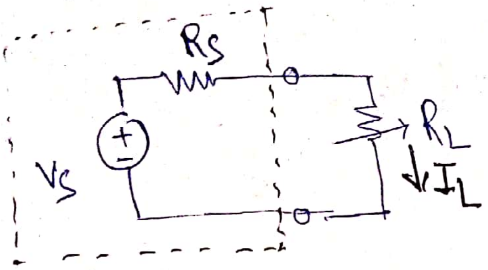
Maximum Power Transfer Theorem in DC Network

Statement: maximum power transfer theorem states that "maximum power is transferred from the source to the load when load resistance is equal to the thevenin's equivalent resistance". i.e. $R_L = R_{th}$



Practical voltage Source

$V_S \rightarrow$ Source voltage

$R_S \rightarrow$ Internal source resistance of voltage source

$R_L \rightarrow$ Load resistance.

From the above figure –

$$\boxed{I_L = \frac{V_S}{R_S + R_L}} \text{ — (i)}$$

$$P = V \cdot I$$

$$P = \frac{V^2}{R}$$

$$P = I^2 R$$

Now, power transferred to load resistance

$$\boxed{P_L = I_L^2 \cdot R_L} \text{ — (ii)}$$

Put the value of I_L in the equation (ii)

$$\boxed{P_L = \left(\frac{V_S}{R_S + R_L}\right)^2 \cdot R_L} \text{ — (iii)}$$

For max^m power across the load, apply max^m and min^m –

To get maximum power transfer, differentiate equation (iii) w.r. to R_L, here R_L is variable

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{V_S^2 R_L}{(R_S + R_L)^2}\right]$$

For maximum power — $\boxed{\dfrac{dP_L}{dR_L} = 0}$

i.e. $\dfrac{dP_L}{dR_L} = \dfrac{d}{dR_L}\left(\dfrac{V_s^2 R_L}{(R_s+R_L)^2}\right) = 0$ { Here R_s and V_s are fixed quantity

$$= V_s^2\left[\frac{(R_s+R_L)^2 \times 1 - R_L \times 2(R_s+R_L)}{(R_s+R_L)^4}\right] = 0$$

$V_s \neq 0$; $(R_s+R_L)[R_s+R_L-2R_L] = 0$

$\boxed{R_s = R_L}$ V.V.

i.e. load resistance (R_L) = Internal resistance of source voltage (R_s)

↓

This is a condition of maximum Power transfer.

What is the value of maximum power —

Sustitute the value of $R_s = R_L$ in the equation (iii)

we get

$$P_L = \frac{V_s^2 \cdot R_L}{(R_L+R_L)^2} = \frac{V_s^2 \cdot R_L}{(2R_L)^2} = \frac{V_s^2}{4R_L} = \frac{V_s^2}{4R_s}$$

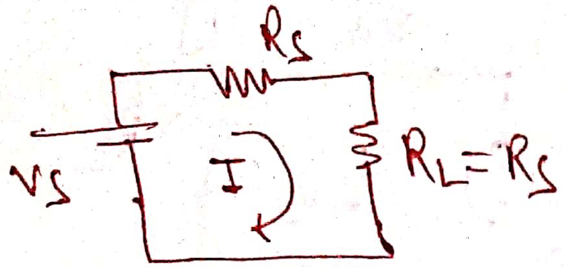
$\boxed{P_{max} = \dfrac{V_s^2}{4R_L} \text{ watt}}$ V.V.

~~Note~~ and maximum current $I_{Lmax} = \dfrac{V_s}{R_s+R_s} = \dfrac{V_s}{2R_s}$ Amp

[$\because R_s = R_L$]

