Constructing Lagrangian torus fibrations

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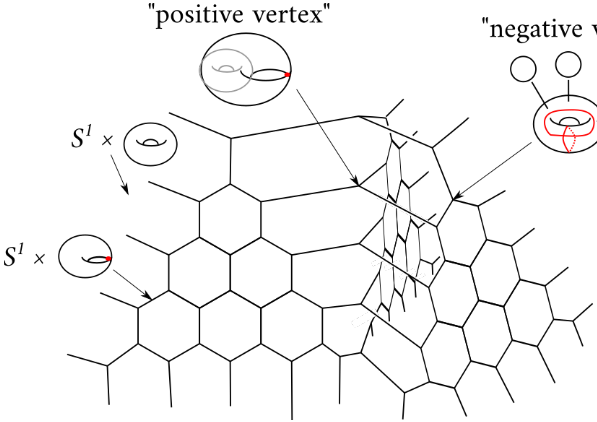
joint with Mirko Mauri

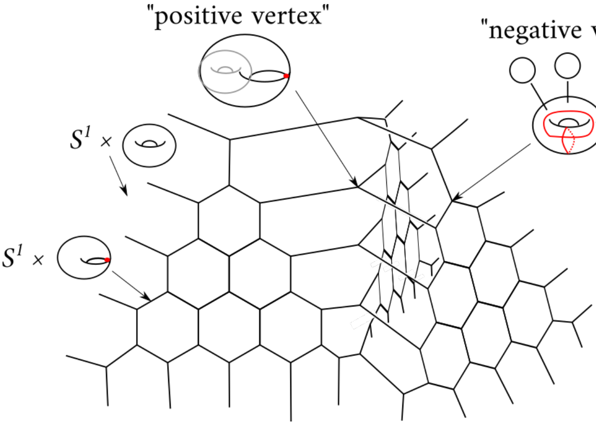
Theorem (Gross 1999)

Many Calabi-Yau 3-folds admit topological torus fibrations over S^3 with singular fibres over a trivalent graph.



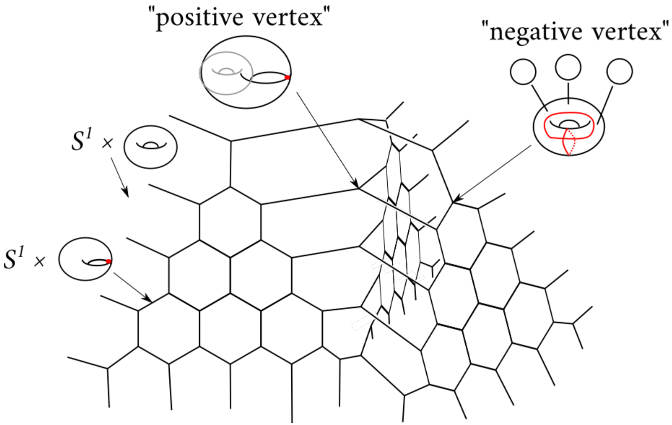
Question

- Can we make these fibrations Lagrangian?
- Even just (semi-)locally?



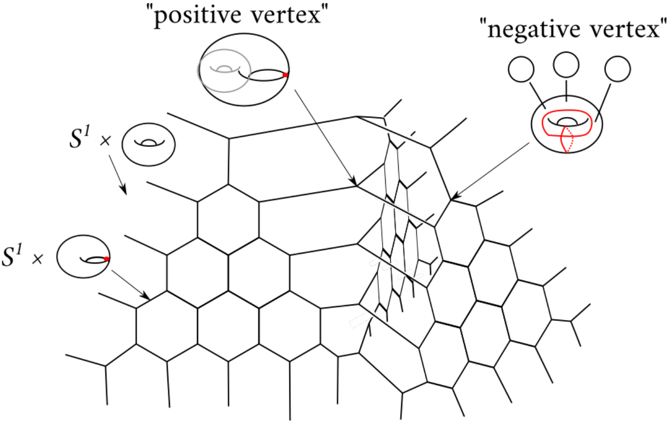
Positive vertex: Yes!

$$F\colon \mathbb{C}^3 \setminus \{xyz - 1 = 0\} \to \mathbb{R}^3$$
$$F(x, y, z) = \big(|x|^2 - |y|^2, |x|^2 - |z|^2, |xyz - 1|\big)$$



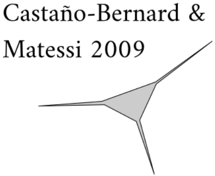
Negative vertex: ?

