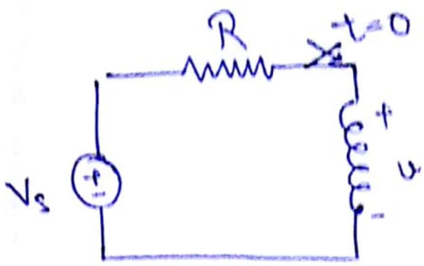
Step Response of RL circuit



Let the response of the ckt be the sum of transient response & steady state response

$$i = i_t + i_{ss} \quad \text{—— (1)}$$

The transient response is a decaying exponential

$$i_t = A e^{-t/\tau} \quad \text{—— (2) where } \tau = \frac{L}{R}$$

Now during steady state, inductor is going to behave like short-circuit ∴ Voltage is zero

$$\therefore i_{ss} = \frac{V_s}{R} \quad \text{—— (3)}$$

∴ In eqⁿ (1)

$$i = A e^{-\frac{t}{\tau}} + \frac{V_s}{R} \quad \text{—— (4)}$$

Finding the value of A

Let I_0 be the initial current through the inductor

$$\therefore i_{0^-} = i_{0^+} = I_0$$

Now at time t=0

$$I_0 = A + \frac{V_s}{R}$$

$$\therefore A = I_0 - \frac{V_s}{R}$$

Substituting value of A in eqⁿ (4)

$$\boxed{\therefore i = \left(I_0 - \frac{V_s}{R}\right) e^{-t/\tau} + \frac{V_s}{R}}$$

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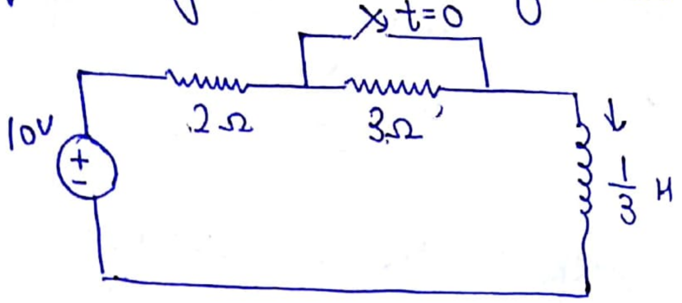
$$\boxed{i(t) = [i(0) - i(\infty)]\, e^{-t/\tau} + i(\infty)}$$

general eqⁿ

Ex. Find $i(t)$ for $t > 0$. Assume the switch is closed for long duration of time



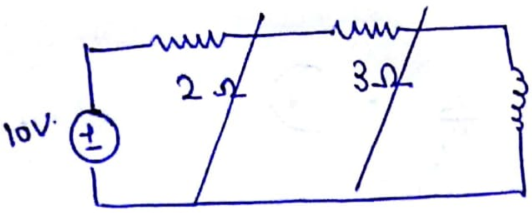
$$i(t) = [i(0) - i(\infty)]\, e^{-t/\tau} + i(\infty)$$

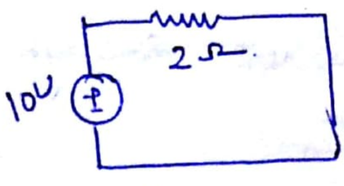
At $t = 0$

$i(t) = \cancel{I_0}$

$i(\infty) = \frac{10}{2} = 5\,A$

for $t < 0$





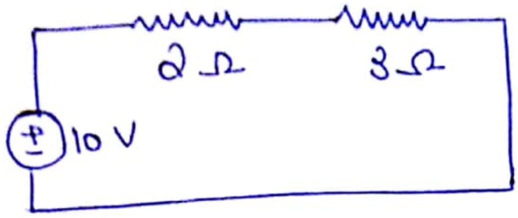
$\therefore i = 5\,A$

The current through the inductor at $t = 0^-$

$$i(0^-) = 5\,A$$

$$i(0^-) = i(0^+) = 5\,A$$

Now for $t > 0$



$$i = \frac{10}{5} = 2A$$

$$i(\infty) = 2A$$

$$\tau = \frac{L}{R}$$

$$\boxed{R_{TH} = 5\,\Omega}$$

$$\therefore \tau = \frac{1}{15}\,s$$

$$i(t) = (5-2)e^{-t/\tau} + 2$$
$$= 3e^{-t/\tau} + 2$$

$$\boxed{\therefore i(t) = 3e^{-15t} + 2 \quad A}$$

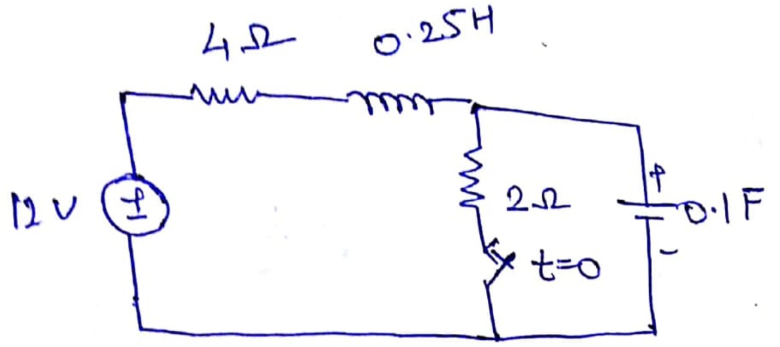
Second Order Circuits

for capacitor,

$$v(0^+) = v(0^-)$$

for inductor,

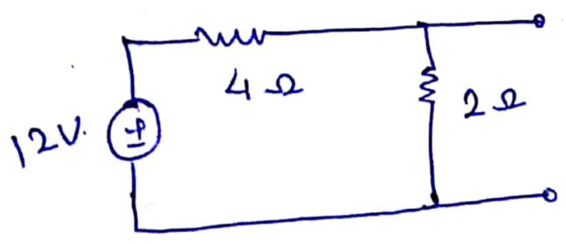
$$i(0^+) = i(0^-)$$



Switch has been closed for a long time and it is opened at t=0. Find $i(0^+)$, $v(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$, $i(\infty)$, $v(\infty)$

Ans.

As the switch was closed for long time



$$\therefore i(0^-) = \frac{12}{6} = 2A$$

a)

$$\therefore v(0^-) = 4V \text{ (across } 2\Omega)$$

$$i(0^+) = i(0^-) = 2A$$

$$v(0^+) = v(0^-) = 4V$$

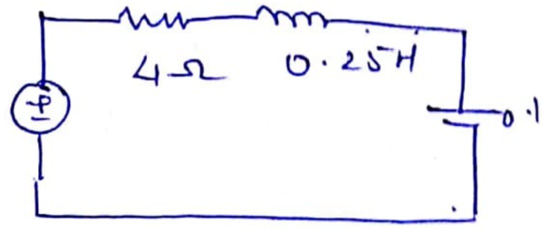
b) at time $t=0^+$ (switch is open)

$$i_c(0^+) = 2\,A$$

$$C\frac{dv}{dt} = i_c$$

$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{2}{0.1} = 20\ V/s$$

Similarly we know,



$$L\frac{di}{dt} = v_L$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

Now to obtain voltage across inductor, applying KVL.

