ELECTRICAL SCIENCE-II
(15B11EC211)

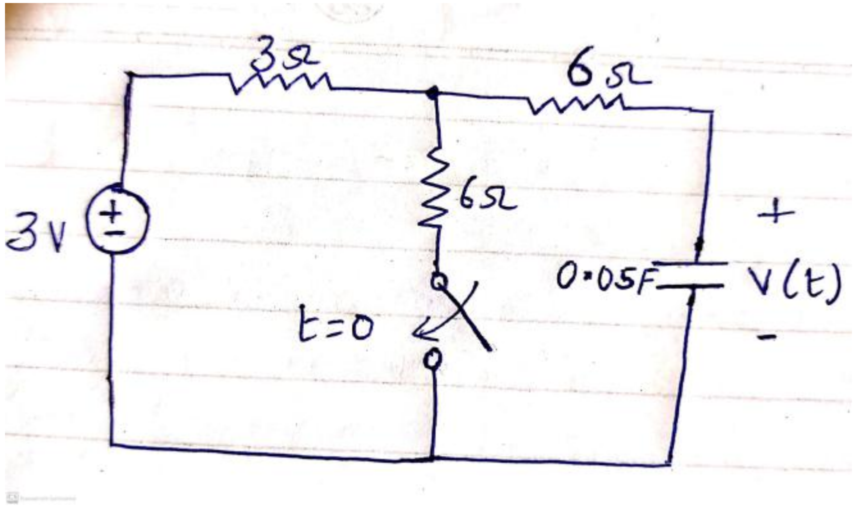
Content

- Example
- Single-Capacitor *RC* Circuit and Single-Inductor *RL* Circuit
- Initial and final value
- Comparison between ***RC*** and ***RL*** Circuits
- Complete Solution by the Differential Equation Approach

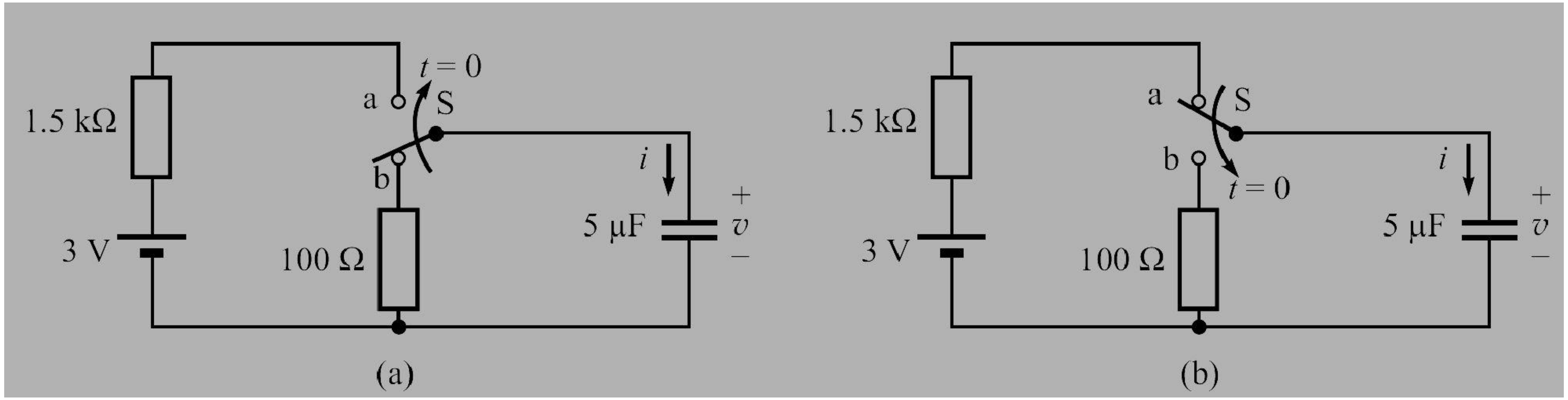
Example

- The circuit shown in Figure is at steady state before the switch closes at time t = 0. Determine the capacitor voltage v(t) for t > 0.

Ans $v(t) = 2 + \ e^{-2.5t}\ V\ for\ t > 0$



Example 3



The single-pole double-throw switch S has been in position b for a long time so that the 5-μF capacitor is fully discharged. Now, at $t = 0$, the switch is thrown to position a. Determine

- (*a*) $v(0+)$,
- (*b*) $i(0+)$,
- (*c*) time constant τ,
- (*d*) v and i at $t = 15$ ms.

