Signals and Systems (10B11EC301)

Objective of the Course

After going through the course, the student shall be able to :

- Understand and analyze any signal and system.
- State how an electronic circuit can be represented as a system and analyzed.
- Understand different types of signal processing tools.
- Understand digital filters.

COURSE CONTENT

1. Classifications of signals

2. Classifications of systems

3. Discrete and Continuous Transforms

4. System characterization in time and frequency domain

5. Introduction to Digital Filters

Text & Reference Books

***Text Book*:**

- A.V.Oppenheim, A. S. Wilsky and S. H. Nawab, *Signals & Systems*, Prentice Hall.

Reference Books:

- Simon Haykin & Barry Van Veen, Signals and Systems, 2nd edition, John Wiley & sons, 2004
- 1. B.P. Lathi, Signals Processing and Linear Systems, Oxford Press.
- 2. J. G. Proakis and D. G. Manolakis, *Digital Signal Processing: Principles, Algorithms, and Applications*, Prentice Hall.
- 3. A.V.Oppenheim and R.W. Schafer, *Discrete-Time Signal Processing*, Prentice Hall.

SIGNALS :

Signals are functions of one or more independent variables that carry information.

For example: A continuous-time *signal* is a function of time, *x(t)*, that we assume is real-valued and defined for all *t*. Eg., let x(t)= 20t+30

- Electrical signals --- voltage v(t) and current i(t) in a circuit
- Acoustic signals --- audio or speech signals (analog or digital)
- Video signals --- intensity variations in an image

Biological signals --- sequence of bases in a gene

THE INDEPENDENT VARIABLES

- Can be continuous: Continuous signal.
- Can be discrete: Discrete signal.
- Can be 1-D, 2-D, … N-D: One dimensional & multi-dimensional signal.
- For this course: We will focus on a single (1-D) independent variable which we call "time".

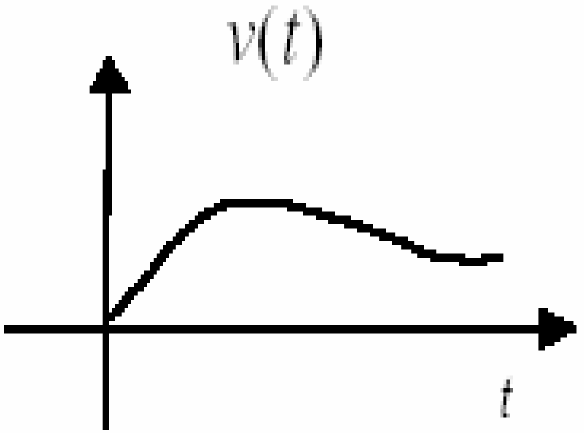
Types of Signals

- Continuous Time and Discrete Time
- Analog and Digital
- Periodic and Aperiodic
- Real and Complex
- Even and Odd
- Energy and Power
- Causal and Non-Causal
- Deterministic and Random

Continuous Time Signals

- Continuous-time signals are functions of a continuous variable (time).

