Steady State and Transient Response

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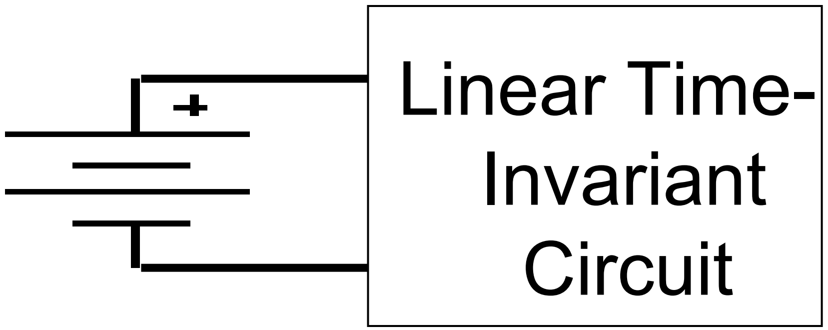
Steady State

- Both the inductance and capacitance are energy-storing elements.
- When connected to a dc source, energy starts flowing to these elements.
- Initially the rate of flow of energy is high, but as more and more energy is stored, the rate of flow decreases.
  - When maximum possible energy has been stored, the flow of energy stops altogether. We say that the circuit has reached its ‘steady state’.

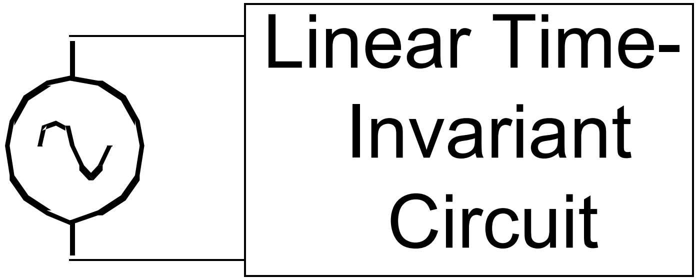
Transient Response

- If we switch off the source, or switch over the network to another source, the circuit starts attaining another 'steady state'.
- The time taken by the circuit to change over from one steady-state condition to another steady-state condition is called *transient time*.
- The response of the circuit during this time is known as *transient response*.

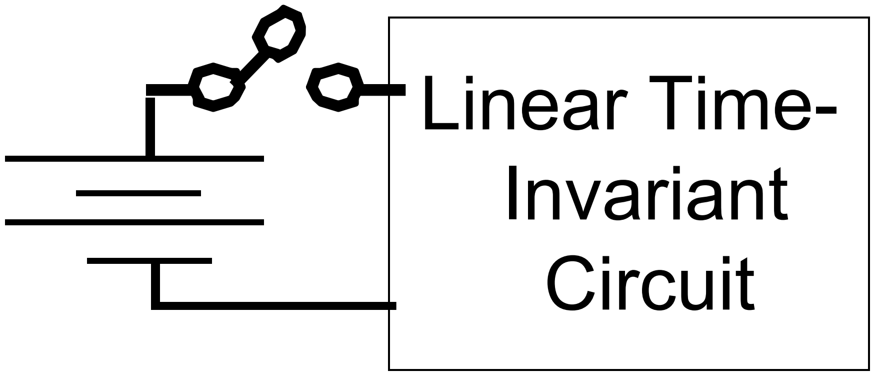
Types of Circuit Excitation



Steady-State Excitation (DC Steady-State)



