Description and Analysis of Systems

UNIT 2

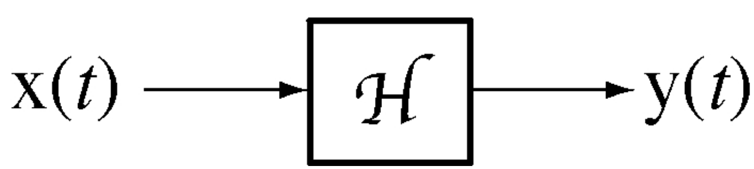
Systems

- Broadly speaking, a system is anything that responds when stimulated or excited
- The systems most commonly analyzed by engineers are artificial systems designed by humans
- Engineering system analysis is the application of mathematical methods to the design and analysis of systems

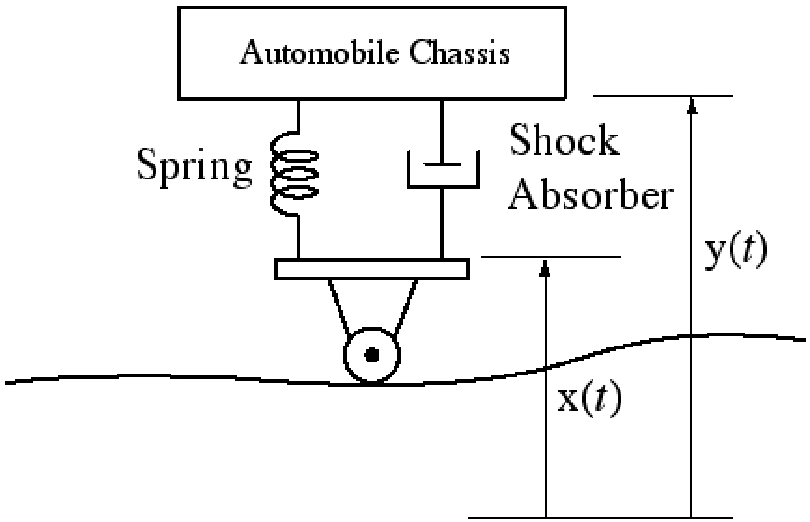
Systems

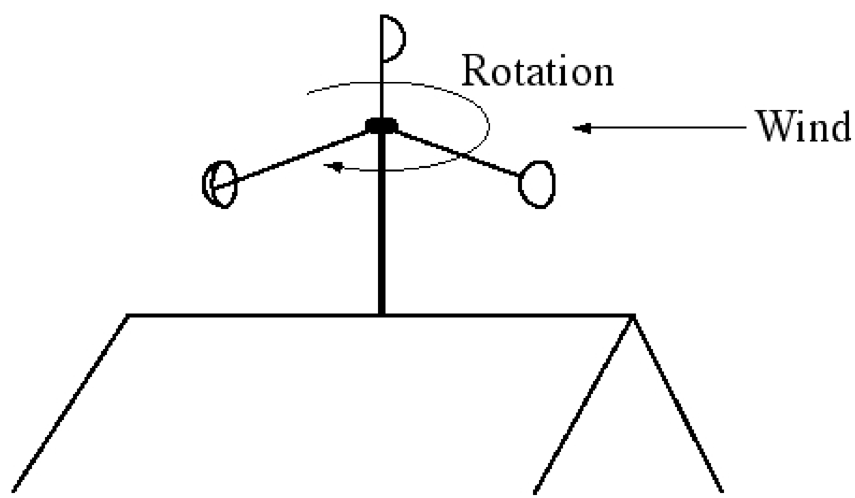
- Systems have inputs and outputs
- Systems accept excitation signals at their inputs and produce response signals at their outputs
- Systems are often usefully represented by *block diagrams*

