STEADY-STATE POWER ANALYSIS

LEARNING GOALS

Instantaneous Power
For the special case of steady state sinusoidal signals

Average Power
Power absorbed or supplied during one cycle

Maximum Average Power Transfer
When the circuit is in sinusoidal steady state

Effective or RMS Values
For the case of sinusoidal signals

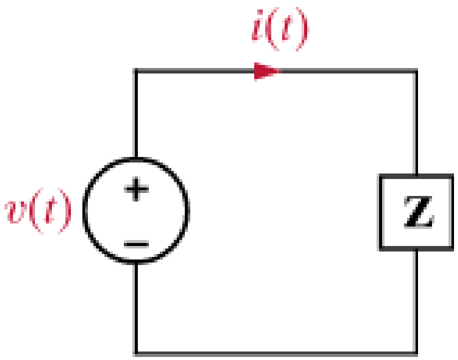
Power Factor
A measure of the angle between current and voltage phasors

Power Factor Correction
How to improve power transfer to a load by "aligning" phasors

Single Phase Three-Wire Circuits
Typical distribution method for households and small loads

GEAUX

INSTANTANEOUS POWER



Instantaneous Power Supplied to Impedance

$$p(t) = v(t)i(t)$$

In steady State

$$v(t) = V_M \cos(\omega t + \theta_v)$$

$$i(t) = I_M \cos(\omega t + \theta_i)$$

$$p(t) = V_M I_M \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$\cos\phi_1 \cos\phi_2 = \frac{1}{2}[\cos(\phi_1 - \phi_2) + \cos(\phi_1 + \phi_2)]$$

$$p(t) = \frac{V_M I_M}{2}[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)]$$

constant

Twice the frequency

LEARNING EXAMPLE

Assume: $v(t) = 4\cos(\omega t + 60°)$,

$$Z = 2\angle 30° \Omega$$

Find: $i(t), p(t)$

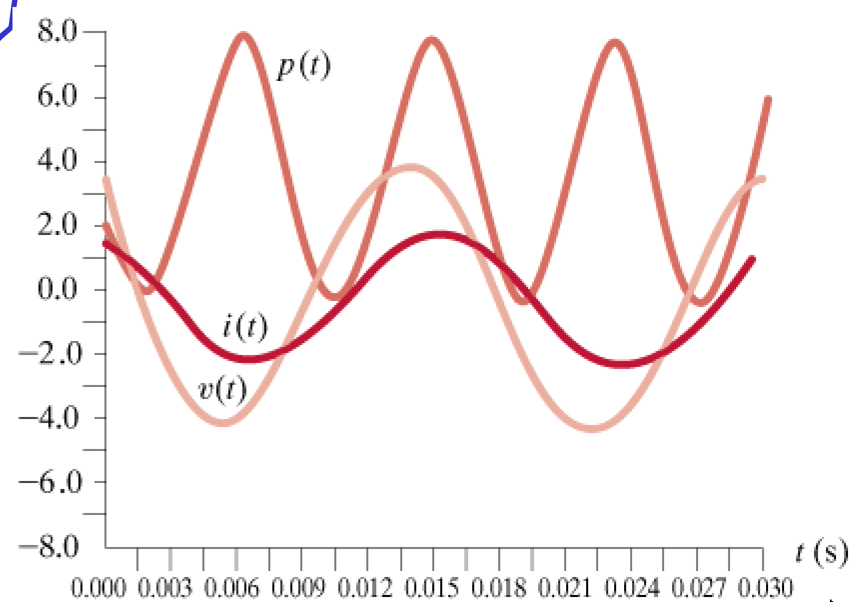
$$I = \frac{V}{Z} = \frac{4\angle 60°}{2\angle 30°} = 2\angle 30° (A)$$

$$i(t) = 2\cos(\omega t + 30°)(A)$$

$$V_M = 4, \theta_v = 60°$$

$$I_M = 2, \theta_i = 30°$$

$$p(t) = 4\cos 30° + 4\cos(2\omega t + 90°)$$



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AVERAGE POWER

For sinusoidal (and other periodic signals) we compute averages over one period

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt \qquad T = \frac{2\pi}{\omega}$$

$$p(t) = \frac{V_M I_M}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

$$P = \frac{V_M I_M}{2}\cos(\theta_v - \theta_i)$$

It does not matter who leads

If voltage and current are in phase

$$\theta_v = \theta_i \Rightarrow P = \frac{1}{2}V_M I_M$$

Purely resistive

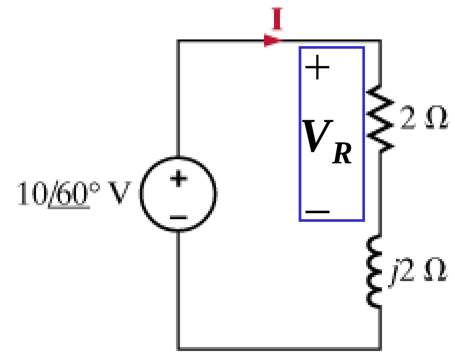
If voltage and current are in quadrature

$$\theta_v - \theta_i = \pm 90° \Rightarrow P = 0$$

Purely inductive or capacitive

LEARNING EXAMPLE

Find the average power absorbed by impedance



$$I = \frac{10\angle 60°}{2+j2} = \frac{10\angle 60°}{2\sqrt{2}\angle 45°} = 3.53\angle 15°(A)$$

$$V_M = 10, I_M = 3.53, \theta_v = 60°, \theta_i = 15°$$

$$P = 35.3\cos(45°) = 12.5W$$

Since inductor does not absorb power one can use voltages and currents across the resistive part

$$V_R = \frac{2}{2+j2}10\angle 60° = 7.06\angle 15°(V)$$

$$P = \frac{1}{2}7.06 \times 3.53W$$

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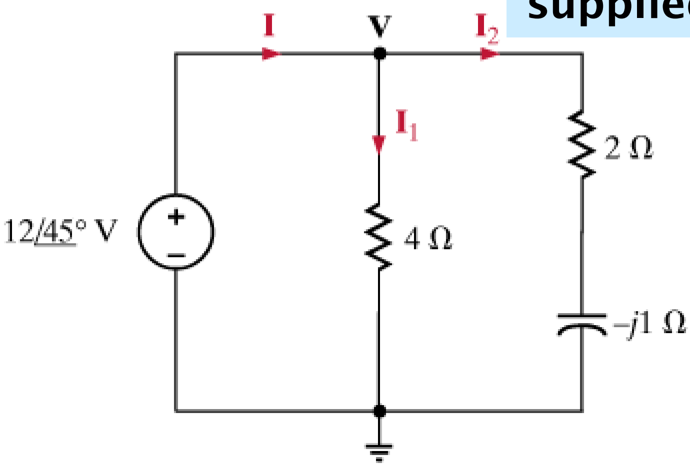
LEARNING EXAMPLE

