Transformers and DC machine

Introduction

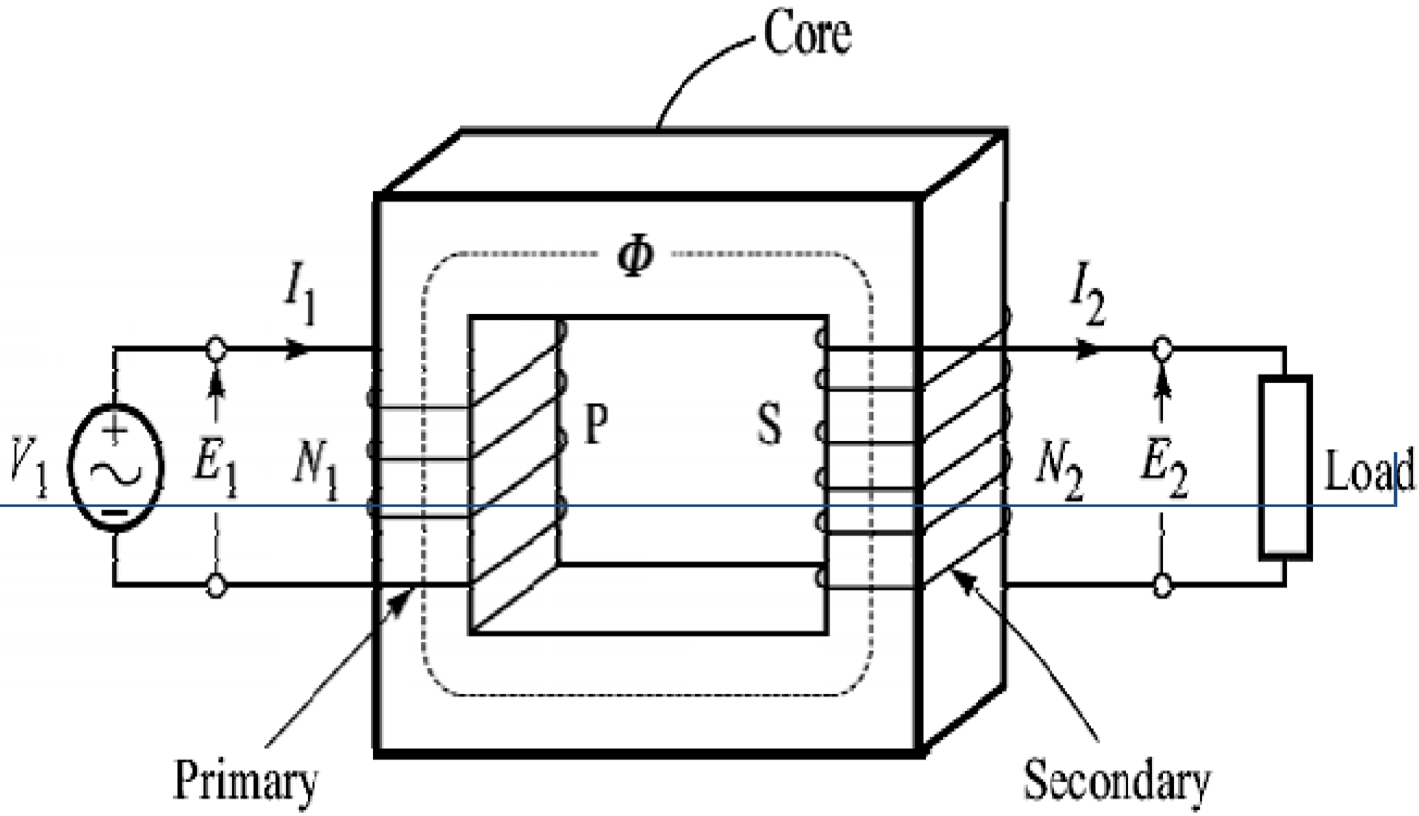
- A transformer is a highly efficient (about 99.5 %) static (non-moving) device.
- It transfers electrical energy form one circuit to another through magnetic coupling (usually from one ac voltage level to another), without any change in its frequency.
- It raises or lowers the voltage in a circuit but with a corresponding decrease or increase in current.
- The product of voltage and current i.e. power remains constant.

- It has two windings, insulated from each other, and wound ona core made up of a magnetic material, because magnetic field can store energy 25000 times more as compared to electric field.
- Transformation of voltage is necessary at different stages of the electrical network consisting of generation, transmission and distribution.
- **Applications:**
  - **Transmission and distribution:** Converts generated voltage of about 11 kV to higher voltages of 132 kV, 220 kV, 400 KV for transmission and to lower voltages up to 440 V for distribution.
  - **Small-sized transformers:** used in communication circuits, radio and TV circuits, telephone circuits, instrumentation and control systems.
  - **Audio transformers**

Principle of Operation

- It operates on the principle of mutual induction between two coils.
- When two coils are inductively coupled and if current in one coil is changed uniformly, then an EMF gets induced in the other coil.
- This EMF can drive a current, when a closed path is provided to it.

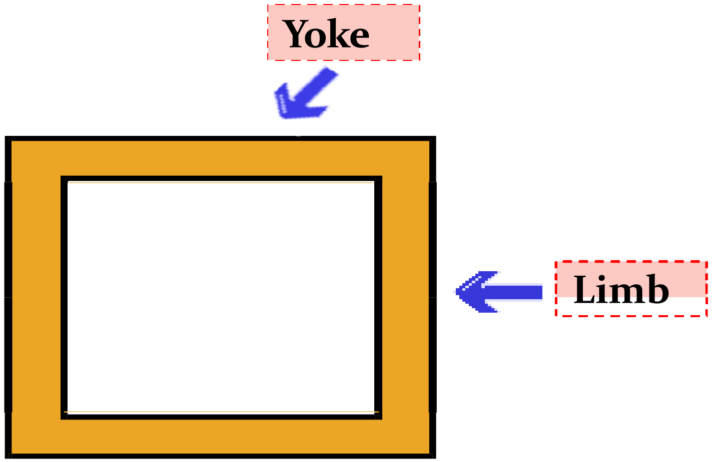
- Construction:



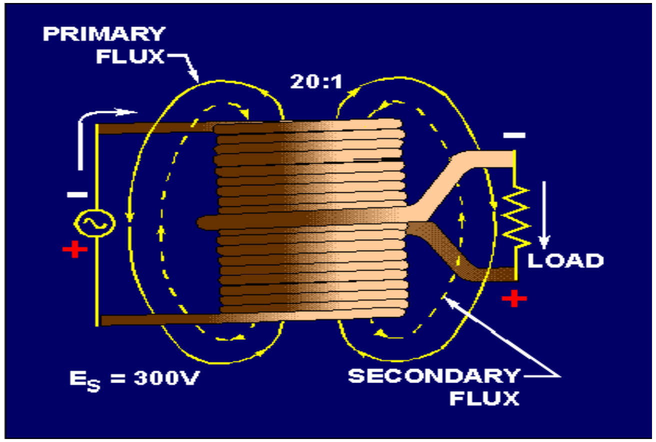
- The main parts are:
  - An iron core that provides a magnetic circuit.
  - Two inductive coils wound on the core. They are suitably insulated from each other and also from the core.
  - A suitable container for assembled core and windings.
  - A suitable medium for insulating the core and winding from the container and cooling windings and core (transformer oil).
  - Suitable brushings (porcelain, oil-filled or capacitor type) for insulating and bringing out the terminals of the windings from the tank.
- The vertical portions of steel core are called Limbs and top and bottom portions are called Yokes.
- Coils P and S (Primary and Secondary) are wound on the two limbs.

– Two main parts: Core and Winding(coils)

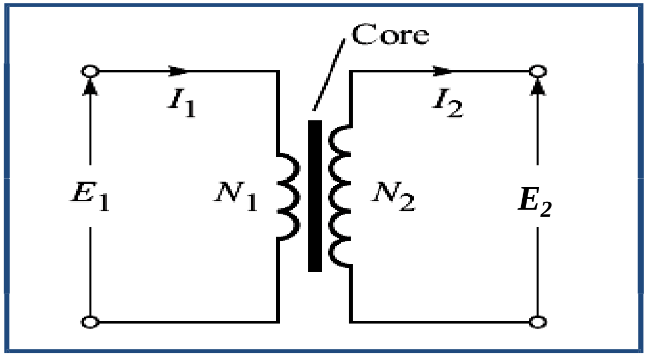
Magnetic core



Winding or coil



- Circuit Symbol:



- N_1 : Number of turns in the Primary
- N_2 : Number of turns in the Secondary
- E_1 : EMF Induced in the Primary
- E_2 : EMF Induced in the Secondary
- I_1: Current through the Primary
- I_2: Current through the Secondary

- Working:
  - There are 2 principles involved:
    - An electric current produces a magnetic field (Electromagnetism).
    - A changing magnetic field within a coil induces an EMF across the ends of the coil (Electromagnetic Induction).
  - In primary circuit, a changing current produces a changing magnetic field.
  - In secondary circuit, voltage is induced by the changing magnetic field produced.
  - There is transfer of energy from one circuit to other.

- EMF Equation:

  - Due to the sinusoidally varying voltage V_1 applied to the primary, the magnetic flux set up in the core is:

$$\Phi = \Phi_m \sin \omega t = \Phi_m \sin 2\pi f t$$

  - According to law of EMI, the resulting induced EMF in a winding of N turns:

$$e = -N \frac{d\Phi}{dt} = -N \frac{d}{dt}(\Phi_m \sin \omega t)$$

$$= -N\omega\Phi_m \cos \omega t = \omega N \Phi_m \sin(\omega t - \pi/2)$$

  - The peak value of the induced EMF is:

$$E_m = \omega N \Phi_m$$

– The RMS value of the induced EMF E:

$$E = \frac{E_m}{\sqrt{2}} = \frac{\omega N \Phi_m}{\sqrt{2}} = \frac{2\pi f N \Phi_m}{\sqrt{2}} = 4.44 \, f N \Phi_m$$

or $$\boxed{E = 4.44 \, f N \Phi_m}$$

This equation is known as **EMF equation** of transformer.

