ELECTRICAL SCIENCE-II (15B11EC211)

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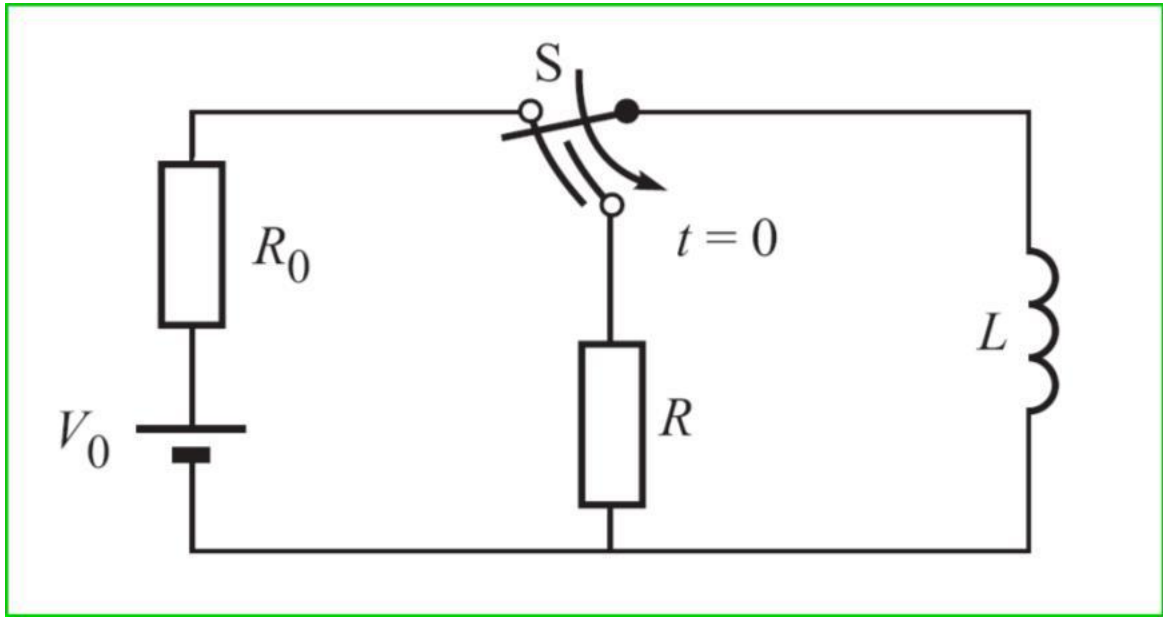
The Simple RL Circuit



At t = 0-, a steady current that has been flowing in the circuit,

$$I_0 = \frac{V_0}{R_0}$$

For $t > 0+$, applying KVL,

$$v_R + v_L = 0 \quad \text{or} \quad Ri + L\frac{di}{dt} = 0 \quad \text{or} \quad \frac{di}{dt} + \frac{R}{L}i = 0$$