Determine the average power absorbed by each resistor, the total average power absorbed and the average power supplied by the source



Inductors and capacitors do not absorb power in the average

$$P_{total} = 18 + 28.7W$$

$$P_{\text{supplied}} = P_{\text{absorbed}} \Rightarrow P_{\text{supplied}} = 46.7W$$

If voltage and current are in phase

$$\theta_v = \theta_i \Rightarrow P = \frac{1}{2}V_M I_M = \frac{1}{2}RI_{1M}^2 = \frac{1}{2}\frac{V_M^2}{R}$$

$$I_1 = \frac{12\angle 45^\circ}{4} = 3\angle 45^\circ (A)$$

$$P_{4\Omega} = \frac{1}{2}12 \times 3 = 18W$$

$$I_2 = \frac{12\angle 45^\circ}{2 - j1} = \frac{12\angle 45^\circ}{\sqrt{5}\angle -26.37^\circ} = 5.36\angle 71.57^\circ (A)$$

$$P_{2\Omega} = \frac{1}{2} \times 2 \times 5.36^2 (W) = 28.7W$$

Verification

$$I = I_1 + I_2 = 3\angle 45^\circ + 5.36\angle 71.57^\circ$$

$$I = 8.15\angle 62.10^\circ (A)$$

$$P = \frac{V_M I_M}{2}\cos(\theta_v - \theta_i)$$

$$P_{\text{supplied}} = \frac{1}{2}12 \times 8.15 \times \cos(45^\circ - 62.10^\circ)$$

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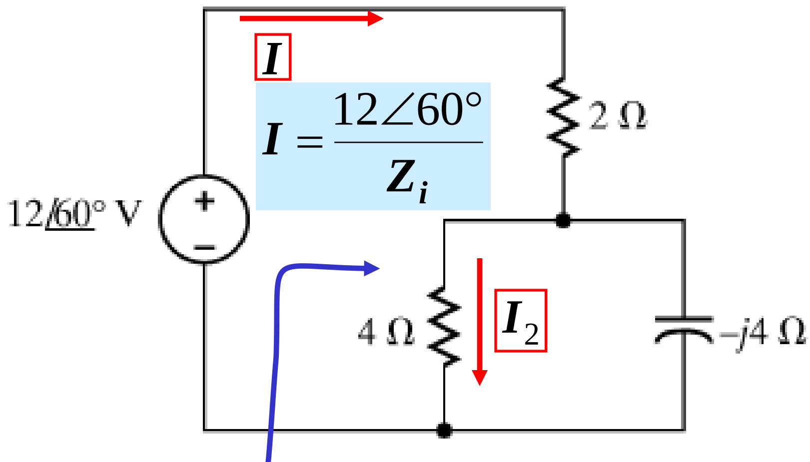
LEARNING EXTENSION

Find average power absorbed by each resistor



$$Z_i = 2 + (4 \parallel -j4)$$

$$Z_i = 2 + \frac{4(-j4)}{4-j4} = \frac{8-j8-j16}{4-j4} = \frac{25.3\angle -71.6^\circ}{4\sqrt{2}\angle -45^\circ}$$

$$Z_i = 4.47\angle -26.6^\circ \Omega$$

$$I = \frac{12\angle 60^\circ}{4.47\angle -26.6^\circ} = 2.68\angle 86.6^\circ (A)$$

$$P_{2\Omega} = \frac{1}{2} R I_M^2 = \frac{1}{2} \times 2 \times 2.68^2 = 7.20W$$

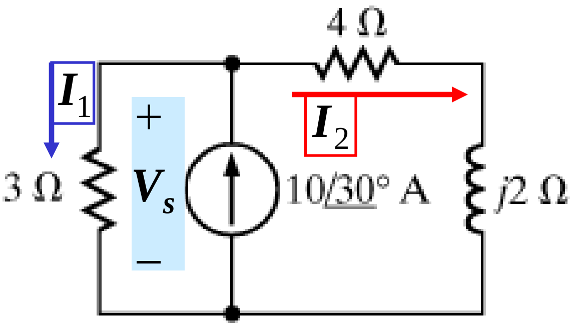
$$I_2 = \frac{-j4}{4-j4} I = \frac{4\angle -90^\circ}{4\sqrt{2}\angle -45^\circ} \times 2.68\angle 86.6^\circ$$

$$I_2 = 1.90\angle 41.6^\circ$$

$$P_{4\Omega} = \frac{1}{2} \times 4 \times 1.90^2 (W)$$

LEARNING EXTENSION

Find the AVERAGE power absorbed by each PASSIVE component and the total power supplied by the source



$$I_1 = \frac{4+j2}{3+4+j2} 10\angle 30°$$

$$I_1 = \frac{4.47\angle 26.57°}{7.28\angle 15.95°} 10\angle 30° = 6.14\angle 40.62° (A)$$

$$P_{3\Omega} = \frac{1}{2} R I_M^2 = \frac{1}{2} \times 3 \times 6.14^2 (W)$$

$$I_2 = 10\angle 30° - 6.14\angle 40.62°$$

$$I_2 = \frac{3}{3+4+j2} 10\angle 30° = \frac{30\angle 30°}{7.28\angle 15.95°}$$
$$= 4.12\angle 14.05° (A)$$

