Department of Mathematics

10B11MA111 **Mathematics-I**

Tutorial Sheet 3 **B.Tech. Core**

Double and Triple Integrals

1. Integrate

   (a) $f(x,y) = x^2 + y^2$ over the triangular region having vertices (0,0),(1,0) and (0,1).

   (b) $f(x,y) = (x+y)^2$ over the region bounded by the ellipse $x^2/a^2 + y^2/b^2 = 1$.

   Ans.(a)1/6, (b) $\pi ab(a^2+b^2)/4$

2. Evaluate the following integrals by sketching the region of integration. Also verify your result by changing the order of integration and evaluating the resulting integral..

   (a) $\int_0^1 \int_2^{4-2x} dydx$, (b) $\int_0^1 \int_y^{\sqrt{y}} dxdy$, (c) $\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3ydxdy$ Ans.(a) 1, (b) 1/6, (c) 2

3. Change the order of integration in the following integrals and evaluate (iii) and (iv).

   (i) $\int_0^{\frac{1}{\sqrt{2}}} \int_x^{\sqrt{1-x^2}} f(x,y)dy\,dx$, (ii) $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} f(r,\theta)dr\,d\theta$. (iii) $\int_0^{\infty}\int_x^{\infty} (e^{-y}/y)dy\,dx$, (iv) $\int_0^1 \int_{2y}^2 e^{-x^2} dx\,dy$

   Ans. (iii) 1 (iv) $(1-e^{-4})/4$

4. Evaluate by changing to polar coordinates:

   (a) $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$, (b) $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}} dxdy$, Ans. (a) $\pi a^4/8$, (b) $a^3\ln\left(\sqrt{2}+1\right)/3$

5. (i) Find by double integration, the area lying inside the cardioid $r = 1+\cos\theta$ and outside the parabola $r(1+\cos\theta)=1$. (ii) Find the volume bounded by the cylinder $x^2+y^2=a^2$ and the cone $x^2+y^2=z^2$. Ans. (i) $\frac{3\pi}{4} - \frac{4}{3}$ (ii) $\frac{4\pi a^3}{3}$.

6. Calculate by double integration the volume generated by the revolution of the cardioid $r = a(1-\cos\theta)$ about its axis. Ans. $8\pi a^3/3$

7. Using suitable transformation, evaluate $\iint_R (x-y)^2 \cos^2(x+y)dxdy$, where *R* is the rhombus with successive vertices at (*л*, 0), (2*л*, *л*), (*л*, 2 *л*) and (0, *л*). Ans. $\pi^4/3$

8. Using the transformation $x + y = u$, $y = u\,v$, show that $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dydx = \frac{e-1}{2}$.

9. Find the volume enclosed by the solid bounded above by the sphere $x^2+y^2+z^2 = 2a^2$ and below by the paraboloid $az = x^2+y^2$. Ans. $\left(\frac{4\sqrt{2}}{3} - \frac{7}{6}\right)\pi a^3$.

10. Find the mass, moment of inertia, radius of gyration and centre gravity of a thin plate bounded by the curves $y = x^2$ and $y = x+2$ whose density $\rho(x,y)$ is constant. (M= $9\rho/2$)

11. (*a*) Find the volume of the solid lying in the first octant bounded by the paraboloid $z = 36 - 4x^2 - 9y^2$. Ans. 27π

    (*b*) Find the volume of the solid enclosed between the surfaces $x^2+y^2=a^2$ and $x^2+z^2=a^2$.

    Ans. $(16/3)a^3$