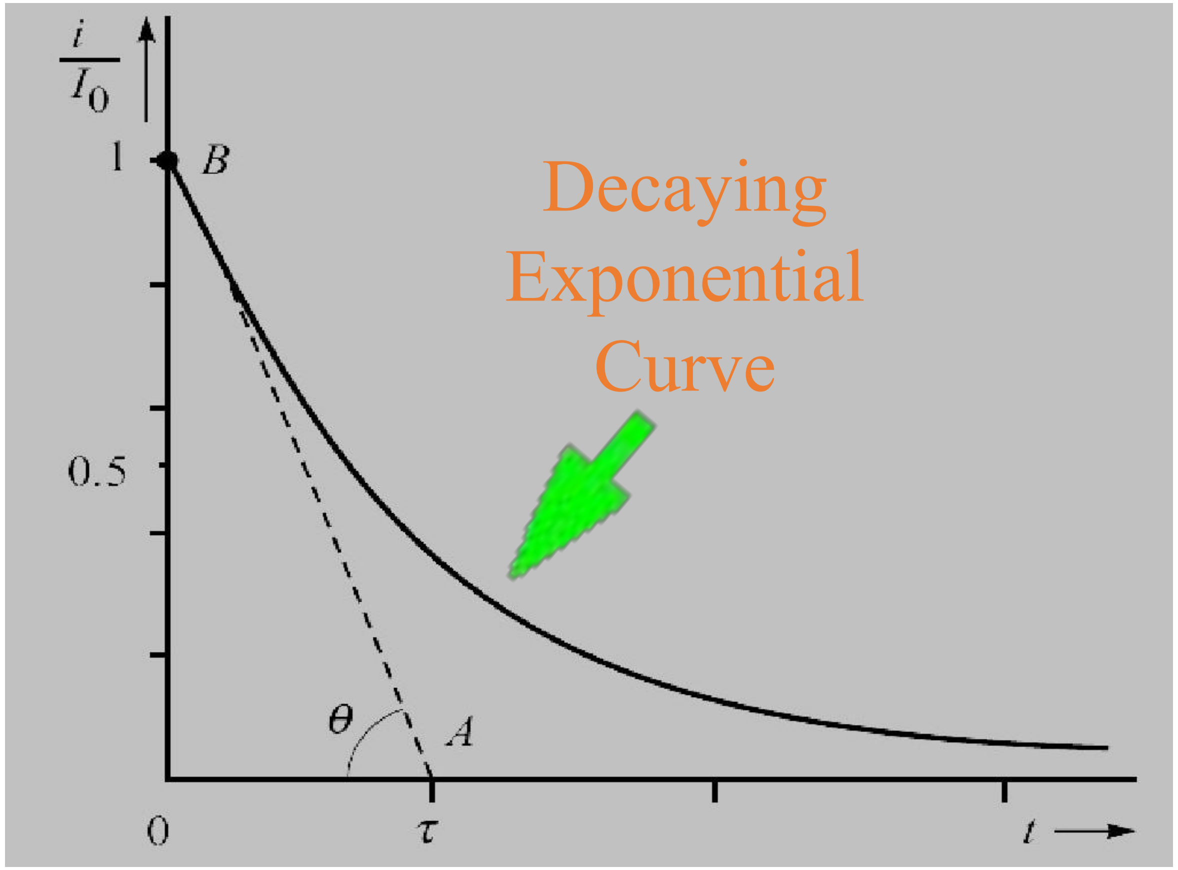
Re-writing the equation to separate variables and then integrating,

$$\frac{di}{i} = -\frac{R}{L}dt$$

$$\int_{I_0}^{i(t)} \frac{1}{i}\,di = \int_0^t \left(-\frac{R}{L}\right)dt \quad \text{or} \quad \ln i\Big|_{I_0}^{i} = -\frac{R}{L}t\Big|_0^t$$

$$\text{or} \quad \ln i - \ln I_0 = -\frac{R}{L}(t-0)$$

or $$i(t) = I_0 e^{-Rt/L}$$



At $t = 0+$, the current is I_0. As time increases, the current decreases and approaches zero.

Concept of Time Constant

- From equation, we see that with larger L/R ratio, the current takes longer to decay.
  - By doubling L/R, the "width" of the curve also doubles.
  - The "width" is proportional to L/R.
  - Instead of "width", we use the concept of "time constant (τ).
  - *It is the time that would be required for the current to drop to zero if it continued to drop at its initial rate.*

The initial rate of decay

$$= \text{ the slope of line AB}$$

$$= \frac{d}{dt}(i/I_0)\Big|_{t=0} = -\frac{R}{L}e^{-Rt/L}\Big|_{t=0} = -\frac{R}{L}$$

From triangle OAB,

$$\tan\theta = \frac{1}{\tau} \qquad \Rightarrow \qquad \frac{1}{\tau} = \frac{R}{L} \qquad \text{or} \qquad \tau = \frac{L}{R}$$

- The ratio L/R must have the units of time.

