Signals and Systems

Signals and their classifications-III

Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2(t)dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

NOTE:A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0

Energy of power signal = ∞

Signal Energy and Power

A signal with finite signal energy is called an **energy signal**.

A signal with infinite signal energy and finite average signal power is called a **power signal**.

Cont..

Ex- $x(t) = e^{-2t} u(t)$

$$E = \int_{-\infty}^{\infty} |x^2(t)| \, dt \quad \text{or} \quad \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{0}^{\infty} |e^{-2t}|^2 \, dt$$

$$= \int_{0}^{\infty} e^{-4t} \, dt$$

$$= \frac{e^{-4t}}{4} \Bigg|_{0}^{\infty}$$

$$= \left| \frac{e^{-\infty} - e^{0}}{4} \right| = \left| \frac{0-1}{4} \right|$$

$$E = \frac{1}{4}$$

Cont..

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)\, dt$$

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{0}^{T} e^{-4t}\, dt$$

$$= \frac{1}{2T} \left| \frac{e^{-4t}}{-4} \right|_{0}^{T} \quad T \to \infty$$

$$= \frac{-e^{-4t} + 1}{8T} \quad T \to \infty$$

$$P = 0$$

$\therefore$ $x(t) = e^{-2t} u(t)$ is an energy signal

BASIC CONTINUOUS-TIME SIGNALS

A. The Unit Step Function:

The *unit step* function $u(t)$, also known as the *Heaviside unit* function, is defined as

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases} \tag{1.18}$$

which is shown in Fig. 1-4(a). Note that it is discontinuous at $t = 0$ and that the value at $t = 0$ is undefined. Similarly, the shifted unit step function $u(t - t_0)$ is defined as

$$u(t - t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases} \tag{1.19}$$

which is shown in Fig. 1-4(b).

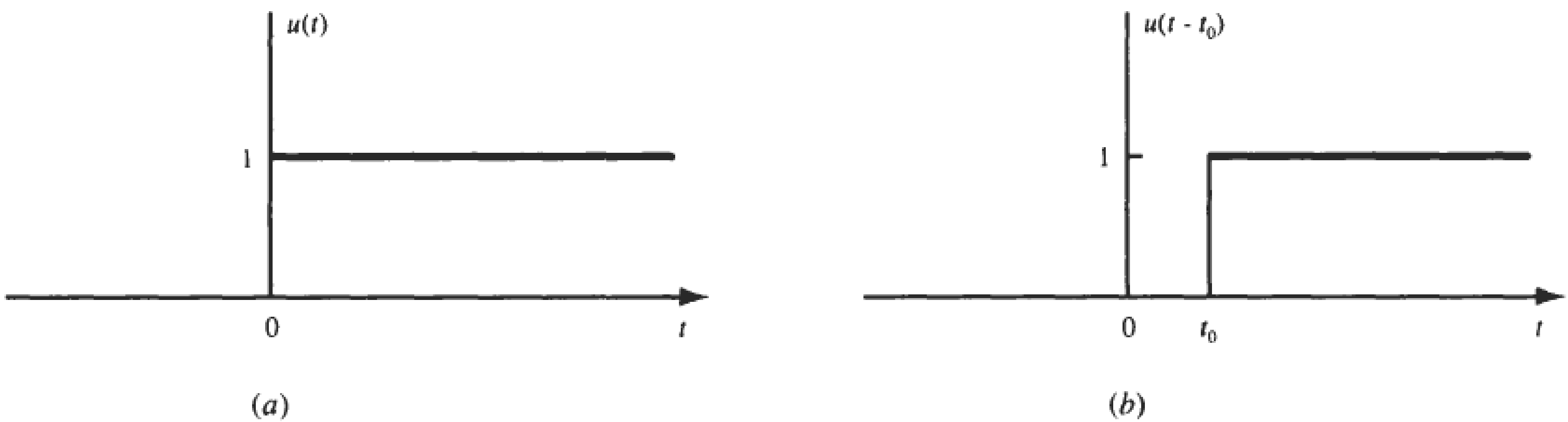


Fig. 1-4 (a) Unit step function; (b) shifted unit step function.