Sinusoidal (Single-Frequency) Excitation □AC Steady-State



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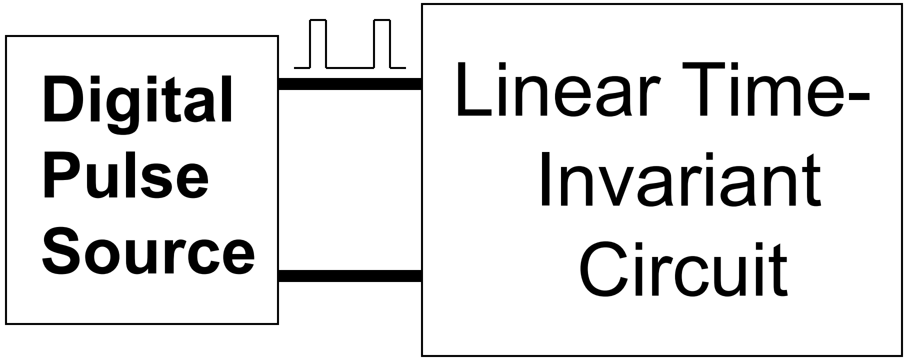
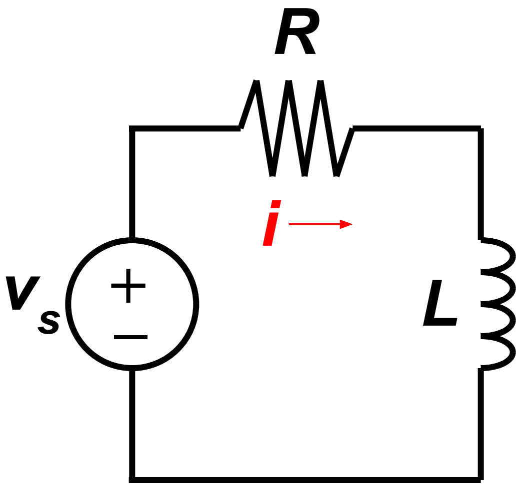
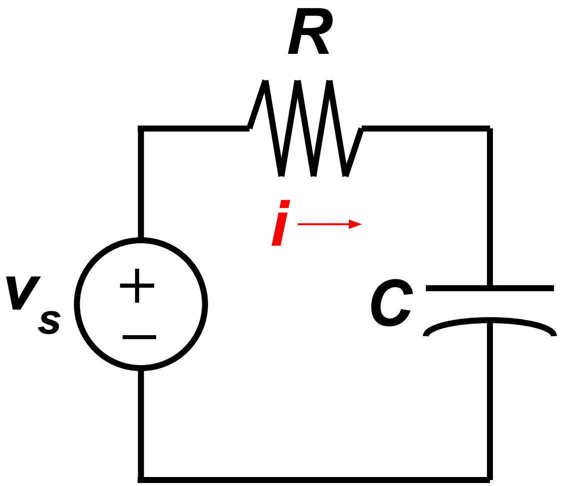


Transient Excitation

First-Order Circuits

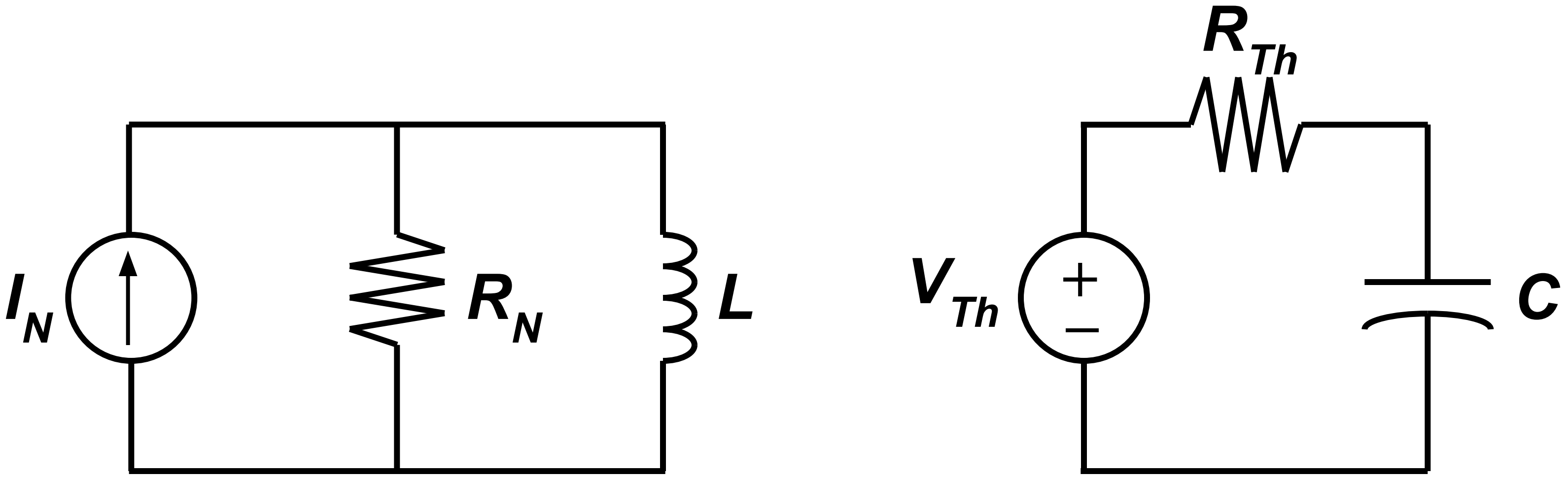
- A circuit that contains only sources, resistors and an inductor is called an ***RL circuit***.
- A circuit that contains only sources, resistors and a capacitor is called an ***RC circuit***.
- RL and RC circuits are called first-order circuits because their voltages and currents are described by first-order differential equations.





