13
Transformers

Objectives : After completing this Chapter, you will be able to :

- State the applications of a transformer in electrical and electronic circuits.
- State the principle of operation of a two-winding transformer.
- Draw the circuit symbol of a transformer.
- Derive the basic emf equation of a transformer.
- State the four conditions for transformer to be ideal.
- State the relations for transformation of voltage, current and impedance by a transformer in terms of its turns-ratio.
- State why a transformer should have no-load current.
- Define the two components of the no-load current.
- Explain why the hysteresis and eddy-current losses occur in the core, and how these can be reduced.
- Explain the construction of core-type and shell-type transformers.
- State what is meant by load component of primary current.
- State why in the equivalent circuit of a transformer we include a resistance and a leakage reactance in both the primary and secondary side.
- State how we obtain a simplified equivalent circuit of a transformer as referred to the primary or to the secondary.
- State the meaning of 'regulation down' and 'regulation up' of a transformer.
- Derive the condition for zero regulation and condition for maximum regulation of a transformer.
- Define 'commercial efficiency' and 'all-day efficiency' of a transformer.
- Derive the condition of maximum efficiency of a transformer.
- Explain how to convert a two-winding transformer into an autotransformer, and state the advantages and disadvantages of doing it.
- Explain how to get the 'equivalent circuit parameters' of a transformer by conducting 'open-circuit test' and 'short-circuit test'.

13.1 INTRODUCTION

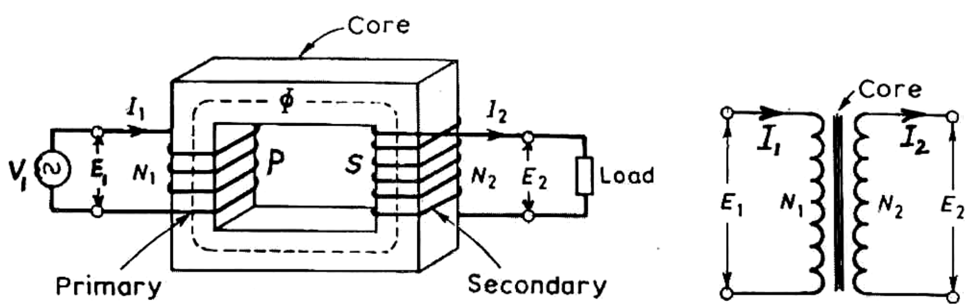
A transformer is a highly efficient device for changing ac voltage from one value to another, without any change in frequency. There exists no simple device that can accomplish such changes in dc voltages. Thus, the transformer has provided a feature to ac power system that lacks in dc power system.

The general practice is to generate ac voltage of about 11 kV, then step up by means of a transformer to higher voltages of 132 kV, 220 kV and 400 kV for the transmission lines. This conversion aids the transmission of huge electrical power at low cost. High-voltage lines carry low currents, and hence the cost of lines and the power loss are tremendously reduced. At distribution points, other transformers are used to step the voltage down to 400 V or 220 V for use in industries, offices and homes. Since there are no moving parts in a transformer, it practically needs almost no maintenance and supervision.

Apart from the above, the transformers are also used in communication circuits, radio and TV circuits, telephone circuits, instrumentation and control systems.

13.2 PRINCIPLE OF OPERATION

A transformer operates on the principle of *mutual induction* between *two coils.* Figure 13.1*a* shows the general construction of a transformer. The vertical portions of the steel-core are termed *limbs*, and the top and bottom portions are called *yokes*. The two coils P and S, having N_1 and N_2 turns, are wound on the limbs. These two windings are electrically unconnected but are linked with one another through a magnetic flux in the core. The coil P is connected to the supply and is therefore called *primary*; coil S is connected to the load and is termed the *secondary*.



(*a*) Construction. (*b*) Circuit symbol.

Fig. 13.1 A transformer.

Figure 13.1*b* shows the circuit symbol of a transformer. The thick line denotes the iron core. By having different ratios N_1/N_2 of the two windings, power at lower or at higher voltage can be obtained. When $N_2 > N_1$, the transformer is called a ***step up*** transformer; and when $N_2 < N_1$, the transformer is called a ***step down*** transformer.

EMF Equation

Consider a sinusoidally varying voltage V_1 applied to the primary of the transformer shown in Fig. 13.1*a*. Due to this voltage, a sinusoidally varying magnetic flux is set up in the core, which can be represented as

$$\Phi = \Phi_m \sin \omega t = \Phi_m \sin 2\pi ft \qquad ...(13.1)$$

where Φ_m is the peak value of the flux and f is the frequency of sinusoidal variation of flux. As per the law of electromagnetic induction, the induced emf in a winding of N turns is given as

$$e = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}(\Phi_m \sin \omega t) = -N\omega\Phi_m \cos \omega t = \omega N\Phi_m \sin(\omega t - \pi/2)$$

...(13.2)

Thus, the peak value of the induced emf is $E_m = \omega N\Phi_m$. Therefore, the rms value of the induced emf E is given as

$$E = \frac{E_m}{\sqrt{2}} = \frac{\omega N\Phi_m}{\sqrt{2}} = \frac{2\pi fN\Phi_m}{\sqrt{2}} = 4.44 fN\Phi_m$$

or

$$\boxed{E = 4.44 fN\Phi_m}$$

...(13.3)

This equation, known as ***emf equation of transformer***, can be used to find the emf induced in any winding (primary or secondary) linking with flux Φ.

Example 13.1 The primary of a 50-Hz step-down transformer has 480 turns and is fed from 6400 V supply. Find (*a*) the peak value of the flux produced in the core, and (*b*) the voltage across the secondary winding if it has 20 turns.

Solution **:** (*a*) Using Eq. 13.3, we get

$$\Phi_m = \frac{E}{4.44 fN_1} = \frac{6400}{4.44 \times 50 \times 480} = 0.06 \text{ Wb} = \mathbf{60\ mWb}$$

(*b*) The voltage induced in the secondary winding is given as

$$E = 4.44 fN_2\Phi_m = 4.44 \times 50 \times 20 \times 0.06 = \mathbf{266.4\ V}$$

13.3 IDEAL TRANSFORMER

We shall describe the physical construction and equivalent circuit of an actual transformer a little later. Here, we define the ***ideal transformer*** as a circuit element. We shall then explore its properties in voltage, current, and impedance transformation. Primary and secondary voltage and current variables are defined in Fig. 13.1*b*, which shows the circuit model of an ideal transformer.

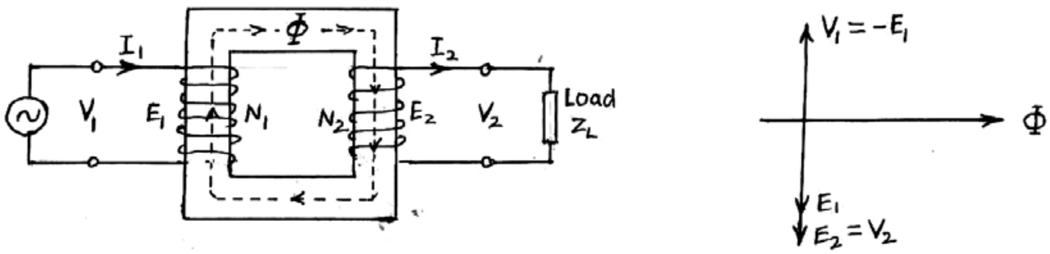
The complete behaviour of a physical transformer can be better understood by initially assuming the transformer to be ideal, and then allowing the imperfections of the actual transformer by suitably introducing some impedances.

Conditions for Ideal Transformer :

(*i*) The permeability (μ) of the magnetic circuit (the core) is infinite, i.e., the magnetic circuit has zero reluctance so that no mmf is needed to set up the flux in the core.
(*ii*) The core of the transformer has no losses.
(*iii*) The resistance of its windings is zero, hence no I^2R losses in the windings.
(*iv*) Entire flux in the core links both the windings, i.e., there is no *leakage flux*.

Thus, an ideal transformer has no losses and stores no energy. Ideal transformer has no physical existence. But the concept of ideal transformer is very helpful in understanding the working of an actual transformer.

Consider an ideal transformer whose secondary is connected to a load Z_L and primary is supplied from an ac source V_1 (Fig. 13.2*a*). The voltage across the load is V_2. The primary and secondary windings of the ideal transformer have zero impedance. Hence, the induced emf E_1 in the primary exactly counter balances the applied voltage V_1, that is, $V_1 = -E_1$. Also, the induced emf E_2 is the same as voltage V_2, that is, $E_2 = V_2$. Here, E_1 is called ***counter emf*** or ***back emf*** induced in the primary, and E_2 called ***mutually induced emf*** in the secondary.



(*a*) The circuit. (*b*) The phasor diagram.

Fig. 13.2 Ideal transformer.

Figure 13.2 *b* shows the phasor diagram of the ideal transformer. We have taken flux Φ as reference phasor, as it is common to both the primary and secondary. As per Eq. 13.2, the induced emfs E_1 and E_2 lag flux Φ by 90°. The voltage V_1 is equal and opposite to emf E_1. Thus, the applied voltage V_1 leads the flux Φ by 90°. According to the first condition of ideality, the reluctance of the magnetic circuit is zero and hence the required magnetizing current to produce flux Φ is also zero.

Transformation Ratio

The *ratio of secondary voltage to the primary voltage is known as* ***transformation ratio*** or ***turns-ratio***. It is denoted by letter *K*. Let N_1 and N_2 be the number of turns in primary and secondary windings, and E_1 and E_2 be the rms values of the primary and secondary induced emfs. Using Eq. 13.3, we can write

$$E_1 = 4.44 f N_1 \Phi_m \quad \text{...(13.4)}$$

and

$$E_2 = 4.44 f N_2 \Phi_m \quad \text{...(13.5)}$$

Then, the *transformation ratio* or *turns-ratio* can be expressed as

$$K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} \quad \text{...(13.6)}$$

Thus, the side of the transformer with the larger number of turns has the larger voltage. Indeed, *the voltage per turn is constant for a given transformer*. By selecting *K* properly, the transformation of voltage can be done from any value to any other convenient value. There can be two[1] cases :

(*i*) When $K > 1$ (i.e., $N_2 > N_1$), $V_2 > V_1$; the device is known as *step-up transformer.*

(*ii*) When $K < 1$ (i.e., $N_2 < N_1$), $V_2 < V_1$; the device is known as *step-down transformer.*

[1] The third case, when $K = 1$ (i.e., $N_1 = N_2$) is not important. We hardly ever use a transformer with unity turns ratio. Such a transformer is used only when you need electrical isolation between two electrical circuits.

In general, a transformer can have more than 2 windings. The windings of a three-winding transformer are called *primary*, *secondary* and *tertiary*. The primary is connected to an ac supply. Different loads may be connected across the secondary and tertiary[2]. The induced emf in a winding is still proportional to its number of turns,

$$E_1 : E_2 : E_3 :: N_1 : N_2 : N_3$$

Volt-Amperes

Consider again the two-winding transformer of Fig. 13.2*a*. For an ideal transformer, the current I_1 in the primary is just sufficient to provide mmf I_1N_1 to overcome the demagnetizing effect of the secondary mmf I_2N_2. Hence,

$$\therefore \qquad I_1N_1 = I_2N_2 \qquad \text{or} \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{K} \qquad \text{...(13.7)}$$

Thus, we find that the current is transformed in the reverse ratio of the voltage. That is, the side of the transformer with the *larger* number of turns has the *smaller* current. For example, a step-up transformer would have a primary with few turns of thick wire (small voltage, large current) and the secondary would have many turns of thin wire (large voltage, small current).

Combining Eqs. 13.5 and 13.7, we have

$$E_1I_1 = E_2I_2$$

Hence, in an ideal transformer the input VA and output VA are identical.

Impedance Transformation

Equations 13.6 and 13.7 reveal a very useful property of transformers, called *impedance transformation*. Figure 13.3 shows an ideal transformer. It has N_1 and N_2 turns in its primary and secondary windings. A load impedance Z_L is connected across its secondary, and an equivalent impedance Z_{eq} is defined at its primary.

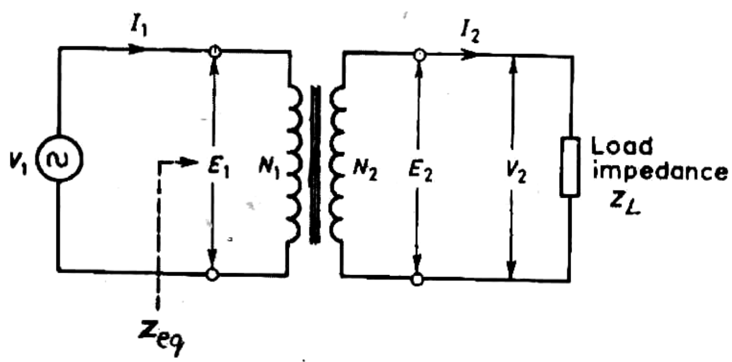


Fig. 13.3 The transformer changes the impedance Z_L to equivalent impedance Z_{eq}.

The equivalent impedance Z_{eq} as faced by a source V_1 is given as

---

[2] Sometimes, the tertiary winding has a centre-tap; the two halves having same number of turns, N_3. The voltage of such a winding is then specified as E_3/0/ E_3, or E_3 - 0 - E_3.

$$Z_{eq} = \frac{V_1}{I_1} = \frac{V_1 \times (V_2 I_2)}{I_1 \times (V_2 I_2)} = \left(\frac{V_1}{V_2}\right) \times \left(\frac{I_2}{I_1}\right) \times \left(\frac{V_2}{I_2}\right) = \left(\frac{1}{K}\right) \times \left(\frac{1}{K}\right) \times Z_L$$

or $$Z_{eq} = Z_L / K^2 \qquad ...(13.8)$$

Therefore, the impedance is transformed in inverse proportion to the square of the turns-ratio. The concept of impedance transformation is used for *impedance matching*. As per maximum power transfer theorem, the load impedance has to be properly matched with the source impedance, as illustrated in Example 13.3 given below.

Example 13.2 A single-phase, 50-Hz transformer has 30 primary turns and 350 secondary turns. The net cross-sectional area of the core is 250 cm^2. If the primary winding is connected to a 230-V, 50-Hz supply, calculate (*i*) the peak value of flux density in the core, (*ii*) the voltage induced in the secondary winding, and (*iii*) the primary current when the secondary current is 100 A (neglect losses).

