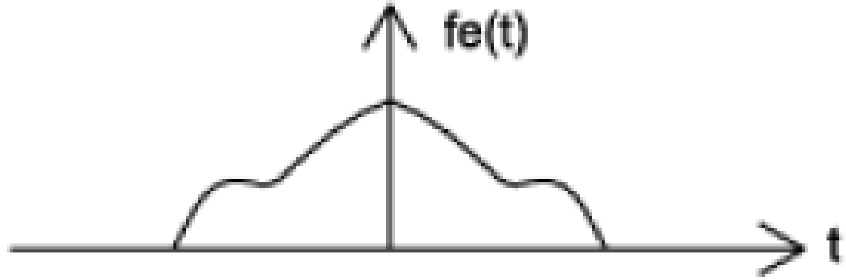
Signals And Systems

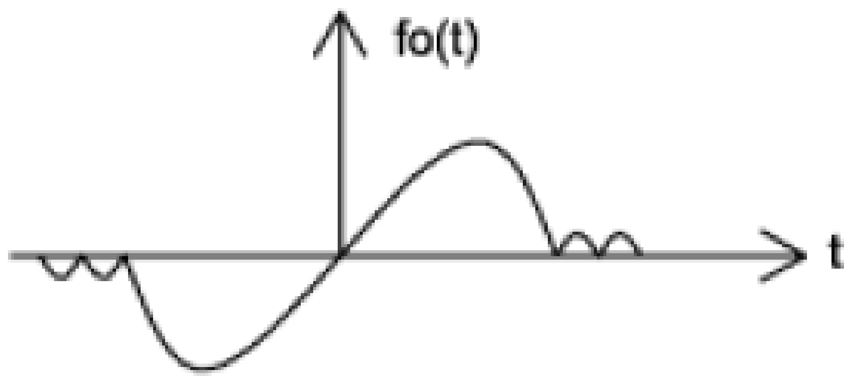
Even and Odd Signals

- An even signal is any signal x such that $x(t) = x(-t)$ or $x[n] = x[-n]$. Even signals can be easily spotted as they are symmetric about the vertical axis.

- An odd signal, on the other hand, is a signal x such that $x(t) = -(x(-t))$ or $x[n]=-x[-n]$.

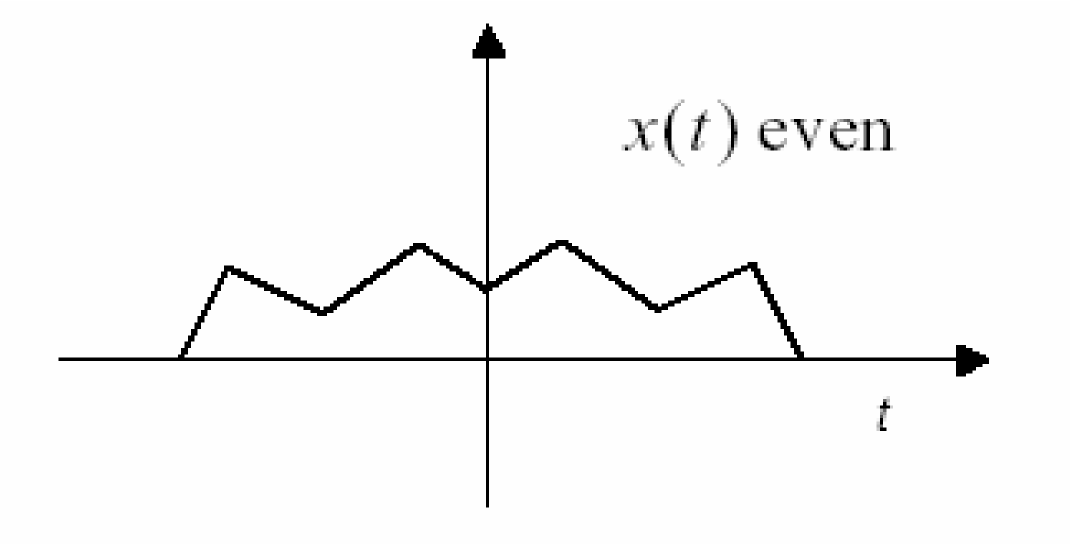


(a)

