Solution of Tutorial 9

Q1. A 40KVA single phase transformer has 400 turns on primary and 100 turns on secondary the primary is
connected to 200V, 50Hz supply, Determine
1) The secondary voltage on open circuit.
2) The current flowing through the two windings on full load.
3) The maximum value of flux.
Solution: Tr, rating=40KVA, N_1=400,N_2=100

1) Secondary voltage on open circuit: V_2

Primary induced voltage V_1=200V,

1) Secondary voltage on open circuit: V_2

$$\frac{V_2}{V_1} = \frac{N_1}{N_2}$$

$$V_2 = \frac{N_1}{N_2} * V_1 = 2000 * \frac{100}{400} = 500volts$$

2)Primary Current (I_1): at full load,

$$I_1 = \frac{KVA*100}{V_1} = 40 * \frac{1000}{200} = 20A$$

$$secondary\ current\ at\ full\ load\ I_2 = \frac{KVA*100}{V_2} = 40 * \frac{1000}{500} = 80A$$

3)Maximum value of flux

$$EMF\ equation \quad E = 4.44 f N_1 \emptyset_m$$

$$\emptyset_m = \frac{V_1}{4.44 * f * N_1} = \frac{200}{4.44 * 50 * 200} = 0.022\ wb$$

2)

Q2

The equivalent parameters of a transformer, having a turns ratio of 5, are R_1=0.5Ω, R_2=0.021Ω, X_1=3.2Ω, X_2=0.12Ω, R_c=350Ω, and X_m=98Ω. Draw the approximate equivalent circuit of the transformer, referred to

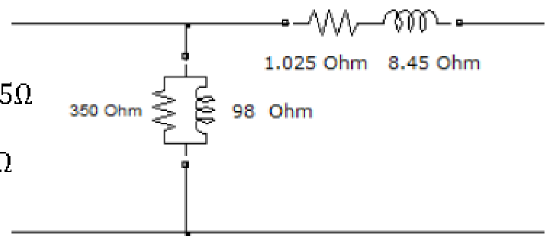
a) The primary
b) The secondary.

Solution

a) Referred to primary

$R_{eq} = R_1 + a^2 R_2 = 0.5 + 5^2(0.021) = 1.025\Omega$

$X_{eq} = X_1 + a^2 X_2 = 3.2 + (5)^2(0.12) = 6.2\ \Omega$



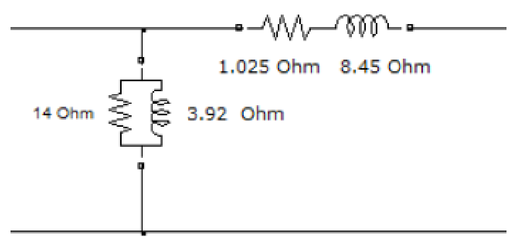
b) Referred to secondary

$$R_{eq} = \left(\frac{1}{a^2}\right) R_1 + R_2 = \left(\frac{1}{25}\right) 0.5 + (0.021) = 0.041\ \Omega$$

$$X_{eq} = \left(\frac{1}{a^2}\right) X_1 + X_2 = \left(\frac{1}{25}\right) 3.2 + (0.12) = 0.248\ \Omega$$

$$R_c^{\backslash} = \left(\frac{1}{a^2}\right) R_c = \left(\frac{1}{25}\right) 350 = 14\ \Omega$$

$$X_m^{\backslash} = \left(\frac{1}{a^2}\right) X_m = \left(\frac{1}{25}\right) 98 = 3.92\ \Omega$$



