bond_project

Charles Dotson

A case study using real world data to formulate optimized dedication, immunization, and other bond portfolios.

LEHIGH UNIVERSITY

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1 Case Study

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ISE 447 - Financial Optimization
Lehigh Univeristy

[31]:
```
'''
Package Imports
'''
import pandas as pd
import numpy as np
import pulp
from pulp import *
import datetime
import matplotlib.pyplot as plt

import FinOpsCodeDeck as finops

from IPython.display import Markdown as md
# %matplotlib widget
```

2 Term Structure

Determine the current term structure of treasury rates (see textbook Section 3.4 or other resources that you can find), and find the present value, duration, and convexity of the stream of liabilities. Please explain the main steps followed in your calculations. Use real world data.

2.1 Deriving Term Structure

In this section, we describe our derivation of the term structure of interest rates. Specifically, we outline our data gathering and transformation techniques and then move to explaining bootstrapping.

2.1.1 Data and Transformations

We begin by pulling current US Treasury issued Bonds and Notes from The Wall Street Journal. We transform this data so we can understand each bonds market. Specifically, we create a bid and ask price for each bond called `'px_bid'` or `'px_ask'`. We also take the maturity of the bond less today's date to get a time to maturity field called `'ttm'`. This time to maturity is a float datatype which represents the years to maturity according to an actual/365 day calendar, the standard calendar of US Treasury Bonds. For sake of simplicity, we use this calendar for the notes as well despite these operating on a 30/360 calendar. Having completed these transformations, we can move to bootstrapping the curve.

2.1.2 Bootstrapping

Bootstrapping is a technique used to find continuous annualized interest rates across all time to maturities. Due to the nature of fixed income securities paying intermediate coupons, bootstrapping is necessary to value a cashflow from one specific point in time to any other. To better understand this, consider the following example.

Example
Let the current market only consist of 2 risk-free bonds that were issued today:
* 1-year zero-coupon bond trading at 99c on the dollar
* 2-year 1.5% annual coupon bond trading at par
To bootstrap the curve, we start with the 1-year zero.

$$99 = 100\exp\{-r\} \implies r = -\log(0.99) \approx 0.01$$

We then use this rate in our calculation with the coupon bond to find the 2-year rate.

$$100 = 1.5\exp\{-0.01(1)\} + 101.5\exp\{-2r\} \implies r \approx 0.0145$$

In this example, we have found the term structure to be given by:
Time to Maturity	Rate
–	–
1	1.00%		2	1.45%

So, doing this over all cashflows of all bonds in our data will allow us to derive a term structure across all maturities. This derived term structure will drive our analysis.

NOTE: For sake of simplicity, we round all time to maturity to the nearest hundredth of a year. From a bond trading perspective, this is essentially every 2.5 trading days representing 1 time period. We do this for simplicity in later sections as not all dates marry exactly together. In the event that a particular liability does not have a term structure rate associated with it, we use the closest prior known date. Additionally, in the event there are multiple calculated yields for a particular time to maturity, we take the arithmetic average of them for that time.

[32]:
```
'''
Data Import for Current Term Structure
---
Imports all active treasury bonds data, time indexes them by year
'''
data_prompt = pd.read_excel('Table.xlsx', sheet_name='PromptUse', index_col =␣
 ↪'DateDue')
data_prompt = data_prompt/1000
term_structure_df = pd.read_excel('TableNew.xlsx', sheet_name='d')
term_structure_df['px_ask'] = [i if i>=5 else 100 - i for i in␣
 ↪term_structure_df['ASKED'].to_list()]
term_structure_df['px_bid'] = [i if i>=5 else 100 - i for i in␣
 ↪term_structure_df['BID'].to_list()]
term_structure_df['ttm'] = [(i - datetime.datetime.now())/datetime.
 ↪timedelta(days=365) for i in term_structure_df['MATURITY']]
```

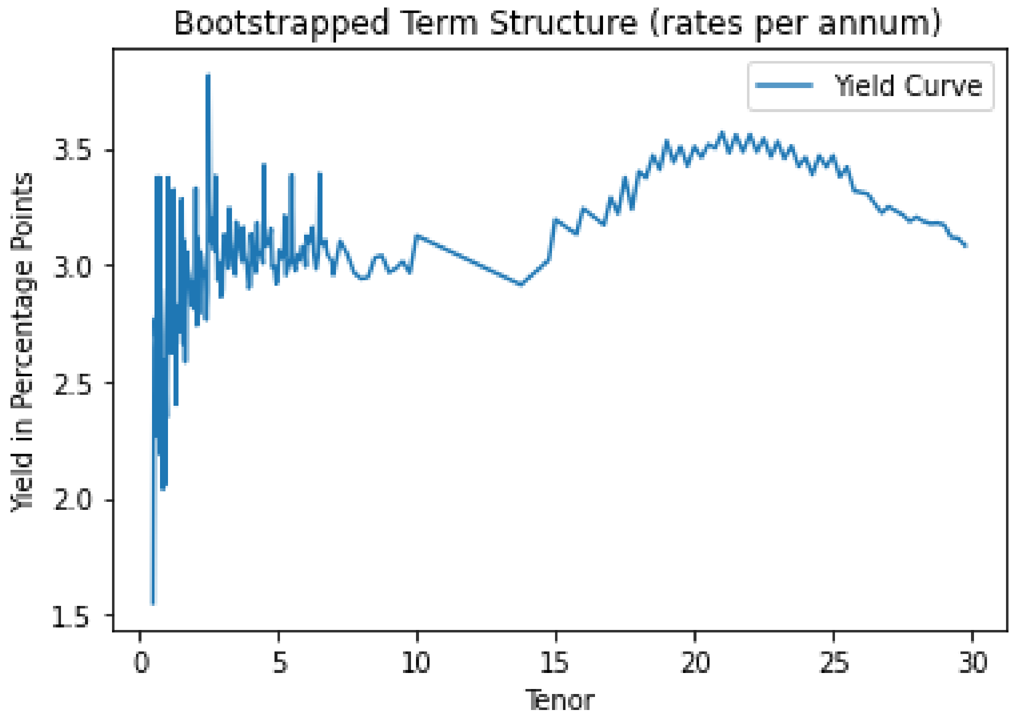
[33]:
```
'''
Bootstrap yield curve
---
begins with zero-coupon bonds to payout (ttm < 0.5 yrs) & calculates yield
```

```
moves to coupon bonds and uses calculated yields to bootstrap further
sorts all bonds into data frame indexed by ttm (by 100th of a year)
NOTE: Averages yields for the same time period
NOTE: assumes yield of period prior if yield for desired period does not exist
'''

'''short term rates'''
mats = []
round_to = 2
for bond_tenor in term_structure_df[term_structure_df['ttm'] <= 0.5].index:
    bond = term_structure_df.loc[bond_tenor]
    cpn = bond['COUPON']/2
    ttm = bond['ttm']
    px = bond['px_ask']
    mats.append([np.round(ttm,round_to),np.log((100 + cpn) / bond['px_ask']) /␣
 ↪bond['ttm']])
rates = pd.DataFrame(mats, columns=['ttm','rate']).set_index('ttm').
 ↪groupby('ttm').mean()

'''longer term rates'''
for bond_tenor in term_structure_df[term_structure_df['ttm']>=0.5].index:
    bond = term_structure_df.loc[bond_tenor]
    px = bond['px_ask']
    ttm = bond['ttm']
    cpn = bond['COUPON']/2
    pmts = int(np.ceil(ttm * 2))
    cfs = [cpn if i+1<pmts else 100 + cpn for i in range(pmts)]
    cfs_idx = [np.round(ttm-i*0.5, round_to) for i in reversed(range(pmts))]
    known_rates = [rates[:cfs_idx[i]].iloc[-1,0] for i in range(pmts-1)]
    val = px - sum([cpn * np.exp((-1) * known_rates[i] * cfs_idx[i]) for i in␣
 ↪range(pmts-1)])
    yld = (-1) * (np.log(val / (100+cpn)) / cfs_idx[pmts-1])
    add_df = pd.DataFrame([np.round(ttm, round_to), yld], index=['ttm','rate']).
 ↪transpose().set_index('ttm')
    rates = pd.concat([rates,add_df],ignore_index=False)
    rates = rates.groupby('ttm').mean()
```

