Linear Algebra Worksheet 4

Jonny Evans

Workshop 4

Exercise 4.1. Find the determinants of the following matrices by using the inductive formula.

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 7 \\ 0 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 5 & 13 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Exercise 4.2. Compute the determinant of the matrix

$$A = \begin{pmatrix} 1 & t & 1 \\ 2 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix}.$$

For which value of t does this matrix fail to be invertible? For this value of t, find an element of $\ker(A)$.

Exercise 4.3. For each matrix below, find its characteristic polynomial, its eigenvalues and its eigenvectors.

$$A = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad E = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$G = \begin{pmatrix} -1 & -2 & 2 \\ -1 & 1 & 1 \\ -5 & -4 & 6 \end{pmatrix}, \quad H = \begin{pmatrix} -2 & 3 & -3 \\ -6 & 7 & -6 \\ -6 & 6 & -5 \end{pmatrix}, \quad J = \begin{pmatrix} 18 & -5 & -6 \\ 81 & -20 & -18 \\ -22 & 6 & 7 \end{pmatrix}$$

Exercise 4.4. Write down two 2-by-2 matrices A, B with $\det(A) = \det(B) = 1$. Find $\det(A + B)$. Repeat twice more with different matrices. Can you get any value for $\det(A + B)$?

Exercise 4.5. Suppose that A has an eigenvector v with eigenvalue λ. Show that $\exp(A)$ has v as an eigenvector and find the eigenvalue.

Exercise 4.6. The *Jacobian* of a differentiable map $F: \mathbb{R}^n \to \mathbb{R}^m$ is the m-by-n matrix

$$Jac(F) = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \cdots & \frac{\partial F_1}{\partial x_{n-1}} & \frac{\partial F_1}{\partial x_n} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \cdots & \frac{\partial F_2}{\partial x_{n-1}} & \frac{\partial F_2}{\partial x_n} \\ \vdots & \vdots & & \vdots & \vdots \\ \frac{\partial F_m}{\partial x_1} & \frac{\partial F_m}{\partial x_2} & \cdots & \frac{\partial F_m}{\partial x_{n-1}} & \frac{\partial F_m}{\partial x_n} \end{pmatrix},$$

where $F(x_1, \ldots, x_n) = \begin{pmatrix} F_1(x_1, \ldots, x_n) \\ \vdots \\ F_m(x_1, \ldots, x_n) \end{pmatrix}$.

Find $\det(Jac(F))$ in the following examples:

1. $m = n = 2$, $F(r, \phi) = \begin{pmatrix} r\cos\phi \\ r\sin\phi \end{pmatrix}$.

2. $m = n = 3$, $F(r, \theta, \phi) = \begin{pmatrix} r \sin\theta \cos\phi \\ r \sin\theta \sin\phi \\ r \cos\theta \end{pmatrix}$

If $m = n$ and F is a change of coordinates $\mathbb{R}^n \to \mathbb{R}^n$ then the determinant of the Jacobian matrix is an important quantity: if $y = F(x)$ then the volume element $dy_1 \cdots dy_n$ is equal to $\det(Jac(F))dx_1 \cdots dx_n$. In the examples we've just computed the volume element in polar and spherical coordinates.

Exercise 4.7. Suppose that A is an n-by-n orthogonal matrix ($AA^T = I$).

1. Show that $\det(A) = \pm 1$.
2. If $\det(A) = 1$, show that $\det(A-I) = (-1)^n \det(A-I)$. (Hint: Use the fact that $A-I = A(I-A^T)$.)
3. Deduce that if n is odd then any orthogonal matrix with determinant one has a fixed vector.

Exercise 4.8. Let A be an n-by-n matrix with characteristic polynomial $\chi_A(t)$; suppose that $\chi_A(t)$ has n distinct roots. By considering $\chi_A(0)$, prove that $\det(A)$ is the product of the eigenvalues of A. One of the coefficients in the polynomial $\chi_A(t)$ is equal to minus the sum of the eigenvalues of A: which coefficient? *(Hint: Recall that if a polynomial $p(t)$ of degree n has roots $\lambda_1, \ldots, \lambda_n$ and the coefficient of t^n is 1 then $p(t) = (t - \lambda_1)(t - \lambda_2) \cdots (t - \lambda_n)$).*