Q3

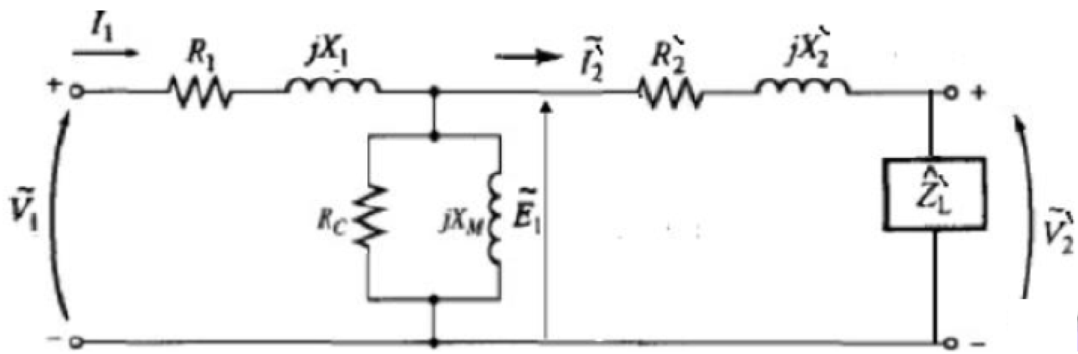
A 15-kVA, 2400:240-V, 60 Hz transformer has the following equivalent circuit parameters:

$R_1 = 2.5\Omega$	$R_2 = 0.025\Omega$	$X_1 = 7\Omega$
$X_2 = 0.07\Omega$	$R_c = 32\ k\Omega$	$X_m = 11.5\ k\Omega$

If the transformer is supplying a 10-kW, 0.8 PF lagging load at rated voltage, assuming the output voltage is the reference, **draw** the transformer's **exact** equivalent circuit referred to the primary (H.V) side and use it to **calculate**:

1. The input current
2. The input voltage
3. The input power factor

Solution



1. **The input current**

$$\overline{I_1} = \overline{I_o} + \overline{I_2'}$$

$$|\overline{I_2'}| = \frac{P_{load}}{|\overline{V_2'}| * pf} = \frac{10 * 10^3}{2400 * 0.8} = 5.2\,Amp$$

$$\overline{I_2'} = |\overline{I_2'}|\angle(-cos^{-1}(p.f) = 5.2\angle - cos^{-1}(0.8) = 5.2\angle - 36.87\,Amp$$

$$\overline{I_o} = \frac{\overline{E_1}}{R_c//X_m}$$

$$\overline{E_1} = \overline{V_2'} + \overline{I_2'}(R_2' + jX_2')$$

$$= 2400\angle 0 + 5.2\angle - 36.87(10^2 * 0.025 + j10^2 * 0.07)$$

$$= 2432.4\angle 0.5\,volt$$

$$R_c//X_m = \frac{R_c * X_m}{R_c + X_m} = \frac{32 * 10^3 * j11.5 * 10^3}{32 * 10^3 + j11.5 * 10^3} = 10.82\angle 70.23\,k\Omega$$

$$\overline{I_o} = \frac{2432.4\angle 0.5}{10822\angle 70.23} = 0.2247\angle - 69.73\,Amp$$

$$\overline{I_1} = 0.2247\angle - 69.73 + 5.2\angle - 36.87 = 5.4\angle - 38.16\,Amp$$

2. The input voltage

$$\overline{V_1} = \overline{E_1} + \overline{I_1}(R_1' + jX_1')$$

$$= 2432.4\angle 0.5 + 5.4\angle - 38.16\ (2.5 + j7)$$

$$= 2466.6\angle 0.99\ volt$$

3. The input power factor

$$pf_{inp} = cos(\angle\overline{V_1} - \angle\overline{I_1})$$

$$= cos(0.99 + 38.16)$$

$$= 0.775$$

Q

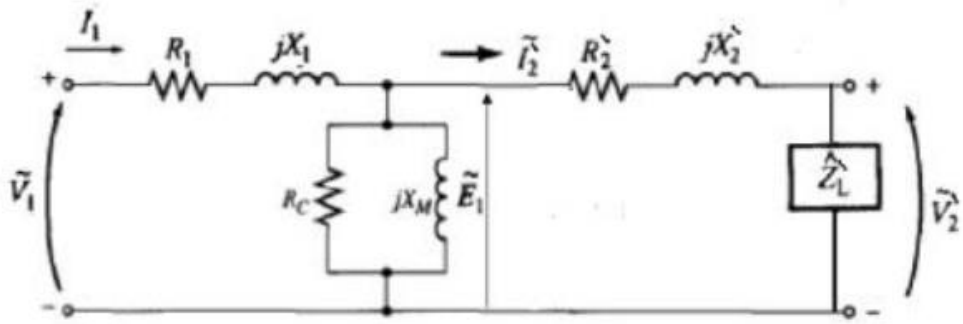
The parameters of a 2300/230 V, 50Hz transformer are given below:

$R_1 = 0.286\ \Omega$	$R_2' = 0.319\ \Omega$	$X_1 = 0.73\ \Omega$
$X_2' = 0.73\ \Omega$	$R_c = 250\ \Omega$	$X_m = 1250\ \Omega$

The secondary load impedance is $Z_L = 0.387 + j\,0.29$. Draw the exact equivalent circuit with the normal voltage across the primary (H.V side) and use it to find:

1. Secondary voltage
2. Input power factor
3. Power input.
4. Power output
5. Primary copper loss
6. Secondary copper loss
7. Core loss

Solution

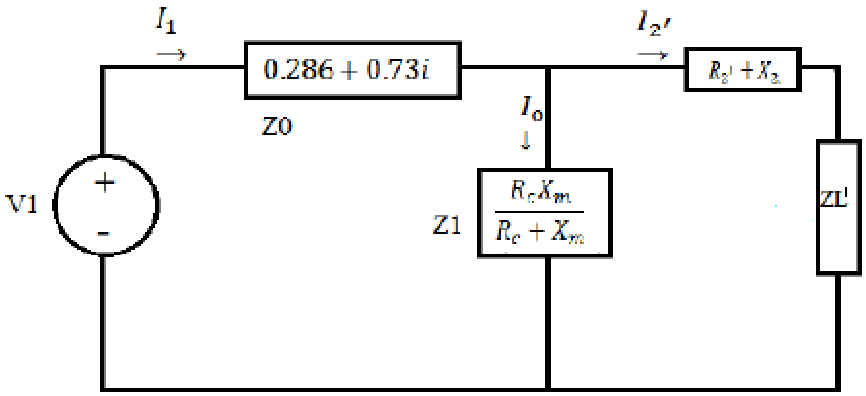


$Z_L = 0.387 + j\ 0.29$　　　a = 2300/230 =10

Since V_1 is rated value:

$V_1 = 2300\ V$

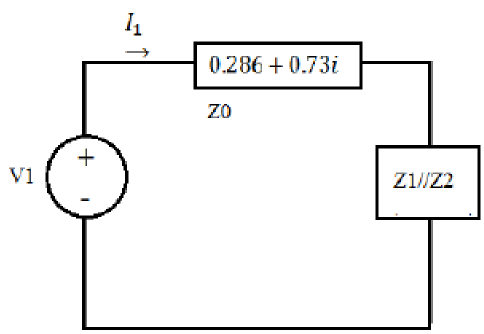
$Z_{L'} = a^2 Z_L = 38.7 + 29i = 48.36∟36.84$