***Solution* :** (*i*) The peak value of the flux in the core is given as

$$\Phi_{\mathrm{m}} = \frac{E_1}{4.44 f N_1} = \frac{230}{4.44 \times 50 \times 30} = 0.034534 \text{ Wb}$$

Therefore, the peak value of the flux density in the core is

$$B_m = \frac{\Phi_m}{A} = \frac{0.034534}{250 \times 10^{-4}} = \mathbf{1.3814\ T}$$

(*ii*) The voltage induced in the secondary winding is

$$E_2 = E_1 \times \frac{N_2}{N_1} = 230 \times \frac{350}{30} \cong 2683.33 \text{ V} = \mathbf{2.683\ kV}$$

(*iii*) The primary current is

$$I_1 = I_2 \left(\frac{N_2}{N_1}\right) = 100 \times \left(\frac{350}{30}\right) = 1166.67 \text{ A} \cong \mathbf{1.167\ kA}$$

Example 13.3 A source with an output resistance of 50 Ω is required to deliver power to a load of 800 Ω. Find the turns-ratio of the transformer to be used for maximizing the load power.

***Solution* :** For delivering maximum power to the load, the equivalent resistance must be equal to the source resistance. This requires a resistance of 50 Ω looking into the primary of the transformer. That is,

$$R_{eq} = R_L / K^2 \quad \text{or} \quad 50 = 800 / K^2 \quad \Rightarrow \quad K = \sqrt{800/50} = \sqrt{16} = 4$$

Thus,

$$K = \frac{N_2}{N_1} = \mathbf{4}$$

Example 13.4 Determine the load current $\mathbf{I}_L$ in the ac circuit shown in Fig. 13.4*a*.

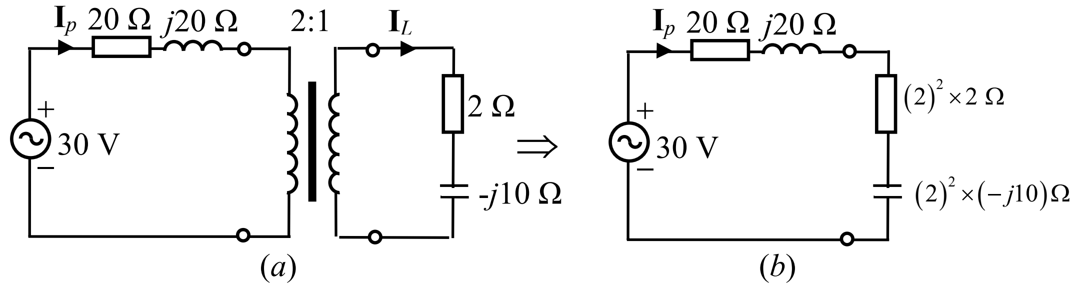


Fig. 13.4

Solution **:** We first transform the load impedance into the primary to simplify the circuit, as shown in Fig. 13.4b. The primary current is then calculated as

$$\mathbf{I}_p = \frac{30\angle 0°}{20 + j20 + 2^2(2 - j10)} = \frac{30\angle 0°}{28 - j20} = \frac{30\angle 0°}{34.4\angle -35.53°} = 0.872\angle 35.53°\ \text{A}$$

The load current, which is the same as the secondary current, is given by Eq. 13.7 as

$$\mathbf{I}_L = 2\times\mathbf{I}_p = 2\times 0.872\angle 35.53° = \mathbf{1.74\angle 35.53°\ \ A}$$

Example 13.5 A single-phase transformer has a core with cross-sectional area of 150 cm^2. It operates at a maximum flux density of 1.1 Wb/m^2 from a 50-Hz supply. If the secondary winding has 66 turns, determine the output in kVA when connected to a load of 4-Ω impedance. Neglect any voltage drop in the transformer.

Solution **:** $\Phi_m = B_m A = 1.1\times 0.015 = 0.0165$ Wb**.** Since the voltage drop in the transformer is negligible, we have

$$V_2 = E_2 = 4.44 f N_2 \Phi_m = 4.44\times 50\times 66\times 0.0165 = 241.76\ \text{V}$$

The output current, $I_2 = \dfrac{V_2}{Z_L} = \dfrac{241.76}{4} = 60.44\ \text{V}$

$\therefore$ Output volt-amperes $= 241.76\times 60.44 = 14\,612\ \text{VA} = \mathbf{14.612\ kVA}$

Example 13.6 A single-phase, 50-Hz transformer has a square core having a net cross-sectional area of 9 cm^2, and three windings designed for the following voltages :

(i) Primary : 230 V; (ii) Secondary : 110 V; and (iii) Tertiary : 6/0/6 V.

Find the number of turns in each winding if the flux density is not to exceed 1 T.

Solution **:** $\Phi_m = B_m A = 1\times 9\times 10^{-4} = 9\times 10^{-4}$ Wb**.**

The tertiary winding is divided into two halves; each half having a voltage E_3 = 6 V. Thus, the number of turns in each half of the tertiary is

$$N_3 = \frac{E_3}{4.44 f\Phi_m} = \frac{6}{4.44\times 50\times 9\times 10^{-4}} = \mathbf{30\ turns}$$

$\therefore$ Total number of turns on the tertiary winding $= 2\times 30 = \mathbf{60\ turns}$.

We have seen that across 30 turns of the tertiary winding, the induced emf is 6 V. Therefore, the number of turns on the primary and secondary can be calculated as follows :

$$\frac{N_1}{N_3} = \frac{E_1}{E_3} \quad \text{or} \quad N_1 = \frac{N_3 E_1}{E_3} = \frac{30 \times 230}{6} = \mathbf{1150\ turns}$$

$$\text{and} \quad \frac{N_2}{N_3} = \frac{E_2}{E_3} \quad \text{or} \quad N_2 = \frac{N_3 E_2}{E_3} = \frac{30 \times 110}{6} = \mathbf{550\ turns}$$

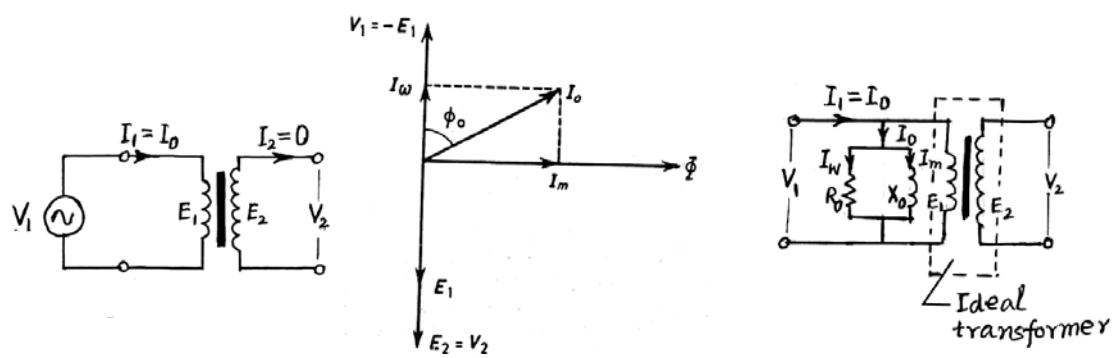
13.4 PRACTICAL TRANSFORMER AT NO LOAD

In actual practice, a transformer can never satisfy any of the conditions specified above for ideal transformer. Nevertheless, the concept of ideal transformer is helpful to understand the working of an actual transformer. We shall consider these conditions one by one, and see in what way a practical transformer deviates from ideal transformer. In this Section, we shall consider only the first two ideality conditions. The remaining two conditions shall be considered in Section 13.7.

Consider a transformer with its primary connected to an alternating voltage source V_1, and no load connected across its secondary (Fig. 13.5*a*). With no closed circuit, the current in the secondary winding is zero. If the transformer were truly ideal, the primary current too would be zero, as per 713.5. But, in practice there does flow a little ***no-load current*** I_0 in the primary. This current I_0 is also called the ***exciting current*** of the transformer.

Effect of Magnetization

Consider the *first ideality condition*. No magnetic material can have infinite permeability so as to offer zero reluctance to the magnetic circuit. Hence, in a practical transformer a finite mmf is needed to establish magnetic flux in the core. As a result, an in-phase ***magnetizing current*** I_m in the primary is needed to set up flux Φ in the core. This current I_m is purely reactive and lags the voltage V_1 by 90°. The flux Φ induces emfs E_1 and E_2 in the primary and the secondary windings. As per Eq. 13.2, both these emfs lag flux Φ by 90°, as shown in Fig. 13.5*b*.



(*a*) The circuit. (*b*) The phasor diagram. (*c*) The equivalent circuit.

Fig. 13.5 Transformer on no load.

As the current I_2 in the secondary is zero (no load connected), the voltage drop in the secondary winding is zero. Hence, $V_2 = E_2$. The induced emf E_1 counter balances the applied voltage V_1 and establishes an electrical equilibrium. If the third and fourth ideality conditions (i.e., the effect of the resistance of the winding and the leakage of flux) are ignored, the magnitude of V_1 will be the same as that of emf E_1. Thus, $V_1 = -E_1$.

Effect of Core Losses

Let us now consider the *second ideality condition*. There exist two reasons (*eddy current* and *hysteresis*) for the energy loss in the core of the transformer. The source must supply enough power to the primary to meet the core losses. Therefore, a ***core-loss current*** I_w (in phase with V_1) flows through the primary, as shown in the phasor diagram of Fig. 13.5*b*.

Thus, we find that the no-load current I_0 has two components, I_m and I_w. The magnetizing current I_m lags voltage V_1 by 90° and the loss component I_w is in phase with voltage V_1. The angle ϕ_0 is the *no-load phase angle*. Thus, from the phasor diagram of Fig. 13.5*b*, we have

$$I_0 = \sqrt{I_w^2 + I_m^2}; \qquad \phi_0 = \tan^{-1}(I_m / I_w) \quad \text{and} \quad \text{Input power } = V_1 I_w = V_1 I_0 \cos\phi_0$$

In the equivalent circuit shown in Fig. 13.5*c*, the no-load current I_0 is shown divided into two parallel branches. The component I_w accounts for the core loss, and hence is shown to flow through a resistance R_0. The component I_m represents magnetizing current. Hence, it is shown to flow through a pure inductive reactance X_0. The R_0-X_0 parallel circuit is called ***exciting circuit*** of the transformer.

Example 13.7 A single-phase, 230-V/110-V, 50-Hz transformer takes an input of 350 volt amperes at no load while working at rated voltage. The core loss is 110 W. Find the loss component of no-load current, the magnetizing component of no-load current and the no-load power factor.

Solution **:** Given : $V_1 I_0 = 350 \text{ VA}$.

$$\therefore \qquad I_0 = \frac{VA}{V_1} = \frac{350}{230} = 1.52 \text{ A}$$

The core loss = Input power at no load, $P_i = V_1 I_0 \cos\phi_0$

Therefore, the power factor at no load is given as

$$pf = \cos\phi_0 = \frac{P_i}{V_1 I_0} = \frac{110 \text{ W}}{350 \text{ VA}} = \mathbf{0.314}$$

The loss component of no-load current is given as

$$I_w = I_0 \cos\phi_0 = 1.52 \times 0.314 = \mathbf{0.478 \ A}$$

The magnetizing component of no-load current is given as

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.52)^2 - (0.478)^2} = \mathbf{1.44 \ A}$$

Since the core losses occur in the iron core, these are also called ***iron losses.*** These losses have two components : (*i*) hysteresis loss, and (*ii*) eddy-current loss

(*i*) Hysteresis Loss : When alternating current flows through the windings, the core material undergoes cyclic process of magnetization and demagnetization. It is found that there is a tendency of the flux density *B* to *lag behind* the field strength *H*. This tendency is called *hysteresis*[3]. The effect of this phenomenon on the core material can be best understood from the *B-H* plot shown in Fig. 13.6.

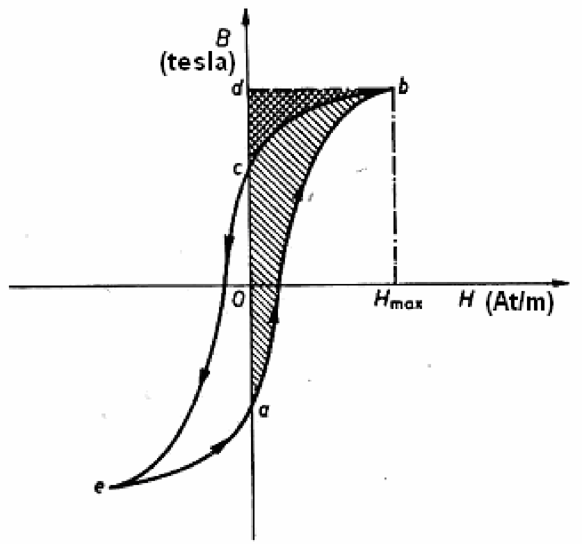


Fig. 13.6 Hysteresis loop and energy relationship per half-cycle.

During positive half-cycle, when H increases from zero to its positive maximum value, the energy is stored in the core and is given by the area *abda*. However, when H decreases from its positive maximum value to zero, the energy is released and is given by the area *bdcb*. The difference between these two energies is the net loss and is dissipated as heat in the core. Thus, as H varies over one complete cycle, the total energy loss (per cubic metre) is represented by the area *abcea* of the hysteresis loop. The hysteresis loss (usually expressed in watts) is given as

$$P_h = K_h B_m^n f V \qquad ...(13.9)$$

where K_h = hysteresis coefficient whose value depends upon the material ($K_h = 0.025$ for cast steel, $K_h = 0.001$ for silicon steel)
B_m = maximum flux density (in tesla)
n = a constant, $1.5 \le n \le 2.5$ depending upon the material
f = frequency (in hertz)
V = volume of the core material (in m^3)

This loss can be minimized by selecting suitable ferromagnetic material for the core.

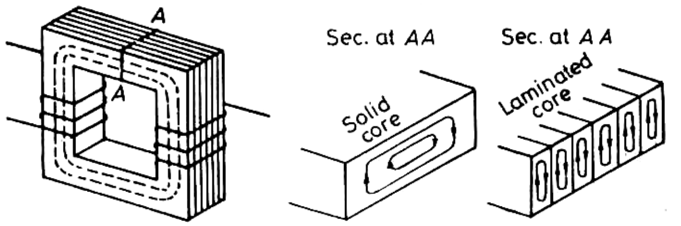
***(ii)* Eddy-Current Loss :** The eddy currents are the circulating currents set up in the core due to alternating magnetic flux (shown with dotted lines in Fig. 13.7*a*). These currents (shown in Fig. 13.7*b*) may be quite high since the resistance of the iron is quite low. This results in unnecessary heating of the core and loss of power. The eddy-current loss (in watts) is given by

$$P_e = K_e B_m^2 f^2 t^2 V \qquad ...(13.10)$$

where K_e = a constant dependent upon the material
t = thickness of laminations (in metre)

[3] In Greek, *hysterein* means 'to lag'.

