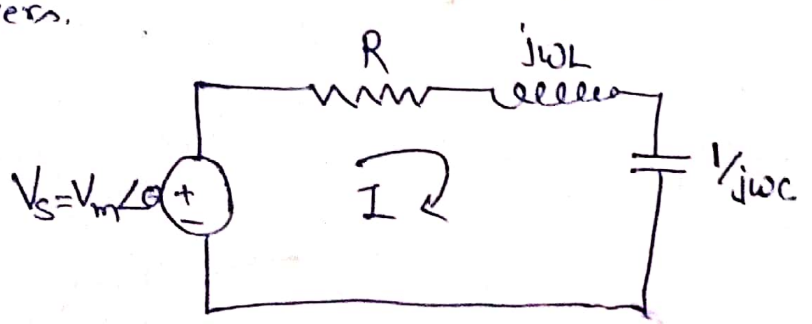
Series Resonance:- The most prominent feature of the frequency response of a Circuit may be the Sharp peak exhibited in its amplitude Characteristic. The Concept of resonance applies in several areas of Science and engineering.

Resonance Occurs in any System that has Complex Conjugate pair of Poles. It is the Cause of oscillations of Stored energy from one form to another. Resonance Occurs in any Circuit that has at least one inductor and one Capacitor.

Resonance is a Condition in an RLC Circuit in which the Capacitive and inductive Reactances are equal in magnitude, thereby resulting in a Purely resistive impedance.

Resonant Circuits are useful for Constructing filters, as their transfer functions Can be highly frequency selective. They are used in many applications such as selecting the desired Stations in TV and receivers.



$$Z = H(\omega) = \frac{V_s}{I} = R + j\omega L + \frac{1}{j\omega C} \quad -(1)$$

or

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) - (2)$$

Resonance occurs, If the Imaginary Part of the transfer function or Impedance is,

$$Im(Z) = \omega L - \frac{1}{\omega C} = 0 \quad -(3)$$

The Value of ω that Satisfies this Condition, is Called the resonant frequency ω_0. Thus, the resonance Condition is-

$$\omega_0 L = \frac{1}{\omega_0 C}$$

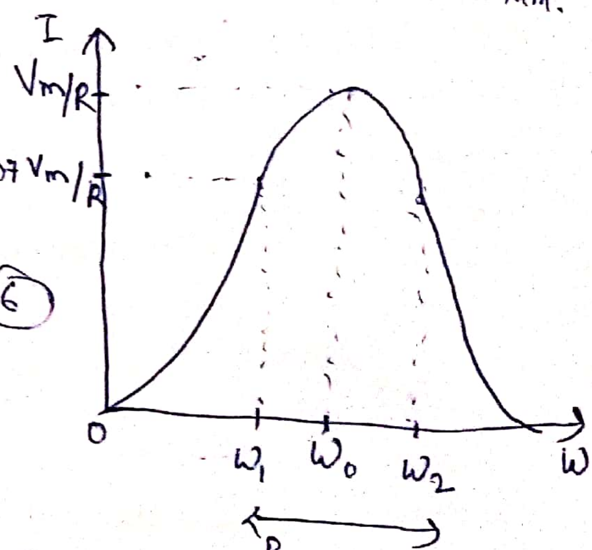
or $$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/Sec}} \quad -(4)$$

or $$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}} \quad -(5)$$

Note that at resonance:-

1. The impedance is purely resistive, thus, $Z = R$. In other words LC combination acts like a short circuit, and the entire voltage is across R.
2. The voltage V_s and I are in phase, so that the power factor is unity.
3. The magnitude of the transfer function $H(\omega) = Z(\omega)$ is minimum.

