Solution
Signals & Systems

Q1 a) $2+3j$

Using $z = x+jy = re^{j\theta}$

$|2+3j| = \sqrt{4+9} = \sqrt{13}$ $\arg(2+3j) = \tan^{-1}(\frac{3}{2})$

$2+3j = \sqrt{13}\, e^{j\tan^{-1}(3/2)}$

b) $(1+j)e^{j\pi/3}$

$= \sqrt{1^2+1^2}\, e^{j\tan^{-1}(1)} e^{j\pi/3} = \sqrt{2}\, e^{j\pi/4} e^{j\pi/3} = \sqrt{2}\, e^{j7\pi/12}$.

c) $(\sqrt{5}+j)^2 e^{-j\pi/3}$

$= (\sqrt{5+1})^2 (e^{j\tan^{-1}(\frac{1}{\sqrt{5}})})^2 e^{-j\pi/3}$

$= (\sqrt{6})^2 e^{j2\tan^{-1}(\frac{1}{\sqrt{5}})} \cdot e^{-j\pi/3}$

$= 6\, e^{j(24.095 - \pi/3)} = 6e^{-j35.91} = 6e^{-j35.91}$

d) $\frac{2-j}{1+j3} = \frac{2-j}{1+j3} \times \frac{1-j3}{1-j3} = \frac{2-j-j6-3}{1+9} = \frac{-1-j7}{10}$

$\left|\frac{-1-j7}{10}\right| = \frac{1}{\sqrt{2}}$ $\arg\left(\frac{-1-7j}{10}\right) = \pi + \tan^{-1}7$

$\frac{2-j}{1+j3} = \frac{1}{\sqrt{2}} e^{j(261)}$

Ques-2

(a) $x(t) = e^{j(2\pi t - \pi)}$

To be periodic, $x(t) = x(t+T) \; \forall t$

$x(t+T) = e^{j2\pi(t+T)} \cdot e^{-j\pi}$

$= e^{j(2\pi t - \pi)} \cdot e^{j2\pi T}$

For periodic, $e^{j2\pi T} = 1$

$\Rightarrow e^{j2\pi T} = e^{j2\pi m}, m \in \mathbb{Z}$

$\Rightarrow 2\pi T = 2\pi m$

$\Rightarrow T = m$ & m is +ve integer

fundamental period = 1 s

fundamental frequency = 1 Hz.

$\therefore$ T satisfies all the condition of period.

$x(t)$ is periodic.

OR

Complex exponentials are always periodic

with $T = \frac{2\pi}{|\omega_0|}$

$T = \frac{2\pi}{2\pi} = 1s. \quad f = 1 Hz.$

(b) $x(t) = 3[\cos(2t)]^2$

$= 3\left[\frac{1 + \cos 4t}{2}\right] = \frac{3}{2} + \frac{3\cos 4t}{2}$

Since, sinusoids are periodic & $\omega_0 = 4 \Rightarrow T = \frac{2\pi}{4} = \frac{\pi}{2}$

Adding dc value will not effect periodicity.

$f_0 = 1/\pi/2 = 2/\pi$ Hz

(c) $x(t) = \cos 4t \cdot \sin(8t)$

$$= \left(\frac{e^{j4t}+e^{-j4t}}{2}\right)\left(\frac{e^{j8t}-e^{-j8t}}{2j}\right)$$

$$= \frac{1}{4j}\left(e^{j4t}e^{j8t}+e^{-j4t}e^{j8t}-e^{j4t}e^{-j8t}-e^{-j4t}e^{-j8t}\right)$$

$$= \frac{1}{4j}\left(e^{j12t}+e^{j4t}-e^{-j4t}-e^{-j12t}\right)$$

$$= \frac{1}{4j}\left(\sin 4t \cdot 2j + \sin 12t \cdot 2j\right)$$

$\omega t = 2\pi f t$

$f = \frac{4}{2\pi}$ or $T = \frac{2\pi}{4}$

$$= \frac{1}{2}\left(\sin 4t + \sin 12t\right)$$

$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}s$ $\qquad T_2 = \frac{2\pi}{12} = \frac{\pi}{6}s$

LCM of $T_1, T_2 = \frac{\pi}{2}s$ & $\frac{T_1}{T_2} = \frac{\pi/2}{\pi/6} = 3$ (integer)

Over all fundamental period, $T_0 = \frac{\pi}{2}s$.