$$P_{4\Omega} = \frac{1}{2} \times 4 \times 4.12^2 (W)$$

$$P_{j2\Omega} = 0 (W)$$

Power supplied by source

Method 1. $P_{\text{supplied}} = P_{\text{absorbed}}$

$$P_{\text{supplied}} = P_{3\Omega} + P_{4\Omega} = 90.50W$$

Method 2: $P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$

$$V_s = 3 I_1 = 18.42\angle 40.62°$$

$$P = \frac{1}{2} \times 18.42 \times 10 \times \cos(40.62° - 30°)$$

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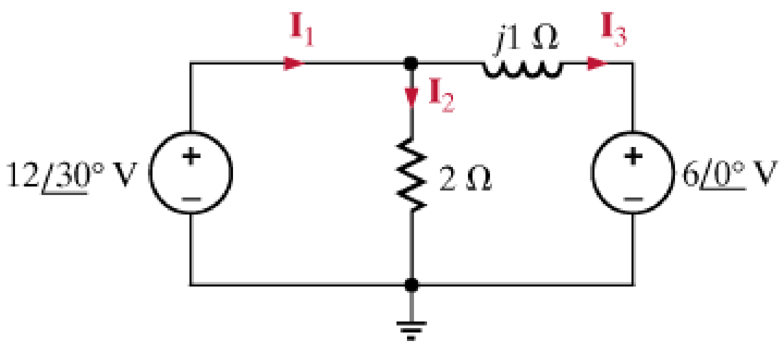
LEARNING EXAMPLE

Determine average power absorbed or supplied by each element



$$I_3 = \frac{12\angle 30^\circ - 6\angle 0^\circ}{j1} = \frac{10.39 + j6 - 6}{j} = 6 - j4.39$$
$$= 7.43\angle -36.19^\circ (A)$$

$$P_{6\angle 0^\circ} = \frac{1}{2} \times 6 \times 7.43 \cos(0 + 36.19^\circ) = 18W$$

Passive sign convention

$$I_2 = \frac{12\angle 30^\circ}{2} = 6\angle 30^\circ (A)$$

$$I_1 = I_2 + I_3 = 5.20 + j3 + 6 - j4.39 = 11.2 - j1.39(A$$
$$= 11.28\angle -7.07^\circ$$

$$P_{2\Omega} = \frac{1}{2} R I_M^2 = \frac{1}{2} \times 2 \times 6^2 = 36(W)$$

$$P_{j1\Omega} = 0$$

$$P_{12\angle 30^\circ} = \frac{1}{2} \times 12 \times 11.28 \times \cos(30^\circ + 7.07^\circ)$$
$$= -54(W) \qquad = 36 + 18$$

To determine power absorbed/supplied by sources we need the currents I1, I2

Average Power

$$P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

For resistors

$$P = \frac{1}{2} R I_M^2 = \frac{1}{2} \frac{V_M^2}{R}$$

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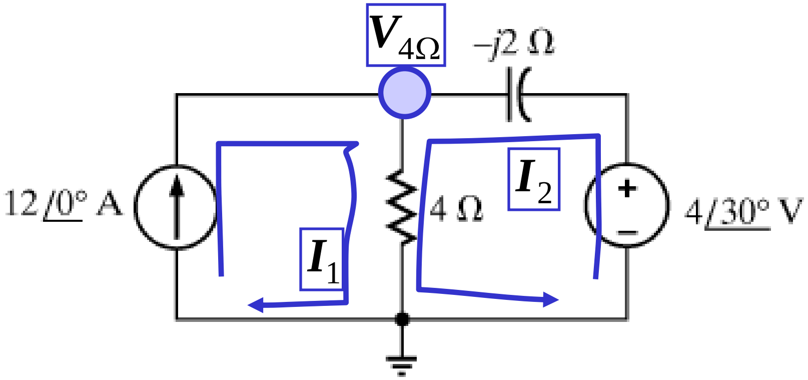
LEARNING EXTENSION

Determine average power absorbed/supplied by each element



$$P_{12\angle 0°} = -\frac{1}{2}\times 19.92\times 12\times\cos(-54.5°-0°) = -69.4(W)$$

$$P_{4\angle 30°} = -\frac{1}{2}\times 4\times(9.97)\cos(30°-204°) = 19.8(W)$$

Check: Power supplied =power absorbed

Loop Equations

$$I_1 = 12\angle 0°$$

$$4\angle 30° = -j2I_2 + 4(I_2 + 12\angle 0°)$$

$$I_2 = \frac{4\angle 30° - 48\angle 0°}{4 - j2} = \frac{3.46 + j2 - 48}{4.47\angle -26.57°}$$

$$I_2 = \frac{44.58\angle 177.43°}{4.47\angle -26.57°} = 9.97\angle 204°(A)$$

$$V_{4\Omega} = 4(I_1 + I_2) = 4(12 + 9.97\angle 204°)(V)$$
$$= 4(12 - 9.108 - j4.055)(V) = 19.92\angle -54.5°(V)$$

$$P_{4\Omega} = \frac{1}{2}\frac{V_M^2}{R} = \frac{1}{2}\times\frac{19.92^2}{4} = 49.6W$$

$$P_{-2j\Omega} = 0(W)$$

Alternative Procedure

Node Equations

$$-12\angle 0° + \frac{V_{4\Omega}}{4} + \frac{V_{4\Omega} - 4\angle 30°}{-j2} = 0$$

$$I_2 = \frac{4\angle 30° - V_{4\Omega}}{-2j}$$

Average Power

$$P = \frac{1}{2}V_M I_M\cos(\theta_v - \theta_i)$$

For resistors

$$P = \frac{1}{2}RI_M^2 = \frac{1}{2}\frac{V_M^2}{R}$$

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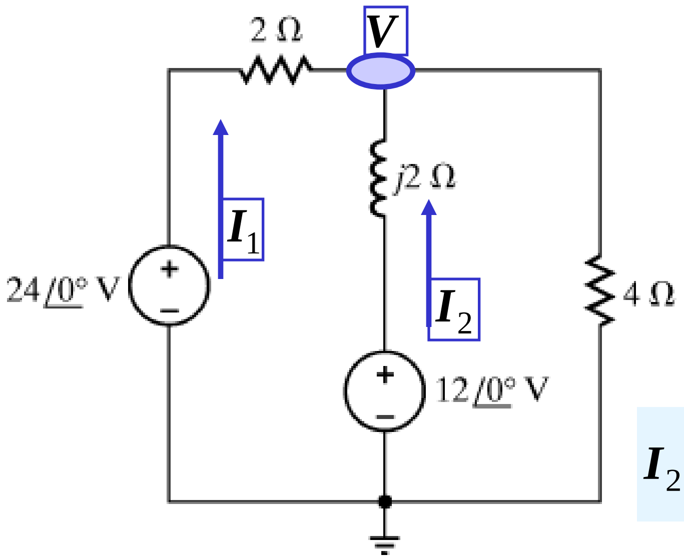
LEARNING EXTENSION

