\ 111110 \\ \hline 100000 \end{array}$$

K-map



$$\begin{aligned}
 \text{Ex: } f(A, B, C, D) &= A(C+D)'(B'+D') + C(B+C'+A'D) \\
 &= AB'C'D' + AC'D' + BC + A'CD \\
 &= AB'C'D' + AC'D'(B+B') + BC(A+A')(D+D') + A'CD(B+B') \\
 &= AB'C'D' + AC'D'B + AC'D'B' + BC[AD + AD' + A'D + A'D'] \\
 &\quad + A'CD B + A'B'CD \\
 &= \cancel{\frac{AB'C'D'}{8}} + \cancel{\frac{AC'D'}{9}} + \cancel{\frac{AB'C'D'}{10}} + \cancel{\frac{ABC'D}{15}} + \cancel{\frac{ABCD'}{13}} + A'BCD \\
 &\quad + \cancel{\frac{A'BCD'}{5}} + \cancel{\frac{A'BCD}{7}} + \cancel{\frac{A'B'CD}{3}}
 \end{aligned}$$

$m_3 \quad m_5 \quad m_7 \quad m_8 \quad m_9 \quad m_{13} \quad m_{15}$

	00	01	11	10
00	.	.	L	
01		1	1	
11		1	1	
10	1	1		

	0	4	12	8
1	5	13	9	
2	7	15	11	
3	6	14	10	

## Quine-McCluskey Method

Taking the corresponding numbers & grouping the nos acc. to the no. of 1's in those nos

$$F(A \bar{B} CDE) = \Sigma m(0, 2, 4, 6, 7, 8, 10, 11, 12, 13, 14, 16, 18, \\ 19, 29, 30)$$

	A	B	C	D	E	
	A	B	C	D	E	
(0)	0	0	0	0	0	
(2)	0	0	0	1	0	
(4)	0	0	1	0	0	
(8)	0	1	0	0	0	
(16)	1	0	0	0	0	
6	0	0	1	1	0	
10	0	1	0	1	0	
12	0	1	1	0	0	
18	1	0	0	1	0	
7	0	0	1	1	1	
11	0	1	0	1	1	
13	0	1	1	0	1	
14	0	1	1	1	0	
19	1	0	0	1	1	
29	1	1	1	0	1	
30	1	1	1	1	0	
	(0,2)	0	0	0	-0	
	(0,4)	0	0	-0	0	
	(0,8)	0	-	0	00	
	(0,16)	-0	0	0	0	
	(2,6)	0	0	-	10	
	(2,10)	0	-	0	10	
	(2,18)	-	0	0	10	
	(4,6)	0	0	1	-0	
	(4,12)	0	-	1	00	
	(8,10)	0	1	0	-0	
	(8,12)	0	1	-	00	
	(16,18)	1	0	0	-0	
		A	B	C	D E	
	(6,7)	0	0	1	1	-
	(6,14)	0	-	1	1	0
	(10,11)	0	1	0	1	-
	(10,14)	0	1	-	1	0
	(12,13)	0	1	1	0	-
	(12,14)	0	1	1	-	0
	(18,19)	1	0	0	1	-
	(13,29)	-	1	1	0	1
	(14,30)	-	1	1	1	0

	A	B	C	D	E
(0, 2, 4, 6)	0	0	-	-	0
(0, 2, 8, 10)	0	-	0	-	0
(0, 2, 16, 18)	-	0	0	-	0

(0, 4, 2, 6)	0	-	-	0	0
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~~(0, 8, 2, 10)~~

~~(0, 8, 4, 12)~~

~~(0, 16, 2, 10)~~

(2, 6, 10, 14)	0	-	-	1	0
----------------	---	---	---	---	---

<del>(2, 10, 6, 14)</del>	0	-	1	-	0
---------------------------	---	---	---	---	---

<del>(2, 10, 6, 14)</del>	0	1	-	-	0
---------------------------	---	---	---	---	---

