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① Given $f(x) = |\sin x|$

$f(x) = \sin x$

$f(-x) = |-\sin x| = \sin x$

$\because f(x) = f(-x)$

$\therefore$ even function

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$(c, c+2l) \longrightarrow (-\pi, \pi)$

$\Rightarrow c = -\pi$ & $l = \pi$

$$\therefore a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{2}{\pi} \int_{0}^{\pi} \sin x\,dx$$

$\left(\because \text{ for even } f^n \int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx\right)$

$$= \frac{2}{\pi} (-\cos x)_0^{\pi} = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx\,dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \sin x \cos nx\,dx$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin(1+n)x + \sin(1-n)x\,dx$$

$\left(\because \sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]\right)$

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$$= \frac{1}{\pi}\left[-\frac{\cos(1+n)x}{1+n} - \frac{\cos(1-n)x}{1-n}\right]_0^\pi$$

$$= \frac{1}{\pi}\left[-\frac{\cos(1+n)\pi}{1+n} - \frac{\cos(1-n)\pi}{1-n} + \frac{1}{1-n} + \frac{1}{1+n}\right]$$

$$= \frac{1}{\pi}\left[-\frac{\cos(\pi+n\pi)}{1+n} - \frac{\cos(\pi-n\pi)}{1-n} + \frac{2}{1-n^2}\right]$$

$$= \frac{1}{\pi}\left[-\frac{(-\cos n\pi)}{1+n} - \frac{(-\cos n\pi)}{1-n} + \frac{2}{1-n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{2\cos n\pi}{1-n^2} + \frac{2}{1-n^2}\right]$$

$$= \frac{2}{\pi(1-n^2)}(\cos n\pi + 1), \quad n \neq 1$$

$$a_n = \frac{2}{\pi(1-n^2)}((-1)^n+1) \quad , n \neq 1$$

For $n \to$ even, $a_n = \frac{4}{\pi(1-n^2)}$

$n \to$ odd $\neq 1$, $a_n = 0$

$$a_1 = \frac{2}{\pi}\int_0^{\pi} \sin x \cos x \, dx = \frac{1}{\pi}\int_0^{\pi} \sin 2x \, dx$$

$$a_1 = 0$$

$$\therefore b_n = \int_{-\pi}^{\pi} \sin x \sin nx \, dx = 0$$

$$\left(\because \int_c^{c+2\pi} \sin nx \sin mx \, dx = 0\right)$$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \frac{4\cos 2nx}{\pi(1-4n^2)}$$

($\because$ Only for $n \to$ even, $a_n \neq 0$ so $n = 2n$)

$$f(x) = \frac{2}{\pi} - \frac{4}{\pi}\sum_{n=1}^{\infty} \left(\frac{\cos 2nx}{4n^2-1}\right)$$

$$= \frac{2}{\pi} - \frac{4}{\pi}\left[\frac{\cos 2x}{3} + \frac{\cos 4x}{15} + \frac{\cos 6x}{35} + \cdots\right]$$

② $f(x) = \cos x \quad , \quad x \in (0, \pi)$

$(0, l) = (0, \pi) \rightarrow l = \pi.$

Half range sine series is given as

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l}$$

$$= \sum_{n=1}^{\infty} b_n \sin nx$$

where

$$b_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi x}{l} dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \cos x \sin nx \, dx$$

$$= \cancel{\frac{1}{\pi} \int_0^{\pi} \sin 2x \, dx}$$

$$= \cancel{\frac{1}{\pi} \left(\frac{-\cos 2x}{2} \right)_0^{\pi}}$$

$$= \cancel{\frac{1}{2\pi} (-1+1)} = 0$$

$\therefore$ $b_n = \frac{2}{\pi} \frac{1}{2} \int_0^{\pi} \sin x(1+n) - \sin x(1-n) dx$

$(\because 2\cos A \sin B = \sin(A+B) - \sin(A-B))$

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$$b_n = \frac{1}{\pi}\left[-\frac{\cos(1+n)x}{1+n} + \frac{\cos(1-n)x}{1-n}\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\cos(1+n)\pi}{1+n} + \frac{\cos(1-n)\pi}{1-n} + \frac{1}{1+n} - \frac{1}{1-n}\right]$$

$$= \frac{1}{\pi}\left[\frac{-\cos(\pi+n\pi)}{1+n} + \frac{\cos(\pi-n\pi)}{1-n} - \frac{2n}{1-n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos n\pi}{1+n} - \frac{\cos n\pi}{1-n} - \frac{2n}{1-n^2}\right]$$

$$= \frac{1}{\pi}\left[\frac{-2n\cos n\pi}{1-n^2} - \frac{2n}{1-n^2}\right]$$

$$= -\frac{2n}{\pi(1-n^2)}(\cos n\pi + 1) \qquad n \neq 1$$

$= n$ $n \in$ odd

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$$b_n = \frac{2}{\pi}\int_0^{\pi} \cos x \sin nx\, dx$$

$n = 1$

$$b_1 = \frac{2}{\pi}\int_0^{\pi} \cos x \sin x\, dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \sin 2x\, dx$$

$$= \frac{-1}{2\pi}\left(\cos 2x\right)_0^{\pi}$$

$$= \frac{-1}{2\pi}\left(1-1\right) = 0$$

$$b_n = \frac{-2n}{\pi(1-n^2)}\left((-1)^n+1\right), \quad n \neq 1.$$

$b_n = 0$, $n \in$ odd

$$b_n = -\frac{4n}{\pi(1-n^2)}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-8n}{\pi(1-4n^2)} \sin 2nx$$

($\because$ Only for even n, $b_n \neq 0 \Rightarrow n = 2n$)

$$= \sum_{n=1}^{\infty} \frac{8n}{\pi(4n^2-1)} \sin 2nx$$