PHYSICS-2

Assignment-5

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F6

Ans-1 Suppose two atoms exert attractive & repulsive force on each other such that the bonding force is.

$$f(r) = \underbrace{\frac{A}{r^M}}_{\text{attractive}} - \underbrace{\frac{B}{r^N}}_{\text{repulsive}} \quad \text{with } \underbrace{N > M}_{\text{due to repulsive}}$$

; r = centre to centre spacing b/w atoms.
A, B, M & N are constants characteristics of the molecule.

At equilibrium, when $r = r_0$

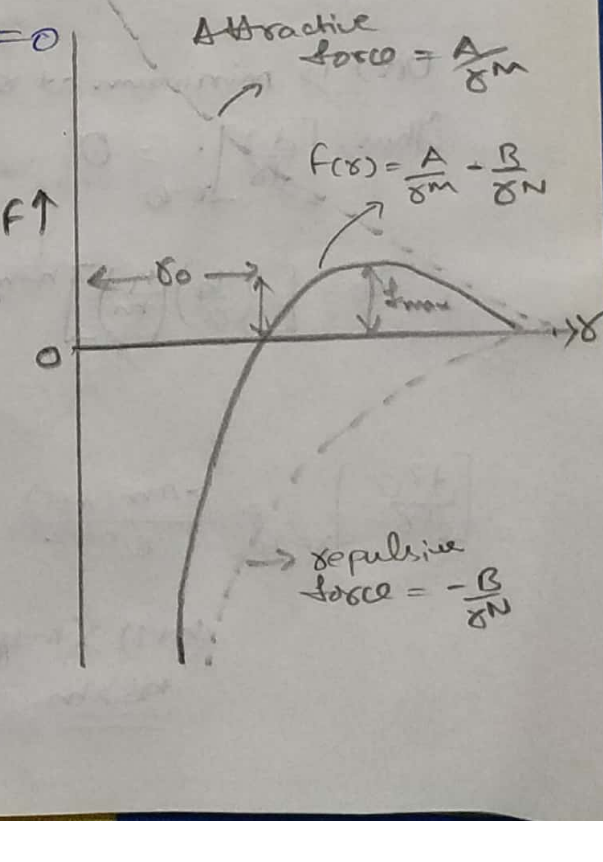
$$f(r) = 0 \implies \frac{A}{r^M} - \frac{A}{r^N} = 0$$

$$\frac{A}{r^M} = \frac{A}{r^N}$$

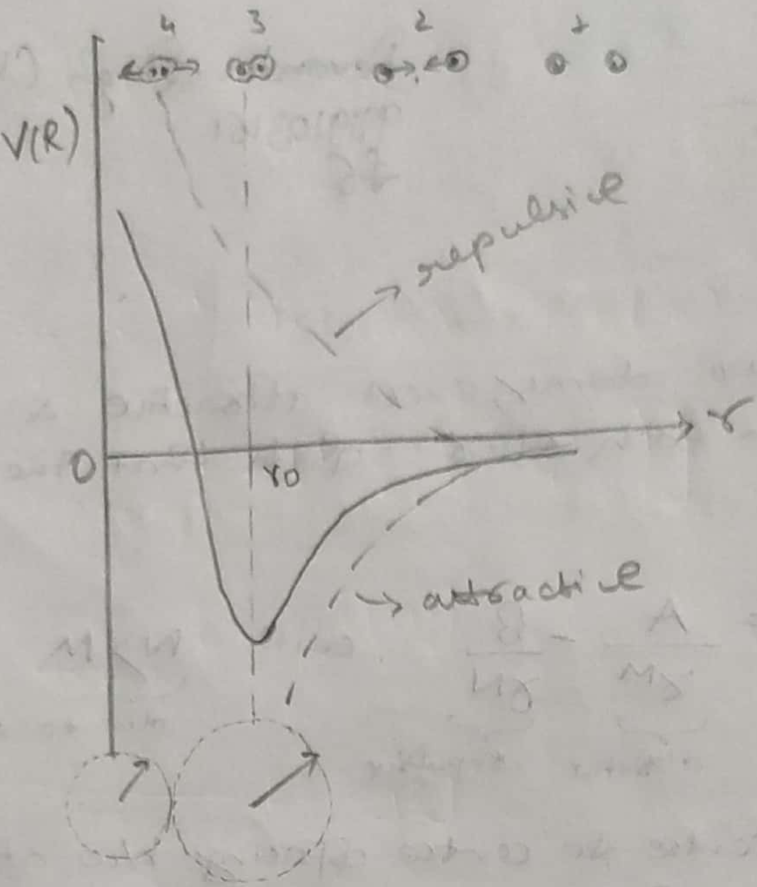
$$r_0^{N-M} = \frac{B}{A}$$

$\cancel{f(r)} =$

$$r_0 = \left(\frac{B}{A}\right)^{\frac{1}{N-M}}$$



Ans 2.



