Number Systems

To be Discussed

- Base or Radix.
- Binary Numbers.
- Octal Numbers.
- Hexadecimal Numbers.
- Inter-system Conversions.

Number System (or Positional Number System)

- We express a number in any base or radix, *X.*
- For binary, the base is 2.
- A number with radix *X* ,

MSD $\boxed{a_m(X)^{m-1} + a_{m-1}(X)^{m-2} + \ldots + a_2(X)^1 + a_1(X)^0 .}$

$\boxed{. + b_1(X)^{-1} + b_2(X)^{-2} + \ldots + b_n(X)^{n}}$ LSD

Number System (or Positional Number System)

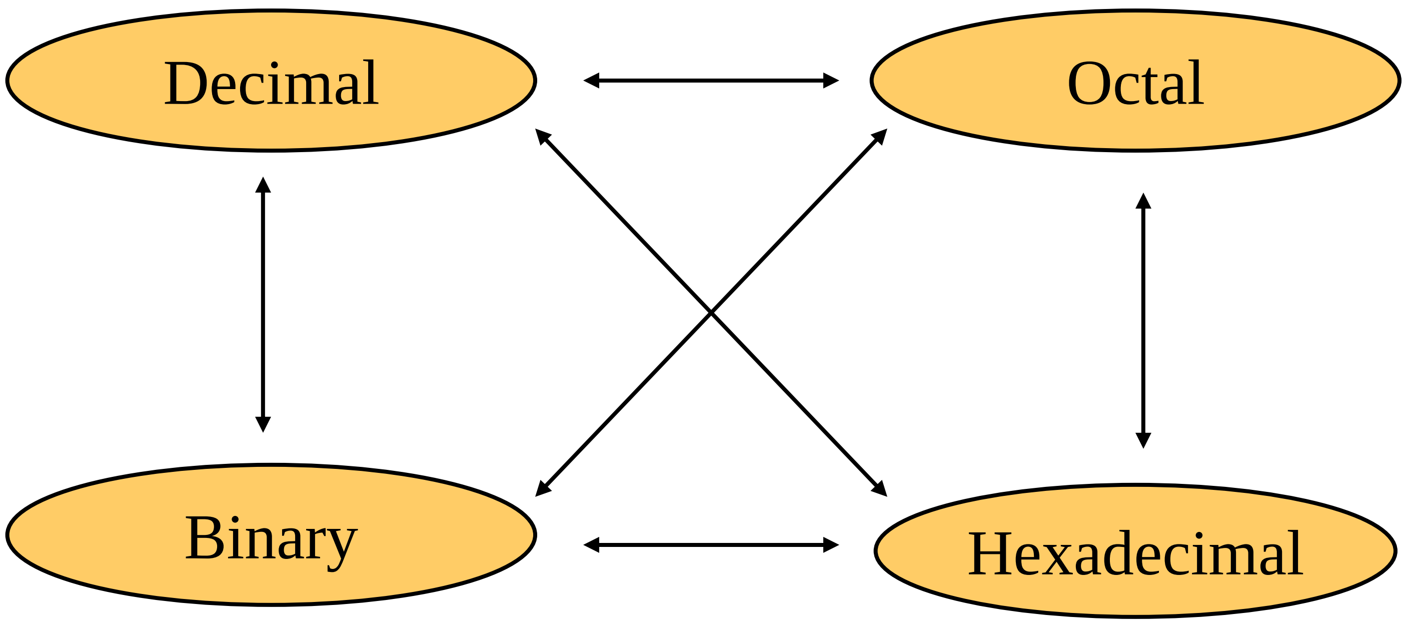
Also known as positional number system since the value of the number are decided by the position of the values like unit place, decimal place, and hundred place e.t.c.
In other words, **the value of number is determined by multiplying each digit by a weight, decided by its position, and then adding the individual products.**
The weight is decided by the location as well as the base in which we want to represent the number.

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Conversion Among Bases

- The possibilities:



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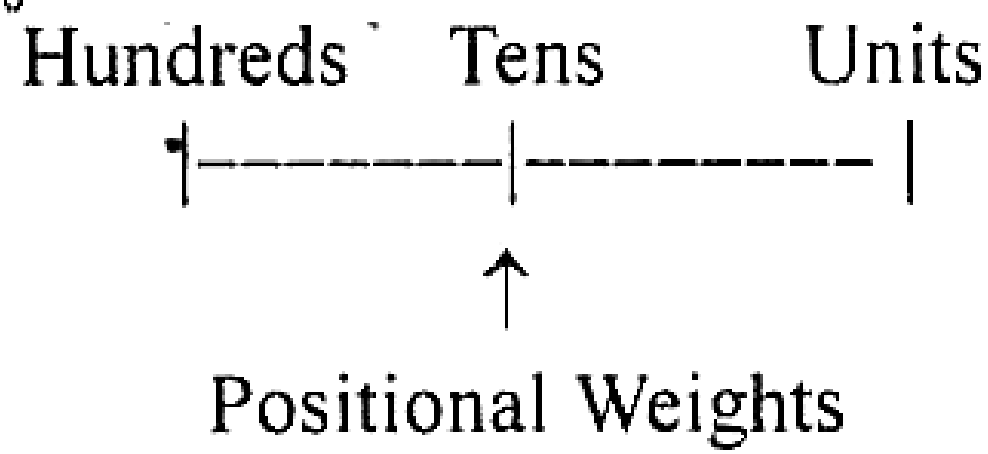
Binary System

- Only two digits (or bits) - 0 and 1.
- The position of 0 or 1 in a number indicates its "weight".
- For example, in decimal system,

$$(198)_{10} = 1 \times 10^2 + 9 \times 10^1 + 8 \times 10^0$$



Similarly, in binary system,

$$
\begin{aligned}
(198)_{10} &= (11000110)_2 \\
&= 1\times2^7 + 1\times2^6 + 0\times2^5 + 0\times2^4 + 0\times2^3 + 1\times2^1 + 0\times2^0 \\
&= 128 + 64 + 0 + 0 + 0 + 4 + 2 + 0 = 198
\end{aligned}
$$

- A group of 4 and 8 bits is called a nibble and a byte.

- With n bits, you can represents number

from 0 to $2^n - 1$

Decimal-Binary Conversion

Double-dabble method :

Example : Convert $(53.625)_{10}$ into binary.

- Take integer and fraction part separately.

Step 1 The integer is 53. The fraction is $0 \cdot 625$.

Step 2 Integer conversion:

Division	Generated remainder
$2\overline{)\,53\,}$	
$2\overline{)\,26\,}$	$\rightarrow 1$
$2\overline{)\,13\,}$	$\rightarrow 0$
$2\overline{)\,6\,}$	$\rightarrow 1$
$2\overline{)\,3\,}$	$\rightarrow 0$
$2\overline{)\,1\,}$	$\rightarrow 1$
$2\overline{)\,0\,}$	$\rightarrow 1 \rightarrow$ MSB

Multiplication	Generated integer
$0.625 \times 2 = 1.25 \rightarrow$	1 $\rightarrow$ MSB
$0.250 \times 2 = 0.50 \rightarrow$	0
$0.500 \times 2 = 1.00 \rightarrow$	1
$0.000 \times 2 = 0.00 \rightarrow$	0

Hence, $(53.625)_{10}$

$= (110101.101)_2$

- Example : Convert $(101111.1101)_2$ into its decimal equivalent.

First, the integer part :

1 0 1 1 1 1

$$
\begin{aligned}
1 \times 2^0 &= 1 \\
1 \times 2^1 &= 2 \\
1 \times 2^2 &= 4 \\
1 \times 2^3 &= 8 \\
0 \times 2^4 &= 0 \\
1 \times 2^5 &= \underline{32} \\
 & \underline{47}
\end{aligned}
$$

Similarly, the fraction part :

$$0 \cdot 1\ 1\ 0\ 1$$

$$1 \times 2^{-4} = 0 \cdot 0625$$
$$0 \times 2^{-3} = 0 \cdot 0000$$
$$1 \times 2^{-2} = 0 \cdot 2500$$
$$1 \times 2^{-1} = 0 \cdot 5000$$
$$\underline{\overline{0 \cdot 8125}}$$

Hence, $(110101.101)_2$

$= (47.8125)_{10}$

Octal Number System

- The base is 8.
- Symbols : 0, 1, 2, 3, 4, 5, 6, and 7.
- Positional weights :

$8^0 = 1$	$8^{-1} = 1/8$
$8^1 = 8$	$8^{-2} = 1/64$
$8^2 = 64$	$8^{-3} = 1/256$
$8^3 = 256$	

Example : Convert $(444.456)_{10}$ to its octal equivalent.

Integer part :

Division		Generated remainder
8) 444		
8) 55	→	4
8) 6	→	7
8) 0	→	6

Fraction Part :

Multiplication	Generated integer
$0 \cdot 456 \times 8 = 3 \cdot 648 \rightarrow$	3
$0 \cdot 648 \times 8 = 5 \cdot 184 \rightarrow$	5
$0 \cdot 184 \times 8 = 1 \cdot 472 \rightarrow$	1
$0 \cdot 472 \times 8 = 3 \cdot 776 \rightarrow$	3
$0 \cdot 776 \times 8 = 6 \cdot 208 \rightarrow$	6

The process is terminated when significant digits are obtained.