<del>(8, 10, 12, 14)</del>	0	1	-	-	0
----------------------------	---	---	---	---	---

~~(8, 12, 10, 14)~~

(16, 18, 12, 14)	0	-	-	-	0
------------------	---	---	---	---	---

A B C D E

(0, 2, 4, 6,  
8, 10, 12, 14)

0 - - - 0

Now we take the pairs which do not combine  
with any other pair

~~(6, 7)~~

x

x

~~(10, 11)~~

x x

x

~~(12, 13)~~

~~(18, 19)~~

x

~~(13, 29)~~

x

~~(14, 30)~~

x

~~(0, 2, 16, 18)~~ x

x

~~(0, 2, 4, 6, 8, 10, 12, 14)~~ x x

x x

8, 10, 12, 14)

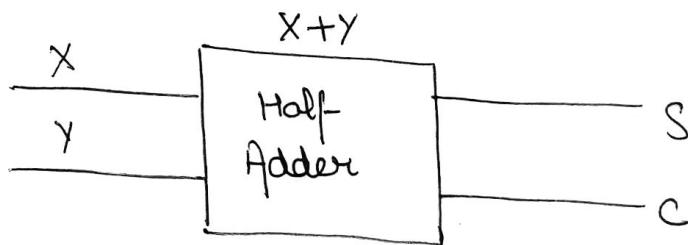
x

Now we select the rows without boxes.

$$(0, 2, 4, 6, 8, 10, 12, 14), (0, 6, 16, 18), (14, 30), (13, 29) \\ (18, 19), (10, 11), (6, 7)$$

$$\therefore \bar{A}\bar{E} + \bar{B}\bar{C}\bar{E} + BC\bar{D}\bar{E} + BC\bar{D}\bar{E} + A\bar{B}\bar{C}D' + \\ AB\bar{C}D + \bar{A}\bar{B}CD \quad \underline{\text{Ans}}$$

SSI  $\rightarrow$  Half Adder



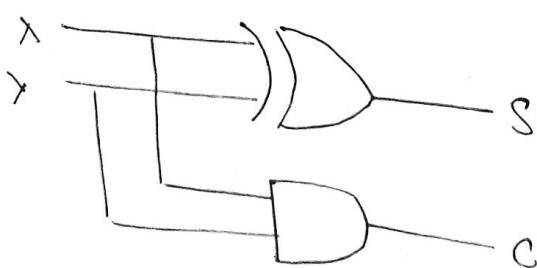
Truth table for a Half Adder:

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Annotations: C is labeled 'Carry bit' and S is labeled 'Sum bit'.

$$S = \bar{X}Y + X\bar{Y} \Rightarrow X \oplus Y$$

$$C = X \cdot Y$$



(2) Full Adder



X	Y	Cin(z)	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	

$$S = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$

$$C = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$$

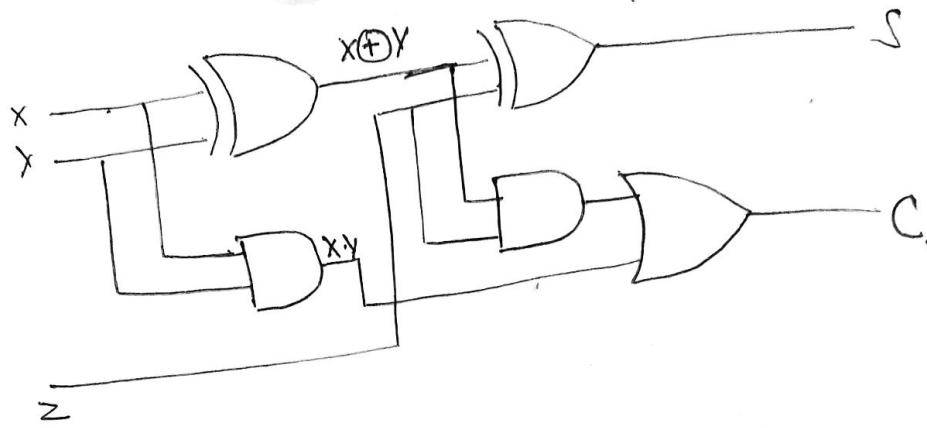
$$S = \bar{y}(\bar{x}z + x\bar{z}) + yz = \bar{y}(x \oplus z) + yz$$