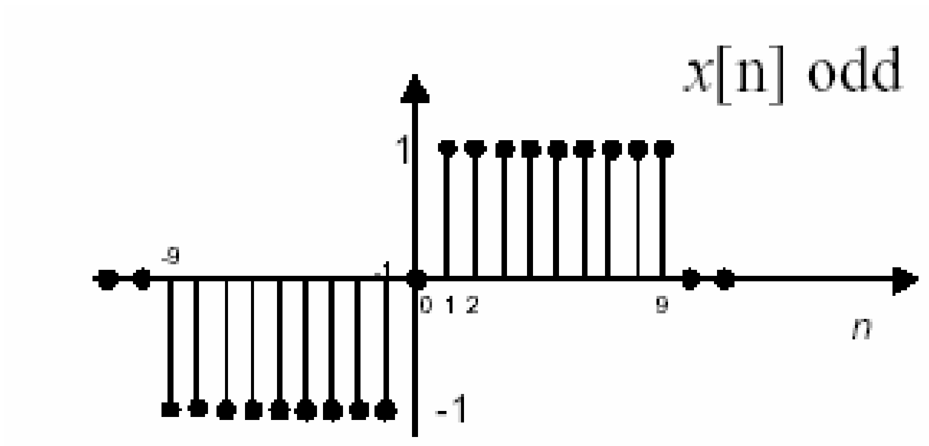


(b)

: (a) An even signal (b) An odd signal



$x(t)$ even
t



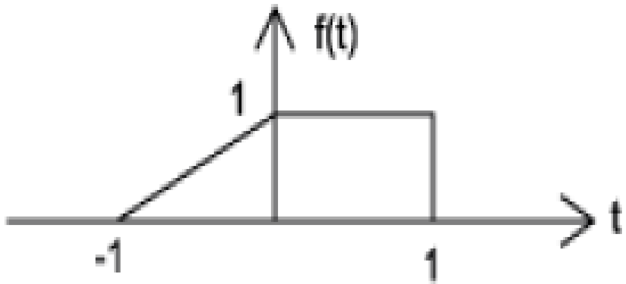
$x[n]$ odd
1
-9
-1
0 1 2
9
n
- 1

- Any signal can be written as a combination of an even and odd signal. That is, any signal can be decomposed into its even part and its odd part as follows:
- x(t) =1/2[x(t) + x(−t)] +1/2[x(t) − x(−t)]