[34]:

```
'''
plot yield curve
---
plots yield curve in percentage points
'''
plt.plot(rates[0.5:] * 100)
plt.title('Bootstrapped Term Structure (rates per annum)')
plt.xlabel('Tenor')
plt.ylabel('Yield in Percentage Points')
plt.legend(['Yield Curve'])
```

[34]: <matplotlib.legend.Legend at 0x19440d14fd0>



[35]:
```
'''
Liability Stream Analysis
---
Calculates time to maturity (in years) of each obligation
Calculates npv, duration, and convexity of liability stream
Prints stats to markdown for viewing
'''
data_prompt['ttm'] = np.round((data_prompt.index - datetime.datetime.now()) /
    datetime.timedelta(days=365), round_to)
r = [rates[:ttm].iloc[-1,0] for ttm in data_prompt['ttm']]
data_prompt['rates'] = r

npv = sum([data_prompt.iloc[i,0]*np.exp((-1)*data_prompt.iloc[i,1]*data_prompt.
    iloc[i,2]) for i in range(len(data_prompt))])
dur = sum([data_prompt.iloc[i,0]*data_prompt.iloc[i,1]*np.exp((-1) *
    (data_prompt.iloc[i,1]+1)*data_prompt.iloc[i,2]) for i in
    range(len(data_prompt))])
con = sum([data_prompt.iloc[i,0]*data_prompt.iloc[i,1]*(data_prompt.
    iloc[i,1]+1)*np.exp((-1)*(data_prompt.iloc[i,1]+2)*data_prompt.iloc[i,2])
    for i in range(len(data_prompt))])
```

```
md('''
<center>

### Solutions

The Net Present Value of the Liabilities is $\${:.2f}$ MM

The Macauley Duration of the Liability stream is ${:.2f}$ years

The Convexity of the Liability stream is ${:.2f}$


'''.format(npv,dur/npv,con/npv))
```

[35]:

2.1.3 Solutions

The Net Present Value of the Liabilities is \$117.68 MM

The Macauley Duration of the Liability stream is 3.83 years

The Convexity of the Liability stream is 23.32

3 Data

Identify *at least* 30 fixed-income assets that are suitable to construct a dedicated bond portfolio for the municipality liabilities that you have been given. Use assets that are considered risk-free; for example, US government non-callable treasury bonds, treasury bills, or treasury notes. Display in an appropriate table the main characteristics of the bonds you choose. Namely, prices, coupon rates, maturity dates, face value).

[36]:
```
'''
code block
'''
ref_data = ['T ' + str(term_structure_df.iloc[bond,1]) + ' ' +␣
↪term_structure_df.iloc[bond,0].strftime('%m/%d/%y') for bond in␣
↪term_structure_df.index]
term_structure_df['ref_data'] = ref_data

bonds_clean = term_structure_df[['ref_data', 'px_ask', 'ASKED YIELD']]
bonds_clean.columns = ['Bond', 'Price', 'Yield']
# bonds_clean = bonds_clean.assign(ttm = term_structure_df.ttm.round(2))
bonds_clean = bonds_clean.set_index('Bond')
```

[37]:
```
# You cannot tabe anything within the string literal for the markdown output,␣
↪See the cell titled "BAD MARKDOWN OUTPUT"
# Also when we finally convert to PDF I plan on using the metadata and hiding␣
↪the markdown output cell because it takes up to much space and is ugly
# Because of this, have all output cells in their own cell as done here
```

```
md('''
<center>

Listed below are all bonds considered in this analysis.  All trade on a face␣
 ↪value of 100 and pay coupons semiannually

{}


'''.format(bonds_clean.to_markdown(colalign = ("center",)))
)
```

[37]: Listed below are all bonds considered in this analysis. All trade on a face value of 100 and pay coupons semiannually

Bond	Price	Yield
T 1.75 05/15/22	100.01	-1.9931
T 2.125 05/15/22	100.012	-2.5447
T 0.0 05/17/22	99.352	0.657
T 0.0 05/19/22	99.34	0.669
T 0.0 05/24/22	99.32	0.69
T 0.0 05/26/22	99.322	0.687
T 0.125 05/31/22	99.312	0.6946
T 1.75 05/31/22	100.014	0.608
T 1.875 05/31/22	100.016	0.544
T 0.0 05/31/22	99.307	0.702
T 0.0 06/02/22	99.327	0.682
T 0.0 06/07/22	99.352	0.657
T 0.0 06/09/22	99.347	0.662
T 0.0 06/14/22	99.35	0.659
T 1.75 06/15/22	100.03	0.608
T 0.0 06/16/22	99.372	0.637
T 0.0 06/21/22	99.342	0.667
T 0.0 06/23/22	99.307	0.703
T 0.0 06/28/22	99.275	0.736
T 0.125 06/30/22	99.294	0.755
T 1.75 06/30/22	100.04	0.74
T 2.125 06/30/22	100.052	0.799
T 0.0 06/30/22	99.275	0.736
T 0.0 07/05/22	99.26	0.751
T 0.0 07/07/22	99.235	0.777
T 0.0 07/12/22	99.237	0.774
T 0.0 07/14/22	99.222	0.789
T 1.75 07/15/22	100.052	0.756
T 0.0 07/21/22	99.21	0.802
T 0.0 07/28/22	99.205	0.807
T 0.125 07/31/22	99.27	0.871