$$C = yz + x(\bar{y}z + y\bar{z}) \\ = yz + x(y \oplus z)$$

$$C = x \cdot y + (x \oplus y) \cdot z$$

$$S = x \oplus (y \oplus z)$$

$$\text{or } (x \oplus y) \oplus z$$

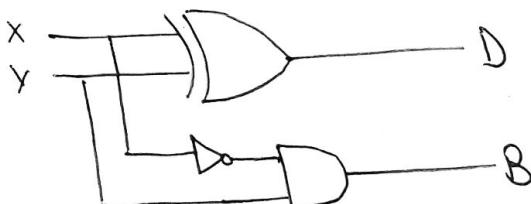


### ③ Half Subtractor

X	Y	Diff (D)	Borrow (B)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = \bar{X}Y + X\bar{Y} = X \oplus Y$$

$$B = \bar{X}Y$$



## Full Subtractor

X	Y	B <sub>1</sub>	D	B <sub>2</sub>
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$D = \bar{x}\bar{y}z + \bar{x}y\bar{z} + x\bar{y}\bar{z} + xyz$$

$$\begin{aligned} B_1 &= \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + xy\bar{z} \\ &= yz + \bar{x}(y \oplus z) \end{aligned}$$

$$\begin{array}{c} \overline{x \oplus z} \\ \hline \overline{xz} + \overline{xz} \end{array}$$

$$\begin{aligned} B_2 &= \bar{y}(x \oplus z) + y(\bar{x}\bar{z} + xz) \\ &= \bar{y}(x \oplus z) + y(\overline{x \oplus z}) \\ &= \underline{\bar{y} \oplus (x \oplus z)} \end{aligned}$$

$$\overline{yz} + \overline{yz}$$

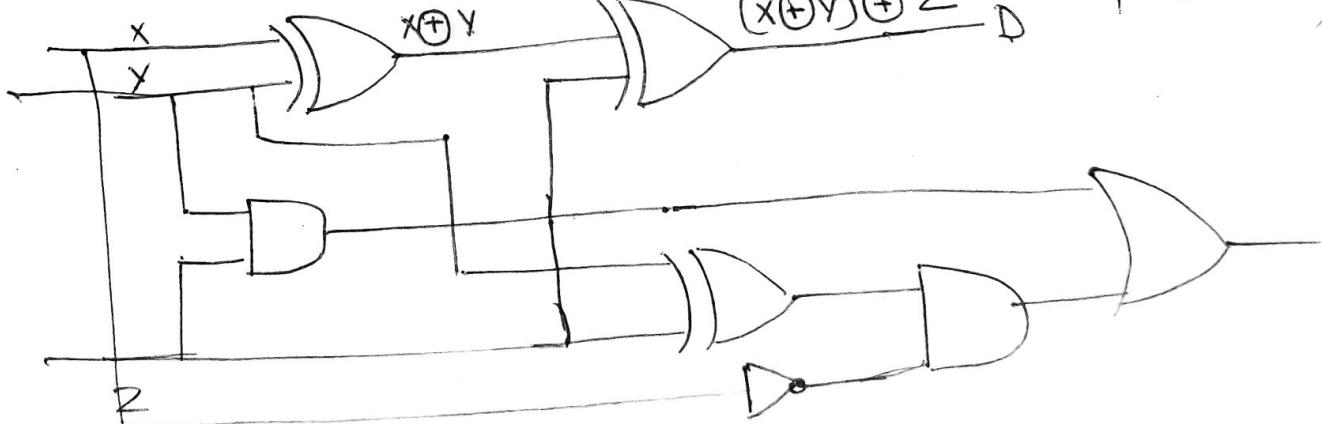
$$\cancel{\overline{yz}} \\ \overline{yz} \cdot \overline{yz}$$