Review (Conceptual)

- Any first-order circuit can be reduced to a Thévenin (or Norton) equivalent connected to either a single equivalent inductor or capacitor.



- In steady state, an inductor behaves like a short circuit
- In steady state, a capacitor behaves like an open circuit

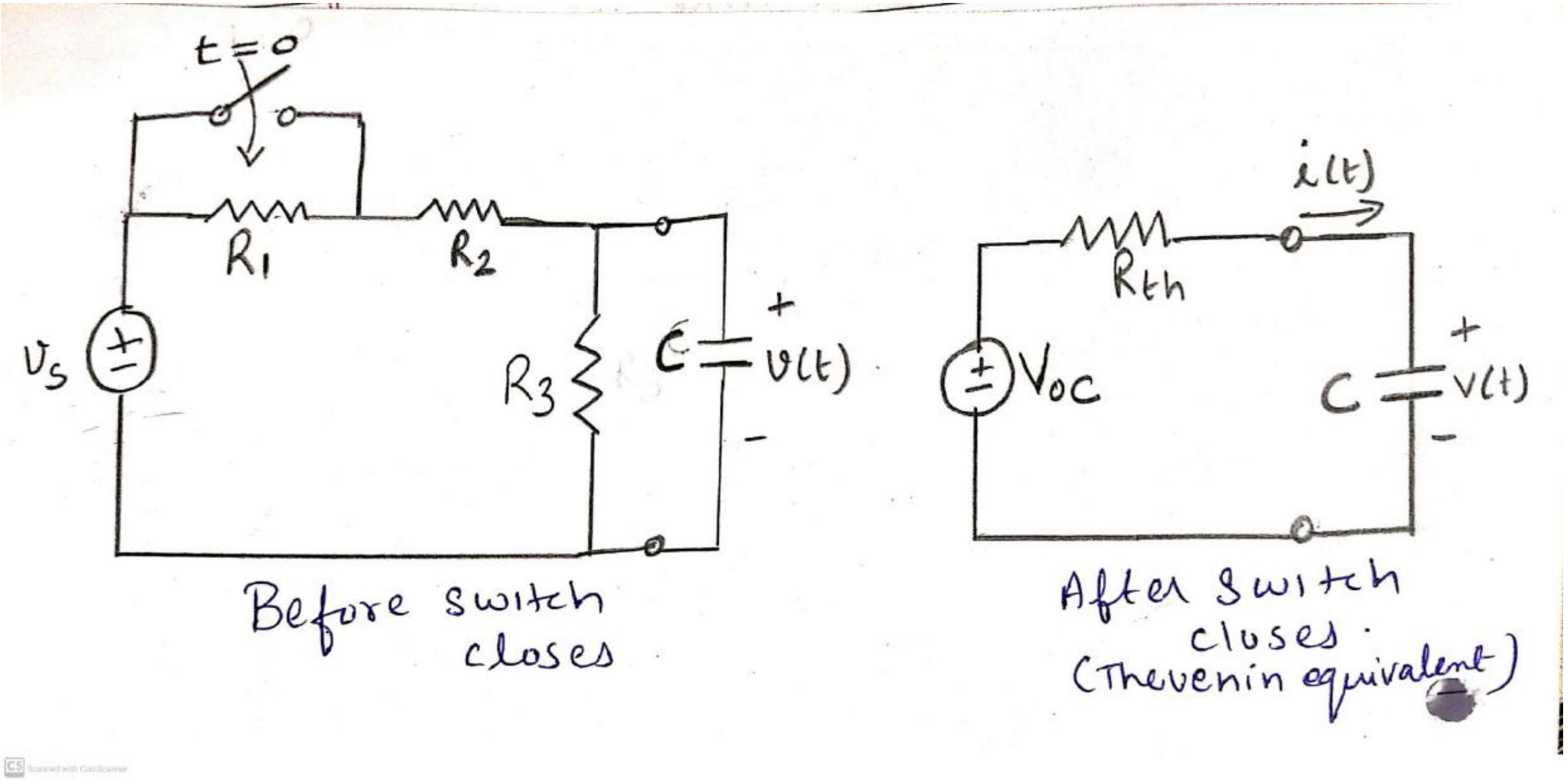
Complete response of a circuit

Complete response = natural response + forced response

- The **natural response** is the general solution of the differential equation representing the first-order circuit, when the input is set to zero.
- The ***step response*** of an RL or RC circuit is its behavior when a voltage or current source **step** is applied to the circuit, or immediately after a switch state is changed.