The eddy-current loss can be minimized by dividing the solid iron core into thin sheets or *laminations* insulated from one another (Fig. 13.7*c*). The path of the induced eddy currents in the core is broken by the insulating material between the sheets. The eddy currents and hence the eddy-current loss is thus substantially reduced.



(*a*) (*b*) (*c*)

Fig. 13.7 Laminated core helps in reducing the eddy currents.

Note that the eddy-current loss varies as the square of the frequency, whereas the hysteresis loss varies directly with the frequency. The total iron loss is given as

$$P_i = P_h + P_e \qquad ...(13.11)$$

Example 13.8 A single-phase, 230-V/110-V, transformer has iron loss of 100 W at 60 Hz, and 60 W at 40 Hz. Determine the hysteresis and eddy-current losses at 50 Hz.

Solution **:** We know that the hysteresis loss, $P_h \propto f \quad \Rightarrow \quad P_h = Af$

and the eddy-current loss, $P_e \propto f^2 \quad \Rightarrow \quad P_e = Bf^2$

Then, the total iron loss, $P_i = P_h + P_e = Af + Bf^2$. Therefore, at the given two frequencies, we have

At 60 Hz : $100 = 60A + 3600B$

At 40 Hz : $60 = 40A + 1600B$

Solving the above two equations, we get A = 1.167 and B = 0.00834. We can now calculate the two losses at 50 Hz,

Hysteresis loss at 50 Hz, $P_h = Af = 1.167 \times 50 = \mathbf{58.35\ W}$

Eddy-current loss at 50 Hz, $P_e = Bf^2 = 0.00834 \times (50)^2 = \mathbf{20.85\ W}$

13.5 CONSTRUCTION OF TRANSFORMER

The main parts of an actual transformer used in power circuits are as follows :

(i) An iron core that provides a magnetic circuit.

(ii) Two inductive coils wound on the core. These are suitably insulated from each other and also from the core. The individual turns of a coil are also insulated from each other.

(iii) A suitable container for the assembled core and windings.

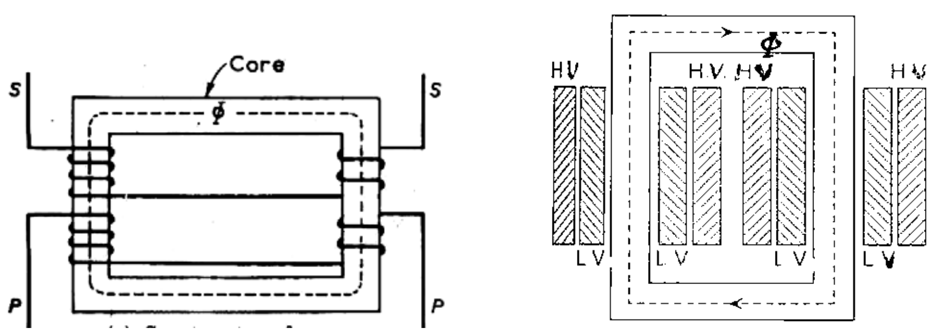
(iv) A suitable medium (called *transformer oil*) for insulating the core and windings from the container. This medium also cools the windings and core of the transformer.

(v) Suitable porcelain bushings for insulating and bringing out winding terminals from the tank.

Core of Transformer

The core is made of steel laminations so as to minimize eddy-current loss. The laminations are about 0.35 mm thick and are insulated from each other by a light coat of varnish on the surface. The laminations are pressed together so as to form a continuous magnetic path, with minimum air gap. Depending upon the construction of the core, there are two types of transformers :

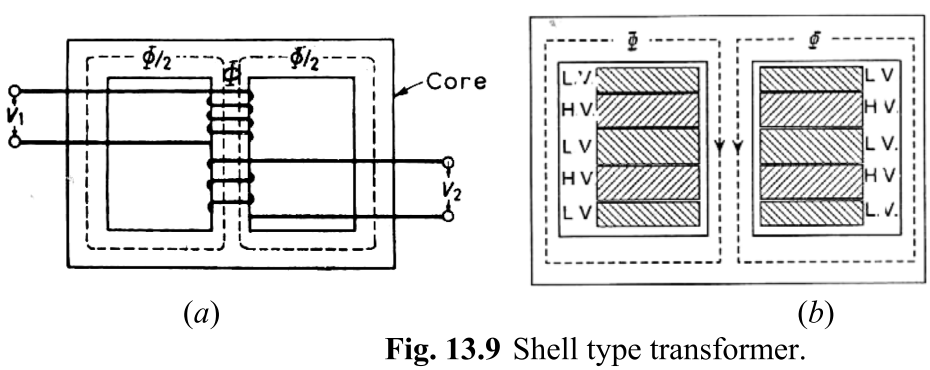
(1) Core Type Transformer : In this type, *the windings surround a considerable part of the core.* Both the windings are divided into two parts and half of each winding is placed on each limb, side by side (shown schematically in Fig. 13.8*a*). This is done to reduce the leakage of the magnetic flux. Furthermore, the low voltage (LV) winding is placed adjacent to the core and high voltage (HV) winding is placed around the LV winding, as shown in Fig. 13.8*b*. This is done to minimize the cost of insulation.



(*a*) (*b*)

Fig. 13.8 Core type transformer.

(2) Shell Type Transformer : It has three limbs. Both the windings are placed on the central limb (shown schematically in Fig. 13.9*a*). The LV and HV windings are sandwiched as shown in Fig. 13.9*b*. Here, *the core surrounds a considerable part of the windings.*



(*a*) (*b*)

Fig. 13.9 Shell type transformer.

In core type transformer, the flux has single path. But in shell type transformer, the flux divides equally in the central limb and returns through the outer two legs. Since there is more space for insulation in the core type transformer, it is preferred for high voltages. On the other hand, the shell type construction is more economical for low voltages.

13.6 TRANSFORMER ON LOAD

Let us examine what happens when a load is connected to the secondary of the transformer. Note that for simplicity we are still considering a partially ideal transformer (i.e., a transformer satisfying only the *ideality conditions* (*iii*) and (*iv*) stated on page 000). Before connecting the load, there exists a flux Φ in the core due to the no-load current I_0 flowing in the primary. On connecting the load, a current I_2 flows through the secondary, as shown in Fig. 13.10. The magnitude and phase of I_2 with respect to the secondary voltage V_2 depends upon the nature of the load.

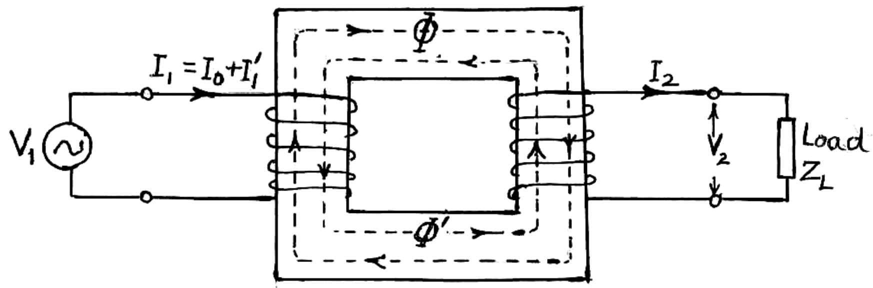


Fig. 13.10 Transformer on load.

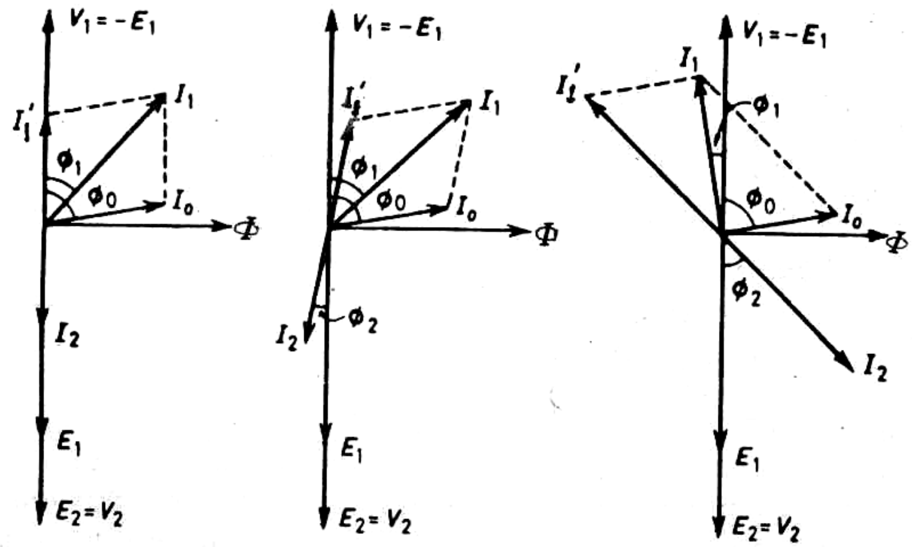
The current I_2 sets up a flux $\Phi^{'}$ in the core, which opposes the main flux Φ. This momentarily weakens the main flux, and the primary back emf E_1 gets reduced. As a result, the difference V_1 - E_1 increases and more current is drawn from the supply. This again increases the back emf E_1, so as to balance the applied voltage V_1. In this process, the primary current increases by I_1'. This current is known as ***primary balancing current***, or ***load component of primary current***. Under such a condition, the secondary ampere-turns must be counterbalanced by the primary ampere-turns. That is, $N_1 I_1^{'} = N_2 I_2$. Hence, we have

$$I_1^{'} = \left(\frac{N_2}{N_1}\right) I_2 = K I_2 \qquad \text{...(13.12)}$$

The total primary current I_1 is the phasor sum of the no-load current I_0 and the primary balancing current I_1'. That is,

$$\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_1^{'} \qquad \text{...(13.13)}$$

The phasor diagrams for the (partial) ideal transformer with resistive, inductive and capacitive loads are shown in Fig. 13.11. Note that the *primary balancing current* I_1' is always in phase opposition to the secondary current I_2. Since the no-load current I_0 is negligibly small (compared to $I_1^{'}$), the primary current I_1 is almost opposite in phase to the secondary current I_2.



(*a*) Resistive load. (*b*) Inductive load. (*c*) Capacitive load.

Fig. 13.11 Phasor diagrams for a transformer on load.

Example 13.9 A single-phase, 440-V/110-V, 50-Hz transformer takes a no-load current of 5A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a power factor of 0.8 lagging to a load, determine the primary current and the primary power factor.

***Solution* :** $\phi_0 = \cos^{-1} 0.2 = 78.46°$ and $\phi_2 = \cos^{-1} 0.8 = 36.87°$

The transformation ratio, $K = \dfrac{V_2}{V_1} = \dfrac{110}{440} = \dfrac{1}{4}$

$\therefore \quad I_1' = K \times I_2 = (1/4) \times 120 = 30 \text{ A}$

The angle between $\mathbf{I}_0$ and $\mathbf{I}_1'$, $\theta = \phi_0 - \phi_2 = 78.46° - 36.87° = 41.59°$

$$\therefore \quad I_1 = \sqrt{I_0^2 + I_1'^2 + 2I_0 I_1' \cos 41.59°}$$

$$= \sqrt{5^2 + 30^2 + 2 \times 5 \times 30 \times \cos 41.59°} = \mathbf{33.9\ A}$$

The phasor diagram is shown in Fig. 13.12. Note that for the sake of clarity, the phasors are not drawn to the scale. The angle between $\mathbf{I}_0$ and $\mathbf{I}_1$ is given as

$$\alpha = \tan^{-1} \frac{I_1' \sin\theta}{I_0 + I_1' \cos\theta} = \tan^{-1} \frac{30 \sin 41.59°}{5 + 30 \cos 41.59°} = 35.97°$$

$\therefore \quad$ angle $\phi_1 = \phi_0 - \alpha = 78.46° - 35.97° = 42.49°$

Thus,

Primary power factor = $\cos\phi_1 = \cos 42.49° = \mathbf{0.737}$

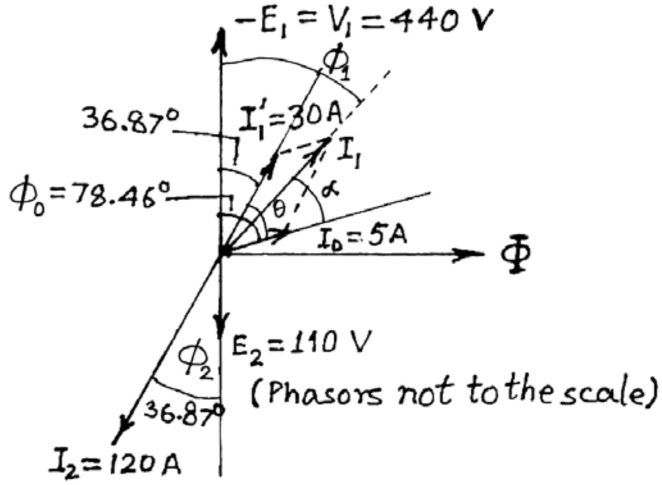


Fig. 13.12 Phasor diagram.

13.7 PRACTICAL TRANSFORMER ON LOAD

We now consider the *ideality conditions* (*iii*) and (*iv*) stated on page ***3***. The effects of deviations from these conditions become more prominent when a practical transformer is put on load.

Effect of Winding Resistance

The windings are made of copper wire, since copper has good conductivity. In actual practice, each winding has some resistance. Current flow through the windings causes not only a voltage drop but also a power loss called *I^2R loss* or *copper loss*. This effect is accounted for by including a resistance R_1 in the primary and resistance R_2 in the secondary, as shown in Fig. 13.13.

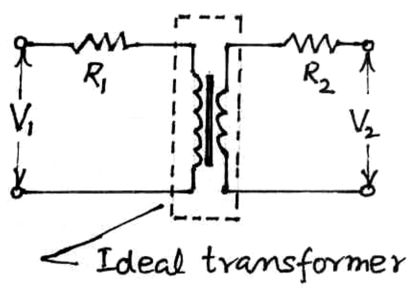
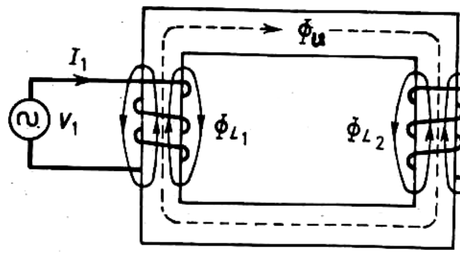


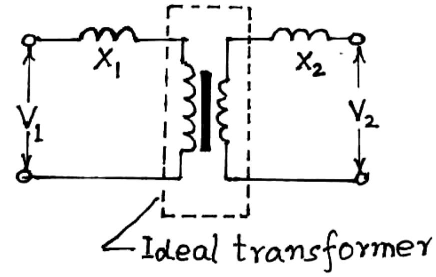
Fig. 13.13 Effect of winding resistance.