$$(y + \bar{z}) \cdot (\bar{y} + z)$$

$$\cancel{yz} + \frac{yz + \bar{z}z}{xz} \\ \overline{xz} \cdot \overline{xz}$$

$$\overline{xz} \cdot \overline{xz}$$

$$(\bar{x} + z) \cdot (x + \bar{z})$$



## Multiplexer

It is a device which has

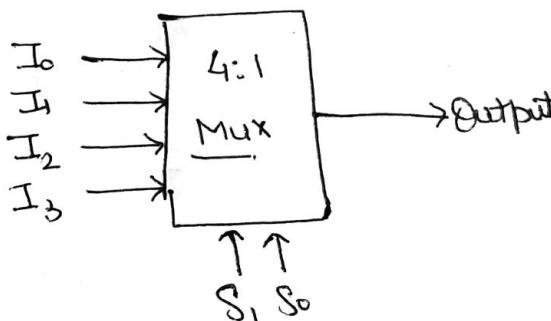
- i) No. of input lines
- ii) No. of selection lines
- iii) Only one output line

It steers one of  $2^n$  input to a single output line in  $n$  selection lines

$$2^n : 1$$

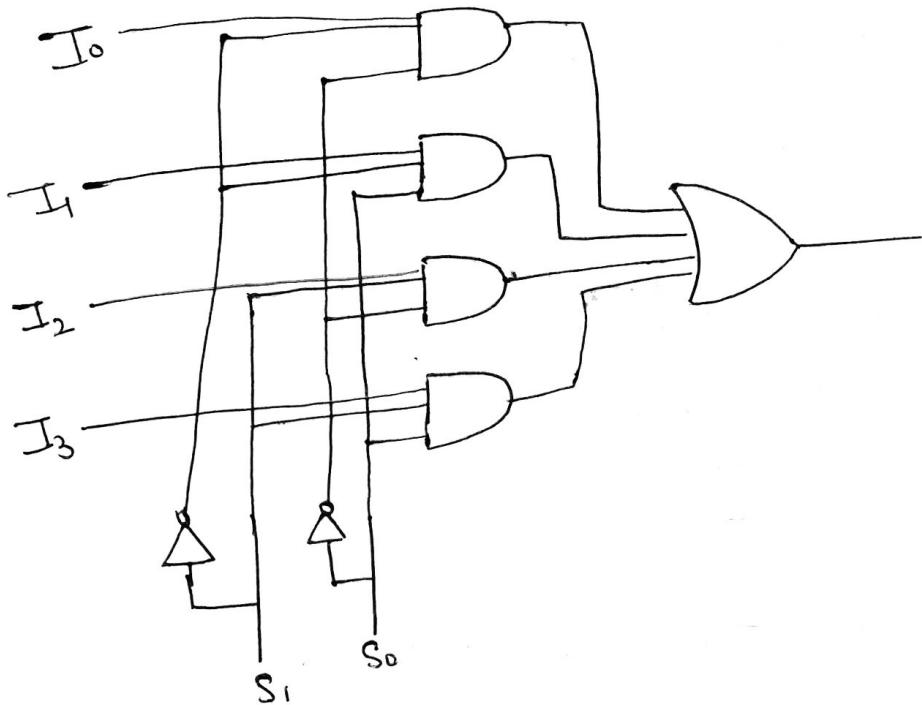
## Data Selector

$I_0$	$I_1$	$I_2$	$I_3$	$S_1$	$S_0$	$Y$
$d_0$	$d_1$	$d_2$	$d_3$	0	0	$d_0$
$d_0$	$d_1$	$d_2$	$d_3$	0	1	$d_1$
$d_0$	$d_1$	$d_2$	$d_3$	1	0	$d_2$
$d_0$	$d_1$	$d_2$	$d_3$	1	1	$d_3$

