Q.1 :- (i) Maxwell's equations, Integral form, Differential form

(a) Gauss's law in electrostatic, $\oint_S \bar{E}.d\bar{S} = \frac{Q_{enc}}{\varepsilon_0}$, $\bar{\nabla}.\bar{E} = \frac{\rho}{\varepsilon_0}$

(b) Gauss's law in magnetostatic, $\oint \bar{B}.d\bar{S} = 0$, $\bar{\nabla}.\bar{B} = 0$

(c) Faraday's law, $\oint_P \bar{E}.d\bar{l} = -\frac{\partial \phi_B}{\partial t}$, $\bar{\nabla}\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$

(d) Modified Ampere's law, $\oint_P \bar{B}.d\bar{l} = \mu_0\left(i + \varepsilon_0 \frac{\partial \phi_E}{\partial t}\right)$, $\bar{\nabla}\times\bar{B} = \mu_0\bar{J} + \mu_0\varepsilon_0\frac{\partial \bar{E}}{\partial t}$

(ii) Boundary conditions for electric and magnetic fields,

a) Normal component of electric displacement $\bar{D}$ is discontinuous across the interface, i.e, $D_{1n} - D_{2n} = \sigma$, where σ = surface charge density.

b) Normal component of magnetic induction $\bar{B}$ is continuous across boundary, i.e; $B_{1n} = B_{2n}$ or $\mu_1 H_{1n} = \mu_2 H_{2n}$

c) Tangential component of $\bar{E}$ is continuous across the interface i.e.

$$E_{1t} = E_{2t}$$

d) Tangential component of magnetic field $\bar{H}$ is discontinuous i.e

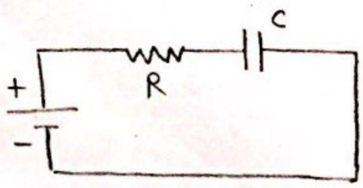
$$H_{1t} - H_{2t} = j \text{ , where } j \text{ is surface current density}$$

Q.2 :- Here, $\bar{B} = \frac{r}{2}\hat{\phi}$

Magnetic flux, $\phi_B = \oint \bar{B}.d\bar{S} = \oint B_\phi\, dS_\phi = \int_{r=1}^{2}\int_{z=0}^{5}\left(\frac{r}{2}\right)(dr\,dz)$

$$\Rightarrow \phi_B = \left(\frac{r^2}{4}\Big|_{r=1}^{2}\right)\left(z\Big|_{z=0}^{5}\right) = \left(1 - \frac{1}{4}\right)(5) = \frac{15}{4}\ \text{Wb}$$

Q.3 :- For instance charge on plates of a capacitor is Q, area of plate is A and distance between plates is d.



So, $V = \frac{Q}{C}$ and $C = \frac{\varepsilon_0 A}{d}$ $\Rightarrow$ $V = \frac{Qd}{\varepsilon_0 A}$

Since, $E = \frac{V}{d} = \frac{Q\cancel{d}}{\varepsilon_0 A\cancel{d}} = \frac{Q}{\varepsilon_0 A} \Rightarrow EA = \frac{Q}{\varepsilon_0} \Rightarrow \phi_E = \frac{Q}{\varepsilon_0}$

and Using $I = \frac{dQ}{dt} \Rightarrow \frac{d\phi_E}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt} \Rightarrow I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

This current I_d (missing in Ampere's law) passes through surface A of capacitor, and is known as Maxwell's displacement current. So Ampere's law is modified by non-steady current.

$$\oint_P \bar{B}.d\bar{l} = \mu_0 (I_{enc} + I_d) = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Here we have, $\frac{V}{d} = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \frac{\varepsilon_0}{d} V A$

$$\Rightarrow I_d = \frac{dQ}{dt} = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} \Rightarrow I_d = 2\varepsilon_0 \times \frac{5\times10^{-4}}{3\times10^{-3}} \frac{d}{dt}(50 \sin 10^3 t)$$

Using $\varepsilon_0 = 8.854\times10^{-12} = \frac{10^{-9}}{36\pi}$, $I_d = 147.4 \cos 10^3 t$ nA

Q.4 :-

$$\bar{S} = \bar{E}\times\bar{H} = EH\sin 90^\circ = EH$$

Given solar energy $= 2$ Cal min^{-1} cm^{-2} $= \frac{2}{60}\times 4.18\times10^4$ $Jm^{-2}sec^{-1} = 1400 Jm^{-2}sec^{-1}$

S = energy flux per unit area per sec.

