15B11PH211 (Physics II-2020) Tutorial Sheet-3

CO2 **1** The charge density $\rho = \rho_o \sin\left(\frac{x}{x_o}\right)$ is given for the region $-\pi/2 \leq x/x_o \leq \pi/2$ and elsewhere $\rho = 0$. Find V and E.

CO2 **2** Given the potential field $V = A \ln(\tan^2 \theta/2) + B$; (a) show that $\nabla^2 V = 0$;(b) select A and B so that V=100 V and $E_\theta = 500$ V/m at P(r = 5, θ = 60°, ϕ = 45°).

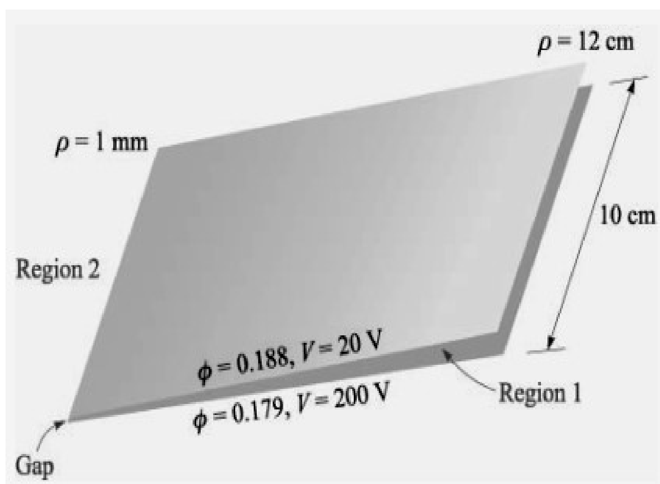
CO3 **3** If $V = 20 \sin\theta / r^3$ V in free space, find: (a) ρ_v at P(r = 2, θ = 30°, ϕ = 0); (b) the total charge within the spherical shell 1 < r < 2 m .

CO3 **4** Coaxial conducting cylinders are located at ρ = 0.5 cm and ρ = 1.2 cm. The region between the cylinders is filled with a homogeneous perfect dielectric. If the inner cylinder is at 100 V and the outer at 0 V , find: (a) the location of the 20 V equi-potential surface ; (b) $E_{\rho\ max}$; (c) ε_R if the charge per meter length on the inner cylinder is 20 nC/ m.

CO3 **5** Two coaxial conducting cones have their vertices at the origin and the z axis as their axis. Cone A has the point A (1, 0, 2) on its surface, while cone B has the point B (0, 3, 2) on its surface. Let $V_A = 100$ V and $V_B = 20$ V. Find: (a) α for each cone; (b) V at P (1, 1, 1).

CO3 **6** Given the potential field $V = (A\rho^4 + B\rho^{-4}) \sin 4\varphi$: (a) show that $\nabla^2 V = 0$; (b) select A and B so that V = 100 V and |**E**| = 500 V/m at P(ρ = 1, φ = 22.5°, z =2).

CO4 **7** The two conducting planes illustrated in figure are defined by 0.001 <ρ< 0.120 m, 0 m < z < 0.1 m, ϕ = 0.179 and 0.188 rad. The medium surrounding the planes is air. For region 1,0.179 <ϕ< 0.188, neglect fringing and find: (a) V(ϕ); (b) E(ρ); (c) D(ρ);(d)ρs on the upper surface of the lower plane;(e) Q on the upper surface of the lower plane (f) Repeat (a) to (c) for region 2 by letting the location of the upper plane be ϕ = 0.188 - 2π, and then find ρs and Q on the lower surface of the lower plane . (g) Find the total charge on the lower plane and the capacitance between the plane.