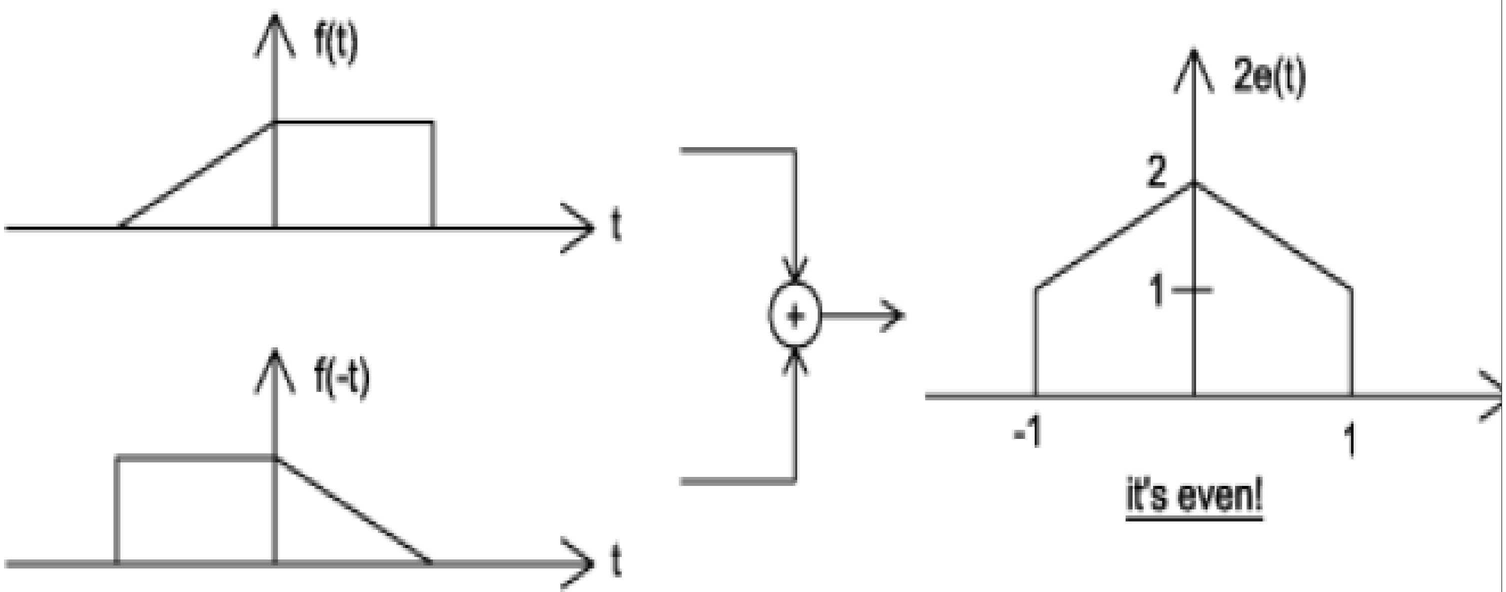
$$x(t) = x_e(t) + x_o(t)$$

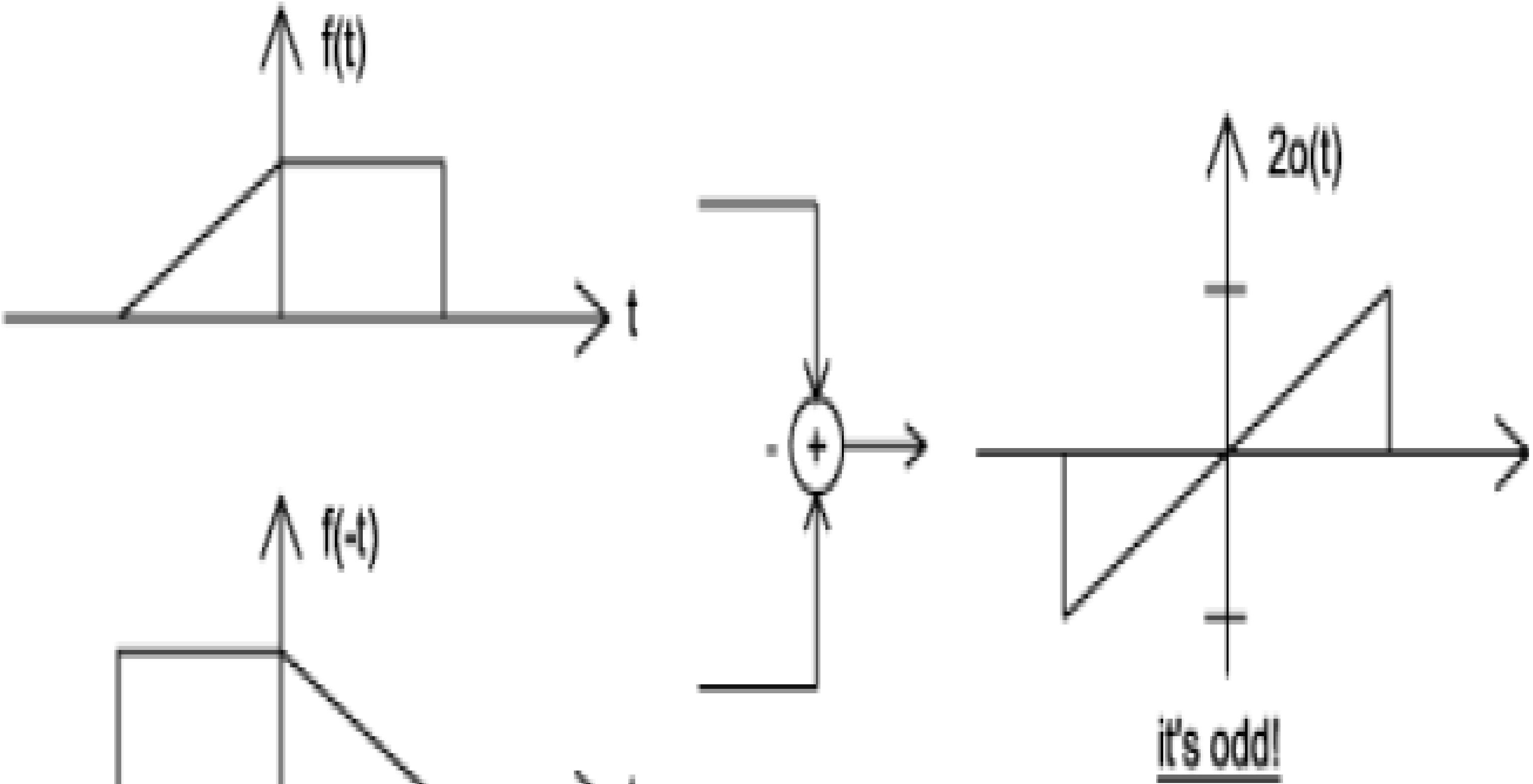
Even part: $$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