- Effect of Frequency:

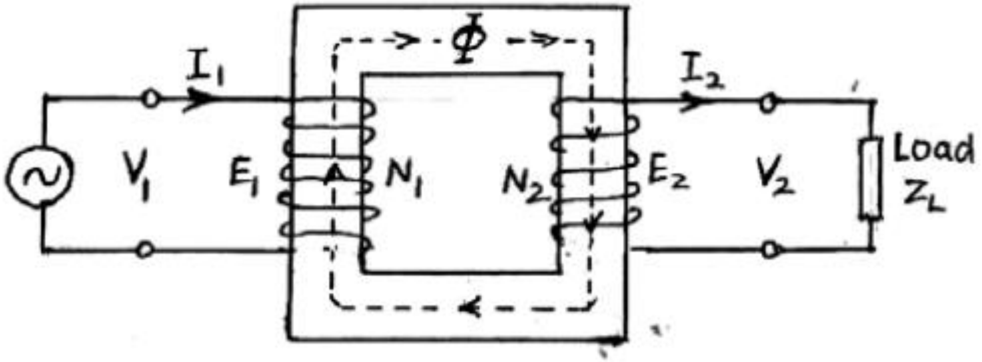
  - At a given flux, EMF of a transformer increases with frequency.
  - By operating at higher frequencies, transformers can be made physically more compact.
  - Because a given core is able to transfer more power without reaching saturation.
  - Fewer turns are needed to achieve same impedance.
  - At higherfrequencies, core losses and skin effect increases, hence, it cannot be increased indefinitely.
  - **E.g.:** Aircraft and military equipments employ 400- Hz power supplies which reduces size and weight.

Ideal Transformer

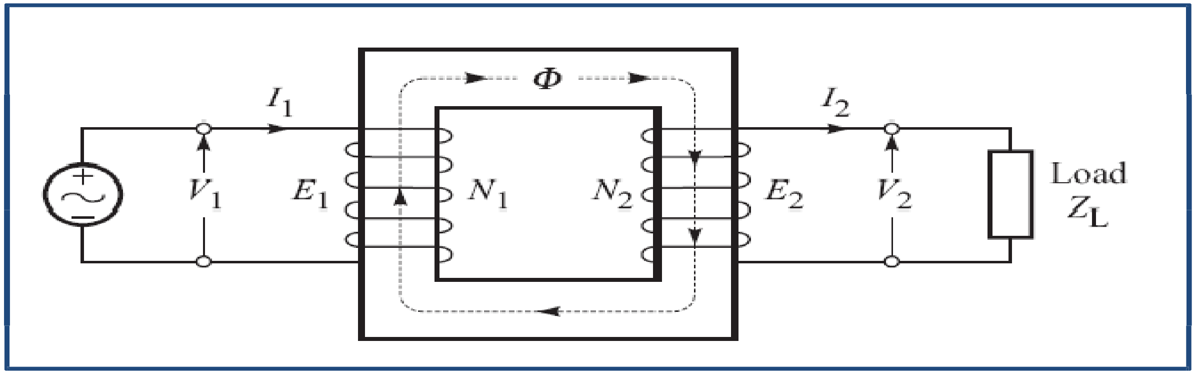
- Has no losses and stores no energy.
- Has no physical existence but is useful in understanding working of actual transformer.
- Conditions:
  - The permeability (μ) of the core is infinite, (i.e., the magnetic circuit has zero reluctance so that no MMF is needed to set up the flux in the core).
  - The core of the transformer has no losses.
  - The resistance of its windings is zero, hence no I^2R losses in the windings.
  - Entire flux in the core links both the windings, i.e., there is no leakage flux.

Conditions for Ideal Transformer

1. **Permeability of core, $\mu_{core} = \infty$** i.e. zero reluctance
2. **Core has no losses** i.e. no hysteresis and eddy current losses
3. **Windings have no resistance** i.e. no ohmic power loss
4. **No flux leakage** i.e. all flux confined in core
5. **Applied voltage,** $V_1 = -E_1$ since no voltage drop in primary windings
6. $E_2 = V_2$ i.e. since no voltage drop in secondary windings

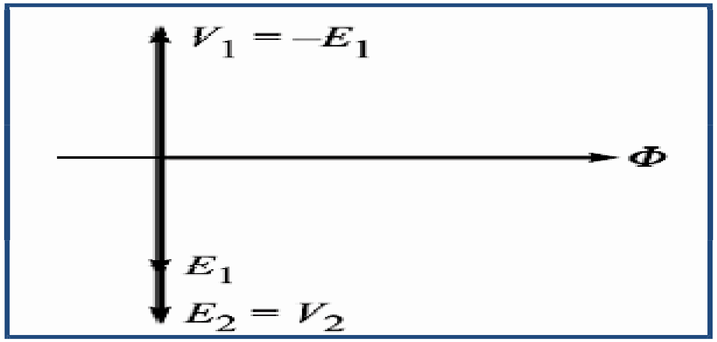


- Circuit Diagram:



- The primary and secondary windings have zero impedance for ideal transformer.
- As reluctance of the magnetic circuit is zero, the required magnetizing current to produce Φ is also zero.

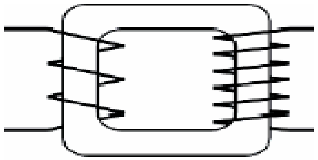
- **Phasor Diagram:**
  - We take flux Φ as reference phasor, as it is common to both the primary and secondary.
  - $V_1 = -E_1$ and $E_2 = V_2$
  - EMF E_1 in the primary exactly counter balances the applied voltage V_1. Hence, E_1 is called *counter emf* or *back emf*.
  - EMF E_2 is called *mutually induced emf*.



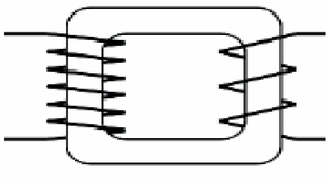
Transformation Ratio

Step-Up and Step-Down Transformer

If $N_1 < N_2$ $\Rightarrow$ $E_1 < E_2$ $\Rightarrow$ Step up



If $N_1 > N_2$ $\Rightarrow$ $E_1 > E_2$ $\Rightarrow$ Step down



The transformation ratio,

$$\boxed{K = \frac{N_2}{N_1} = \frac{E_2}{E_1}}$$

Volt-Amperes

- *Output power* depends on $\cos\phi_2$ (power factor of secondary).
- As *pf* can change depending on the load, the rating is not specified in watts or kilowatts.
- But is indicated as a product of voltage and current called VARATING.
- For ideal transformer : $\boxed{V_1 I_1 = V_2 I_2}$

$$kVA\ rating\ of\ a\ transformer = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

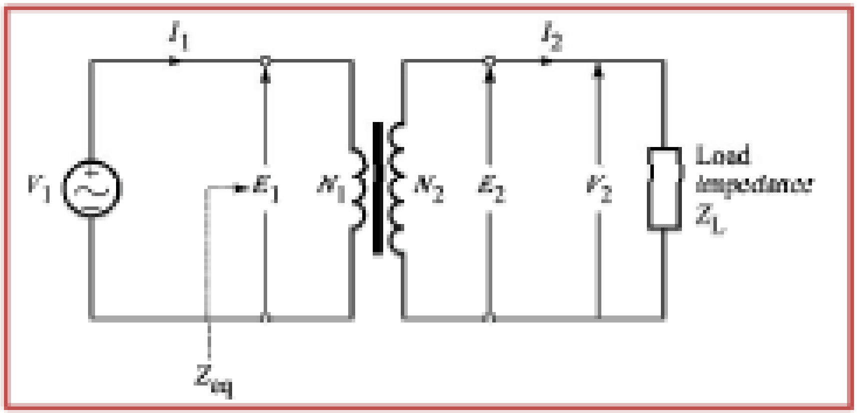
$$\boxed{I_{1\ (\text{full load})} = \frac{\text{kVA rating} \times 1000}{V_1}} \qquad \boxed{I_{2\ (\text{full load})} = \frac{\text{kVA rating} \times 1000}{V_2}}$$

- Why is transformer rating in kVA?

  - Transformers are rated in VA, because the manufacturer does not know the power factor of the load which you are going to connect.
  - So the customer should not exceed the VA rating of the transformer.
  - In case of motors, the manufacturer knows exactly the power factor at full load.
  - That is why motors are rated in kW.

Impedance Transformation

The concept of impedance transformation is used for impedance matching.



$$Z_{eq} = \frac{V_1}{I_1} = \frac{V_1 \times (V_2 I_2)}{I_1 \times (V_2 I_2)} = \left(\frac{V_1}{V_2}\right) \times \left(\frac{I_2}{I_1}\right) \times \left(\frac{V_2}{I_2}\right) = \left(\frac{1}{K}\right) \times \left(\frac{1}{K}\right) \times Z_L$$

or $$\boxed{Z_{eq} = Z_L / K^2}$$

Example 1: A single-phase, 50-Hz transformer has 30 primary turns and 350 secondary turns. The net cross-sectional area of the core is 250 cm^2. If the primary winding is connected to a 230-V, 50-Hz supply, calculate

(*a*) the peak value of flux density in the core,

(*b*) the voltage induced in the secondary winding, and

(*c*) the primary current when the secondary current is 100 A. (Neglect losses.)