Effect of Flux Leakage

In a practical transformer, the entire magnetic flux does not remain confined to the magnetic core. Not all the flux produced by the primary winding links with the secondary. As shown in Fig. 13.14*a*, a part of it leaks through the air paths. The difference between the total flux linking with the primary and the useful mutual flux Φ_U linking with both the windings is called the *primary leakage flux*, Φ_{L1}. Similarly, Φ_{L2} represents the *secondary leakage flux*.



(*a*) Its definition.



(*b*) Its effect accounted for.

Fig. 13.14 Leakage flux in a transformer.

It is the useful mutual flux Φ_U that is responsible for the transformer action. The primary leakage flux Φ_{L1} induces an emf E_{L1} in the primary winding. Similarly, flux Φ_{L2} induces an emf E_{L2} in the secondary. The effect of flux leakage can then be accounted for by including reactances X_1 and X_2 in the primary and secondary windings, respectively (as shown in Fig. 13.14*b*), such that

$$E_{L1} = I_1 X_1 \qquad \text{and} \qquad E_{L2} = I_2 X_2$$

The reactances X_1 and X_2 are called *primary* and *secondary leakage reactances*, respectively. **Note** that these reactances are fictitious quantities. These are introduced just as a convenience to represent the effect of the flux leakage.

The reluctance of the paths of the leakage fluxes Φ_{L1} and Φ_{L2} is almost entirely due to the long air paths and is therefore practically constant. Consequently, the value of the leakage flux is proportional to the current. On the other hand, the value of the useful flux Φ_U remains almost independent of the load. Furthermore, the reluctance of the paths of the leakage flux is very high. Hence, the value of this flux is relatively small even on full load.

13.8 EQUIVALENT CIRCUIT OF A TRANSFORMER

The function of an ideal transformer is to transform electric power from one voltage level to another without incurring any loss and without needing any magnetizing current. For such a transformer, the volt-amperes in the primary are exactly balanced by the volt-amperes in the secondary. An ideal transformer is supposed to operate at 100 percent efficiency.

We stated the four conditions that must be satisfied by a transformer to be ideal. We then examined the effects of each of these conditions and explored how to account for the deviations in a practical transformer. Based on this, we can now draw the *equivalent circuit* of a practical transformer (Fig. 13.15). This circuit is merely a representation of the following KVL equations for the primary and secondary sides of the transformer.

$$\mathbf{V}_1 = I_1 R_1 + jI_1 X_1 - \mathbf{E}_1 = I_1(R_1 + jX_1) - \mathbf{E}_1 \qquad \text{...(13.14)}$$

and

$$\mathbf{E}_2 = I_2 R_2 + jI_2 X_2 + \mathbf{V}_2 = I_2(R_2 + jX_2) + \mathbf{V}_2 \qquad \text{...(13.15)}$$

Equation 13.14 states that the applied voltage $\mathbf{V}_1$ is the phasor sum of the negative of induced emf $\mathbf{E}_1$ and the voltage drops in primary *resistance*, R_1, and *leakage reactance*, X_1, due to the flow of current I_1. The induced emf $\mathbf{E}_2$ forces a current I_2 in the secondary circuit. Hence, Eq. 13.15 states that the induced emf $\mathbf{E}_2$ is phasor sum of the load voltage $\mathbf{V}_2$ and the voltage drops in secondary *resistance*, R_2, and *leakage reactance*, X_2, due to the flow of current I_2.

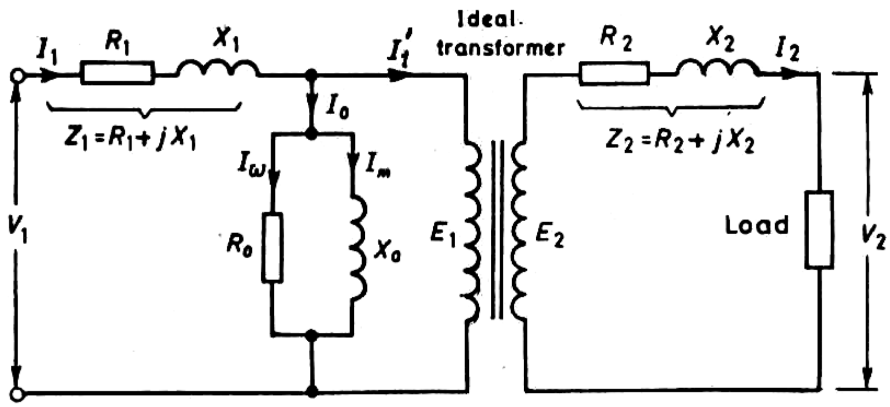


Fig. 13.15 Equivalent circuit of a transformer.

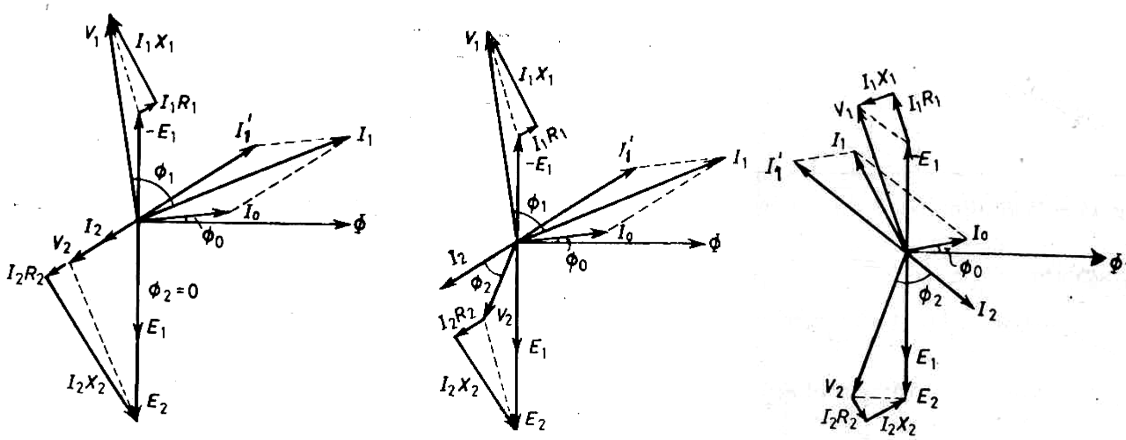
Furthermore, it can be seen from Fig. 13.15 that the primary current I_1 is composed of two components, the no-load current I_0 and the load component of primary current I_2'. Moreover, the current I_0 consists of two components I_w and I_m. The current I_w flows through resistance R_0 and accounts for the iron loss of the transformer. The current I_m, called ***magnetizing current***, is required to establish working magnetic flux in the core. The ac voltage source connected to the primary winding does not have to supply any power in making this current flow. Hence, in the equivalent circuit, it is shown to flow through a pure reactance X_0.

Phasor Diagram

We can draw the phasor diagram of a transformer circuit with a given load, provided all its parameters (as used in the equivalent circuit of Fig. 13.15) are known. While drawing the phasor diagram, we should keep the following points in mind :

i) It is most convenient to commence the phasor diagram with the phasor representing the quantity that is common to the two windings, namely, the flux Φ.
ii) The induced emfs E_1 and E_2 lag behind flux Φ by 90°.
iii) The values of emfs E_1 and E_2 are proportional to the number of turns on the primary and secondary windings.
iv) The magnitude and phase of current I_2 is decided by the load.
v) The resistive voltage drops are always in phase with the respective current phasor.
vi) The inductive voltage drops lead the respective current phasor by 90°.
vii) The secondary induced emf E_2 is obtained by vector sum of the terminal voltage V_2 and the impedance drop I_2Z_2. Hence, V_2 must be drawn such that the phasor sum of V_2 and I_2Z_2 is E_2.
viii) The primary balancing current I_1' and the secondary current I_2 are in inverse proportion to the number of turns on the primary and secondary windings.
ix) The primary current I_1 is the vector sum of the no-load current I_0 and the primary balancing current I_1'.
x) The primary voltage V_1 is obtained by adding vectorially the impedance drop I_1Z_1 to negative of E_1.
xi) The phase angle f_1 between V_1 and I_1 is the power factor angle of the transformer.

The phasor diagrams for different types of loads (resistive, inductive and capacitive) are shown in Fig. 13.16.

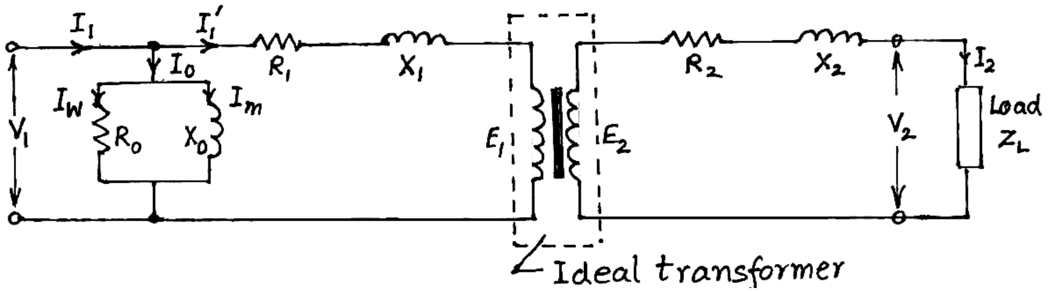


(*a*) Resistive load. (*b*) Inductive load. (*c*) Capacitive load.

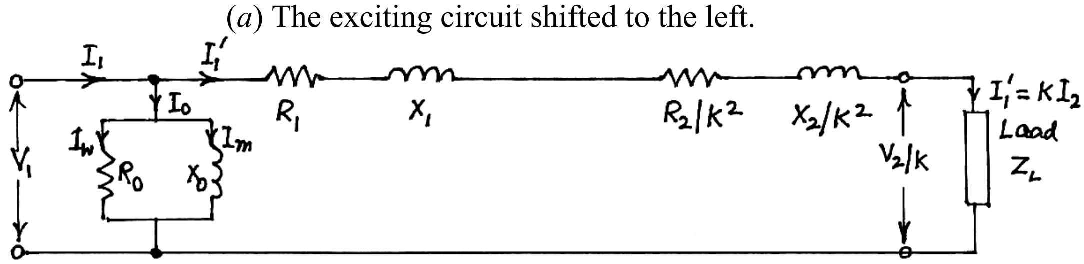
Fig. 13.16 Phasor diagrams for different types of loads.

Simplified Equivalent Circuit

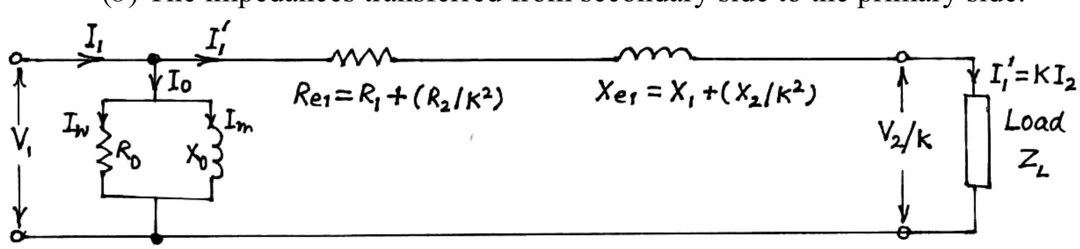
Since the no-load current I_0 of a transformer is only about 3-5 percent of the full-load primary current, not much error will be introduced if the exciting circuit R_0-X_0 in Fig. 13.15 is shifted to the left of impedance R_1-X_1. This results in a circuit shown in Fig. 13.17*a*.



(*a*) The exciting circuit shifted to the left.



(*b*) The impedances transferred from secondary side to the primary side.



(*c*) Equivalent resistance and reactance referred to the primary side.

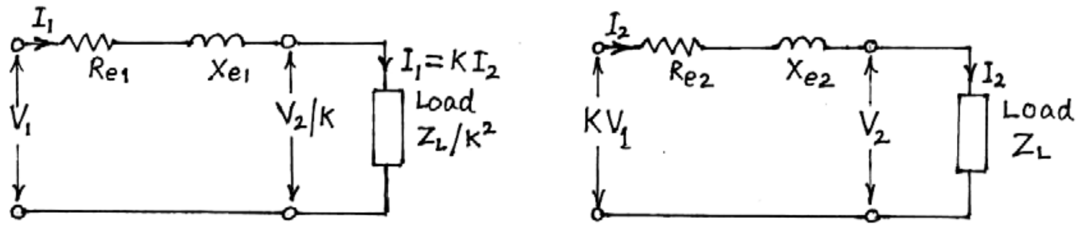
Fig. 13.17 Simplified equivalent circuit of a transformer as referred to the primary side.

Using the impedance transformation, we can now transform the impedances from the secondary side to the primary side and remove the ideal transformer from the circuit, as shown in Fig. 13.17*b*. This can further be simplified by combining the two resistances together and the two leakage reactances together, as shown in Fig. 13.17*c*. Here, the total resistance and total leakage reactance *as referred to primary* are given as

$$R_{e1} = R_1 + (R_2 / K^2) \qquad \text{and} \qquad X_{e1} = X_1 + (X_2 / K^2) \qquad ...(13.16)$$

Approximate Equivalent Circuit

Compared to the full-load primary current, the no-load current of a transformer is very small (only 3-5 percent). Therefore, while considering the behaviour of a transformer on full-load, we can omit the exciting circuit R_0-X_0 without introducing an appreciable error. The resulting approximate equivalent circuit is shown in Fig. 13.18*a*.



(*a*) As referred to primary side. (*b*) As referred to secondary side.

Fig. 13.18 Approximate equivalent circuit of a transformer.

Alternatively, we could transfer the impedances from primary side to the secondary side, so as to get the approximate equivalent circuit as shown in Fig. 13.18*b*. Here, the total resistance and total leakage reactance *as referred to secondary* are given as

$$R_{e2} = K^2 R_1 + R_2 \qquad \text{and} \qquad X_{e2} = K^2 X_1 + X_2 \qquad ...(13.17)$$

Example 13.10 A single-phase, 50-kVA, 4400-V/220-V, 50-Hz transformer has primary and secondary resistance R_1 = 3.45 Ω and R_2 = 0.009 Ω, respectively. The values of the leakage reactances are X_1 = 5.2 Ω and X_2 = 0.015 Ω. Calculate for this transformer (*a*) the equivalent resistance as referred to the primary, (*b*) the equivalent resistance as referred to the secondary, (*c*) the equivalent reactance as referred to the primary, (*d*) the equivalent reactance as referred to the secondary, (*e*) the equivalent impedance as referred to the primary, (*f*) the equivalent impedance as referred to the secondary, (*g*) the total copper loss first by using the individual resistances of the two windings and then by using the equivalent resistances as referred to each side.