Thus, the octal equivalent of $(444 \cdot 456)_{10}$ is $(674.35136)_8$.

Example : Convert $(237)_8$ and $(120)_8$ to their decimal numbers.

$$\begin{aligned}(237)_8 &= 2\times 8^2 + 3\times 8^1 + 7\times 8^0\\ &= 2\times 64 + 3\times 8 + 7\times 1\\ &= 128 + 24 + 7\\ &= (159)_{10}\end{aligned}$$

$$\begin{aligned}(120)_8 &= 1\times 8^2 + 2\times 8^1 + 0\times 8^0\\ &= 1\times 64 + 2\times 8 + 0\times 1\\ &= 64 + 16 + 0\\ &= (80)_{10}\end{aligned}$$

What is the conversion of $(293)_8$ into decimal number ?

Ans. : The Question is wrong.

Octal-Binary Conversion

- Replace each digit by its binary 3-bit equivalent. For example, $(705)_8 = (?)_2$

$$\begin{array}{ccc} 7 & 0 & 5 \\ \downarrow & \downarrow & \downarrow \\ 111 & 000 & 101 \end{array}$$

$$(705)_8 = (111000101)_2$$

Binary-Octal Conversion

- Group bits in threes, starting on right
- Convert to octal digits

Example : $(1011010111)_2 = (?)_8$

001 011 010 111

1 3 2 7

$(1011010111)_2 = (1327)_8$

Hexadecimal Numbers

- Radix of 16.
- Sixteen symbols:

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.
- Positional weights :

  $\ldots, \; 16^3, 16^2, 16^1, 16^0, 16^{-1}$

Example : Convert $(115)_{10}$ to hexadecimal.

Division	Remainder
$16\overline{)\,115}$	–
$16\overline{)\,7}$	3
$16\overline{)\,0}$	7

Hence, $(115)_{10} = (73)_{16}$

Exercise : Convert $(235)_{10}$ to hexadecimal.

Ans : $(235)_{10} = (EB)_{16}$

Hexadecimal to Decimal Conversion

$$
\begin{aligned}
A3BH = (A3B)_{16} &= A \times 16^2 + 3 \times 16^1 + B \times 16^0 \\
&= 10 \times 16^2 + 3 \times 16^1 + 11 \times 16^0 \\
&= 10 \times 256 + 3 \times 16 + 11 \times 1 \\
&= 2560 + 48 + 11 \\
&= (2619)_{10}
\end{aligned}
$$

Exercise : Convert 2F3H into decimal number

Ans. : $(755)_{10}$

Binary-Hexadecimal Conversion

- Group bits in fours, starting on right for integer part
- From pt. group bits in fours from left to right
- Convert to hexadecimal digits

Example : $(1010111011.111011)_2 = (?)_{16}$

0010 1011 1011   1110 1100

2   B   B   E   C

$$(1010111011.111011)_2 = (2BB.EC)_{16}$$

Hexadecimal to Binary

– Convert each hexadecimal digit to a 4-bit equivalent binary representation

$$(\mathbf{10AF})_{16} = (?)_2$$

1 0 A F

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

0001 0000 1010 1111

$$(\mathbf{10AF})_{16} = (\mathbf{1000010101111})_2$$

Hexadecimal-Octal Conversion

(i) Convert the given hexadecimal number to its binary equivalent.

(ii) Form groups of 3 bits, starting from the LSB (least significant digit).

(iii) Write the equivalent octal number for each group of 3 bits.

$(1F0C)_{16} = (?)_8$

1	F	0	C
0001	1111	0000	1100

1	7	4	1	4
001	111	100	001	100

$(1F0C)_{16} = (17414)_8$

Octal to Hexadecimal Conversion

Use binary as an intermediary

$(1076)_8 = (?)_{16}$

1	0	7	6
001	000	111	110

2 3 E

$(1076)_8 = (23E)_{16}$

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Exercise – Convert ...

Decimal	Binary	Octal	Hexa- decimal
29.8			
	101.1101		
		3.07	
			C.82

Exercise – Convert …

Answer

Decimal	Binary	Octal	Hexa-decimal
29.8	11101.110011…	35.63…	1D.CC…
5.8125	101.1101	5.64	5.D
3.109375	11.000111	3.07	3.1C
12.5078125	1100.10000010	14.404	C.82