Bond	Price	Yield
T 1.875 07/31/22	100.064	0.902
T 2.0 07/31/22	100.072	0.915
T 0.0 08/04/22	99.085	0.93
T 0.0 08/11/22	99.055	0.96
T 1.5 08/15/22	100.04	0.999
T 1.625 08/15/22	100.05	0.999
T 7.25 08/15/22	101.184	0.942
T 0.0 08/18/22	99.01	1.006
T 0.0 08/25/22	99.01	1.007
T 0.125 08/31/22	99.234	1.042
T 1.625 08/31/22	100.052	1.057
T 1.875 08/31/22	100.076	1.037
T 0.0 09/01/22	98.957	1.06
T 0.0 09/08/22	98.912	1.106
T 1.5 09/15/22	100.03	1.214
T 0.0 09/15/22	98.9	1.12
T 0.0 09/22/22	98.885	1.135
T 0.0 09/29/22	98.887	1.133
T 0.125 09/30/22	99.184	1.258
T 1.75 09/30/22	100.056	1.266
T 1.875 09/30/22	100.074	1.244
T 0.0 10/06/22	98.845	1.176
T 0.0 10/13/22	98.757	1.266
T 1.375 10/15/22	100.004	1.336
T 0.0 10/20/22	98.732	1.292
T 0.0 10/27/22	98.697	1.329
T 0.125 10/31/22	99.14	1.365
T 1.875 10/31/22	100.072	1.375
T 2.0 10/31/22	100.09	1.379
T 0.0 11/03/22	98.675	1.352
T 0.0 11/10/22	98.605	1.424
T 1.625 11/15/22	100.022	1.483
T 7.625 11/15/22	103.032	1.346
T 0.125 11/30/22	99.08	1.522
T 2.0 11/30/22	100.084	1.505
T 0.0 12/01/22	98.64	1.389
T 1.625 12/15/22	100.012	1.557
T 0.0 12/29/22	98.515	1.52
T 0.137 12/31/22	99.016	1.653
T 2.125 12/31/22	100.09	1.669
T 1.5 01/15/23	99.274	1.712
T 0.0 01/26/23	98.452	1.586
T 0.125 01/31/23	98.27	1.771
T 1.75 01/31/23	100	1.749
T 2.375 01/31/23	100.142	1.739
T 1.375 02/15/23	99.22	1.795

Bond	Price	Yield
T 2.0 02/15/23	100.05	1.788
T 7.125 02/15/23	104.006	1.708
T 0.0 02/23/23	98.332	1.713
T 0.125 02/28/23	98.206	1.855
T 1.5 02/28/23	99.232	1.849
T 2.625 02/28/23	100.192	1.853
T 0.5 03/15/23	98.276	1.879
T 0.0 03/23/23	98.282	1.772
T 0.125 03/31/23	98.134	1.954
T 1.5 03/31/23	99.19	1.97
T 2.5 03/31/23	100.146	1.964
T 0.25 04/15/23	98.13	2.016
T 0.0 04/20/23	98.127	1.933
T 0.125 04/30/23	98.056	2.057
T 1.625 04/30/23	99.19	2.056
T 2.75 04/30/23	100.206	2.061
T 0.125 05/15/23	98.03	2.066
T 1.75 05/15/23	99.22	2.068
T 0.125 05/31/23	97.306	2.115
T 1.625 05/31/23	99.156	2.12
T 2.75 05/31/23	100.204	2.124
T 0.25 06/15/23	97.316	2.136
T 0.125 06/30/23	97.23	2.19
T 1.375 06/30/23	99.034	2.181
T 2.625 06/30/23	100.156	2.178
T 0.125 07/15/23	97.19	2.227
T 0.125 07/31/23	97.144	2.271
T 1.25 07/31/23	98.262	2.243
T 2.75 07/31/23	100.19	2.248
T 0.125 08/15/23	97.114	2.277
T 2.5 08/15/23	100.094	2.257
T 6.25 08/15/23	104.296	2.23
T 0.125 08/31/23	97.062	2.344
T 1.375 08/31/23	98.26	2.313
T 2.75 08/31/23	100.176	2.31
T 0.125 09/15/23	97.04	2.33
T 0.25 09/30/23	97.056	2.347
T 1.375 09/30/23	98.222	2.345
T 2.875 09/30/23	100.23	2.339
T 0.125 10/15/23	96.276	2.389
T 0.375 10/31/23	97.032	2.412
T 1.625 10/31/23	98.286	2.399
T 2.875 10/31/23	100.21	2.413
T 0.25 11/15/23	96.262	2.425
T 2.75 11/15/23	100.156	2.413
T 0.5 11/30/23	97.01	2.475

Bond	Price	Yield
T 2.125 11/30/23	99.172	2.431
T 2.875 11/30/23	100.202	2.454
T 0.125 12/15/23	96.126	2.46
T 0.75 12/31/23	97.07	2.508
T 2.25 12/31/23	99.202	2.481
T 2.625 12/31/23	100.08	2.466
T 0.125 01/15/24	96.046	2.5
T 0.875 01/31/24	97.072	2.542
T 2.5 01/31/24	99.302	2.532
T 0.125 02/15/24	95.286	2.534
T 2.75 02/15/24	100.112	2.542
T 1.5 02/29/24	98.056	2.546
T 2.125 02/29/24	99.092	2.533
T 2.375 02/29/24	99.232	2.531
T 0.25 03/15/24	95.286	2.556
T 2.125 03/31/24	99.07	2.554
T 2.25 03/31/24	99.13	2.576
T 0.375 04/15/24	95.286	2.584
T 2.0 04/30/24	98.286	2.581
T 2.25 04/30/24	99.12	2.579
T 2.5 04/30/24	99.27	2.582
T 0.25 05/15/24	95.144	2.601
T 2.5 05/15/24	99.25	2.613
T 2.0 05/31/24	98.264	2.593
T 0.25 06/15/24	95.066	2.628
T 1.75 06/30/24	98.06	2.632
T 2.0 06/30/24	98.214	2.647
T 0.375 07/15/24	95.064	2.669
T 1.75 07/31/24	98.014	2.665
T 2.125 07/31/24	98.264	2.674
T 0.375 08/15/24	94.31	2.693
T 2.375 08/15/24	99.092	2.702
T 1.25 08/31/24	96.25	2.708
T 1.875 08/31/24	98.042	2.721
T 0.375 09/15/24	94.224	2.735
T 1.5 09/30/24	97.07	2.717
T 2.125 09/30/24	98.21	2.713
T 0.625 10/15/24	95.034	2.723
T 1.5 10/31/24	97.02	2.745
T 2.25 10/31/24	98.274	2.733
T 0.75 11/15/24	95.054	2.752
T 2.25 11/15/24	98.244	2.765
T 7.5 11/15/24	111.176	2.685
T 1.5 11/30/24	96.292	2.766
T 2.125 11/30/24	98.142	2.763
T 1.0 12/15/24	95.204	2.758

Bond	Price	Yield
T 1.75 12/31/24	97.134	2.775
T 2.25 12/31/24	98.22	2.771
T 1.125 01/15/25	95.25	2.777
T 1.375 01/31/25	96.114	2.779
T 2.5 01/31/25	99.09	2.776
T 1.5 02/15/25	96.184	2.798
T 2.0 02/15/25	97.296	2.786
T 7.625 02/15/25	112.286	2.725
T 1.125 02/28/25	95.186	2.779
T 2.75 02/28/25	99.286	2.787
T 1.75 03/15/25	97.052	2.798
T 0.5 03/31/25	93.222	2.798
T 2.625 03/31/25	99.176	2.787
T 2.625 04/15/25	99.164	2.799
T 0.375 04/30/25	93.04	2.815
T 2.875 04/30/25	100.066	2.8
T 2.125 05/15/25	98.016	2.806
T 2.75 05/15/25	99.282	2.791
T 0.25 05/31/25	92.186	2.811
T 2.875 05/31/25	100.066	2.802
T 0.25 06/30/25	92.116	2.819
T 2.75 06/30/25	99.266	2.805
T 0.25 07/31/25	92.046	2.826
T 2.875 07/31/25	100.06	2.813
T 2.0 08/15/25	97.136	2.833
T 6.875 08/15/25	112.206	2.778
T 0.25 08/31/25	91.29	2.844
T 2.75 08/31/25	99.23	2.839
T 0.25 09/30/25	91.224	2.846
T 3.0 09/30/25	100.166	2.836
T 0.25 10/31/25	91.16	2.85
T 3.0 10/31/25	100.156	2.849
T 2.25 11/15/25	98.004	2.85
T 0.375 11/30/25	91.224	2.855
T 2.875 11/30/25	100.026	2.849
T 0.375 12/31/25	91.166	2.853
T 2.625 12/31/25	99.09	2.835
T 0.375 01/31/26	91.094	2.864
T 2.625 01/31/26	99.062	2.855
T 1.625 02/15/26	95.194	2.867
T 6.0 02/15/26	111.052	2.84
T 0.5 02/28/26	91.18	2.865
T 2.5 02/28/26	98.232	2.856
T 0.75 03/31/26	92.092	2.867
T 2.25 03/31/26	97.24	2.867
T 0.75 04/30/26	92.042	2.868