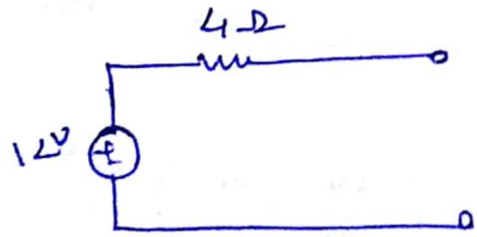
$$12 - 4i(0^+) - V_L(0^+) - V_C(0^+) = 0$$

$$\therefore V_L(0^+) = 0$$

$$\therefore \frac{di(0^+)}{dt} = 0$$

c) $i(\infty) = 0$

$v(\infty) = 12$



$i(\infty) \longrightarrow$ across inductor
$v(\infty) \longrightarrow$ across capacitor

★ The response of first order ckt to a non constant source:

The general form of RL & RC ckt is represented by

$$\frac{dx(t)}{dt} + ax(t) = y(t) \quad \text{—— (1)}$$

where, $y(t)$ is a const. for a const voltage or current source and $a = \frac{1}{\tau}$ the reciprocal of time const.

To make LHS a perfect derivative we use a method called as (IF method).

Let us consider, a derivative of product of two terms.

$$\frac{d(xe^{at})}{dt} = \frac{dx}{dt}e^{at} + axe^{at}$$

$$\frac{d(xe^{at})}{dt} = e^{at}\left(\frac{dx}{dt} + ax\right) \quad \text{—— (2)}$$

RHS of eqn (2) is same as LHS of eqn (1)

∴ multiplying e^{at} on both the sides then LHS can be represented by the perfect derivative, $\frac{dxe^{at}}{dt}$

$$\left(\frac{dx}{dt} + ax\right)e^{at} = ye^{at}$$

Integrating both the sides we have

$$xe^{at} = \int ye^{at}dt + k \quad \text{—— (3)}$$

where, k is a const

Solving for $x(t)$, multiply by e^{-at} on both sides of eqn (3)

$$x = e^{-at}\int y e^{at} dt + k e^{-at}$$

where $y(t)$ is source can be represented by $\underline{M}$

$$x = M e^{-at}\int e^{at} dt + k e^{at}$$

$$\boxed{x = \frac{M}{a} + k e^{-at}}$$

$$\boxed{x = x_f + x_n}$$

forced response   Natural response

Natural response $x_n = k e^{-at}$

forced response $x_f = e^{-at}\int y(t) e^{at} dt$

If $y(t)$ is exponential function then

$$y(t) = e^{bt} \quad \& \quad a+b \neq 0$$

$$x_f = e^{-at}\int e^{bt} e^{at} dt$$

$$= e^{-at}\int e^{(a+b)t} dt$$

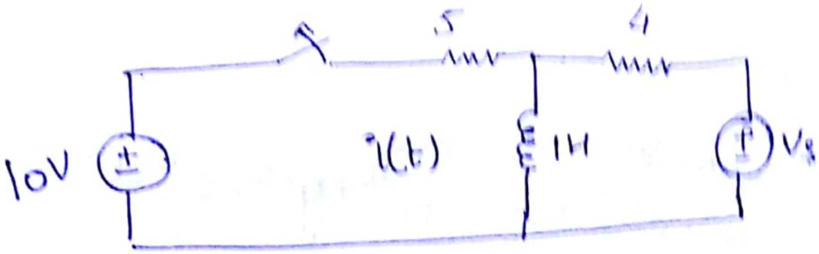
$$= \frac{1}{a+b} e^{-at} e^{(a+b)t}$$

$$x_f = \frac{1}{a+b} e^{bt}$$

Forced response of a forcing function

Forcing function $y(t)$	Forced response $x(t)$
① Constt $y(t)=M$	$x_f = N$ a constt
② Exponential $y(t)=Me^{bt}$	$x_f = Ne^{bt}$
③ Sinusoidal $y(t)=M\sin(\omega t+\theta)$	$x_f = A\sin\omega t + B\cos\omega t$

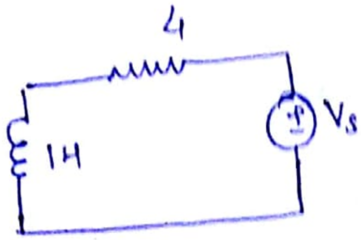
Ex. Find the current i for the ckt shown here for $t>0$ when the value of $V_s(t)=10e^{-2t}u(t)$



Since the forcing function is an exponential
∴ the forced response shall be of the form

$$i_f = Be^{-2t} \quad \text{—①}$$

for $t\geq 0$ ~~at~~ writing KVL.



$$V_s - 4i(t) - V_L = 0$$

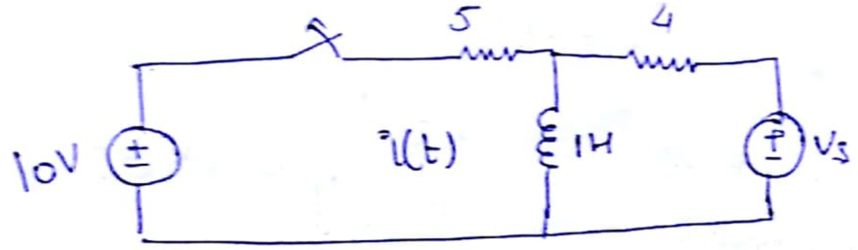
$$10e^{-2t} - 4i(t) - V_L = 0$$

$$L\frac{di}{dt} + 4i = 10e^{-2t} \quad \text{—②}$$

Forced response of a forcing function

Forcing function $y(t)$	forced response $x(t)$
① Const. $y(t) = M$	$x_f = N$ a const.
② Exponential $y(t) = Me^{-bt}$	$x_f = Ne^{-bt}$
③ Sinusoidal $y(t) = M\sin(\omega t + \theta)$	$x_f = A\sin\omega t + B\cos\omega t$

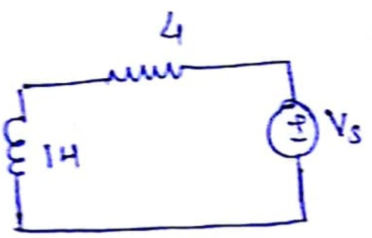
Ex. Find the current i for the ckt shown here for $t > 0$ when the value of $V_s(t) = 10e^{-2t}$



Since the forcing function is an exponential ∴ the forced response shall be of the form.

$$i_f = Be^{-2t} \quad \text{—①}$$

for $t \geq 0$ ~~ret~~ writing KVL.



$$V_s - 4i(t) - V_L = 0$$

$$10e^{-2t} - 4i(t) - V_L = 0$$

$$L\frac{di}{dt} + 4i = 6e^{-2t} \quad \text{—②}$$

$$\frac{di}{dt} + \frac{4i}{L} = \frac{10e^{-2t}}{L}$$

taking $i = i_f$

for time $t>0$ substituting the value of i_f in eqn ②

$$\frac{d(Be^{-2t})}{dt} + 4Be^{-2t} = 10e^{-2t}$$

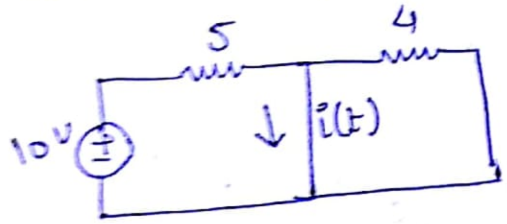
$$-2Be^{-2t} + 4B\cancel{e^{-2t}} = 10\cancel{e^{-2t}}$$