The frequency response of the circuit's current magnitude shows the symmetry in this graph.



$$I = |I| = \frac{V_m}{\sqrt{R^2 + \left(\omega L - 1/\omega C\right)^2}} \quad -(6)$$

The average power dissipated by the RLC circuit is:-

$$P(\omega) = \frac{1}{2} I^2 R \quad -(7)$$

Or $$P(\omega_0) = \frac{1}{2} \frac{V_m^2}{R} \quad -(8)$$

At certain frequencies $\omega = \omega_1, \omega_2$, the dissipated power is half of the maximum value, that is,

$$P(\omega_1) = P(\omega_2) = \frac{\left(V_m/\sqrt{2}\right)^2}{2R} = \frac{V_m^2}{4R} \quad -(9)$$

Hence ω_1 and ω_2 are called half power frequencies.

The half power frequencies are obtained by setting Z equal to $\sqrt{2}R$.

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\,R$$

$$\Rightarrow \quad \left.\begin{aligned} \omega_1 &= -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \\ \text{and } \omega_2 &= \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \end{aligned}\right\} \quad -(10)$$

We can relate the half-power frequencies to the resonant frequency,

$$\omega_0 = \sqrt{\omega_1 \omega_2} \quad -(11)$$

Eqn (11) showing that the resonant frequency is the geometric mean of the half-power frequencies.

The bandwidth BW is defined as the difference b/w the two half power frequencies, i.e.-

$$BW = \omega_2 - \omega_1 \quad -(12)$$

<u>Quality factor</u>:- The "Sharpness" of the resonance in a resonance circuit is measured quantitatively by Q-factor.

At resonance, the reactive energy in the circuit oscillates b/w the inductor and the capacitor. For any resonant circuit, the quality factor is the ratio of the reactive power to the average power at the resonant frequency, that is,

$$Q = \frac{\text{Reactive Power}}{\text{Average Power}} \quad -(13)$$

In the Series RLC Circuit,

$$Q = \frac{I^2 X_L}{I^2 R} = \frac{I^2 X_C}{I^2 R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C}$$

Or

$$\boxed{Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 R C}} \quad -(14)$$

Notice that quality factor is dimensionless.

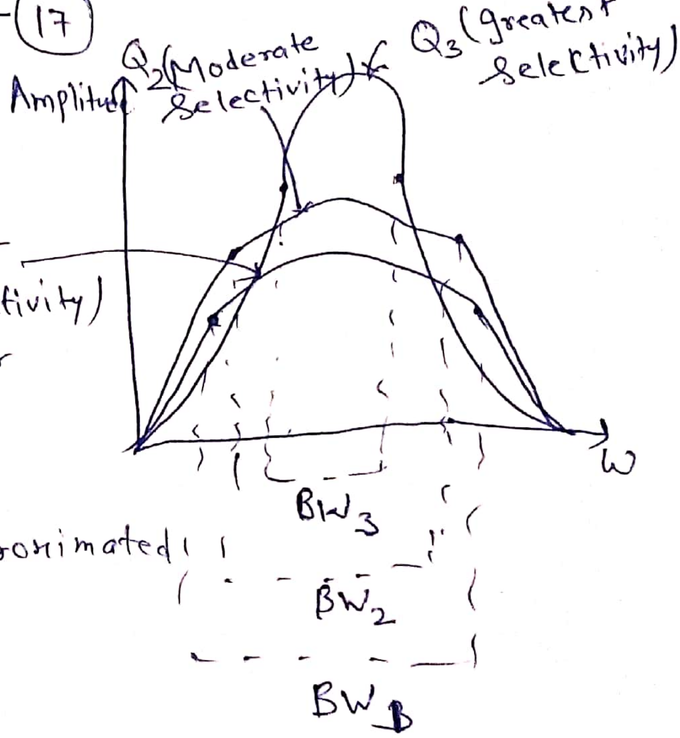
Also, Band Width BW is -

$$BW = \frac{R}{L} \quad (15) \quad \{\because \text{from Eqn. } (10) \ \& \ (12)\}$$

therefore Q factor can be written as in terms of B.W.

$$B.W. = \frac{R}{L} = \frac{\omega_0}{Q} \Rightarrow Q = \frac{\omega_0}{B.W.} \quad (16)$$

or $$B.W = \omega_0^2 RC \quad (17)$$



A resonant circuit is designed to operate at or near its resonant frequency. It is said to be high-Q circuit when its Quality factor is equal to or greater than 10.