A single-input, single-output system block diagram

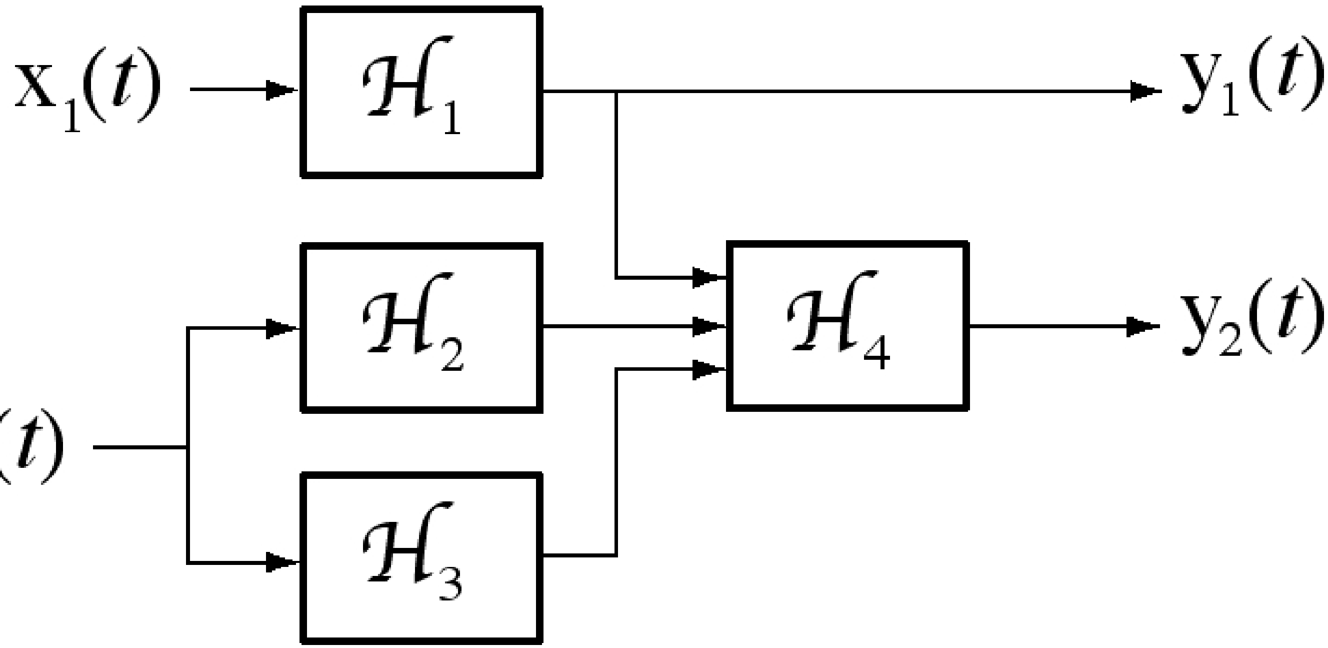


System Examples



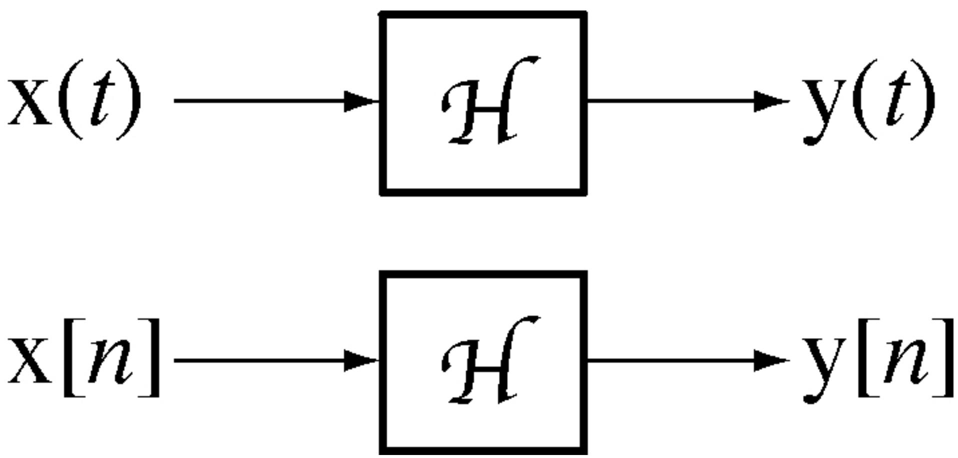


A Multiple-Input, Multiple-Output System Block Diagram



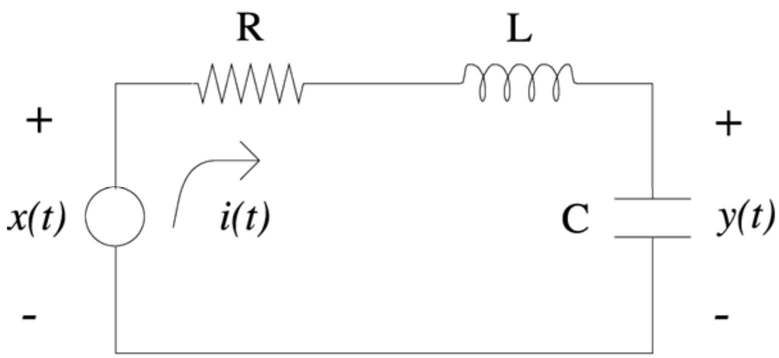
CT and DT Systems

CT systems respond to and produce CT signals



DT systems respond to and produce DT signals

Ex.1 RLC circuit, example of a CT system.