Solution **:** Full-load primary current, $I_1 = \dfrac{\text{kVA}}{V_1} = \dfrac{50\,000}{4400} = 11.36 \text{ A}$

Full-load secondary current, $I_2 = \frac{\text{kVA}}{V_2} = \frac{50\,000}{220} = 227.27 \text{ A}$

Transformation ratio, $K = \frac{V_2}{V_1} = \frac{220}{4400} = \frac{1}{20} = 0.05$

(a) $R_{e1} = R_1 + (R_2 / K^2) = 3.45 + [0.009/(0.05)^2] = \mathbf{7.05\ \Omega}$

(b) $R_{e2} = K^2 R_1 + R_2 = (0.05)^2 \times 3.45 + 0.009 = \mathbf{0.0176\ \Omega}$

(c) $X_{e1} = X_1 + (X_2 / K^2) = 5.2 + [0.015/(0.05)^2] = \mathbf{11.2\ \Omega}$

(d) $X_{e2} = K^2 X_1 + X_2 = (0.05)^2 \times 5.2 + 0.015 = \mathbf{0.028\ \Omega}$

(e) $Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{(7.05)^2 + (11.2)^2} = \mathbf{13.23\ \Omega}$

(f) $Z_{e2} = \sqrt{R_{e2}^2 + X_{e2}^2} = \sqrt{(0.0176)^2 + (0.028)^2} = \mathbf{0.0331\ \Omega}$

(g) Total copper loss = $I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (227)^2 \times 0.009 = \mathbf{909\ W}$

By considering equivalent resistances,

Total copper loss = $I_1^2 R_{e1} = (11.36)^2 \times 7.05 = \mathbf{909.8\ W}$

Total copper loss = $I_2^2 R_{e2} = (227.27)^2 \times 0.0176 = \mathbf{909\ W}$

13.9 VOLTAGE REGULATION OF A TRANSFORMER

With the increase in the load on a transformer, there is a change in its secondary terminal voltage. The voltage falls if the load power is lagging. It increases if the power factor is leading. The *voltage regulation of a transformer is defined as the change in its secondary terminal voltage from no load to full load*, the primary voltage being assumed constant. Let

$V_{2(0)}$ = secondary terminal voltage at no load,

and V_2 = secondary terminal voltage at full load.

Then, the voltage drop $V_{2(0)}$ - V_2 is called the ***inherent regulation.*** We can compare this change in voltage with respect to either the no-load voltage or the full-load voltage, and can be expressed as either per unit basis or percentage basis. Thus, we have

i) Per unit **regulation down** $= \frac{V_{2(0)} - V_2}{V_{2(0)}}$

% **regulation down** $= \frac{V_{2(0)} - V_2}{V_{2(0)}} \times 100$

ii) Per unit **regulation up** $= \frac{V_{2(0)} - V_2}{V_2}$

% **regulation up** $= \frac{V_{2(0)} - V_2}{V_2} \times 100$

Normally, when nothing is specified, '*regulation*' means '*regulation down*'.

Approximate Voltage Drop

At no load, $V_1 \approx E_1$ and $E_2 = V_{2(0)}$. Also, if K is the transformation ratio, the secondary terminal voltage at no load, $V_{2(0)} = E_2 = KE_1 = KV_1$. Thus, using the equivalent circuit referred to the secondary (shown in Fig. 13.18*b*), we can draw the phasor diagram of the transformer, as given in Fig. 13.19. In this diagram,

OA represents the full-load terminal voltage, V_2
AB represents the resistive voltage drop, I_2R_{e2}
BC represents the reactive voltage drop, I_2X_{e2}
AC represents the total impedance voltage drop, I_2Z_{e2}
OC represents the no-load terminal voltage, $V_{2(0)}$
(same as the input voltage referred to the secondary, KV_1)

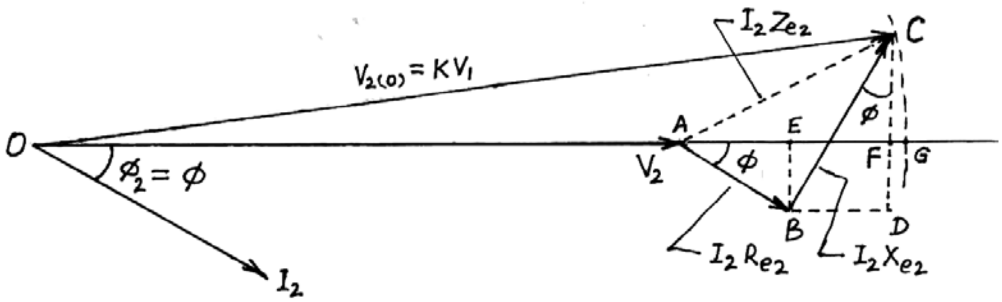


Fig. 13.19 Phasor diagram of a transformer referred to the secondary.

With point O as centre and OC as radius, draw an arc which intersects the extended OA at point G, so that $OC = OG$. From point C drop a perpendicular on OG, which intersects it at F. Thus, we have

$$\text{Exact voltage drop} = V_{2(0)} - V_2 = OC - OA = OG - OA = AG = AF + FG$$

Here, AF is the approximate voltage drop, which we intend to determine. The error in this approximation is FG. Since the impedance voltage drop I_2Z_{e2} (= AC) is very small compared to the full-load voltage V_2 (= OA), the error committed in the determination of the approximate voltage drop would be negligibly small.

From point B, drop a perpendicular BE on OG, and draw BD parallel to AF. We can now write

$$\text{Approximate voltage drop, } AF = AE + EF = AE + BD$$
$$= I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi \qquad ...(13.18)$$

In case of *leading power factor* of the load (instead of lagging), we can determine the approximate voltage drop either by putting ***-f*** in place of ***f*** in Eq. 13.18 or by redrawing the phasor diagram for leading power factor, to get

$$\text{Approximate voltage drop, } AF = AE - EF = AE - BD$$
$$= I_2R_{e2}\cos\phi - I_2X_{e2}\sin\phi \qquad ...(13.19)$$

Thus, in general, we can write

$$\text{Approximate voltage drop} = I_2 R_{e2} \cos\phi \pm I_2 X_{e2} \sin\phi \qquad ...(13.20)$$

in which + sign is to be used for lagging power factor and – sign for leading power factor. The voltage drop expressed in percentage is the ***percent regulation***, and is given as

$$\% \text{ Regulation} = \frac{I_2 R_{e2} \cos\phi \pm I_2 X_{e2} \sin\phi}{V_{2(0)}} \times 100$$

$$= V_r \cos\phi \pm V_x \sin\phi \qquad ...(13.21)$$

where $\quad V_r = \% \text{ resistive drop} = \dfrac{I_2 R_{e2}}{V_{2(0)}} \times 100 \qquad ...(13.22)$

and $\quad V_x = \% \text{ reactive drop} = \dfrac{I_2 X_{e2}}{V_{2(0)}} \times 100 \qquad ...(13.23)$

Exact Voltage Drop

Referring to Fig. 13.19, the exact voltage drop is AG. We have already determined the approximate value of voltage drop given by AF. To determine FG, we proceed as follows. For the right angle triangle OFC, we can write

$$OC^2 = OF^2 + FC^2$$

or $\quad OC^2 - OF^2 = FC^2$

or $\quad (OC - OF)(OC + OF) = FC^2$

or $\quad (OG - OF)(OC + OF) = FC^2$

or $\quad FG(2OC) = FC^2 \quad [\text{Taking } OF \approx OC]$

$$\therefore \quad FG = \frac{FC^2}{2OC} = \frac{(DC - DF)^2}{2OC} = \frac{(DC - BE)^2}{2OC} = \frac{(I_2 X_{e2} \cos\phi - I_2 R_{e2} \sin\phi)^2}{2V_{2(0)}}$$

Thus, for lagging power factor,

$$\text{Exact voltage drop} = AF + FG$$

$$= (I_2 R_{e2} \cos\phi + I_2 X_{e2} \sin\phi) + \frac{(I_2 X_{e2} \cos\phi - I_2 R_{e2} \sin\phi)^2}{2V_{2(0)}}$$

For leading power factor,

$$\text{Exact voltage drop} = (I_2 R_{e2} \cos\phi - I_2 X_{e2} \sin\phi) + \frac{(I_2 X_{e2} \cos\phi + I_2 R_{e2} \sin\phi)^2}{2V_{2(0)}}$$

Thus, in general, we have

$$\text{Exact voltage drop} = (I_2R_{e2}\cos\phi \pm I_2X_{e2}\sin\phi) + \frac{(I_2X_{e2}\cos\phi \mp I_2R_{e2}\sin\phi)^2}{2V_{2(0)}}$$

...(13.24)

or

$$\%\,\text{Exact voltage drop} = \frac{(I_2R_{e2}\cos\phi \pm I_2X_{e2}\sin\phi)}{V_{2(0)}}\times 100 + \frac{(I_2X_{e2}\cos\phi \mp I_2R_{e2}\sin\phi)^2}{2V_{2(0)}\times V_{2(0)}}\times 100$$

$$= (V_r\cos\phi \pm V_x\sin\phi) + \frac{1}{2V_{2(0)}}(V_x\cos\phi \mp V_r\sin\phi)^2$$

...(13.25)

Keep in mind that upper signs are to be used for lagging power factor, and the lower signs for leading power factor.

Condition for Zero Regulation

It is possible to obtain zero regulation for a transformer. For this, the voltage drop (as given by Eq. 13.20) from no-load to full-load should be zero. It is possible only if the sign in Eq. 13.20 is negative, i.e., ***only if the load has leading power factor***. Thus, the ***condition of zero regulation*** is given as

$$I_2R_{e2}\cos\phi - I_2X_{e2}\sin\phi = 0 \qquad \text{or} \qquad \tan\phi = \frac{R_{e2}}{X_{e2}}$$

...(13.26)

Also, note that for leading power factor, if the magnitude of the phase angle ***f*** is high, the magnitude of $I_2X_{e2}\sin\phi$ may become greater than that of $I_2R_{e2}\cos\phi$. The regulation then becomes negative. It means that on increasing the load the terminal voltage increases.

Condition for Maximum Regulation

We can derive the condition for maximum regulation (the worst case) using Eq. 13.20. Maximum value of regulation occurs when the voltage drop is maximum (for which we use + sign). Therefore, the condition of maximum regulation can be obtained by differentiating Eq. 13.20 with respect to the phase angle ***f*** and equating it to zero,

$$\frac{d}{d\phi}(I_2R_{e2}\cos\phi + I_2X_{e2}\sin\phi) = 0 \quad \Rightarrow \quad (-I_2R_{e2}\sin\phi + I_2X_{e2}\cos\phi) = 0$$

or $$\tan\phi = \frac{X_{e2}}{R_{e2}}$$

...(13.27)

Example 13.11 A single-phase, 40-kVA, 6600-V/250-V, transformer has primary and secondary resistances R_1 = 10 Ω and R_2 = 0.02 Ω, respectively. The equivalent leakage reactance as referred to the primary is 35 Ω. Find the full-load regulation for the load power factor of (*a*) unity, (*b*) 0.8 lagging, and (*c*) 0.8 leading.

Solution **:** Given : R_1 = 10 Ω; R_2 = 0.02 Ω; X_{e1} = 35 Ω

$$\text{the turns-ratio}, K = \frac{250}{6600} = 0.0379$$

$$\text{the full-load current}, I_2 = \frac{40\,000}{250} = 160 \text{ A}$$

$\therefore \qquad R_{e2} = K^2 R_1 + R_2 = (0.0379)^2 \times 10 + 0.02 = 0.0343\ \Omega$

and $\qquad X_{e2} = K^2 X_{e1} = (0.0379)^2 \times 35 = 0.0502\ \Omega$

(*a*) For power factor, $\cos\phi = 1$; $\sin\phi = 0$. Hence,

$$\therefore \quad \%\ \text{Regulation} = \frac{I_2 R_{e2}\cos\phi + I_2 X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 1 + 0}{250} \times 100 = \mathbf{2.195\ \%}$$

(*b*) For power factor, $\cos\phi = 0.8$ (lagging, ϕ positive); $\sin\phi = \sqrt{1-\cos^2\phi} = 0.6$. Hence,

$$\therefore \quad \%\ \text{Regulation} = \frac{I_2 R_{e2}\cos\phi + I_2 X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 0.8 + 160 \times 0.0502 \times 0.6}{250} \times 100 = \mathbf{3.68\ \%}$$

(*c*) For power factor, $\cos\phi = 0.8$ (leading, ϕ negative); $\sin\phi = -0.6$. Hence,

$$\therefore \quad \%\ \text{Regulation} = \frac{I_2 R_{e2}\cos\phi - I_2 X_{e2}\sin\phi}{V_{2(0)}} \times 100$$

$$= \frac{160 \times 0.0343 \times 0.8 - 160 \times 0.0502 \times 0.6}{250} \times 100 = \mathbf{-0.172\ \%}$$

13.10 EFFICIENCY OF A TRANSFORMER

Like any other machine, the efficiency of a transformer is defined as

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Power output}}{\text{Power output +Power loss}} = \frac{P_o}{P_o + P_l}$$

...(13.26)

There are two types of losses in a transformer :

(*a*) *Copper losses* or *I^2R losses* in the primary and secondary windings, given as

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{e1} = I_2^2 R_{e2}$$

...(13.27)

The copper losses are variable with current. Let us see by what factor the copper losses get reduced when the load on the transformer decreases. We know that the output power is given as

$$P_o = VI\cos\phi = VI \times pf \qquad \Rightarrow \qquad VI = \frac{P_o}{pf}$$

...(13.28)

Thus, for a given load we can find the volt-ampere (VA) of the transformer. Assuming the voltage to remain constant, the current is proportional to the VA of the transformer. Since the copper losses are proportional to the square of current, the value of the copper losses for a given load (and hence for given VA) can be calculated from

$$P_c = \left(\frac{\text{VA}}{\text{VA}_{\text{FL}}}\right)^2 P_{c(\text{FL})}$$

...(13.29)

(*b*) *Iron losses* or *core losses*, due to hysteresis and eddy-currents, given by Eqs. 13.7 and 13.8, respectively. That is, $P_i = P_h + P_e$. Since the maximum value of the flux Φ_m in a normal transformer does not vary more than about 2 % between no load and full load, it is usual to assume *the core losses constant at all loads*.