OR

Using $2\cos A \sin B = \sin(A+B) - \sin(A-B)$

$\cos 4t \sin 8t = \frac{1}{2}\left(\sin(12t) + \sin(+4t)\right)$

$T_1 = \frac{\pi}{6}s \qquad T_2 = \pi/2\, s$

fundamental period $=$ LCM$(T_1, T_2) = \pi/2\, s$

fundamental freq $=$ HCF$\left(\frac{1}{T_1}, \frac{1}{T_2}\right) = \frac{2}{\pi}$ Hz or $\frac{1}{\pi/2}$ Hz

(d) $x(t) = 4u(t) + 2\sin 8t$

$$= \begin{cases} 2\sin 8t, & t<0 \\ 4+2\sin 8t, & t>0 \end{cases}$$

It is independently periodic for $-ve$ & $+ve$ time.

In general, it is an aperiodic signal.

(e) $x(t) = 2\cos(2\pi t/3) - 2\sin(\pi t/8) + 2\cos(2\pi t + \pi/6)$

$$T_1 = \frac{2\pi}{2\pi/3} \qquad T_2 = \frac{2\pi}{\pi/8} \qquad T_3 = \frac{2\pi}{2\pi/\cancel{6}} = 1$$

$T_3 = 1$

LCM of T_1, T_2, T_3 = LCM$(3, 16, \frac{1}{8})$

$= 48$

$$\frac{T_1}{T_2} = \frac{3}{16}, \quad \frac{T_2}{T_3} = \frac{16}{1}, \quad \frac{T_3}{T_1} = \frac{1}{3}$$

All ratios are ~~rational~~ rational numbers, hence sum of sinusoids is ~~always~~ also periodic with period = ~~48~~ 336

fundamental frequency = $\frac{1}{\cancel{48}}$ ~~1/3~~ $\frac{1}{336}$ Hz

$|e^{j\theta}| = 1$

Ques 3

(a) $x(t) = e^{-j2t}u(t)$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{0}^{\infty} |e^{-j2t}|^2 dt$$

$= \infty$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} |x(t)|^2 dt$$

$$= \frac{1}{\pi}\int_0^{\pi/2} dt = \frac{1}{\pi}\cdot\frac{\pi}{2} = \frac{1}{2}W$$

$\therefore$ $0 < P < \infty$, the signal is power signal.

(b) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2+t, & 1 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$

$E = \int_0^1 t^2 dt + \int_1^2 (2+t)^2 dt$

$= \left[\frac{t^3}{3}\right]_0^1 + \left[\frac{t^3}{3} + 4t + \frac{4t^2}{2}\right]_1^2$

$= \frac{1}{3} + \frac{37}{3}$

$= \frac{1}{3} + \frac{37}{3} = \frac{38}{3}$ J

$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$

$\because T = \infty$

$P = 0$

(c) $x(t) = \cos(4t + \pi/3)$

$E = \int_{-\infty}^{\infty} |\cos(4t + \pi/3)|^2 dt$

$= \int_{-\infty}^{\infty} \frac{1}{2} + \frac{\cos(8t + 2\pi/3)}{2} dt$

$= \infty$

$P = \frac{1}{\pi/2} \int_{-\pi/4}^{\pi/4} |\cos(4t + \pi/3)|^2 dt$

$= \frac{1}{\pi/2} \left(\int_{-\pi/4}^{\pi/4} \frac{1}{2} dt + \int_{-\pi/4}^{\pi/4} \frac{\cos(8t + 2\pi/3)}{2} dt \right)$

$= \frac{1}{\pi/2} \left[\frac{2\pi}{4}\right] \cdot \frac{1}{2} + 0$

$= \frac{1}{2}$ W

(d) $x(t) = (\cos(4t + \pi/3))^2$

$= \frac{1 + \cos(8t + 2\pi/3)}{2}$

$E = \int_{-\infty}^{\infty} \left| \frac{1 + \cos(8t + 2\pi/3)}{2} \right|^2 dt$

