Department of Mathematics

10B11MA111 **Mathematics-I**

Tutorial Sheet 2 **B.Tech. Core**

Taylor's series, Maxima and Minima, Jacobians

1. Expand the following functions in a Taylors'series upto third degree terms: (i) $e^x \cos y$ about (0, 0) (ii) $\tan^{-1}(y/x)$ about (1, 1) and (iii) $x^3+3y^3-xy^2$ about the point (1,-1).
2. Obtain the second order Taylor's series approximation to the function $f(x,y)=e^x \sin y$ about the point (0, 0). Find the maximum absolute error in the region $|x|\le 0.1,\ |y|\le 0.1$.
3. Expand $f(x,y)=\sqrt{x+y}$ in Taylor's series up to second order terms about the point (1, 3) and hence evaluate $f(1.1,\ 2.9)$. Estimate the maximum absolute error in the region $|x-1|\le 0.2,\ |y-3|\le 0.1.$
4. Find all the extreme points of the given functions and classify them:
   (i) $x^4+y^4-y^2-x^2+1$ (ii) $(x^2+y^2)e^{6x+2x^2}$ (iii) $6x^2-2x^3+3y^2+6xy$.
5. Find the absolute maxima and minima of the $f(x,y)=2x^2-4x+y^2-4y+1$ on the closed triangular plate bounded by the lines $x=0, y=2, y=2x$.
6. The temperature at a point (x, y) on a metal plate is $T(x,y)=4x^2-4xy+y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the ant?
7. Use Lagrange's method to
   (a) Find the largest possible value of the product $\{xyz, x,y,z>0\}$ if $x+y+z^2=16$.
   (b) Find the point closest to the origin on the line of intersection of the planes $y+2z=12$ and $x+y=6$.
   (c) minimum value of $x^2+y^2+z^2$ subject to the conditions $ax+by+cz=1, \alpha x+\beta y+\gamma z=1$.
8. Show that $\dfrac{\partial(u,v)}{\partial(x,y)}\times\dfrac{\partial(x,y)}{\partial(u,v)}=1.$
9. If $u=\dfrac{(x+y)}{(1-xy)}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\dfrac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find the relationship.
10. Using Jacobians, show that the functions $u=(x+y)/z$, $v=(y+z)/x$, $w=y(x+y+z)/xz$ are not independent. Also find a relation among them.

Answers: **1.(i)** $1+x+(x^2-y^2)/2+(x^3-3xy^2)/6+\ldots$, **(ii)** $(\pi/4)+\left(-(x-1)/2+(y-1)/2\right)+\left\{(x-1)^2/4-(y-1)^2/4\right\}+1/12\left\{-(x-1)^3-3(x-1)^2(y-1)+3(x-1)(y-1)^2+(y-1)^3\right\}$
(iii) $-3+\{2(x-1)+11(y+1)\}+\{3(x-1)^2+2(x-1)(y+1)-10(y+1)^2\}+\{(x-1)^3-1/3(x-1)(y+1)^2+3(y+1)^3\}$
(2) 0.00147 or 0.000814 (without using B) **(3)** $|Error|\le 0.64\times10^{-4}$, **(4) (i)** Loc.max at (0,0), loc.min. at $(\pm1/\sqrt{2},\pm1/\sqrt{2})$, **(ii)** Loc. Min. at (0,0) and (-1,0), loc. Max. at (-1/2,0). **(iii)** Loc. Min. at (0,0) and saddle point at (1,-1), **(5)** Min at (1,2) and max. at (0,0), **(6)** Lowest temp. is $T\left(\sqrt{5},2\sqrt{5}\right)=0^0=T\left(-\sqrt{5},-2\sqrt{5}\right)$ and highest temp. is $T\left(2\sqrt{5},-\sqrt{5}\right)=125^0=T\left(-2\sqrt{5},\sqrt{5}\right)$, **(7) (a)** $f(32/5,32/5,4/\sqrt{5})=4096/25\sqrt{5}$, **(b)** (2, 4, 4), 6, **(c)** $\sum(a-\alpha)^2/\sum(a\beta-b\alpha)$, **(9)** 0, Yes, $u=\tan v$ **(10)** $uv=w+1$.