The efficiency of a transformer can thus be written as

$$\eta = \frac{P_o}{P_o + P_l} = \frac{P_o}{P_o + P_c + P_i} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e2} + P_i}$$

...(13.30)

Condition for Maximum Efficiency

Assuming that the transformer is operating at a constant terminal voltage and a constant power factor, we are interested to know for what load (i.e., what value of I_2) the efficiency becomes maximum. To determine this, we first divide the numerator and denominator of Eq. 13.30 by I_2, to get

$$\eta = \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + I_2 R_{e2} + P_i / I_2}$$

Obviously, the efficiency will be maximum when the denominator of the above equation is minimum, for which we must have

$$\frac{d}{dI_2}(V_2 \cos\phi_2 + I_2 R_{e2} + P_i / I_2) = 0 \qquad \text{or} \qquad R_{e2} - \frac{P_i}{I_2^2} = 0$$

$$\text{or} \qquad I_2^2 R_{e2} = P_i \qquad \text{or} \qquad P_c = P_i \qquad ...(13.31)$$

Thus, the efficiency at a given terminal voltage and load power factor is maximum for such a load current I_2 which makes the variable losses (copper losses) equal to the constant losses (iron losses).

All-day Efficiency

The efficiency defined in Eq. 13.26 is called ***commercial efficiency.*** This efficiency is not of much use in case of a distribution transformer. The primary of a distribution transformer remains energized all the time. But the load on the secondary is intermittent and variable during the day. It means that the core losses occur throughout the day, but the copper losses occur only when the transformer is loaded. Such transformers, therefore, are designed to have minimum core losses. This gives them better ***all-day efficiency***, defined below.

$$\eta_{\text{all-day}} = \frac{\text{Output energy (in kW h) in a cycle of 24 hours}}{\text{Total input energy (in kW h)}} \qquad ...(13.32)$$

Example 13.12 For a single-phase, 150-kVA, transformer, the required no-load voltage ratio is 5000-V/250-V. Find

(*a*) the number of turns in each winding for a maximum core flux of 0.06 Wb.
(*b*) the efficiency at half rated kVA, and unity power factor.
(*c*) the efficiency at full load, and 0.8 power factor lagging.
(*d*) the kVA load for maximum efficiency, if the full-load copper losses are 1800 W and core losses are 1500 W.

Solution **:** (*a*) Using the emf equation, we have

$$E_2 = 4.44 f N_2 \Phi_m \quad \Rightarrow \quad N_2 = \frac{E_2}{4.44 f \Phi_m} = \frac{250}{4.44 \times 50 \times 0.06} = 18.8 \text{ (say, } \mathbf{19\ turns}\text{)}$$

and
$$N_1 = \frac{E_1}{E_2} N_2 = \frac{5000}{250} \times 19 = \mathbf{380\ turns}$$

(*b*) At half rated kVA, the current is half the full-load current, and hence the output power too reduces by 0.5. Thus,

Output power, $P_o = 0.5 \times (\text{kVA}) \times (\text{power factor}) = 0.5 \times 150 \times 1 = 75\ \text{kW}$

Since copper losses is proportional to the square of current, we have

Copper losses, $P_c = (0.5)^2 \times (\text{full-load copper loss}) = (0.5)^2 \times 1800\ \text{W} = 0.45\ \text{kW}$

Iron losses being fixed, we have

Iron losses, $P_i = 1500\ \text{W} = 1.5\ \text{kW}$

$$\therefore \qquad \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{75}{75 + 0.45 + 1.5} \times 100 = \mathbf{97.47\ \%}$$

(*c*) At full-load and 0.8 power factor,

Output power, $P_o = \text{(kVA)} \times \text{(power factor)} = 150 \times 0.8 = 120 \text{ kW}$
Copper losses, P_c = 1800 W = 1.8 kW
Iron losses, P_i = 1500 W = 1.5 kW

$$\therefore \qquad \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{120}{120 + 1.8 + 1.5} \times 100 = \mathbf{97.3\ \%}$$

(*d*) Let *x* be the fraction of full-load kVA at which the efficiency becomes maximum (that is, when the variable copper losses are equal to the fixed iron losses). Then

$$P_c = P_i \quad \text{or} \qquad x^2 \times 1800 = 1500 \qquad x = \sqrt{1500/1800} = 0.913$$

Therefore, the load kVA under the condition of maximum efficiency is

$$\text{Load kVA} = \text{(Full-load kVA)} \times x = 150 \times 0.913 = \mathbf{137\ kVA}$$

Example 13.13 For a single-phase, 200-kVA, distribution transformer has full-load copper losses of 3.02 kW and iron losses of 1.6 kW. It has following load distribution over a 24-hour day :

(*a*) 80 kW at unity power factor, for 6 hours.
(*b*) 160 kW at 0.8 power factor (lagging), for 8 hours.
(*c*) no load for the remaining 10 hours.

Determine its all-day efficiency.

Solution **:** (*a*) For 80 kW load at unity power factor (for 6 hours) :

Output energy = $80 \times 6 = 480 \text{ kW h}$

$$\text{kVA} = \frac{P_o}{pf} = \frac{80}{1} = 80\,\text{kVA}$$

Using Eq. 13.29, we have

$$\text{Copper losses, } P_c = \left(\frac{\text{kVA}}{\text{kVA}_{\text{FL}}}\right)^2 P_{c(\text{FL})} = \left(\frac{80}{200}\right)^2 \times (3.02) = 0.4832 \text{ kW}$$

Iron losses, P_i = 1.6 kW
Total losses, $P_l = P_c + P_i$ = 0.4832 kW + 1.6 kW = 2.0832 kW
$\therefore$ Total energy losses in 6 hours $= 2.0832 \times 6 = 12.50 \text{ kW h}$

(*b*) For 160 kW load at 0.8 power factor (for 8 hours) :

Output energy = $160 \times 8 = 1280 \text{ kW h}$

$$\text{kVA} = \frac{P_o}{pf} = \frac{160}{0.8} = 200\,\text{kVA} = \text{kVA}_{\text{FL}}$$

$\therefore$ Copper losses, $P_c = P_{c(\text{FL})} = 3.02 \text{ kW}$
Iron losses, P_i = 1.6 kW
Total losses, $P_l = P_c + P_i$ = 3.02 kW + 1.6 kW = 4.62 kW
$\therefore$ Total energy losses in 8 hours $= 4.62 \times 8 = 36.96 \text{ kW h}$

(*c*) For the no-load period of 10 hours :

Output energy = 0
Copper losses, P_c = 0
Iron losses, P_i = 1.6 kW

Total losses, $P_l = P_c + P_i = 0 + 1.6 = 1.6$ kW

$\therefore$ Total energy losses in 10 hours $= 1.6 \times 10 = 16$ kW h

For 24-hour period :

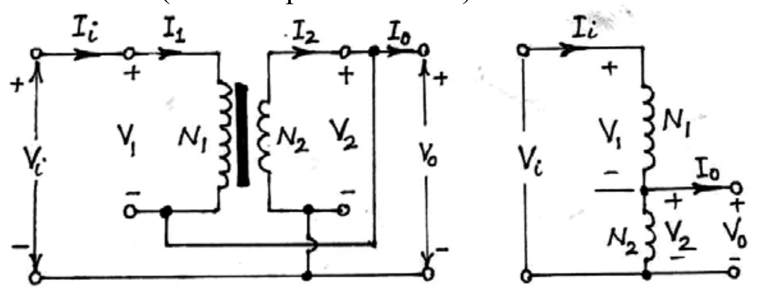
Total output energy, $W_o = 480 + 1280 = 1760$ kW h

Total energy losses, $W_l = 12.50 + 36.96 + 16 = 65.46$ kW h

$$\therefore \quad \text{All-day efficiency, } \eta_{\text{all-day}} = \frac{W_o}{W_o + W_l} \times 100 = \frac{1760}{1760 + 65.46} \times 100 = \mathbf{96.41\%}$$

13.11 AUTOTRANSFORMERS

An autotransformer is a special transformer-connection that is useful in power systems, motor starters, variable ac sources, and other applications. Figure 13.20*a* shows the special connection with the primary and secondary drawn in the usual position. Figure 13.20*b* shows the autotransformer (in the step-down mode) drawn in a manner that clarifies its function.



(*a*) Special connection of a transformer. (*b*) Redrawn in the standard way.

Fig. 13.20 A two-winding transformer converted into an autotransformer.

Note that the primary and secondary windings are connected in series for the new primary; the secondary is the new secondary. Also, the primary and secondary are not electrically isolated from each other. Obviously, the voltage $V_2 = V_o$. From Fig. 13.20*b*, it is obvious that

$$V_i = V_1 + V_2 = \frac{N_1}{N_2} V_2 + V_2 = \frac{N_1 + N_2}{N_2} V_o$$

or

$$V_o = \frac{N_2}{N_1 + N_2} V_i \qquad \text{...(13.33)}$$

Hence, the new turns-ratio becomes $N_2 : (N_1 + N_2)$. Thus, we find that an autotransformer works like a ***potential divider*** circuit, except that numbers of turns are to be used instead of resistances.

The apparent power rating (kVA rating) of the transformer is increased by the special connection, as is illustrated in Example 13.14, given below.

Example 13.14 A single phase, 12-kVA, 120-V/120-V transformer is connected as an autotransformer to make a 240-V/120-V transformer. What is the apparent power rating of the autotransformer ?

Solution **:** Figure 13.21 shows the transformer connection with rated voltage and current. The current rating on both primary and secondary windings is

$$I_1 = I_2 = \frac{12 \text{ kVA}}{120 \text{ V}} = 100 \text{ A}$$

In the autotransformer mode, the input apparent power is $240 \times 100 = \mathbf{24\ kVA}$, and the output apparent power is $120 \times 200 = \mathbf{24\ kVA}$. Thus, the apparent power capacity of the 12-kVA transformer is doubled by the autotransformer connection. In effect, half the apparent power is transformed and half is conducted directly to the secondary side.

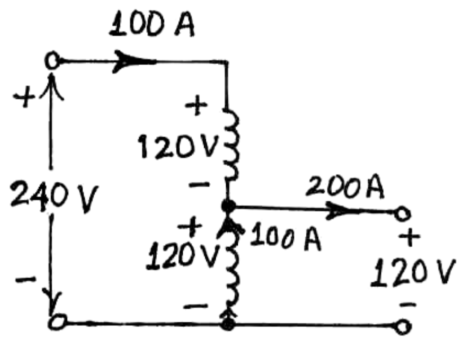


Fig. 13.21

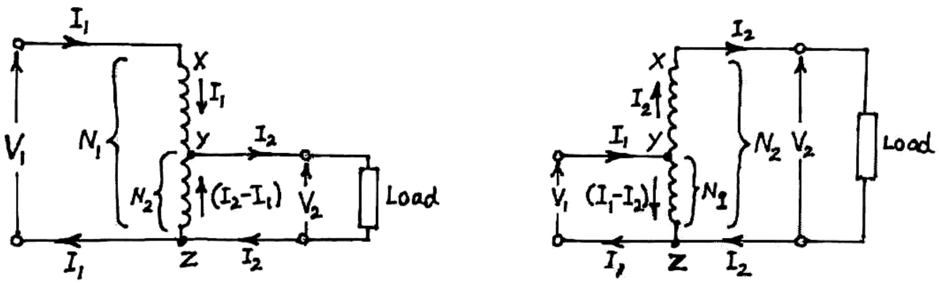
Practical Autotransformers

In practice, an autotransformer is made by winding a single coil XZ of N_1 turns on a magnetic core, as shown in Fig. 13.22*a*. This winding is excited by a voltage V_1, so that a flux is set up in the core and an emf E_1 is induced in the winding. The winding is tapped at point Y, such that there are N_2 turns between Y and Z. An emf E_2 exists between the terminals Y and Z such that the ratio $E_2/E_1 = N_2/N_1 = K$ becomes the turns-ratio of the autotransformer. For ideal conditions, the turns-ratio is given as

$$K = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

If a load is connected across terminals Y and Z, a load-current I_2 flows due to the emf E_2. The mmf due to current I_2 is counterbalanced by the mmf due to current I_1.

The portion YZ of the winding is common to both the primary and secondary sides. Hence, it is called ***common winding***. The portion XY is called ***series winding***. In ***variacs*** (***variable autotransformers***), point Y is made a sliding contact so as to give a variable output voltage.



(*a*) A step-down autotransformer. (*b*) A step-up autotransformer.

Fig. 13.22 In practice, a single winding is used in making an autotransformer.

(*a*) Step-down Autotransformer : In Fig. 13.22*a*, $N_2 < N_1$, hence $V_2 < V_1$. Hence, this arrangement is a step-down autotransformer. The output current I_2 is greater than the input current I_1. The distribution of currents in the winding is shown in the figure.

The apparent power (volt-amperes) on the two sides must be the same $V_1 I_1 = V_2 I_2$. We can write the volt-amperes delivered to the load as

$$V_2 I_2 = V_2 I_1 + V_2 (I_2 - I_1)$$

The part $V_2 I_1$ represents the volt-amperes conductively transferred form ac source to the load through the winding portion *XY*. Only the remaining part $V_2(I_2 - I_1)$ of the total volt-amperes is inductively transferred form ac source to the load through the winding portion *YZ*.

(*b*) Step-up Autotransformer : In Fig. 13.22*b*, $N_2 > N_1$, hence $V_2 > V_1$. Hence, this arrangement is a step-up autotransformer. The output current I_2 is less than the input current I_1. The volt-amperes drawn from the ac source at the input of the autotransformer can be written as

$$V_1 I_1 = V_1 I_2 + V_1 (I_1 - I_2)$$

The part $V_1 I_2$ represents the volt-amperes conductively transferred to the load through the winding portion *XY*. The remaining part $V_1(I_1 - I_2)$ is inductively transferred to the load through the winding portion *YZ*.