Bond	Price	Yield
T 2.375 04/30/26	98.062	2.861
T 1.625 05/15/26	95.094	2.879
T 0.75 05/31/26	91.292	2.884
T 2.125 05/31/26	97.05	2.875
T 0.875 06/30/26	92.074	2.886
T 1.875 06/30/26	96.052	2.868
T 0.625 07/31/26	91.024	2.891
T 1.875 07/31/26	96.024	2.87
T 1.5 08/15/26	94.144	2.896
T 6.75 08/15/26	115.122	2.878
T 0.75 08/31/26	91.134	2.89
T 1.375 08/31/26	93.302	2.885
T 0.875 09/30/26	91.246	2.89
T 1.625 09/30/26	94.262	2.894
T 1.125 10/31/26	92.21	2.893
T 1.625 10/31/26	94.234	2.893
T 2.0 11/15/26	96.07	2.903
T 6.5 11/15/26	115.034	2.893
T 1.25 11/30/26	93.032	2.882
T 1.625 11/30/26	94.222	2.879
T 1.25 12/31/26	92.304	2.889
T 1.75 12/31/26	95.034	2.887
T 1.5 01/31/27	93.292	2.891
T 2.25 02/15/27	97.042	2.9
T 6.625 02/15/27	116.114	2.912
T 1.125 02/28/27	92.052	2.888
T 1.875 02/28/27	95.172	2.878
T 0.625 03/31/27	89.244	2.891
T 2.5 03/31/27	98.09	2.88
T 0.5 04/30/27	89.002	2.897
T 2.75 04/30/27	99.126	2.881
T 2.375 05/15/27	97.164	2.913
T 0.5 05/31/27	88.24	2.916
T 0.5 06/30/27	88.184	2.916
T 0.375 07/31/27	87.274	2.903
T 2.25 08/15/27	96.24	2.922
T 6.375 08/15/27	116.224	2.92
T 0.5 08/31/27	88.06	2.926
T 0.5 10/31/27	87.262	2.932
T 2.25 11/15/27	96.186	2.927
T 6.125 11/15/27	116.04	2.928
T 0.625 11/30/27	88.082	2.937
T 0.625 12/31/27	88.02	2.944
T 0.75 01/31/28	88.166	2.949
T 2.75 02/15/28	99.004	2.937
T 1.125 02/29/28	90.136	2.934

Bond	Price	Yield
T 1.25 03/31/28	90.284	2.95
T 1.25 04/30/28	90.24	2.955
T 2.875 05/15/28	99.192	2.948
T 1.25 05/31/28	90.2	2.956
T 1.25 06/30/28	90.146	2.965
T 1.0 07/31/28	88.296	2.965
T 2.875 08/15/28	99.162	2.961
T 5.5 08/15/28	114.156	2.944
T 1.125 08/31/28	89.164	2.964
T 1.25 09/30/28	90.026	2.968
T 1.375 10/31/28	90.226	2.966
T 3.125 11/15/28	100.312	2.959
T 5.25 11/15/28	113.192	2.935
T 1.5 11/30/28	91.112	2.964
T 1.375 12/31/28	90.194	2.946
T 1.75 01/31/29	92.234	2.951
T 2.625 02/15/29	98.01	2.948
T 5.25 02/15/29	114.03	2.933
T 1.875 02/28/29	93.152	2.942
T 2.375 03/31/29	96.14	2.951
T 2.875 04/30/29	99.172	2.949
T 2.375 05/15/29	96.134	2.945
T 1.625 08/15/29	91.192	2.918
T 6.125 08/15/29	120.284	2.908
T 1.75 11/15/29	92.076	2.909
T 1.5 02/15/30	90.076	2.915
T 0.625 05/15/30	83.23	2.923
T 6.25 05/15/30	123.232	2.904
T 0.625 08/15/30	83.09	2.92
T 0.875 11/15/30	84.224	2.921
T 1.125 02/15/31	86.076	2.918
T 5.375 02/15/31	118.256	2.923
T 1.625 05/15/31	89.25	2.925
T 1.25 08/15/31	86.162	2.925
T 1.375 11/15/31	87.046	2.935
T 1.875 02/15/32	91.02	2.936
T 2.875 05/15/32	99.176	2.927
T 4.5 02/15/36	118.09	2.88
T 4.75 02/15/37	121.096	2.957
T 5.0 05/15/37	124.052	2.989
T 4.375 02/15/38	116.29	3.018
T 4.5 05/15/38	118.154	3.033
T 3.5 02/15/39	105.026	3.108
T 4.25 05/15/39	115.006	3.106
T 4.5 08/15/39	118.124	3.114
T 4.375 11/15/39	116.14	3.146

Bond	Price	Yield
T 4.625 02/15/40	120.014	3.143
T 1.125 05/15/40	70.18	3.309
T 4.375 05/15/40	116.1	3.178
T 1.125 08/15/40	70.03	3.324
T 3.875 08/15/40	108.282	3.227
T 1.375 11/15/40	73.066	3.326
T 4.25 11/15/40	113.294	3.243
T 1.875 02/15/41	79.282	3.325
T 4.75 02/15/41	121.106	3.225
T 2.25 05/15/41	84.262	3.335
T 4.375 05/15/41	115.164	3.272
T 1.75 08/15/41	77.146	3.348
T 3.75 08/15/41	106.144	3.294
T 3.125 11/15/41	97.11	3.311
T 2.0 11/30/41	80.292	3.34
T 2.375 02/15/42	86.136	3.316
T 3.125 02/15/42	97.094	3.312
T 3.0 05/15/42	95.104	3.322
T 2.75 08/15/42	91.074	3.349
T 2.75 11/15/42	91.022	3.356
T 3.125 02/15/43	96.232	3.345
T 2.875 05/15/43	92.264	3.354
T 3.625 08/15/43	104.124	3.335
T 3.75 11/15/43	106.112	3.334
T 3.625 02/15/44	104.09	3.346
T 3.375 05/15/44	100.094	3.356
T 3.125 08/15/44	96.11	3.359
T 3.0 11/15/44	94.084	3.366
T 2.5 02/15/45	86.084	3.369
T 3.0 05/15/45	94.094	3.358
T 2.875 08/15/45	92.116	3.35
T 3.0 11/15/45	94.174	3.337
T 2.5 02/15/46	86.076	3.344
T 2.5 05/15/46	86.074	3.338
T 2.25 08/15/46	82.016	3.335
T 2.875 11/15/46	92.234	3.31
T 3.0 02/15/47	94.266	3.307
T 3.0 05/15/47	94.282	3.302
T 2.75 08/15/47	90.276	3.285
T 2.75 11/15/47	90.302	3.277
T 3.0 02/15/48	95.21	3.25
T 3.125 05/15/48	98.04	3.232
T 3.0 08/15/48	95.242	3.241
T 3.375 11/15/48	103.004	3.205
T 3.0 02/15/49	96.142	3.199
T 2.875 05/15/49	94.084	3.194

