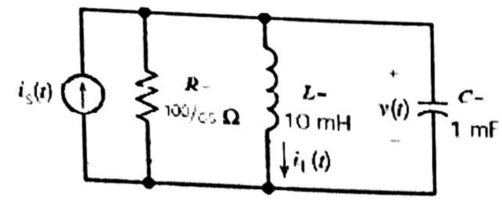
1. Solution of ques.1

After $t=0$



$$\text{KCL}: i_s(t)=\frac{v(t)}{R}+i_L(t)+C\frac{dv(t)}{dt}$$

$$\text{KVL}: v(t) = L\frac{di_L(t)}{dt}$$

$$i_s(t)=\frac{L}{R}\frac{di_L(t)}{dt}+i_L(t)+LC\,\frac{d^2 i_L(t)}{dt^2}$$

$$\frac{d^2 i_L(t)}{dt^2}+\frac{1}{RC}\frac{di_L(t)}{dt}+\frac{1}{LC}i_L(t)=\frac{1}{LC}i_s(t)$$

$$\frac{d^2 i_L(t)}{dt^2}+(650)\frac{di_L(t)}{dt}+(10^5)i_L(t)=(10^5)i_s(t)$$

(a) Try a forced response of the form $i_f(t)=A$. Substituting into the differential equations gives

$$0+0+A\frac{1}{(.01)(1\times10^{-3})}=\frac{1}{(.01)(1\times10^{-3})} \Rightarrow A=1. \text{ Therefore } i_f(t)=1\text{ A}.$$

(b) Try a forced response of the form $i_f(t)=At+B$. Substituting into the differential equations gives $0+A\dfrac{65}{(100)\ (0.001)}+(At+B)\dfrac{1}{(0.01)(0.001)}=0.5t$. Therefore $A=0.5$ and $B=-3.25\times10^{-3}$. Finally $i_f(t)=5t-3.25\times10^{-3}$ A.

(c) Try a forced response of the form $i_f(t)=Ae^{-250t}$. It doesn't work so try a forced response of the form $i_f(t)=Bte^{-250t}$. Substituting into the differential equation gives

$$\left[(-250)^2 Be^{-250t}-500Be^{-250t}\right]+650\left[(-250)Bte^{-250t}+Be^{-250t}\right]+10^5 Bte^{-250t}=2e^{-250t}.$$

Equating coefficients gives

$$(250)^2 B+650(-250)B+10^5 B=0 \Rightarrow \left[(250)^2+650(-250)+10^5\right]B=0 \Rightarrow [0]B=0$$

and

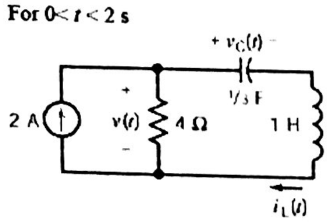
$$-500B+650B=2 \Rightarrow B=0.0133$$

Finally $i_f(t)=0.0133\,te^{-250t}$ A.

2. Solution of ques.2

Use superposition. Find the response to inputs $2u(t)$ and $-2u(t-2)$ and then add the two responses. First, consider the input $2u(t)$:

For $0<t<2$ s



Using the operator $s=\frac{d}{dt}$ we have

KVL:

$$v_c(t)+s\,i_L(t)+4\left[i_L(t)-2\right]=0 \qquad (1)$$

KCL:

$$i_L(t)=\frac{1}{3}s\,v_c(t) \Rightarrow v_c(t)=\frac{3}{s}i_L(t) \qquad (2)$$

Plugging (2) into (1) yields the characteristic equation: $(s^2+4s+3)=0$. The natural frequencies are $s_{1,2}=-1\,,-3$. The inductor current can be expressed as

$$i_L(t)=i_n(t)+i_f(t)=\left(A_1\,e^{-t}+A_2\,e^{-3t}\right)+0=A_1\,e^{-t}+A_2\,e^{-3t}.$$

Assume that the circuit is at steady state before $t=0$. Then $v_c(0^+)=0$ and $i_L(0^+)=0$.