Saving in Copper

For the same voltage ratio and capacity (volt-ampere rating), an autotransformer needs much less copper (or aluminium) material compared to a two-winding transformer. The cross-sectional area of a conductor is proportional to the current carried by it, and its length is proportional to the number of turns. Therefore,

$$\text{Weight of copper in a winding} \propto NI = kNI$$

For a two-winding transformer :

$$\text{Weight of copper in primary} = kN_1 I_1$$

$$\text{Weight of copper in secondary} = kN_2 I_2$$

$$\text{Total weight of copper} = k(N_1 I_1 + N_2 I_2)$$

For an autotransformer (see Fig. 13.22*a*) :

The portion *XY* of the winding has $N_1 - N_2$ turns and carries current I_1. The portion *YZ* of the winding has N_2 turns and carries current $I_2 - I_1$. Therefore,

$$\text{Weight of copper in portion } XY = k(N_1 - N_2)I_1$$

$$\text{Weight of copper in portion } YZ = kN_2(I_2 - I_1)$$

$$\text{Total weight of copper } = k(N_1 - N_2)I_1 + kN_2(I_2 - I_1) = k[(N_1 - 2N_2)I_1 + N_2 I_2]$$

Therefore, the ratio of copper- weights for the two cases is

$$\frac{k[(N_1 - 2N_2)I_1 + N_2 I_2]}{k(N_1 I_1 + N_2 I_2)} = \frac{\left[1 - 2\left(\frac{N_2}{N_1}\right)\right]\left(\frac{I_1}{I_2}\right) + \left(\frac{N_2}{N_1}\right)}{\left(\frac{I_1}{I_2}\right) + \left(\frac{N_2}{N_1}\right)} = \frac{[1 - 2K]K + K}{K + K} = 1 - K$$

Evidently, the saving is large if K is close to unity. A unity transformation ratio means that no copper is needed at all for the autotransformer. The winding can be removed all together. The volt-amperes are conductively transformed directly to the load !

Disadvantages : The use of autotransformer has following disadvantages :

1. No electrical isolation between the two sides.
2. Should an open-circuit occur between points *Y* and *Z*, full primary high voltage appears across the load.
3. The short-circuit current is larger than that in two-winding transformer.

Applications : The autotransformers find applications in following areas :

1. Boosting or buckling of supply voltage by a small amount.
2. Starting of ac machines, where the voltage is raised in two or more steps.
3. Continuously varying ac supply as in variacs.

13.12 THREE-PHASE TRANSFORMERS

Modern large transformers are usually of the three-phase core type, schematically shown in fig. 13.23. Three similar limbs are connected by top and bottom yokes. Each limb has primary and secondary windings arranged concentrically. In Fig. 13.23, the primary is shown star-connected and the secondary delta-connected. In actual practice, the windings may be connected star/delta, delta/star, star/star or delta/delta, depending upon the conditions under which the transformer is to be used.

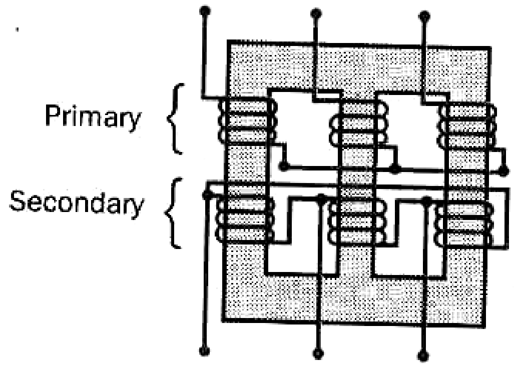


Fig. 13.23 Three-phase core-type star/delta connected transformer

---

Example 13.15 A three-phase, 50-Hz transformer has 840 turns on the primary and 72 turns on the secondary winding. The supply voltage is 3300 V. Determine the secondary line voltage on no load when the windings are connected (*a*) star/delta, (*b*) delta/star.

Solution **:** (*a*) For star/delta connection :

$$\text{Primary phase voltage, } V_{ph1} = \frac{V_{L1}}{\sqrt{3}} = \frac{3300}{\sqrt{3}} = 1905.3 \text{ V}$$

$$\text{Secondary phase voltage, } V_{ph2} = 1905.3 \times \frac{72}{840} = 163.3 \text{ V}$$

$$\therefore \quad \text{Secondary line voltage, } V_{L2} = V_{ph2} = \mathbf{163.3\ V}$$

(*b*) For delta/star connection :

$$\text{Primary phase voltage, } V_{ph1} = V_{L1} = 3300 \text{ V}$$

$$\text{Secondary phase voltage, } V_{ph2} = 3300 \times \frac{72}{840} = 283 \text{ V}$$

$$\therefore \quad \text{Secondary line voltage, } V_{L2} = V_{ph2} \times \sqrt{3} = 283 \times \sqrt{3} = \mathbf{490\ V}$$

13.13 TRANSFORMER TESTING

There are two simple tests that may be conducted on a transformer to determine its efficiency and regulation. These are called open-circuit test and short-circuit test. The power required to carry out these tests is very small compared with the full-load output of the transformer.

Open-Circuit Test

This test determines the no load current and the parameters of the exciting circuit of the transformer. The transformer is connected as shown in Fig. 13.24. Generally, the low voltage (LV) side is supplied rated voltage and frequency through an autotransformer (also called a ***variac***). The high voltage (HV) side is left open. The ratio of the voltmeter readings, V_2/ V_1, gives the transformation ratio of the transformer. The reading of ammeter A, I_o, gives the no-load current I_0, and its reading is a check on the magnetic quality of the ferromagnetic core and joints.

The primary current on no load is usually less than 5 per cent of the full-load current. Hence, the I^2R loss on no load is less than 1/400 of the primary I^2R loss of full load and is therefore negligible compared with the core loss. Hence the wattmeter reading, W_o, can be assumed to give the core loss of the transformer.

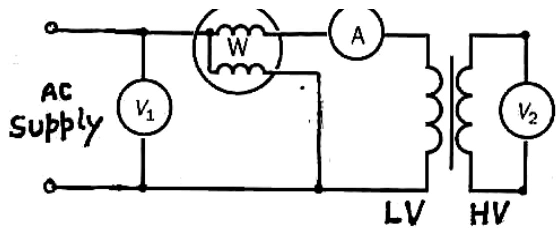


Fig. 13.24 Open-circuit test on a transformer.

Various parameters of the transformer can be calculated as under.

$$P_i = W_o; \qquad I_0 = I_o; \qquad I_w = \frac{W_o}{V_1}; \qquad I_m = \sqrt{I_0^2 - I_w^2}; \qquad R_0 = \frac{V_1}{I_w}; \qquad X_0 = \frac{V_1}{I_m}$$

Short-Circuit Test

This test determines the equivalent resistance and leakage reactance of the transformer. The connections are made as shown in Fig. 13.25. Generally, the LV side of the transformer is short-circuited through a suitable ammeter A_2. A *low* voltage is applied to the primary (HV) side. This voltage is adjusted with the help of a variac so as to circulate full-load currents in the primary and secondary circuits. The reading of ammeter A_1, I_{sc}, gives the full-load current in the primary winding. On the other hand, the core loss is negligibly small, since the applied voltage (and hence the flux) is less than about one-twentieth of the rated voltage. Hence the wattmeter reading, W_{sc}, gives the copper loss (P_c).

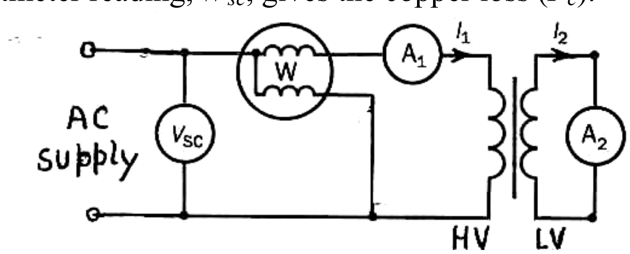


Fig. 13.25 Short-circuit test on a transformer.

Equivalent resistance, reactance and impedance as referred to the primary side can be calculated as under.

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2}; \qquad Z_{e1} = \frac{V_{sc}}{I_{sc}}; \qquad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2}$$

Example 13.16 A single-phase, 50-Hz, 12-kVA, 200-V/400-V transformer gives the following test results :

(i) Open-circuit test (with HV winding open) : 200 V, 1.3 A, 120 W
(ii) Short-circuit test (with LV winding short-circuited) : 22 V, 30 A, 200 W

Calculate (a) the magnetizing current and the core-loss current, and (b) the parameters of equivalent circuit as referred to the low voltage winding.

Solution **:** (a) The wattmeter reading, 120 W, in the open-circuit test gives the core loss. Therefore, the core-loss current is given as

$$I_w = \frac{W_o}{V_1} = \frac{120\ \text{W}}{200\ \text{V}} = \mathbf{0.6\ A}$$

Hence the magnetizing current is given as

$$I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1.3)^2 - (0.6)^2} = \mathbf{1.15\ A}$$

(b) The parameters of the exciting circuit are given by the open-circuit test, as

$$R_0 = \frac{V_1}{I_w} = \frac{200\ \text{V}}{0.6\ \text{A}} = \mathbf{333\ \Omega} \qquad \text{and} \qquad X_0 = \frac{V_1}{I_m} = \frac{200\ \text{V}}{1.15\ \text{A}} = \mathbf{174\ \Omega}$$

The short-circuit test gives the equivalent resistance and reactance as referred to the primary side (high voltage winding). From the given specification of the transformer,

The transformation ratio, $K = \dfrac{V_2}{V_1} = \dfrac{200\ \text{V}}{400\ \text{V}} = \dfrac{1}{2}$

The rated full-load current in the high voltage side, $I_{\text{FL}} = \dfrac{12\ \text{kVA}}{400\ \text{V}} = 30\ \text{A}$

This confirms that the short-circuit test has been done at the rated full-load. Thus,

$$R_{e1} = \frac{W_{sc}}{I_{sc}^2} = \frac{200\ \text{W}}{(30\ \text{A})^2} = 0.222\ \Omega \qquad \text{and} \qquad Z_{e1} = \frac{V_{sc}}{I_{sc}} = \frac{22\ \text{V}}{30\ \text{A}} = 0.733\ \Omega$$

$$\therefore \qquad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(0.733)^2 - (0.222)^2} = \mathbf{0.699\ \Omega}$$

We can now determine the equivalent resistance and reactance as referred to the secondary side (low voltage winding), as

$$R_{e2} = K^2 R_{e1} = \left(\frac{1}{2}\right)^2 \times 0.222 = \mathbf{0.055\ \Omega}$$

and $$X_{e2} = K^2 X_{e1} = \left(\frac{1}{2}\right)^2 \times 0.699 = \mathbf{0.175\ \Omega}$$

ADDITIONAL SOLVED EXAMPLES

Example 13.17 A 25-kVA transformer has 500 turns on the primary and 40 turns on the secondary winding. The primary winding is connected to a 3-kV, 50-Hz ac source. Calculate (*a*) the secondary emf, (*b*) the primary and secondary currents on full load, and (*c*) the maximum flux in the core.

***Solution* :** (*a*) The transformation ratio is given as

$$K = \frac{N_2}{N_1} = \frac{40}{500} = 0.08$$

$\therefore$ Secondary emf, $E_2 = KE_1 \approx KV_1 = 0.08 \times 3000 = \mathbf{240\ V}$

(*b*) The primary and secondary full-load currents are given as

$$I_1 = \frac{\text{kVA}}{V_1} = \frac{25\ \text{kVA}}{3\ \text{kV}} = \mathbf{8.33\ A} \qquad \text{and} \qquad I_2 = \frac{I_1}{K} = \frac{8.33\ \text{A}}{0.08} = \mathbf{104.125\ A}$$

(*c*) The maximum flux in the core is given by emf equation,

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{3000}{4.44 \times 50 \times 500} = \mathbf{0.027\ Wb}$$

Example 13.18 A 230-V, 50-Hz, single-phase transformer has 50 turns on its primary. It is required to operate with a maximum flux density of 1 T. Calculate the active cross-sectional area of the core. Find suitable dimensions for a square core.

***Solution* :** From the emf equation, we have

$$\Phi_m = \frac{E_1}{4.44 f N_1} = \frac{230}{4.44 \times 50 \times 50} = 0.02072\ \text{Wb}$$

$\therefore$ Active core area, $A = \dfrac{\Phi_m}{B_m} = \dfrac{0.02072}{1} = 0.02072\ \text{m}^2 = \mathbf{207.2\ cm^2}$

Due to the insulation of laminations from each other, the gross area is about 10 % greater than the active area. Thus,

$$\text{Gross area} = 207.2 \times 1.1 = 227.92\ \text{cm}^2$$

If the core has square cross-section, the side of the square is

$$a = \sqrt{227.92} = 15.09 \approx \mathbf{15\ cm}$$

Example 13.19 A single-phase transformer has a core whose cross-sectional area is 150 cm^2, operates at a maximum flux density of 1.1 Wb/m^2 from a 50-Hz supply. If the secondary winding has 66 turns, determine the output in kVA when connected to a load of 4-Ω impedance. Neglect any voltage drop in the transformer.

***Solution*:** $\Phi_m = B_m A = 1.1 \times 0.015 = 0.0165\ \text{Wb}$.

$$E_2 = 4.44 \Phi_m f N_2 = 4.44 \times 0.0165 \times 50 \times 66 = 240\,\text{V} = V_2$$

(Neglecting the voltage drops)

Secondary current, $I_2 = \dfrac{V_2}{Z_L} = \dfrac{240}{4} = 60\ \text{A}$

$\therefore$ output in kVA $= \dfrac{V_2 I_2}{1000} = \dfrac{240 \times 60}{1000} = \mathbf{14.4\ kVA}$

Example 13.20 A 11-kV/400-V distribution transformer takes a no-load primary current of 1 A at a power factor of 0.24 lagging. Find (*i*) the core-loss current, (*ii*) the magnetizing current, and (*iii*) the iron loss.

Solution **:** (*i*) The core-loss current, $I_w = I_0 \cos\phi_0 = 1.0 \times 0.24 = \mathbf{0.24\ A}$

(*ii*) The magnetizing current, $I_m = \sqrt{I_0^2 - I_w^2} = \sqrt{(1)^2 - (0.24)^2} = \mathbf{0.971\,A}$

(*iii*) The iron loss, $P_i = V_1 I_0 \cos\phi_0 = 11000 \times 1.0 \times 0.24 = \mathbf{2640\ W}$

Example 13.21 A two-winding, step-down transformer has a turns-ratio (N_2/N_1) of 0.5. The primary winding resistance and reactance are 2.5 Ω and 6 Ω, whereas the secondary winding resistance and reactance are 0.25 Ω and 1 Ω, respectively. Its magnetizing current and core-loss current are 51.5 mA and 20.6 mA, respectively. While in operation, the output voltage for a load of $25\angle 30°\,\Omega$ is found to be 50 V. Determine the supply voltage, the current drawn from the supply and the power factor.