for $Q \geq 10$ (high Q-circuits) ~~B.W~~ half power frequency can be approximated as-

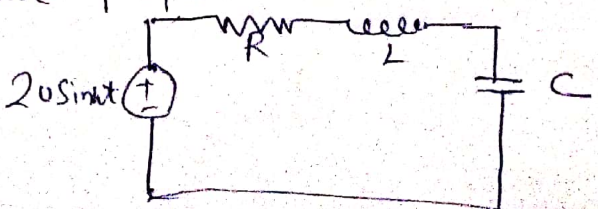
$$\left.\begin{aligned} \omega_1 &\simeq \omega_0 - \frac{BW}{2} \\ \& \ \omega_2 &= \omega_0 + \frac{BW}{2} \end{aligned}\right\} \quad (18)$$

High-Q circuits are used in communication n/ws.

Prob. In the circuit of given fig., $R = 2\Omega$, $L = 1mH$, and $C = 0.4\mu F$

(a) Find the resonant frequency & half power frequencies.

(b) B.W. (c) Q-factor (d) Determine the amplitude at ω_0, ω_1 & ω_2. (e) Calculate the power dissipated at resonance and at the half power frequencies.



Soln.:-

(a) Resonant frequency $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 0.4 \times 10^{-6}}}$

$$\omega_0 = 50k \text{ rad/sec.}$$

Half Power frequency:-

Method 1:- $\omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

$$= \frac{-\cancel{2}}{\cancel{2} \times 10^{-3}} + \sqrt{(10^3)^2 + (50 \times 10^3)^2}$$

$$= -1 + \sqrt{1 + 2500}$$

$$= 49 \text{ krad/sec}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$= 1 + \sqrt{1 + 2500}$$

$$\boxed{\omega_2 = 51 \text{ krad/Sec}}$$

Method 2:- * Should have B.W Value already which is 51 - 49 = 2 krad/sec.

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$\Rightarrow \omega_1 \omega_2 = \omega_0^2 = 2500 \times 10^6 \quad - (1)$$

Also, $\omega_2 - \omega_1 = B.W \Rightarrow \omega_2 - \omega_1 = 2 \times 10^3 \quad - (2)$

Solving (1) & (2)

$$\boxed{\omega_1 = 49 \text{ krad/sec}, \ \omega_2 = 51 \text{ krad/sec}}$$

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Method 3:- for this we should have Q value already which is $Q = 25 (Q \geq 10)$, therefore,

$$\omega_1 = \omega_0 - \frac{BW}{2}$$

$$\omega_1 = 50 - \frac{2}{2} = 49 \text{ k rad/sec}$$

$$\omega_2 = \omega_0 + \frac{BW}{2} = 50 + \frac{2}{2} = 51^{k} \text{ rad/sec}$$

(b) $B.W. = \omega_2 - \omega_1 = 51 - 49 = 2 \text{ k rad/sec}$

Method 2 :- $B.W = \frac{R}{L} = \frac{2}{10^{-3}} = 2 \text{ K rad/sec.}$

(d) Current at $\omega_0 = \frac{V_m}{R} = \frac{20}{2} = 10 \text{ A.}$

Current at half power frequencies $(\omega = \omega_1, \omega_2)$

$$I = \frac{V_m}{\sqrt{2}R} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

(e) Power at $\omega_0 = \frac{V_m^2}{2R} = \frac{(20)^2}{2 \times 2} = 100 \text{ W}$

Power at half power frequencies $(\omega = \omega_1, \omega_2)$

$$\boxed{P = \frac{V_m^2}{4R} = 50 \text{ W}}$$

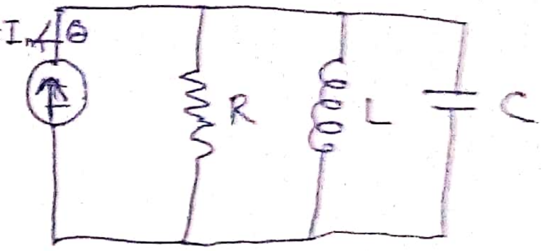
(c) Q. factor

$$\boxed{Q = \frac{\omega_0}{B.W} = \frac{50}{2} = 25}$$

Parallel Resonance:- The parallel RLC Circuit in fig. is the dual of the Series RLC Circuit.