$$2B = 10$$

$$\boxed{B = 5}$$

$$\therefore \boxed{i_f = 5e^{-2t}}$$

for natural response $t<0$



$R_L = 4\,\Omega$

$i(0) = 2\,A$

$$i_n = Ae^{-\left(\frac{R_1}{L}\right)t}$$

$$\tau = \frac{L}{R} = \frac{1}{4}$$

$$i = Ae^{-4t} + 5e^{-2t}$$

for inductor $i(0^+) = i(0^-) = i(0)$

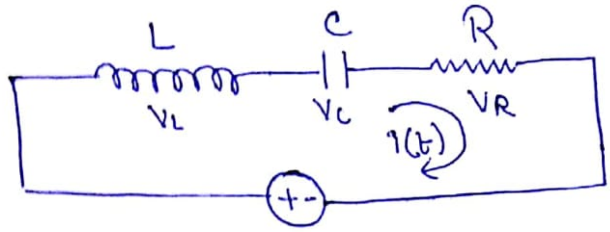
$$2 = A + 5$$

$$A = 3$$

$$\therefore \boxed{i = 3e^{-4t} + 5e^{-2t}\ A}$$

Second Order Ckts

① Series RLC Ckt



Apply KVL in the ckt

$$V_s(t) = V_L + V_C + V_R$$

$$= L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(t)dt + V_C(0) + Ri(t)$$

differentiating w.r.t. time t

$$\frac{dV_s(t)}{dt} = L\frac{d^2i}{dt^2} + \frac{1}{C}i(t) + \frac{dRi(t)}{dt}$$

$$\frac{dV_s}{dt} = L\frac{d^2i}{dt} + R\frac{di(t)}{dt} + \frac{1}{C}i(t)$$

$$\frac{1}{L}\frac{dV_s(t)}{dt} = \frac{d^2i}{dt} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) \quad \text{—①}$$

general 2nd order diff. eqn

$$\frac{d^2x(t)}{dt} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t) \quad \text{—②}$$

Comparing eqn ① & ②

$$2\alpha = \frac{R}{L} \qquad \omega_0^2 = \frac{1}{LC}$$

$$\Rightarrow \boxed{\underbrace{\alpha}_{\text{damping coeff. factor}} = \frac{R}{2L}} \qquad \boxed{\underbrace{\omega_0}_{\text{Resonant frequency}} = \frac{1}{\sqrt{LC}}}$$

damping coeff./factor

Resonant frequency

$$f(t) = \frac{1}{L}\frac{dV_s(t)}{dt}$$

forcing function.

We know that,

$$x(t) = x_f + x_n$$

To calculate natural response, Let us assume that soln of diff eqn

$$\underline{x_n(t) = ke^{st}}$$

$$\frac{d^2ke^{st}}{dt^2} + 2\alpha\frac{dke^{st}}{dt} + \omega_0^2ke^{st} = 0$$

$$ks^2e^{st} + 2\alpha ske^{st} + \omega_0^2ke^{st} = 0$$

$$\underbrace{ke^{st}}(s^2 + 2\alpha s + \omega_0^2) = 0$$

where $\neq 0$

$$\therefore \; s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s = \frac{-2\alpha \pm \sqrt{4\alpha^2 - 4\omega_0^2}}{2}$$

$$\boxed{s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}}$$

$$0.4 = R_f \cos\theta + 0.1$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

where, $\omega_0 \rightarrow$ resonant frequency or undamped natural frequency

s_1 & s_2 are natural frequencies

These can have 3 types of soln depending upon the value of
$\left.\begin{array}{l}\alpha > \omega_0 \\ \alpha < \omega_0 \\ \alpha = \omega_0\end{array}\right\}$

$x_n(t)$

The natural response of RLC ckt where k_1 & k_2 are consts & are determined by initial values of $i(0)$, $\frac{di(0)}{dt}$

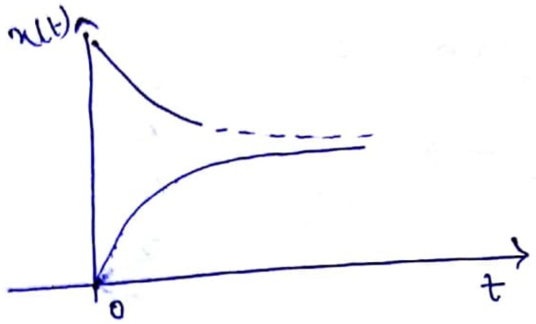
$$\boxed{x_n = k_1 e^{s_1 t} + k_2 e^{s_2 t}}$$

Case 1: when $\alpha > \omega_0$

$\therefore$ The response is over damped

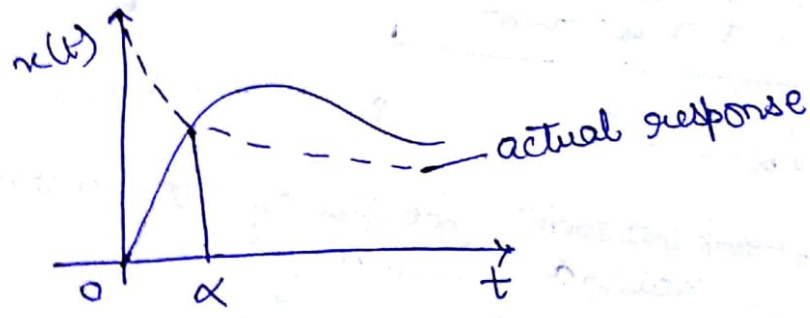
So the roots are real & distinct

$\therefore x_n = k_1 e^{s_1 t} + k_2 e^{s_2 t}$



Case 2: when $\alpha = \omega_0$

$\therefore$ The response is critically damped



roots of real & equal

$\therefore x_n = k_1 e^{s_1 t} + k_2 e^{s_1 t}$

$= k_3 e^{s_1 t}$ $\qquad k_3 = k_1 + k_2$

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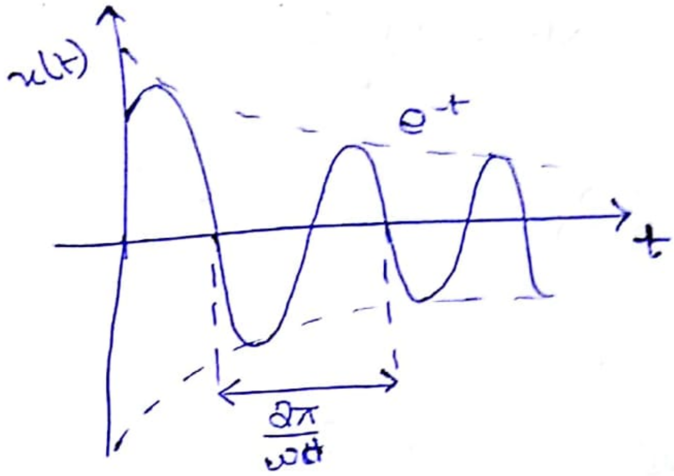
Case 3: when $\alpha < \omega_0$

$\therefore$ The response is underdamped



oscillatory response

The roots are imaginary

$$x_n(t) = A\cos\omega_d t + B\sin\omega_d t$$

$A = k_1 e^{s_1 t}$

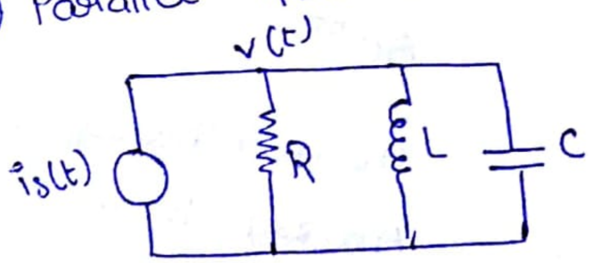
$B = k_2 e^{s_2 t}$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

damped natural frequency

$$\boxed{x_n(t) = k_1 e^{-s_1 t}\cos(\omega_d t) + k_2 e^{-s_2 t}\sin(\omega_d t)}$$