Bond	Price	Yield
T 2.25 08/15/49	82.304	3.191
T 2.375 11/15/49	85.126	3.175
T 2.0 02/15/50	78.066	3.189
T 1.25 05/15/50	64.106	3.184
T 1.375 08/15/50	66.176	3.177
T 1.625 11/15/50	71.07	3.166
T 1.875 02/15/51	75.31	3.152
T 2.375 05/15/51	85.156	3.141
T 2.0 08/15/51	78.106	3.137
T 1.875 11/15/51	76.014	3.124
T 2.25 02/15/52	83.134	3.108

4 Dedication Portfolio

Formulate a linear programming model to find the lowest cost bond dedicated portfolio that covers the stream of liabilities. To eliminate the possibility of any interest risk, assume that a 0% reinvestment rate on cash balances carried out from one date to the next. Assume no short selling of bonds is allowed. What is the cost of your portfolio? How does this cost compares with the NPV of the liabilities? What is the composition of the portfolio?

4.1 Mathematical Formulation

$$\begin{aligned} \min \quad & z_0 + \sum_{i=1}^{N} P_i x_i \\ \text{s.t.} \quad & \sum_{i=1,\dots,n: M_i \geq t-1} C_i x_i + \sum_{i=1,\dots,n: M_i \geq t} 100 x_i + z_{t-1} - z_t = L_t \\ & \text{All variables are non-negative} \end{aligned}$$

4.2 Code

[38]:
```
'''
Data Manipulation
'''
term_by_maturity = term_structure_df.set_index('MATURITY')
possibilities = term_by_maturity.drop(
    index=[i for i in term_by_maturity.index.to_list() if i > data_prompt.index.
 ↪to_list()[-1]],
    columns=['BID', 'ASKED', 'ASKED YIELD']
    )

'''List of bond maturities less than liability maturity'''
date_lists_to_change_to_periods = [
    [i for i in possibilities.index.to_list() if i <= t]
    for t in data_prompt.index.tolist()
    ]
```

```
'''Removing the duplicates from each one'''
for i in reversed(range(1,len(date_lists_to_change_to_periods))):
    for j in range(0,len(date_lists_to_change_to_periods[i-1])):
        date_lists_to_change_to_periods[i].
 ↪remove(date_lists_to_change_to_periods[i-1][j])

for i in range(0,len(date_lists_to_change_to_periods)):
    possibilities.loc[date_lists_to_change_to_periods[i],'period'] = i+1

possibilities['face'] = 100
possibilities['bond#'] = range(1,len(possibilities)+1)
possibilities = possibilities.set_index('bond#')

'''for labeling later'''
dec_var_names = possibilities['ref_data']
```

[39]:
```
'''Getting data ready for the solver'''

'''Exmpty Array'''
cfs = np.zeros((len(possibilities),len(date_lists_to_change_to_periods)))

'''CF Matrix'''
'''Will make function later'''
for i in range(0, len(cfs)):
    for j in range(1, len(cfs[0])+1):
        if possibilities.loc[i+1,'period'] == j and possibilities.
 ↪loc[i+1,'COUPON'] == 0:
            cfs[i][j-1] = possibilities.loc[i+1,'face']
        elif possibilities.loc[i+1,'period'] == j and possibilities.
 ↪loc[i+1,'COUPON'] != 0:
            cfs[i][0:j-1] = possibilities.loc[i+1,'COUPON']/2
            cfs[i][j-1] = possibilities.loc[i+1,'face'] + possibilities.
 ↪loc[i+1,'COUPON']/2

cf_matrix = cfs.tolist()
prices = possibilities['px_ask'].values.tolist()
liabilities = data_prompt['Amount'].values.tolist()
```

[40]:
```
'''Solving for the dedicated portfolio'''

# Making variable list of strings
periods = [i for i in range(0,len(cf_matrix[0])+1)]

# Dictionary of period constraints
period_dict = {}
for i in range(0,len(cf_matrix[0])):
```

```
    period_dict['Period {}'.format(i+1)] =␣
↪dict(zip(dec_var_names,[cf_matrix[j][i] for j in range(0,len(cf_matrix))]))

objective = dict(zip(dec_var_names, prices))

# Decision Vars
quantity = LpVariable.dict('', dec_var_names, lowBound=0)
excess = LpVariable.dict('carryover', periods, lowBound=0)

# Intializing the Problem
dedication_1 = LpProblem('Dedicated', LpMinimize)

# Objective function
dedication_1 += excess[0]+lpSum([objective[i]*quantity[i] for i in␣
↪dec_var_names])

# Constraints
for i in range(0,len(cf_matrix[0])):
    dedication_1 += lpSum([period_dict['Period {}'.format(i+1)][j]*quantity[j]␣
↪for j in dec_var_names]) + excess[i]- excess[i+1] == liabilities[i]

dedication_1.solve()
```

[40]: 1

[41]:
```
composition = pd.DataFrame(
    [v.varValue for v in dedication_1.variables() if v.name[0] != 'c' and v.
↪varValue > 0],
    index=[v.name[1:] for v in dedication_1.variables() if v.name[0] != 'c' and␣
↪v.varValue > 0],
    columns=['Quantity']
    )
```

4.3 Results

[42]:
```
# Just have to match bonds with the tickers Jack created instead

'''Printing Solutions'''

md('''
##### <center> Dedication Portfolio Cost & Composition  </center>

<center>

Portfolio Cost $ = \${:.2f} $ MM <br>
```

```
</center>

<center>

{}

'''.format(dedication_1.objective.value(),composition.to_markdown(colalign =␣
↪("right",))))
```

[42]: #####

Dedication Portfolio Cost & Composition

Portfolio Cost $ = $117.77 $ MM

	Quantity
T_0.625_05_15_30	0.0697819
T_0.75_05_31_26	0.107674
T_1.25_05_31_28	0.0637436
T_1.5_11_30_24	0.0714316
T_2.0_05_31_24	0.0509223
T_2.875_04_30_29	0.0664559
T_5.5_08_15_28	0.084142
T_6.125_08_15_29	0.0774112
T_6.25_08_15_23	0.0590762
T_6.375_08_15_27	0.0423924
T_6.625_02_15_27	0.0507125
T_6.75_08_15_26	0.0780774
T_6.875_08_15_25	0.0750923
T_7.125_02_15_23	0.0667
T_7.25_08_15_22	0.0836671
T_7.625_02_15_25	0.0819673

5 Sensitivity Analysis

Use the linear programming sensitivity analysis information to determine the term structure of interest rates implied by the optimal bond portfolio you found in the previous question. Use a plot to compare these rates with the current term structure of treasury rates you found in the first question.

