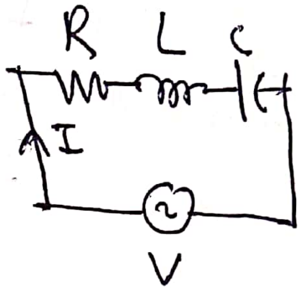
Summary of Resonance (Series RLC)

① Resonance frequency $\omega_0 = \frac{1}{\sqrt{LC}}$



② Quality factor $(Q) = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Selectivity

$$\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\text{Resonance freq}^n}{\omega_2 - \omega_1} = \frac{\omega_0}{BW}}$$

$$\left|\frac{V_C}{V}\right| = \left|\frac{V_L}{V}\right| = M_C = M_L = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

At resonance

magnification factor

③

$$\boxed{R,\ L,\ C,\ \omega_0,\ BW,\ S,\ Q}$$

$\omega_0 = \frac{1}{\sqrt{LC}}$ $\quad BW = \frac{\omega_0}{Q = S} = \frac{R}{L}$ rad/sec

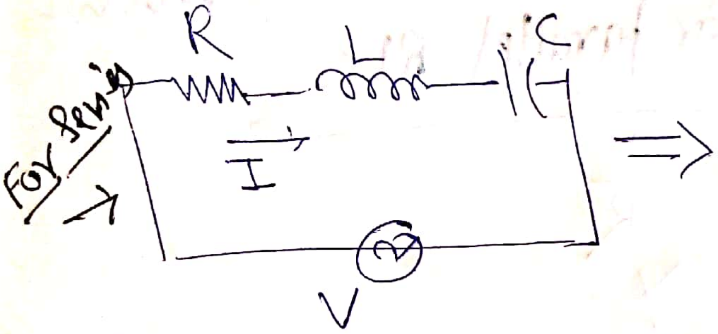
$$\cancel{BW = \frac{\omega_0}{\frac{\omega_0 R}{\omega_0}}}$$

$$BW = \frac{\omega_0}{\frac{\omega_0 L}{R}} = \frac{R}{L} \text{ or } \frac{\omega_0}{\frac{1}{\omega_0 RC}} = \omega_0^2 RC = \frac{1}{L\cancel{C}} \cdot R\cancel{C} = \frac{R}{L}$$

④ $\omega < \omega_0$ → R_C ; $\omega_0 = \omega$ → R ; $\omega > \omega_0$ → R_L } Series resonance

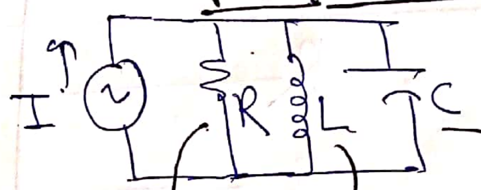
Parallel RLC Resonance Circuit

For series



⟹

For parallel



$$\boxed{Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Z → Impedance

$$\boxed{Y = \frac{1}{Z}}$$

Y → Admittance

$Z_1 = R \rightarrow \frac{1}{Y_1}$ $\quad Z_2 = i\omega L \rightarrow \frac{1}{Y_2}$ $\quad Z_3 = \frac{1}{i\omega C} \rightarrow \frac{1}{Y_3}$

$$Y = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

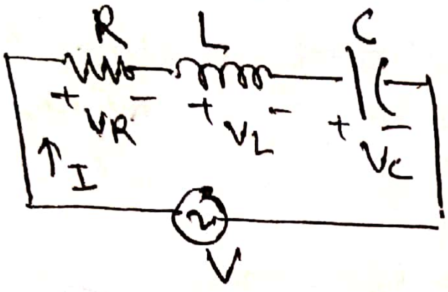
$$Y = \frac{1}{R} + \frac{1}{i\omega L} + \frac{i\omega C}{1}$$