② Parallel RLC Ckt



Applying KCL

$i_s(t) = i_R + i_C + i_L$

$(t-2)$ $(t-0)$ $(t-2)$ $t>0$ $t>0$

$$i_s(t) = \frac{V(t)}{R} + \frac{1}{L}\int_0^t v(t)\,dt + i_L(0) + \frac{c\,dv(t)}{dt}$$

On diff. w.r.t t

$$\frac{di_s(t)}{dt} = \frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) + \frac{c\,d^2v(t)}{dt^2}$$

$$\frac{c\,di_s(t)}{dt} = \frac{d^2v(t)}{dt} + \frac{1}{RC}\cdot\frac{dv(t)}{dt} + \frac{1}{LC}v(t) \quad -(3)$$

General eqn

$$\frac{d^2x(t)}{dt} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t) \quad -(4)$$

Comparing eqn (3) & (4)

$$2\alpha = \frac{1}{RC}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Forced Response (for RLC)

Input fn $f(t)$	Forced Response (x_f)
(1) k	A
kt	$At + B$
(2) ke^{bt} where $b \neq s_1, s_2$	Ae^{bt}
(3) $k\sin\omega t$	$A\sin\omega t + B\cos\omega t$
(4) $k\cos\omega t$	$A\cos\omega t + B\sin\omega t$

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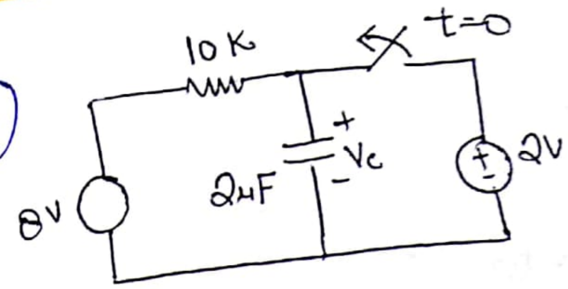
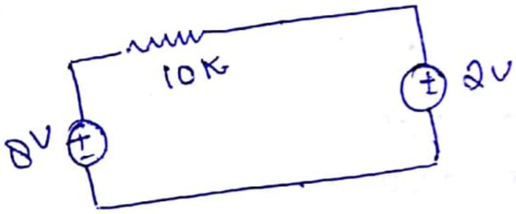


a) Find V_c when switch is open

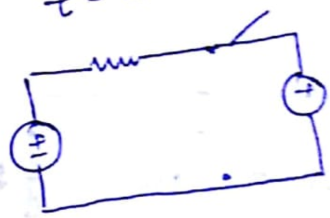
b) Value of capacitor voltage after 50 ms of switch opens

$t<0$

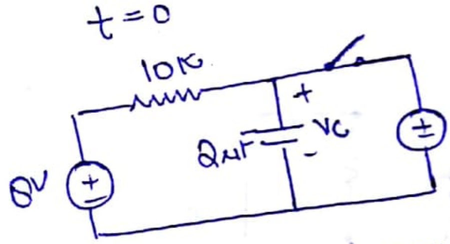


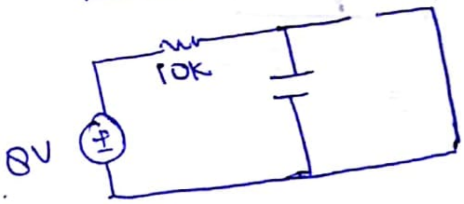
$i(0^-) = \frac{8-2}{10K} = \frac{6}{10K}$

$t=\infty$



$t=0$





$8 - 10K \times \frac{6}{10K} = V_c$

a) $\underline{2\ V = V_c}$

b) $V = V_0 e^{-t/RC}$

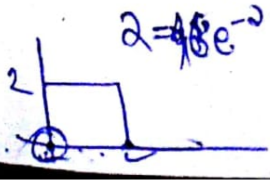
$2 = \cancel{16}\, e^{-\frac{\cancel{50}\ 2.5 \times 10^{-3}\ 10^{3}}{10 \times 2 \times 10^{-6}}}$

$= \cancel{16}\, e^{-2.5}$

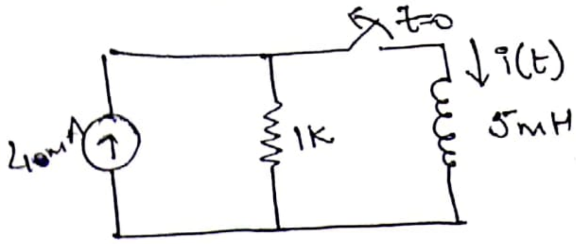
$2 = \cancel{16} e^{-2.5}\ V$

$\underline{v(t) = 8 - 6e^{-2.5}}$

$v(t) = 8 + (2-8)\, e^{-t/20}$



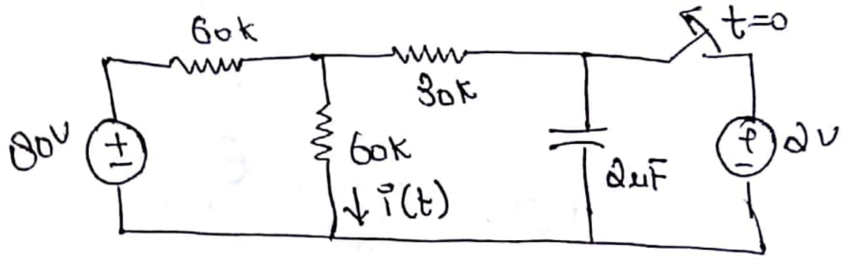
(2)



a) Find Inductor current after the switch until is ~~open~~ closed

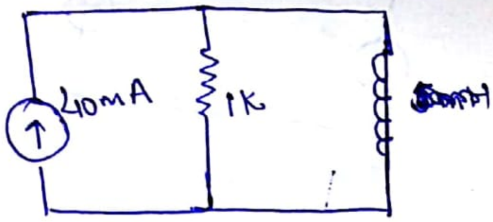
b) Find time to reach 2 mA across the inductor.