Solution **:** Given : $\mathbf{Z}_1 = (2.5 + j6)\ \Omega = 6.5\angle 67.38°\ \Omega\,;\mathbf{Z}_2 = (0.25 + j1)\ \Omega = 1.03\angle 75.96°\ \Omega$

Let us take V_2 as the reference phasor, i.e., $\mathbf{V}_2 = 50\angle 0°\ \text{V}$.

$$\therefore\ \mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_L} = \frac{50\angle 0°\ \text{V}}{25\angle 30°\ \Omega} = 2\angle -30°\ \text{A}$$

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_2 = (50 + j0) + (2\angle -30°)(1.03\angle 75.96°)$$
$$= (50 + j0) + (1.432 + j1.48) = (51.432 + j1.48)\ \text{V} = 51.45\angle 1.65°\ \text{V}$$

$$\mathbf{E}_1 = \mathbf{E}_2\left(\frac{N_1}{N_2}\right) = (51.45\angle 1.65°) \times 2 = 102.9\angle 1.65°\ \text{V};\quad -\mathbf{E}_1 = 102.9\angle 181.65°\ \text{V}$$

$$\mathbf{I}_1^{'} = -\mathbf{I}_2\left(\frac{N_2}{N_1}\right) = -(2\angle -30°) \times 0.5 = 1\angle 150°\ \text{A}$$

As seen from the phasor diagram of Fig. 13.5*b*, $\mathbf{I}_m$ lags $-\mathbf{E}_1$ by 90° and $\mathbf{I}_w$ is in phase with $-\mathbf{E}_1$.

$$\therefore\ \mathbf{I}_m = 0.0515\angle(181.65° - 90°) = 0.0515\angle 91.65°\,\text{A}\ \text{and}\qquad \mathbf{I}_w = 0.0206\angle 181.65°\ \text{A}$$

$$\therefore\ \mathbf{I}_1 = \mathbf{I}_1^{'} + \mathbf{I}_m + \mathbf{I}_w = (1\angle 150° + 0.0515\angle 91.65° + 0.0206\angle 181.65°)$$
$$= (-0.866 + j0.5) + (-0.00148 + j0.05148) + (-0.020 - j0.000593)$$
$$= -0.88748 + j0.550887 = \mathbf{1.044556\angle 148.17°\,A}$$

$$\mathbf{V}_1 = -\mathbf{E}_1 + \mathbf{I}_1\mathbf{Z}_1 = 102.9\angle 181.65° + (1.044556\angle 148.17°)(6.5\angle 67.38°)$$
$$= 102.9\angle 181.65° + 6.7896\angle 215.55° = (-102.857 - j2.9629) + (-5.524 - j3.947)$$
$$= -108.381 - j6.9099 = \mathbf{108.601\angle 183.647°\ V}$$

The angle between $\mathbf{V}_1$ and $\mathbf{I}_1 = 183.647° - 148.17° = 35.477°$

$\therefore$ Primary power factor $= \cos(35.477°) = \mathbf{0.814\ lagging}$

Example 13.22 A single-phase, step-down transformer has turns-ratio of 4. The resistance and reactance of the primary winding are 1.4 Ω and 5.5 Ω, respectively, and those of the secondary winding are 0.06 Ω and 0.04 Ω, respectively. If the LV winding is short-circuited and the HV winding is connected to a 24-V, 50-Hz source, determine (*a*) the current in the LV winding, (*b*) the copper loss in the transformer, and (*c*) the power factor. Ignore the no-load current I_0.

Solution **:** Given : $K = (1/4) = 0.25$. The equivalent resistance and reactance as referred to HV side are given as

$$R_{e1} = R_1 + R_2 / K^2 = 1.4 + (0.06)/(0.25)^2 = 2.36\ \Omega$$

and $$X_{e1} = X_1 + X_2 / K^2 = 5.5 + (0.04)/(0.25)^2 = 6.14\ \Omega$$

$$\therefore \quad Z_{e1} = \sqrt{R_{e1}^2 + X_{e1}^2} = \sqrt{(2.36)^2 + (6.14)^2} = 6.578\ \Omega$$

(*a*) The current in the HV winding,

$$I_{SC} = \frac{V_{SC}}{Z_{e1}} = \frac{24}{6.578} = 3.648\ \text{A}$$

Therefore, ignoring I_0, we have $I_1 = I_1' = 3.648\ \mathbf{A}$

Thus, the current in the LV winding, $I_2 = I_1' / K = 3.648/(0.25) = \mathbf{14.592\ A}$

(*b*) The copper loss in transformer $= I_1^2 R_{e1} = (3.648)^2 \times 2.36 = \mathbf{31.4\ W}$

(*c*) The power factor $= \cos\phi_1 = \dfrac{P}{V_1 I_1} = \dfrac{31.4}{24 \times 3.648} = \mathbf{0.3586}$

Example 13.23 The results of test conducted on a single-phase, 20-kVA, 2200-V/220-V, 50-Hz transformer are given as under :

OC test (HV winding open) : 220 V, 4.2 A, 148 W.
SC test (LV winding short-circuited) : 86 V, 10.5 A, 360 W.

Determine (*a*) the regulation and efficiency at 0.8 pf lagging at full load, and (*b*) the power factor on short-circuit.

Solution **:** (*a*) From the short-circuit test, we have V_{SC} = 86 V, I_{SC} = 10.5 A, P_{SC} = 360 W. Then,

$$Z_{e1} = \frac{V_{SC}}{I_{SC}} = \frac{86\ \text{V}}{10.5\ \text{A}} = 8.19\ \Omega; \quad R_{e1} = \frac{P_{SC}}{I_{SC}^2} = \frac{360\ \text{W}}{(10.5\ \text{A})^2} = 3.265\ \Omega$$

$$\therefore \quad X_{e1} = \sqrt{Z_{e1}^2 - R_{e1}^2} = \sqrt{(8.19)^2 - (3.265)^2} = 7.51\ \Omega$$

The full-load primary current, $I_1 = \dfrac{\text{VA}}{V_1} = \dfrac{20\,000}{2200} = 9.09\ \text{A}$

Since power factor is 0.8, we have $\cos\phi = 0.8$, and $\sin\phi = \sin[\cos^{-1} 0.8] = 0.6$

Using Eq. 13.18, we can determine regulation in terms of quantities referred to the primary side,

$$\%\ \text{Regulation} = \frac{I_1 (R_{e1}\cos\phi + X_{e1}\sin\phi)}{V_1} \times 100$$

$$= \frac{9.09(3.265 \times 0.8 + 7.51 \times 0.6)}{2200} \times 100 = \mathbf{2.94\ \%}$$

The short-circuit test has been conducted for a short-circuit primary current of 10.5 A, whereas full-load primary current is only 9.09 A. Therefore, the full-load copper loss is given as

$$\text{Full-load copper loss } = \left(\frac{9.09}{10.5}\right)^2 \times 360 = 269.58 \text{ W}$$

The open-circuit test gives the core loss. Hence, P_i = 148 W.

The full-load output power, $P_o = VA \times pf = 20\,000 \times 0.8 = 16\,000$ W

$$\therefore \quad \% \text{ Efficiency, } \eta = \frac{P_o}{P_o + P_c + P_i} \times 100 = \frac{16\,000}{16\,000 + 269.8 + 148} \times 100 = \mathbf{97.45\ \%}$$

(*b*) The power factor on short-circuit is given as

$$pf = \cos\phi_{SC} = \frac{R_{e1}}{Z_{e1}} = \frac{3.265}{8.19} = \mathbf{0.399\,(lagging)}$$

Example 13.24 A single-phase, 200-kVA transformer has an efficiency of 98 % at full-load. If the maximum efficiency occurs at three-quarter of full-load, calculate the efficiency at half of full-load current, assuming the power factor to be 0.8 lagging.

Solution **:** At power factor of 0.8, the full-load output power,

$$P_{out} = (\text{kVA}) \times (pf) = (200\,\text{kVA}) \times 0.8 = 160\,\text{kW}$$

Since the efficiency is only 98 %, the input power, $P_{in} = \frac{P_{out}}{\eta} = \frac{160 \text{ kW}}{0.98} = 163.26\,\text{kW}$

Therefore, total losses (copper loss and iron loss) on full load is

$$P_c + P_i = P_{in} - P_{out} = 163.26 - 160 = 3.26\,\text{kW} \qquad \ldots(i)$$

We know that maximum efficiency occurs when the variable loss (copper loss) equals the fixed loss (iron loss). Hence, we must have

$$\left(\frac{3}{4}\right)^2 P_c = P_i \qquad \ldots(ii)$$

Solving Eqs (*i*) and (*ii*), we get $P_c = 2.09$ kW and $P_i = 1.17$ kW

Now, at half load, the total loss is given as

$$\text{Total losses, } P_l = \left(\frac{1}{2}\right)^2 P_c + P_i = \left(\frac{1}{2}\right)^2 \times 2.09 + 1.17 = 1.69 \text{ kW}$$

$$\therefore \quad \% \text{ Efficiency, } \eta_{\text{half-load}} = \frac{\text{Output power}}{\text{Output power + Total losses}} \times 100 = \frac{(160/2)}{(160/2) + 1.69} = \mathbf{97.93\ \%}$$

Example 13.25 A single-phase, 150-kVA, 5000-V/250-V transformer has the full-load copper losses of 1.8 kW and core losses of 1.5 kW. Find

(*a*) The number of turns in each winding for a maximum core flux of 60 mWb.
(*b*) The efficiency at full rated kVA, with power factor of 0.8 lagging.
(*c*) The efficiency at half the rated kVA, with unity power factor.
(*d*) The kVA load for maximum efficiency.

Solution **:** (*a*) Since $E = 4.44 fN\Phi_m$, the number of turns on the secondary winding,

$$N_2 = \frac{E_2}{4.44 f\Phi_m} = \frac{250}{4.44\times 50\times 0.06} = \mathbf{19\ turns}$$

$$\therefore \qquad N_1 = N_2\frac{E_1}{E_2} = 19\times\frac{5000}{250} = \mathbf{380\ turns}$$

(*b*) At full rated kVA, the current is also full-load. Therefore,

Power output, $P_o = V_2 I_2 \times \cos\phi = (150\,\text{kVA})\times 0.8 = 120$ kW

Copper losses, $P_c = 1.8$ kW and the iron losses, $P_i = 1.5$ kW

$$\therefore \quad \%\ \text{Efficiency}, \eta = \frac{P_o}{P_o + P_{oc} + P_i}\times 100 = \frac{120}{120+1.8+1.5}\times 100 = \mathbf{97.32\ \%}$$

(*c*) At half the rated kVA, the current is half the full-load current. Thus,

Power output, $P_o = 0.5\times V_2 I_2\times\cos\phi = 0.5\times(150\,\text{kVA})\times 1 = 75$ kW

Copper losses, $P_c = (0.5 I_2)^2 R_{e2} = 0.25 I_2^2 R_{e2} = 0.25\times(1.8\ \text{kW}) = 0.45\,\text{kW}$

Iron losses, $P_i = 1.5$ kW (Iron losses do not change with load)

$$\therefore \quad \%\ \text{Efficiency}, \eta = \frac{P_o}{P_o + P_{oc} + P_i}\times 100 = \frac{75}{75+0.45+1.5}\times 100 = \mathbf{97.47\ \%}$$

(*d*) Let *x* be the fraction of the full-load kVA at which maximum efficiency occurs. Then, according to the condition for maximum efficiency, we should have

$$x^2(\text{copper losses at full load}) = P_i$$

$$\text{or} \quad x^2\times 1.8 = 1.5 \quad \Rightarrow \quad x = \sqrt{\frac{1.5}{1.8}} = 0.913$$

Hence, the required kVA load for maximum efficiency $= (150\ \text{kVA})\times 0.913 = \mathbf{137\ kVA}$

Example 13.26 A single-phase, 50-kVA, 2400-V/240-V, 50-Hz transformer is used to step down the voltage of a distribution system. The low tension voltage is required to be kept constant at 240 V.

(*a*) What load impedance connected to the LV side will be loading the transformer fully at 0.8 power factor lagging ?
(*b*) What is the value of this impedance referred to the high voltage side ?
(*c*) What is the value of the current referred to the high voltage side ?

Solution **:** (*a*) Since the secondary voltage is required to be constant, the secondary current remains the same whatever be the value of the power factor. The full-load secondary current I_2 can be calculated from the kVA ratings,

$$I_2 = \frac{50\,\text{kVA}}{240\,\text{V}} = 208.33 \text{ A}$$

Therefore, the load impedance, $Z_L = \frac{V_2}{I_2} = \frac{240 \text{ V}}{208.33 \text{ A}} = \mathbf{1.152\ \Omega}$

(*b*) Transformation ratio, $K = \frac{V_2}{V_1} = \frac{240}{2400} = 0.1$

The load impedance referred to the primary side,

$$Z_{eq} = Z_L / K^2 = 1.152/(0.1)^2 = \mathbf{115.2\ \Omega}$$

(*c*) The current referred to the high voltage side,

$$I_1' = KI_2 = 0.1 \times 208.33 = \mathbf{20.833\ A}$$

Example 13.27 A single-phase, 10-kVA, 2300-V/230-V, 50-Hz transformer is connected as an autotransformer with LT winding in series with the HT winding as shown in Fig. 13.26. The autotransformer is excited from a 2530-V source, and it is fully loaded such that the rated currents of the windings are not exceeded. Determine (*a*) the current distribution in the windings, (*b*) the kVA output, (*c*) the volt-amperes transferred conductively and inductively from the input to the output, and (*d*) the saving in copper as compared to the two-winding transformer for the same output.

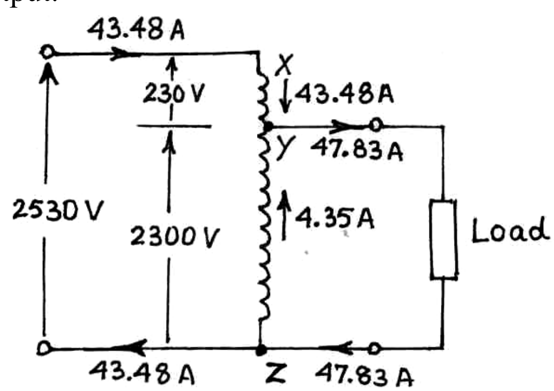


Fig. 13.26 A two-winding transformer connected as autotransformer.

Solution **:**

$$\text{Current rating of HT winding} = \frac{10\,000}{2300} = 4.35 \text{ A}$$

$$\text{Current rating of LT winding} = \frac{10\,000}{230} = 43.48 \text{ A}$$

(*a*) The autotransformer can supply a load current I_2 = 43.48 + 4.35 = 47.83 A at 2300 V, as shown in Fig. 13.26. The input current I_1 = 43.48 A. The current distribution is shown in the figure.

(*b*) The kVA output $= \frac{2300 \times 47.83}{1000} = \mathbf{110\,kVA}$