Signals And Systems

For $x(t)$ indicated in Figure P1.6, sketch the following:

(a) $x(-t)$

(b) $x(t+2)$

(c) $x(2t+2)$

(d) $x(1-3t)$

Figure P1.6

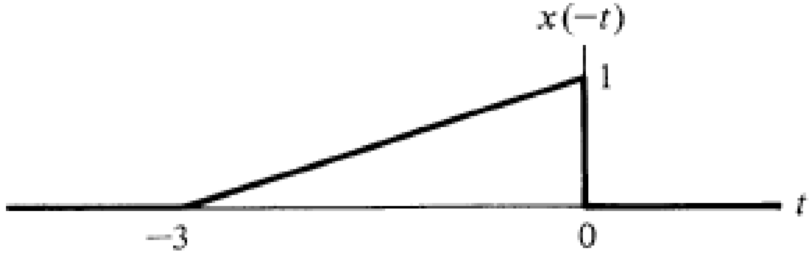


Figure S1.6-1

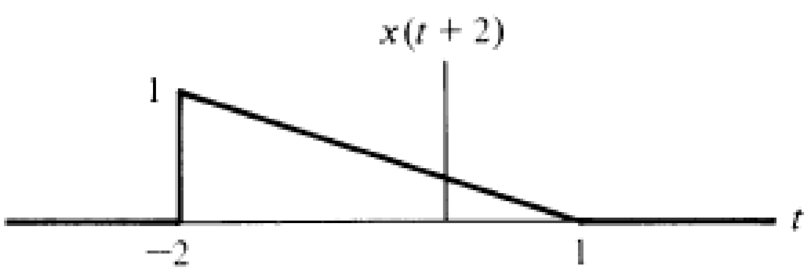


Figure S1.6-2

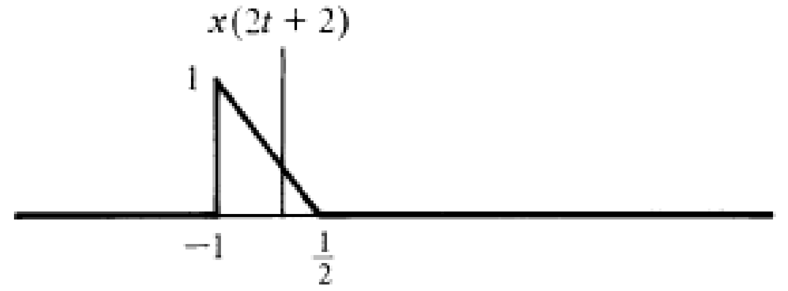


Figure S1.6-3

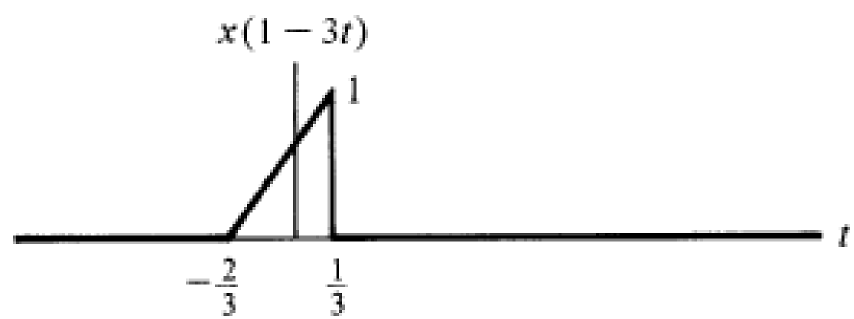


Figure S1.6-4

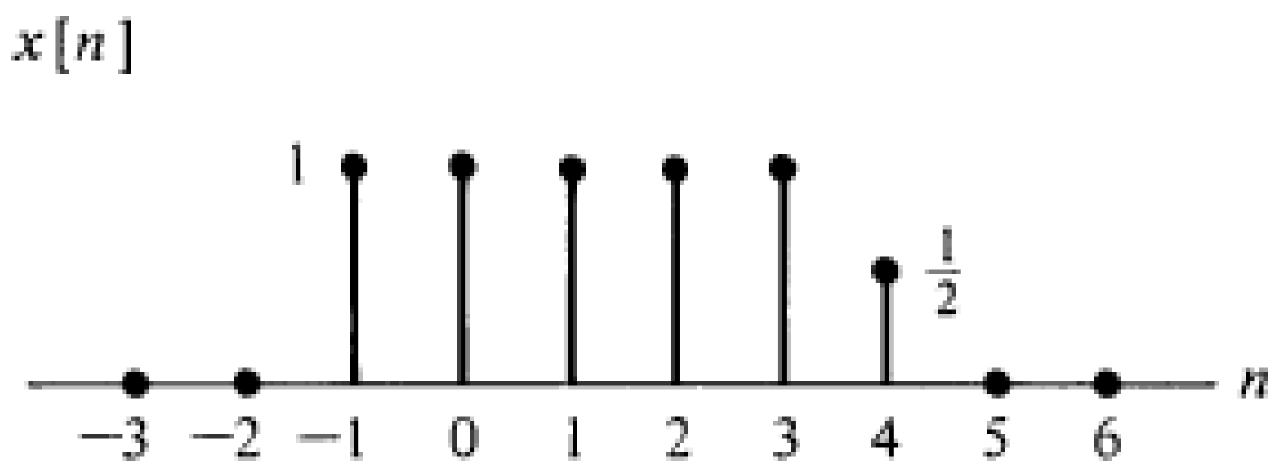


Figure P2.3

Sketch and carefully label each of the following signals:

(i) $x[n-2]$

(ii) $x[4-n]$

(iii) $x[2n]$

What difficulty arises when we try to define a signal as $x[n/2]$?

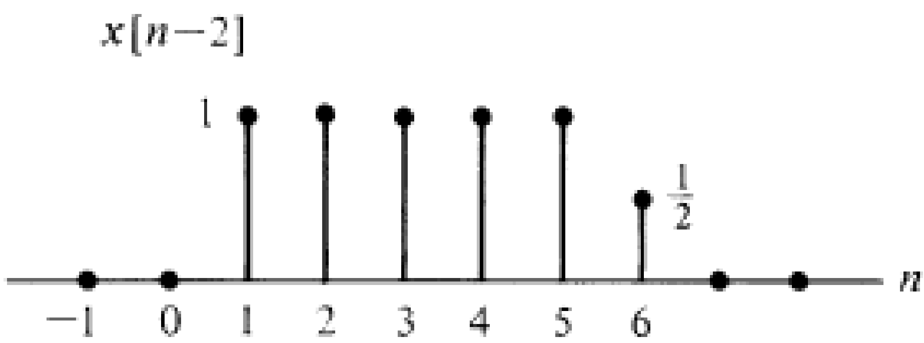


Figure S2.3-1

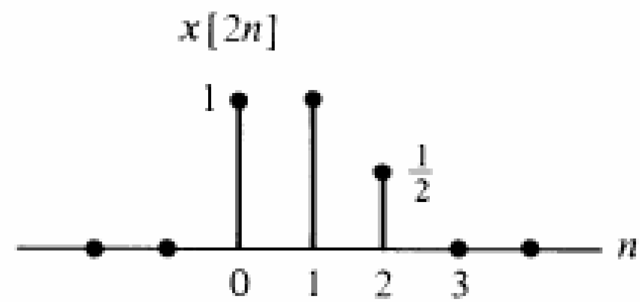


Figure S2.3-3

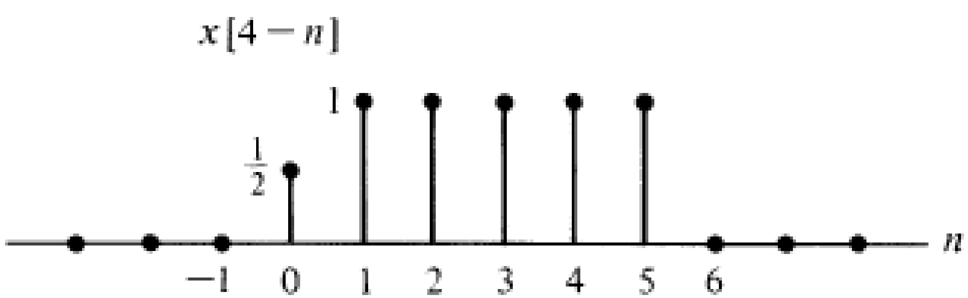


Figure S2.3-2

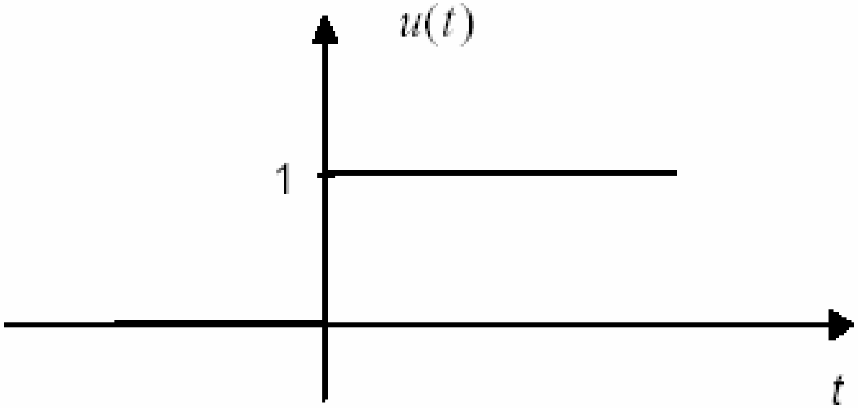
The difficulty arises when we try to evaluate $x[n/2]$ at $n = 1$, for example (or generally for n an odd integer). Since $x[\frac{1}{2}]$ is not defined, the signal $x[n/2]$ does not exist.