Determine average power absorbed/supplied by each element



$$I_1 = \frac{24\angle 0° - V}{2} = \frac{24 - 14.77 - j1.85}{2} = 4.62 - j0.925$$

$$I_1 = 4.71\angle -11.32°(A)$$

$$I_2 = \frac{12\angle 0° - V}{j2} = \frac{12 - 14.77 + j1.85}{j2} \times \frac{-j}{-j}$$

$$I_2 = \frac{-1.85 + j2.77}{2} = -0.925 + j1.385(A) = 1.67\angle 123.73°(A)$$

Node Equation

$$\frac{V - 24\angle 0°}{2} + \frac{V - 12\angle 0°}{j2} + \frac{V}{4} = 0 \quad \times j4$$

$$2j(V - 24) + 2(V - 12) + jV = 0$$

$$(2 + 3j)V = 24 + j48$$

$$V = \frac{24 + j48}{2 + j3} \times \frac{2 - j3}{2 - j3} = \frac{192 + j24}{13}$$

$$= 14.88\angle 7.125°(V)$$

$$= 14.77 + j1.85(V)$$

$$P_{2\Omega} = \frac{1}{2} \times 2 \times 4.71^2 = 22.18(W)$$

$$P_{4\Omega} = \frac{1}{2} \times \frac{14.88^2}{4} = 27.67(W)$$

For resistors

$$P = \frac{1}{2} R I_M^2 = \frac{1}{2} \frac{V_M^2}{R}$$

$$P_{12\angle 0°} = -\frac{1}{2} \times 12 \times 1.67 \cos(0° - 123.73°) = 5.565(W)$$

$$P_{24\angle 0°} = -\frac{1}{2} \times 24 \times 4.71 \times \cos(0° + 11.32°) = -55.42(W)$$

Average Power

$$P = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$$

Check :

$$P_{\text{absorbed}} = 22.18 + 27.67 + 5.565(W)$$

$$P_{\text{supplied}} = 55.42(W)$$

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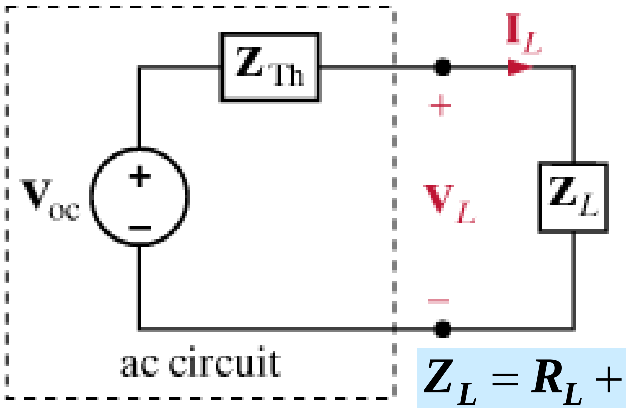
MAXIMUM AVERAGE POWER TRANSFER

$$Z_{TH} = R_{TH} + jX_{TH}$$



$$Z_L = R_L + jX_L$$

$$P_L = \frac{1}{2} V_{LM} I_{LM} \cos(\theta_{V_L} - \theta_{I_L})$$

$$= \frac{1}{2} |V_L| |I_L| \cos(\theta_{V_L} - \theta_{I_L})$$

$$V_L = \frac{Z_L}{Z_L + Z_{TH}} V_{OC} \qquad \Rightarrow |V_L| = \left| \frac{Z_L}{Z_L + Z_{TH}} \right| |V_{OC}|$$

$$I_L = \frac{V_L}{Z_L} \quad \Rightarrow \angle I_L = \angle V_L - \angle Z_L \qquad \Rightarrow |I_L| = \frac{|V_L|}{|Z_L|}$$

$$\Rightarrow \theta_{V_L} - \theta_{I_L} = \angle Z_L$$

$$Z_L = R_L + jX_L \quad \Rightarrow \tan(\angle Z_L) = \frac{X_L}{R_L}$$

$$\cos\theta = \frac{1}{\sqrt{1 + \tan^2\theta}} \qquad \therefore \cos(\theta_{V_L} - \theta_{I_L}) = \frac{R_L}{\sqrt{R_L^2 + X_L^2}}$$

$$P_L = \frac{1}{2} \frac{\cancel{|Z_L|} |V_{OC}|^2}{|Z_L + Z_{TH}|^2} \frac{R_L}{\cancel{\sqrt{R_L^2 + X_L^2}}}$$

$$Z_L + Z_{TH} = (R_L + R_{TH}) + j(X_L + X_{TH})$$

$$|Z_L + Z_{TH}|^2 = (R_L + R_{TH})^2 + (X_L + X_{TH})^2$$

$$P_L = \frac{1}{2} \frac{|V_{OC}|^2 R_L}{(R_L + R_{TH})^2 + (X_L + X_{TH})^2}$$

$$\left. \begin{array}{l} \dfrac{\partial P_L}{\partial X_L} = 0 \\ \dfrac{\partial P_L}{\partial R_L} = 0 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} X_L = -X_{TH} \\ R_L = R_{TH} \end{array} \right.$$

$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2} \left(\frac{|V_{OC}|^2}{4R_{TH}} \right)$$

