

For this,  $Z = R + j(\omega L - \frac{1}{\omega C})$

Impedance  $\rightarrow |Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

At resonance  $\rightarrow \boxed{\omega L = \frac{1}{\omega C}}$

$\boxed{|Z|_{\min} = R}$

$\rightarrow$  Minimum ~~resistance~~ impedance is formed at resonance

$\rightarrow$  From the circuit  $I = \frac{V}{Z}$

$\rightarrow$  At resonance  $I = \frac{|V|}{|Z|_{\min}}$

Note  $\odot$  if  $|Z|_{\min}$  is minimum value then  $I$  would be maximum.

Therefore

$\boxed{I_{\max} = \frac{|V|}{|Z|_{\min}}}$

$\rightarrow$  From the above expression at resonance condition in series RLC circuit  $\rightarrow I \rightarrow \text{maximum}$

$\rightarrow |Z| \rightarrow \text{minimum}$

$\boxed{I_{\max} = \frac{|V|}{|Z|_{\min}} = \frac{|V|}{R}}$

① At resonance -  
across R

$$V = IZ$$

$$\boxed{|V| = I_{\max} R}$$

② At resonance -  
across L

$$V_L = I_{\max} X_L = I_{\max} \omega L \cdot j$$

$$|V_L| = |I_{\max} \omega L \cdot j| = I_{\max} \omega L$$

$$\boxed{V_L = I_{\max} \omega L} \quad \{ |j| = 1 \}$$

③ At resonance -  
across C

$$V_C = I_{\max} X_C = I_{\max} \frac{1}{j\omega C}$$

$$|V_C| = |I_{\max} \frac{1}{j\omega C}|$$

$$\boxed{|V_C| = \frac{I_{\max}}{\omega C}}$$

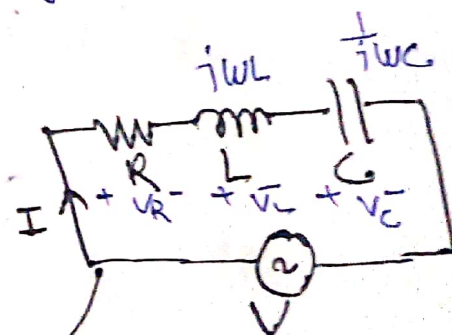
$$\boxed{\omega_0 \rightarrow \text{Resonance frequency} = \frac{1}{\sqrt{LC}}}$$

Quality factor or magnification factor:

Quality factor is denoted by Q while magnification factor is denoted by M.

Quality factor (Q):  $\frac{|V_L|}{V} = \frac{|V_C|}{V}$

$$\boxed{\begin{aligned} V_R &= IR \\ V_L &= I \cdot j\omega L \\ V_C &= \frac{I \cdot 1}{j\omega C} \end{aligned}}$$





At resonance - current is  $I_{\max}$  and impedance is  $Z_{\min}$ .

$$\begin{aligned} V_R &= I_{\max} \cdot R \\ V_L &= I_{\max} \cdot j\omega_0 L \\ V_C &= \frac{I_{\max}}{j\omega_0 C} \end{aligned}$$

$\omega_0 \rightarrow$  resonance frequency

Quality factor at resonance condition -

① Q for inductor :  $Q_L = \left| \frac{V_L}{V_R} \right|$

$$Q_L = \left| \frac{I_{\max} \cdot j\omega_0 L}{I_{\max} \cdot R} \right| = \frac{\omega_0 L}{R}$$

$$\because |j| = 1$$

$$Q_L = \frac{\omega_0 L}{R}$$

We know that  $\omega_0 = \frac{1}{\sqrt{LC}}$  at resonance condition

Put the value of  $\omega_0$  in above expression -

$$Q_L = \frac{\omega_0 L}{R} = \frac{1}{\sqrt{LC}} \cdot \frac{L}{R} = \sqrt{\frac{L}{C}} \cdot \frac{1}{R}$$

$$Q_L = \frac{1}{R} \sqrt{\frac{L}{C}}$$

① Quality factor for capacitor:  $Q_c = \left| \frac{V_c}{V_R} \right|$

$$Q_c = \left| \frac{\cancel{I_{\max}} \cdot 1}{\cancel{I_{\max}} \cdot R \cdot i\omega C} \right| = \frac{1}{\omega C R} \quad \left[ \because \left| \frac{1}{i} \right| = 1 \right]$$

$$Q_c = \frac{1}{\omega C R}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_c = \frac{\sqrt{LC}}{1} \cdot \frac{1}{CR} = \sqrt{\frac{CL}{C^2}} \cdot \frac{1}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q_c = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Therefore,

$$Q = Q_c = Q_L = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Note: Series resonance circuit is also called as magnification.  
at resonance  $\rightarrow |V_L| = |V_c|$  which is cancel  
to each other only  $|V| = V_R$  exists