4 Solutions

Solution 4.1. (a) We start with the matrix $A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 0 & 7 \\ 0 & 1 & 0 \end{pmatrix}$. Note that the final row only has one nonzero entry, so we should expand around that. There is only one term that contributes:

$$-\det \begin{pmatrix} 3 & 1 \\ 1 & 7 \end{pmatrix} = -(21-1) = -20.$$

(b) We start with the matrix $B = \begin{pmatrix} 1 & 2 & 5 & 13 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 3 \\ 1 & 1 & 1 & 1 \end{pmatrix}$. Expanding down the leftmost column, we get

$$\det(B) = \det \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} - 0 + \det \begin{pmatrix} 2 & 5 & 13 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \det \begin{pmatrix} 2 & 5 & 13 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}.$$

We have

$$\begin{aligned}
\det \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} &= 1(1 \times 1 - 3 \times 1) - 0 + 1(2 \times 3 - 1 \times 1) \\
&= 3, \\
\det \begin{pmatrix} 2 & 5 & 13 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} &= 2(2 \times 1 - 1 \times 1) - 5(1 \times 1 - 1 \times 1) + 13(1 \times 1 - 2 \times 1) \\
&= -11, \\
\det \begin{pmatrix} 2 & 5 & 13 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix} &= 2(2 \times 3 - 1 \times 1) - (5 \times 3 - 1 \times 13) + 0 \\
&= 8,
\end{aligned}$$

so $\det(B) = -16$.

(c) We start with the matrix $C = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$. Expanding down the second column gives

$$\det(C) = -\det \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

Expanding this subdeterminant down the first column gives

$$\det(C) = -\left(-\det \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) = 1.$$

Solution 4.2. We have

$$\det \begin{pmatrix} 1 & t & 1 \\ 2 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix} = \det \begin{pmatrix} t & 1 \\ 3 & 1 \end{pmatrix} - 0 + 2 \begin{pmatrix} 1 & t \\ 2 & 3 \end{pmatrix},$$

by expanding along the bottom row, which gives

$$\det \begin{pmatrix} 1 & t & 1 \\ 2 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix} = 3 - 3t.$$

This is zero (i.e. the matrix is not invertible) if and only if $t = 1$. If we set $t = 1$, an element of $\ker(A)$ is then a solution to $Av = 0$, that is a vector $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = Av = \begin{pmatrix} x + y + z \\ 2x + 3y + z \\ x + 2z \end{pmatrix}.$$

These equations imply that $x = -2z$ and $y = -x - z = z$, so the vector $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ is in (in fact spans) the kernel of A when $t = 1$.

Solution 4.3. (a) If $A = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix}$ then the characteristic polynomial is

$$\det \begin{pmatrix} 1 - t & 5 \\ 2 & 4 - t \end{pmatrix} = t^2 - 5t - 6.$$

The roots are $\frac{5 \pm \sqrt{25+24}}{2} = -1, 6$. The corresponding eigenvectors are $\begin{pmatrix} 1 \\ -2/5 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. For example, you get the first one by solving

$$\begin{pmatrix} -x \\ -y \end{pmatrix} = \begin{pmatrix} 1 & 5 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + 5y \\ 2x + 4y \end{pmatrix},$$

which gives $y = -2x/5$.

(b) If $B = \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix}$ then the characteristic polynomial is

$$\det \begin{pmatrix} 3 - t & -2 \\ 1 & 3 - t \end{pmatrix} = t^2 - 6t + 11.$$

The roots are $\frac{6 \pm \sqrt{36-44}}{2} = 3 \pm i\sqrt{2}$. The corresponding eigenvectors are $\begin{pmatrix} 1 \\ \pm i\sqrt{2} \end{pmatrix}$. To see this, you need to solve

$$\begin{pmatrix} (3 \pm i\sqrt{2})x \\ (3 \pm i\sqrt{2})y \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3x - 2y \\ x + 3y \end{pmatrix},$$

which gives $y = \pm ix/\sqrt{2}$.

(c) If $C = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ then the characteristic polynomial is

$$\det \begin{pmatrix} -t & i \\ i & -t \end{pmatrix} = t^2 + 1,$$

so the roots are $t = \pm i$. The corresponding eigenvectors are $\begin{pmatrix} \pm 1 \\ 1 \end{pmatrix}$.

(d) If $D = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ then we have

$$\det(D - tI) = \det \begin{pmatrix} -t & 1 & 0 \\ 0 & -t & 1 \\ 1 & 0 & -t \end{pmatrix} = -t^3 + 1.$$

The solutions of $t^3 = 1$ are $1, \mu, \mu^2$, where $\mu = e^{2\pi i/3}$. The corresponding eigenvectors are:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ \mu \\ \mu^2 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ \mu^2 \\ \mu \end{pmatrix}.$$

For example, you can find the second one by solving

$$\begin{pmatrix} y \\ z \\ x \end{pmatrix} = D \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \mu x \\ \mu y \\ \mu z \end{pmatrix},$$

which gives $y = \mu x$ and $z = \mu^2 x$.