(3)

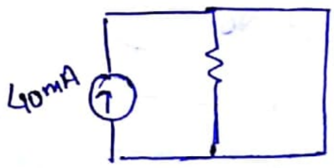


Find the current i(t) for t>0 ckt is in steady state for t<0

(2)

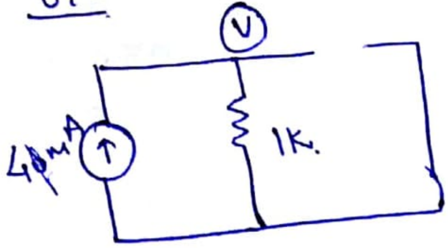


t<0



i=

t>0

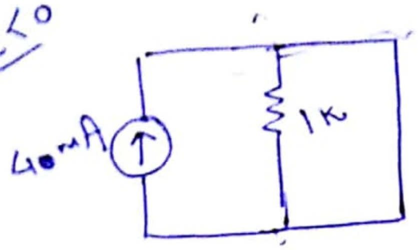


$v(0^+) = 40\ V$

$v(0^-) = 40V$

$$v(t)=v(\infty)+[v(0)-v(\infty)]e^{-t/\tau}$$

$t<0$



$v(0^-)=40\ V$

$\cancel{v(0^+)=4...}$

$(1-e^{-\frac{R}{L}t})$

at $t=0$

$i(0)=0$

$\tau=\frac{L}{R}$

$=5\mu s.$

$\cancel{t=t_0}$

$$i=i(\infty)+[i(0)-i(\infty)]e^{-t/\tau}$$

$$=4mA+[0-4]e^{-t/}$$

$$2\ mA=[4-4e^{-t/5\mu s}]\ mA$$

$$1-\frac{1}{2}=e^{-t/5\ \mu s}$$

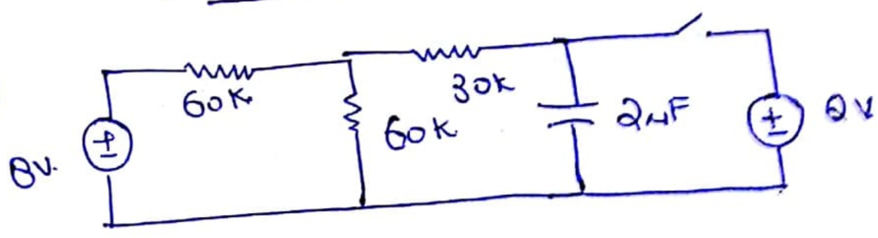
$$\frac{1}{2}=e^{-t/5}$$

$$e^{+t/5}=2$$

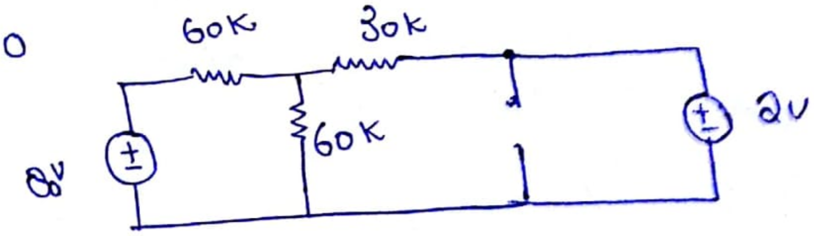
$$\frac{+t}{5}=\ln 2$$

$$t=5\ln 2\ \mu s$$

3



$t<0$

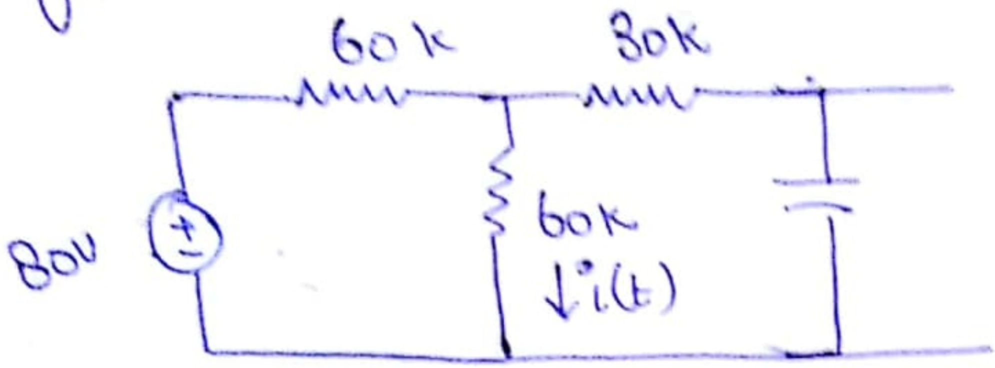


$R=60\ k$

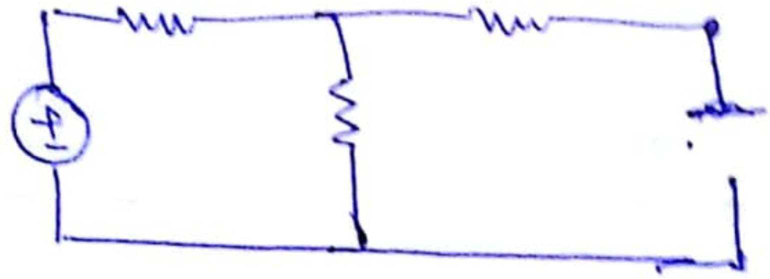
$$\tau=RC=60\times 2=120\times 10^{3}\times 10^{-6}$$

$$=120\ mS$$

for $t=0$



$t>0$



Source free series RLC ckt

$$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

a) when $(\alpha > \omega_0)$ Overdamped

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

b) Critically damped $(\alpha = \omega_0)$ $\qquad S_1 = S_2 = -\alpha = -\frac{R}{2L}$, $C = \frac{4L}{R^2}$

$$i(t) = A_1 e^{S_1 t} + A_2 t e^{S_1 t}$$

$$= (A_1 + A_2 t) e^{-\alpha t}$$

c) Underdamped $(\alpha < \omega_0)$

$$S_1 = -\alpha + j\omega_d$$

$$S_2 = -\alpha - j\omega_d$$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} \longrightarrow$ damping frequency

$$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

$$B_1 = A_1 + A_2$$

$$B_2 = j(A_1 + A_2)$$

Source free parallel RLC ckt

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\alpha = \frac{1}{2RC} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

a) Overdamped $(\alpha > \omega_0)$

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

b) Critically damped $(\alpha = \omega_0)$

$$v(t) = (A_1 + A_2 t) e^{-\alpha t}$$

c) Underdamped $(\alpha < \omega_0)$ $\qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$$S_1, S_2 = -\alpha \pm j\omega_d$$

$V(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t)$

Step response of a series RLC ckt

$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (Overdamped)

$v(t) = V_s + (A_1 + A_2 t)e^{-\alpha t}$ (Critically damped)

$v(t) = V_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t}$ (Underdamped)

Step response of parallel RLC ckt

$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (overdamped)

$i(t) = I_s + (A_1 + A_2 t)e^{-\alpha t}$ (Critically damped)

$i(t) = I_s + (A_1 \cos \omega_d t + A_2 \sin \omega_d t)e^{-\alpha t}$ (underdamped)

Qn $R = 40\,\Omega$ $L = 4H$ $C = \frac{1}{4}F$

Calculate the characteristic roots of the ckt. Is the natural response O, C, U?