$U(r) = \frac{-a}{r^m} + \frac{b}{r^n}$ — (1) ; r_0 = distance b/w the atoms for this minimum U(r) to occur

$U(r=r_0)_{min} \rightarrow$ -ive

U(r) is minimum at $r=r_0$

Thus, $\left[\frac{dU}{dr}\right]_{r=r_0} = 0$

$r_0 = \left[\left(\frac{b}{a}\right)\left(\frac{n}{m}\right)\right]^{\frac{1}{n-m}}$

The minimum U(r) occur only if $n>m$.

$\Rightarrow$ The attractive force should vary more slowly with r than repulsive force.

$\left[\frac{d^2U}{dr^2}\right]_{r=r_0} = \frac{-am(m+1)}{r_0^{m+2}} + \frac{bn(n+1)}{r_0^{n+2}} > 0$

$(n+1) > (m+1)$

$n > m$

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$$\left[\frac{dU}{dr}\right]_{r=r_0} = \frac{ma}{r_0^{m+1}} - \frac{nb}{r_0^{n+1}} = 0 \quad \rightarrow 1$$

$$r_0^n = r_0^m \left[\left(\frac{b}{a}\right)\frac{n}{m}\right]$$

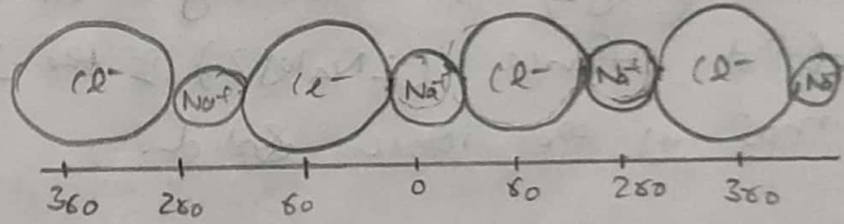
Putting 'r_0^n' in (1)

$$U_{min} = \frac{-a}{r_0^m} + b\left(\frac{a}{b}\right)\left(\frac{m}{n}\right)\frac{1}{r_0^m}$$

$$U_{min} = \frac{a}{r_0^m}\left(\frac{m}{n}\right) - \left[\frac{a}{r_0^m}\right]$$

$$\boxed{U_{min} = \frac{-a}{r_0^m}\left(1 - \frac{m}{n}\right)}$$

Ans. 3



Madelung constant (A) in a linear chain of ions of alternative signs.

The attractive Coulomb energy due to nearest neighbours is

$$\frac{-e^2}{4\pi\epsilon_0 r_0} + \left[-\frac{e^2}{4\pi\epsilon_0 r_0}\right] = \frac{-2e^2}{4\pi\epsilon_0 r_0}$$

The repulsive energy due to two next neighbors at a distance of $2r_0$ is $\dfrac{2e^2}{4\pi\epsilon_0 (2r_0)}$

.. of $3r_0$ is $\dfrac{-2e^2}{4\pi\epsilon_0 (3r_0)}$ and so on

Thus, the total energy due to all the ions in the linear array is

$$\frac{-2e^2}{4\pi\epsilon_0 r_0} + \frac{2e^2}{4\pi\epsilon_0(2r_0)} - \frac{2e^2}{4\pi\epsilon_0(3r_0)} + \cdots$$

$$= -\frac{e^2}{4\pi\epsilon_0 r_0}\left[2\left(1-\tfrac{1}{2}+\tfrac{1}{3}-\tfrac{1}{4}+\cdots\right)\right]$$

$$= -\frac{e^2}{4\pi\epsilon_0 r_0}\left[2\log(1+1)\right]$$

$$= \frac{-e^2}{4\pi\epsilon_0 r_0}\left[2\log 2\right]$$

Thus $(2\log 2)$ is Madelung constt. per molecule of the ionic solid. Hence $(2N_A \log 2)$ is the Madelung constant per mol of the ionic solid.