Using KVL we see that $(1)\dfrac{di_L(0^+)}{dt}=4\left[2-i_L(0^+)\right]-v_C(0^+)=8$ A/s. Then

$$\left.\begin{aligned} i_L(0)&=0=A_1+A_2\\ \frac{di_L(0)}{dt}&=8=-A_1-3A_2\end{aligned}\right\}\; A_1=4\,,\;A_2=-4.$$

Therefore $i_L(t)=4e^{-t}-4e^{-3t}$ A. The response to 2u(t) is

$$\begin{aligned} v_1(t)=8-4\,i_L(t)&=\begin{cases}0 & t<0\\ 8-16\,e^{-t}+16\,e^{-3t}\text{ V} & t>0\end{cases}\\ &=\left[8-16\,e^{-t}+16\,e^{-3t}\right]u(t)\text{ V}\end{aligned}$$

The response to –2u(t-2) can be obtained from the response to 2u(t) by first replacing t by t-2 everywhere is appears and the multiplying by –1. Therefore, the response to –2u(t-2) is

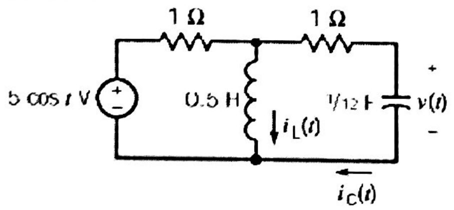
$$v_2(t)=\left[-8+16e^{-(t-2)}-16\,e^{-3(t-2)}\right]u(t-2)\text{ V}.$$

By superposition, $v(t)=v_1(t)+v_2(t)$. Therefore

$$v(t)=\left[8-16e^{-t}+16e^{-3t}\right]u(t)+\left[-8+16e^{-(t-2)}-16\,e^{-3(t-2)}\right]u\,(t-2)\text{ V}$$

3. Solution of ques.3

For $t>0$



KCL at top node:

$$\left(0.5\frac{di_L(t)}{dt}-5\cos t\right)+i_L(t)+\frac{1}{12}\frac{dv(t)}{dt}=0 \qquad (1)$$

KVL for right mesh:

$$0.5\frac{di_L(t)}{dt}=\frac{1}{12}\frac{dv(t)}{dt}+v(t) \qquad (2)$$

Taking the derivative of these equations gives:

$$\tfrac{d}{dt}\text{ of }(1)\Rightarrow 0.5\frac{d^2i_L(t)}{dt^2}+\frac{di_L(t)}{dt}+\frac{1}{12}\frac{d^2v(t)}{dt^2}=-5\sin t \qquad (3)$$

$$\tfrac{d}{dt}\text{ of }(2)\Rightarrow 0.5\frac{d^2i_L(t)}{dt^2}=\frac{1}{12}\frac{d^2v(t)}{dt^2}+\frac{dv(t)}{dt} \qquad (4)$$

Solving for $\frac{d^2 i_L(t)}{dt^2}$ in (4) and $\frac{di_L(t)}{dt}$ in (2) & plugging into (3) gives

$$\frac{d^2v(t)}{dt^2}+7\frac{dv(t)}{dt}+12v(t)=-30\sin t$$

The characteristic equation is: $s^2+7s+12=0$.

The natural frequencies are $s_{1,2}=-3,-4$.

The natural response is of the form $v_n(t) = A_1e^{-3t}+A_2e^{-4t}$. Try a forced response of the form $v_f(t)=B_1\cos t+B_2\sin t$. Substituting the forced response into the differential equation and equating like terms gives $B_1=\frac{21}{17}$ and $B_2=-\frac{33}{17}$.

$$v(t)=v_n(t)+v_f(t)=A_1e^{-3t}+A_2e^{-4t}+\frac{21}{17}\cos t-\frac{33}{17}\sin t$$

We will use the initial conditions to evaluate A_1 and A_2. We are given $i_L(0)=0$ and $v(0)=1$ V. Apply KVL to the outside loop to get

$$1\left[i_C(t)+i_L(t)\right]+1(i_C(t))+v(t)-5\cos t=0$$

At t = 0+

$$i_C(0)=\frac{5\cos(0)+i_L(0)-v(0)}{2}=\frac{5+0-1}{2}=2 \text{ A}$$

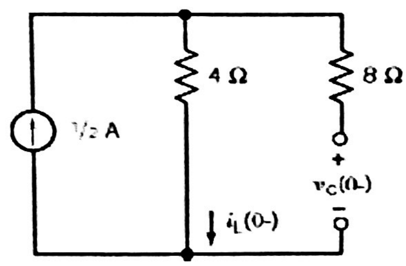
$$\frac{dv(0)}{dt}=\frac{i_C(0)}{1/12}=\frac{2}{1/12}=24 \text{ V/s}$$

$$\left.\begin{array}{l} v(0^+)=1=A_1+A_2+\frac{21}{17} \\ \frac{dv(0^+)}{dt}=24=-3A_1-4A_2-\frac{33}{17} \end{array}\right\} \Rightarrow \begin{array}{l} A_1=25 \\ A_2=-\frac{429}{17} \end{array}$$