Solution :

(*a*) Since the voltage across a capacitor cannot change instantaneously, we have $v(0+) = v(0-) = \mathbf{0\ V}$

(*b*) $$i(0^+) = I_0 = \frac{V_0}{R} = \frac{3\,\text{V}}{1.5\,\text{k}\Omega} = \mathbf{2\ mA}$$

(*c*) $$\tau = RC = (1.5\ \text{k}\Omega)(5\ \mu\text{F}) = \mathbf{7.5\ ms}$$

(*d*) At $t = 15$ ms :

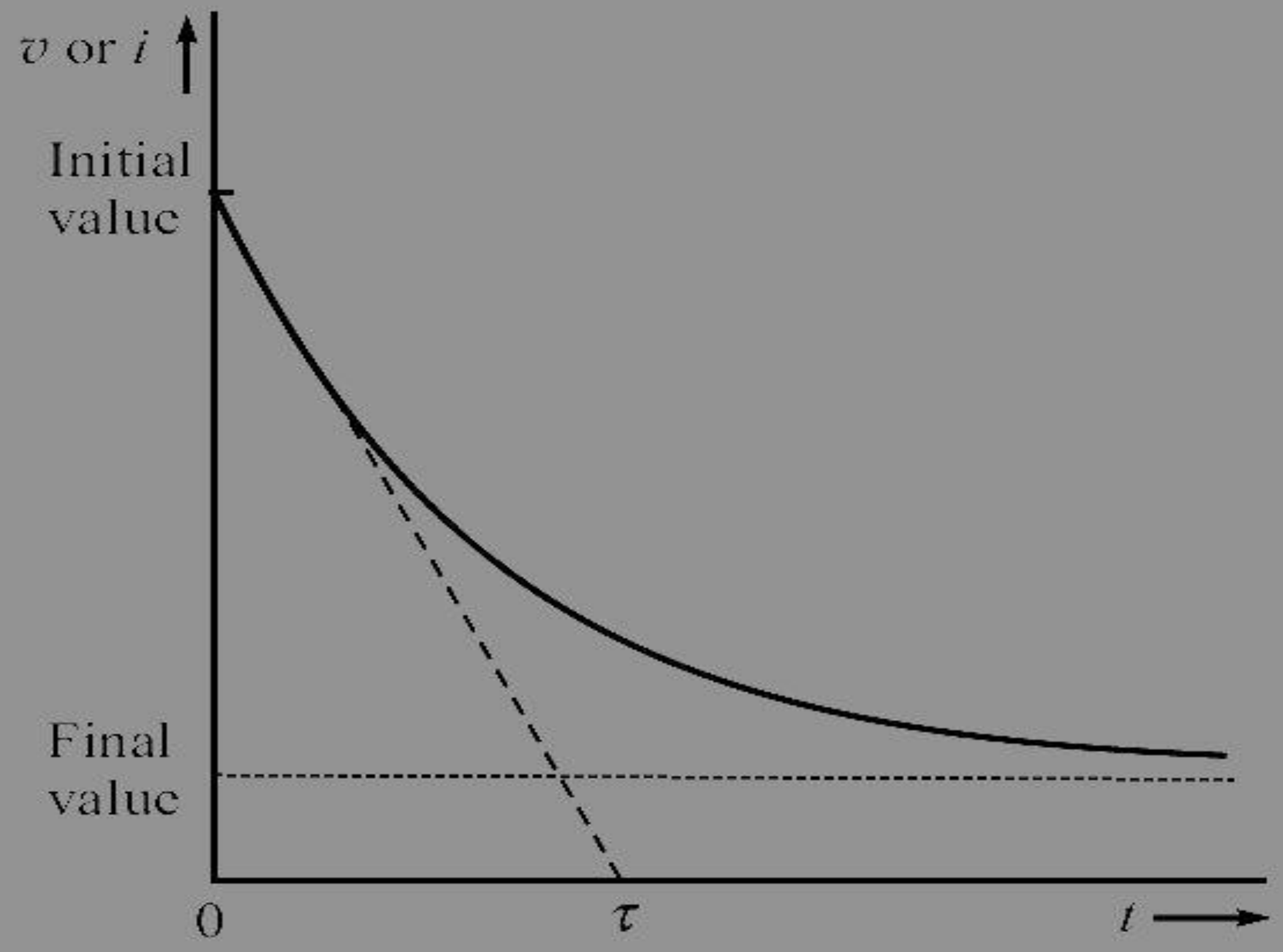
$$v = V_0(1 - e^{-t/\tau}) = 3(1 - e^{-(15\text{ ms})/(7.5\text{ ms})}) = \mathbf{2.594\ V}$$

$$i = I_0 e^{-t/\tau} = (2\ \text{mA})\, e^{-(15\text{ms})/(7.5\text{ ms})} = \mathbf{0.27\ mA}$$