Meaning of Time Constant

Determining the value of $i(t)/I_0$ at $t = \tau$, we have

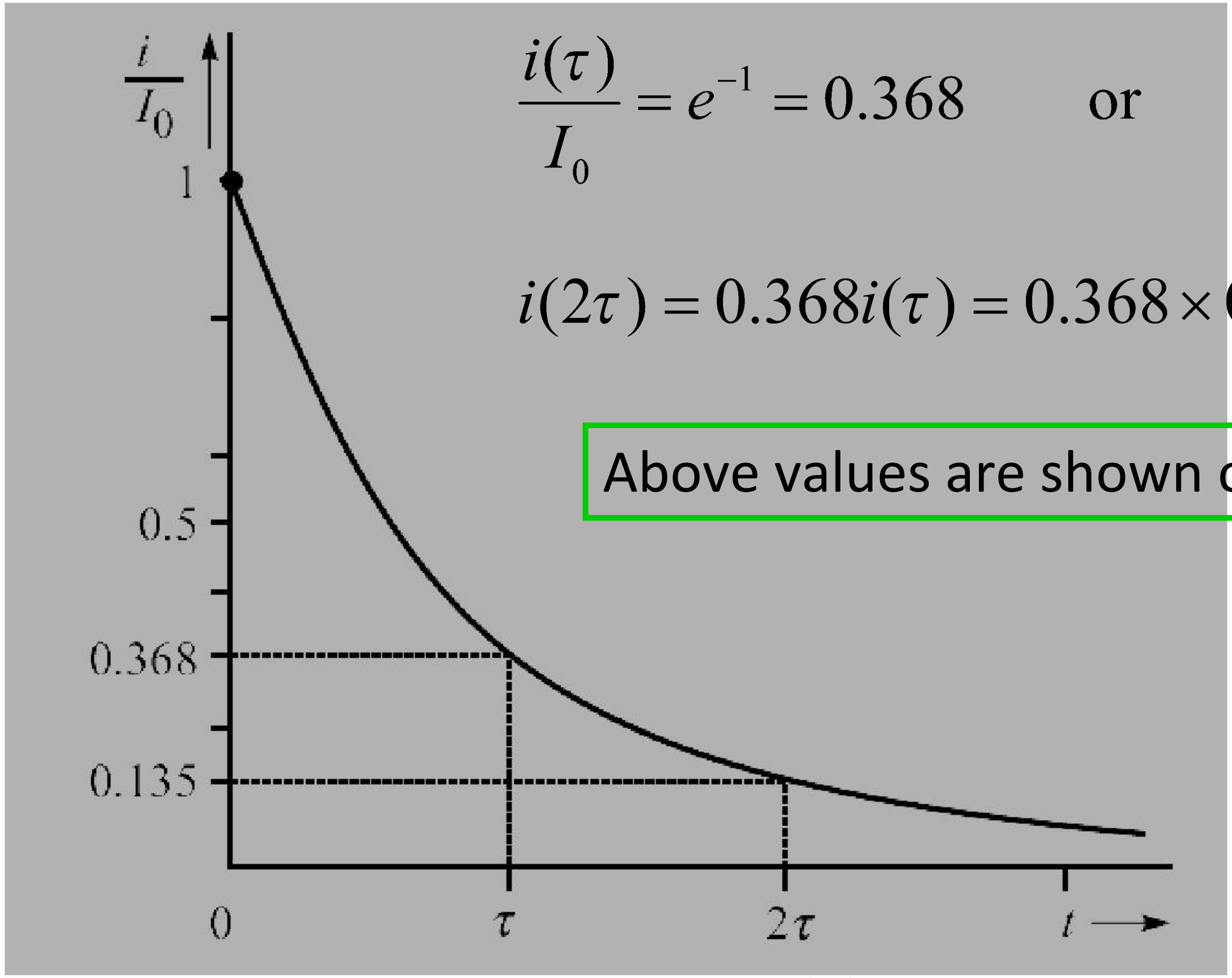
$$\frac{i(\tau)}{I_0} = e^{-1} = 0.368 \qquad \text{or} \qquad i(\tau) = 0.368 I_0$$

Thus, in one time constant the response drops to 36.8 % of its initial value. Hence,

$$i(2\tau) = 0.368 i(\tau) = 0.368 \times 0.368 I_0 = 0.1354 I_0$$

How long does it take for the current to decay to zero ?

Ans. : To answer this question, let us calculate



$$\frac{i(\tau)}{I_0} = e^{-1} = 0.368 \qquad \text{or} \qquad i(\tau) = 0.368 I_0$$