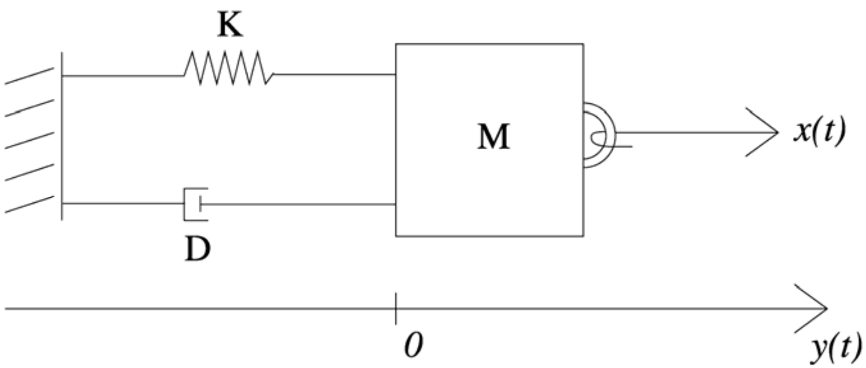
$$\mathrm{R}\; i(t) + \mathrm{L}\frac{di(t)}{dt} + y(t) = x(t)$$

$$i(t) = \mathrm{C}\frac{dy(t)}{dt}$$

$$\Downarrow$$

$$\mathrm{LC}\frac{d^2y(t)}{dt^2} + \mathrm{RC}\frac{dy(t)}{dt} + y(t) = x(t)$$

Ex. 2 Mechanical system, example of a CT system.



$x(t)$ - applied force

K - spring constant

D - damping constant

$y(t)$ - displacement from rest

Force Balance:

$$\mathrm{M}\frac{d^2y(t)}{dt^2} = x(t) - \mathrm{K}y(t) - \mathrm{D}\frac{dy(t)}{dt}$$

$$\Downarrow$$

$$\mathrm{M}\frac{d^2y(t)}{dt^2} + \mathrm{D}\frac{dy(t)}{dt} + \mathrm{K}y(t) = x(t)$$

Observation: Very different physical systems may be modeled mathematically in very similar ways.

Ex.3. Example of a DT system.

Balance in a bank account from month to month

y[n] = Balance at end of the nth month

x[n] = net deposit in nth month (deposits - withdrawals)

1% interest each month

$$y[n] = 1.01y[n-1] + x[n]$$

Observations:

1) **Systems are described by differential and difference equations.**

2) **Such an equation, by itself, does not completely describe the input-output behavior of a system: we need auxiliary conditions (initial conditions, boundary conditions).**

3) **In some cases the system of interest has time as the natural independent variable and is causal. However, that is not always the case.**

4) **Very different physical systems may have very similar mathematical descriptions.**

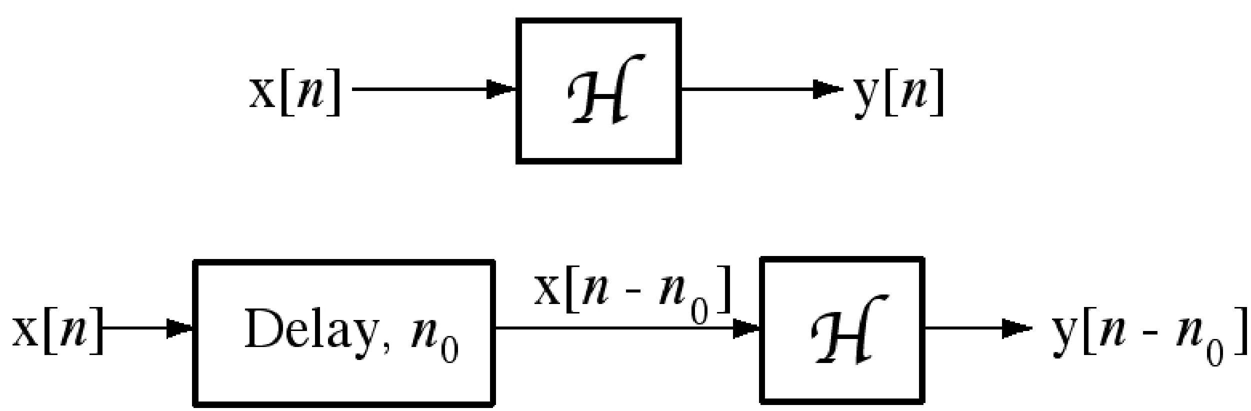
Different Properties of Systems

- Time Invariance
- Stability
- Causality
- Memory
- Invertibility
- Homogeneity
- Additivity
- Linearity