(e) If $E = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ then we have

$$\det(E - tI) = \det \begin{pmatrix} 1-t & 1 & 0 \\ 1 & -t & 1 \\ 1 & 1 & -t \end{pmatrix} = (1-t)(-t(1-t)-1) - (1-t) = -t^3 + 2t^2 + t - 2.$$

It is easy to check that $t = 1$ is a solution. By long division of polynomials, we get that $-t^3 + 2t^2 + t - 2 = (t-1)(-t^2 + t + 2)$, and the roots of $-t^2 + t + 2 = 0$ are $\frac{-1 \pm \sqrt{1+8}}{2} = -1, 2$. Therefore the eigenvalues of E are $-1, 1, 2$. The corresponding eigenvectors are $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. For example, to find the first one, we need to solve

$$\begin{pmatrix} x+y \\ x+z \\ y+z \end{pmatrix} = E \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ -y \\ -z \end{pmatrix},$$

which means $y = -2x = -2z$, so $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ works.

(f) If $F = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ then the characteristic polynomial is

$$\chi_F(t) = \det(F - tI) = \det \begin{pmatrix} -t & 1 & 0 \\ 1 & -t & 1 \\ 0 & 1 & -t \end{pmatrix} = -t(t^2 - 1) - (-t) = -t^3 + 2t.$$

This factors as $-t(t^2 - 2)$ so the eigenvalues are $0, \pm\sqrt{2}$. The eigenvectors are $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ \sqrt{2} \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$.

For example, the first one comes from solving the equation

$$\begin{pmatrix} y \\ x+z \\ y \end{pmatrix} = F \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

which implies $y = 0$ and $x = -z$.

Solution 4.4. If we use the matrix $A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$ and $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ then $v^T A v = 1$ is our ellipsoid.

The characteristic polynomial of A is

$$\chi_A(t) = \det \begin{pmatrix} 2-t & -1 & 0 \\ -1 & 2-t & -1 \\ 0 & -1 & 2-t \end{pmatrix} = (2-t)((2-t)^2 - 1) - (2-t) = (2-t)(t^2 - 4t + 2).$$

The eigenvalues are therefore 2, $2\pm\sqrt{2}$, which are all positive. The corresponding eigenvectors are u, v, w solving $Au = 2u$, $Av = (2+\sqrt{2})v$ and $Aw = (2-\sqrt{2})w$. For example, if $u = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ then

$$2u = \begin{pmatrix} 2x \\ 2y \\ 2z \end{pmatrix} = Au = \begin{pmatrix} 2x - y \\ -x + 2y - z \\ 2z - y \end{pmatrix},$$

so $y = 0$ and $x = -z$, so $u = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ up to scale. Similarly, we find $v = \begin{pmatrix} -1/\sqrt{2} \\ 1 \\ -1/\sqrt{2} \end{pmatrix}$ and $w = \begin{pmatrix} 1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{pmatrix}$. These are then the principal directions and the principal radii are $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2\pm\sqrt{2}}}$.

(g) If $G = \begin{pmatrix} -1 & -2 & 2 \\ -1 & 1 & 1 \\ -5 & -4 & 6 \end{pmatrix}$ then the characteristic polynomial is

$$\det \begin{pmatrix} -1-t & -2 & 2 \\ -1 & 1-t & 1 \\ -5 & -4 & 6-t \end{pmatrix} = -t^3 + 6t^2 - 11t + 6,$$

which has roots $1, 2, 3$. You can guess that 1 is a root, and then find the other factors by polynomial long division. The eigenvectors are respectively $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$.

(h) If $H = \begin{pmatrix} -2 & 3 & -3 \\ -6 & 7 & -6 \\ -6 & 6 & -5 \end{pmatrix}$ then the characteristic polynomial turns out to be

$$-t^3 + 3t - 2,$$

which has roots $t = 1, 2$ (1 is a repeated root). The eigenvector for 2 is $\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$. For the eigenvalue 1, we solve

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 & 3 & -3 \\ -6 & 7 & -6 \\ -6 & 6 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2x + 3y - 3z \\ -6x + 7y - 6z \\ -6x + 6y - 5z \end{pmatrix}.$$