$$Z_1 = \frac{R_c X_m}{R_c + X_m} = 240.58 + 48i = 245.32\llcorner 11.28$$

$$Z_2 = (R_{2'} + X_2) + Z_{L'} = 39.02 + 29.73i = 49.055\llcorner 37.3$$

$$Z_0 = 0.286 + 0.73i = 0.784\llcorner 68.6$$



$$I_1 = \frac{V_1}{Z_{eq}} = 45.46 - 30.29i = 54.6\llcorner - 33.67A$$

$$I_0 = I_1 \frac{Z_2}{Z_2 + Z_1} = 9 - 1.9i = 21\llcorner - 64.65$$

$$I_{2'} = I_1 - I_0 = 36.42 - 28.38i = 46.17\llcorner - 37.9$$

1. Secondary Voltage

$V_{2'} = I_{2'} Z_{l'} = 2232.678 - 41.856i = 2233 \llcorner -1.07 V$

$V_2 = \frac{V_{2'}}{a} = 223.3 - 4.1698i = 223.3 \llcorner 1.07^{\circ} V$

2. Input p.f

$p.f = \cos\left(V_{1angle} - I_{1angle}\right) = \cos(33.6) = 0.8329\ lag$

3. Power Input:

$|I_1||V_1| \cos(-33.6) = 104.4\, kW$

4. Power Output:

$|I_2||V_2| \cos(\varphi_2) = 80.1 kW$

5. Power loss (copper):

Primary: $|I_1{}^2| R_1 = 858.8W$

Secondary $|I_{2'}{}^2| R_{2'} = 683.3W$

6. Power loss (core):

$P = |I_0|^2 R_c = 20.5 kW$

Q

The equivalent parameters of a 150kVA, 2400V/240V transformer, are R_1=0.2Ω, R_2=2mΩ, X_1=0.45Ω, X_2=4.5mΩ, R_c=10kΩ, and X_m=1.55kΩ. The transformer is operating at rated load and rated voltage with 0.8 lagging power factor. Using the approximate equivalent circuit referred to the primary side, determine:

1. Voltage regulation.
2. The transformer power loss.
3. Efficiency.

Solution

Using the approximate circuit we can get:

$$R_{eq} = R_1 + a^2 R_2 = 0.2 + (10)^2 * (2) * 10^{-3} = 0.4\ \Omega$$

$$X_{eq} = X_1 + a^2 X_2 = 0.45 + (10)^2 * (4.5) * 10^{-3} = 0.9\,\Omega$$

$$P_{core} = \frac{V_1^2}{R_c} = \frac{2400^2}{10 * 10^3} = 5.76\ w$$

Ac
Go

$$P_{cu\,fl} = (I^{\backslash}_{2\,fl})^2 R_{eq} = (\frac{S_{fl}}{V_2^{\backslash}})^2 R_{eq} = \left(\frac{150 * 10^3}{2400}\right)^2 (0.4) = 1562.5\ \text{w}$$

$$X = load\ factor = 1\ \ (because\ stated\ at\ rated\ load)$$

$$pf = power\ factor = \cos(\phi) = 0.8\ lag$$

1. Voltage Regulation

$$\mathcal{E} = \frac{X.\,S_{rated}}{\left|V_2^{\backslash}\right|^2}\left[R_{eq}\cos\phi + X_{eq}\sin\phi\right]$$

$$= \frac{(1)(150 * 10^3)}{(2400)^2}[(0.4)(0.8) + (0.9)\sin((\cos^{-1} 0.8))]$$

$$= 2.24\ \%$$

2. Transformer Power loss

$$P_{loss} = P_{core} + X^2 P_{cu\,fl}$$

$$= 576 + 1^2(1562.5)$$

$$= 2.1385\ Kw$$

3. Efficiency

$$\eta = \frac{XS_{rated}\cos\phi}{XS_{rated}\cos\phi + P_{core} + X^2 P_{cu\,fl}}$$

$$= \frac{(1)(150 * 10^3)(0.8)}{(1)(150 * 10^3)(0.8) + 576 + 1^2(1562.5)}$$

$$= 98.25\ \%$$

Q

The equivalent parameters of a 110kVA, 2200V/110V transformer, are R_1=0.22Ω, R_2=0.5mΩ, X_1=2Ω, X_2=5mΩ, R_c=5494.5Ω, and X_m=1099Ω. Using the approximate equivalent circuit referred to the primary side, when the transformer is operating at 80% full load with unity power factor determines:

1. Voltage regulation.
2. The transformer power loss.
3. Efficiency.

Solution

Using the approximate circuit we can get:

$$R_{eq} = R_1 + a^2 R_2 = 0.22 + (20)^2(0.5 * 10^{-3}) = 0.42\ \Omega$$

$$X_{eq} = X_1 + a^2 X_2 = 2 + (20)^2(5 * 10^{-3}) = 4\ \ \Omega$$

$$P_{core} = \frac{V_1^2}{R_c} = \frac{2200^2}{5494.5} = 880.88\ \ w$$

$$P_{cu\,fl} = \left(I_{2\,fl}^{\backslash}\right)^2 R_{eq} = \left(\frac{S_{fl}}{V_2^{\backslash}}\right)^2 R_{eq} = \left(\frac{110 * 10^3}{2200}\right)^2 (0.42) = 1050\ \text{w}$$

$$X = load\ factor = 0.8\ \ (because\ stated\ 80\%\ full\ load)$$

$$pf = power\ factor = \cos(\phi) = 1\ (because\ stated\ unity\ power\ factor)$$

1. **Voltage Regulation**

$$\varepsilon = \frac{X.\,S_{rated}}{\left|V_2^{\backslash}\right|^2}\left[R_{eq}\cos\phi + X_{eq}\sin\phi\right]$$

$$\varepsilon = \frac{(0.8 * 110 * 10^3)}{(2200)^2}\left[(0.42)(1) + (4)(0)\right] = 0.76\ \%$$

2. **Transformer Power loss**

$$P_{loss} = P_{core} + X^2 P_{cu\,fl}$$

$$= 880.88 + 0.8^2 * 1050$$

$$= 1.55\ Kw$$

3. **Efficiency**

$$\eta = \frac{XS_{rated}\cos\emptyset}{XS_{rated}\cos\emptyset + P_{core} + X^2 P_{cu\,fl}}$$

$$\eta = \frac{0.8 * (110 * 10^3)}{0.8 * (110 * 10^3) + 880.88 + 0.8^2 * 1050}$$

$$= 98.26\%$$

Q

A 120 KVA, 2400/240 volt transformer has the following parameters:
R_1=0.75 ohm, X_1=0.8 ohm, R_2=0.0045 ohm, X_2=0.008 ohm
The total transformer losses at full load is 4 kW and the load that achieve the transformer maximum efficiency is 57.73% of the rated load. calculate:

1. The equivalent impedance referred to the primary.
2. The iron and full load copper losses.
3. The transformer maximum efficiency at 0.8 p.f lag.
4. The transformer voltage regulation at the loading conditions mentioned in 3.

Solution

1. **The equivalent impedance referred to the primary.**

$$R_{eq} = R_1 + a^2 R_2 = 0.75 + (10)^2(0.0045) = 1.2\ \Omega$$

$$X_{eq} = X_1 + a^2 X_2 = 0.8 + (10)^2(0.008) = 1.6\ \Omega$$

2. **The iron and full load copper losses.**

$$P_{loss\,fl} = P_{core} + P_{cu\,fl} = 4\,Kw \quad \rightarrow (1)$$

$$X_{\eta_{max}} = \sqrt{\frac{P_{core}}{P_{cu\,fl}}} = 0.5773 \quad \rightarrow (2)$$

Solving (1) and (2) we get

$$P_{core} = 1\ Kw \quad P_{cu\,fl} = 3\ Kw$$

3. **The transformer maximum efficiency at 0.8 p.f lag.**

$$\eta_{max} = \frac{X_{\eta_{max}} S_{rated} \cos\emptyset}{X_{\eta_{max}} S_{rated} \cos\emptyset + P_{core} + X_{\eta_{max}}{}^2 P_{cu\,fl}}$$

$$\eta_{max} = \frac{(0.5773)(120 * 10^3)(0.8)}{(0.5773)(120 * 10^3)(0.8) + 1 * 10^3 + (0.5773)^2(3 * 10^3)}$$

$$= 96.57\%$$

4. **The transformer voltage regulation at the loading conditions mentioned in 3.**

$$\varepsilon = \frac{X_{\eta_{max}} \cdot S_{rated}}{\left|V_2^{\backslash}\right|^2}\left[R_{eq}\cos\phi + X_{eq}\sin\phi\right]$$

$$= \frac{(0.5773)(120 * 10^3)}{(2400)^2}[(1.2)(0.8) + (1.6)\sin((\cos^{-1}0.8))]$$

$$= 2.3\ \%$$