B. The Unit Impulse Function:

The *unit impulse* function $\delta(t)$, also known as the *Dirac delta* function, plays a central role in system analysis. Traditionally, $\delta(t)$ is often defined as the limit of a suitably chosen conventional function having unity area over an infinitesimal time interval as shown in Fig. 1-5 and possesses the following properties:

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

$$\int_{-\varepsilon}^{\varepsilon} \delta(t)\, dt = 1$$

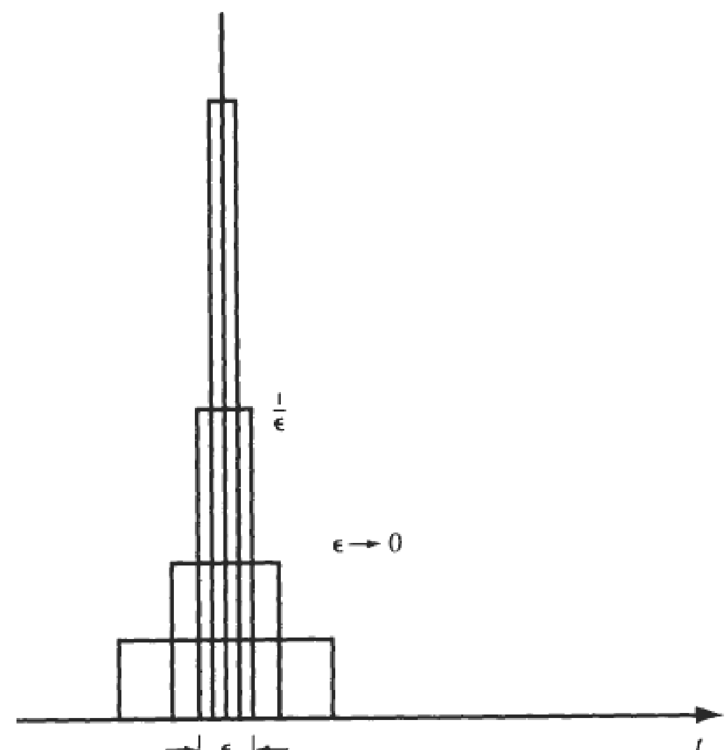
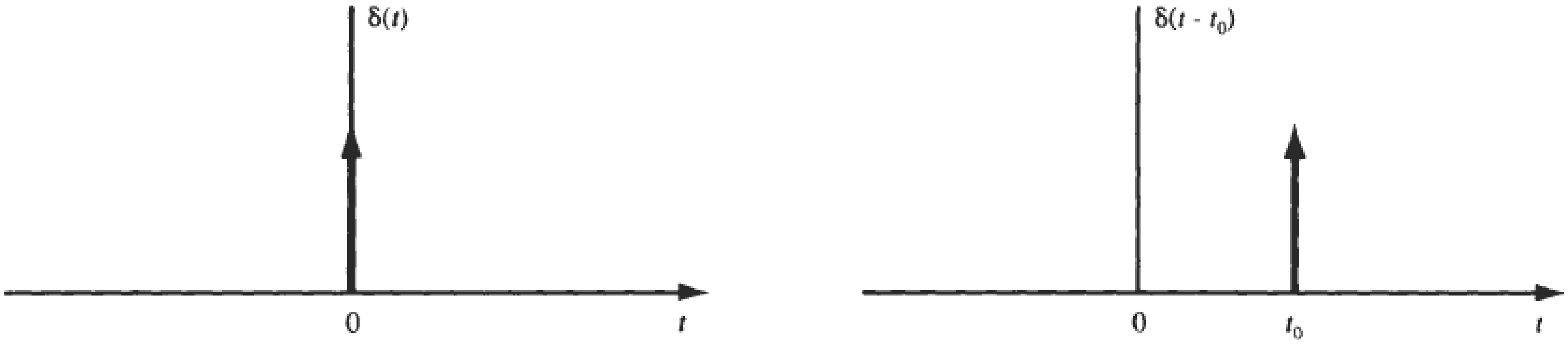


Fig. 1-5

Cont..