$= \left(\int_{-\infty}^{\infty} \left| \frac{1}{2} + \frac{\cos(8t + 2\pi/3)}{2} \right| \left| \frac{1}{2} + \frac{\cos(8t + 2\pi/3)}{2} \right| dt \right.$

$= \left(\int_{-\infty}^{\infty} \frac{1}{4} + \frac{\cos^2(8t + 2\pi/3)}{2^2} + \frac{\cos(8t + 2\pi/3)}{2} \right) dt$

$= \int_{-\infty}^{\infty} \left[\frac{1}{4} + \frac{1}{8} + \frac{1}{8}\cos(16t + 4\pi/3) + \frac{\cos(8t + 2\pi/3)}{2} \right) dt = \infty$

(d) $x(t) = (\cos(4t + \pi/3))^2$ ⑥

$$= \frac{1}{2} + \frac{\cos(8t + 2\pi/3)}{2}$$

$$= \frac{1}{2} + \frac{e^{j(8t+2\pi/3)} + e^{-j(8t+2\pi/3)}}{2\cdot 2}$$

$$= \frac{1}{2} + \frac{1}{2^2}e^{j(8t+2\pi/3)} + \frac{1}{2^2}e^{-j(8t+2\pi/3)}$$

$$E = \int_{-\infty}^{\infty} \left| \frac{1}{2} + \frac{1}{2^2}e^{j(8t+2\pi/3)} + \frac{1}{2^2}e^{-j(8t+2\pi/3)} \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{2} + \frac{1}{2^2}e^{+j(8t+2\pi/3)} + \frac{1}{2^2}e^{-j(8t+2\pi/3)}\right)$$

$$\left(\frac{1}{2} + \frac{1}{2^2}e^{-j(8t+2\pi/3)} + \frac{1}{2^2}e^{j(8t+2\pi/3)}\right) dt$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(\frac{1}{4} + \frac{1}{8}e^{j(8t+2\pi/3)} + \frac{1}{8}e^{-j(8t+2\pi/3)}\right.$$

$$+ \frac{1}{8}e^{-j(8t+2\pi/3)} + \frac{1}{4^2} + \frac{1}{4^2}e^{-j2(8t+2\pi/3)} + \frac{1}{8}e^{j(8t+2\pi/3)}$$

$$\left. + \frac{1}{4^2}e^{j2(8t+2\pi/3)} + \frac{1}{4^2}\right) dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} \left(\frac{3}{8} + \frac{2}{8}e^{j(8t+2\pi/3)} + \frac{1}{4^2}e^{j2(8t+2\pi/3)}\right.$$

$$\left. + \frac{1}{2^2}e^{-j(8t+2\pi/3)} + \frac{1}{4^2}e^{-j2(8t+2\pi/3)}\right) dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} \left(\frac{3}{8} + \frac{1}{2}\cos(8t + 2\pi/3) + \frac{1}{2^3}\cos((8t+2\pi/3)2)\right) dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} \frac{3}{8}\,dt = \frac{3}{8}$$ Ans $P = 3/8\,W$

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Ques-7 (a) $g(t) = 2t^2 - 3t + 6$

$$g_e(t) = \frac{2t^2 - 3t + 6 + 2(-t)^2 - 3(-t) + 6}{2}$$

$$= \frac{4t^2 + 12}{2} = 2t^2 + 6$$

$$g_o(t) = \frac{2t^2 - 3t + 6 - (2(-t)^2 - 3(-t) + 6)}{2}$$

$$= -\frac{6t}{2} = -3t$$

(b) $g(t) = 20\cos(40\pi t - \frac{\pi}{4})$

$$g_e(t) = \frac{20\cos(40\pi t - \pi/4) + 20\cos(-40\pi t - \pi/4)}{2}$$

$$= 20[\cos 40\pi t \cos \pi/4 + \sin 40\pi t \sin \pi/4]/2$$

$$+ 20[\cos 40\pi t \cos \pi/4 - \sin(40\pi t) \sin \pi/4]/2$$

$$g_e(t) = 20 \cos 40\pi t \cos \frac{\pi}{4} = \frac{20}{\sqrt{2}} \cos 40\pi t$$