15B11PH211 (Physics II-2020) Tutorial Sheet-3 (Solution)

1. $$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} = -\frac{\rho_o}{\varepsilon_o}\sin\frac{x}{x_o}$$

$$\frac{\partial V}{\partial x} = \frac{\rho_o}{\varepsilon_o}x_o\cos\frac{x}{x_o} + C_1$$

$$V = \frac{\rho_o}{\varepsilon_o}x_o^2\sin\frac{x}{x_o} + C_1 x + C_2$$

$$E = -\frac{\partial V}{\partial x} = -\frac{\rho_o}{\varepsilon_o}x_o\cos\frac{x}{x_o} - C_1$$

2. Given $V = A\ln\left(\frac{\tan^2\theta}{2}\right) + B$

(a) Show that $\nabla^2 V = 0$

$$\nabla^2 V = \frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dV}{d\theta}\right)$$

$$\text{where}\ \frac{dV}{d\theta} = \frac{d}{d\theta}\left(A\ln\left(\frac{\tan^2\theta}{2}\right) + B\right) = \frac{2A}{\sin\theta}$$

$$\text{Then}\ \nabla^2 V = \frac{1}{r^2\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{2A}{\sin\theta}\right) = 0$$

(b) $V = 100\ V$ and

$$E_\theta = 500\frac{V}{m}\ \text{at}\ P(r = 5, \theta = 60^o, \phi = 45^o$$

$$E_\theta = -\nabla V = \frac{2A}{5\sin 60} = -0.462A = 500$$

$\Rightarrow A = -1082.5\ V$,

$$\text{Then}, V_p = -1082.5\ln\left(\frac{\tan^2\theta}{2}\right) + B$$

$\Rightarrow B = -1089.3\ V$,

Summerizing

$$V = -1082.5\ln\left(\frac{\tan^2\theta}{2}\right) - 1089.3$$

3. $V = 20\frac{\sin\theta}{r^3}V$

$$(a)\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) = -\frac{\rho}{\varepsilon_o}$$

at $P(r = 2, \theta = 30^o, \phi = 0)$

$$\nabla^2 V = \frac{120\sin\theta}{r^5} + \frac{20\cos 2\theta}{r^5\sin\theta} = \frac{20(4\sin^2\theta + 1)}{r^5\sin\theta} = -\frac{\rho}{\varepsilon_o}$$

ρ_v at P = $-2.5\varepsilon_o = -22.1 pC/m^3$

(b) Total charge within the sphere $1 < r < 2$m

$$Q = -\varepsilon_o\int_0^{\pi}\int_0^{2\pi}\int_1^{2}\frac{20(4\sin^2\theta + 1)}{r^5\sin\theta}r^2\sin\theta\, dr\, d\theta\, d\phi = -3.9nC$$

4 (a) $$V(\rho) = 100\frac{\ln(0.012/\rho)}{\ln(0.012/0.005)} = 20$$

$\rho = 1.01\,cm$

(b) $$E_\rho = -\frac{\partial V}{\partial\rho} = \frac{100}{\rho\ln(2.4)}$$

maximum will occur at inner cylinder, or at $\rho = 0.5$ cm

$$E_{\rho,\max} = \frac{100}{0.005\ln(2.4)} = 22.8kV/m$$

(c) The capacitance per meter length is

$$C = \frac{2\pi\varepsilon_o\varepsilon_R}{\ln(2.4)} = \frac{Q}{V_o}$$

$$\varepsilon_R = \frac{(20\times 10^{-9})\ln(2.4)}{2\pi\varepsilon_o(100)} = 3.15$$

5. *Cone A has the point* $A(1,0,2)$ *on its surface and cone B has the point* $B(0,3,2)$ *on its surface.* $V_A = 100\ V$ *and* $V_B = 20\ V$.

(a) α *for each cone*: $\alpha_A = tan^{-1}\left(\frac{1}{2}\right) = 26.57^o$

and $\alpha_B = tan^{-1}\left(\frac{3}{2}\right) = 56.31^o$.