(a) (b)

Fig. 1-6 (*a*) Unit impulse function; (*b*) shifted unit impulse function.

Some additional properties of $\delta(t)$ are

$$\delta(at) = \frac{1}{|a|}\delta(t) \tag{1.23}$$

$$\delta(-t) = \delta(t) \tag{1.24}$$

$$x(t)\delta(t) = x(0)\delta(t) \tag{1.25}$$

if $x(t)$ is continuous at $t = 0$.

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0) \tag{1.26}$$

if $x(t)$ is continuous at $t = t_0$.

Using Eqs. (*1.22*) and (*1.24*), any continuous-time signal $x(t)$ can be expressed as

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau \tag{1.27}$$

Relation between unit step function and unit impulse function

$$\delta(t) = u'(t) = \frac{du(t)}{dt}$$

Then the unit step function $u(t)$ can be expressed as

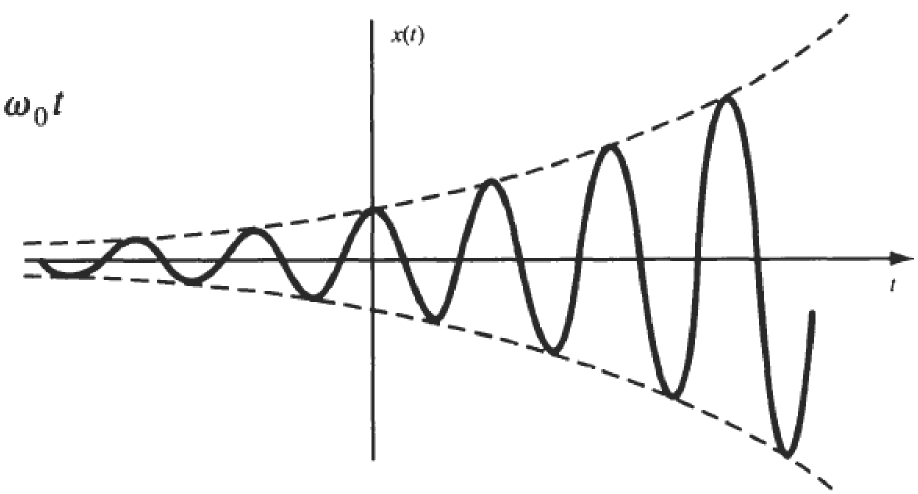
$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

C. Complex Exponential Signals:

The *complex exponential* signal

$$x(t) = e^{j\omega_0 t} \qquad (1.32)$$

$$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$$



BASIC DISCRETE-TIME SIGNALS

The *unit step* sequence $u[n]$ is defined as

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{1.43}$$

which is shown in Fig. 1-10(a). Note that the value of $u[n]$ at $n = 0$ is defined [unlike the continuous-time step function $u(t)$ at $t = 0$] and equals unity. Similarly, the shifted unit step sequence $u[n - k]$ is defined as

$$u[n-k] = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases} \tag{1.44}$$

which is shown in Fig. 1-10(b).