Solution :

(*a*) The peak value of the flux,

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{230}{4.44 \times 50 \times 30} = 0.034534 \text{ Wb}$$

$$\therefore \quad B_m = \frac{\Phi_m}{A} = \frac{0.034534}{250 \times 10^{-4}} = \mathbf{1.3814\ T}$$

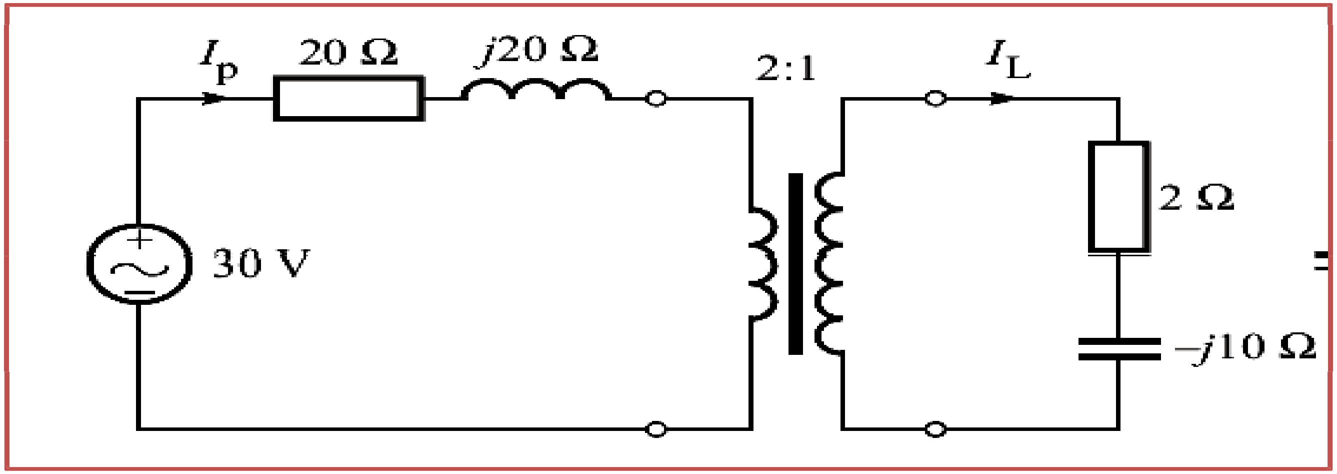
(*b*) The voltage induced in the secondary,

$$E_2 = E_1 \times \frac{N_2}{N_1} = 230 \times \frac{350}{30} = 2683.33 \text{ V} \approx \mathbf{2.683\ kV}$$

(*c*) The primary current,

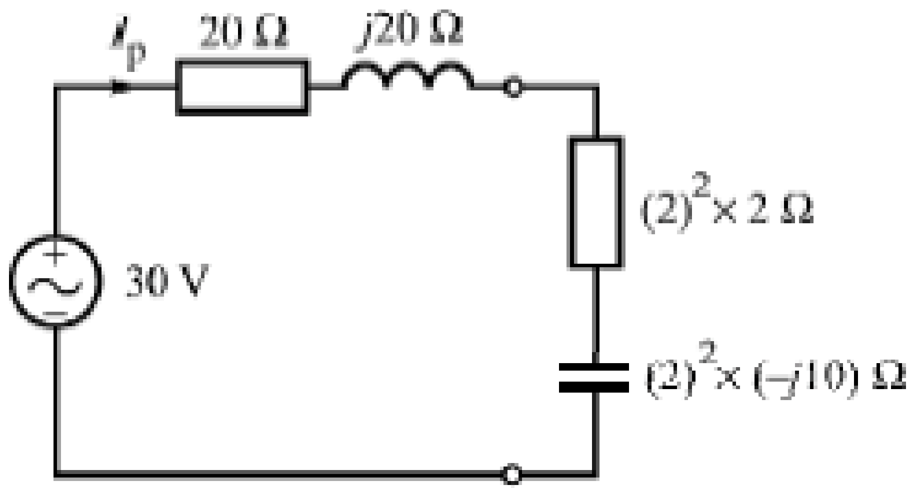
$$I_1 = I_2\left(\frac{N_2}{N_1}\right) = 100 \times \left(\frac{350}{30}\right) = 1166.67 \text{ A} \approx \mathbf{1.167\ kA}$$

Example 2: Determine the load current I_L in the ac circuit shown:



Solution :

Transforming the load impedance into the primary:

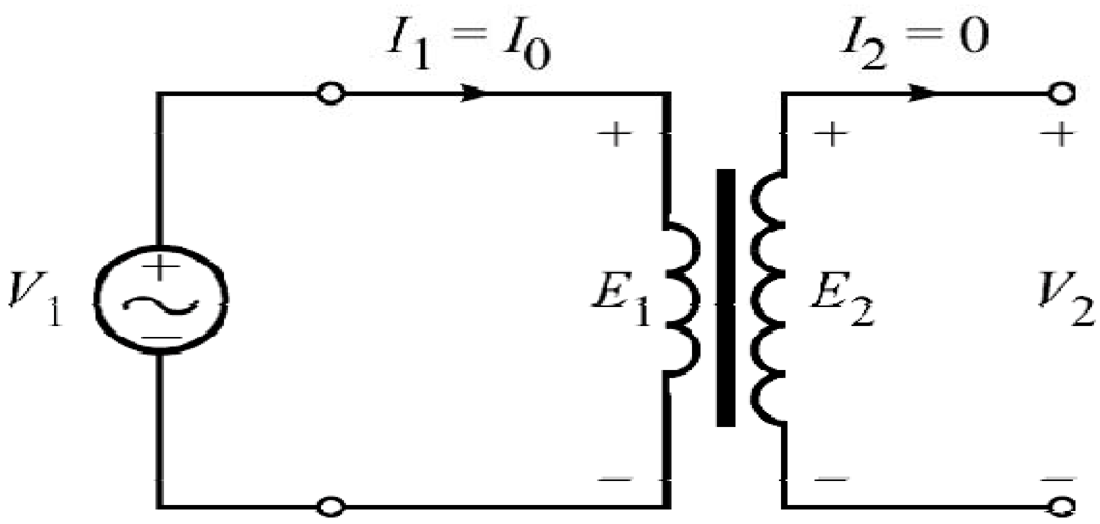


$$\mathbf{I}_\mathrm{p} = \frac{30\angle 0^\circ}{20 + j20 + 2^2(2 - j10)} = 0.872\angle 35.53^\circ \text{ A}$$

$$\therefore \quad \mathbf{I}_\mathrm{L} = 2\times \mathbf{I}_\mathrm{p} = 2\times 0.872\angle 35.53^\circ = \mathbf{1.74\angle 35.53^\circ \text{ A}}$$

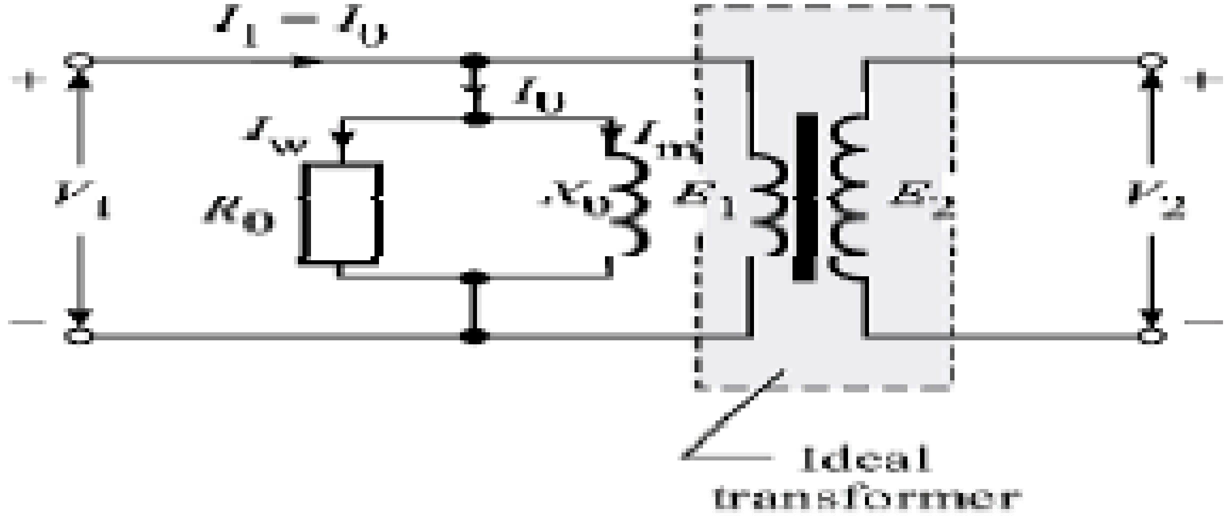
Practical Transformer at no Load

- Let primary be connected to a sinusoidal alternating voltage V_1.
- Let I_0 be the ***no-load primary current*** (also called ***exciting current***) .i.e. is the resultant of two components:
  - Magnetizing current component (I_m) due to effect of Magnetisation.
  - Iron loss component (I_w) due to the effect of Core Losses.

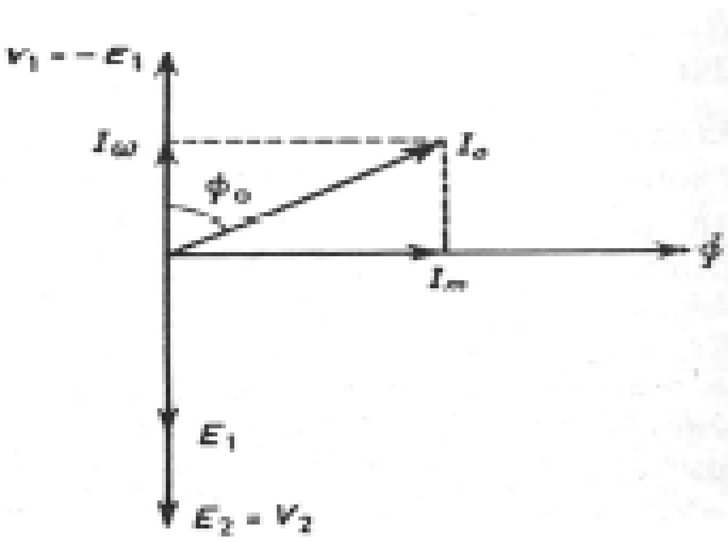


- Effect of Magnetisation:
  - No magnetic material can have infinite permeability.
  - A finite mmf is needed to establish magnetic flux in the core.
  - An in-phase ***magnetizing current*** I_m in the primary is needed to set up flux Φ in the core.
  - I_m is purely reactive (current I_m lags voltage V_1 by 90°).
  - This effect is modeled by putting reactance X_0 in parallel with the ideal transformer.
- **Effect of Core Losses :**
  - There exist hysteresis and eddy current losses for the energy loss in the core.
  - The source must supply enough power to the primary to meet the core losses.
  - These can be represented by putting a resistance R_0 in parallel with the ideal transformer.
  - The core-loss current I_w flowing through R_0 is in phase with the applied voltage V_1.

- **Equivalent Circuit:**



- **Phasor Diagram:**



- Angle Φ_0 is called ***no-load phase angle***.
- From phasor diagram:

$$I_0 = \sqrt{I_w^2 + I_m^2}\,; \qquad \phi_0 = \tan^{-1}(I_m / I_w);$$

$$\begin{aligned} \text{and} \quad \text{Input power} &= \text{Iron loss} \\ &= V_1 I_w = V_1 I_0 \cos\phi_0 \end{aligned}$$

- The R_0-X_0 circuit is called ***exciting circuit***.