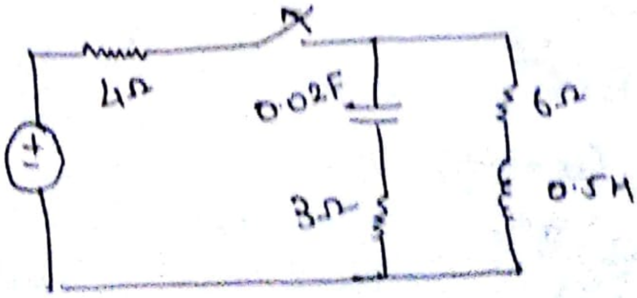
$\alpha = \frac{R}{2L} = \frac{40}{2 \times 4} = 5$ $\omega_0 = \frac{1}{\sqrt{LC}} = 1$

$s_1 = -5 \pm \sqrt{25-1}$
$= -5 \pm \sqrt{24}$

$s_1 = -0.101$
$s_2 = -9.89$

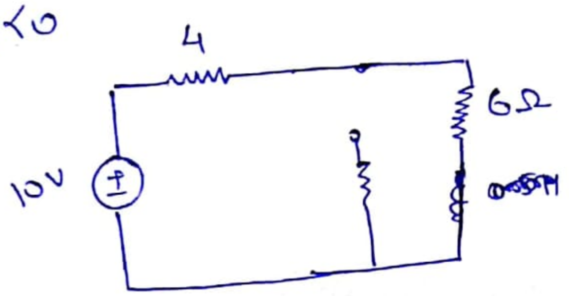
$i(t) = A_1 e^{(-5+\sqrt{24})t} + A_2 e^{(-5-\sqrt{24})t}$
$= A_1 e^{-0.101t} + A_2 e^{-9.89t}$

Qn



Find $i(t)$ assuming ckt has reached steady state at $t=0^-$

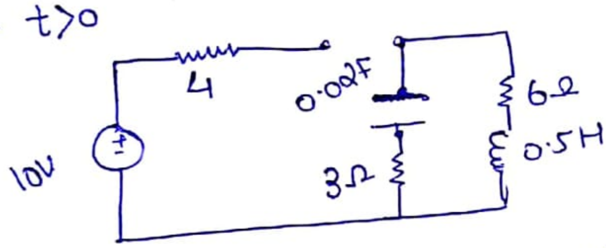
for $t<0$



$i(t)0^- = 1$ Amp.

$v(0^-) = 6$ V.

for $t>0$



$\alpha = \frac{R}{2L} = \frac{9}{1} = 9$

$\omega_0 = \frac{1}{\sqrt{\frac{2}{100} \times \frac{8}{16}}}$

$= 10$

$\alpha = 9$

$\omega_d = \sqrt{81-100}$

$= \sqrt{+19}$

~~$i(t) =$~~

$S_{1,2} = -9 \pm j\omega_d$

$S_{1,2} = -9 \pm j\sqrt{19}$

$i(t) = e^{-9t}(B_1 \cos\sqrt{19}t + B_2 \sin\sqrt{19}t)$

At $t=0$ $\quad i(0) = 1A$

$\underline{1 = B_1}$

for ckt, applying KVL

$6i + L\frac{di}{dt} + 3i + \underbrace{\frac{1}{C}\int i\,dt}_{=V_0} = 0$

$\frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$

$= -2(9\times1 - 6)$ Polarity reversed as now the cap. is discharging

$= -6$ A/s

$\frac{di(t)}{dt} = (B_1\cos\sqrt{19}t + B_2\sin\sqrt{19}t)(-9)e^{-9t} +$

$e^{-9t}(-\sqrt{19}B_1 \sin\sqrt{19}t + B_2\sqrt{19}\cos\sqrt{19}t)$

$$\frac{di(0)}{dt} = -9B_1 + \sqrt{19}B_2$$

$$-6 = -9 \times 1 + \sqrt{19}B_2$$

$$\frac{3}{\sqrt{19}} = B_2$$

$$\underline{B_2 = 0.6882}$$

$$\therefore \underline{i(t) = e^{-9t}\left(\cos\sqrt{19}t + 0.6882\sin\sqrt{19}t\right)}$$

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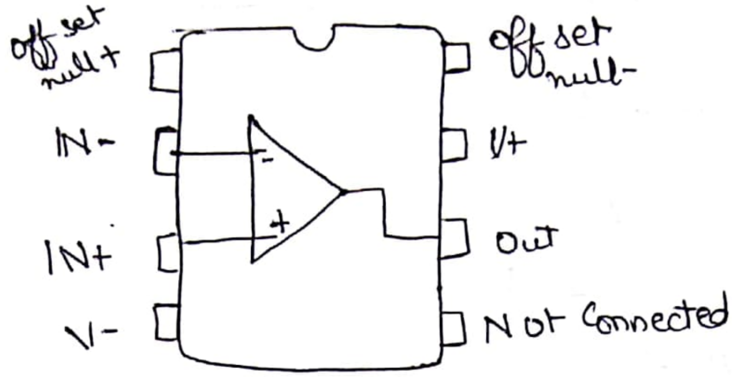
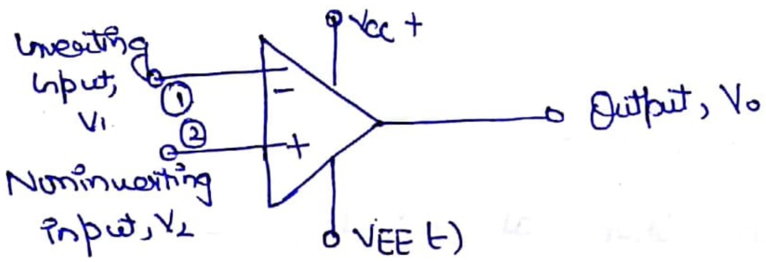
Operational Amplifiers (Op-amp)



Dual in-line package - 8 pin

741

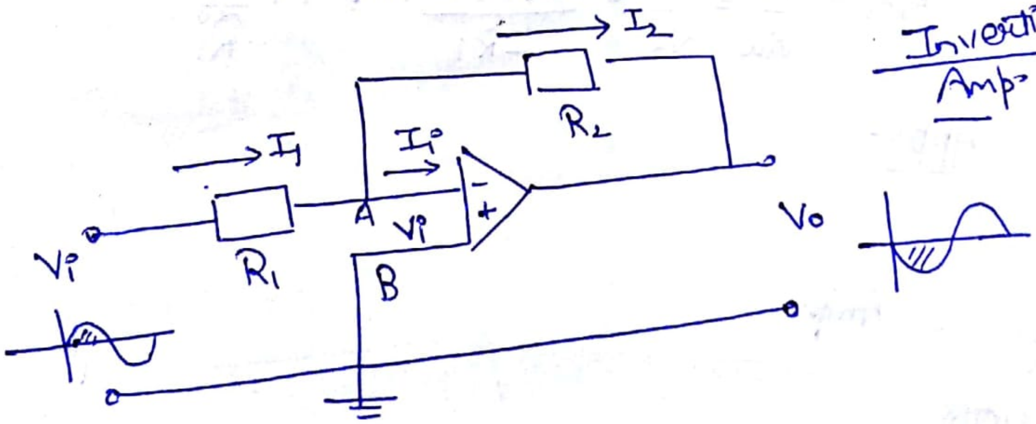
Schematic symbol of op-amp



Open loop Voltag gain

$$A_{OL} = \frac{V_o}{V_d}$$

Inverting Amp.



$I_1 = \frac{V_1}{R_1}$ $\quad I_2 = \frac{-V_o}{R_f}$ $\quad$ Since $I_i = 0$

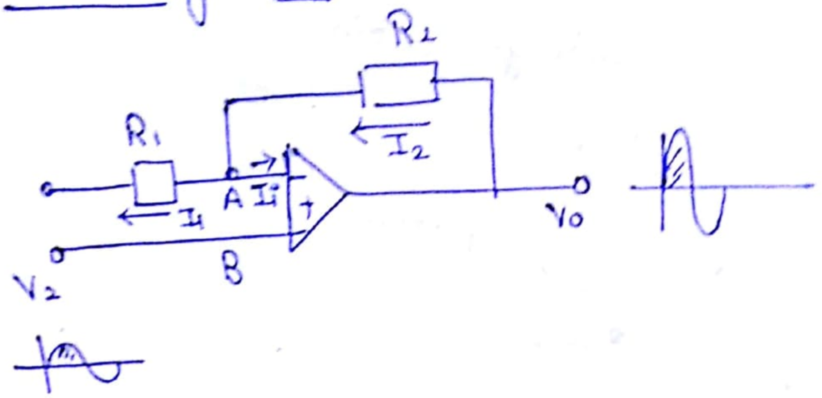
$\therefore \underline{I_1 = I_2}$

$$\frac{V_i}{R_1} = -\frac{V_o}{R_f} \quad \therefore \boxed{\frac{V_o}{V_i} = -\frac{R_f}{R_1}} \sim \text{amplification factor}$$

Closed loop gain :

$$\boxed{A_{CL} = -\frac{R_f}{R_1}}$$

Noninverting Amp.