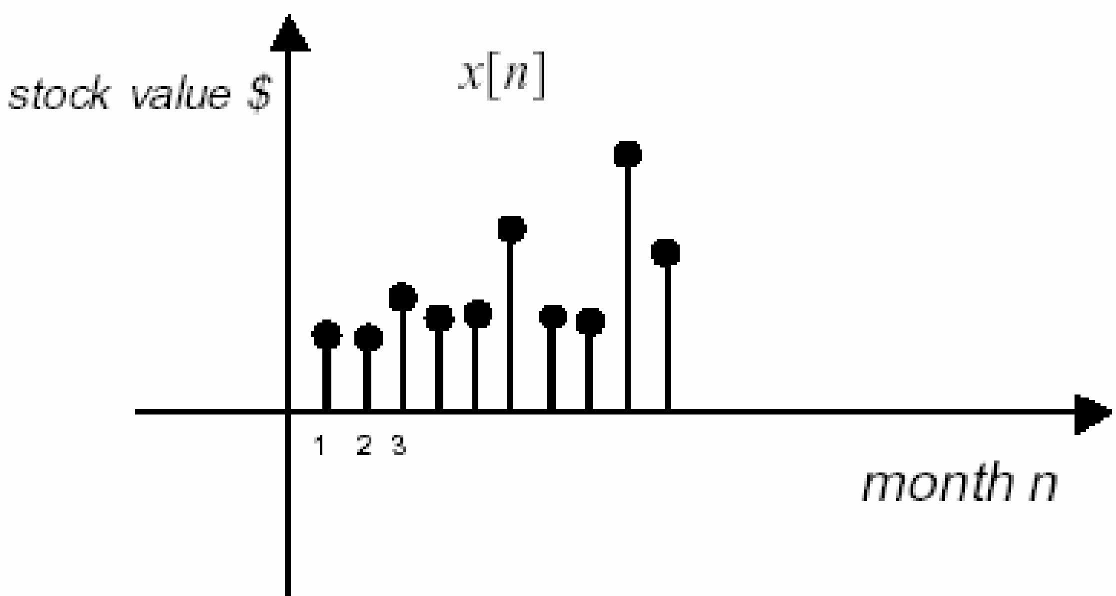
Example: The speed of a car $v(t)$



Discrete Time Signals

•Discrete-time signals are functions of a discrete variable, i.e., they are defined only for integer values of the independent variable (time steps).

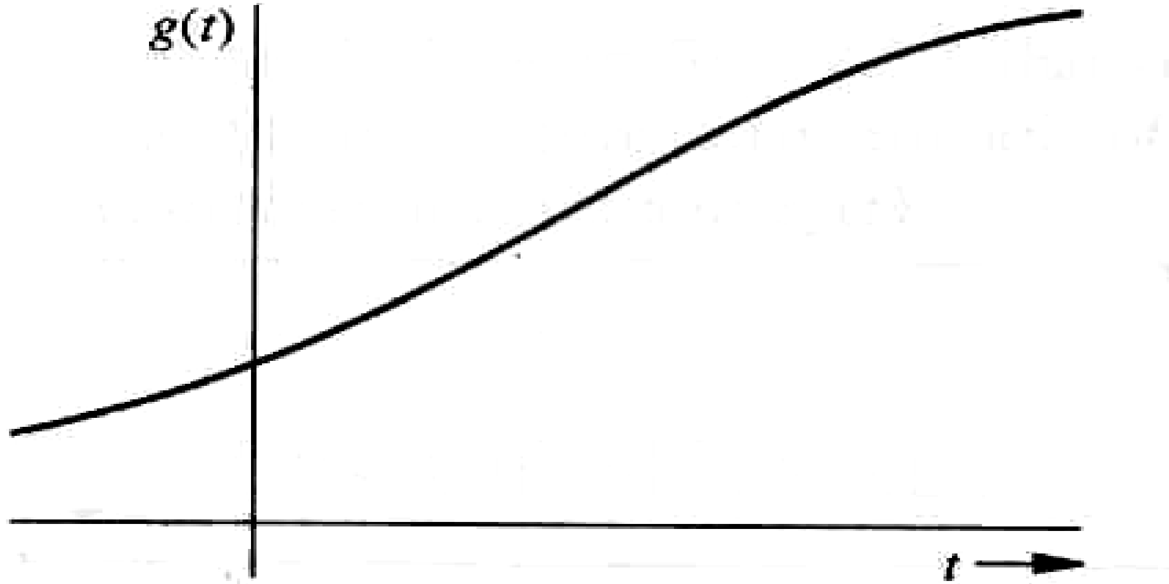
Example: The value of a stock at the end of each month



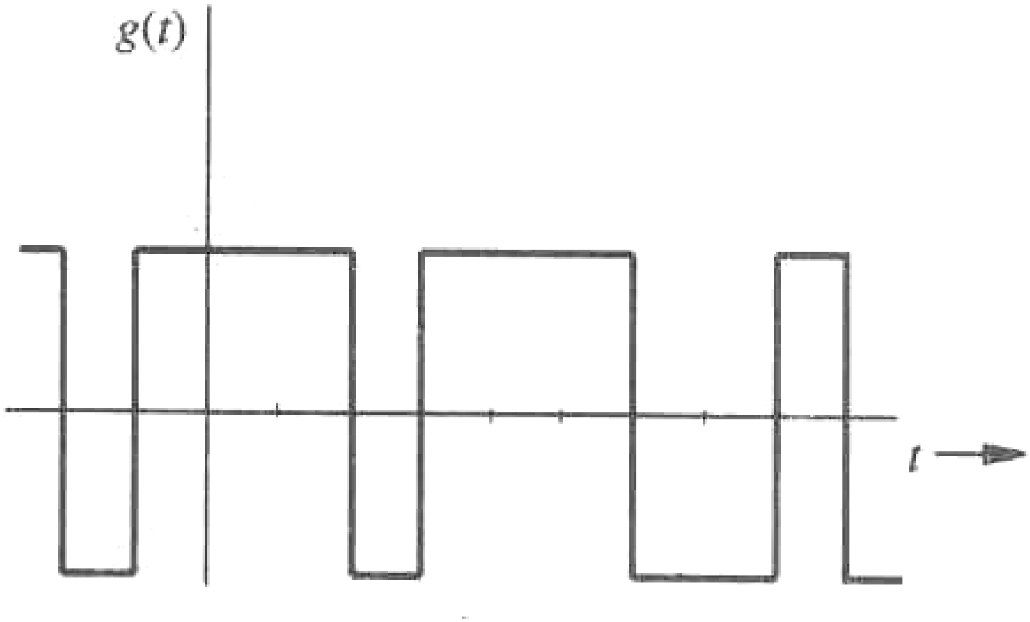
Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