$$g_o(t) = \frac{20\cos(40\pi t - \pi/4) - 20\cos(-40\pi t - \pi/4)}{2}$$

$$= 20[\cos 40\pi t \cos \frac{\pi}{4} + \sin 40\pi t \sin \pi/4]/2$$

$$- 20[\cos 40\pi t \cos \frac{\pi}{4} - \sin 40\pi t \sin \frac{\pi}{4}]/2$$

$$= 20 \sin 40\pi t \sin \frac{\pi}{4} = \frac{20}{\sqrt{2}} \sin 40\pi t.$$

(c) $g(t) = \frac{2t^2 - 3t + 6}{1+t}$

$$g_e(t) = \frac{\frac{2t^2-3t+6}{1+t} + \frac{2t^2+3t+6}{1-t}}{2} = \frac{6+5t^2}{1-t^2}$$

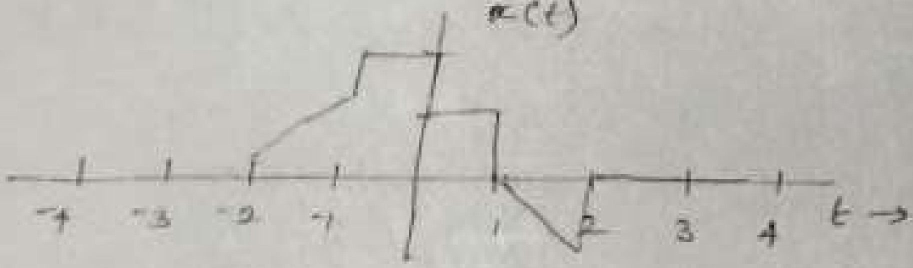
$$g_o(t) = \frac{\frac{2t^2-3t+6}{1+t} - \frac{2t^2+3t+6}{1-t}}{2} = -t\,\frac{2t^2+9}{1-t^2}$$

(d) $g(t) = \text{sinc}(t)$

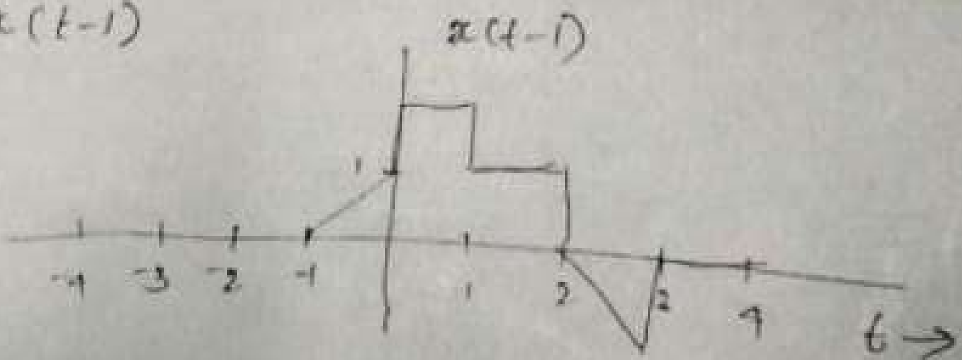
$$g_e(t) = \frac{\frac{\sin \pi t}{\pi t} + \frac{\sin(-\pi t)}{-\pi t}}{2} = \frac{\sin \pi t}{\pi t}$$