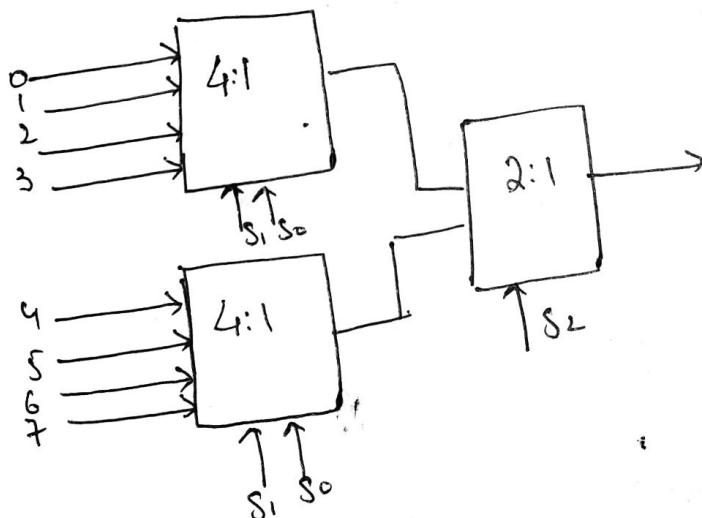


$S_1$	$S_0$	$Y$
0	0	$I_0$
0	1	$I_1$
1	0	$I_2$
1	1	$I_3$

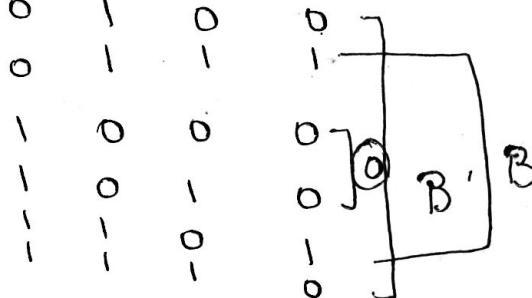
$$Y = I_0 \bar{S}_1 \bar{S}_0 + I_1 \bar{S}_1 S_0 + I_2 S_1 \bar{S}_0 + I_3 S_1 S_0$$



Larger Multiplexer  
from smaller multiplexers



F	B	C	F	Mux Input
0	0	0	1	
0	0	1	1	(1)
0	1	0	0	
0	1	1	1	
1	0	0	0	
1	0	1	0	
1	1	0	1	

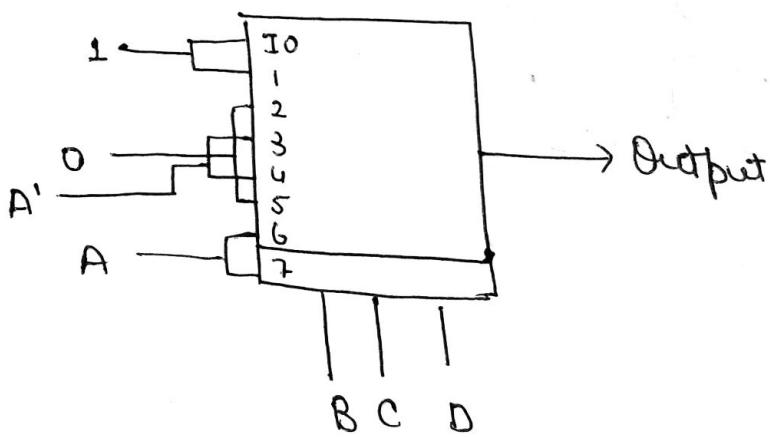


using B as input

$$F(A, B, C, D) = \sum m(0, 1, 3, 4, 8, 9, 14, 15)$$

Implement using 8:1 MUX

	I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>	I <sub>3</sub>	I <sub>4</sub>	I <sub>5</sub>	I <sub>6</sub>	I <sub>7</sub>
A'(0)	0	1	2	3	4	5	6	7
A(0)	0	1	10	11	12	13	14	15
	1	1	0	A'	A'	0	A	A