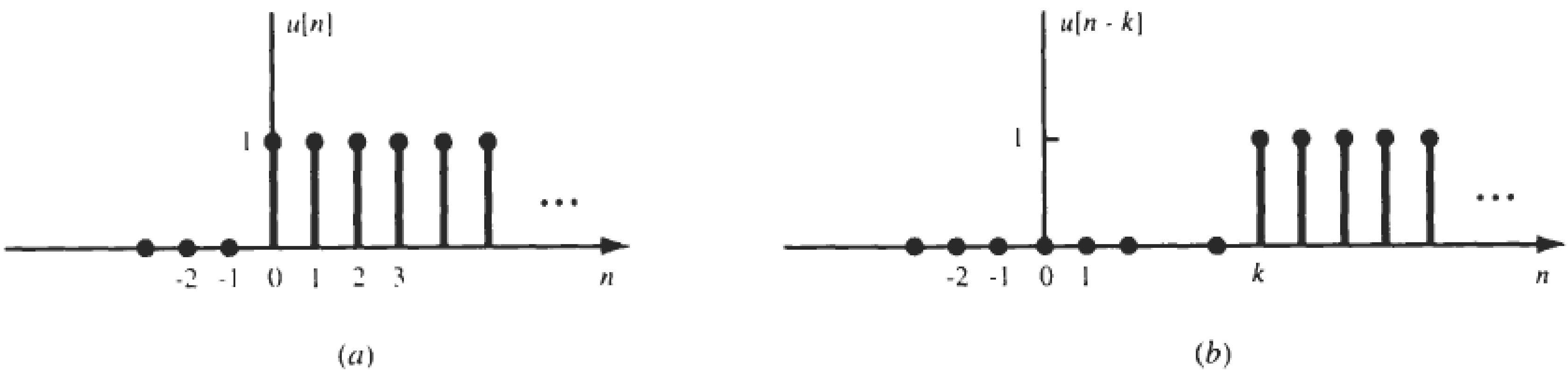


Fig. 1-10 (a) Unit step sequence; (b) shifted unit step sequence.

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B. The Unit Impulse Sequence:

The *unit impulse* (or *unit sample*) sequence $\delta[n]$ is defined as

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \tag{1.45}$$

which is shown in Fig. 1-11(a). Similarly, the shifted unit impulse (or sample) sequence $\delta[n-k]$ is defined as

$$\delta[n-k] = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases} \tag{1.46}$$

which is shown in Fig. 1-11(b).

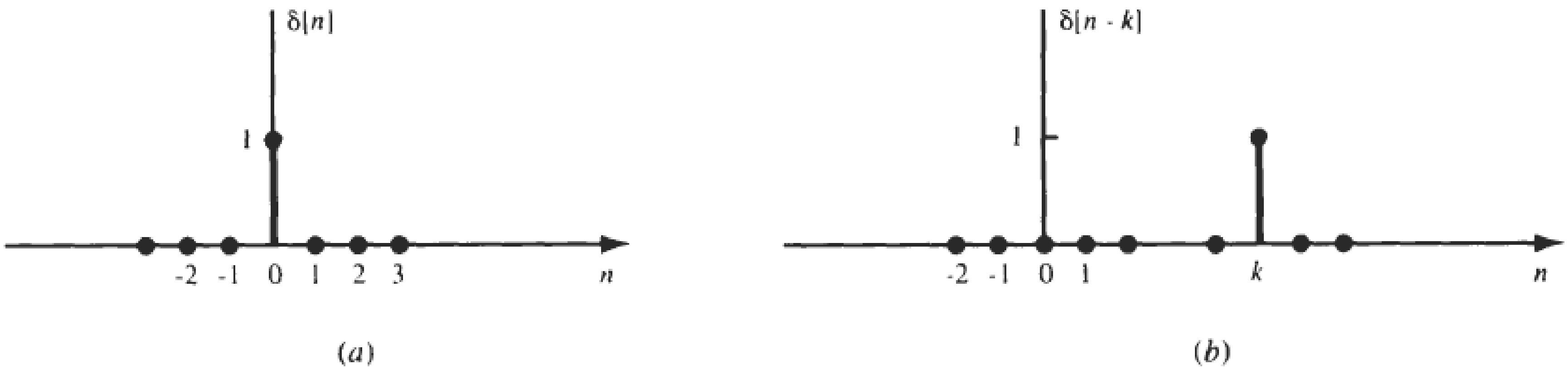
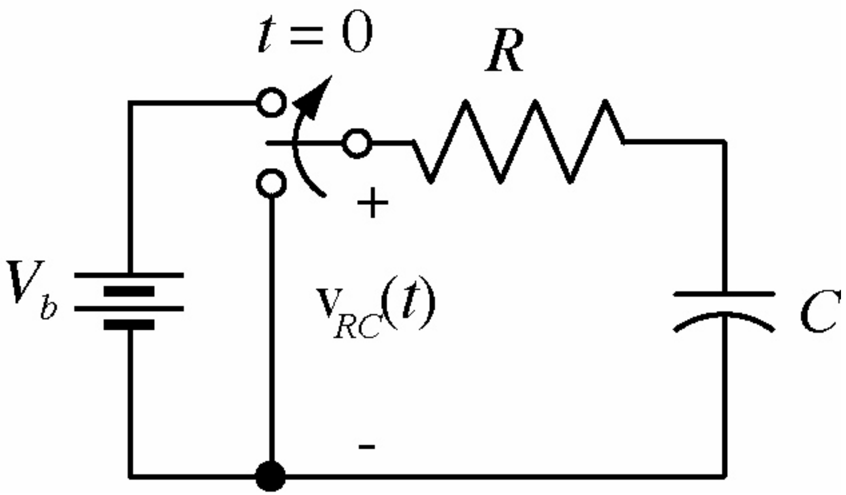