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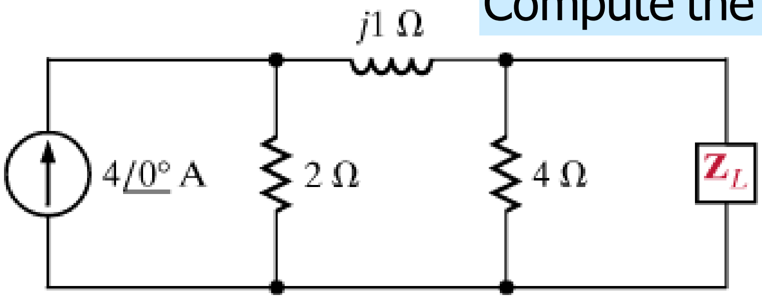
LEARNING EXAMPLE

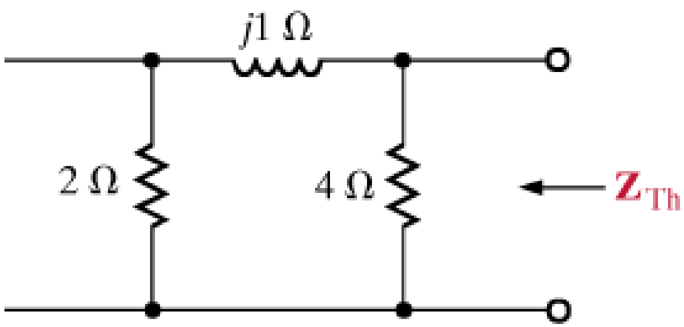
Find Z_L for maximum average power transfer.
Compute the maximum average power supplied to the load



$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

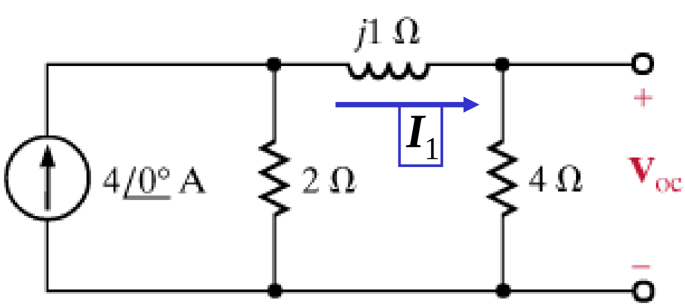
Remove the load and determine the Thevenin equivalent of remaining circuit



$$Z_{TH} = 4 \,\|\, (2 + j1) = \frac{8 + j4}{6 + j1} = \frac{(8 + j4)(6 - j1)}{37} = \frac{52 + j16}{37}\Omega$$

$$= \frac{8 + j4}{6 + j1} = \frac{8.94\angle 26.57^\circ}{6.08\angle 9.64^\circ} = 1.47\angle 16.93^\circ \Omega$$

$$Z_L^* = 1.47\angle -16.93^\circ = 1.41 - j0.43\Omega$$



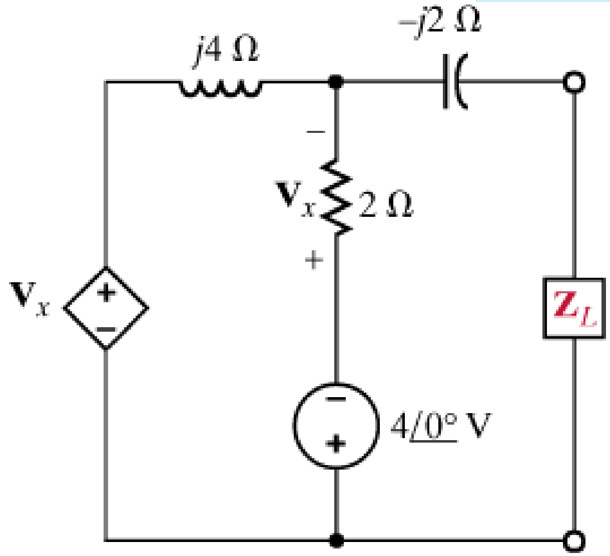
$$V_{OC} = 4 \times \frac{2}{6 + j1} 4\angle 0^\circ = \frac{32\angle 0^\circ}{6.08\angle 9.64^\circ} = 5.26\angle -9.64^\circ$$

$$P_L^{\max} = \frac{1}{2} \times \frac{5.26^2}{4 \times 1.41} = 2.45(W)$$

We are asked for the value of the power. We need the Thevenin voltage

LEARNING EXAMPLE Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load

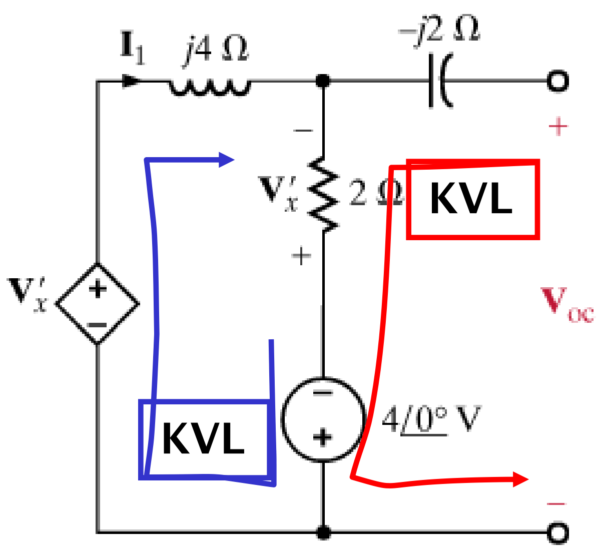


$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

Circuit with dependent sources!

$$Z_{TH} = \frac{V_{OC}}{I_{SC}}$$



$$4\angle 0° = -V_x' + (2 + j4)I_1$$

$$V_X' = -2I_1$$

$$4\angle 0° = (4 + j4)I_1 = (4\sqrt{2}\angle 45°)I_1$$

$$I_1 = \frac{4\angle 0°}{4\sqrt{2}\angle 45°} = 0.707\angle -45°(A)$$

$$V_{OC} = 2I_1 - 4\angle 0° = 1 - j1 - 4 = -3 - j1 = \sqrt{10}\angle -161.5°$$

Next: the short circuit current ...