The admittance is -

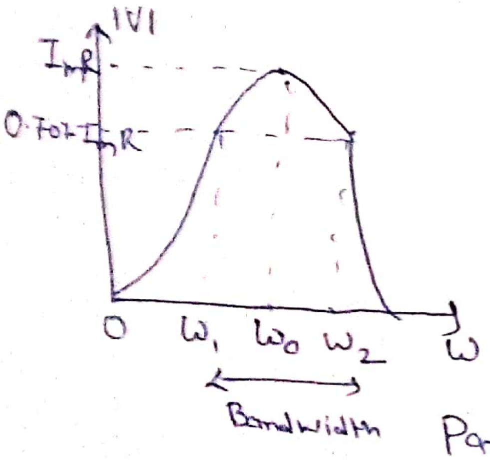
$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$$



$$\Rightarrow \quad Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) \quad \text{—} (1)$$

Resonance Occurs When the imaginary Part of Y is zero, i.e

$$\omega C - \frac{1}{\omega L} = 0$$



or

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}} \quad \text{—} (2)$$

(Resonance frequency)

The resonance Condition for the parallel resonance is the same as series resonance. The Voltage |V| is sketched in fig. as a function of frequency. Notice that at resonance, the Parallel LC Combination acts Like an open Circuit so that the entire Circuit flows through R.

Now By replacing the R, L and C in Series Circuit {in Eqn (10)} to the $\frac{1}{R}$, C and L respectively, the half power frequencies are.

$$\left.\begin{aligned} \omega_1 &= \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \\ \omega_2 &= \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} \end{aligned}\right\} \quad \text{—} (3)$$

$$\boxed{BW = \omega_2 - \omega_1 = \frac{1}{RC}} \quad \text{—} (4)$$

$$\boxed{Q = \frac{\omega_0}{B.W} = \omega_0 RC = \frac{R}{\omega_0 L}} \quad \text{—} (5)$$

Again for High Circuits ($Q \geq 10$),

$$\left.\begin{aligned} W_1 &\simeq W_0 - BW/2 \\ W_2 &\simeq W_0 + BW/2 \end{aligned}\right\} \quad (6)$$

Prob.:- In the parallel RLC circuit of fig., Let $R = 8\,k\Omega$, $L = 0.2\,mH$ and $C = 8\,\mu F$.

(a) Calculate W_0, Q and BW, (b) Find W_1 and W_2.

(c) Determine the power dissipated at W_0, W_1 and W_2.

Soln:- (a) $$W_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.2\times10^{-3}\times8\times10^{-6}}} = 25\,k\,rad/sec.$$

$$B.W = \frac{1}{RC} = \frac{1}{8\times8\times10^{-6}\times10^{3}} = 15.625\,rad/sec$$

$$\boxed{Q = \frac{W_0}{B.W} = \frac{25\times10^3}{15.625} = 1600}$$

(b) $\because$ $Q \geq 10$, Therefore,

$$W_2 = W_0 + \frac{B.W}{2} = 25000 + \frac{15.625}{2} = 25.007\,k\,rad/sec$$

$$W_1 = W_0 - \frac{B.W.}{2} = 25000 - \frac{15.625}{2} = 24.992\,k\,rad/sec$$

(c) Power Dissipated at W_0,

$$P = \frac{V_m^2}{2R} = \frac{10^2}{2\times8\times10^3} = 6.25\,mW$$

and Power dissipated at W_1, W_2,

$$P = \frac{V_m^2}{4R} = \frac{10^2}{4\times8\times10^3} = \cancel{12.5\,m}\ 3.125\,mW$$