Response of first order circuit to a constant input

- RC Circuit



- Thevenins Equivalent

$$V_{oc} = \frac{R3}{R2 + R3} V_s$$

$$R_t = \frac{R2R3}{R2 + R3}$$

- Capacitor Current

$$i(t)=C\frac{d}{dt}v(t)$$

Apply KVL to Thevenin equivalent

$$V_{oc} = R_t i(t) + v(t)$$

Putting value of i(t)

$$V_{oc} = R_t C \frac{d}{dt} v(t) + v(t)$$

$$\frac{d}{dt}v(t) + \frac{v(t)}{R_t C} = \frac{V_{oc}}{R_t C}$$

- The highest-order derivative in this equation is first order, so this is a first-order differential equation.

Equation has form

$$\frac{d}{dt} x(t) + \frac{x(t)}{\tau} = K$$

where τ = time constant

To solve the above equation the variable are seperated and then integrated

rewrite the equation as

$$\frac{dx}{dt} = \frac{K\tau - x}{\tau}$$

or

$$\frac{dx}{K\tau - x} = -\frac{dt}{\tau}$$

forming the indefinite integral, we have

$$\int \frac{dx}{K\tau - x} = -\frac{1}{\tau}\int dt + D$$

where D is constant of integration

$$\ln(K\tau - x) = -\frac{t}{\tau} + D$$

Solving for x gives.

$$x(t) = k\tau + Ae^{-t/\tau} \quad — (1)$$

where $A = e^{D}$, which is determined from initial condn, $x(0)$

To find A, let $t = 0$, then.

$$x(0) = k\tau + Ae^{-0/\tau} = k\tau + A$$

$$A = x(0) - k\tau$$

Now putting value of A in eq (1)

$$x(t) = k\tau + [x(0) - k\tau]\, e^{-t/\tau} \quad — (2)$$

Similarly in eq (1)

$$x(\infty) = \lim_{t \to \infty} x(t) = k\tau$$

Equation (2) can be written as.

$$x(t) = x(\infty) + [x(0) - x(\infty)]\, e^{-t/\tau}$$

Taking the derivative of $x(t)$ with respect to t will give time constant

$$\frac{d}{dt} x(t) = -\frac{1}{\tau} [x(0) - x(\infty)]\, e^{-t/\tau} \quad — (3)$$

Now let $t = 0$ in eq (3)

$$\left.\frac{d}{dt} x(t)\right|_{t=0} = -\frac{1}{\tau} [x(0) - x(\infty)]$$

$$\tau = \frac{x(\infty) - x(0)}{\left.\frac{d}{dt} x(t)\right|_{t=0}}$$

Differential equation for RC circuit was

$$\frac{d}{dt} v(t) + \frac{v(t)}{R_t C} = \frac{V_{oc}}{R_t C}$$

Comparing with eq

$$\frac{d}{dt} x(t) + \frac{x(t)}{\tau} = K$$

We see that

$x(t) = v(t)$, $\tau = R_t C$ and $K = \frac{V_{oc}}{R_t C}$

Making these substitution.

$\cancel{v(t) = V_{oc} + [v(0) - v(\infty)]}$

$$v(t) = V_{oc} + [v(0) - V_{oc}] e^{-t/R_t C} \quad (4)$$

The second term on RHS dies as t increases. This is transient or natural response.

At $t = 0$, in eqn (4) $e^{-0} = 1$

so

$v(0) = v(0)$ as required.

when $t = 5\tau$, $e^{-5} = 0.0067 \approx 0$

so at $t = 5\tau$

$v(5\tau) = 0.9933 V_{oc} + 0.0067 v(0)$

$v(5\tau) \approx V_{oc}$

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This is steady state or forced response.

Complete response = NR + FR

$$NR = [v(0) - V_{oc}]\, e^{-t/(R_t C)}$$

$$FR = V_{oc}$$

so
complete response is :

$$v(t) = V_{oc} + [v(0) - V_{oc}]\, e^{-t/R_t C}$$

RL Circuit