## Decoders

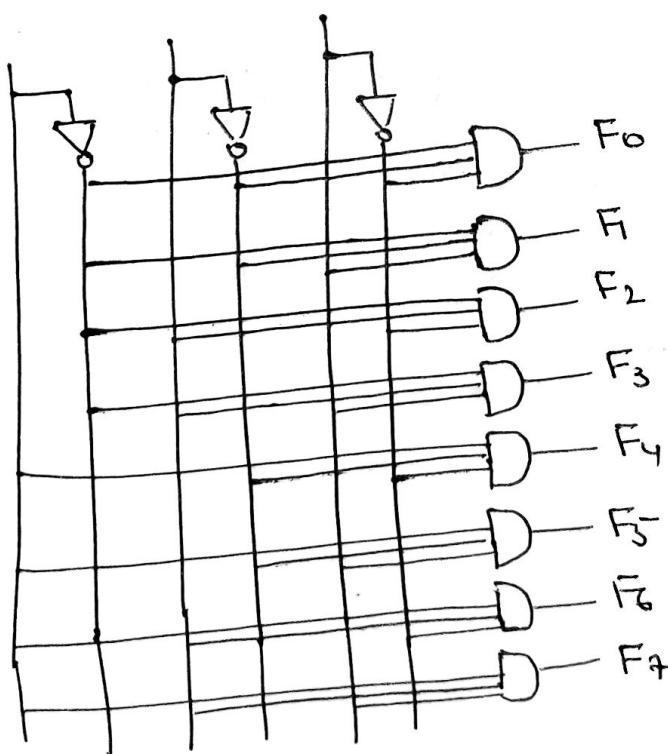
for  $n$  input lines there must be  $2^n$  output lines.

Eg. Taking 2:4 decoder

x	y	$F_0$	$F_1$	$F_2$	$F_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	1
1	1	0	0	0	0

3:8 Decoder

x	y	z	$F_0$	$F_1$	$F_2$	$F_3$	$F_4$	$F_5$	$F_6$	$F_7$
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	1
1	1	0	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	0