Fig. 1-11 (a) Unit impulse (sample) sequence; (b) shifted unit impulse sequence.

The Unit Step Function

The unit step function can mathematically describe a signal that is zero up to some point in time and non-zero after that.

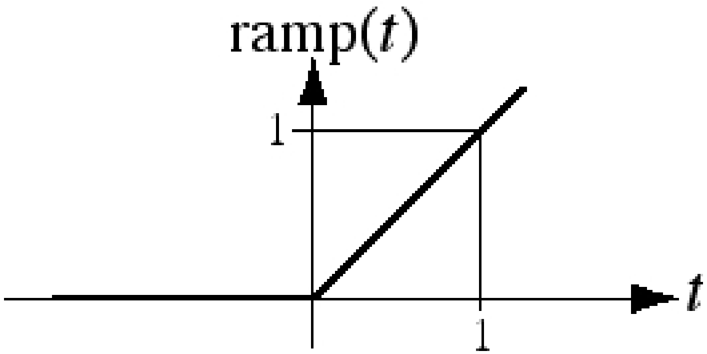


$$\mathrm{v}_{RC}(t) = V_b\,\mathrm{u}(t)$$

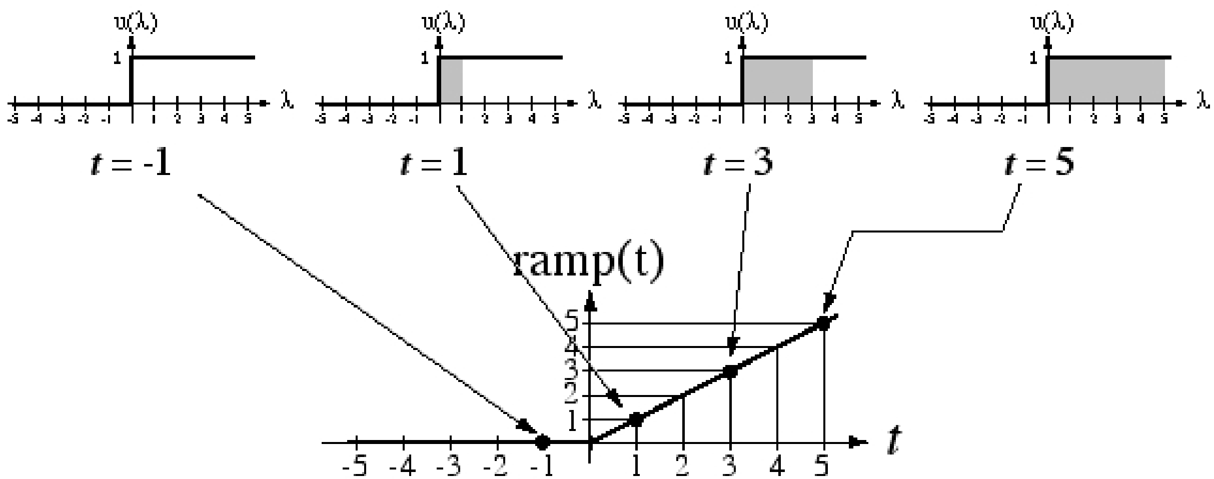
$$\mathrm{i}(t) = \left(V_b / R\right)e^{-t/RC}\,\mathrm{u}(t)$$