5.1 Shadow Prices

[43]:
```
'''
Pull sensitivity analysis
---
https://s3.amazonaws.com/assets.datacamp.com/production/course_8835/slides/
↪chapter4.pdf
```

```
'''
o = [{'name':name, 'shadow price':c.pi} for name, c in dedication_1.constraints.
 ↪items()]
shadow_px = pd.DataFrame(o).set_index(data_prompt.index).drop('name',axis=1)
clean_shadow_px = pd.DataFrame(o).set_index(data_prompt.index.strftime('%m/%d/
 ↪%y')).drop('name',axis=1)
md('''

<center>

{}

'''.format(clean_shadow_px.to_markdown(colalign = ("right",)))
)
```

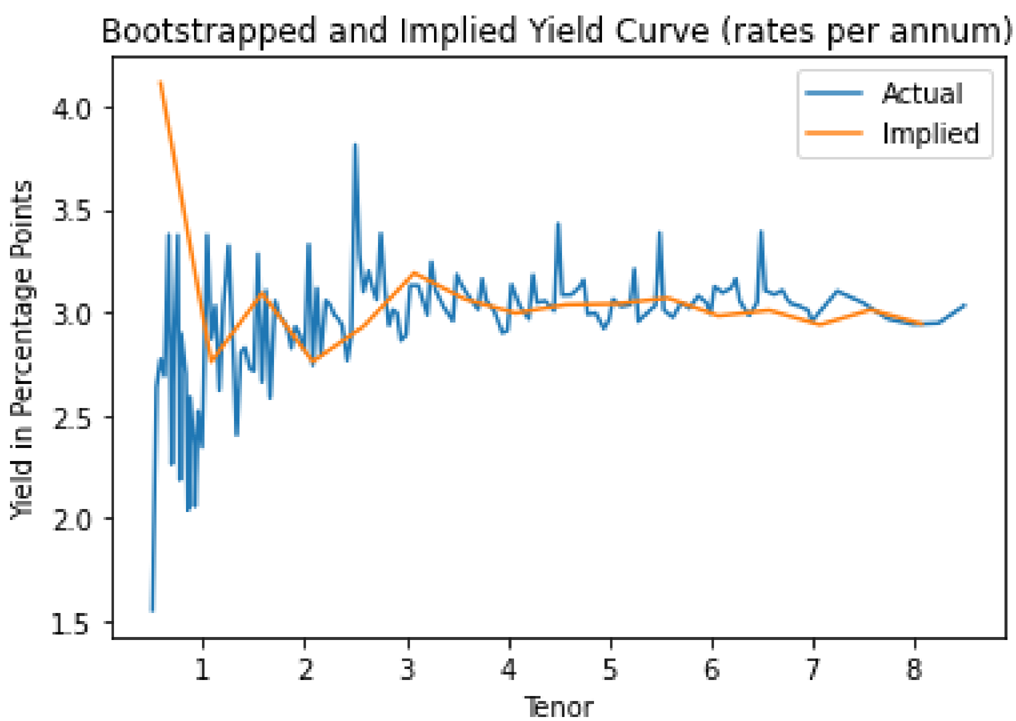
[43]:

DateDue	shadow price
12/15/22	0.976444
06/15/23	0.970693
12/15/23	0.952351
06/15/24	0.944203
12/15/24	0.927139
06/15/25	0.906415
12/15/25	0.896101
06/15/26	0.884951
12/15/26	0.870135
06/15/27	0.856877
12/15/27	0.8426
06/15/28	0.834112
12/15/28	0.820296
06/15/29	0.812113
12/15/29	0.795826
06/15/30	0.788304

[44]:
```
'''
Presents implied and actual yield curve as a plot
'''
shadow_ttm = ((shadow_px.index - datetime.datetime.now()) / datetime.
 ↪timedelta(days=365)).to_list()
shadow_factors = shadow_px['shadow price'].to_list()
implied_rates = [-np.log(shadow_factors[i]) / shadow_ttm[i] for i in␣
 ↪range(len(shadow_ttm))]

implied_rates_df = pd.DataFrame(
                        data = [shadow_ttm, implied_rates],
                        index=['ttm','implied_rate']
```

```
                    )

implied_rates_df = (implied_rates_df
                         .transpose()
                         .round({'ttm':round_to})
                         .set_index('ttm')
                    )

plt.plot(rates[0.5:8.5] * 100)
plt.plot(implied_rates_df * 100)
plt.title('Bootstrapped and Implied Yield Curve (rates per annum)')
plt.xlabel('Tenor')
plt.ylabel('Yield in Percentage Points')
plt.legend(['Actual', 'Implied'])
```

[44]: <matplotlib.legend.Legend at 0x19440c09d30>



5.2 Implied Rates

[45]:
```
md('''


<center>
```

```
{}

'''.format((implied_rates_df*100).to_markdown(colalign = ("right",)))
)
```

[45]:

ttm	implied_rate
0.58	4.11341
1.08	2.75888
1.58	3.09092
2.08	2.75911
2.58	2.92969
3.08	3.18928
3.58	3.06237
4.08	2.995
4.58	3.03576
5.08	3.04005
5.58	3.06798
6.08	2.98156
6.58	3.0082
7.08	2.93799
7.58	3.01088
8.08	2.94262

6 Immunization Portfolios

Formulate a linear programming model to find the lowest cost bond immunized portfolio that matches the present value, duration, and convexity of a stream of liabilities. Assume that no short rates are allowed. What is the cost of your portfolio? How much would you save by using this immunization strategy instead of the dedication one? Is your portfolio immunized against non-parallel shifts in the term structure? Explain why or why not.

6.1 Mathematical Formulation

6.2 Code

[46]:

```
'''
Aggregates cashflow matrix and ref data for immunization
---
Puts cashflow matrix into a dataframe for merging
merges possible bond ref data with cashflow matrix
cleans resulting dataframe

NOTE: MATH NEEDS WORK HERE BUT WE CAN FIGURE OUT
from here: use ttm and col_num against calculated curve to find appropriate␣
↪measure
```

```
        pv_factor = exp{-rt} = exp{- () * (ttm)}
'''
cf_df = pd.DataFrame(cf_matrix, index=dec_var_names)

cf_df = pd.merge(                                                        #␣
 ↪Combines possible bonds with cashflow matrix
    left = possibilities,                                                #␣
 ↪possible bonds - SAME DF AS DEDICATION
    right = cf_df,                                                       #␣
 ↪Cashflow matrix - NP ARRAY FROM DEDICATION AS DF FOR MERGING
    how='inner',                                                         #␣
 ↪Catches any missed bonds on merge
    left_on='ref_data',                                                  #␣
 ↪possibilities not indexed by bond name - CHAZ IS THIS SOMETHING WE CAN␣
 ↪ADJUST OR NO?????????????????????
    right_index=True                                                     #␣
 ↪Casflow df indexed by bond name
)

cf_df = (cf_df
            .drop(['COUPON','period','face'],axis=1)                     #␣
 ↪Drops unnecessary ref data
            .set_index('ref_data')                                       #␣
 ↪Sets index to bond name
            .round({'ttm':round_to})                                     #␣
 ↪rounds time to maturity to 2 decimal places -- allows use of derived term␣
 ↪structure (indexed by hundredths)
        )

cf_df
```