Some Important Continuous and Discrete Time Signals

Continuous Time Impulse and Step functions

The continuous-time *unit step* function $u(t)$ is defined as follows:

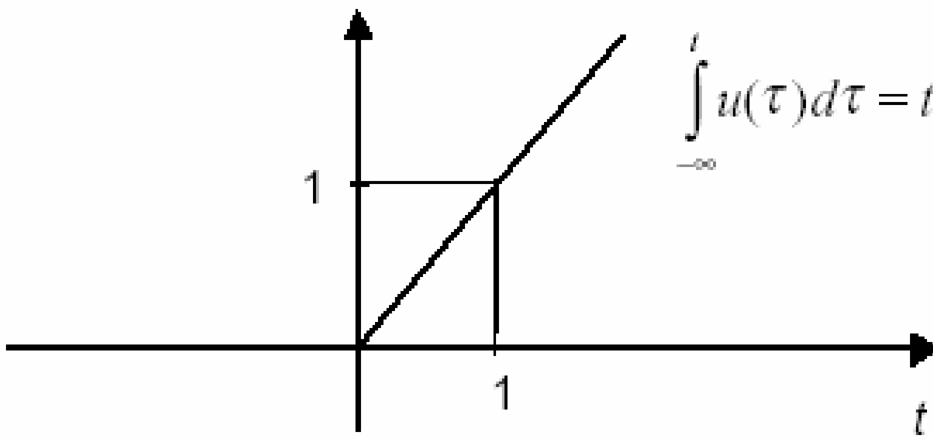
$$u(t) := \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



Note that since *u(t)* is discontinuous at the origin, it can't be formally differentiated.

Integrals of $u(t)$

The first integral of $u(t)$ is a unit ramp function starting a t=0.



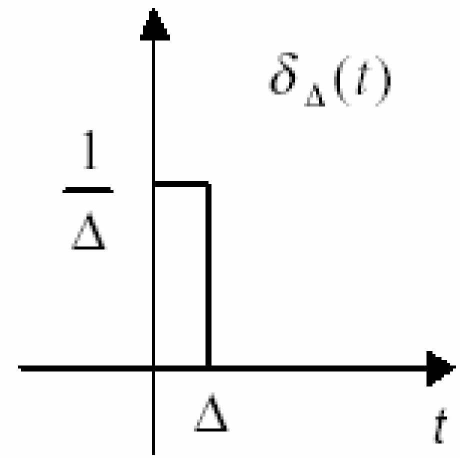
$$\int_{-\infty}^{t} u(\tau)d\tau = tu(t)$$

Successive integrals of $u(t)$ are:

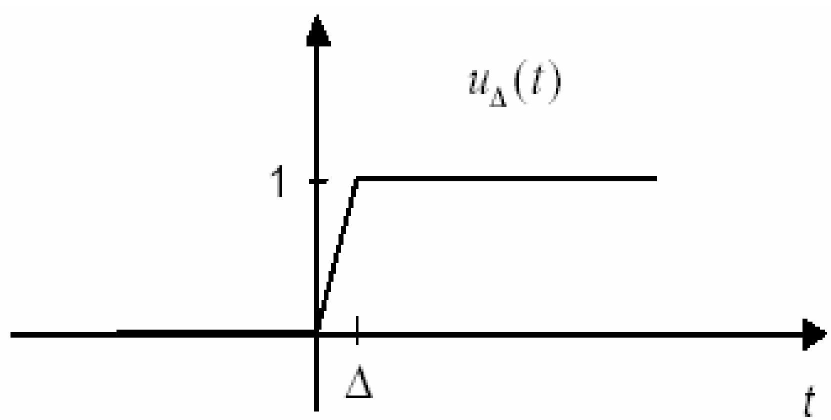
$$\int_{-\infty}^{t}\int_{-\infty}^{\tau_{k-1}}\cdots\int_{-\infty}^{\tau_1} u(\tau)d\tau d\tau_1 \cdots d\tau_{k-1} = \frac{1}{k!}t^k u(t)$$

- The unit impulse, a generalized function, can be defined as follows. Consider a pulse function of unit area:

$$\delta_\Delta(t) := \begin{cases} \dfrac{1}{\Delta}, & 0 \le t < \Delta \\ 0, & \text{otherwise} \end{cases}$$



The integral of this pulse is an approximation to the unit step:



As Δ tends to 0, The pulse $\delta_\Delta(t)$ gets taller and thinner, but keeps its unit area, while $u_\Delta(t)$ approaches a unit step function. At the limit,

$$\delta(t) := \lim_{\Delta \to 0} \delta_\Delta(t)$$

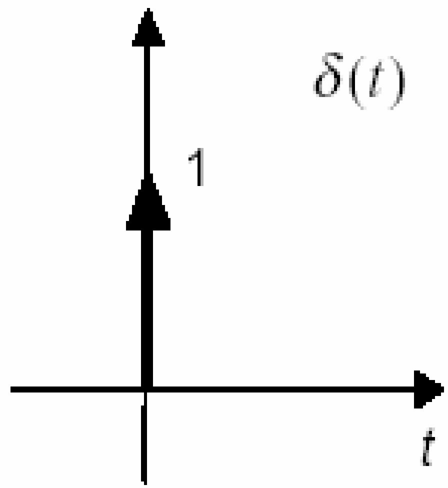
$$u(t) := \lim_{\Delta \to 0} u_\Delta(t)$$

Note that $\delta_\Delta(t) = \frac{d}{dt} u_\Delta(t)$, and in this sense we can write $\delta(t) = \frac{d}{dt} u(t)$ at the limit.

Conversely, we have the important relationship

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

Graphically, $\delta(t)$ is represented by an arrow "pointing to infinity" at t=0 with its length equal to its area.

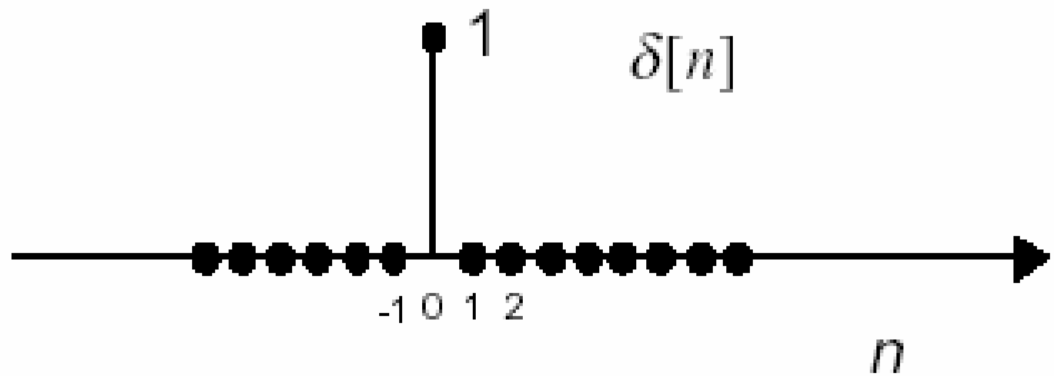


Note that with definition, the area of the impulse is "to the right" of t=0, so that integrating $A\delta(t)$ from t=0 will give A, i.e., $\int_0^\infty A\delta(t)dt = A$. Had we defined the impulse as the limit of the pulse $\tilde{\delta}_\Delta(t) = \frac{1}{\Delta}[u(t+\Delta) - u(t)]$ whose area lies to the left of t=0, then we would have obtained $\int_0^\infty A\delta(t)dt = 0$. To "catch the impulse" the trick is then to integrate from the left of the y-axis, but infinitesimally close to it. This time is denoted as t=0^-. There is a similar definition for t=0^+ to the right of the y-axis, so that for our original definition of $\delta(t)$, the above integral would have evaluated to $\int_{0^+}^\infty A\delta(t)dt = 0$.

Discrete Time Impulse and Step functions

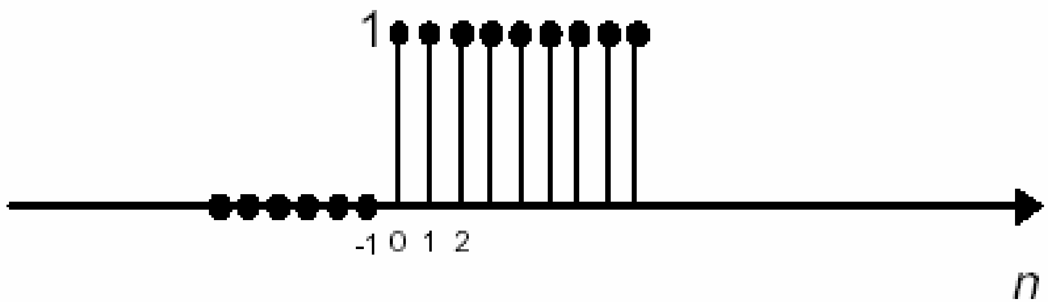
- One of the simplest discrete-time signals is the unit impulse, defined by

$$\delta[n] := \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



- The discrete-time unit step is defined by

$$u[n] := \begin{cases} 1, n \geq 0 \\ 0, n < 0 \end{cases}$$



The unit step is the running sum of an impulse:

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

and conversely, the impulse is the first-difference of a unit step

$$\delta[n] = u[n] - u[n-1]$$ Also, $$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