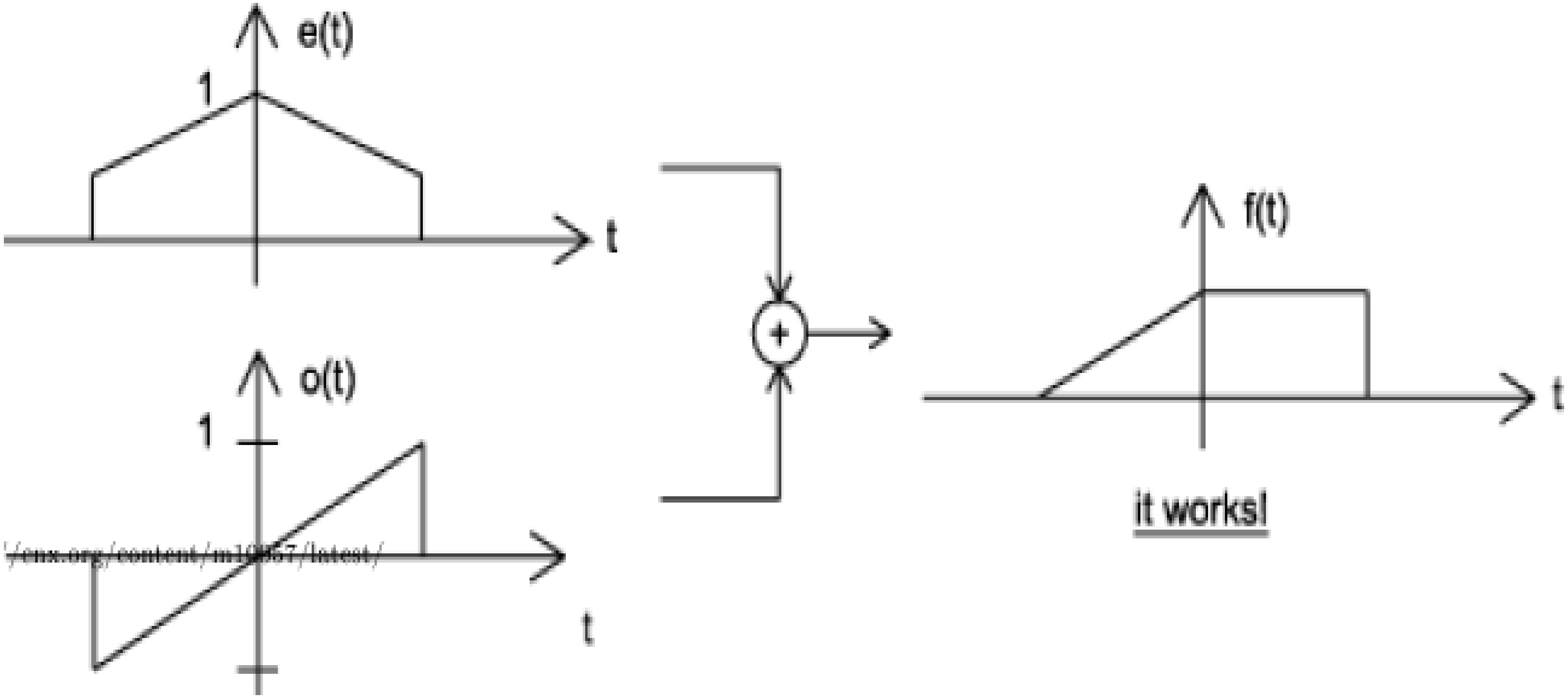
Odd part: $$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$