Max^m Power transfer theorem is applicable when load is variable

$$\text{Efficiency } (\eta) = \frac{P_{out}}{P_{in}} \times 100\%$$



$$I = \frac{V_S}{2R_S}$$

$$P_{out} = \frac{V_S^2}{4R_S}$$ → output power

What is the power given by the battery –

$$P_{in} = -V_S \times I = -\frac{V_S \times V_S}{2R_S}$$

$$\boxed{P_{in} = -\frac{V_S^2}{2R_S}}$$

minus sign means power given by the battery

- → Power given
+ → Power absorb

Therefore,

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% = \frac{\frac{\cancel{V_S^2}}{4\cancel{R_S}}}{\frac{\cancel{V_S^2}}{2\cancel{R_S}}} \times 100\%$$

$$\eta = \frac{2}{4} \times 100\%$$

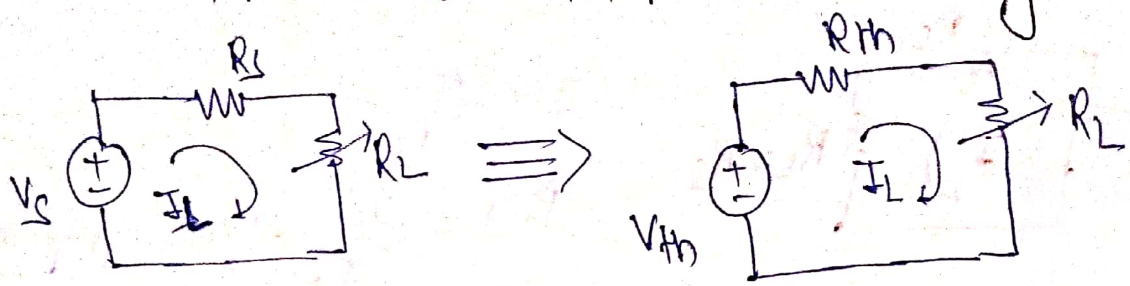
$$\boxed{\eta = 50\%}$$

Note

For max^n power across the load, the $\eta = 50\%$.

remain power loss across the R_S

Maximum Power transfer based problem solve using thevenin's theorem



$$P_{max} = \frac{V_s^2}{4R_s} = \frac{V_s^2}{4R_L}$$

and

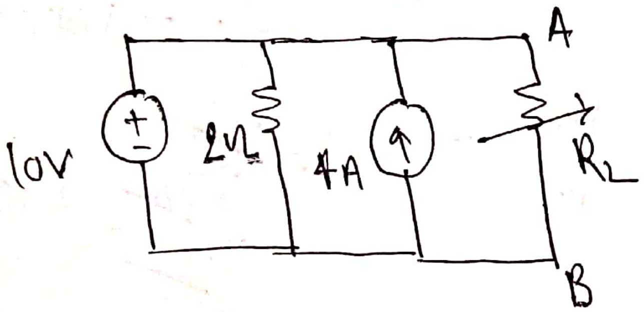
$$R_s = R_L$$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

and

$$R_L = R_{th}$$

Prob: 1 Find the value of R_L and P_{max} of the given circuit.



Sol^n: Apply the thevenin's theorem - Remove load resistance (R_L)