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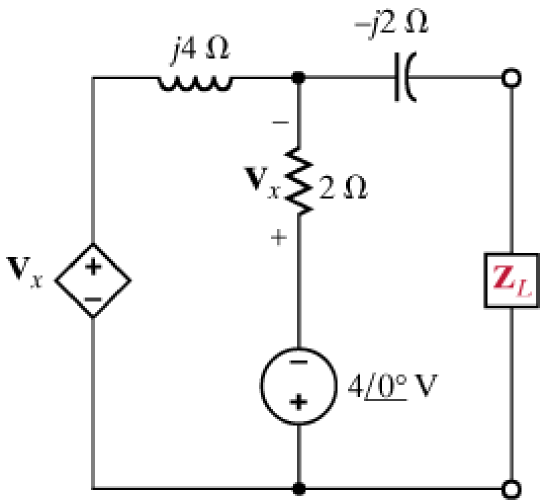
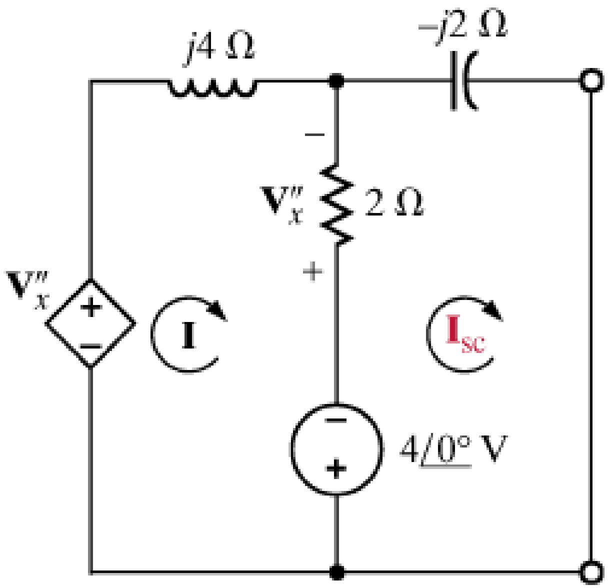
LEARNING EXAMPLE (continued)...

$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$

Original circuit





LOOP EQUATIONS FOR SHORT CIRCUIT CURRENT

$$-V_x'' + j4I + 2(I - I_{SC}) - 4\angle° = 0$$

$$4\angle 0° + 2(I_{SC} - I) - j2I_{SC} = 0$$

CONTROLLING VARIABLE

$$V_x'' = 2(I_{SC} - I)$$

Substitute and rearrange

$$(4 + j4)I - 4I_{SC} = 4$$

$$-2I + (2 - j2)I_{SC} = -4 \Rightarrow I = (1 - j1)I_{SC} + 2$$

$$4(1 + j)[(1 - j)I_{SC} + 2] - 4I_{SC} = 4$$

$$I_{SC} = -1 - j2(A) = \sqrt{5}\angle -116.57°$$

$$V_{OC} = 2I_1 - 4\angle 0° = 1 - j1 - 4 = -3 - j1 = \sqrt{10}\angle -161.57°$$

$$Z_{TH} = \sqrt{2}\angle -45° = 1 - j1\Omega \Rightarrow Z_L^{opt} = 1 + j1\Omega$$

$$P_L^{\max} = \frac{1}{2}\times\frac{(\sqrt{10})^2}{4} = 1.25(W)$$

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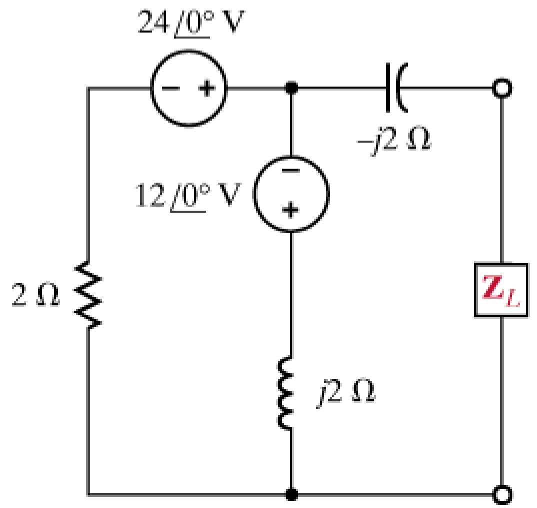
LEARNING EXTENSION

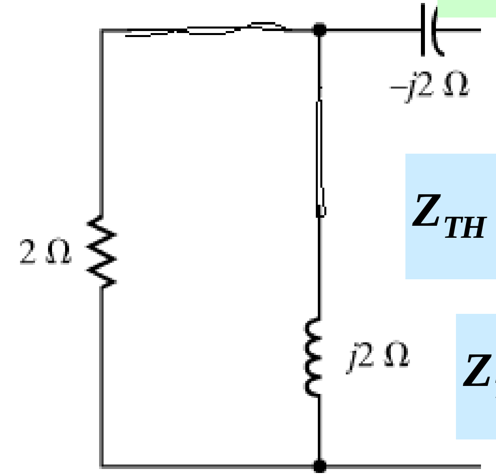
Find Z_L for maximum average power transfer.

Compute the maximum average power supplied to the load



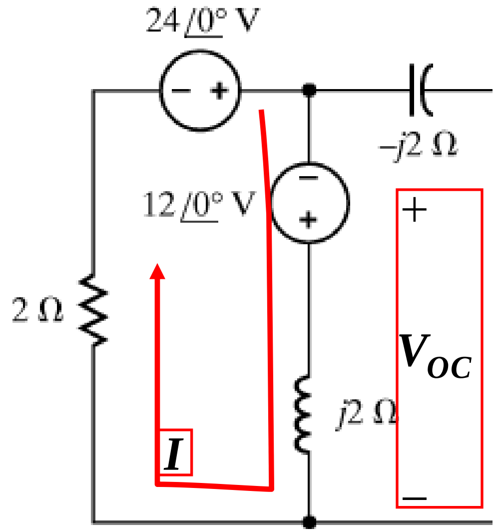
$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$