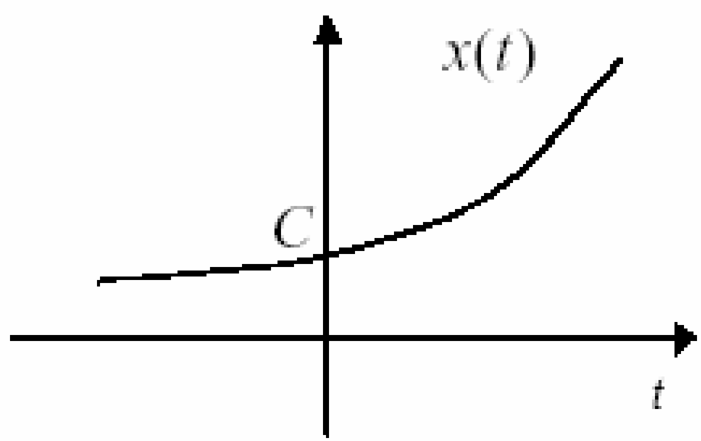
Real Exponential signals

- Continuous-Time

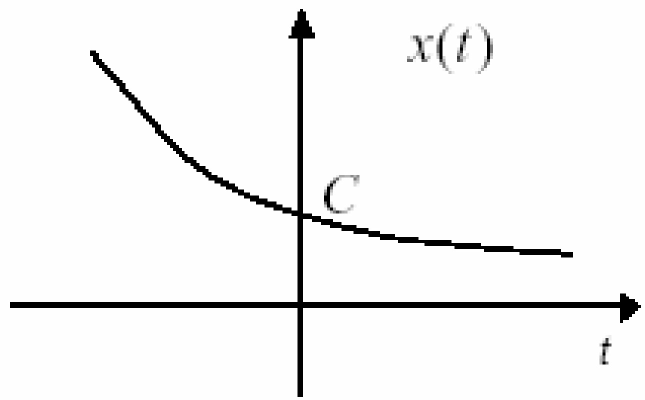
$$x(t) = Ce^{at}, \quad C, a \text{ real}$$

Case a=0: We simply get the constant signal $x(t) = C$.

Case a>0: The exponential tends to infinity as $t \to \infty$ (here C>0).



Case a<0: The exponential tends to zero as $t \to \infty$ (here C>0).



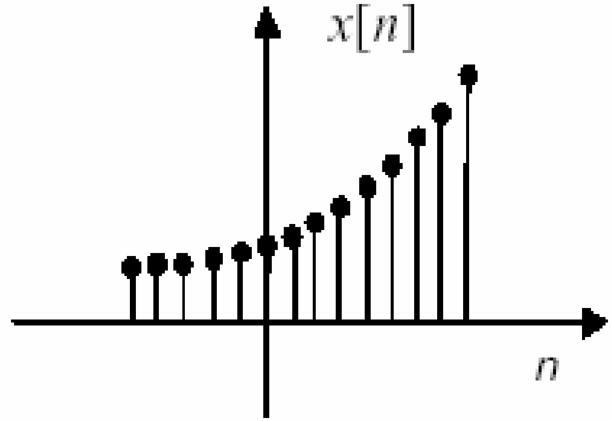
- Discrete-Time

$$x[n] = C\alpha^n, \quad C, \alpha \text{ real}$$

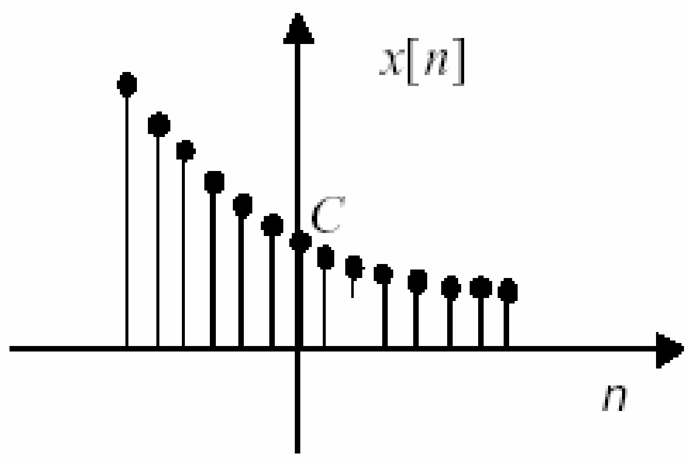
There are six cases to consider (apart from the trivial case $\alpha = 0$): $\alpha = 1$, $\alpha > 1$, $0 < \alpha < 1$, $\alpha < -1$, $\alpha = -1$ and $-1 < \alpha < 0$. Here we assume that C>0.

Case $\alpha = 1$: We get a constant signal $x[n] = C$.