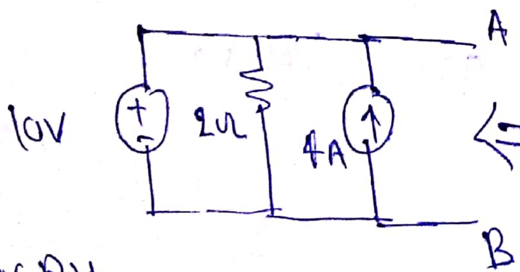
For V_{th} → open terminal voltage

R_{th} → Resistance across open terminal

→ Voltage source → S.C

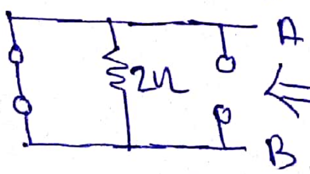
→ Current source → O.C

For V_{th}:-

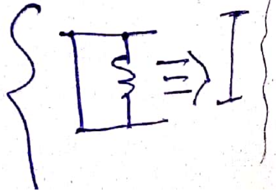


⟸ $V_{AB} = V_{th} = 10$ Volt

For R_{th}

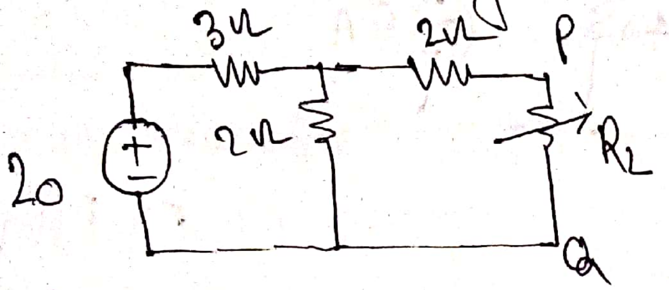


⟸ $R_{AB} = R_{th} = 0$



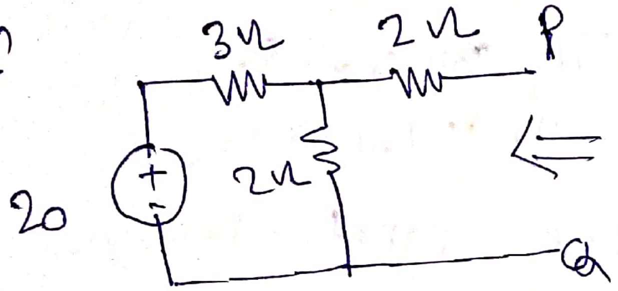
For $P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{(10)^2}{4\times 0} = \infty$ watt.

Prob: 2 Calculate the value of R_L and P_{max} of the following circuits.



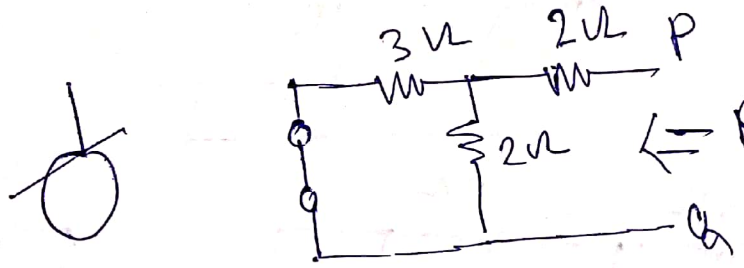
Sol^n: Apply the thevenin's theorem to find the value of R_{th} and V_{th}.

For V_{th}



$\Leftarrow V_{PQ} = V_{th} = \frac{20\times 2}{5} = 8$ volt

for R_{th}



$\Leftarrow R_{th} = 2 + 3||2$

$= 2 + \frac{3\times 2}{5}$

$= 2 + 1.2$

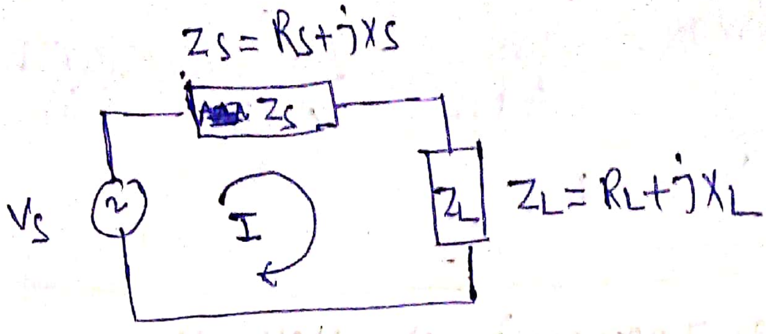
$= 3.2\Omega$

Therefore, the value of $P_{max} = \frac{V_{th}^2}{4R_{th}}$

$$P_{max} = \frac{(8)^2}{4\times 3.2} = \frac{64}{4\times 3.2}$$

$$\boxed{P_{max} = \frac{64\times 10}{4\times 32} = 5 \text{ watt}}$$

and $\boxed{R_L = R_{th} = 3.2\Omega}$



$$I = \frac{V_S}{Z_S + Z_L} = \frac{V_S}{(R_S + jX_S) + (R_L + jX_S)}$$

$$I = \frac{V_S}{(R_S + R_L) + j(X_S + X_L)}$$

$$|I| = \left| \frac{V_S}{(R_S + R_L) + j(X_S + X_L)} \right| = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}$$

$$\boxed{|I| = \frac{V_S}{\sqrt{(R_S + R_L)^2 + (X_S + X_L)^2}}} \quad \text{—— (i)}$$

Power across the load (R_L) $\equiv$ Active power across load (R_L)

__Note__ Active power is always flow across the R_L but not across the ~~reactive power~~ reactance.

$$P_L = I^2 . R_L \quad \text{—— (ii)}$$

$\boxed{P = I^2 Z_L \rightarrow \text{gives complex power}}$

$\downarrow$

Also called the Apparent Power

Substitute the value of (i) in the (ii)

$$\boxed{P_L = I^2 R_L = \frac{V_S^2 \cdot R_L}{(R_L+R_S)^2 + (X_S+X_L)^2}} \quad \text{--(ii)}$$

It is a power across the load.