$$Y = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)$$

$$\boxed{|Y| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

Parallel RLC Resonance using Series RLC Resonance

For Series RLC



Apply KVL —

$$V = V_R + i(V_L - V_C)$$

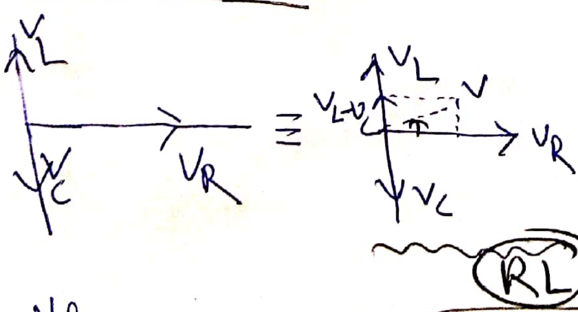
$$V = IR + i(IX_L - IX_C)$$

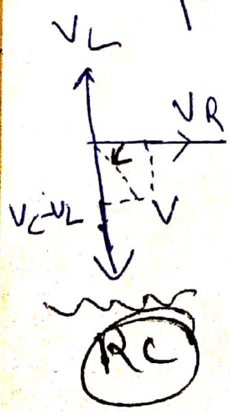
$$\frac{V}{I} = R + i(X_L - X_C)$$

$$Z = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

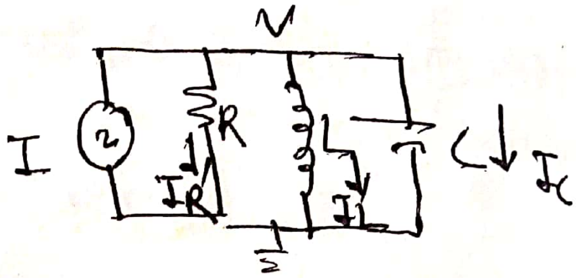
$$\boxed{|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(a) Phasor diagram :-





For parallel RLC



Apply KCL

$$I = I_R + I_L + I_C$$

$$I = \frac{V}{R} + \frac{V}{X_L} + \frac{V}{X_C}$$

$$\frac{I}{V} = \frac{1}{R} + \frac{1}{X_L} + \frac{1}{X_C}$$

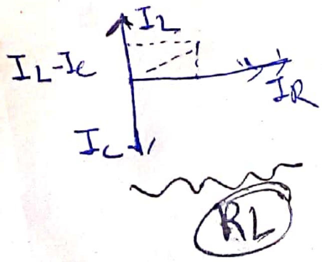
$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} + \frac{1}{\frac{1}{i\omega C}}$$

$$\frac{1}{Z} = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)$$

$$Y = \frac{1}{R} + i\left(\omega C - \frac{1}{\omega L}\right)$$

$$\boxed{|Y| = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

(b)



