**Department of Mathematics**

**15B11MA211** **Mathematics-II**

**Tutorial Sheet 4** **B.Tech. Core**

**Fourier Series**

1. Expand the function $f(x) = x\sin x$ in a Fourier series in the interval $-\pi \le x < \pi$. Use the series obtained to show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} = ........ = \frac{\pi - 2}{4}$.

2. Given $f(x) = \begin{cases} -x+1 \; for -\pi < x \le 0 \\ x+1, \; for \, 0 \le x \le \pi \end{cases}$, find a Fourier series for $f(x)$ and hence show that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + .......... = \frac{\pi^2}{8}$$

3. Find the Fourier series expansion of the function $f(x) = \begin{cases} \pi x \; for \, 0 \le x < 1 \\ 0 \quad x = 1 \\ \pi(x-2) \; for \; 1 < x < 2. \end{cases}$ in the interval [0,2].

4. Find the Fourier series expansion of the function $f(x)=e^{-4x}$ in the interval [-2,2].

5. Find the Fourier series expansion of the function $f(x) = x - x^2$ in the interval $-1 < x \le 1$.

6. Find the half range sine series for the function $f(x) = x^2$ for $0 < x < \pi$.

7. Find the half range cosine series for the function $f(x) = 2x - 1$ for $0 < x < 1$.

8. Find the Fourier series expansion of the function $f(x) = \begin{cases} 0 \text{ for } 0 \le x < l \\ x \text{ for } l \le x < 2l \end{cases}$ in the interval $[0, 2l]$.

Answers. (3) $f(x) = 2(\sin \pi x - \frac{\sin 2\pi x}{2} + \frac{\sin 3\pi x}{3} - ...)$,

(4) $a_0=(e^8-e^{-8})/8$, $a_n=(e^8-e^{-8})\, 8\, (-1)^n/(64+pi^2\, n^2)$, $b_n=(e^8-e^{-8})\, n^*\, pi\, (-1)^n/(64+pi^2\, n^2)$

(5) $f(x) = -\frac{1}{3} + \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi x}{(2n+1)^2} + \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{\sin(2n+1)\pi x}{(2n+1)}$,

(6) $f(x) = \frac{2}{\pi}\left\{(\pi^2 - 4)\sin x - \frac{\pi^2 \sin 2x}{2} + \frac{1}{3}(\pi^2 - \frac{4}{3^2})\sin 3x - ......\right\}$

(7) $f(x) = -\frac{8}{\pi^2}\left(\cos \pi x + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + ....\right)$

(8) $f(x) = \frac{3l}{4} + \frac{l}{\pi^2}\sum_{n=0}^{\infty}\frac{\cos(2n+1)\pi x / l}{(2n+1)^2}$

$$-\frac{l}{\pi}\left\{\frac{3\sin \pi x / l}{1} + \frac{\sin 2\pi x / l}{2} + \frac{3\sin 3\pi x / l}{3} + \frac{\sin 4\pi x / l}{4} + ....\right\}$$