Loss in Transformer

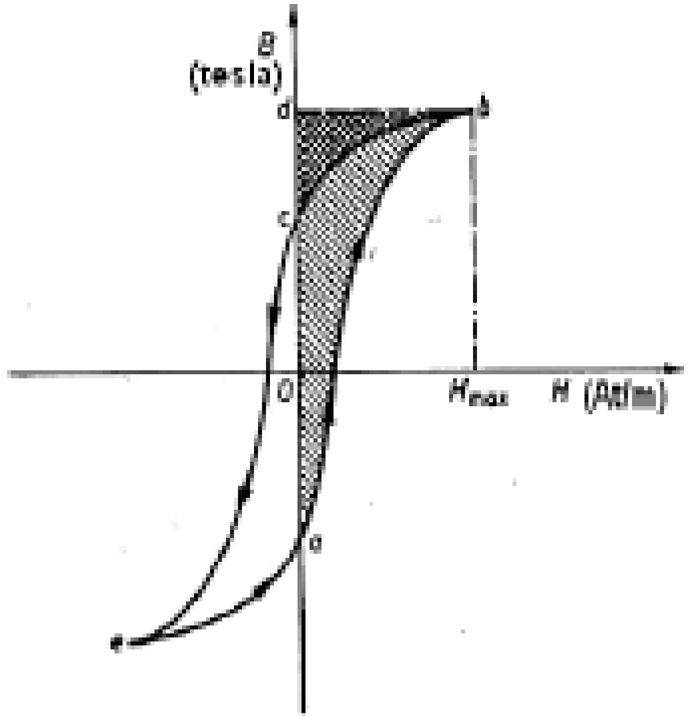
- A) Hysteresis Loss
- B) Eddy Current Loss

a) Hysteresis Loss:

When alternating current flows through the windings, the core material undergoes cyclic process of magnetization and demagnetization. It is found that there is a tendency of the flux density B to *lag behind* the field strength H. It is called *hysteresis*.

Total energy loss (per cubic metre) is represented by the area *abcea* of the hysteresis loop. The hysteresis loss (usually expressed in watts) is given as :

$$P_h = K_h B_m^n f\, V$$



where:

- K_h = hysteresis coefficient whose value depends upon the material (K_h = 0.025 for cast steel, K_h = 0.001 for silicon steel)
- B_m = maximum flux density (in tesla)
- n = a constant, depending upon the material = Stein Metz's constant
- f = frequency (in hertz)
- V = volume of the core material (in m^3)

Remedy: This loss can be minimized by selecting suitable ferromagnetic material for the core (Cold Rolled Grain Oriented Steel-CRGOS).

b) Eddy-Current Losses:

The eddy currents are the circulating currents set up in the core due to alternating magnetic flux. These currents may be quite high since the resistance of the iron is quite low. This results in unnecessary heating of the core and loss of power. The eddy-current loss (in watts) is given by:

$$P_e = K_e B_m^2 f^2 t^2 V$$

where:

- K_e = a constant dependent upon the material
- t = thickness of laminations (in meters)

Remedy: To reduce Eddy current losses laminated sheets of CRGO steel are used to make the transformer core.

Example 3: A single-phase, 230-V/110-V, 50- Hz transformer takes an input of 350 volt amperes at no load while working at rated voltage. The core loss is 110 W. Find

(a) the no-load power factor,

(b) the loss component of no-load current, and

(*c*)the magnetizing component of no-load current.

Solution : (*a*) Given :

$$V_1 I_0 = 350 \text{ VA}$$

$$\therefore \quad I_0 = \frac{VA}{V_1} = \frac{350}{230} = 1.52 \text{ A}$$

The core loss = Input power at no load,

$$P_i = V_1 I_0 \cos\phi_0$$

$$\therefore \quad pf = \cos\phi_0 = \frac{P_i}{V_1 I_0} = \frac{110 \text{ W}}{350 \text{ VA}} = \mathbf{0.314}$$

(*b*) The loss component of no-load current,

$$I_w = I_0 \cos\phi_0 = 1.52 \times 0.314 = \mathbf{0.478 \text{ A}}$$

(*c*) The magnetizing component of no-load current,

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.52)^2 - (0.478)^2} = \mathbf{1.44 \text{ A}}$$

Example 4: A 100-kVA, 4000-V/200-V, 50-Hz, single-phase transformer has 100 secondary turns. Determine:

(a) the primary and secondary currents,

(b) the number of primary turns, and

(c) the maximum value of the flux.

Solution :

(a) The kVA rating = $V_1I_1 = V_2I_2$ = 100 kVA.

$$\therefore\ I_1 = \frac{\text{kVA rating}}{V_1} = \frac{100\,000}{4000} = \mathbf{25\ A}$$

$$I_2 = \frac{\text{kVA rating}}{V_2} = \frac{100\,000}{200} = \mathbf{500\ A}$$

(b) Since $$\frac{N_1}{N_2} = \frac{V_1}{V_2}$$

$$\therefore\quad N_1 = \left(\frac{V_1}{V_2}\right)N_2 = \left(\frac{4000}{200}\right)\times 100 = \mathbf{2000}$$

(c) $$E_2 = 4.44 f \Phi_m N_2$$

$$\therefore\quad \Phi_m = \frac{E_2}{4.44 f N_2} = \frac{200}{4.44\times 50\times 100}$$

$$= \mathbf{9.01\ mWb}$$

Example 5: A single-phase, 440-V/110-V, 50-Hz transformer takes a no-load current of 5 A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a power factor of 0.8 lagging to a load, determine the primary current and the primary power factor. Also, draw the phasor diagram.

Solution:

$$\phi_0 = \cos^{-1} 0.2 = 78.46^\circ \quad \text{and} \quad \phi_2 = \cos^{-1} 0.8 = 36.87^\circ$$

$$K = \frac{V_2}{V_1} = \frac{110}{440} = \frac{1}{4}$$

$$\therefore \quad I_1^{'} = K \times I_2 = (1/4) \times 120 = 30 \text{ A}; \quad \mathbf{I}_1^{'} = 30\angle -36.87^\circ \text{ A}$$

$$\mathbf{I}_1 = \mathbf{I}_1^{'} + \mathbf{I}_0 = 30\angle -36.87^\circ + 5\angle -78.46^\circ = \mathbf{33.9\angle -42.49^\circ \text{ A}}$$

Primary power factor:

$$pf = \cos\phi_1 = \cos 42.49^\circ = \mathbf{0.737 \text{ (lagging)}}$$

Practical Transformer with Load

Till now we have considered an ideal transformer:

1. **No resistance and no reactance of the windings.**
2. **All the flux produced by the primary links with the secondary.**

However, in practical transformer the above conditions are not true and there is:

1. **Effect of Winding Resistance:** The windings of the transformer cause power loss called I^2R loss or copper loss. This effect is accounted for by including a resistance R1 in the primary and resistance R2 in the secondary, as shown in figure.

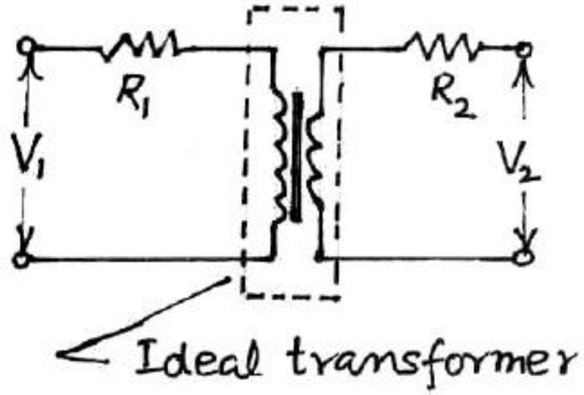


Figure: Effect of winding resistance

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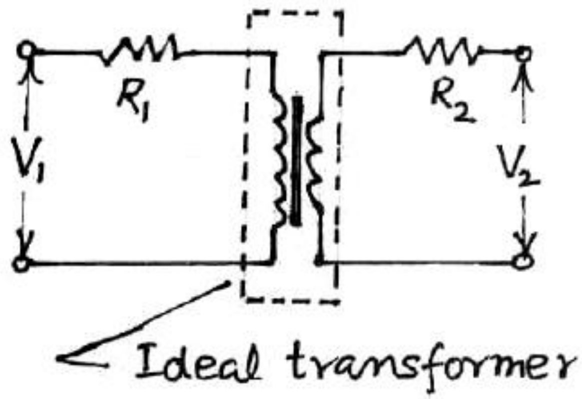
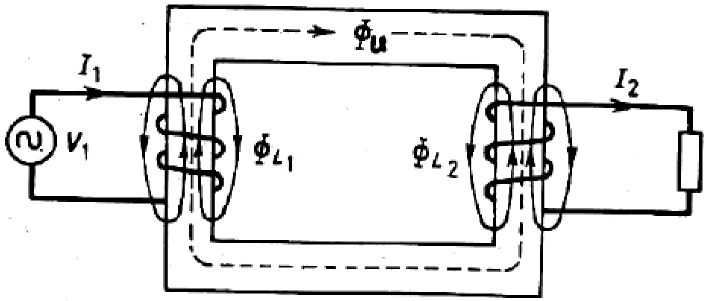


Figure: Effect of winding resistance

2. Effect of Flux Leakage: The entire magnetic flux does not remain confined to the magnetic core and thus not all the flux produced by the primary winding links with the secondary.



Φ_{L1}: ***Primary leakage flux***
Φ_{L2}: ***Secondary leakage flux***
Φ_U: **Useful mutual flux**

The difference between the total flux linking with the primary winding and the useful mutual flux linking with both the windings, is called the primary leakage flux, Φ_{L1}. This induces a voltage E_{L1} in the primary winding (leading the current by $\pi/2$). Hence, the concept of leakage reactances, X_1 and X_2.