$$Z_{TH} = -j2 + (2 \parallel j2) = -j2 + \frac{4j}{2+j2}\Omega$$

$$Z_{TH} = \frac{4}{2+j2} = \frac{8-j8}{8} = 1 - j(\Omega)$$



$$V_{OC} = -12\angle 0° + j2I$$
$$= -12 + j2 \times 9(1-j)$$
$$= 6 + j18$$

$$V_{OC} = 18.974\angle 71.57°(V)$$

$$|V_{OC}|^2 = 6^2 + 18^2 = 360$$

$$Z_L^{opt} = 1 + j(\Omega)$$

$$P_L^{\max} = \frac{1}{2} \times \frac{360}{4} = 45(W)$$

$$36\angle 0° = (2 + j2)I$$

$$I = \frac{36(2-j2)}{8} = 9(1-j) = 12.73\angle -45°$$

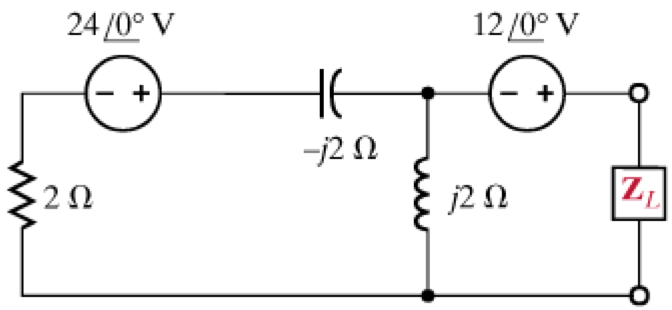
GEAUX

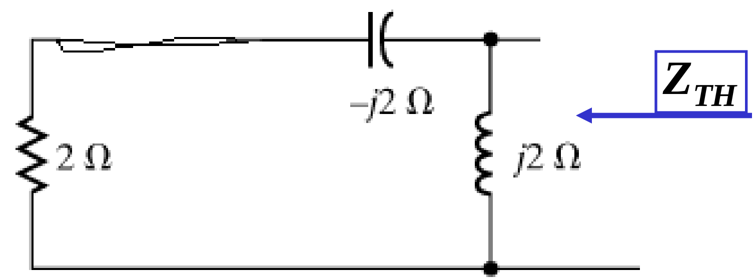
LEARNING EXTENSION

Find Z_L for maximum average power transfer.
Compute the maximum average power supplied to the load

$$\therefore Z_L^{opt} = Z_{TH}^*$$

$$P_L^{\max} = \frac{1}{2}\left(\frac{|V_{OC}|^2}{4R_{TH}}\right)$$



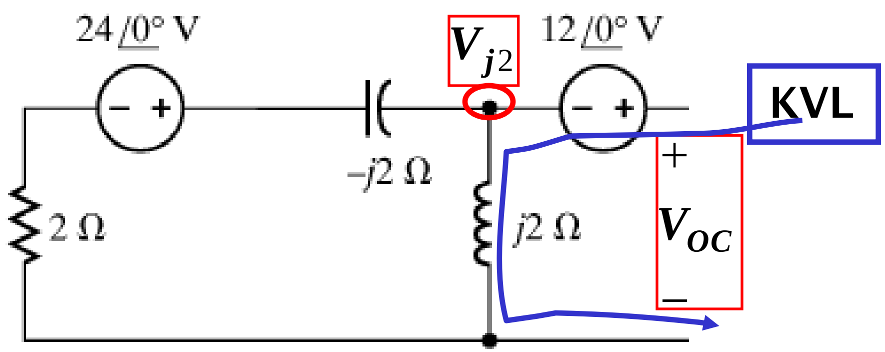


$$Z_{TH} = j2 \,\|\, (2 - j2) = \frac{j2(2-j2)}{2 + j2 - j2}$$

$$Z_{TH} = 2 + j2(\Omega)$$

$$Z_L^{opt} = 2 - j2(\Omega)$$

$$P_L^{\max} = \frac{1}{2} \times \frac{720}{4 \times 2} = 45(W)$$



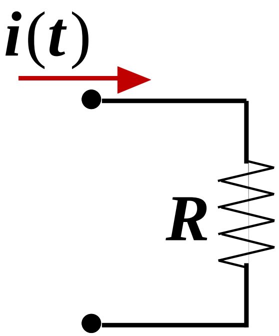
$$V_{j2} = \frac{j2}{j2 - j2 + 2} 24\angle 0^\circ = 24\angle 90^\circ$$

$$V_{OC} = 12\angle 0^\circ + 24\angle 90^\circ = 12 + j24(V)$$

$$|V_{OC}|^2 = 12^2 + 24^2 = 720$$

GEAUX

EFFECTIVE OR RMS VALUES



Instantaneous power

$$p(t) = i^2(t)R$$

The effective value is the equivalent DC value that supplies the same average power

If current is periodic with period T

$$P_{av} = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = R\left(\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt\right)$$

If current is DC ($i(t) = I_{dc}$) then

$$P_{dc} = RI_{dc}^2$$

$$I_{eff} : P_{av} = P_{dc}$$

$$I_{eff}^2 = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)dt}$$

Definition is valid for ANY periodic signal with period T

If the current is sinusoidal the average power is known to be $P_{av} = \frac{1}{2} I_M^2 R$

$$\therefore I_{eff}^2 = \frac{1}{2} I_M^2$$

For a sinusoidal signal

$$x(t) = X_M \cos(\omega t + \theta)$$

the effective value is

$$X_{eff} = \frac{X_M}{\sqrt{2}}$$

For sinusoidal case $P_{av} = \frac{1}{2} V_M I_M \cos(\theta_v - \theta_i)$

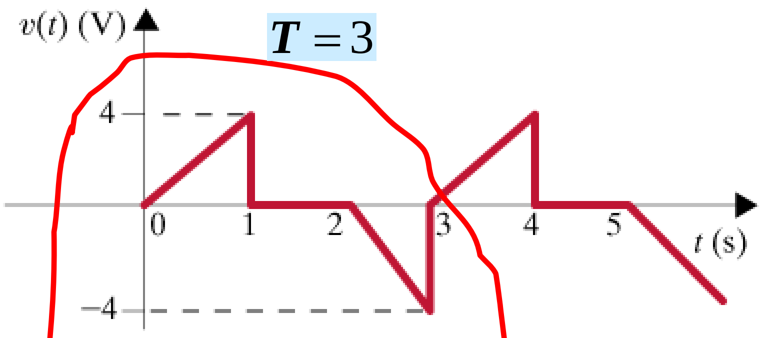
$$P_{av} = V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

effective $\approx$ rms (root mean square)