$\Rightarrow EH = 1400\ Jm^{-2}sec^{-1}$

Since $\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377$ in free space

$\Rightarrow EH \times \frac{E}{H} = 1400 \times 377 \Rightarrow E = 726.5$ V/m and $H = \frac{E}{377} = 1.927 \frac{A}{m}$

Amplitude of electric and magnetic fields are

$$E_0 = E\sqrt{2} = 102.3 \text{ V/m}$$

$$H_0 = H\sqrt{2} = 2.717 \text{ A/m}$$

Q5: $\omega = 2\pi \times 10^7 = 2\pi \nu \Rightarrow \nu = 10^7\ Hz$

$c = 3 \times 10^8\ m/s \Rightarrow \lambda = \frac{c}{\nu} = 30\ m$

and $k = \frac{2\pi}{\lambda} = 0.20944\ rad/m$

Here, $\bar{E} = (10\hat{z} + 20\hat{y}) \cos(2\pi \times 10^7 t - kx)$

Since, $\bar{\nabla} \times \bar{E} = -\frac{\partial(\mu_0 \bar{H})}{\partial t}$

$$\Rightarrow \bar{\nabla} \times \bar{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & E_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{y}\left(\frac{\partial E_z}{\partial x} - 0\right) + \hat{z}\left(\frac{\partial E_y}{\partial x}\right)$$

$$= \hat{x}(0 - 0) - \hat{y}\frac{\partial}{\partial x}\left[10\cos(2\pi \times 10^7 t - kx)\right] + \hat{z}\frac{\partial}{\partial x}\left[20\cos(2\pi \times 10^7 t - kx)\right]$$

$$\Rightarrow -\frac{\partial(\mu_0 \bar{H})}{\partial t} = -\hat{y}(+k)\left[10\sin(2\pi \times 10^7 t - kx)\right] + \hat{z}(+k)\left[20\sin(2\pi \times 10^7 t - kx)\right]$$

Integrating w.r.t. 't',

$$-\mu_0 \bar{H} = +\frac{k\hat{y}}{2\pi \times 10^7}\left[10\cos(2\pi \times 10^7 t - kx)\right] - \frac{k\hat{z}}{2\pi \times 10^7}\left[20\cos(2\pi \times 10^7 t - kx)\right]$$

$$\Rightarrow \bar{H} = \frac{(\omega/c)}{\mu_0 \times 2\pi \times 10^7}(-10\hat{y} + 20\hat{z})\cos(2\pi \times 10^7 t - kx)$$

Since $\omega = 2\pi \times 10^7$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

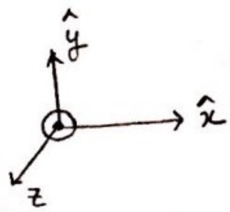
$$\Rightarrow \bar{H} = \sqrt{\frac{\varepsilon_0}{\mu_0}}\,(-10\hat{y} + 20\hat{z})\cos(2\pi \times 10^7 t - kx)$$

$$\Rightarrow \bar{H} = \frac{1}{377}(-10\hat{y} + 20\hat{z})\cos(2\pi \times 10^7 t - kx)$$

where, impedence, $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\ \Omega$

Q.6:- $\bar{E} = E_0 \hat{y} \cos(\omega t - kx)$

$E_0 = 8000\ V/m$, $\omega = 10^{10}\ rad/s$, $\nu = \frac{\omega}{2\pi} = \frac{10^{10}}{2\pi}\ sec^{-1}$



i) $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8 \times 2\pi}{10^{10}} = 18.8 \times 10^{-2}\ m$

ii) $\nu = \frac{\omega}{2\pi} = 1591.3\ MHz$

iii) $T = \frac{1}{\nu} = 0.63\ ns$

iv) $E_0 = cB_0$ and $H_0 = \frac{E_0}{\mu_0 c} = \frac{8000}{4\pi \times 10^{-7} \times 3 \times 10^8} = 21.22\ A/m$ and $\bar{H} = H_0 \hat{z} \cos(\omega t - kx)$.

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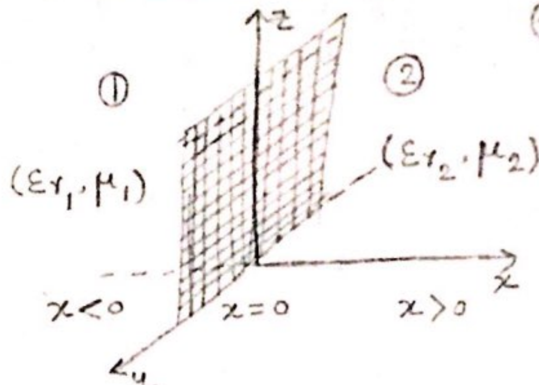
Q.7: $\bar{E}_1 = 4\hat{i} + 3\hat{j} + 5\hat{k}$, $x<0$

$E_{1t} = 3\hat{j} + 5\hat{k}$ (yz-plane)