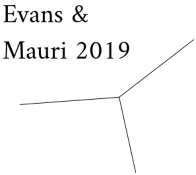
$$F : \mathbb{C}^3 \setminus \{(xy - z - 1)z = 0\} \to \mathbb{R}^3?$$



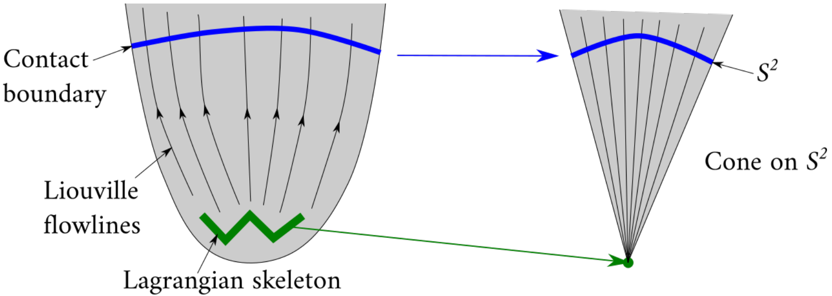
Negative vertex: Yes!

$$F \colon \mathbb{C}^3 \setminus \{(xy - z - 1)z = 0\} \to \mathbb{R}^3$$



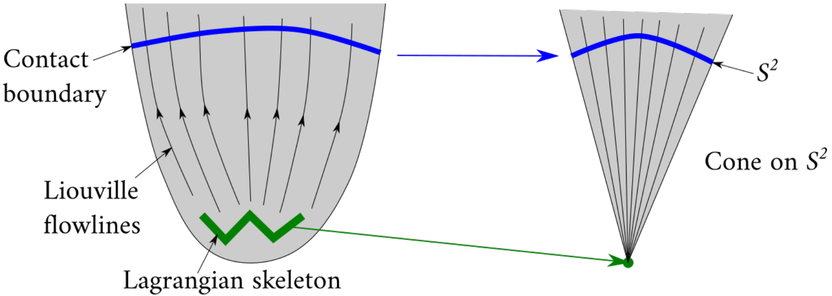


Proof

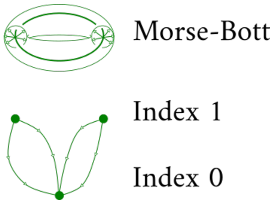


Contact boundary
Liouville flowlines
Lagrangian skeleton
S^2
Cone on S^2

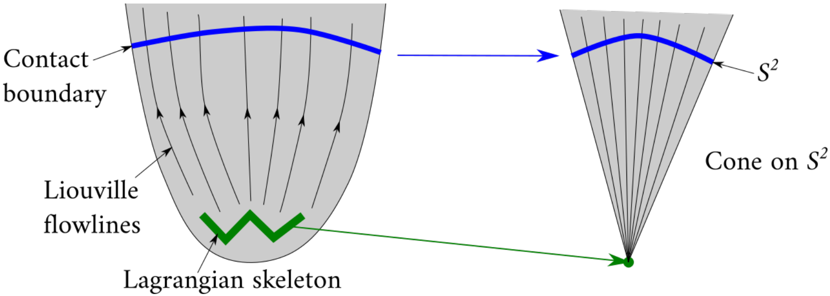
Proof



1. Lagrangian skeleton is Gross fibre.



Proof



1. Lagrangian skeleton is Gross fibre.
2. Contact boundary has LTF with three nodal fibres.