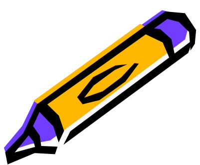
# Signals and systems



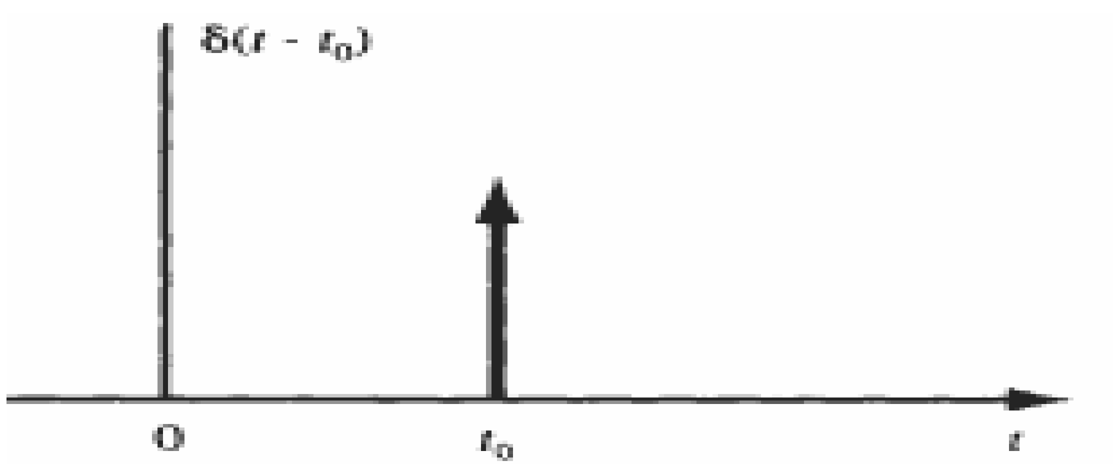
# Properties of Impulse Function:

## 1. Shifting Property:

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)\, dt = \phi(0)$$

**Similarly, for the delayed delta function δ(t - *t$_o$*) *it* is**

$$\int_{-\infty}^{\infty} \phi(t)\delta(t - t_0)\, dt = \phi(t_0)$$



2. Scaling Property:

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

3. It is an even function:

$$\delta(-t) = \delta(t)$$

4. $x(t)\,\delta(t) = x(0)\,\delta(t)$

or, $x(t)\,\delta(t - t_o) = x(t_o)\,\delta(t - t_o)$

Contd...

## 5. Convolution property

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$$

**6.** $$\int_{-\infty}^{\infty} \phi(t)\delta'(t)\,dt = -\phi'(0)$$

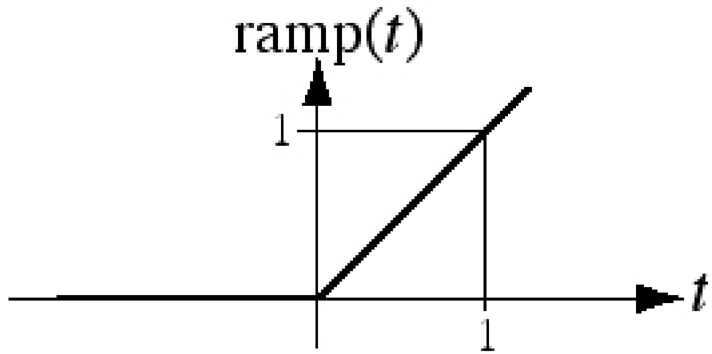
**7.** $$\delta(t) = u'(t) = \frac{du(t)}{dt}$$

