CSED342 Assignment 8

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Problem 2a

Knowledge Base in CNF

First, we convert each formula in the knowledge base $KB = \{(A \vee B) \to \neg C, \neg(\neg A \vee C) \to D, A\}$ into CNF.

1. Convert $(A \vee B) \to \neg C$:

   $$(A \vee B) \to \neg C \equiv \neg(A \vee B) \vee \neg C.$$

   Using De Morgan's laws:

   $$\neg(A \vee B) \equiv \neg A \wedge \neg B.$$

   Thus:

   $$\neg(A \vee B) \vee \neg C \equiv (\neg A \wedge \neg B) \vee \neg C \equiv (\neg A \vee \neg C) \wedge (\neg B \vee \neg C).$$

2. Convert $\neg(\neg A \vee C) \to D$:

   $$\neg(\neg A \vee C) \to D \equiv \neg\neg(\neg A \vee C) \vee D \equiv (\neg A \vee C) \vee D \equiv \neg A \vee C \vee D$$

3. The formula A is already in CNF.

Thus, the CNF form of the knowledge base is:

$$KB = \{\neg A \vee \neg C, \neg B \vee \neg C, \neg A \vee C \vee D, A\}.$$

Derivation using Modus Ponens

Now, we use Modus Ponens to derive D.

1. From A, apply to $\neg A \vee \neg C$:

   $$\frac{A, \neg A \vee \neg C}{\neg C} \quad \text{(Modus Ponens)}$$

2. From $\neg C$, apply to $\neg A \vee C \vee D$:

   $$\frac{\neg C, C \vee \neg A \vee D}{\neg A \vee D} \quad \text{(Modus Ponens)}$$

3. From A, apply to $\neg A \vee D$:

$$\frac{A, \neg A \vee D}{D} \quad \text{(Modus Ponens)}$$

Thus, we derive D.

Problem 2b

Knowledge Base in CNF

Convert the knowledge base $KB = \{A \vee B, B \to C, (A \vee C) \to D\}$ into CNF.

1. The formula $A \vee B$ is already in CNF.
2. Convert $B \to C$:
$$B \to C \equiv \neg B \vee C.$$
3. Convert $(A \vee C) \to D$:
$$(A \vee C) \to D \equiv \neg(A \vee C) \vee D \equiv (\neg A \wedge \neg C) \vee D \equiv (\neg A \vee D) \wedge (\neg C \vee D).$$

Thus, the CNF of the knowledge base is:

$$KB = \{A \vee B, \neg B \vee C, \neg A \vee D, \neg C \vee D\}.$$

Derivation using the Resolution Rule

We use the resolution rule to derive D.

1. Resolve $A \vee B$ and $\neg B \vee C$:
$$\frac{A \vee B, \neg B \vee C}{A \vee C} \quad \text{(Resolution)}$$
2. Resolve $A \vee C$ and $\neg A \vee D$:
$$\frac{C \vee A, \neg A \vee D}{C \vee D} \quad \text{(Resolution)}$$
3. Resolve $C \vee D$ and $\neg C \vee D$:
$$\frac{D \vee C, \neg C \vee D}{D} \quad \text{(Resolution)}$$

Thus, we derive D.