For maximum power $\rightarrow$ Differentiate power w.r.to variable element

Possible cases for maximum power:

<u>case-1</u> When R_L is variable and X_L is fixed.

$$\frac{dP_L}{dR_L} = 0 \quad \text{(Apply max and min.)}$$

From-(ii)

$$\frac{dP_L}{dR_L} = 0 = \frac{\left[(R_L+R_S)^2 + (X_S+X_L)^2\right] \cdot V_S^2 - V_S^2 \cdot R_L \times \left[2(R_L+R_S)\right]}{\left[(R_L+R_S)^2 + (X_S+X_L)^2\right]^2}$$

$$R_L^2 + R_S^2 + \cancel{2R_L R_S} + (X_S+X_L)^2 = 2R_L^2 + \cancel{2R_L R_{th}}$$

$$R_L^2 = R_S^2 + (X_L+X_S)^2$$

$$\boxed{R_L = \sqrt{R_S^2 + (X_L+X_S)^2}} \quad \text{--(A)}$$

<u>case-2</u> only X_L is variable, R_L is fixed.

$$\frac{dP_L}{dX_L} = 0 = \left[(R_L+R_S)^2 + (X_L+X_S)^2\right] \times 0 - V_S^2 R_L \left[2(X_L+X_S)\right] =$$

$$X_L + X_S = 0; \quad \boxed{X_L = -X_S} \quad \text{--(B)}$$

Case-3 when R_L and X_L are variable.

It means that both condition (A) and (B) should be satisfied.

i.e. $R_L = \sqrt{R_S^2 + (X_L + X_S)^2}$

and

$X_L = -X_S$

From equation (A) and (B). Substitute the value of X_L in of B into A.

$R_L = \sqrt{R_S^2 + (X_S - X_S)^2} = R_S$

$$\boxed{R_L = R_S}$$

So for case-III $\boxed{R_L = R_S \text{ and } X_S = -X_L}$

Given in the circuit —

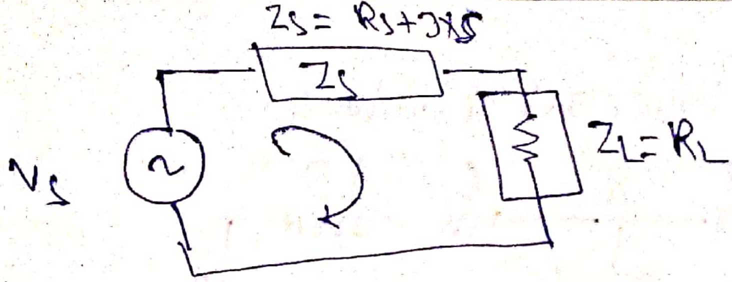
$Z_L = R_L + j X_L$

$= R_L - j X_L$

$Z_L = R_S - j X_S$

$$\boxed{Z_L = Z_S^*}$$

Case-4 when the load is only resistive. i.e. $Z_L = R_L$



when know that - $I = \dfrac{V_S}{\sqrt{(R_S+R_L)^2+(X_S+X_L)^2}}$

But given circuit is only R_L and $X_L = 0$

we get $I = \dfrac{V_S}{\sqrt{(R_S+R_L)^2 + X_S^2}}$

$$\boxed{P = I^2 R_L = \frac{V_S^2 \times R_L}{(R_L+R_S)^2 + (X_S)^2}}$$

Apply the same procedure —

$$\frac{dP_L}{dR_L} = 0$$

$$\boxed{R_L = \sqrt{R_S^2 + X_S^2}}$$

Conclusion: (i) only R_L is variable.