Finally,

$$\therefore v(t)=25e^{-3t}-\frac{429e^{-4t}-21\cos t+33\sin t}{17} \text{ V}$$

4. Solution of ques.4



The circuit will be at steady state for t<0:

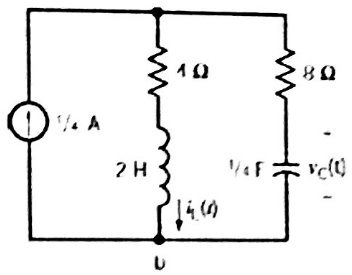
so

$$i_L(0^+)=i_L(0^-)=0.5 \text{ A}$$

and

$$v_C(0^+)=v_C(0^-)=2 \text{ V}.$$

For t>0:



Apply KCL at node b to get:

$$\frac{1}{4}=i_L(t)+\frac{1}{4}\frac{d}{dt}v_C(t) \Rightarrow i_L(t)=\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)$$

Apply KVL at the right-most mesh to get

$$4\,i_L(t)+2\frac{d}{dt}i_L(t)=8\left(\frac{1}{4}\frac{d}{dt}v_C(t)\right)+v_c(t)$$

Use the substitution method to get

$$4\left(\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)\right)+2\frac{d}{dt}\left(\frac{1}{4}-\frac{1}{4}\frac{d}{dt}v_C(t)\right)=8\left(\frac{1}{4}\frac{d}{dt}v_c(t)\right)+v_c(t)$$

or

$$2=\frac{d^2}{dt^2}v_C(t)+6\frac{d}{dt}v_C(t)+2v_C(t)$$

The forced response will be a constant, $v_C=B$ so $2=\frac{d^2}{dt^2}B+6\frac{d}{dt}B+2B \Rightarrow B=1\text{ V}$.

To find the natural response, consider the characteristic equation:

$$0=s^2+6s+2=(s+5.65)(s+0.35)$$

The natural response is

$$v_n=A_1\,e^{-5.65t}+A_2\,e^{-0.35t}$$

so

$$v_c(t)=A_1\,e^{-5.65t}+A_2\,e^{-0.35t}+1$$

Then

$$i_L(t)=\frac{1}{4}+\frac{1}{4}\frac{d}{dt}v_c(t)=\frac{1}{4}+1.41A_1\,e^{-5.65t}+0.0875A_2\,e^{-0.35t}$$

At t=0+

$$2=v_c(0+)=A_1+A_2+1$$

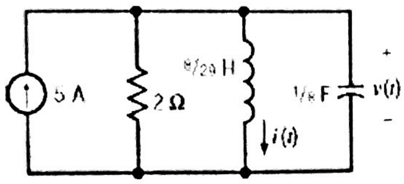
$$\frac{1}{2}=i_L(0+)=\frac{1}{4}+1.41A_1+0.0875A_2$$

so $A_1=0.123$ and $A_2=0.877$. Finally

$$v_c(t)=0.123\,e^{-5.65t}+0.877\,e^{-0.35t}+1\text{ V}$$

5. Solution of ques.5

After $t = 0$



The inductor current and voltage are related be

$$v(t) = L\frac{di(t)}{dt} \quad (1)$$

Apply KCL at the top node to get

$$C\frac{dv(t)}{dt} + i(t) + \frac{v(t)}{2} = 5 \quad (2)$$

Using the operator $s = \frac{d}{dt}$, and substituting (1) into (2) yields $(s^2 + 4s + 29)\, i(t) = 5$.

The characteristic equation is $s^2 + 4s + 29 = 0$. The characteristic roots are $s_{1,2} = -2 \pm j5$.

The natural response is of the form $i_n(t) = e^{-2t}\left[A\cos 5t + B\sin 5t\right]$.

Try a forced response of the form $i_f(t) = A$. Substituting into the differential equation gives $A = 5$. Therefore $i_f(t) = 5$ A.

The complete response is $i(t) = 5 + e^{-2t}\left[A\cos 5t + B\sin 5t\right]$ where the constants A and B are yet to be evaluated using the initial condition:

$$i(0) = 0 = A + 5 \Rightarrow A = -5$$

$$0 = v(0) = L\frac{di(0)}{dt} \Rightarrow \frac{di(0)}{dt} = 0 = -2A + 5B \Rightarrow B = \frac{2A}{5} = -2$$

Finally, $\underline{i(t) = 5 + e^{-2t}\left[-5\cos 5t - 2\sin 5t\right]\ \text{A}}$.