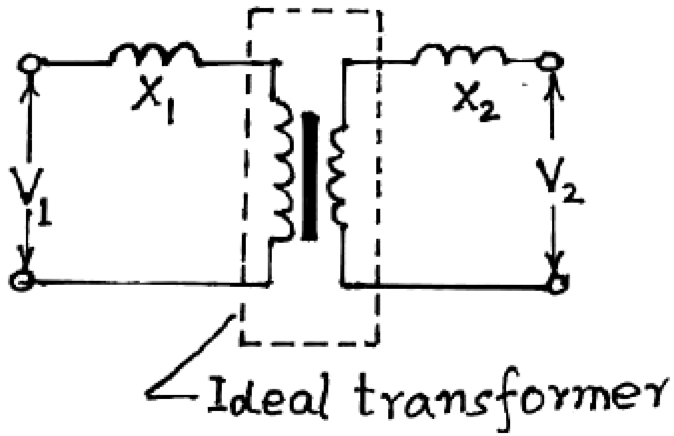


Figure: Effect of leakage flux in a transformer

$$\boxed{E_{L1} = I_1 X_1 \quad \text{and} \quad E_{L2} = I_2 X_2}$$

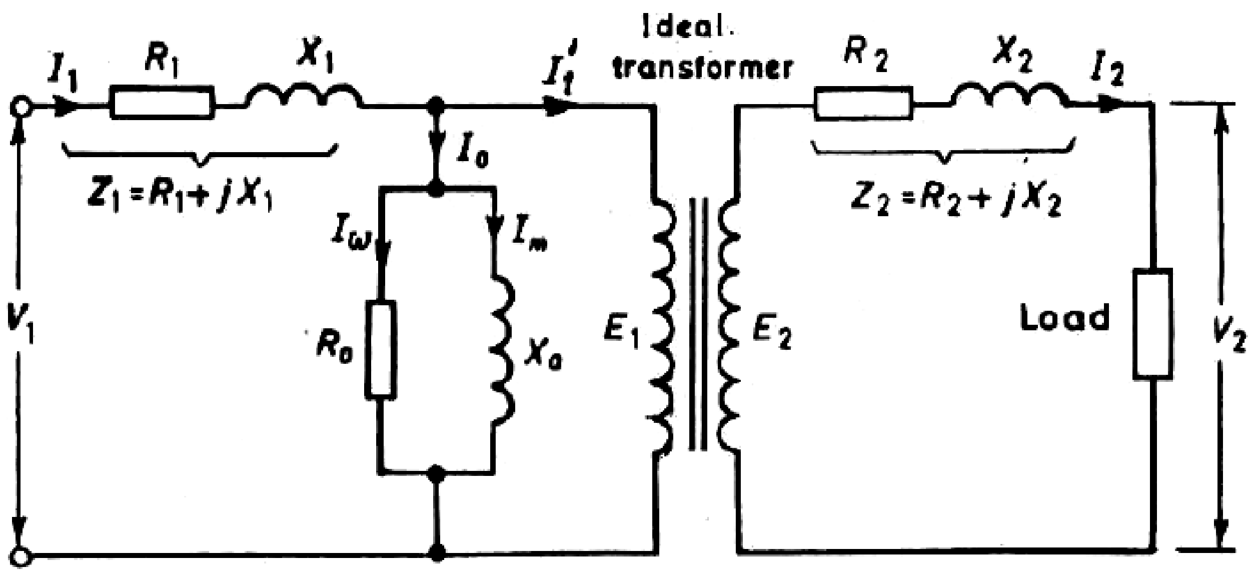
EQUIVALENT CIRCUIT OF A TRANSFORMER

The equivalent circuit is merely a circuit representation of the equations that describe the behavior of the device.

Writing the KVL equations for the two sides,

$$\mathbf{V}_1 = I_1 R_1 + jI_1 X_1 - \mathbf{E}_1 = I_1 (R_1 + jX_1) - \mathbf{E}_1$$

$$\mathbf{E}_2 = I_2 R_2 + jI_2 X_2 + \mathbf{V}_2 = I_2 (R_2 + jX_2) + \mathbf{V}_2$$



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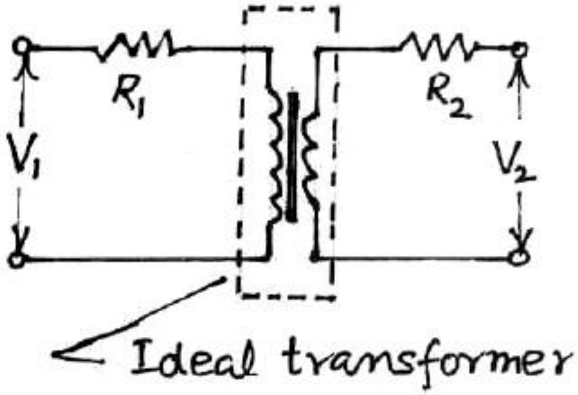

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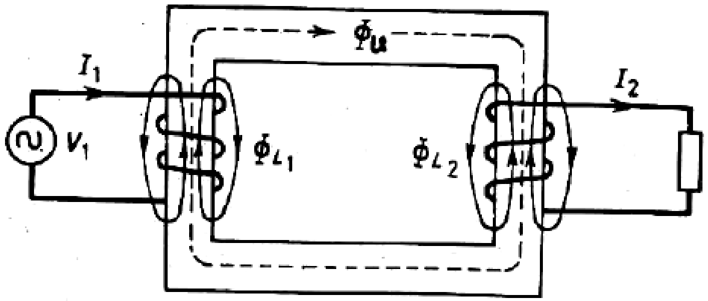

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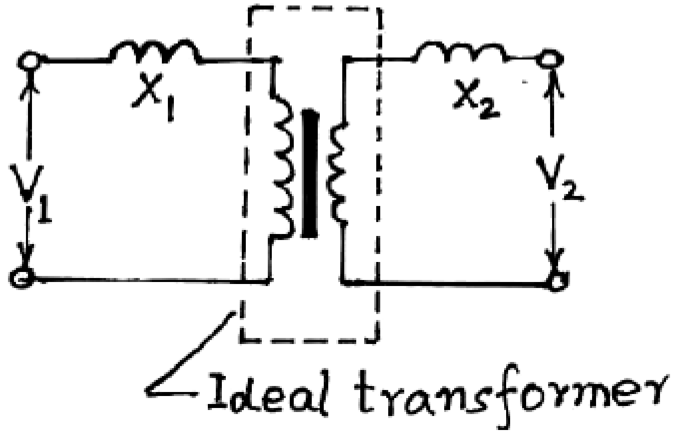

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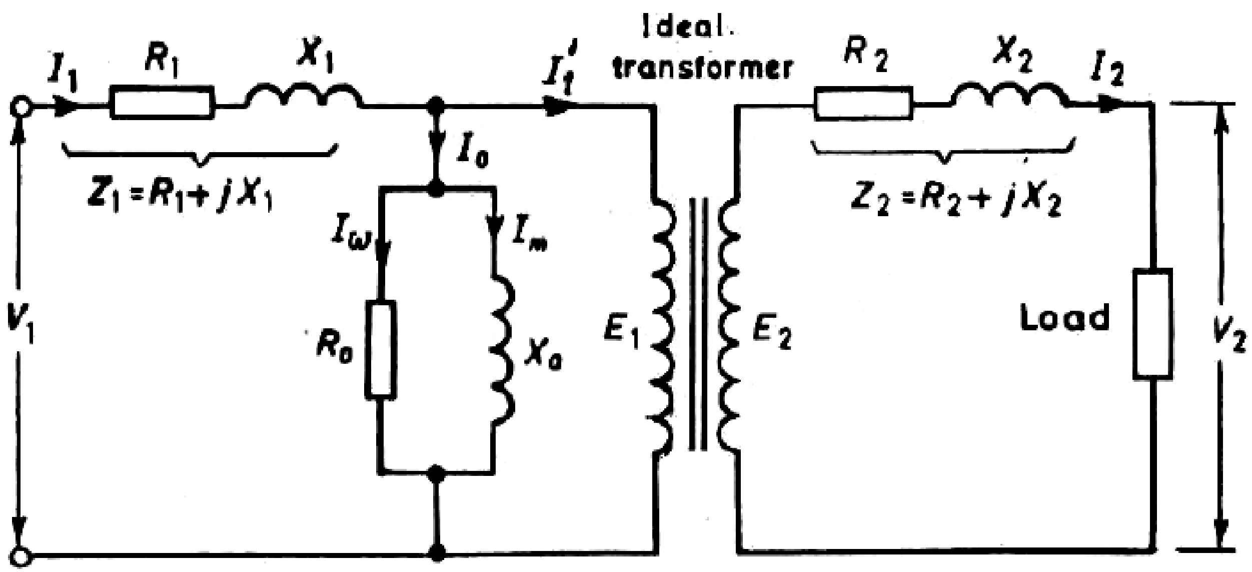
The equivalent circuit is merely a circuit representation of the equations that describe the behavior of the device.

Writing the KVL equations for the two sides,

$$\mathbf{V}_1 = I_1 R_1 + jI_1 X_1 - \mathbf{E}_1 = I_1 (R_1 + jX_1) - \mathbf{E}_1$$

$$\mathbf{E}_2 = I_2 R_2 + jI_2 X_2 + \mathbf{V}_2 = I_2 (R_2 + jX_2) + \mathbf{V}_2$$



Example 6: A single-phase, 50-kVA, 4400-V/220-V, 50-Hz transformer has $R_1 = 3.45\ \Omega$, $R_2 = 0.009\ \Omega$, $X_1 = 5.2\ \Omega$ and $X_2 = 0.015\ \Omega$. Calculate

(a) the R_e as referred to the primary,

(b) the R_e as referred to the secondary,

(c) the X_e as referred to the primary,

(d) the X_e as referred to the secondary,

(e) the Z_e as referred to the primary,

(f) the Z_e as referred to the secondary, and

(g) the total copper loss.