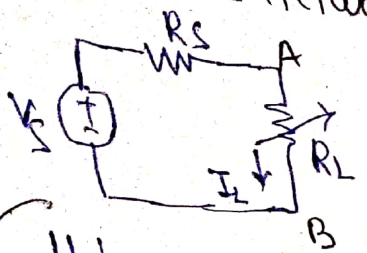
$R_L = |\text{Remaining element}|$

(ii) only X_L is variable

$X_L = -(\text{Remaining Reactance})$

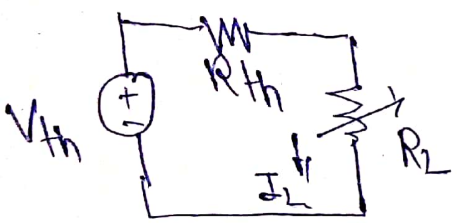
Maximum Power Transfer Theorem in AC circuits

For DC Network



It has only resistive part

Convert into Thevenin's equivalent



⊙ Condition for max^m Power Transferred-

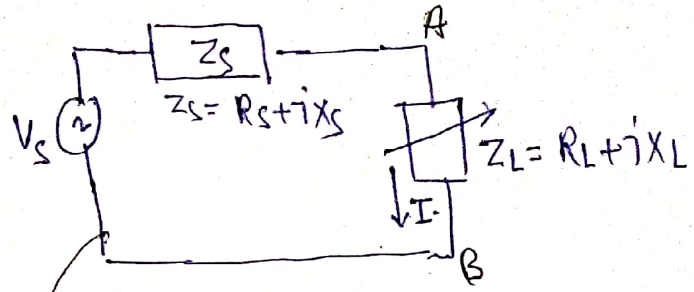
(1) $R_L = R_{th}$

(2) $P_{max} = \dfrac{V_{th}^2}{4R_{th}}$

<u>Note</u> :- Max^m power transferred theorem is applicable when the load is variable.

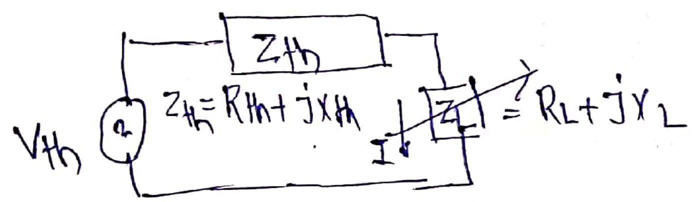
$$\boxed{\text{load} \begin{cases} \text{In DC} \rightarrow R_L \\ \text{In AC} \rightarrow Z_L = R_L + jX_L \end{cases}}$$

For AC Network



It has both resistive and reactive part

Convert into Thevenin's equivalent



⊙ Condition for max^m power transferred

There are four cases

<u>Case-1</u> R_L is variable and X_L is fixed

$$\boxed{R_L = \sqrt{R_{th}^2 + (X_L + X_{th})^2}}$$

⊙ <u>Case-2</u> X_L is variable and R_L is fixed.

$$\boxed{\begin{aligned} X_L &= -X_{th} \\ Z_L &= R_L - jX_{th} \end{aligned}}$$

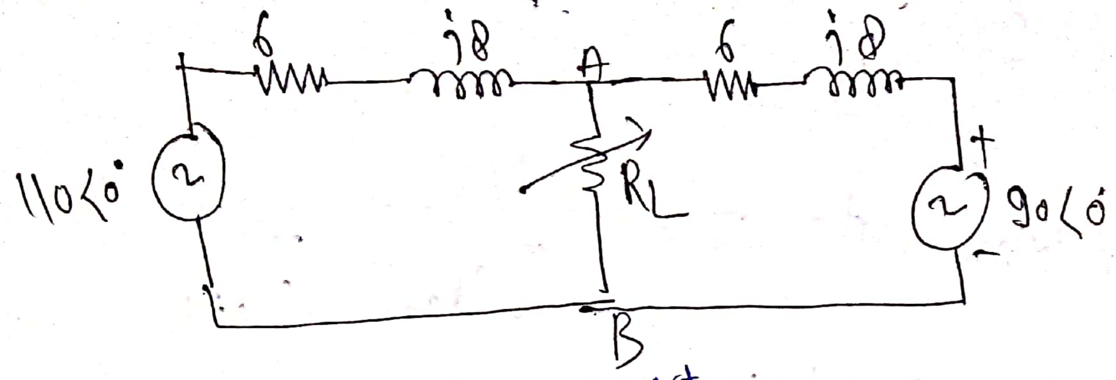
⊙ <u>Case-3</u> Both R_L and X_L are variable

$$\boxed{\begin{aligned} R_L &= R_{th}, \quad X_L = -X_{th} \\ Z_L &= R_{th} - jX_{th} \\ Z_L &= Z_{th}^* \end{aligned}}$$