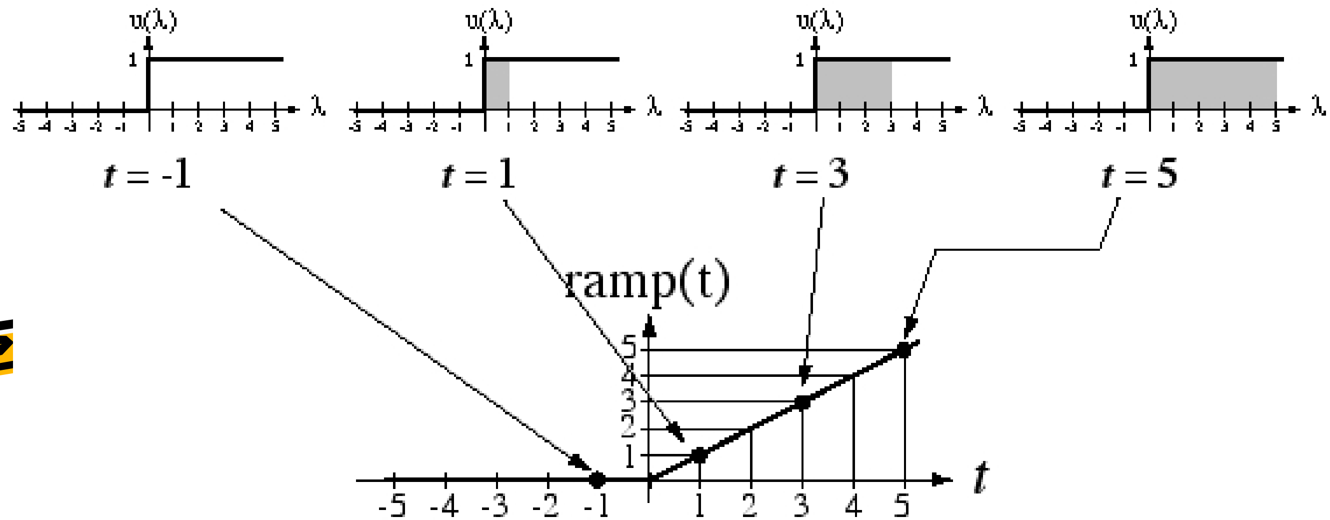
**Or,**

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

# The CT Unit Ramp Function

$$\operatorname{ramp}(t) = \begin{cases} t \;, & t > 0 \\ 0 \;, & t \le 0 \end{cases} = \int_{-\infty}^{t} \mathrm{u}(\lambda)\,d\lambda = t\,\mathrm{u}(t)$$