[46]:

```
                    px_ask   px_bid   ttm        0       1       2       3  \
ref_data
T 1.75 05/15/22    100.010  100.004 -0.01  100.8750  0.0000  0.0000  0.0000
T 2.125 05/15/22   100.012  100.006 -0.01  101.0625  0.0000  0.0000  0.0000
T 0.0 05/17/22      99.352   99.342 -0.00  100.0000  0.0000  0.0000  0.0000
T 0.0 05/19/22      99.340   99.330  0.00  100.0000  0.0000  0.0000  0.0000
T 0.0 05/24/22      99.320   99.310  0.02  100.0000  0.0000  0.0000  0.0000
…                     …        …      …        …       …       …       …
T 6.125 08/15/29   120.284  120.274  7.25    3.0625  3.0625  3.0625  3.0625
T 1.75 11/15/29     92.076   92.066  7.50    0.8750  0.8750  0.8750  0.8750
T 1.5 02/15/30      90.076   90.066  7.75    0.7500  0.7500  0.7500  0.7500
T 0.625 05/15/30    83.230   83.220  8.00    0.3125  0.3125  0.3125  0.3125
T 6.25 05/15/30    123.232  123.222  8.00    3.1250  3.1250  3.1250  3.1250

                        4       5       6       7       8       9      10  \
```

```
ref_data
T 1.75 05/15/22   0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
T 2.125 05/15/22  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
T 0.0 05/17/22    0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
T 0.0 05/19/22    0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
T 0.0 05/24/22    0.0000  0.0000  0.0000  0.0000  0.0000  0.0000  0.0000
…                    …       …       …       …       …       …       …
T 6.125 08/15/29  3.0625  3.0625  3.0625  3.0625  3.0625  3.0625  3.0625
T 1.75 11/15/29   0.8750  0.8750  0.8750  0.8750  0.8750  0.8750  0.8750
T 1.5 02/15/30    0.7500  0.7500  0.7500  0.7500  0.7500  0.7500  0.7500
T 0.625 05/15/30  0.3125  0.3125  0.3125  0.3125  0.3125  0.3125  0.3125
T 6.25 05/15/30   3.1250  3.1250  3.1250  3.1250  3.1250  3.1250  3.1250

                      11      12      13        14        15
ref_data
T 1.75 05/15/22   0.0000  0.0000  0.0000    0.0000    0.0000
T 2.125 05/15/22  0.0000  0.0000  0.0000    0.0000    0.0000
T 0.0 05/17/22    0.0000  0.0000  0.0000    0.0000    0.0000
T 0.0 05/19/22    0.0000  0.0000  0.0000    0.0000    0.0000
T 0.0 05/24/22    0.0000  0.0000  0.0000    0.0000    0.0000
…                    …       …       …         …         …
T 6.125 08/15/29  3.0625  3.0625  3.0625  103.0625    0.0000
T 1.75 11/15/29   0.8750  0.8750  0.8750  100.8750    0.0000
T 1.5 02/15/30    0.7500  0.7500  0.7500    0.7500  100.7500
T 0.625 05/15/30  0.3125  0.3125  0.3125    0.3125  100.3125
T 6.25 05/15/30   3.1250  3.1250  3.1250    3.1250  103.1250

[289 rows x 19 columns]
```

[47]:
```python
'''
Create Present Value, Duration, and Convexity factors for all possible time␣
↪index based on derived rates curve
'''
t = rates.index
r = rates['rate']
npv_factor = np.exp(-r*t)
dur_factor = t*np.exp(-r*(t+1))
con_factor = t*(t+1)*np.exp(-r*(t+2))
```

[48]:
```python
'''
Calculates npv, duration, and convexity terms for all bonds consiuder in problem
'''
npvs=[]
durs=[]
cons=[]
for bond in cf_df.index:
    bond_df = cf_df.loc[bond]
```

```
    bond_ttm = bond_df.loc['ttm']
    bond_cf_stream = bond_df.loc[0:]
    eo_cfs = bond_cf_stream.idxmax()
    cpn_ttm = [(bond_ttm - 0.5*i).round(round_to) for i in range(eo_cfs+1)]
    bond_cf_ttm = pd.Series(data=bond_df.loc[0:eo_cfs].to_list(),␣
 ↪index=reversed(cpn_ttm))

    bond_npv = sum([bond_cf_ttm.loc[i] * npv_factor.loc[:i].iloc[-1] for i in␣
 ↪bond_cf_ttm.index])
    bond_dur = sum([bond_cf_ttm.loc[i] * dur_factor.loc[:i].iloc[-1] for i in␣
 ↪bond_cf_ttm.index])
    bond_con = sum([bond_cf_ttm.loc[i] * con_factor.loc[:i].iloc[-1] for i in␣
 ↪bond_cf_ttm.index])

    npvs.append(bond_npv)
    durs.append(bond_dur)
    cons.append(bond_con)

immunization_df = pd.DataFrame([npvs, durs, cons], columns=cf_df.index,␣
 ↪index=['npv','duration','convexity']).transpose()
```

[49]:
```
'''
Solves immunization portfolio
'''
bond_count = LpVariable.dicts('Bonds',dec_var_names,lowBound=0)

immunization = LpProblem('immunization', LpMinimize)

immunization += lpSum([cf_df['px_ask'].loc[i] * bond_count[i] for i in␣
 ↪dec_var_names])
immunization += lpSum([immunization_df['npv'].loc[i] * bond_count[i] for i in␣
 ↪dec_var_names]) == npv
immunization += lpSum([immunization_df['duration'].loc[i] * bond_count[i] for i␣
 ↪in dec_var_names]) == dur
immunization += lpSum([immunization_df['convexity'].loc[i] * bond_count[i] for␣
 ↪i in dec_var_names]) == con

immunization.solve()
```

[49]: 1

[50]:
```
'''
Print Solution to Immunized portfolio
'''
```

```
bonds_fin = pd.DataFrame([[v.name[6:] for v in immunization.variables()], [v.
 ↪varValue for v in immunization.variables()]], index=['bond','amt']).
 ↪transpose().set_index('bond')
bonds_fin[bonds_fin['amt']>0]
```

[50]:

```
                         amt
bond
T_6.125_08_15_29  0.227666
T_6.625_02_15_27  0.610606
T_7.25_08_15_22   0.184973
```

6.3 Portfolio Allocation

[51]:

```
md('''

<center> Immunized Portfolio Value of ${:.2f} MM  </center> <br>



<center>

{}

'''.format(value(immunization.objective), bonds_fin[bonds_fin['amt']>0].
 ↪to_markdown(colalign = ("right",)))
)
```

[51]: Immunized Portfolio Value of $117.00 MM

bond	amt
T_6.125_08_15_29	0.227666
T_6.625_02_15_27	0.610606
T_7.25_08_15_22	0.184973

6.4 Immunization with Dedication Constraint

Combine a cash matching strategy (dedication) for the liabilities for the first three years and an immunization strategy based on matching present value, duration and convexity for the liabilities during the last five years. Compare the characteristics of the three bond portfolios you have obtained. Explain which one you think is the best one and why.