⊙ <u>Case-4</u> Load is purely resistive $(X_L = 0)$

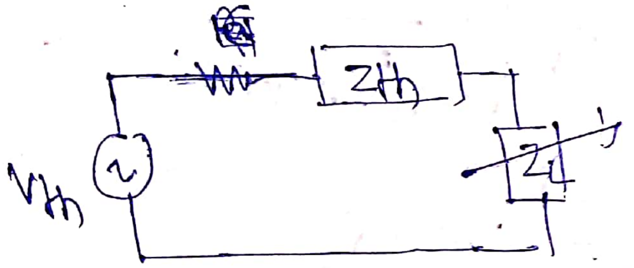
$R_L = \sqrt{R_{th}^2 + X_{th}^2}$

Prob: Find the maximum power across the load.

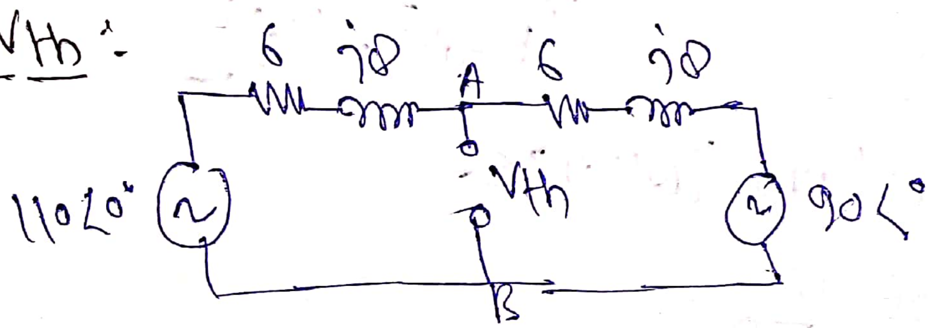


Sol^n

First given circuit convert into thevenin's equivalent circuit then apply max^m power transfer theorem.



For Vth :-



Apply node at A (KCL) —

$$\frac{V_{th} - 110\angle 0^\circ}{6+j8} + \frac{V_{th} - 90\angle 0^\circ}{6+j8} = 0$$

$$2V_{th} = 200$$

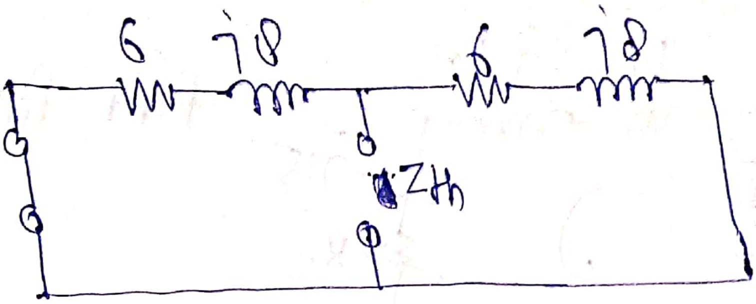
$$\boxed{V_{th} = 100 \text{ volt}}$$

For Rth : Step-1 - Remove load

Step-2 Voltage source → S.C

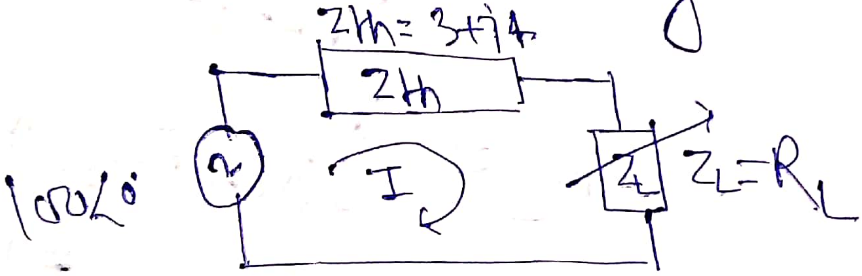
~~Step~~ current source → O.C

Step-3 find Rth across open terminal.