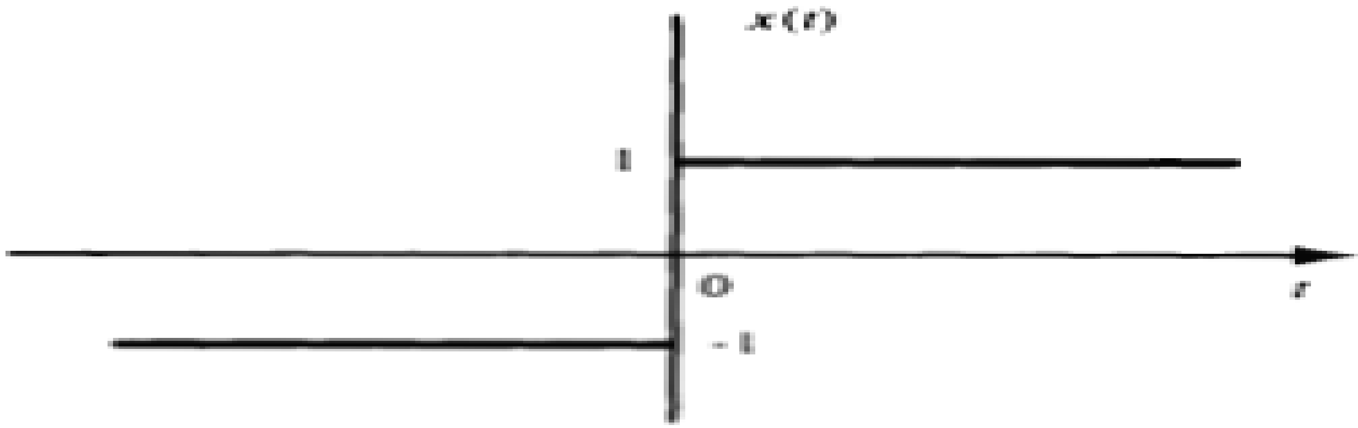


# Some more CT functions:

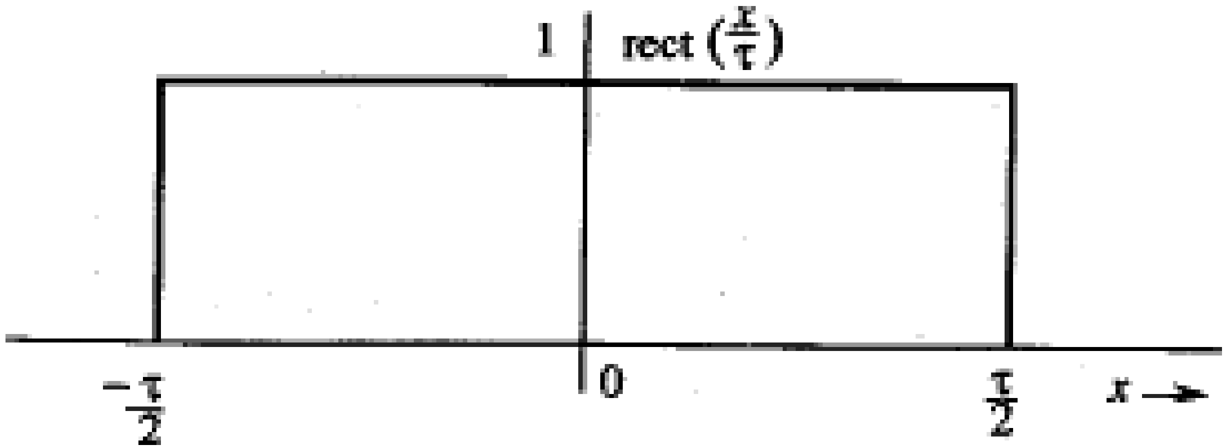
## Signum Function:

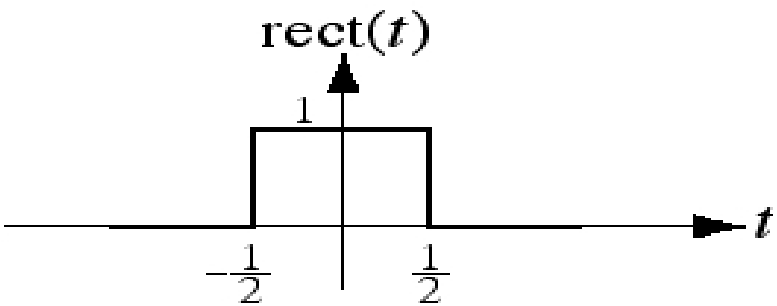
$$x(t) = \operatorname{sgn} t = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$