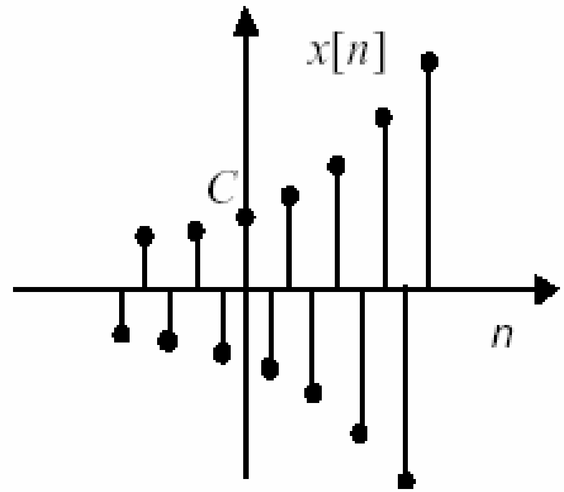
Case $\alpha > 1$: We get a positive signal that grows exponentially.



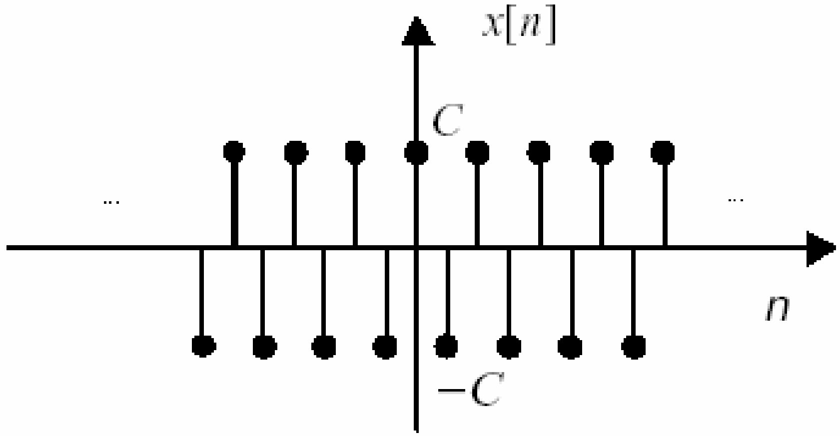
Case $0 < \alpha < 1$: The signal is positive and decays exponentially.



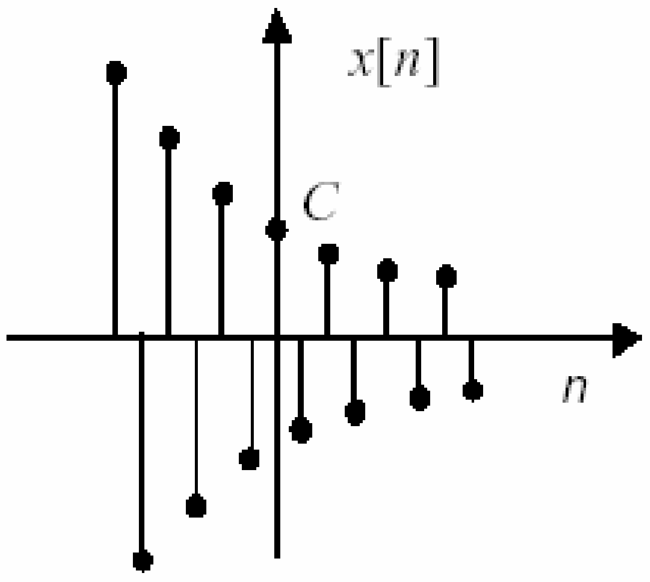
Case $\alpha < -1$: The signal alternates between positive and negative values and grows exponentially.



Case $\alpha = -1$: The signal alternates between +C and -C.



Case $-1 < \alpha < 0$: The signal alternates between positive and negative values and decays exponentially.