$$Z_{th} = \frac{(6+j8) \times (\cancel{6+j8})}{2(\cancel{6+j8})} = 3+j4$$

Replace the entire circuit by V_{th} and Z_{th}



$$R_L = |\text{Remaining element}| = |3+j4| = 5$$

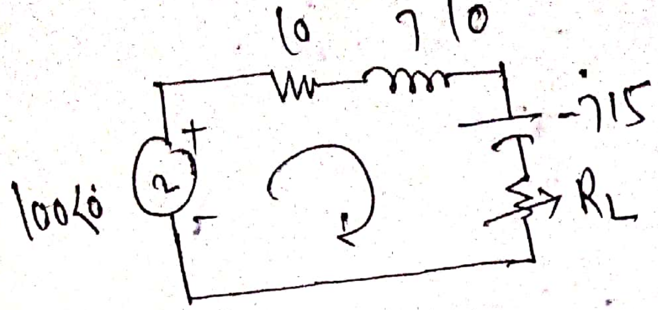
$$R_L = 5\,\Omega$$

$$I = \frac{100}{\sqrt{(8)^2+16}} = \frac{100}{\sqrt{64+16}} = \frac{100}{\sqrt{80}}$$

$$P = I^2 R_L = \frac{(100)^2}{80} \times 5 = \frac{(100)^2}{16} = \frac{10^4}{16}$$

$$\boxed{P = 625 \text{ watt}}$$

Prob! find P_{max} across R_L.



Soln

$$R_L = \left| \text{Remaining element} \right| = \left| 10 + j10 - j15 \right|$$

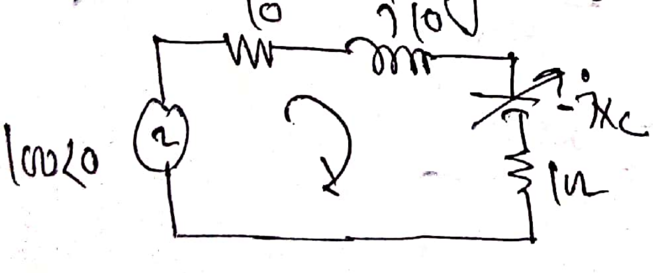
$$= \left| 10 - j5 \right|$$

$$R_L = \sqrt{100 + 25} = \sqrt{125}$$
$$= 11.18$$

$$\boxed{I = \frac{100\angle 0}{\sqrt{(10+R_L)^2 + (10-15)^2}}}$$

$$\boxed{P = I^2 \times R_L = I^2 \times 11.18} = 54.68 \text{ watt}$$

Prob! find the value of X_C for the max^m Power load.



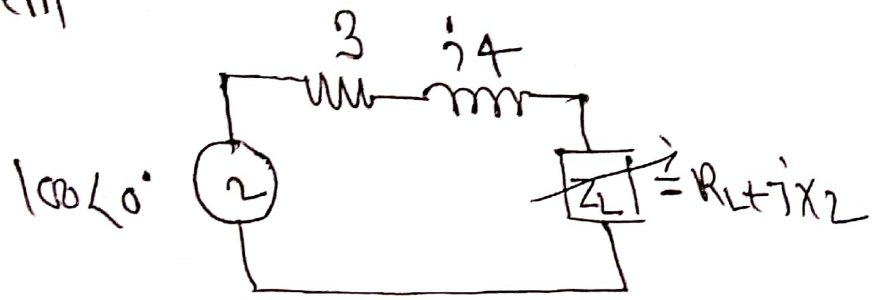
Soln When X_L is variable —

$$X_L = -X_S$$

$$-jX_C = -j \times 10$$

$$\boxed{X_C = 10 \text{ ohm}}$$

Prob: Determine the value of Z_L in maximum power transfer theorem



Soln: When Z_L is variable or both R_L and X_L are varia

$R_L = R_{th}, \quad X_L = -X_{th}$

$Z_L = R_L + jX_L$

$Z_L = R_{th} - jX_{th}$

$$\boxed{Z_L = Z_{th}^*}$$

$$\boxed{Z_L = 3 - j4}$$