$g_o(t) = 0$

Q5

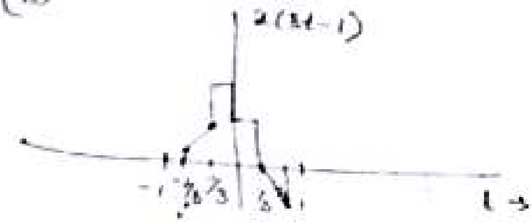


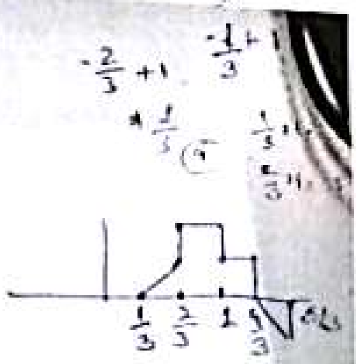
a. $x(t-1)$



$x(a(t-1/a))$

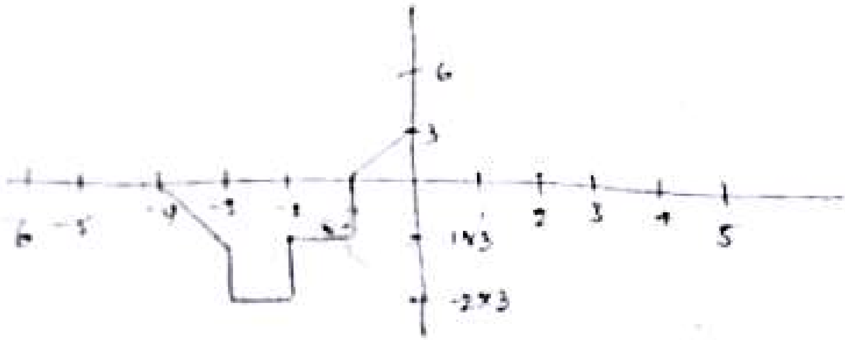
(b)





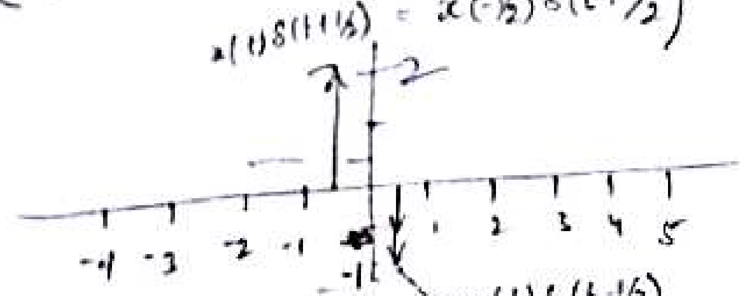
$x(t+2)$

(c) $-3x(t+2)$



(d) $x(t)\delta(t+1/2) - \delta(t-1/2)$

$x(t)\delta(t+1/2) = x(-1/2)\delta(t+1/2)$



$x(t)\delta(t-1/2)$

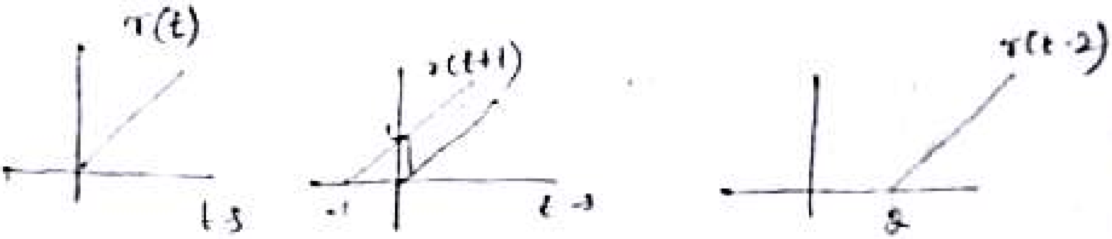
$= x(1/2)\delta(t-1/2)$

Ques 6

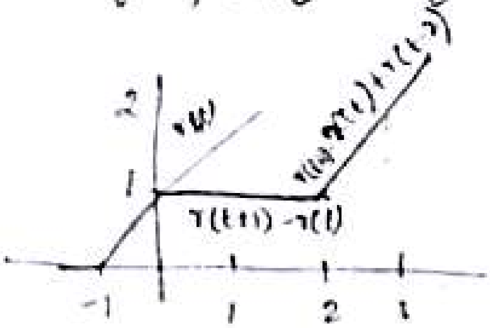
(a) $x(t) = u(t) - u(t-1)$

$= \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$ $\quad = \begin{cases} 1 & t > 1 \\ 0, & t < 1 \end{cases}$ $\quad = \begin{cases} 1, & 0 < t < 1 \\ 0, & \text{elsewhere} \end{cases}$