But  $|V_L| > |V|$   
 $|V_c| > |V|$

or

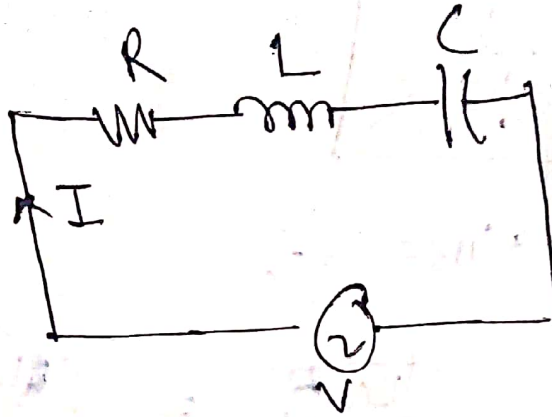
$$|V_L| = |V_c| > |V|$$

$V_L$  and  $V_c$  are magnified  
w.r. to  $V$

$$M = \frac{V_L}{V} \text{ or } \frac{V_c}{V}$$



Bandwidth: Bandwidth represent the range of the frequency for which the power level in the signal is at least half of the maximum power.



We know that

$$P = V \cdot I$$

$$= \frac{V^2}{R}$$

$$= I^2 R$$

At resonance condition -  $|Z|_{\min} = R$  and  $I_{\max}$

Therefore

$\text{power} = I_{\max}^2 \cdot R$

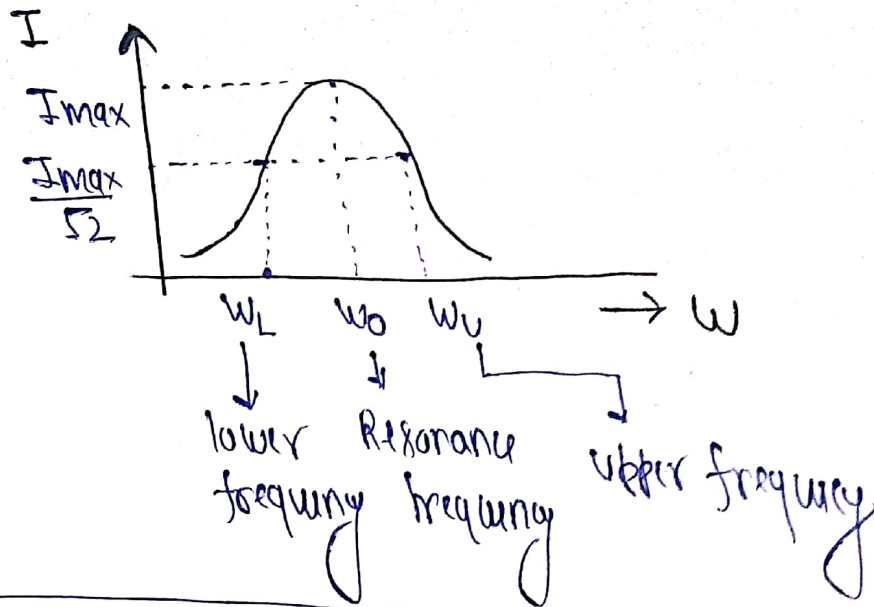
the power does not exist across the L and C due to cancel each other at resonance condition

Half maximum power is defined as -

$P = \frac{P_{\max}}{2}$

$$I^2 R = \frac{I_{\max}^2 R}{2}$$

$$I = \frac{I_{\max}}{\sqrt{2}}$$



$$\text{Band width (BW)} = \omega_U - \omega_L$$

Relation between quality factor and Bandwidth :

$$B.W = \frac{\omega_0}{Q} \quad \text{or} \quad Q = \frac{\omega_0}{B.W}$$

$$\omega_0 \rightarrow \text{resonance frequency} = \frac{1}{\sqrt{LC}}$$

$$L \rightarrow B.W. = \frac{\omega_0}{\frac{\omega_0 L}{R}} = \frac{R}{L} \rightarrow \text{for inductor}$$

$$C \rightarrow B.W = \frac{\omega_0}{\frac{1}{\omega_0 C R}} = \omega_0^2 \cdot C R = \frac{1}{L C} \cdot C R = \frac{R}{L}$$

↓  
for capacitor