All three equations reduce to $y = x + z$, which cuts out a plane in $\mathbb{R}^3$. In other words, any vector of the form $\begin{pmatrix} x \\ x+z \\ z \end{pmatrix}$ is an eigenvector. We can get away with specifying two linearly independent eigenvectors spanning this plane, say $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ anda $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

(j) If $J = \begin{pmatrix} 18 & -5 & -6 \\ 81 & -20 & -18 \\ -22 & 6 & 7 \end{pmatrix}$ then the characteristic polynomial turns out to be

$$-t^3 + 5t^2 - 7t + 3,$$

which has roots $1, 3$ (where 1 is repeated). The eigenvector for 3 is $\begin{pmatrix} -4 \\ -18 \\ 5 \end{pmatrix}$. The eigenvector for 1 is $\begin{pmatrix} -3 \\ -15 \\ 4 \end{pmatrix}$ (although the eigenvalue is repeated, there is only one eigenvector).

Solution 4.5. There are many possible answers, for example $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \det \begin{pmatrix} 2 & 0 \\ 0 & 1/2 \end{pmatrix}$, or $A = \det \begin{pmatrix} 1 & 4 \\ 1 & 5 \end{pmatrix}$ $B = \det \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} = 1$. These give $\det(A+B) = 9/2$ and $\det(A+B) = -6$ respectively. The point is, there is no way to predict $\det(A+B)$ just from knowing $\det(A)$ and $\det(B)$. For example, $\det \begin{pmatrix} t & 0 \\ 0 & 1/t \end{pmatrix} = 1$ so if you take $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} t & 0 \\ 0 & 1/t \end{pmatrix}$ then you get $\det(A+B) = 2 + t + 1/t$ and this takes on every possible value as t varies. For example, if you want to get $\det(A+B) = D$ then you pick t satisfying $2 + t + 1/t = D$ (which is equivalent to $t^2 + (2-D)t + 1 = 0$ or $t = \frac{2-D\pm\sqrt{D^2-4D}}{2}$).

Solution 4.6. If $Av = \lambda v$ then $\exp(A)v = \sum_{n\geq 0} \frac{1}{n!} A^n v = \sum_{n\geq 0} \frac{1}{n!} \lambda^n v = e^\lambda v$ so $\exp(A)$ has v as an eigenvector with eigenvalue e^λ.

Solution 4.7. 1. The Jacobian matrix is

$$\begin{pmatrix} \frac{\partial(r\cos\phi)}{\partial r} & \frac{\partial(r\cos\phi)}{\partial\phi} \\ \frac{\partial(r\sin\phi)}{\partial r} & \frac{\partial(r\sin\phi)}{\partial\phi} \end{pmatrix} = \begin{pmatrix} \cos\phi & -r\sin\phi \\ \sin\phi & r\cos\phi \end{pmatrix},$$

whose determinant is $r\cos^2\phi + r\sin^2\phi = r$. ! The Jacobian matrix is

$$\begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix},$$

whose determinant is (using the inductive formula, expanding along the bottom row):

$$\cos\theta(r^2\cos\theta\sin\theta\cos^2\phi + r^2\cos\theta\sin\theta\sin^2\phi) - (-r\sin\theta)(r\sin^2\theta\cos^2\phi + r\sin^2\theta\sin^2\phi),$$

which simplifies to $r^2\sin\theta$.

Solution 4.8. 1. We have $1 = \det(I) = \det(AA^T) = \det(A)\det(A^T) = \det(A)^2$, so $\det(A) = \pm 1$.

2. We have

$$\begin{aligned} \det(A-I) &= \det(A(I-A^T)) \\ &= \det(A)\det(I-A^T) \\ &= \det(A)\det(I-A) \\ &= \det(A)\det(-I)\det(A-I) \\ &= (-1)^n\det(A-I), \end{aligned}$$

so if n is odd then $\det(A-I) = -\det(A-I)$, which means $\det(A-I) = 0$.

3. Since $\det(A-I) = 0$, 1 is a root of the characteristic polynomial, so A has an eigenvector with eigenvalue 1. Such a vector is a fixed vector.

Solution 4.9. The characteristic polynomial is both $\det(A-tI)$ and $(t-\lambda_1)\cdots(t-\lambda_n)$ where $\lambda_1,\ldots,\lambda_n$ are the eigenvalues. The constant coefficient is therefore both $\chi_A(0)$ and $\lambda_1\cdots\lambda_n$, so $\chi_A(0) = \lambda_1\cdots\lambda_n$, and the coefficient of t^{n-1} is $-\lambda_1 - \cdots - \lambda_n$.