$$\mathrm{v}_C(t) = V_b\left(1 - e^{-t/RC}\right)\mathrm{u}(t)$$

The Unit Ramp Function

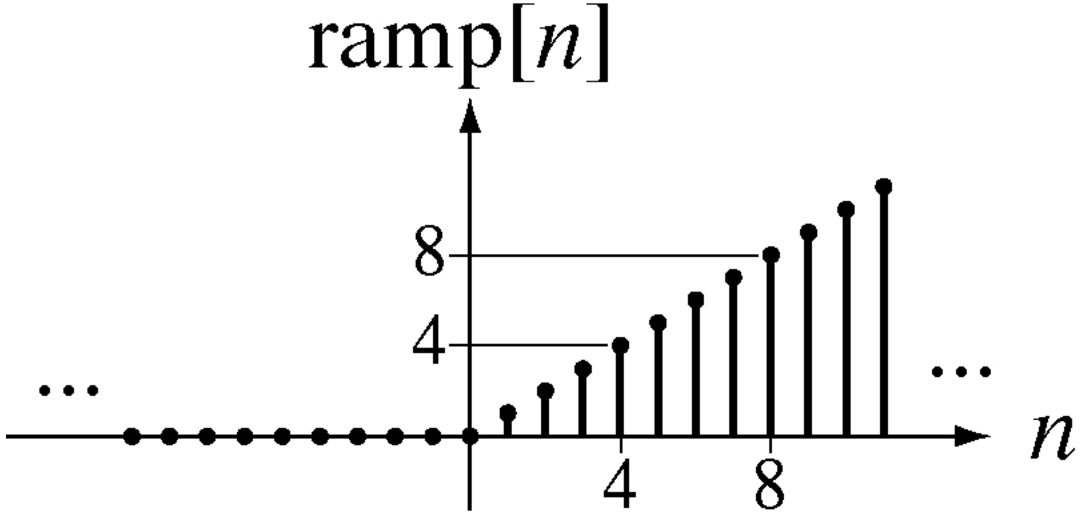


$$\mathrm{ramp}(t) = \begin{cases} t \;, & t > 0 \\ 0 \;, & t \le 0 \end{cases} = \int_{-\infty}^{t} \mathrm{u}(\lambda)\, d\lambda = t\,\mathrm{u}(t)$$



The Unit Ramp Function

$$\mathrm{ramp}[n] = \begin{cases} n \ , & n \geq 0 \\ 0 \ , & n < 0 \end{cases} = n\,\mathrm{u}[n] = \sum_{m=-\infty}^{n} \mathrm{u}[m-1]$$



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C. Complex Exponential Sequences:

The *complex exponential* sequence is of the form

$$x[n] = e^{j\Omega_0 n} \tag{1.52}$$

Again, using Euler's formula, $x[n]$ can be expressed as

$$x[n] = e^{j\Omega_0 n} = \cos \Omega_0 n + j \sin \Omega_0 n \tag{1.53}$$

Thus $x[n]$ is a complex sequence whose real part is $\cos \Omega_0 n$ and imaginary part is $\sin \Omega_0 n$.