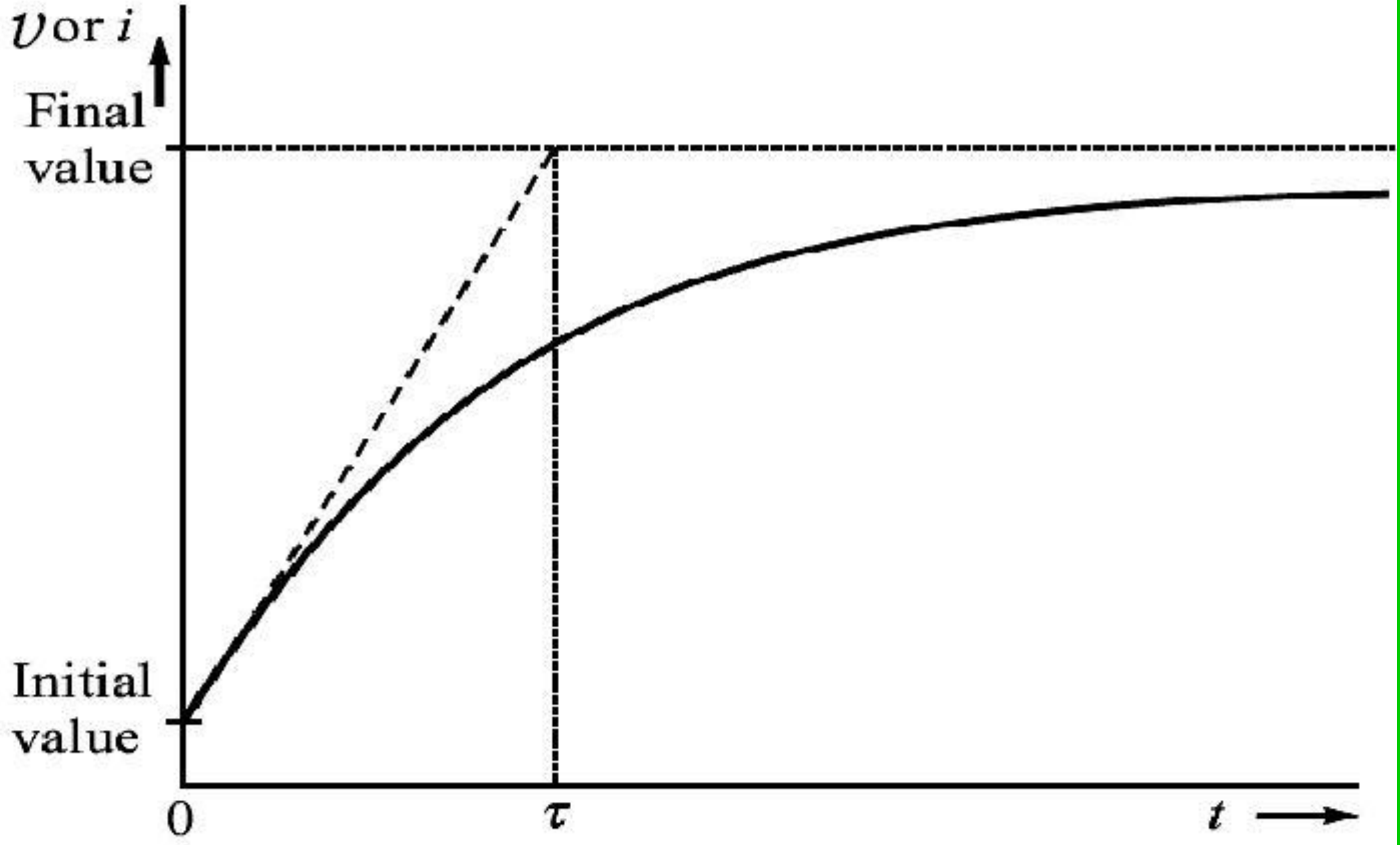
Single-Capacitor *RC* Circuit and Single-Inductor *RL* Circuit

- The ***time constant*** for
  - For *LR* circuit, $\tau = L / R_{\mathrm{Th}}$
  - For *CR* circuit, $\tau = CR_{\mathrm{Th}}$

Here, R_{Th} is Thevenin resistance as “seen” by the capacitor or inductor.



(a) Exponentially decreasing with time.



(*b*) Exponentially increasing with time.

- The voltages and currents approach their final values asymptotically.
- It means that they never actually reach them.
- However, after *five time-constants* they change by 99.3 % of their total change.

Important Point

(For solving Problems)

If immediately after switching,

$v(0+)$ and $i(0+)$ are *initial values*

and $v(\infty)$ and $i(\infty)$ are *final values*.

Then, the expressions for all the voltages and currents in the circuit for any time t are given as

$$\boxed{\begin{aligned} v(t) &= v(\infty) + [v(0^+) - v(\infty)]e^{-t/\tau} \text{ V} \\ i(t) &= i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau} \text{ A} \end{aligned}}$$

Comparison between *RC* and *RL* Circuits

Though both give similar response, but we prefer *RC* over *RL* circuit, because

- Inductors are not as nearly ideal as capacitors.
- Inductors are relatively bulky, heavy and difficult to fabricate, especially using integrated-circuit techniques.
- Inductors are relatively costlier.
- The magnetic field emanating from the inductors can induce unwanted voltages in other components.

Complete Solution by the Differential Equation Approach

5 major steps to find the complete solution:

- Determine initial conditions on capacitor voltages and/or inductor currents.
- Find the differential equation for either capacitor voltage or inductor current (mesh/loop/nodal …. analysis).
- Determine the natural solution (complementary solution).
- Determine the forced solution (particular solution).
- Apply initial conditions to the complete solution to determine the unknown coefficients in the natural solution.

Here we will consider three cases for the input to the circuit.

- First case

$$v_s(t) = V_0$$

- Second case

$$v_s(t) = V_0 e^{-t/\tau}$$

- Third case

$$v_s(t) = V_0 \cos(\omega t + \theta)$$

These three cases are special because the forced response will have the same form as the input.