Short cut

$R \longrightarrow \frac{1}{R}$

$Z \longrightarrow Y$

$I \longrightarrow V$

$L \longrightarrow C$

Short cut

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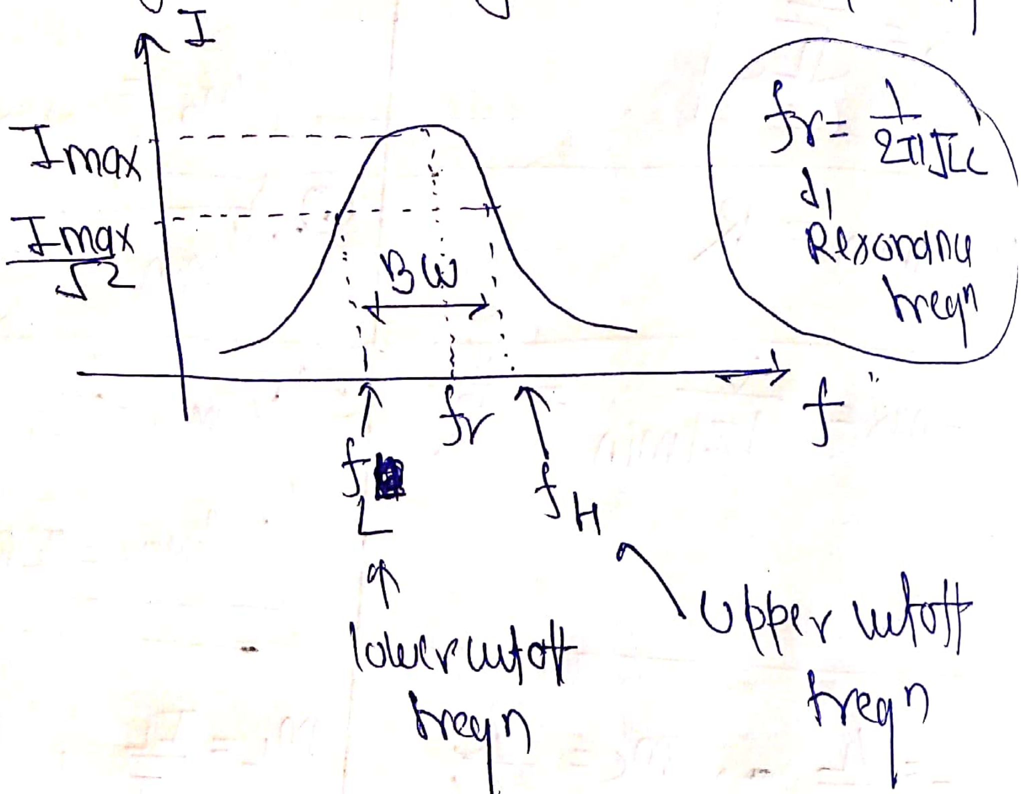
At resonance (Series)	At resonance (Parallels)
$\omega L = \frac{1}{\omega C}$	$\omega C = \frac{1}{\omega L}$
$\omega = \frac{1}{\sqrt{LC}}$	$\omega^2 = \frac{1}{LC}$
$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$	$\omega = \frac{1}{\sqrt{LC}}$
	$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$
① $\|Z\|_{min} = R$	① $\|Y\|_{min} = \frac{1}{R}$ $\left\{Y_{min} = \frac{1}{Z_{max}}\right\}$
② $I_{max} = \frac{V}{\|Z\|_{min}} = \frac{V}{R}$	② $\|V\|_{max} = \frac{I}{\|Y\|_{min}} = \frac{I}{Y_R} = \frac{I}{\frac{1}{R}} = IR$
	$\boxed{\|V\|_{max} = IR}$
③ $m_L = \frac{V_L}{V}$, $m_C = \frac{V_C}{V}$	$m_L = \frac{I_L}{I}$, $m_C = \frac{I_C}{I}$
At resonance	At resonance
$m_L = m_C = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R}\sqrt{\frac{L}{C}}$	$m_L = m_C = \omega_0 RC = \frac{R}{\omega_0 L} = R\sqrt{\frac{C}{L}}$
$= Q = S = \frac{\omega_0}{BW} = \frac{\omega_0}{\omega_2 - \omega_1}$	$= Q = S = \frac{\omega_0}{BW} = \frac{\omega_0}{\omega_2 - \omega_1}$
④ $BW = \frac{\omega_0}{Q} = \frac{\omega_0}{\frac{\omega_0 L}{R}} = \frac{R}{L}$	④ $BW = \frac{\omega_0}{Q} = \frac{\omega_0}{\omega_0 RC} = \frac{1}{RC}$
rad/sec	rad/sec

Bandwidth:

The difference between the lower cutoff frequency and upper cutoff frequency at which current is 0.707 (corresponding to half of maximum power)



$$\text{Bandwidth (BW)} = (f_H - f_L)\ Hz$$

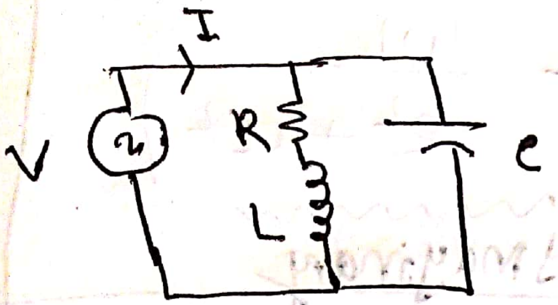
$$\text{Bandwidth (BW)} = \frac{R}{2\pi L}\ \cancel{rad/sec}\ Hz$$

$$\text{Lower cutoff freq}^n\ (f_L) = f_r - \frac{B.W.}{2}$$

$$\text{Upper cutoff freq}^n\ (f_H) = f_r + \frac{B.W.}{2}$$

$$\boxed{f_r \rightarrow \text{Resonance freq}^n = \frac{1}{2\pi\sqrt{LC}}\ Hz}$$

Prob: Find the resonance condition of the following circuit.