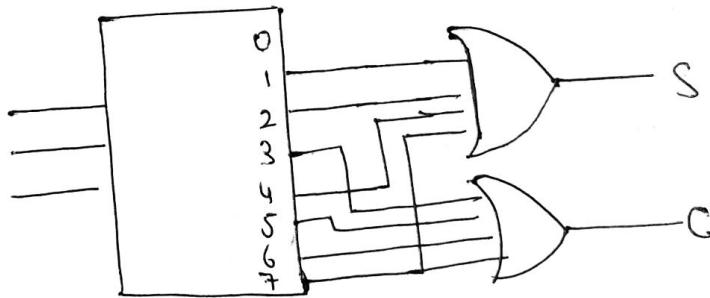
# Decoders - Implementing functions

For Full adder

$$S(x,y,z) = \sum m(1,2,4,7)$$

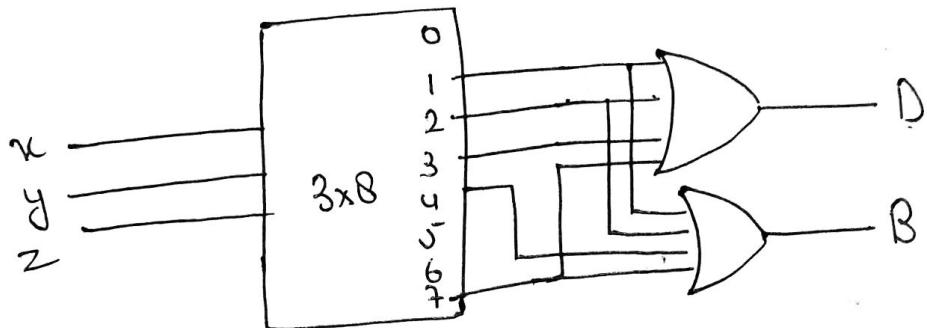
$$C(x,y,z) = \sum m(3,5,6,7)$$

x	y	z	C	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Full Subtractor

x	y	z	B	D	D = $\sum m(1,2,3,7)$
0	0	0	0	0	$B = \sum m(1,2,4,7)$
0	0	1	1	1	
0	1	0	1	1	
0	1	1	0	1	
1	0	0	1	0	
1	0	1	0	0	
1	1	0	0	0	
1	1	1	1	1	



Full Subtractor

Decoders with enable

If the device gets activated if  $E=1$

$E$	$x$	$y$	1	0	0	0
1	0	0	0	1	0	0
1	0	1	0	0	1	0
1	1	0	0	0	1	0
1	1	1	0	0	0	1
<hr/>			0	0	0	0

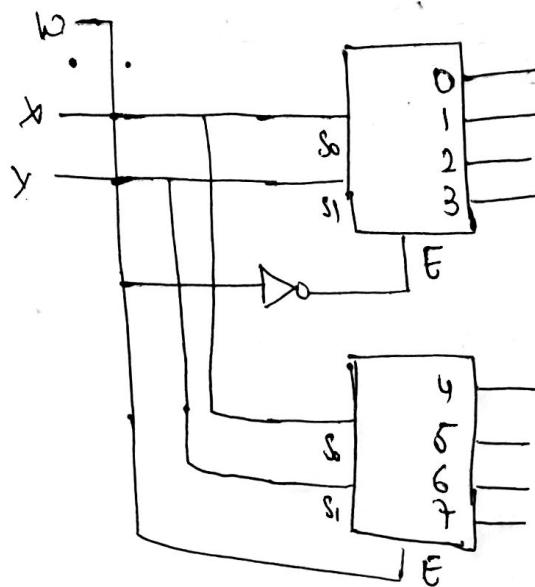
Decoder with enable 1

$E$	$x$	$y$	1	0	0	0
0	0	0	0	1	0	0
0	0	1	0	0	1	0
0	1	0	0	0	0	1
0	1	1	0	0	0	0
<hr/>			0	0	0	0

Decoder with enable 0

## Larger decoders

3x8 with 2x4 decoders

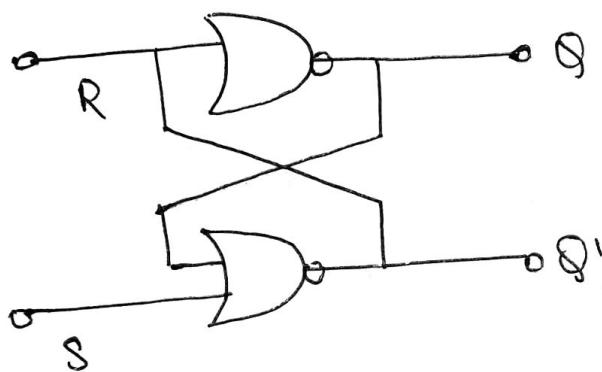


If upper E works then the lower one gets disabled & vice versa.

## SR Latch

flip flop - Non-transparent element  
 latches - transparent / decision taken by it

Active-High input S-R latch



S	R	Q	Q'	
1	0	0	0	initial (after S=1, R=0))
0	0	1	0	
0	1	0	1	(after S=0, R=1)
0	0	0	1	
1	1	0	0	invalid

Gated - SR latch

characteristic eq<sup>n</sup>

$$Q(t+1) = S + R'Q(t)$$

$$\underline{R \cdot S = 0}$$