**Sgn(t) = 2 u(t) - 1**

- **Gate function (Rectangular function):**



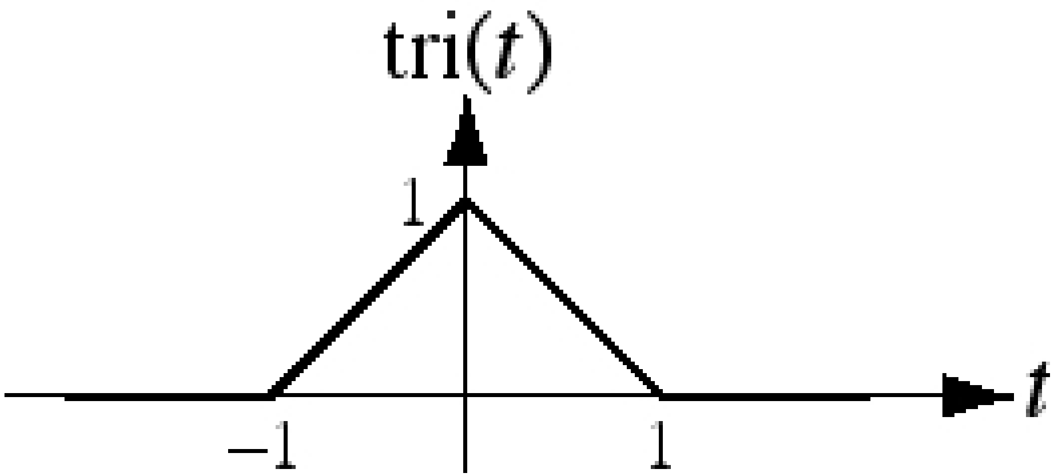




$$rect(t) = \begin{cases} 1 & -1/2 \le t \le 1/2 \\ 0 & elsewhere \end{cases}$$

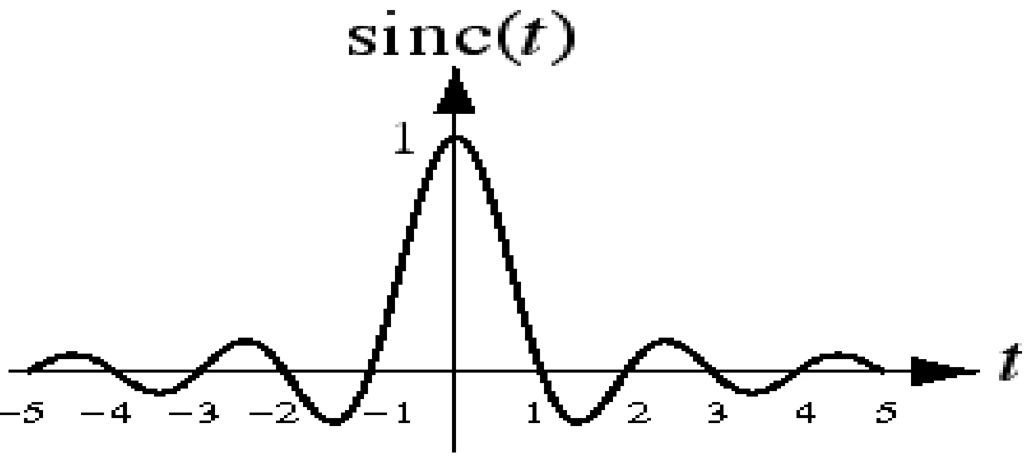
$$rect(t) = u\,(t + 1/2) - u\,(t - 1/2)$$

- **Triangular function:**

$$\mathrm{tri}(t) = \begin{cases} 1-|t| & , \quad |t|<1 \\ 0 & , \quad |t|\geq 1 \end{cases}$$



# The CT Unit Sinc Function




$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\lim_{t \to 0} \operatorname{sinc}(t) = \lim_{t \to 0} \frac{\frac{d}{dt}\left(\sin(\pi t)\right)}{\frac{d}{dt}(\pi t)} = \lim_{t \to 0} \frac{\pi \cos(\pi t)}{\pi} = 1$$

# Properties of Sinc function:

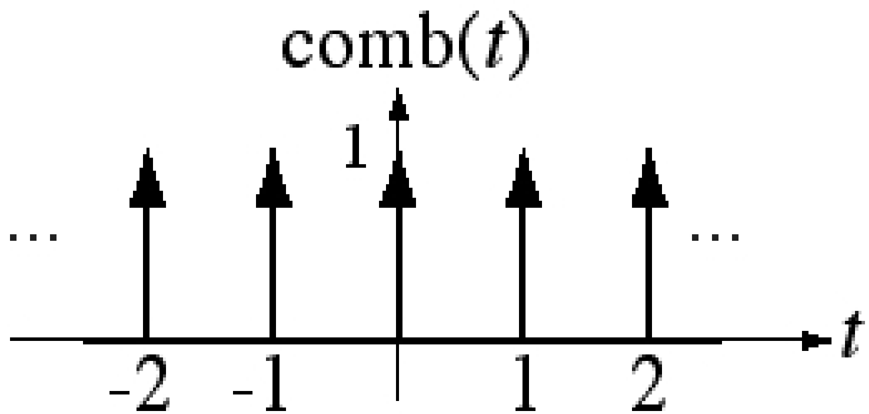
- sinc(t) is an even function of t.
- sinc(t) = 0, when sin (t) = 0. This means that sinc(t) = 0 for t = ±π, ±2π, ±3π ,±4π,……..
- Using L'Hopital's rule, we find sinc(0) = 1.
- sinc(t) is the product of an oscillating signal sin(t) [of period 2 π] and a monotonically decreasing function 1/t.