Solution : Full-load primary current:

$$I_1 = \frac{\text{kVA}}{V_1} = \frac{50\,000}{4400} = 11.36 \text{ A}$$

Full-load secondary current:

$$I_2 = \frac{\text{kVA}}{V_2} = \frac{50\,000}{220} = 227.27 \text{ A}$$

Transformer ratio:

$$K = \frac{V_2}{V_1} = \frac{220}{4400} = \frac{1}{20} = 0.05$$

(a) $R_{e1} = R_1 + (R_2 / K^2) = 3.45 + [0.009 /(0.05)^2] = \mathbf{7.05\ \Omega}$

(b) $R_{e2} = K^2 R_1 + R_2 = (0.05)^2 \times 3.45 + 0.009 = \mathbf{0.0176\ \Omega}$

(c) $X_{e1} = X_1 + (X_2 / K^2) = 5.2 + [0.015 /(0.05)^2] = \mathbf{11.2\ \Omega}$

(d) $X_{e2} = K^2 X_1 + X_2 = (0.05)^2 \times 5.2 + 0.015 = \mathbf{0.028\ \Omega}$

(e) $Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{(7.05)^2 + (11.2)^2} = \mathbf{13.23\ \Omega}$

(f) $Z_{e2} = \sqrt{R_{e2}^2 + X_{e2}^2} = \sqrt{(0.0176)^2 + (0.028)^2} = \mathbf{0.0331\ \Omega}$

(g) Total copper loss:

$$= I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (227)^2 \times 0.009 = \mathbf{909\ W}$$

Alternatively, by considering equivalent resistances, total copper loss:

$$= I_1^2 R_{e1} = (11.36)^2 \times 7.05 = \mathbf{909.8\ W}$$

$$= I_2^2 R_{e2} = (227.27)^2 \times 0.0176 = \mathbf{909\ W}$$

Voltage Regulation of transformer

- The voltage regulation of a transformer is defined as the change in its secondary terminal voltage from no load to full load, the primary voltage being assumed constant.
- $V_{2(0)}$ = secondary terminal voltage at no load and V_2 = secondary terminal voltage at full load.
- The voltage drop $V_{2(0)}$-V_2 is called the **inherent regulation.**

(i) **Per unit regulation down** $= \dfrac{V_{2(0)} - V_2}{V_{2(0)}}$

% regulation down $= \dfrac{V_{2(0)} - V_2}{V_{2(0)}} \times 100$

(ii) **Per unit regulation up** $= \dfrac{V_{2(0)} - V_2}{V_2}$

% regulation up $= \dfrac{V_{2(0)} - V_2}{V_2} \times 100$

Normally, when nothing is specified, '*regulation*' means '*regulation down*'.

Example 8: The open circuit voltage of a transformer is 240 V. A tap changing device is set to operate when the percentage regulation drops below 2.5%. Determine the load voltage at which the mechanism operates.

Solution :

$$\text{Regulation} = \frac{(\text{no load voltage} - \text{terminal load voltage})}{\text{no load voltage}} \times 100\%$$

$$\text{Hence } 2.5 = \left[\frac{240 - V_2}{240}\right] 100\%$$

$$\therefore \quad \text{the load voltage, } V_2 = 240 - 6 = \boxed{\mathbf{234\,V}}$$

VOLTAGE REGULATION OF A TRANSFORMER

$$\% \text{ Regulation} = \frac{I_2 R_{e2} \cos\phi \pm I_2 X_{e2} \sin\phi}{V_{2(0)}} \times 100$$

in which + sign is to be used for lagging power factor and – sign for leading power factor.

Condition for Zero Regulation

$$I_2 R_{e2} \cos\phi - I_2 X_{e2} \sin\phi = 0 \qquad \text{or} \qquad \tan\phi = \frac{R_{e2}}{X_{e2}}$$

Condition for Maximum Regulation

$$\frac{d}{d\phi}(I_2 R_{e2} \cos\phi + I_2 X_{e2} \sin\phi) = 0 \quad \Rightarrow \quad (-I_2 R_{e2} \sin\phi + I_2 X_{e2} \cos\phi) = 0$$

$$\tan\phi = \frac{X_{e2}}{R_{e2}}$$

Example 9: A single-phase, 40-kVA, 6600-V/250-V, transformer has primary and secondary resistances $R_1 = 10\ \Omega$ and $R_2 = 0.02\ \Omega$, respectively. The equivalent leakage reactance as referred to the primary is $35\ \Omega$. Find the full-load regulation for the load power factor of

(*a*) unity,

(*b*) 0.8 lagging, and

(*c*) 0.8 leading.

Solution : Given : $R_1 = 10\ \Omega$; $R_2 = 0.02\ \Omega$; $X_{e1} = 35\ \Omega$

$$\text{the turns-ratio, } K = \frac{250}{6600} = 0.0379$$

$$\text{the full-load current, } I_2 = \frac{40000}{250} = 160\ \text{A}$$

$$\therefore \quad R_{e2} = K^2 R_1 + R_2 = (0.0379)^2 \times 10 + 0.02 = 0.0343\ \Omega$$

$$\text{and} \quad X_{e2} = K^2 X_{e1} = (0.0379)^2 \times 35 = 0.0502\ \Omega$$

(a) For power factor, $\cos\phi = 1$; $\sin\phi = 0$. Hence,

$$\therefore \quad \%\ \text{Regulation} = \frac{I_2 R_{e2} \cos\phi + I_2 X_{e2} \sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 1 + 0}{250} \times 100 = \mathbf{2.195\ \%}$$

(b) For power factor, $\cos\phi = 0.8$ (lagging, ϕ positive):

$$\sin\phi = \sqrt{1-\cos^2\phi} = 0.6$$

$$\therefore \quad \%\ \text{Regulation} = \frac{I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi}{V_{2(0)}}\times 100$$

$$= \frac{160\times 0.0343\times 0.8 + 160\times 0.0502\times 0.6}{250}\times 100 = \mathbf{3.68\ \%}$$

(c) For power factor, $\cos\phi = 0.8$ (leading, ϕ negative):

$$\sin\phi = 0.6$$

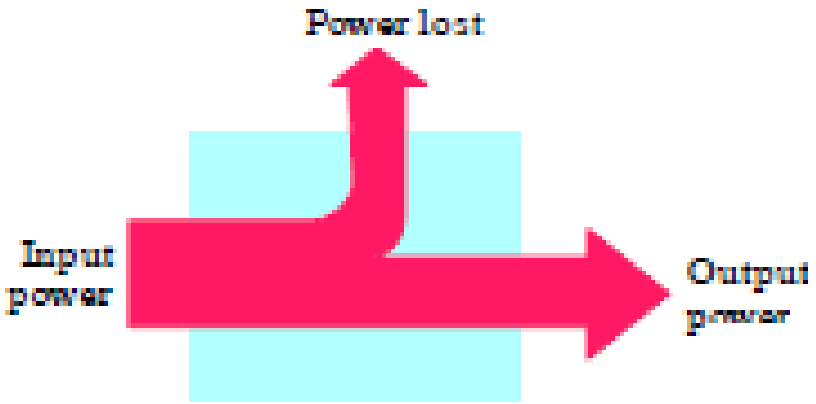
$$\therefore \quad \%\ \text{Regulation} = \frac{I_2R_{e2}\cos\phi - I_2X_{e2}\sin\phi}{V_{2(0)}}\times 100$$

$$= \frac{160\times 0.0343\times 0.8 - 160\times 0.0502\times 0.6}{250}\times 100 = \mathbf{-0.172\ \%}$$

Efficiency of a transformer

- Like any other machine, the efficiency of a transformer is defined as:

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power output}}{\text{Power output} + \text{Power loss}} = \frac{P_o}{P_o + P_l}$$



- Ideal transformer is 100% efficient.
- **Large-size transformers** are designed to be more efficient ($\eta > 98$ %)
- But, the efficiency of **small transformers** (used in power adapters for charging mobile phones) is not more than 85 %.

- **Power losses in transformer:**

1. Copper losses or I^2R losses :

– In the primary and secondary windings, given as:

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{e1} = I_2^2 R_{e2}$$

– *The copper losses are variable* with current (square of current).

$$P_c \, \alpha \, I^2$$

– Copper losses for a given load (and hence for given VA) can be calculated as (assuming voltage to be constant):

$$P_c = \left(\frac{\mathrm{VA}}{\mathrm{VA_{FL}}} \right)^2 P_{c(\mathrm{FL})}$$

2. Iron losses or core losses :

- Due to hysteresis and eddy-currents.
- Given by: $\boldsymbol{P_i = P_h + P_e}$
- Since the flux Φ_m does not vary more than about 2 % between no load and full load, it is usual to assume *the core losses constant at all loads*.

- The efficiency of a transformer can thus be written as:

$$\eta = \frac{P_o}{P_o + P_l} = \frac{P_o}{P_o + P_c + P_i} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e2} + P_i}$$

Condition for maximum efficiency

- Taking I_2 common, $\eta = \frac{V_2 \cos\phi}{V_2 \cos\phi + x^2 I_2 R_{e2} + P_i/I_2}$
- For maximum η, the denominator should be minimum $\frac{d}{dI_2}(V_2 \cos\phi + x^2 I_2 R_{e2} + P_i/I_2) = 0$
- We obtain, $x^2 I_2 R_{e2} = P_i$ or $x^2 P_c = P_i$
- Hence, efficiency is maximum when iron loss=copper loss.

Example 10: For a single-phase, 50-Hz, 150-kVA transformer, the required no-load voltage ratio is 5000-V/250-V and the full-load copper losses are 1800 W and core losses are 1500 W. Find

(*a*) the number of turns in each winding for a maximum core flux of 0.06 Wb,

(*b*) the efficiency at half rated kVA, and unity power factor,

(*c*) the efficiency at full load, and 0.8 power factor lagging, and

(*d*) the kVA load for maximum efficiency.

Solution: (a) Using the emf equation, we have:

$$E_2 = 4.44 f N_2 \Phi_m$$

$$\Rightarrow \quad N_2 = \frac{E_2}{4.44 f \Phi_m} = \frac{250}{4.44 \times 50 \times 0.06} = 18.8 \text{ (say, 19 turns)}$$

and $\quad N_1 = \dfrac{E_1}{E_2} N_2 = \dfrac{5000}{250} \times 19 = 380 \text{ turns}$

(b) At half rated-kVA, the current is half the full-load current, and hence the output power too reduces by 0.5. Thus:

$$P_o = 0.5 \times (\text{kVA}) \times (\text{power factor}) = 0.5 \times 150 \times 1 = 75 \text{ kW}$$

$$P_c = (0.5)^2 \times (\text{full-load copper loss}) = (0.5)^2 \times 1800 \text{ W} = 0.45 \text{ kW}$$

Iron losses (fixed), $P_i = 1500 \text{ W} = 1.5 \text{ kW}$

$$\therefore \qquad \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{75}{75 + 0.45 + 1.5} \times 100 = \mathbf{97.47\ \%}$$

(c) At full load and 0.8 power factor:

$$P_o = (\text{kVA}) \times (\text{power factor}) = 150 \times 0.8 = 120 \text{ kW}$$

$$P_c = 1800 \text{ W} = 1.8 \text{ kW}; \quad \text{and} \quad P_i = 1500 \text{ W} = 1.5 \text{ kW}$$

$$\therefore \qquad \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{120}{120 + 1.8 + 1.5} \times 100 = \mathbf{97.3\ \%}$$

(d) Let x be the fraction of full-load kVA at which the efficiency becomes maximum

$$P_c = P_i \quad \text{or} \qquad x^2 \times 1800 = 1500 \qquad x = \sqrt{1500/1800} = 0.913$$

Therefore, the load kVA under the condition of maximum efficiency,

$$= (\text{Full-load kVA}) \times x = 150 \times 0.913 = \mathbf{137\ kVA}$$

Example 11: For a single-phase, 200-kVA, distribution transformer has full-load copper losses of 3.02 kW and iron losses of 1.6 kW. It has following load distribution over a 24-hour day :

(i) 80 kW at unity power factor, for 6 hours.