(b) *V at* $P(1,1,1)$ $V(\theta) = C_1 \ln \tan\left(\frac{\theta}{2}\right) + C_2$

$$20 = C_1 \ln \tan\left(\frac{56.31}{2}\right) + C_2$$

$$100 = C_1 \ln \tan\left(\frac{26.57}{2}\right) + C_2$$

Solving these two equation for $C_1 = -97.7$

and $C_2 = -41.1$.

Now at P, $\theta = tan^{-1}(\sqrt{2}) = 54.7^o$.

Thus $V_p = 23.3\ V$

6. *Given the potential field* $V = (A\rho^4 + B\rho^{-4})sin4\varphi$

(a) *Show that* $\nabla^2 V = 0$

In cylinderical coordinates

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\varphi^2}$$

$$= \frac{16}{\rho}(A\rho^3 + B\rho^{-5})sin4\varphi$$

$$-\frac{16}{\rho^2}(A\rho^4 + B\rho^{-4})sin4\varphi = 0$$

(b) $E = -\nabla V = -\frac{\partial V}{\partial\rho}\boldsymbol{a}_\rho - \frac{1}{\rho}\frac{\partial V}{\partial\varphi}\boldsymbol{a}_\varphi$

at P, $E_P = -4(A-B)\boldsymbol{a}_\rho$

Thus $|E_P| = \pm 4(A-B)$ *and* $V_P = A + B$

$\Rightarrow A = 112.5, B = -12.5$ *or*

$A = -12.5\ ,\ B = 112.5$

7. (a) $V(\phi)$: *the general solution to the Laplace'sequation will be* $V = C_1\phi + C_2$

$20 = 0.188C_1 + C_2$ *and* $200 = 0.179C_1 + C_2$, $\Rightarrow$ $C_1 = -2.00\times10^4;\ C_2 = 3.78\times10^3$

Finally, $V(\phi) = -2.00\times10^4\phi + 3.78\times10^3$

(b) $E(\rho) = -\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial\phi} = \frac{2\times10^4}{\rho}\boldsymbol{a}_\phi\ \frac{V}{m}$

(c) $D(\rho) = \varepsilon_o E(\rho) = \frac{2\times10^4\varepsilon_o}{\rho}\boldsymbol{a}_\phi\ \frac{C}{m^2}$

(d) $\rho_s = \boldsymbol{D}.\boldsymbol{n}|_{surface} = \frac{2\times10^4\varepsilon_o}{\rho}\frac{C}{m^2}$

(e) Q_t *on the upper surface of the lower plane*: $Q_t = \int_0^1\int_{0.001}^{0.120}\frac{2\times10^4\varepsilon_o}{\rho}d\rho dz = 84.7\ nC$

(f) *Repeat (a) to (c) for region 2* $\phi = 0.188 - 2\pi$,

and then ρ *and Q onthe lower surface of the lopwer plane.*

$220 = (0.188 - 2\pi)C_1 + C_2$ *and* $200 = 0.179C_1 + C_2$,

Finally,

$V(\phi) = 28.7\ \phi + 194.9$ *in region 2*

$$E(\rho) = -\nabla V = -\frac{1}{\rho}\frac{\partial V}{\partial\phi} = -\frac{28.7}{\rho}\boldsymbol{a}_\phi\ \frac{V}{m}$$

$$D(\rho) = \varepsilon_o E(\rho) = \frac{-28.7\varepsilon_o}{\rho}\boldsymbol{a}_\phi\ \frac{C}{m^2}$$

$$\rho_s = \boldsymbol{D}.\boldsymbol{n}|_{surface} = \frac{28.7\varepsilon_o}{\rho}\frac{C}{m^2}$$

(g) *Total charge on the lower plane and the capacitance between the planes.*

Total charge will be $Q_{net} = Q_t + Q_b = 84.7\text{nC} + 0.122\text{nC}$.

The capacitance will be $C = \frac{Q_{net}}{\Delta V} = \frac{84.8}{200 - 20} = 471\ \text{pF}$