$E_{1n} = 4\hat{i}$



Using boundary conditions,

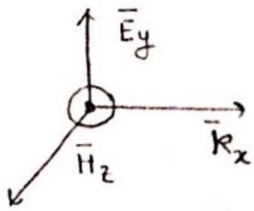
$$E_{1t} = E_{2t} \quad \text{and} \quad D_{1n} = D_{2n} \Rightarrow \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$$

$$\Rightarrow E_{2t} = 3\hat{j} + 5\hat{k} \quad \text{and} \quad E_{2n} = \frac{\varepsilon_2}{\varepsilon_1} E_{1n} = \frac{\cancel{\varepsilon_0}\varepsilon r_2}{\cancel{\varepsilon_0}\varepsilon r_1} E_{2n} = \frac{\varepsilon r_2}{\varepsilon r_1} E_{2n} = \frac{3}{\cancel{5}} \times \cancel{5}\hat{i}$$

$$\Rightarrow \bar{E}_2 = 3\hat{i} + 3\hat{j} + 5\hat{k} \text{ V/m}$$

Q.8: $E_0 = 0.05$ V/m , $\nu = 6$ MHz and $c = 3\times10^8$ m/s

$$\lambda = \frac{c}{\nu} = \frac{3\times10^8}{6\times10^6} = 50 \text{ m} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 0.1256 \text{ m}^{-1}$$



$$\omega = 2\pi\nu = 3.76\times10^7 \text{ rad/s}$$

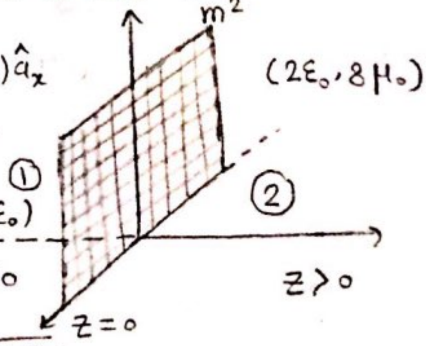
$$H_0 = \frac{E_0}{\mu_0 c} = 1.33\times10^{-4} \text{ A/m} \quad \text{and} \quad B_0 = \mu_0 H_0 = 1.67\times10^{-10} \text{ wb/m}^2$$

$$\Rightarrow \bar{E} = E_0\hat{y}\cos(kx-\omega t)\ \frac{\text{V}}{\text{m}} , \quad \bar{H} = H_0\hat{z}\cos(kx-\omega t) \text{ A/m}$$

$$\bar{B} = B_0\hat{z}\cos(kx-\omega t)\ \frac{\text{wb}}{\text{m}^2}$$

and $\bar{P} = \bar{E}\times\bar{H} \Rightarrow P_x = E_0 H_0 \langle \cos^2(kx-\omega t)\rangle = \frac{E_0 H_0}{2} = 3.325\ \frac{\mu\omega}{\text{m}^2}$

Q.9: $\bar{H}_i = 10\cos(10^8 t - k_1 z)\hat{a}_x$ mA/m $= 10\cos(\omega t - k_1 z)\hat{a}_x$



In region ①, $k_1 = \frac{\omega}{c} = \frac{10^8}{3\times10^8} = \frac{1}{3}$, $\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi$

$$k_2 = \frac{\omega}{v_2} = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$$

$$\Rightarrow k_2 = \frac{\omega}{c}\sqrt{2\times8} = 4\frac{\omega}{c} = 4k_1 = \frac{4}{3} , \quad \eta_2 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = 2\eta_0$$

Using. $\bar{\nabla}\times\bar{E}_i = -\frac{\partial(\mu_0\bar{H}_i)}{\partial t} = 10\mu_0\omega\sin(\omega t - k_1 z)\hat{a}_x$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \cancel{\frac{\partial}{\partial x}} & \cancel{\frac{\partial}{\partial y}} & \frac{\partial}{\partial z} \\ E_{ix} & E_{iy} & E_{iz} \end{vmatrix} = \hat{a}_x\left(-\frac{\partial E_{iy}}{\partial z}\right) = 10\mu_0\omega\sin(\omega t - k_1 z)\hat{a}_x$$

$$\Rightarrow E_{iy} = \frac{10\mu_0\omega\cos(\omega t - k_1 z)}{(-k_1)} \Rightarrow \bar{E}_i = -\frac{10\mu_0}{c}\cos(\omega t - k_1 z)\hat{a}_y$$

$$\Rightarrow \bar{E}_i = -10\eta_0\cos(10^8 t - k_1 z)\hat{a}_y \text{ mA/m}$$

Now $$\frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\eta_0 - \eta_0}{2\eta_0 + \eta_0} = \frac{1}{3}$$

$$\Rightarrow E_{r0} = \frac{1}{3} E_{i0}$$

Thus $$\bar{E}_r = -\frac{10}{3}\eta_0 \cos\left(10^8 t + \frac{1}{3} z\right) \hat{a}_y \ \frac{mV}{m}$$

From which we can calculate, $\bar{H}_r = -\frac{10}{3} \cos\left(10^8 t + \frac{1}{3} \hat{z}\right) a_x \ \frac{mA}{m}$

Similarly. $$\frac{E_{t0}}{E_{i0}} = 1 + \frac{E_{r0}}{E_{i0}} = 1 + \frac{1}{3} = \frac{4}{3}$$

Thus, $$\bar{E}_t = -E_{t0} \cos\left(10^8 t - \beta_2 z\right) \hat{a}_y \ mA/m$$

or $$\bar{E}_t = -\frac{40}{3}\eta_0 \cos\left(10^8 t - \frac{4}{3} z\right) \hat{a}_y \ mA/m$$

and from which we can obtain,

$$\bar{H}_t = \frac{20}{3} \cos\left(10^8 t - \frac{4}{3} z\right) a_x \ mA/m$$

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