(ii) 160 kW at 0.8 power factor (lagging), for 8 hours.

(iii) No load, for the remaining 10 hours.

Determine its all-day efficiency.

Solution:

(i) For 80 kW load at unity power factor (for 6 hours) :

Output energy $= 80 \times 6 = 480$ kW h

$$\text{kVA} = \frac{P_o}{pf} = \frac{80}{1} = 80\,\text{kVA}$$

$$\therefore P_c = \left(\frac{\text{kVA}}{\text{kVA}_{FL}}\right)^2 P_{c(FL)} = \left(\frac{80}{200}\right)^2 \times (3.02) = 0.4832\ \text{kW}$$

Iron losses, Pi = 1.6 kW

Total losses, Pl = Pc + Pi = 0.4832 kW + 1.6 kW = 2.0832 kW

$$\therefore \text{Total energy losses in 6 hours} = 2.0832 \times 6 = 12.50\ \text{kWh}$$

(ii) For 160-kW load at 0.8 power factor (for 8 hours) :

Output energy $= 160 \times 8 = 1280$ kW h

$$kVA = \frac{P_o}{pf} = \frac{160}{0.8} = 200\,\text{kVA} = \text{kVA}_{\text{FL}}$$

$\therefore$ Copper losses, $P_c = P_{c(FL)} = 3.02$ kW

Iron losses, $P_i = 1.6$ kW

Total losses, $Pl - P_c + P_i - 3.02\text{ kW} + 1.6\text{ kW} - 4.62\text{ kW}$

$\therefore$ Total energy losses in 8 hours $= 4.62 \times 8 = 36.96$ kW h (*iii*) For the no-load period of 10 hours :

Output energy $P_o = 0$

Copper losses, $P_c = 0$

Iron losses, $P_i = 1.6$ kW

Total losses, $P_l = P_c + P_i = 0 + 1.6 = 1.6$ kW

$\therefore$ Total energy losses in 10 hours $= 1.6 \times 10 = 16$ kWh

Thus, for 24-hour period :

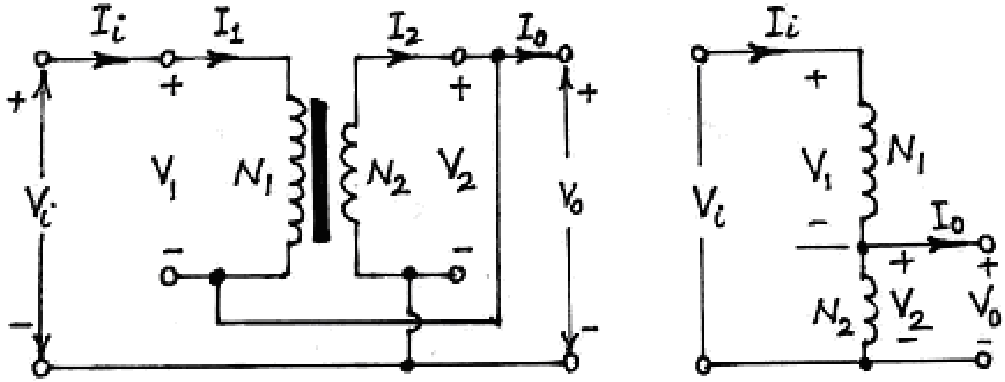
- Total output energy, $W_o = 480 + 1280 = 1760$ kW h
- Total energy losses, $W_l = 12.50 + 36.96 + 16 = 65.46$ kW h

$\therefore$ All-day efficiency,

$$\eta_{\text{all-day}} = \frac{W_o}{W_o + W_l} \times 100 = \frac{1760}{1760 + 65.46} \times 100 = 96.41\,\%$$

AUTOTRANSFORMERS

An autotransformer is a special transformer-connection that is useful in power systems, motor starters, variable ac sources, and other applications.

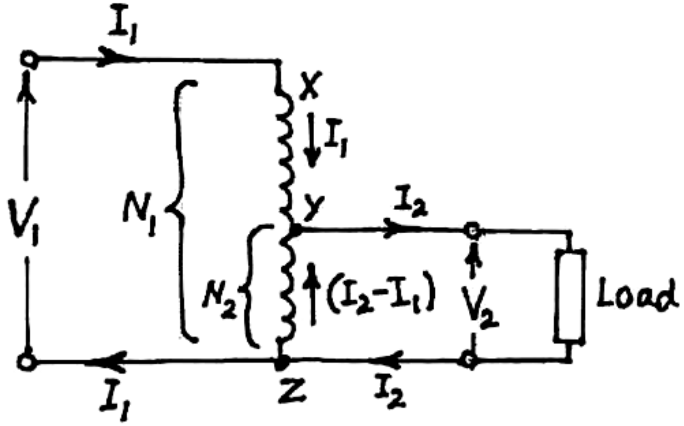


- The primary and secondary windings are connected in series for the new primary; the secondary is the new secondary.
- The primary and secondary are not electrically isolated from each other. Obviously, the voltage $V_2 = V_o$.

$$V_i = V_1 + V_2 = \frac{N_1}{N_2} V_2 + V_2 = \frac{N_1 + N_2}{N_2} V_o \quad \text{or} \quad V_o = \frac{N_2}{N_1 + N_2} V_i$$

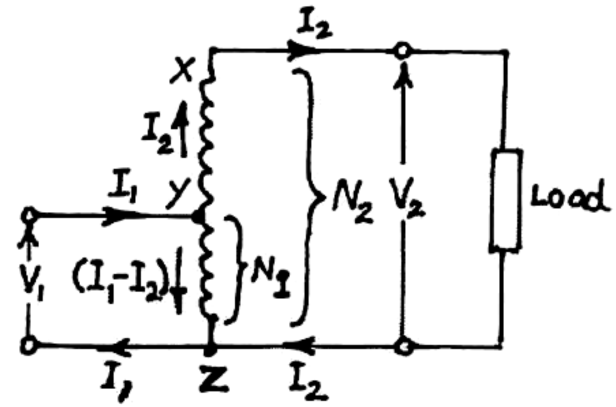
- The new turns-ratio becomes $N_2 : (N_1 + N_2)$

Thus, we find that an autotransformer works like a ***potential divider*** circuit, except that numbers of turns are to be used instead of resistances. **The apparent power rating (kVA rating) of the transformer is increased by the special connection.**



(*a*) A step-down autotransformer

$$V_2 I_2 = V_2 I_1 + V_2 (I_2 - I_1)$$



(*b*) A step-up autotransformer.

$$V_1 I_1 = V_1 I_2 + V_1 (I_1 - I_2)$$

Savings in Copper

- For the same voltage ratio and capacity (volt-ampere rating), an autotransformer needs much less copper (or aluminum) material compared to a two-winding transformer.
- The cross-sectional area of a conductor is proportional to the current carried by it, and its length is proportional to the number of turns. Therefore, weight of copper in a winding

$$\propto NI = kNI$$

1. **For a two-winding transformer:**

$$\text{Weight of copper in primary} = kN_1 I_1$$

$$\text{Weight of copper in secondary} = kN_2 I_2$$

$$\text{Total weight of copper} = k(N_1 I_1 + N_2 I_2)$$

2. <u>For an autotransformer:</u>

Weight of copper in portion $XY = k(N_1 - N_2)I_1$

Weight of copper in portion $YZ = kN_2(I_2 - I_1)$

$$\text{Total weight of copper } = k(N_1 - N_2)I_1 + kN_2(I_2 - I_1) = k[(N_1 - 2N_2)I_1 + N_2 I_2]$$

- Therefore, the ratio of copper- weights for the case 1 and case 2 is:

$$\frac{k[(N_1 - 2N_2)I_1 + N_2 I_2]}{k(N_1 I_1 + N_2 I_2)} = \frac{\left[1 - 2\left(\frac{N_2}{N_1}\right)\right]\left(\frac{I_1}{I_2}\right) + \left(\frac{N_2}{N_1}\right)}{\left(\frac{I_1}{I_2}\right) + \left(\frac{N_2}{N_1}\right)} = \frac{[1 - 2K]K + K}{K + K} = 1 - K$$

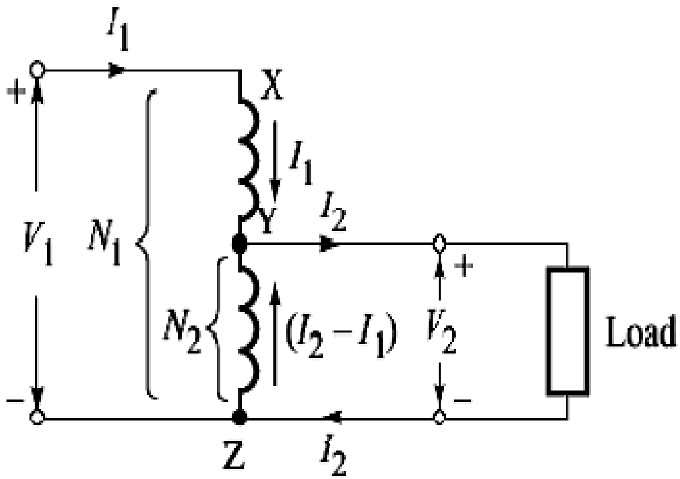
$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

The saving is large if K is close to unity. A unity transformation ratio means that no copper is needed at all for the autotransformer. The winding can be removed all together. The volt-amperes are conductively transformed directly to the load.