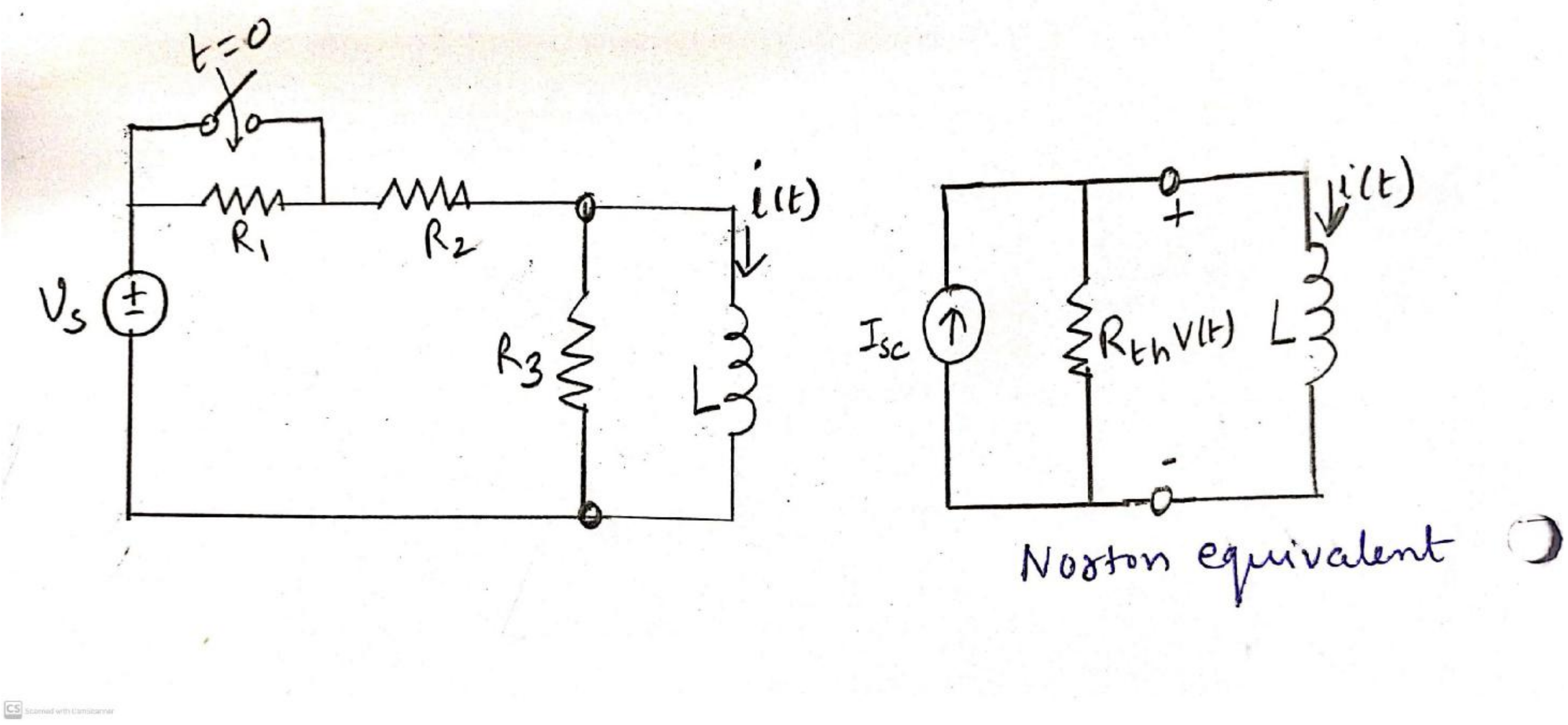
$$i(2\tau) = 0.368 i(\tau) = 0.368 \times 0.368 I_0 = 0.1354 I_0$$

Above values are shown on the graph.

$$i(3\tau) = 0.0498I_0,$$
$$i(4\tau) = 0.0183I_0,$$
$$i(5\tau) = 0.0067I_0,$$
$$\dots\ \dots\ \dots$$

- It takes about **five time constants** for the current to decay to zero.
- At the end of this time interval, the current is less than one percent of its original value.

RL Circuit



Norton's Equivalent

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$$I_{sc} = \frac{V_s}{R2}$$

$$R_t = \frac{R2R3}{R2 + R3}$$

Inductor voltage is given as

$$v(t) = L\frac{di(t)}{d(t)}$$

- Apply KCL to the top node of Nortons equivalent

$$I_{sc} = i(t) + \frac{v(t)}{R_t}$$

After putting the value of v(t) in above equation

$$I_{sc} = i(t) + \frac{L\frac{di(t)}{d(t)}}{R_t}$$

$$\frac{d}{dt} i(t) + \frac{R_t}{L} i(t) = \frac{R_t}{L} I_{sc}$$

- Compare with the differential equation
- $\frac{d}{dt}x(t) + \frac{x(t)}{\tau} = K$
- $\frac{d}{dt}i(t) + \frac{R_t}{L}i(t) = \frac{L}{R_t}I_{sc}$

Here

$$x(t) = i(t),\ \tau = \frac{L}{R_t},\ and\ K = \frac{L}{R_t}I_{sc}$$

Making substitution in equation

- $x(t) = x(\infty) + (x(0) - x(\infty))e^{-t/\tau}$

In RL circuit

- $i(\infty) = I_{sc}$

- $\mathrm{i}(t) = I_{sc} + (i(0) - I_{sc})e^{\frac{-R}{L}t}$t

So Complete response $\mathrm{i}(t)$ is sum of natural response and forced response

- Natural response = $(i(0) - I_{sc})e^{\frac{-R}{L}t}$t
- Forced response = I_{sc}