★ Since $I_i = 0 \quad \therefore I_1 = I_2 = I$

★ Due to virtual short at input terminal

$$V_2 = V_B = V_A = IR_1$$

$$\boxed{V_o = I(R_1 + R_2)}$$

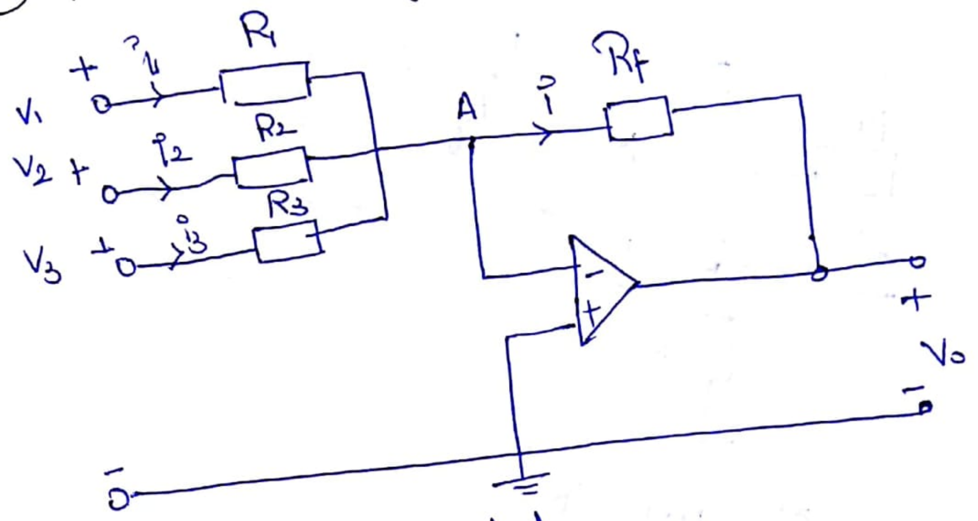
$R_f = R_2$

$$A_{CL} = \frac{V_o}{V_{in}} = \frac{V_o}{V_2} = \frac{I(R_1+R_2)}{IR_1} = 1 + \frac{R_f}{R_1}$$

Simple Apps-

1. Adder
2. Difference Amp.
3. Subtractor
4. Voltage follower

① Adder or Summing ckt



A is virtually grounded

$i_1 = \frac{V_1}{R_1}$ - - - - $i_n = \frac{V_n}{R_n}$

$i = -\frac{V_o}{R_f} = 0 - \frac{V_o}{R_f}$

Applying KCL

$i_1 + i_2 + i_3 - - i_n = i$

$\therefore \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_o}{R_f}$ —— (three)

$\therefore V_o = -\left\{ \frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 \right\}$

$V_o = -\boxed{K_1 V_1 + K_2 V_2 + K_3 V_3}$

if $R_1 = R_2 = R_3 = R_f$

$\therefore \boxed{V_o = -(V_1 + V_2 + V_3)}$

if $R_1 = R_2 = R_3 = R$

$V_o = -\frac{R_f}{R}(V_1 + V_2 + V_3)$

$\boxed{V_o = -A_{CL}(V_1 + V_2 + V_3)}$

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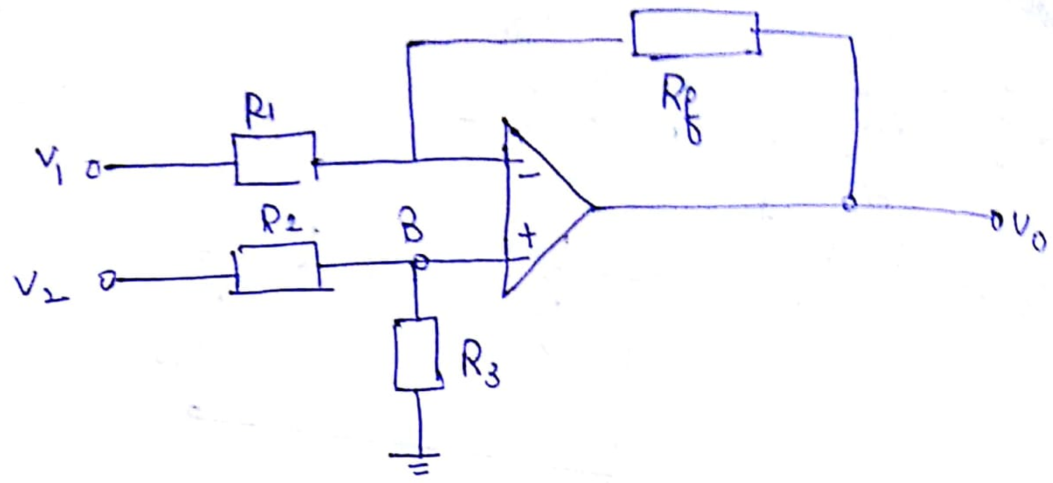
(2) Difference Amp.



Voltage at point B

$$V_B = \frac{R_3}{R_2+R_3} V_2$$

★ It is a combination of inverting & non-inverting amplifiers

★ ~~wh~~ when $V_1 = 0 \longrightarrow$ non-inverting

★ When $V_2 = 0$, $\longrightarrow$ inverting

★ We shall use superposition theorem to determine output whe both the inputs are present

When $V_2 = 0$, R_2 & R_3 are in ||, carrying no current

$$\boxed{\therefore V_{o1} = -\frac{R_f}{R_1} v_1}$$

When $V_1 = 0$, $\therefore \boxed{V_B = \frac{R_3}{R_2+R_3} V_2}$

$$V_{o2} = \left(1+\frac{R_f}{R_1}\right) V_B$$

$$= \left(1+\frac{R_f}{R_1}\right)\left(\frac{R_3}{R_2+R_3}\right) V_2$$

we make $R_1 = R_2$ & $R_f = R_3$

$$V_{o2} = \frac{R_f}{R_1} V_2$$

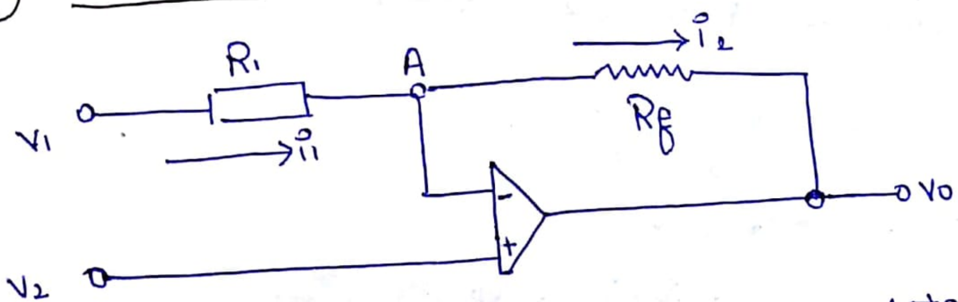
$\therefore \quad V_o = V_{o1} + V_{o2}$