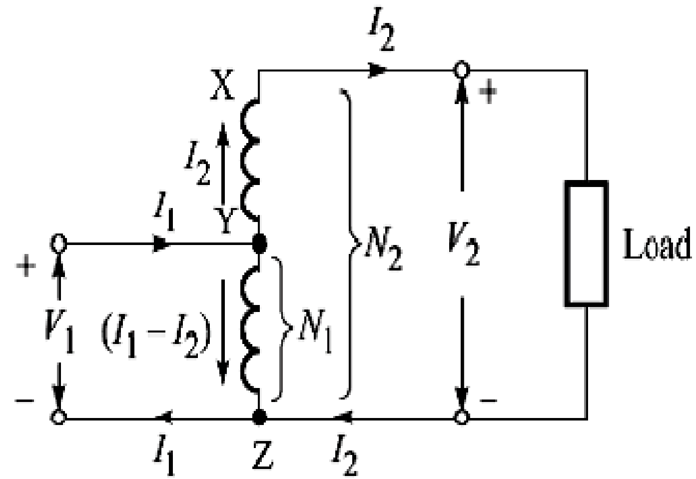
Autotransformers

- It is a special transformer that is useful in power systems, motor starters, variable ac sources, etc.
- It has a part of its winding common to the primary and secondary circuits.

- Types:



(*a*) Step-down



(*b*) Step-up

- The portion YZ of the winding is called ***common winding***.
- The portion XY is called ***series winding***.

In ***variacs*** (***variable autotransformers***), point Y is made a sliding contact so as to give a variable output voltage.

- **Advantages of Autotransformers:**
  - A saving in cost since less copper is needed.
  - Less volume, hence less weight.
  - A higher efficiency, resulting from lower I^2R losses.
  - A continuously variable output voltage is achievable if a sliding contact is used.
  - A smaller percentage voltage regulation.
  - Higher VA Rating.

Transformer Testing

- There are two simple tests to determine the equivalent-circuit parameters and its efficiency and regulation:
  - Open-circuit test (OC Test)
  - Short-circuit test (SC Test)

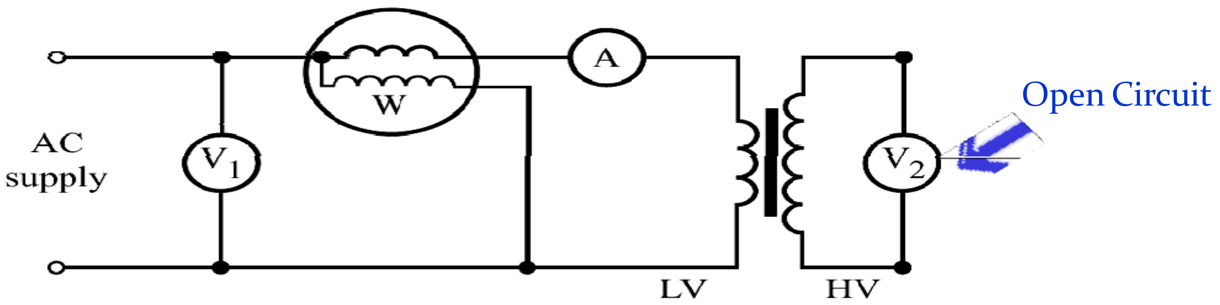
- **Advantage** of these tests is without actually loading the transformers, we can determine the Losses and Regulation, for full-load.

- Open Circuit Test:

  – This test determines the no-load current and the parameters of the exciting circuit of the transformer.

  – Generally, the low voltage (LV) side is supplied rated voltage through a ***variac***.

  – The high voltage (HV) side is left **open.**



– The reading of ammeter A_1, I_{sc}, gives the full-load current in the primary winding.

– Since the applied voltage (and hence the flux) is small, the core loss is negligibly small.

– Hence, the wattmeter reading, W_{sc}, gives the copper loss (P_c).

– **Calculations:**

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2}; \qquad Z_{e1} = \frac{V_{sc}}{I_{sc}}; \qquad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

Example 12: A single-phase, 50-Hz, 12-kVA, 200-V/400-V transformer gives the following test results :

(*i*) Open-circuit test (with HV winding open) : 200 V, 1.3 A, 120 W

(*ii*) Short-circuit test (with LV winding short-circuited) : 22 V, 30 A, 200 W

Calculate :

(*a*) the magnetizing current and the core-loss current, and

(*b*) the parameters of equivalent circuit as referred to the low voltage winding.

Solution :

(*a*) The wattmeter reading, 120 W, in the open-circuit test gives the core losses. Therefore, the core-loss current is given as

$$I_w = \frac{W_o}{V_1} = \frac{120\ \text{W}}{200\ \text{V}} = \mathbf{0.6\ A}$$

$$\therefore \quad I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.3)^2 - (0.6)^2} = \mathbf{1.15\ A}$$

(*b*) The parameters of the exciting circuit are given by the open-circuit test, as

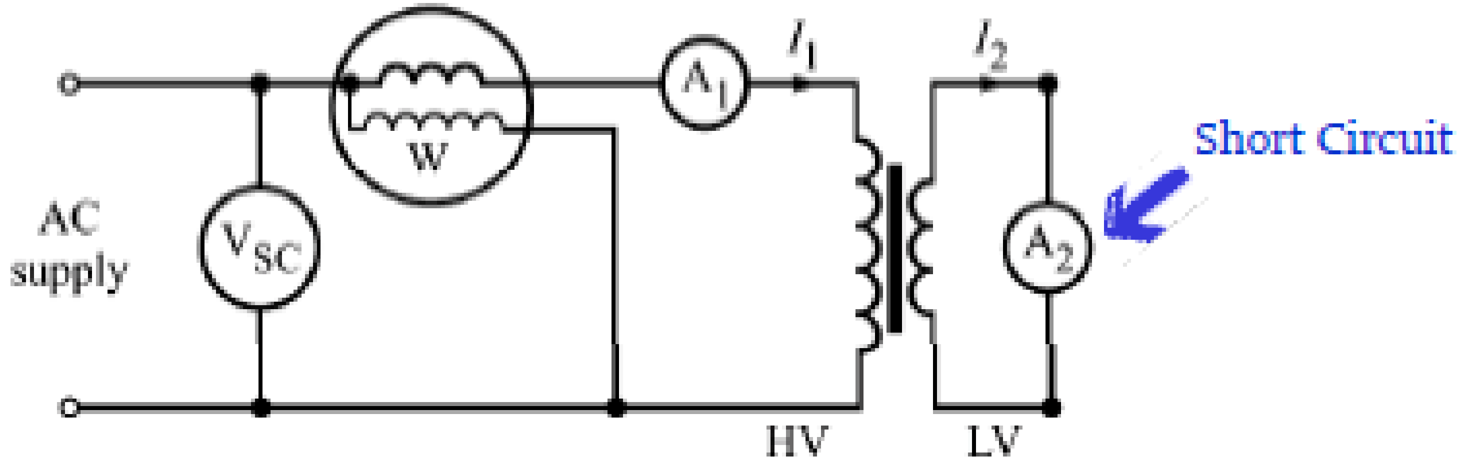
$$R_0 = \frac{V_1}{I_w} = \frac{200\ \text{V}}{0.6\ \text{A}} = \mathbf{333\ \Omega} \quad \text{and} \quad X_0 = \frac{V_1}{I_m} = \frac{200\ \text{V}}{1.15\ \text{A}} = \mathbf{174\ \Omega}$$

$$\text{Now,} \quad K = \frac{V_2}{V_1} = \frac{200\ \text{V}}{400\ \text{V}} = \frac{1}{2} \quad \text{and} \quad I_{FL} = \frac{12\ \text{kVA}}{400\ \text{V}} = 30\ \text{A}$$

This confirms that the short-circuit test has been done at the rated full-load .

Short-Circuit Test:

- This test determines the equivalent resistance and leakage reactance.
- Generally, the LV side of the transformer is short-circuited through a suitable ammeter A_2.
- A *low* voltage is applied to the primary (HV) side.
- This voltage is adjusted with the help of a variac so as to circulate full-load currents in the primary and secondary circuits.



- The reading of ammeter A_1, I_{sc}, gives the full-load current in the primary winding.
- Since the applied voltage (and hence the flux) is small, the core loss is negligibly small.
- Hence, the wattmeter reading, W_{sc}, gives the copper loss (P_c).
- **Calculations:**

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2}; \qquad Z_{e1} = \frac{V_{sc}}{I_{sc}}; \qquad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

Example 12: A single-phase, 50-Hz, 12-kVA, 200-V/400-V transformer gives the following test results :

(*i*) Open-circuit test (with HV winding open) : 200 V, 1.3 A, 120 W

(*ii*) Short-circuit test (with LV winding short-circuited) : 22 V, 30 A, 200 W

Calculate :

(*a*) the magnetizing current and the core-loss current, and

(*b*) the parameters of equivalent circuit as referred to the low voltage winding.

Solution :

(*a*) The wattmeter reading, 120 W, in the open-circuit test gives the core losses. Therefore, the core-loss current is given as

$$I_w = \frac{W_0}{V_1} = \frac{120\ \text{W}}{200\ \text{V}} = \mathbf{0.6\ A}$$

$$\therefore\quad I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.3)^2 - (0.6)^2} = \mathbf{1.15\ A}$$

(*b*) The parameters of the exciting circuit are given by the open-circuit test, as

$$R_0 = \frac{V_1}{I_w} = \frac{200\ \text{V}}{0.6\ \text{A}} = \mathbf{333\ \Omega} \quad \text{and} \quad X_0 = \frac{V_1}{I_m} = \frac{200\ \text{V}}{1.15\ \text{A}} = \mathbf{174\ \Omega}$$

$$\text{Now,}\quad K = \frac{V_2}{V_1} = \frac{200\ \text{V}}{400\ \text{V}} = \frac{1}{2} \quad \text{and} \quad I_{FL} = \frac{12\ \text{kVA}}{400\ \text{V}} = 30\ \text{A}$$

This confirms that the short-circuit test has been done at the rated full-load .

$$\therefore\ R_{e1} = \frac{W_{sc}}{I_{sc}^2} = \frac{200\text{ W}}{(30\text{ A})^2} = 0.222\ \Omega \quad \text{and} \quad Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{22\text{ V}}{30\text{ A}} = 0.733\ \Omega$$

The equivalent resistance and reactance as referred to the secondary side (low voltage winding):

$$R_{e2} = K^2 R_{e1} = \left(\frac{1}{2}\right)^2 \times 0.222 = \mathbf{0.055\ \Omega}$$

and $$X_{e2} = K^2 X_{e1} = \left(\frac{1}{2}\right)^2 \times 0.699 = \mathbf{0.175\ \Omega}$$