Suburban bus routing problem

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ABSTRACT

Rural and suburban areas have a low density of population. The bus services for these areas have unique features such as no transfer exists as there is only one bus route for each area and nearly all passengers travel to the same destination of city center. We formulate such a suburban bus route design problem as an optimization model. We subsequently prove that the problem is NP-hard. In view of the problem structure, a dynamic programming approach is developed to obtain the optimal solution efficiently for practical-size problems.

Problem description and Modelling

- Consider the bus starts at a suburban bus station $\rightarrow$ node 0
- Let the destination be the city centre $\rightarrow$ node $N + 1$
- Let there be N locations for bus stops from the bus station and city centre

Each location has 2 stops (i.e), 1, -1, 2, -2, . . . N, -N , where

i, -i $\rightarrow$ same geographical location ; opposite directions

AIM - to find the minimum distance between nodes 0 and N+1 visiting all i or -i,

$\forall\ i \in \{1, 2, 3, \ldots N\}$

Hardness of the Problem

- This problem can be restated as a decision making problem. How?
- Is the bus route shorter than L ? Polynomial time
- Proving NP completeness
  - NP Hard
  - Every NP hard problem should be reducible to this in polynomial time
- It is an NP hard problem - decision making - polynomial time

NP Completeness

- We show the NP completeness of the problem by reducing a well-known NP-complete problem
- Consider the TSP problem
- Assume
  - The nodes 0 and N+1 coincide
  - The nodes i and -i coincide for i = 1 , 2 , . . . , N
- The problem now resembles a TSP

Algorithm prerequisites

- The algorithm starts with 0 and terminates at N+1
- f(i , S) definition →

  the least cost to go from i to N + 1 passing through all the vertices in S

- If both k ∈ S and -k ∈ S them, the bus needs to visit any one of the two
- $d_{i,j}$ is the distance between bus stops i → j

Algorithm

Procedure f(i, S)

if $S = \varnothing$

$f(i,S) = d_{i,N+1}$

else if $S = \{ k, -k \}$

$f(i,S) = \min(d_{i,k} + d_{k,N+1} , d_{i,-k} + d_{-k,N+1})$

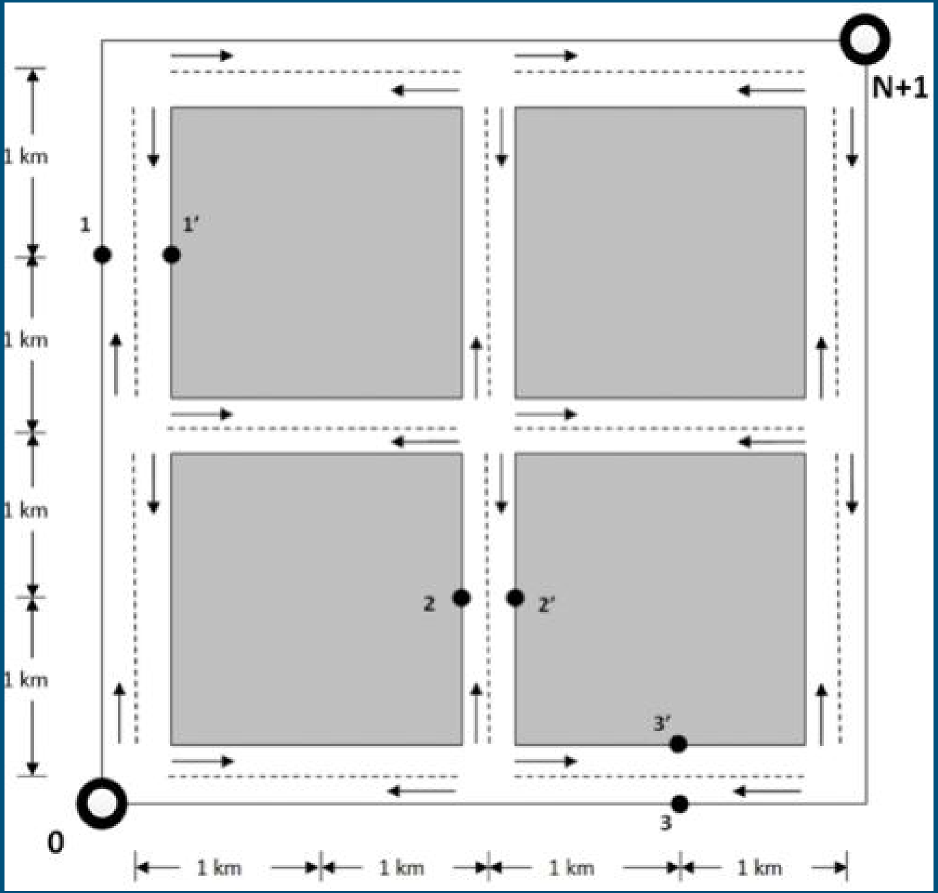
else

$f(i, S) = \min_{\forall j \in S} (d_{i,j} + f(j , S - \{ j, -j\}))$

Example

A square block model. Problem constraints

- Buses can make U turns only at the end of roads
- Time for turning is taken to be 0



Adjacency matrix

	0	1	1'	2	2'	3	3'	4
0	0	3	5	3	5	5	3	8
1	5	0	2	8	6	10	8	5
1'	3	2	0	6	4	8	6	7
2	5	4	6	0	2	6	4	5
2'	3	6	8	2	0	4	2	7
3	3	6	8	2	4	0	2	7
3'	5	6	8	4	6	2	0	5
4	8	7	5	7	5	5	7	0

References

Research paper - Shuaian Wang and Xiaobo Qu (2014) “Rural bus route design Problem: Model Development and Case Studies” *KSCE Journal of Civil Engineering (2015)*

CLRS - Dynamic Programming, theory of computation