Analog or Digital Signals

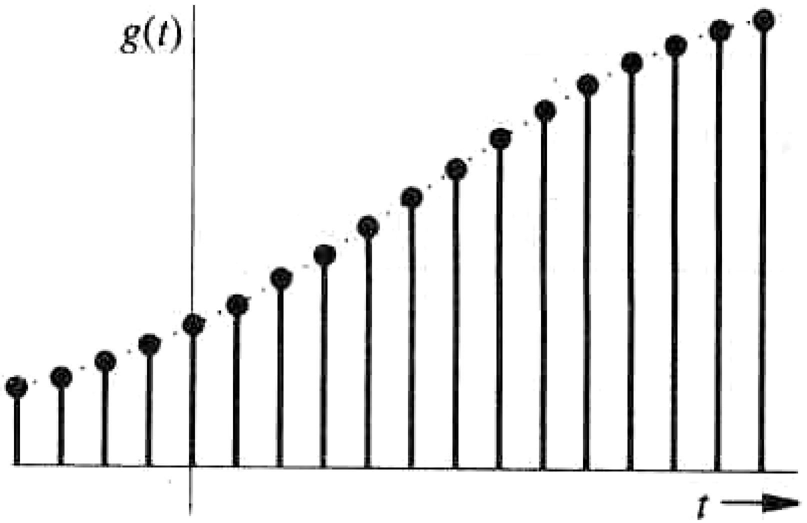
- **Analog Signals:** A signal whose amplitude can take any value in a continuous range.
- **Digital Signal:** A signal whose amplitude can take on only finite number of values.



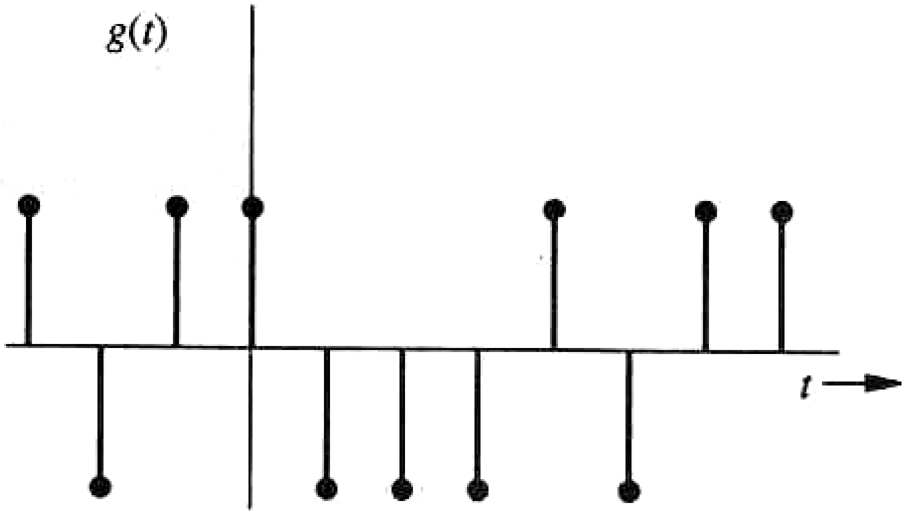
Continuous-Time Analog Signal



Continuous-Time Digital Signal



Discrete-Time Analog Signal



Discrete-Time Digital Signal

Periodic & Aperiodic Signals

- A continuous-time signal $x(t)$ is periodic if there exists a T for which:

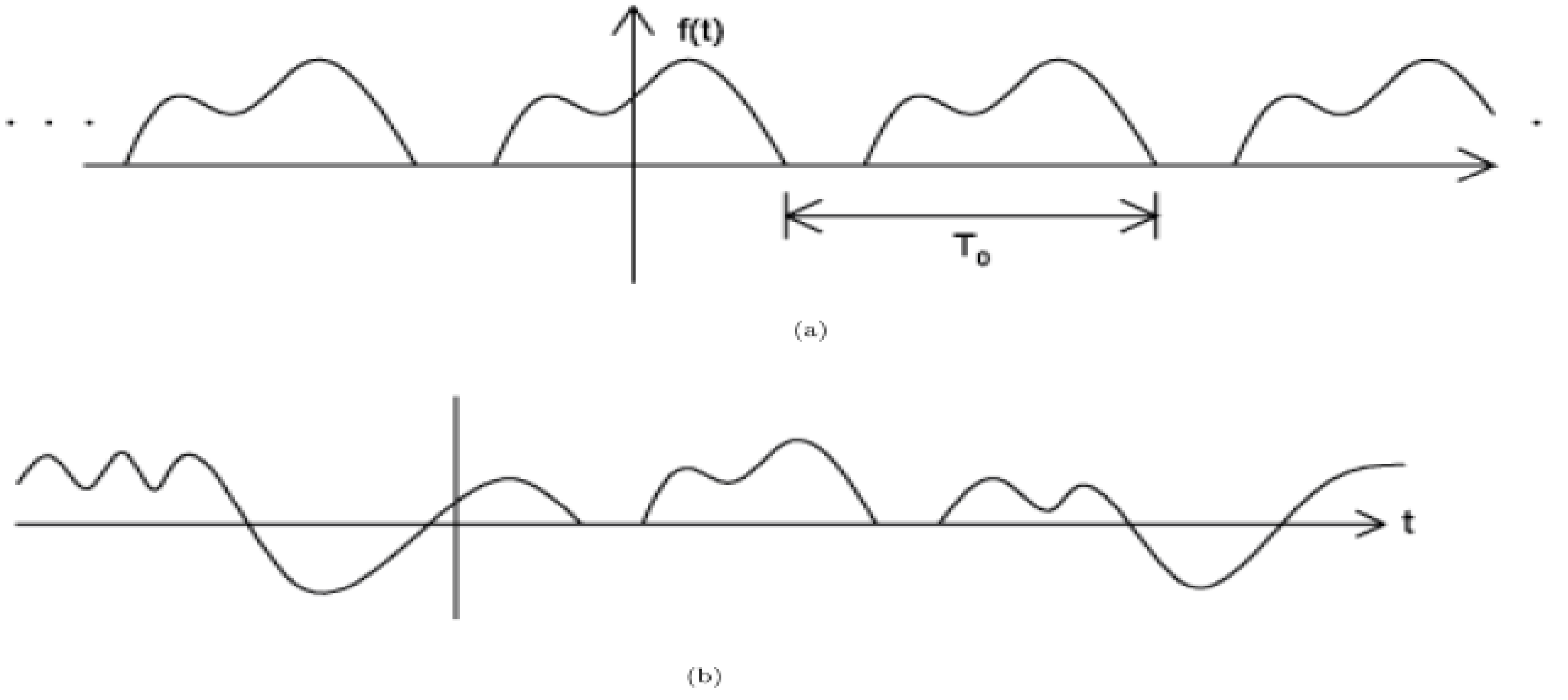
$$x(t) = x(t+T)$$

- A discrete-time signal $x[n]$ is periodic if there exists an N for which:

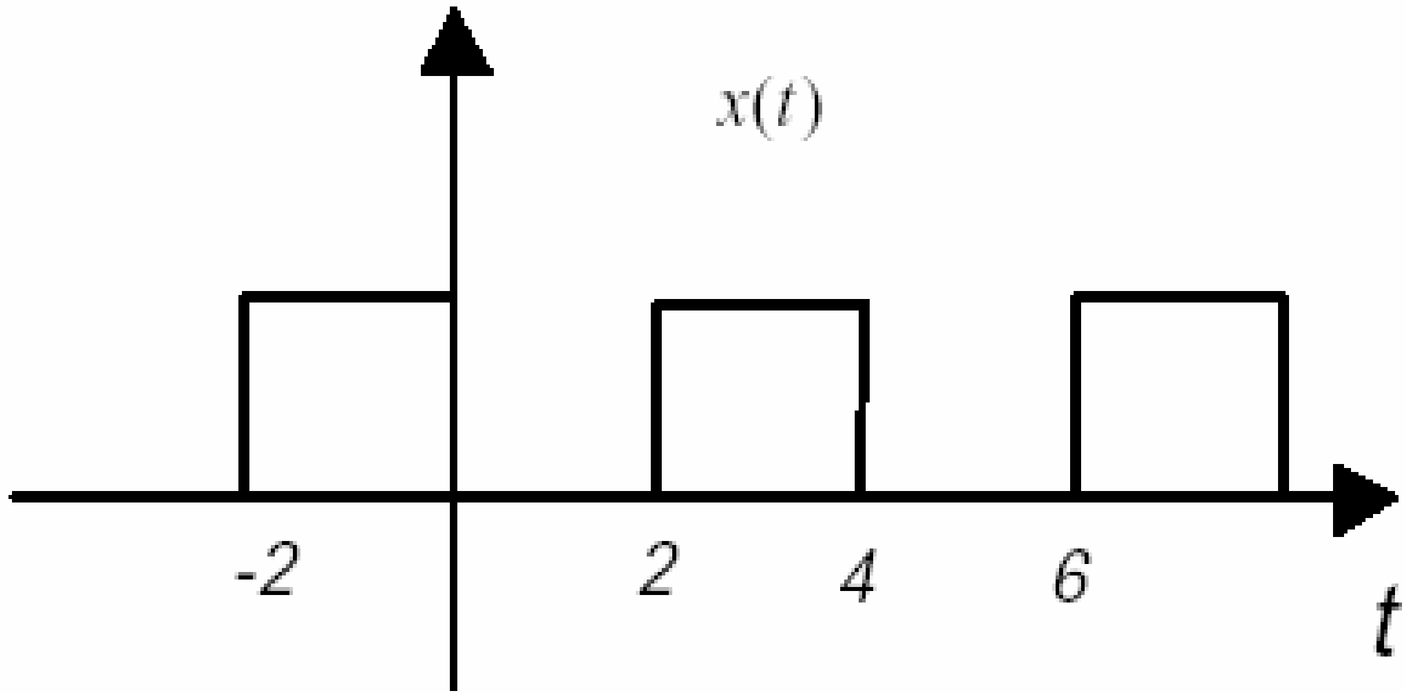
$$x[n] = x[n+N]$$

The smallest such T or N is called the fundamental period.

(a) Periodic and (b) aperiodic signals



Examples of periodic signals



The fundamental period of this square wave signal is $T = 4$, but 8, 12 and 16 are also periods of the signal.