GEAUX

LEARNING EXAMPLE

Compute the rms value of the voltage waveform



$T = 3$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

One period

$$v(t) = \begin{cases} 4t & 0 < t \le 1 \\ 0 & 1 < t \le 2 \\ -4(t-2) & 2 < t \le 3 \end{cases}$$

The two integrals have the same value

$$\int_0^T v^2(t)dt = \int_0^1 (4t)^2 dt + \int_2^3 (4(t-2))^2 dt$$

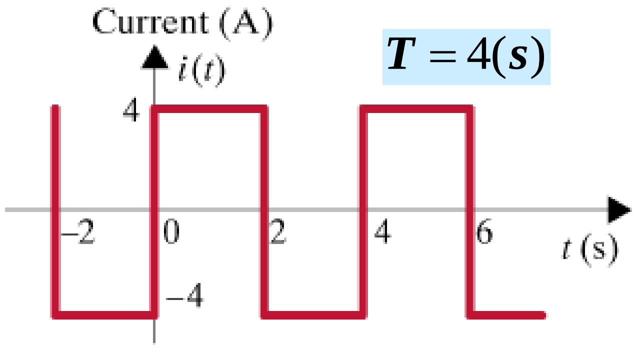
$$\int_0^3 v^2(t)dt = 2\times\left[\frac{16}{3}t^3\right]_0^1 = \frac{32}{3}$$

$$V_{rms} = \sqrt{\frac{1}{3}\times\frac{32}{3}} = 1.89(V)$$

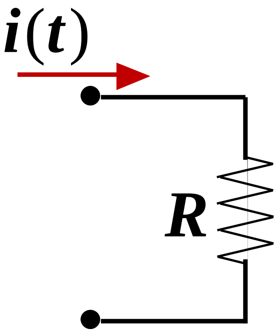
GEAUX

LEARNING EXAMPLE

Compute the rms value of the voltage waveform and use it to determine the average power supplied to the resistor



$T = 4(s)$



$R = 2\Omega$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

$$i^2(t) = 16;\ 0 \le t < 4$$

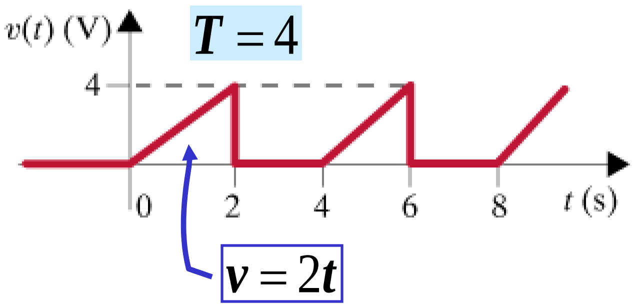
$$I_{rms} = 4(A)$$

$$P_{av} = RI_{rms}^2 = 32(W)$$

GEAUX

LEARNING EXTENSION

Compute rms value of the voltage waveform



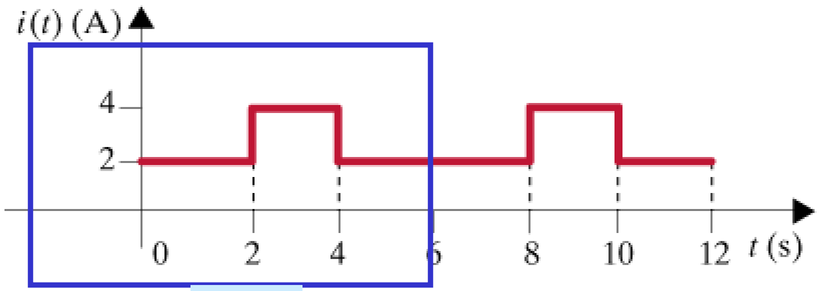
$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)\,dt}$$

$$V_{rms} = \sqrt{\frac{1}{4}\int_0^2 (2t)^2\,dt}$$

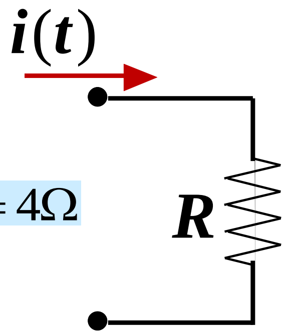
$$= \left[\frac{1}{3}t^3\right]_0^2 = \frac{8}{3}(V)$$

LEARNING EXTENSION

Compute the rms value for the current waveforms and use them to determine average power supplied to the resistor



$T = 6$



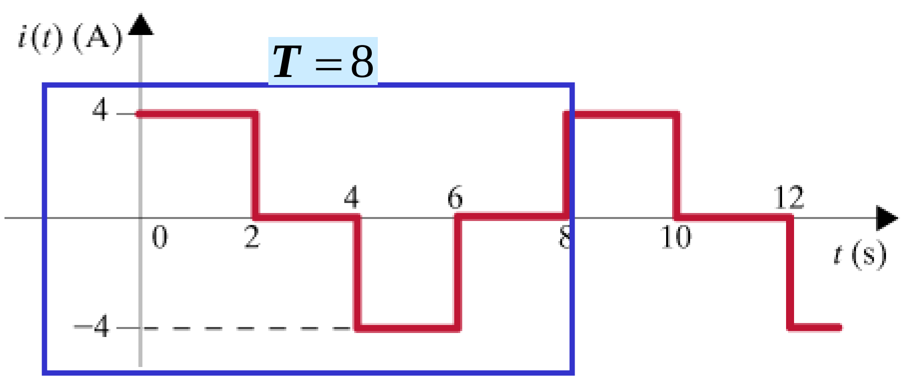
$R = 4\Omega$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} x^2(t)dt}$$

$$P_{av} = I_{rms}^2 R$$

$$I_{rms}^2 = \frac{1}{6}\left[\int_0^2 4dt + \int_2^4 16dt + \int_4^6 4dt\right] = \frac{8+32+8}{6} = 8$$

$$P = 8 \times 4 = 32(W)$$



$T = 8$

$$I_{rms}^2 = \frac{1}{8}\left[\int_0^2 16dt + \int_4^6 16dt\right] = 8$$

$$P = 32(W)$$

GEAUX