Periodicity of $e^{j\Omega_0 n}$:

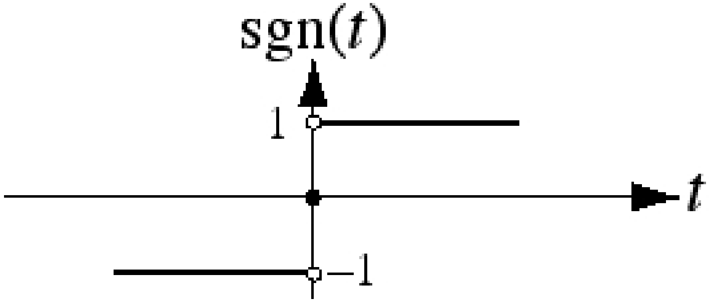
In order for $e^{j\Omega_0 n}$ to be periodic with period N (> 0), Ω_0 must satisfy the following condition (Prob. 1.11):

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} \qquad m = \text{positive integer} \tag{1.54}$$

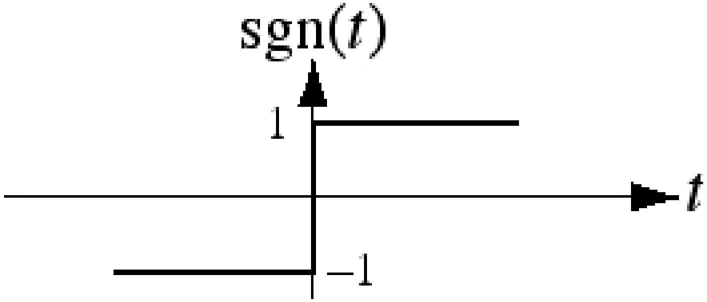
The Signum Function

$$\operatorname{sgn}(t) = \begin{Bmatrix} 1 & , & t > 0 \\ 0 & , & t = 0 \\ -1 & , & t < 0 \end{Bmatrix}$$

Precise Graph



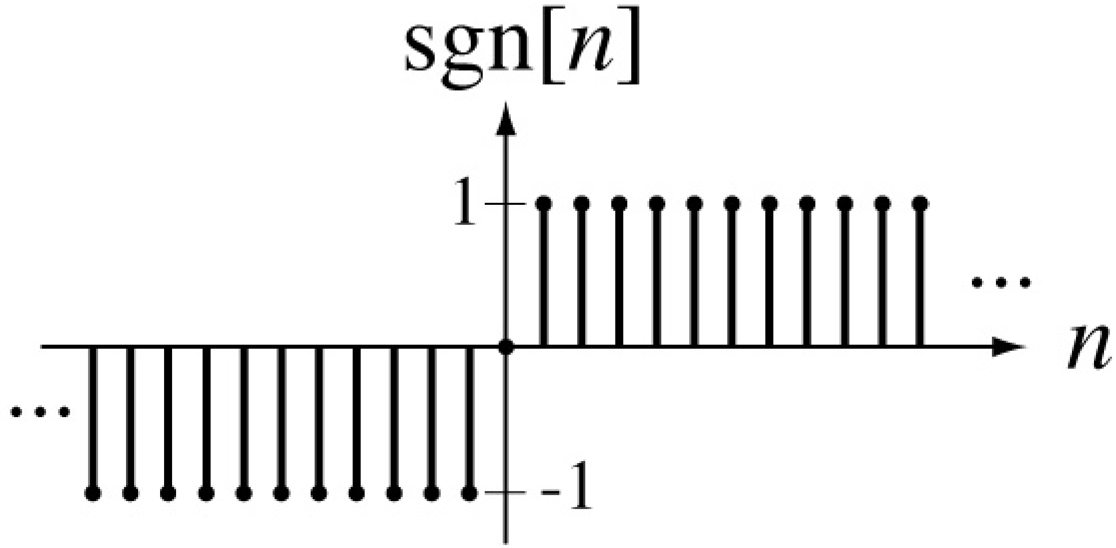
Commonly-Used Graph



The signum function, in a sense, returns an indication of the sign of its argument.

The Signum Function

$$\operatorname{sgn}[n] = \begin{cases} 1 \quad , & n > 0 \\ 0 \quad , & n = 0 \\ -1 , & n < 0 \end{cases}$$



Thank You