Assignment 1 - Physics

Q-1:- Determine E_{avg}, E_{rms}, and E_{mp} by using molecular Energy distribution with energies between E and $(E+dE)$ in a sample of an ideal gas that contains N molecules and whose absolute temperature is T.

Soln:- As, $n(\varepsilon)\,d\varepsilon = g(\varepsilon)\, f(\varepsilon)\, d\varepsilon$

$$= g(\varepsilon)\, A e^{-\varepsilon/kT}\, d\varepsilon$$

$$\therefore\ n(\varepsilon)\,d\varepsilon = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{\varepsilon}\, e^{-\varepsilon/kT}\, d\varepsilon$$

(1) Average Energy:-

$$(\bar{\varepsilon}) = \frac{\text{Total Energy}}{\text{Total no. of Particle}}$$

$$= \frac{\int_0^\infty \varepsilon\, n(\varepsilon)\, d\varepsilon}{N}$$

$$= \frac{\int_0^\infty \frac{2\pi N\, \varepsilon\sqrt{\varepsilon}\, e^{-\varepsilon/kT}}{(\pi kT)^{3/2}}}{N}$$

$$= \frac{2\pi}{(\pi kT)^{3/2}} \int_0^\infty \varepsilon^{3/2} e^{-\varepsilon/kT}\, d\varepsilon$$

using $\int_0^\infty x^n e^{-\alpha x}\, dx = \frac{n!}{\alpha^{n+1}}$

$$\therefore \int_0^\infty x^{3/2} e^{-\alpha x}\, dx = \frac{3}{4\alpha^2}\sqrt{\pi/\alpha}$$

$$\Rightarrow\ \bar{\varepsilon} = \frac{2\pi}{(\pi kT)^{3/2}} \times \frac{3}{4\left(\frac{1}{kT}\right)^2}\sqrt{\pi kT}$$

$$\boxed{(\bar{\varepsilon}) = \frac{3}{2} kT}$$

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(ii) rms value of Energy:-

$$\sqrt{\overline{\varepsilon^2}} = \sqrt{\frac{\int_0^\infty \varepsilon^2 n(\varepsilon)\, d\varepsilon}{N}}$$

$$= \sqrt{\int_0^\infty \frac{2\pi \cancel{N} \varepsilon^2 \sqrt{\varepsilon} e^{-\varepsilon/kT}}{\cancel{N}(\pi kT)^{3/2}} d\varepsilon}$$

$$= \sqrt{\int_0^\infty \frac{2\pi}{(\pi kT)^{3/2}} \varepsilon^{5/2} e^{-\varepsilon/kT} d\varepsilon}$$

using $\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$

$$\sqrt{\overline{\varepsilon^2}} = \sqrt{\frac{15}{4}(kT)^2}$$

$$\sqrt{\overline{\varepsilon^2}} = \sqrt{15}\,\frac{kT}{2}$$

(iii) Most probable Energy

$$\frac{dn(\varepsilon)}{d\varepsilon} = 0$$

$$\therefore \frac{2\pi N}{(\pi kT)^{3/2}}\left[\frac{1}{2}\varepsilon^{-1/2} e^{-\varepsilon/kT} + \varepsilon^{1/2}\left(-\frac{1}{kT}\right)e^{-\varepsilon/kT}\right] = 0$$

$$\Rightarrow \varepsilon^{1/2} e^{-\varepsilon/kT}\left[\frac{1}{2\varepsilon} - \frac{1}{kT}\right] = 0$$

$$\therefore \frac{\varepsilon}{kT} = \frac{1}{2}$$

$$\Rightarrow \boxed{\varepsilon = \frac{1}{2}kT}$$

Q.2:- Write down the number of particles with velocities v and v + dv from molecular energy and calculate Ratio between $E_{avg} : E_{rms} : E_{mp}$

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$\underline{Sol^n - 2}$:- as $n(v)\,dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}\,dv$

Ratio b/n $E_{avg} : E_{rms} : E_{mp} = \frac{3}{2}\cancel{kT} : \frac{\sqrt{15}}{2}\cancel{kT} : \frac{1}{2}\cancel{kT}$

$= 3 : \sqrt{15} : 1$

as $n(\varepsilon)\,d\varepsilon = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{\varepsilon}\, e^{-\varepsilon/kT}\,d\varepsilon$

Now, $\varepsilon = \frac{1}{2}mv^2$

$\therefore d\varepsilon = mv\,dv$

$\therefore n(v)\,dv = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{\frac{1}{2}mv^2}\; e^{-\frac{1}{2}\frac{mv^2}{kT}}\, mv\,dv$

$= \frac{4\pi N}{(2kT\pi)^{3/2}} (m)^{3/2} v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}\,dv$

$\underline{Q-3}$:- Prove average energy of a free e^- gas at $T=0$ is $3/5$ of fermi energy $\left[g(\varepsilon) = \frac{3N}{2}\varepsilon_F^{-3/2}\varepsilon^{1/2}\,d\varepsilon\right]$

$\underline{Sol^n}$ – $n(\varepsilon)\,d\varepsilon = \frac{3N}{2}\frac{\varepsilon_F^{-3/2}\varepsilon^{1/2}}{e^{(\varepsilon-\varepsilon_F)/kT}+1}\,d\varepsilon \longrightarrow$ FD energy distribution

$\boxed{\text{as, } n\varepsilon = g(\varepsilon) f_{FD}(\varepsilon)\,d\varepsilon}$

Average Energy, $\bar{\varepsilon} = \frac{\int_0^\infty \varepsilon\, n(\varepsilon)\,d\varepsilon}{N}$

$\bar{\varepsilon}_0 = 3\varepsilon_F^{-3/2} \int_0^{\varepsilon_f} \frac{\varepsilon^{3/2}}{e^{(\varepsilon-\varepsilon_F)/kT}+1}\,d\varepsilon$

at $T=0$; $e^{(\varepsilon-\varepsilon_F)/kT} = e^{-\infty} = 0$

Hence $\boxed{\bar{\varepsilon}_0 = \frac{3}{5}\varepsilon_F}$

Thus, average energy of a free e^- gas at $T=0$ is $\frac{3}{5}$ of fermi energy

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