(a)




f(t)
t
f(-t)
t
-
+
2o(t)
it's odd!


e(t)
1
t
o(t)
1
t
+
f(t)
t
it works!

Energy and Power signals

Consider v(t) to be the voltage across a resistor R producing a current i(t).

The power dissipated in a resistor is

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R}$$

and the *total energy* dissipated during a time interval $[t_1, t_2]$ is

$$E = \int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{v^2(t)}{R} dt$$

The *average power* dissipated over that interval is just

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{v^2(t)}{R} dt$$

Analogously, the total energy and average power over $[t_1, t_2]$ or $[n_1, n_2]$ of an arbitrary signal are defined as follows.

$$E := \int_{t_1}^{t_2} |x(t)|^2 \, dt \qquad E := \sum_{n=n_1}^{n_2} |x[n]|^2$$

$$P := \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt \qquad P := \frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} |x[n]|^2$$

The total energy and average power of a signal defined over -∞ < t, n < ∞ are defined as:

$$E_{\infty} := \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$E_{\infty} := \lim_{N \to \infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P_{\infty} := \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$P_{\infty} := \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Class of <u>Finite-Energy Signals</u>: signals for which $E_\infty < \infty$.

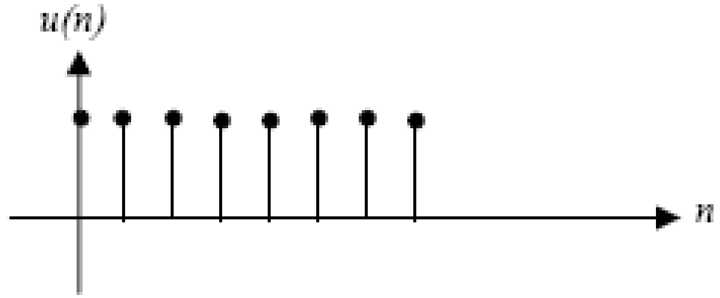
Class of <u>Finite-Power Signals</u>: signals for which $P_\infty < \infty$.

