Department of Mathematics

15B11MA211 | **Mathematics-2**

Tutorial Sheet 6 | **B.Tech. Core**

Analytic Functions and Complex Integration

1. Evaluate the following limits: (a) $\lim\limits_{z\to -i} \dfrac{z^2+1}{z+i}$ (b) $\lim\limits_{z\to \frac{1+i\sqrt{3}}{2}} \dfrac{z^3+1}{z^4+z^2+1}$

2. (a) Show that f(z) = $\bar{z}$ is continuous but not differentiable at any point.
(b) If f(z) = $x^2 + iy^2$, does $f'(z)$ exist at any point?

3. Determine whether C-R equations are satisfied for (a) 1/z (b) cosh2z

4. (a) Show that f(z) = $|z^2|$ is differentiable at z = 0 but not analytic there.
(b) Show that $u(x,y) = 2x + y^3 - 3x^2 y$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function f(z)=u+iv.

5. Show that for the function $f(z) = \dfrac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$, $z \neq 0$ and $f(0) = 0$

C-R equations are satisfied at origin, but function is not analytic at the point.

6. Determine the analytic function f(z) = u + iv, where

(a) $u + v = \dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ (b) $u(r,\theta) = r^2 \cos 2\theta$ (c) $v = (x - y)/(x^2 + y^2)$.

7. Integrate $\int_C (z + 2\bar{z})dz$ from $z = 0$ to $z = 1 + i$ along the following two paths

(a) line joining (0,0) and (1,1) (b) the curve $x = t, y = t^2, 0 \le t \le 1$.

8. Integrate f(z) = z in the positive sense around the squares with corners at (1,1), (2,1), (2,2) and (1,2).

9. Evaluate $\int_C |z| dz$, where C is the contour (a) straight line from z = -i to z = i; (b) the unit circle $|z-1| = 1$.

10. Let m be an integer and C the circle $|z - z_0| = R$. Show that the integral of $(z - z_0)^m$ over C in the anticlockwise direction vanishes if $m \neq -1$ and is equal to $2\pi i$ if m = -1. Hence evaluate $\int_C [P(z)/z]dz,$ where $P(z) = 2 - z + 3z^2 + z^3$ and C is the unit circle $|z| = 1$.

11. Using Cauchy theorem or otherwise show that (a) $\int_C \dfrac{dz}{z-2} = 0,$ where C is the circle $|z| = 1$
(b) $\int_C \dfrac{dz}{z} = 2\pi i,$ where C is a closed contour enclosing z = 0. (c) $\int_C \dfrac{dz}{(z+1)^2} = 0,$ where C is the circle $|z| = 2$.

12. Using the Cauchy integral formula or otherwise show that

(a) $\int_C \dfrac{e^{-z}}{z+1} dz = 2\pi e i,$ where C is the circle $|z| = 2$.

(b) $\int_C \dfrac{e^{2z}}{(z+1)^4} dz = 8\pi i/(3e^2),$ where C is the circle $|z| = 2$.

(c) $\int_C \dfrac{\cos \pi z^2}{(z-1)(z-2)} dz = 2\pi i, 0, 4\pi i$ according as C is the circle $|z| = 3/2, 1/2$ or 3.

(d) $\int_C \dfrac{e^{-z}}{z^2} dz = -2\pi i,$ where C is the ellipse $2x^2 + y^2 = 2.$