(b) $x(t) = r(t+1) - r(t) + r(t-2)$

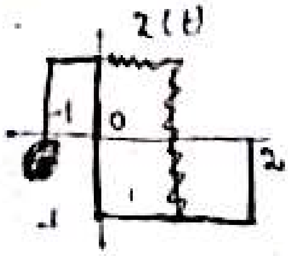


$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$ $\quad = \begin{cases} t+1, & t+1 \geq 0 \\ 0, & t+1 < 0 \end{cases}$ $\quad = \begin{cases} t-2, & t-2 \geq 0 \\ 0, & t-2 < 0 \end{cases}$



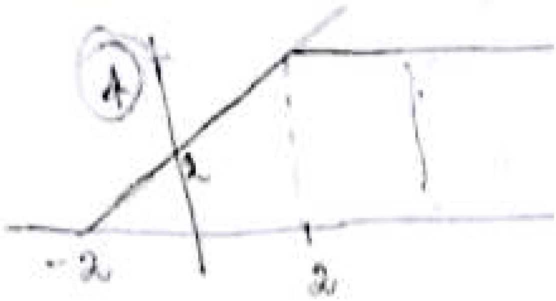
$= \begin{cases} t+1, & -1 \leq t \leq 0 \\ 1, & 0 \leq t \leq 2 \\ t-1, & t \geq 2 \\ 0, & \text{elsewhere.} \end{cases}$

(c) $x(t) = u(t+1) - 2u(t) + u(t-2)$



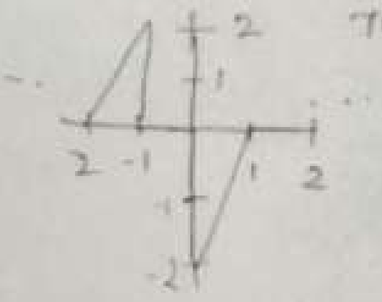
$= \begin{cases} 0 & t < -1 \\ 1 & -1 < t < 0 \\ -1 & 0 < t < 2 \\ 0 & t > 2 \end{cases}$

(d) $x(t) = r(t+2) - r(t-2)$ (12)

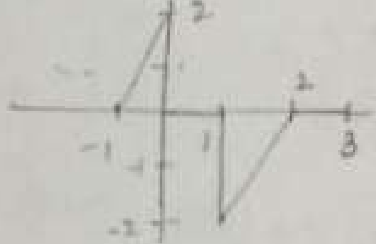


$$= \begin{cases} 0 & t < -2 \\ t+2 & |t| \le 2 \\ t+2-t+2 = 4 & t \ge 2 \end{cases}$$

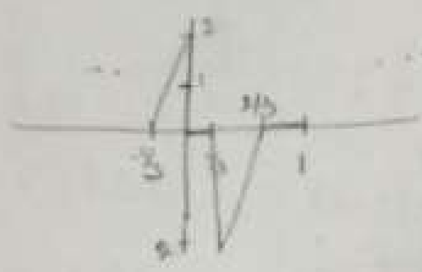
$x(t)$ $\qquad T = 4$



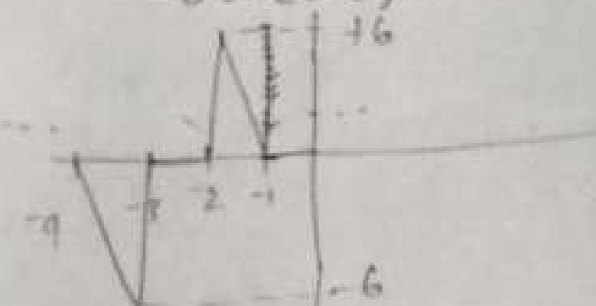
$x(t-1)$ $\qquad T = 4$



$x(3t-1)$ $\qquad T = 4/3$



$-3x(t+2)$ $\qquad T = 4/3$



$x(t)\,\delta(t+1/2) - x(t)\,\delta(t-1/2) = x(p)\,\delta(t-1/2)$

Aperiodic