Example: $x[n] := \begin{cases} 1, 0 \le n \le 10 \\ 0, \text{otherwise} \end{cases}$ $E_\infty = 11$

$x(t) = 4$ infinite energy

$$P_\infty := \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 4^2\,dt = \lim_{T\to\infty}\frac{4^2}{2T}2T = 16$$

Example – Unit Step Sequence



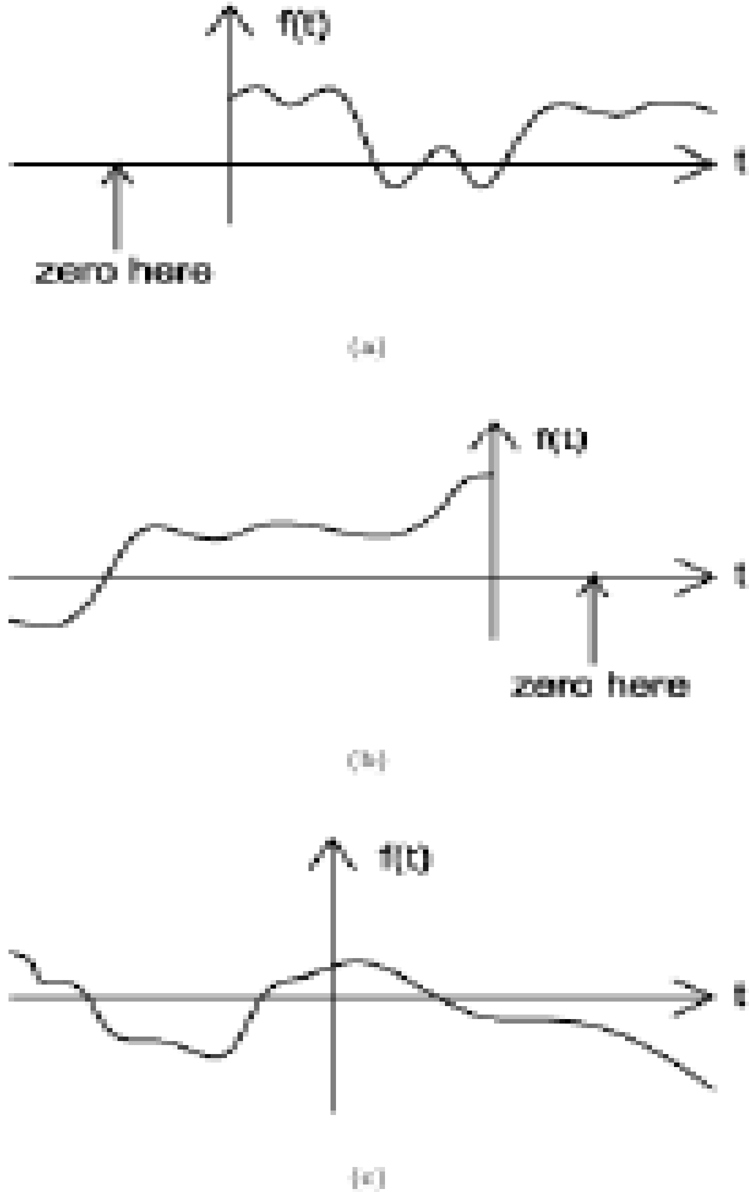
Obviously, it is not an energy signal but it is a power signal.

$$p = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{-N}^{N} \left| x(n) \right|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (1)^2$$

$$= \lim_{N\to\infty} \frac{N+1}{2N+1} = \frac{1}{2} \Rightarrow$$ it is a power signal!

Causal/ anti-causal/non-causal signals:

- **Causal signals** are signals that are zero for all negative time.
- **Anticausal signals** are signals that are zero for all positive time.
- **Noncausal signals** are signals that have nonzero values in both positive and negative time.



(a) A causal signal (b) An anticausal signal (c) A noncausal signal

Deterministic vs. Random Signals:

- **A deterministic signal** is a signal in which each value of the signal is fixed and can be determined by a mathematical expression, rule, or table. Because of this the future values of the signal can be calculated from past values with complete confidence.
- **A random signal** has a lot of uncertainty about its behavior. The future values ofa random signal cannot be accurately predicted and can usually only be guessed based on the averages of sets of signals