Time Invariance

- If an excitation causes a response and delaying the excitation simply delays the response by the same amount of time, regardless of the amount of delay, then the system is *time invariant*

Time Invariant System



Example:

$$y(t) = \sin(x(t))$$

$$x_1(t) = x(t - t_0)\ \text{(Delayed Input)}$$

$$y_1(t) = \sin(x_1(t)) = \sin(x(t - t_0))$$

$$\text{Delayed output}$$

$$\mathrm{y}_2(t) = y(t - t_0) = \sin(x(t - t_0))$$

$$As, y_1(t) = y_2(t)$$

$$\therefore \text{Time Invarient}$$

Example:

$$y[n] = nx[n]$$

$$x_1[n] = x[n-n_0]\ (\text{Delayed input})$$

$$\mathrm{y}_1[n] = nx[n-n_0]$$

Delayed Output

$$\mathrm{y}_2[n] = y[n-n_0] = [n-n_0](x[n-n_0])$$

$$y_1[n] \neq y_2[n]$$

Time Varying System

Stability

- Any system for which the response is bounded for any arbitrary bounded excitation, is called a *bounded-input-bounded-output* (BIBO) stable system

$$y(t) = tx(t) \quad -\mathrm{B} < \mathrm{x(t)} < \mathrm{B}$$

$$\text{As } t \to \infty,\ y(t) \to \infty \quad \therefore \text{Unstable system}$$

$$y(t) = e^{x(t)} \quad -\mathrm{B} < \mathrm{x(t)} < \mathrm{B}$$

$$\text{As } t \to \infty,\ e^{-B} < y(t) < e^{B} \quad \therefore \text{stable system}$$

Causality

- Any system for which the response occurs only during or after the time in which the excitation is applied is called a *causal* system.
- Strictly speaking, all real physical systems are causal

$$y[n] = x[n-1] \rightarrow \textit{causal} \text{ system}$$

$$y[n] = x[n] - x[n+1] \rightarrow \text{non causal system}$$

Memory

- If a system's response at any arbitrary time depends only on the excitation at that same time and not on the excitation or response at any other time is called a *static* system and is said to have no *memory*
- A system whose response at some arbitrary time does depend on the excitation or response at another time is called a *dynamic* system and is said to have memory.

$$y[n] = (2x[n] - x^2[n])^2 \rightarrow \text{Memoryless system}$$

$$\mathrm{y[n]} = \mathrm{y[n-1]} + \mathrm{x[n]} \rightarrow \text{System with memory}$$

Invertibility

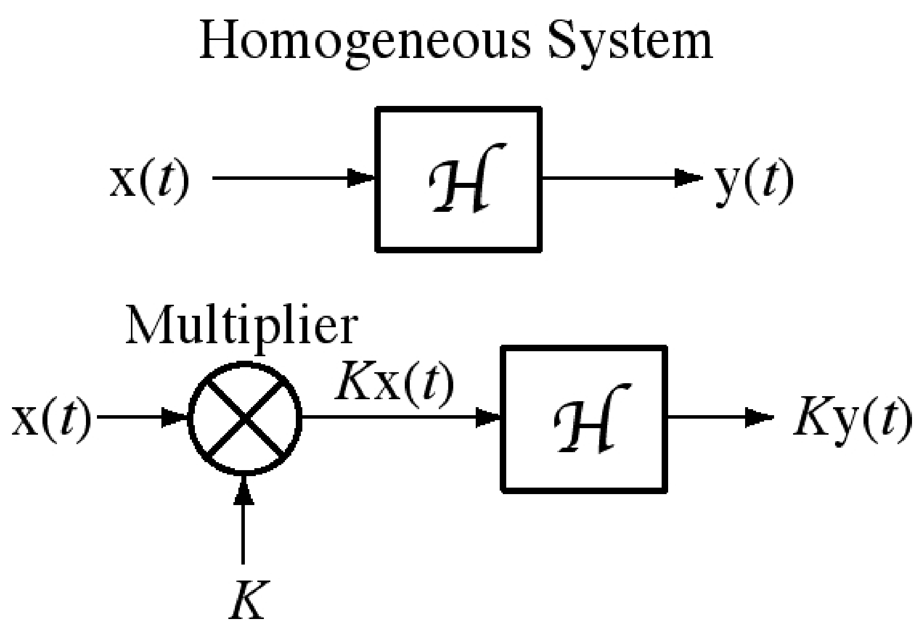
- A system is said to be invertible if unique excitations produce unique responses.

