Department of Mathematics

15B11MA111 **Mathematics-I**

Tutorial Sheet 1 **B.Tech. Core**

Partial Differentiation

1. Determine the following limits if they exist.

   (a) $\lim\limits_{(x,y)\to(0,0)} (1-x-y)/(x^2+y^2)$ (b) $\lim\limits_{(x,y)\to(0,0)} xy/|xy|$ (c) $\lim\limits_{(x,y)\to(0,0)} \cot^{-1}\left(\frac{1}{\sqrt{x^2+y^2}}\right)$

2. Show that the function $f(x,y)=\begin{cases}\frac{x^2+y^2}{|x|+|y|}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

   is **continuous** at $(0,0)$ but its partial derivatives f_x and f_y **do not exist** at $(0,0)$.

3. Show that the function $f(x,y)=\begin{cases}\frac{xy}{x^2+5y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

   is **not continuous** at $(0,0)$ but its partial derivatives f_x and f_y **exist** at $(0,0)$.

4. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function $f(x,y)=\begin{cases}\frac{x^3y}{x^2+y}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$..

   Also discuss the continuity of f_{xy} and f_{yx} at $(0,0)$.

5. If $x^y y^x z^z = c$, then find $\partial^2 z/\partial x\partial y$ at $x=y=z$.

6. If $u(x,y)=\tan^{-1}\left(xy/\sqrt{1+x^2+y^2}\right)$, show that $\frac{\partial^2 u}{\partial x\partial y}=\frac{1}{\left(1+x^2+y^2\right)^{3/2}}$.

7. If $u=\tan^{-1}\left\{(x^3+y^3)/(x-y)\right\}$, prove that

   (i) $x\,\partial u/\partial x + y\,\partial u/\partial y = \sin 2u$, (ii) $x^2\,\partial^2 u/\partial x^2 + 2xy\,\partial^2 u/\partial x\partial y + y^2\,\partial^2 u/\partial y^2 = (2\cos 2u-1)\sin 2u$

8. If $u=x^2-y^2+\sin yz$, where $y=e^x$, and $z=\log x$, find $\frac{du}{dx}$.

9. If $u=F(x-y,y-z,z-x)$, prove that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$.

10. If $z=uv$, and $u^2+v^2-x-y=0$, $u^2-v^2+3x+y=0$, find $\frac{\partial z}{\partial x}$.

11. Transform the equation $\left(\frac{\partial u}{\partial x}\right)^2-\left(\frac{\partial u}{\partial y}\right)^2=0$ into polar coordinates.

12. If $u=x^2-y^2$, $v=2xy$ and $f(x,y)=\phi(u,v)$, show that $\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}=4\left(x^2+y^2\right)\left(\frac{\partial^2\phi}{\partial u^2}+\frac{\partial^2\phi}{\partial v^2}\right)$

Answers: 1. (a) Limit exists and unbounded (b) Limit DNE (c) limit=0. **(3)** f_x & f_y = 0. **(4)** $f_{xy}(0,0)$=1, $f_{yx}(0,0)$=0. **(5)** $-3/z(1+\log z)$. **(8)** $2x+\left(-2y+z\cos yz\right)e^x+\left(y\cos yz\right)/x$. **(10)** $\left(2u^2-v^2\right)/2uv$.

(11) $\left(\frac{\partial u}{\partial r}\right)^2-\frac{1}{r^2}\left(\frac{\partial u}{\partial\theta}\right)^2=\frac{2\tan 2\theta}{r}\frac{\partial u}{\partial r}\frac{\partial u}{\partial\theta}$.