$$\boxed{V_o = -\frac{R_f}{R_1}(V_1 - V_2)}$$

$$A_D = \frac{V_o}{V_1 - V_2} = -\frac{R_f}{R_1}$$

differential gain

③ Subtractor



Bcoz of virtual short A is at same potential as V_2

$$i_1 = i_2$$

$$\frac{V_1 - V_2}{R_1} = \frac{V_2 - V_o}{R_f}$$

$$\boxed{\frac{V_o}{R_f} = \frac{V_2}{R_f} + \frac{V_2 - V_1}{R_1}}$$

$$V_o = V_2\left(1 + \frac{R_f}{R_1}\right) - \frac{R_f}{R_1} V_1$$

If $R_f >> R_1$

$$\therefore V_o = \frac{R_f}{R_1}(V_2 - V_1)$$

$$\boxed{V_o = A_{CL}(V_2 - V_1)}$$

Q. $R_1 = 20\,k\Omega$ $\quad R_f = 2\,M\Omega$

$$A_{OL} = -\frac{R_f}{R_1} = -\frac{\cancel{2}\times 10^{\cancel{6}3}}{\cancel{20}\times 10^{\cancel{3}}} = -100$$

Input Resistance $= \infty$

Output " $= 0$

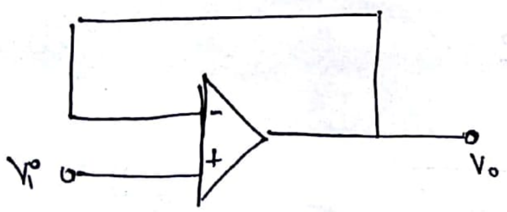
Q. $R_f = 200\,k\Omega$ $\quad R_1 = 2\,k\Omega$ $\quad V_1 = 4V$ $\quad V_2 = 8.8V$

$$V_o = -\frac{200}{\cancel{2}}\left(\frac{0.\cancel{2}}{10}\right)$$

$$= -20\ V$$

$$V_o = V_2\left(1 + \frac{R_f}{R_1}\right) - V_1\frac{R_f}{R_1}$$

<u>Voltage follower</u>

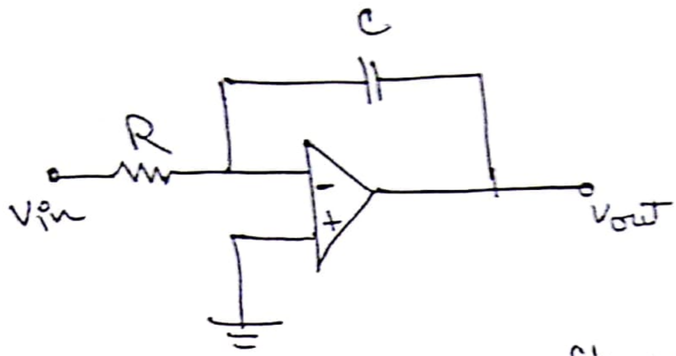


$$A_{CL} = 1 + \frac{R_f}{R_1}$$

The lowest gain possible is 1, for which $R_f = 0$ (shorting) & $R_1 = \infty$ (opening)

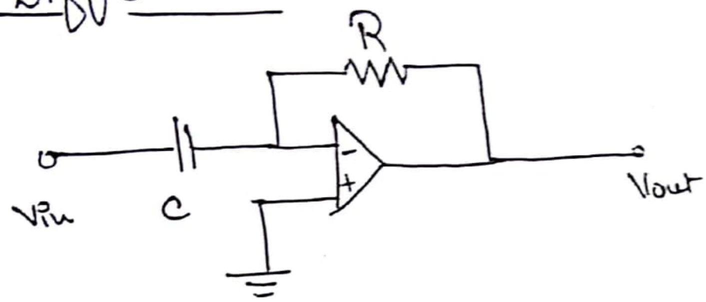
(Buffer amplifier)

Integrating using op-amp.



$$V_{out} = \int_0^t -\frac{V_{in}}{RC}\, dt + V_{initial}$$

Differentiator



$$V_{out} = -RC\left(\frac{dV_{in}}{dt}\right)$$

Common Mode Rejection Ratio (CMRR)

$V_0 = A_D V_D + A_C V_C$ — Common mode gain

A_D ↓ differential gain

$\therefore$ $CMRR = \frac{A_d}{A_c}$ (ideally $A_c = 0$)

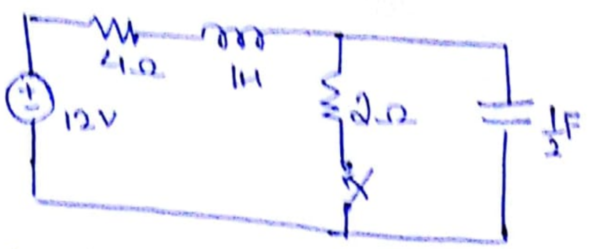
$\therefore$ Ideally $CMRR = \infty$ but this does not happen.

$$V_0 = A_d V_d\left(1 + \frac{A_c V_c}{A_d V_d}\right) = A_d V_d\left[1 + \frac{V_c}{V_d\, CMRR}\right]$$

★ Slew rate

★ Input bias current

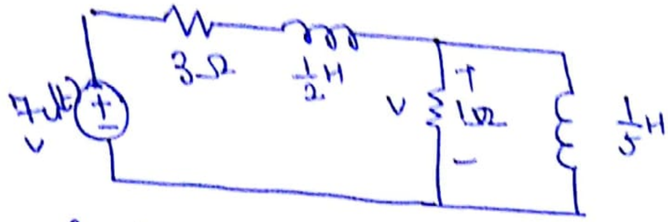
(1)



$k_1 + k_2 e^{-t/\tau}$

Find the complete response of the ckt 'v' & then i for $t>0$

(2)



find $v_o(t)$ for $t>0$

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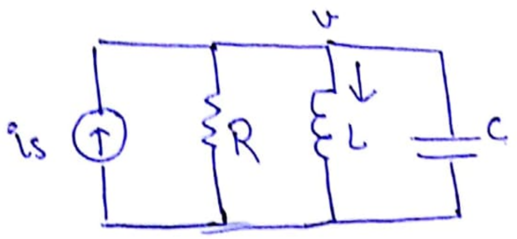
(1)



$$i_R + i_L + i_C = i_s$$

$$\frac{V}{R} + \frac{1}{L}\int_{t_0}^{t} v\,dt + i_L(t_0) + C\frac{dv}{dt} = i_s$$

$$\frac{dv}{R\,dt} + \frac{1}{L}v + C\frac{dv^2}{dt^2} = \frac{di_s}{dt}$$

0^-

$$C\frac{dv^2}{dt^2} + R\frac{dv}{dt} + \frac{v}{L} = \frac{di_s}{dt}$$

$$\frac{dv^2}{dt^2} + \frac{R}{C}\frac{dv}{dt} + \frac{v}{LC} = \frac{di_s}{dt}$$

for $i_s = 1$ $\quad \frac{di_s}{dt} = 0$

$$s^2 + 2\alpha s + \omega^2 = 0$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad \alpha = \frac{R}{2C}$ $\quad \omega_0 = \frac{1}{\sqrt{10\times10^{-3}\times1\times10^{-3}}} = \frac{1}{\sqrt{10^{-5}}}$