$$y(t) = 2x(t), w(t) = \frac{1}{2} y(t) \rightarrow \text{combination gives invertible system}$$

$$\mathrm{y(t)} = \mathrm{x}^2(t) \rightarrow \text{non invertible system}$$

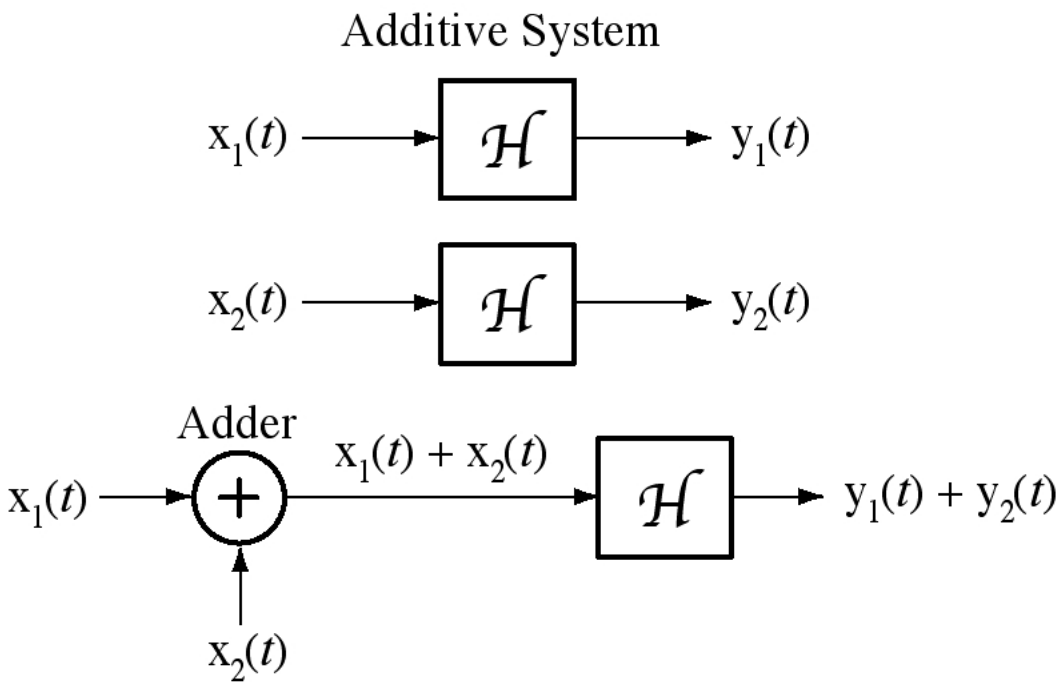
Homogeneity

- In a *homogeneous* system, multiplying the excitation by any constant (including *complex* constants), multiplies the response by the same constant.



Additivity

- If one excitation causes a response and another excitation causes another response and if, for any arbitrary excitations, the sum of the two excitations causes a response which is the sum of the two responses, the system is said to be *additive*



Linearity and LTI Systems

- If a system is both homogeneous and additive it is *linear*.
- If a system is both linear and time-invariant it is called an *LTI* system
- Some systems which are non-linear can be accurately approximated for analytical purposes by a linear system for small excitations

Example -

$$y(t) = tx(t)$$

$$y_1(t) = tax_1(t)$$

$$y_2(t) = tbx_2(t)$$

$$x_3(t) = ax_1(t) + bx_2(t)$$

$$y_3(t) = tx_3(t) = t[ax_1(t) + bx_2(t)]$$

$$y_3(t) = y_1(t) + y_2(t) \rightarrow \text{Linear System}$$

Example -

$$y[n] = \mathrm{Re}\{x[n]\}$$

$$x_1[n] = r[n] + js[n]$$

$$y_1[n] = r_1[n]$$

$$x_2[n] = jx_1[n] = jr[n] - s[n]$$

$$y_2[n] = \mathrm{Re}\{x_2[n]\} = -s[n] \neq jy_1[n]$$

System violates homogeinity property.

Hence, it is non linear system.