Complex Exponential Signals

- Continuous-Time

$$x(t) = Ce^{at}$$

C, a complex, $\quad C = |C|e^{j\theta}, \quad a = r + j\omega_0$

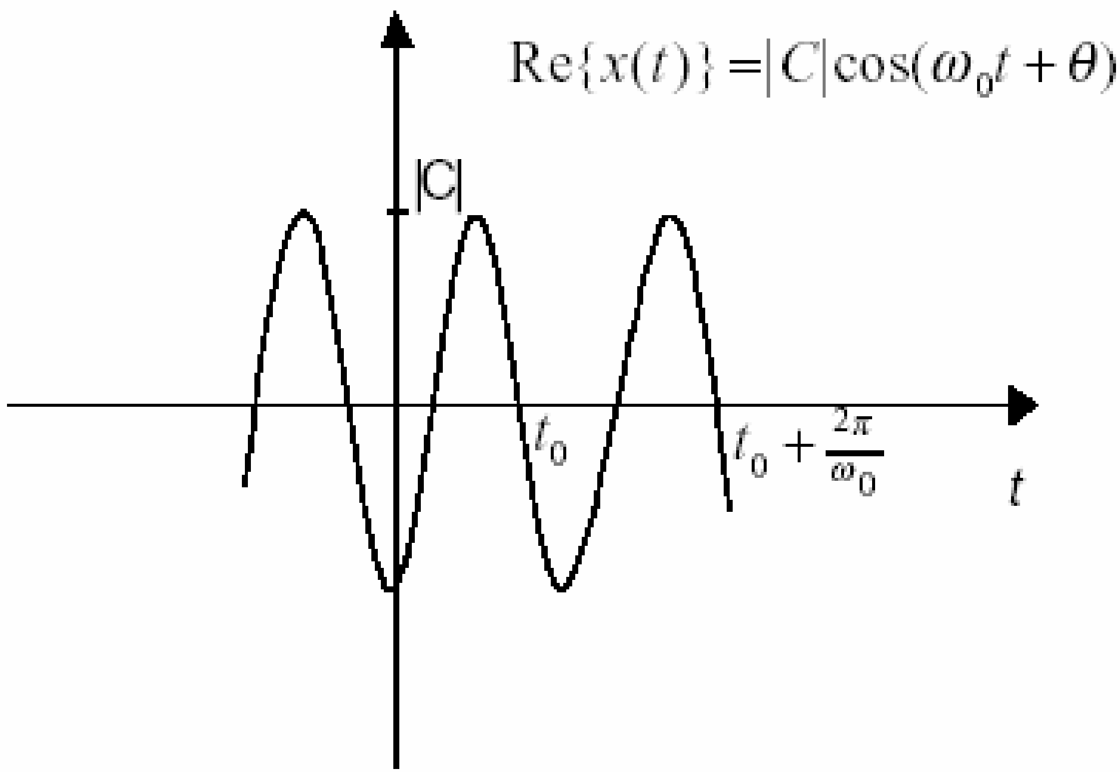
$$x(t) = Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = |C|e^{rt}e^{j(\omega_0 t+\theta)}$$

Using Euler's relation, we get

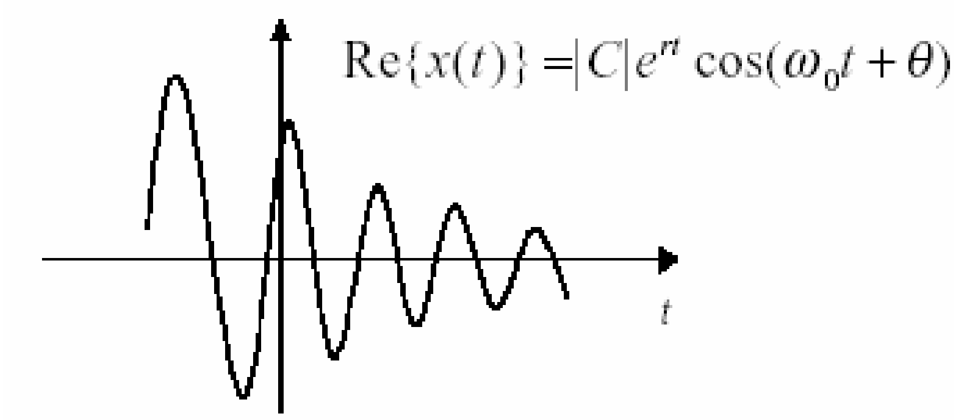
$$x(t) = |C|e^{rt}\cos(\omega_0 t + \theta) + j|C|e^{rt}\sin(\omega_0 t + \theta)$$

For $r=0$, we obtain a complex periodic signal of period $T=\dfrac{2\pi}{\omega_0}$ whose real and imaginary parts are sinusoidal

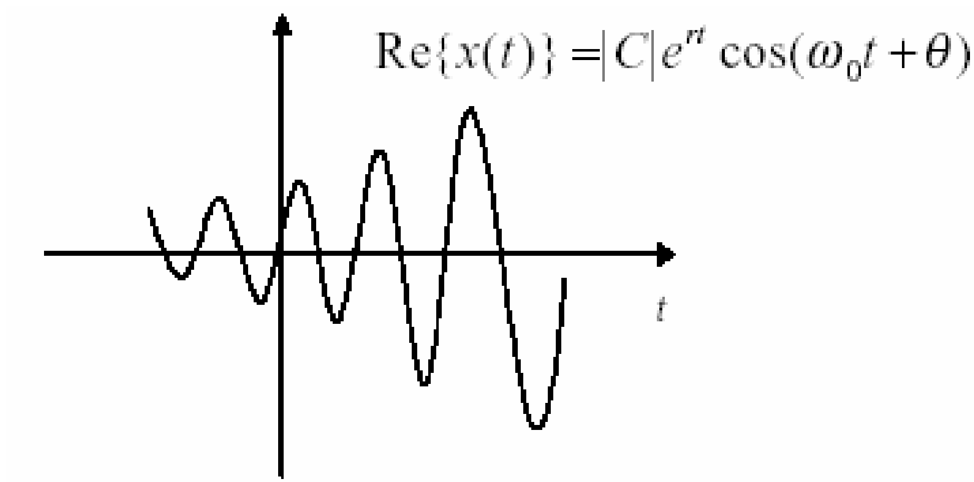
$$x(t)=|C|e^{j\theta}e^{j\omega_0 t}=|C|\cos(\omega_0 t+\theta)+j|C|\sin(\omega_0 t+\theta)$$