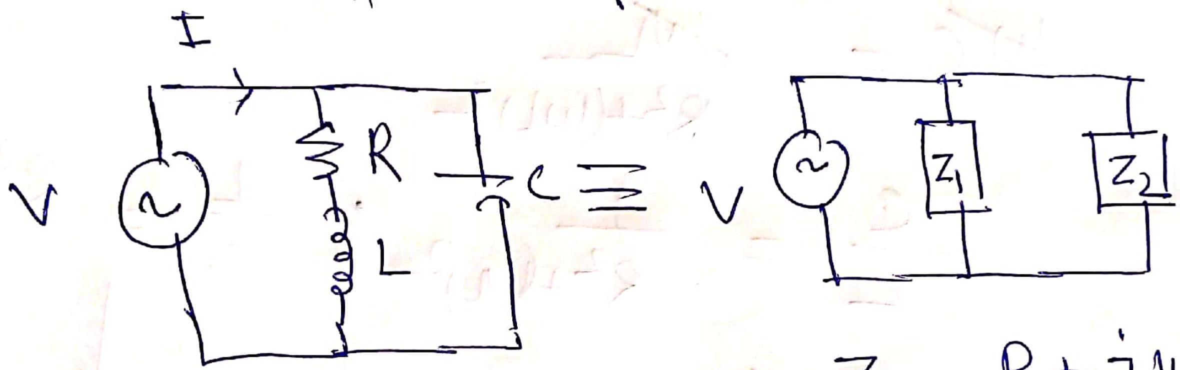


Soln $L \rightarrow X_L = jwL = 2\pi f L$ ohm

↑ Inductive Reactance

$C \rightarrow X_C = \frac{1}{jwc} = \frac{1}{j2\pi fc}$ ohm

↑ Capacitive Reactance.



$Z_1 = R + jwL$

$Z_2 = \frac{1}{jwc}$

Since the circuit is in parallel combination-

$$Y = \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} = \frac{1}{R+jwL} + \frac{1}{\frac{1}{jwc}}$$

↑ Admittance

↑ Impedance

$$= \frac{R-jwL}{(R+jwL)(R-jwL)} + jwc$$

$$= \frac{R-jwL}{R^2+w^2L^2} + jwc$$

$$\boxed{Y = \underbrace{\frac{R}{R^2+(\omega L)^2}}_{\text{Real}} + j\underbrace{\left(\omega C - \frac{\omega L}{R^2+(\omega L)^2}\right)}_{\text{Imaginary}}}$$

At resonance, the imaginary part $= 0$

$$\omega C - \frac{\omega L}{R^2+(\omega L)^2} = 0$$

$$\cancel{\omega} C = \frac{\cancel{\omega} L}{R^2+(\omega L)^2}$$

$$\frac{C}{L} = \frac{1}{R^2+(\omega L)^2} \quad ; \quad \frac{L}{C} = R^2+(\omega L)^2$$

$$(\omega L)^2 = \frac{L}{C} - R^2$$

$$\omega L = \sqrt{\frac{L}{C} - R^2}$$

$$\omega = \frac{1}{L}\sqrt{\frac{L}{C} - R^2}$$

$$\boxed{\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

rad/sec

$$\boxed{f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right) - \frac{R^2}{L^2}}} \text{ Hz}$$

and $\boxed{Y = \frac{R}{R^2+(\omega L)^2} \text{ ohm}^{-1}}$ **Ans**

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