  Therefore sinc (t) exhibits sinusoidal oscillations of period 2 π with amplitude decreasing continuously as 1/t.

# The CT Unit Comb

The CT unit comb is defined by

$$\text{comb}(t) = \sum_{n=-\infty}^{\infty} \delta(t-n) \;,\quad n \text{ an integer}$$




The comb is a sum of uniformly-spaced impulses.

# The Unit Impulse Sequence:

**The unit impulse (or unit sample) sequence δ[n] is defined as**

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases}$$


(a) (b)

**( a ) Unit impulse (sample) sequence; (b) shifted unit impulse sequence.**

# Properties of Impulse Sequence :

## 1. Shifting Property:

$$x[n]\delta[n] = x[0]\delta[n]$$

$$x[n]\delta[n-k] = x[k]\delta[n-k]$$

## 2.

$$\delta[n] = u[n] - u[n-1]$$

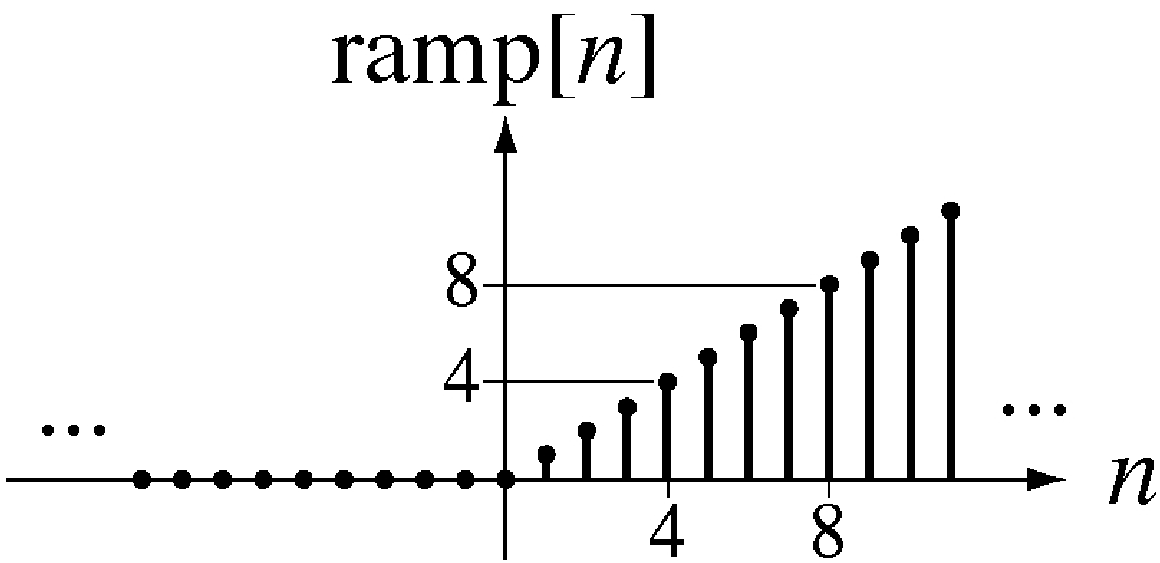
$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