- For r<0, we get a complex periodic signal multiplied by a decaying exponential whose real and imaginary parts are "damped sinusoids"



For r>0, we get a complex periodic signal multiplied by a growing exponential



Discrete-Time

$$x[n] = C\alpha^n$$

C, α complex, $\quad C = |C|e^{j\theta}, \quad \alpha = |\alpha|e^{j\omega_0}$

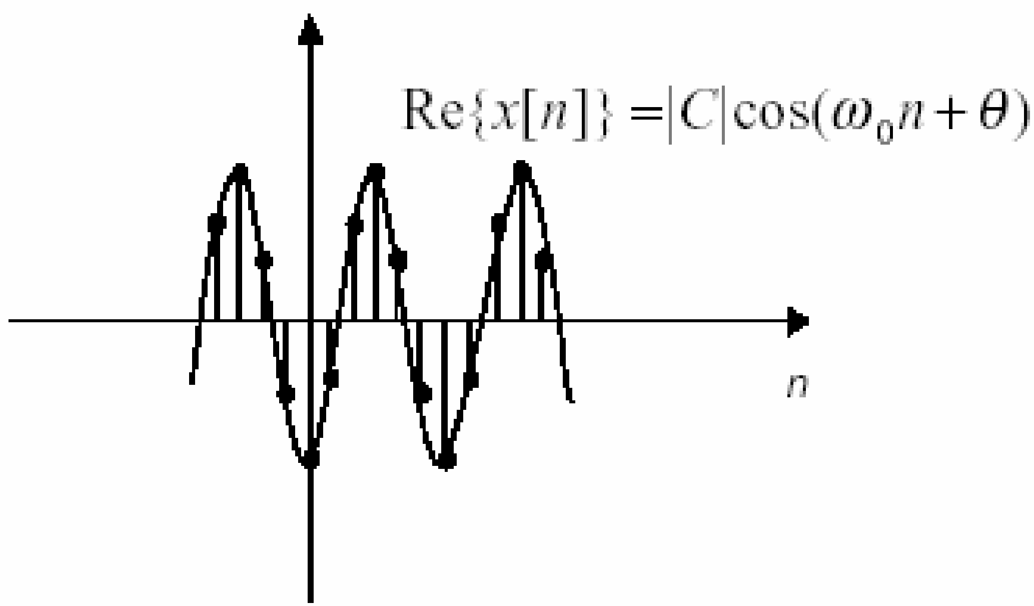
$$x[n] = C\alpha^n = |C|e^{j\theta}|\alpha|^n e^{j\omega_0 n} = |C||\alpha|^n e^{j(\omega_0 n + \theta)}$$

Using Euler's relation, we obtain

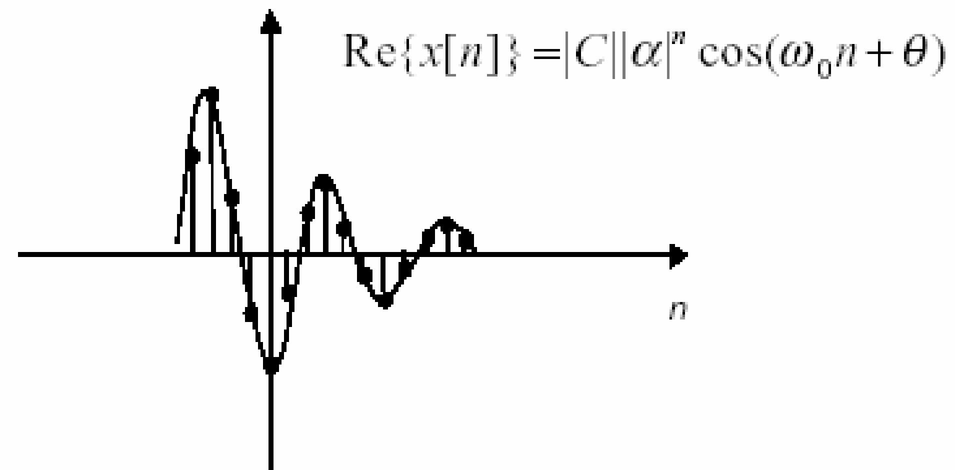
$$x[n] = |C||\alpha|^n \cos(\omega_0 n + \theta) + j|C||\alpha|^n \sin(\omega_0 n + \theta)$$

- For |a|=1, we obtain a complex signal whose real and imaginary parts are sinusoidal, but not necessarily periodic!

$$x[n] = |C|\cos(\omega_0 n + \theta) + j|C|\sin(\omega_0 n + \theta)$$



- For | a|<1, we get a complex signal whose real and imaginary parts are damped sinusoidal sequences.



For | a|>1, we obtain a complex signal whose real and imaginary parts are growing sinusoidal sequences