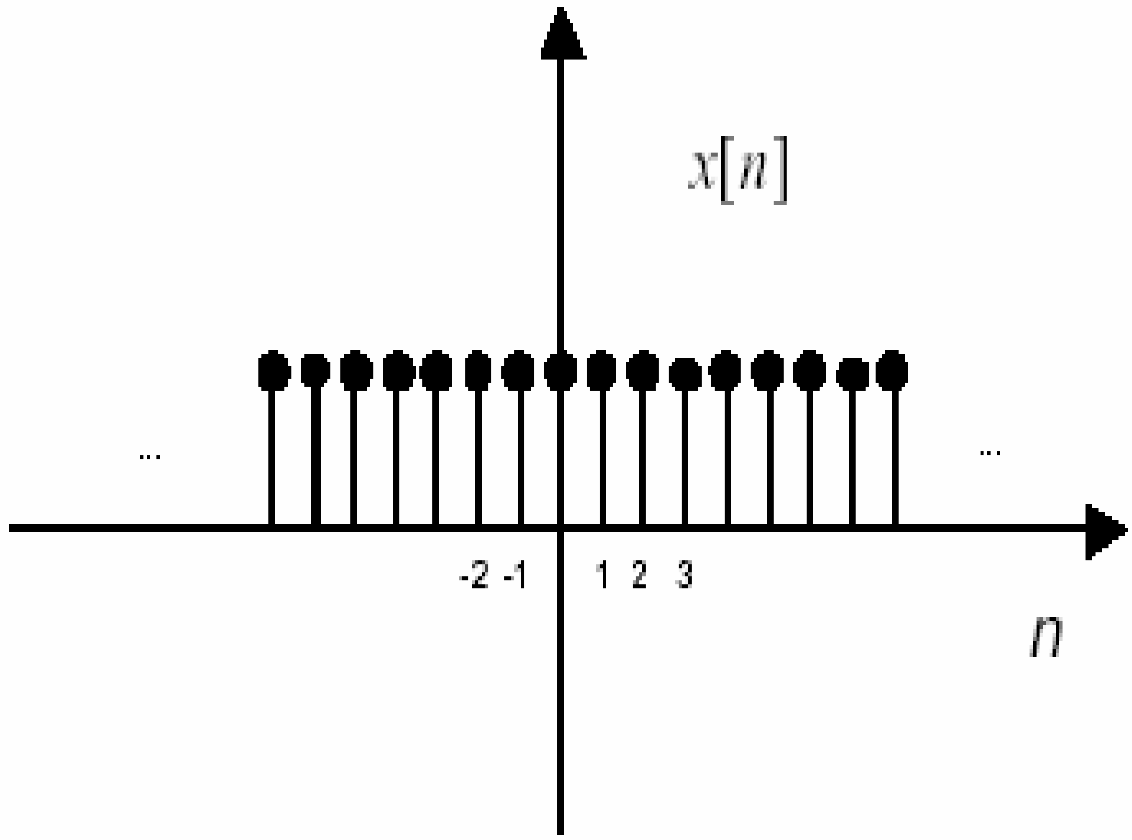
Complex exponential

$$x(t) = e^{j\omega_0 t}:$$

$$x(t+T) = e^{j\omega_0 (t+T)} = e^{j\omega_0 t} e^{j\omega_0 T}$$

The right-hand side is equal to $x(t) = e^{j\omega_0 t}$ if $T = \dfrac{2\pi k}{\omega_0}, k = \pm 1, \pm 2, \ldots$ so these are all periods of the complex exponential. The fundamental period is $T = \dfrac{2\pi}{\omega_0}$.

Discrete-time signal $x[n]=1$ is periodic with fundamental period $N=1$



Periodic signals

Question: Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods T_1 and T_2 respectively.

Under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic, and what is the fundamental period of $x(t)$ if it is periodic?

Answer: If the ratio T_1/T_2 is rational then x(t) is periodic, and the period of x(t) is given by the LCM of T_1 and T_2.

Example : Tell whether the following signal is periodic or not? If yes , find its fundamental period.

$$a)\quad x(t) = \cos(\pi/3)t + \sin(\pi/4)t$$

$$b)\quad x(t) = \cos t + \sin\sqrt{2}\,t$$

$$T_1 = 2\pi/\omega_1 = 6$$

$$T_2 = 2\pi/\omega_2 = 8$$

$$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4}, \text{ is rational}$$

$$\therefore\ \mathrm{T_o} = 4T_1 = 3T_2 = 24$$

Real And Complex Signals

- A signal $x(t)$ is a real signal if its value is a real number and signal $x(t)$ is a complex signal if its value is a complex number.
- A general complex signal is a function of the form - $x(t) = x_1(t) + jx_2(t)$, where $x_1(t)$ & $x_2(t)$ are real signals and $j=\sqrt{-1}$.
- Polar form representation of complex numbers is : $x(t)=r(t)e^{j\theta(t)}$ where $r(t)$ and $\theta(t)$ are real numbers.

Write the following complex signals in polar form -

$$\text{(a)} \quad x_1(t) = (1+j)e^2 e^{-j(1+3t)}$$

$$x_1(t) = (1+j)e^2 e^{-j(1+3t)} = \sqrt{2}e^2 e^{j\frac{\pi}{4}} e^{-j(1+3t)} = \sqrt{2}e^2 e^{-j(1-\frac{\pi}{4}+3t)}$$

$$\Rightarrow \quad r_1(t) = \sqrt{2}e^2, \;\; \theta_1(t) = -1 + \frac{\pi}{4} - 3t$$

$$\text{(c)} \quad x_1[n] = \frac{1}{n^j}, \; n > 0$$

$$x_1[n] = \frac{1}{n^j} = n^{-j} = e^{-j\ln n}, \; n > 0$$

$$\Rightarrow \quad r_1[n] = 1, \;\; \theta_1[n] = -\ln n$$