## 3. Convolution:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

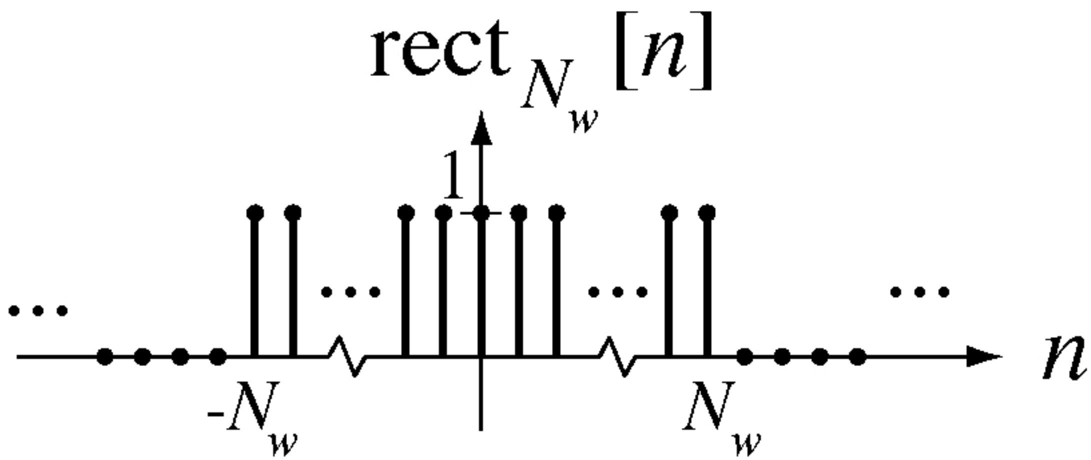
# The DT Unit Ramp Function

$$\mathrm{ramp}[n] = \begin{cases} n \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases} = \sum_{m=-\infty}^{n} \mathrm{u}[m-1]$$



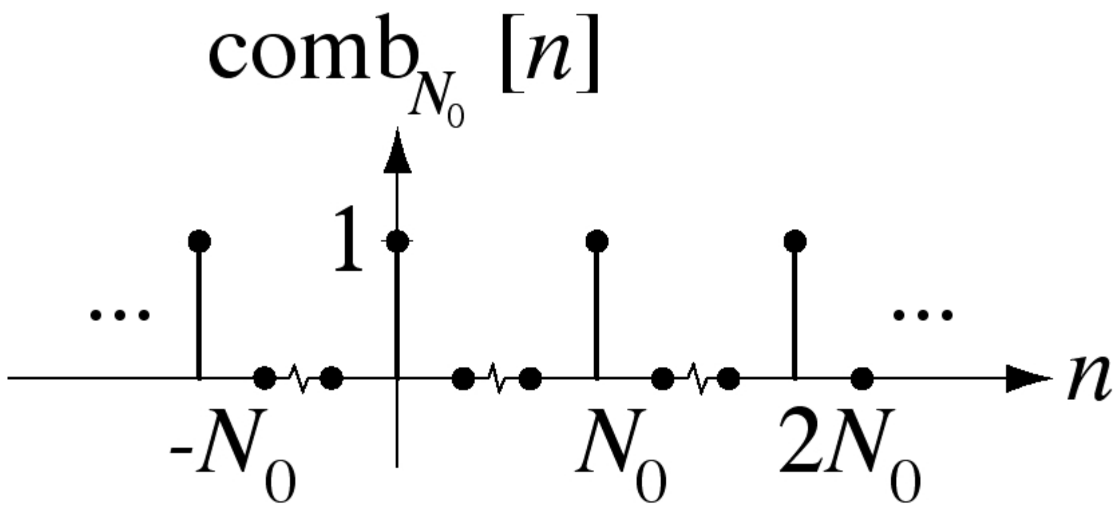
# The DT Rectangle Function

$$\mathrm{rect}_{N_w}[n] = \begin{cases} 1 \; , & |n| \le N_w \\ 0 \; , & |n| > N_w \end{cases} , \; N_w \ge 0 \; , \; N_w \text{ an integer}$$



# The DT Comb Function

$$\mathrm{comb}_{N_0}[n] = \sum_{m=-\infty}^{\infty} \delta[n - mN_0]$$



# Plot the following functions

$$3\,rect\left(\frac{t+1}{4}\right)$$

$$-5\,ramp\,(0.1t)$$

$$2\,sinc\,(5t)$$

$$-3\,sgn\,(2t)$$

$$-7\,tri\left(\frac{t-4}{8}\right)$$