6.5 Mathematical Formulation

6.6 Code

[52]:

```
'''
Immunization part
```

```
Calculates npv, duration, and convexity terms for all bonds considered in the␣
↪problem
FROM period 7-16
'''

ded_period = 6
imm_period = 5
imm_start_period = len(data_prompt) - imm_period*2
imm_end_period = len(data_prompt)
```

[53]:
```
'''
Solves combined portfolio
'''
bond_q = LpVariable.dicts('Bond',dec_var_names,lowBound=0)
excess_cf = LpVariable.dicts('ExcessCf', periods[:ded_period+1], lowBound=0)


combined = LpProblem('Combined', LpMinimize)

combined += lpSum([cf_df['px_ask'][i] * bond_q[i] for i in dec_var_names]+␣
↪excess_cf[0])

for i in range(0,ded_period):
    combined += lpSum([cf_df[i][j]*bond_q[j] for j in dec_var_names]) +␣
↪excess_cf[i]- excess_cf[i+1] == liabilities[i]

combined += lpSum([immunization_df['npv'][i] * bond_q[i] for i in␣
↪dec_var_names]) == npv
combined += lpSum([immunization_df['duration'][i] * bond_q[i] for i in␣
↪dec_var_names]) == dur
combined += lpSum([immunization_df['convexity'][i] * bond_q[i] for i in␣
↪dec_var_names]) == con

combined.solve()
```

[53]: 1

6.7 Portfolio Allocation

[54]:
```
'''
Print Solution to Combined portfolio
'''
bonds_comb = pd.DataFrame(
            [v.varValue for v in combined.variables() if v.varValue > 0],
            index=[str(v.name[:-8].replace('Bond', '').replace('_', ' ') + v.
↪name[-8:].replace('_', '/'))
```

```
            if v.name[0] == 'B' else str(r'$\text{Excess}_' + v.name[-1] + '$')
            for v in combined.variables() if v.varValue > 0],
            columns=['Quantity'])

bonds_comb.index.name = 'Bonds/Excess Cashflows'


md('''

<center> Combined Portfolio Value of ${:.2f} MM  </center> <br>



<center>

{}

'''.format(value(combined.objective), bonds_comb.to_markdown(colalign =␣
↪("right",)))
)
```

[54]: Combined Portfolio Value of $117.28 MM

Bonds/Excess Cashflows	Quantity
T 1.75 05/15/23	0.0657302
T 2.25 10/31/24	0.0686657
T 2.5 05/15/24	0.0480649
T 2.75 05/15/25	0.0794382
T 5.5 08/15/28	0.398787
T 6.125 08/15/29	0.0674905
T 6.25 08/15/23	0.0563053
T 6.375 08/15/27	0.201915
T 7.25 08/15/22	0.0827312

7 Part 7: Dedication Portfolio with Short Selling

The municipality would like to make a second bid (find a different portfolio of bonds). What is your best dedicated portfolio of risk-free bonds you can create *if short sales are allowed*? Did you find arbitrage opportunities? Did you take into consideration the bid-ask spread of the bonds? How would you take them in consideration and what is the result? Did you set limits in the transaction amounts? Discuss the practical feasibility of your solutions.

7.1 Without Transaction Limits

7.1.1 Mathematical Formulation

input here

7.1.2 Code

[55]:
```
'''
Solves short portfolio
'''
short_limit = 0.5
long_q = LpVariable.dicts('Long',dec_var_names,lowBound=0)
short_q = LpVariable.dicts('Short',dec_var_names,lowBound=0)
excess_cfs = LpVariable.dicts('ExcessCf', periods, lowBound=0)

short = LpProblem('Short', LpMinimize)

#Objective
short += lpSum([cf_df['px_ask'][i] * long_q[i] - cf_df['px_bid'][i] *␣
 ↪short_q[i] for i in dec_var_names]+ excess_cfs[0])

#Bounds Objective to be NonNegative - Municipality can't profit from short␣
 ↪trading - At best they get their dedication portfolio free
short += lpSum([cf_df['px_ask'][i] * long_q[i] - cf_df['px_bid'][i] *␣
 ↪short_q[i] for i in dec_var_names]+ excess_cfs[0]) >= 0

#Liabilities Constraints
for i in range(0,len(cf_matrix[0])):
    short += lpSum([cf_df[i][j]*long_q[j] - cf_df[i][j]*short_q[j] for j in␣
 ↪dec_var_names]) + excess_cfs[i] - excess_cfs[i+1] == liabilities[i]

short.solve()
```

[55]: 1

7.1.3 Portfolio Allocation

[56]:
```
'''
Print Solution to Short portfolio
'''
bonds_short = pd.DataFrame(
    [v.varValue for v in short.variables() if v.varValue != 0],
    index=[str(v.name[:-8].replace('_', ' ')+v.name[-8:].replace('_', '/')) for␣
 ↪v in short.variables() if v.varValue != 0],
    columns=['Quantity']
    )

md('''

<center> Portfolio Value of ${:.2f} MM  </center> <br>

<center>
```

```
{}

'''.format(value(short.objective), bonds_short[bonds_short['Quantity']!=0].
↪to_markdown(colalign = ("right",)))
)
```

[56]: Portfolio Value of $-0.00 MM

	Quantity
ExcessCf/3	-1.38988e-09
ExcessCf/6	7.49844
Long T 0.75 05/31/26	0.104984
Long T 1.5 11/30/24	0.0685951
Long T 2.0 05/31/24	0.0481139
Long T 2.875 05/15/28	0.0607689
Long T 5.25 02/15/29	0.0638876
Long T 5.5 08/15/28	0.0816424
Long T 6.125 08/15/29	0.0755646
Long T 6.25 05/15/30	0.0678788
Long T 6.25 08/15/23	0.0563529
Long T 6.375 08/15/27	0.0395095
Long T 6.625 02/15/27	0.0479221
Long T 6.75 08/15/26	0.0753781
Long T 7.125 02/15/23	61.1449
Long T 7.625 02/15/25	0.154094
Short T 0.0 06/16/22	2.04863
Short T 0.137 12/31/22	63.2136

7.1.4 Discussion

Answer to questions here.

7.2 With Transaction Limits

7.2.1 Mathematical Formulation

7.2.2 Code

[57]:
```
'''
Solves short portfolio with a Shorting Limit
'''
short_limit = 0.5
long_q = LpVariable.dicts('Long',dec_var_names,lowBound=0)
short_q = LpVariable.dicts('Short',dec_var_names,lowBound=0)
excess_cfs = LpVariable.dicts('ExcessCf', periods, lowBound=0)

short_l = LpProblem('Short', LpMinimize)
```

```python

#Objective
short_l += lpSum([cf_df['px_ask'][i] * long_q[i] - cf_df['px_bid'][i] * short_q[i] for i in dec_var_names]+ excess_cfs[0])

#Bounds Objective to be Positive - Municipality can't profit from short trading - At best they get their dedication portfolio free
short_l += lpSum([cf_df['px_ask'][i] * long_q[i] - cf_df['px_bid'][i] * short_q[i] for i in dec_var_names]+ excess_cfs[0]) >= 0

#Adds a Short Limit as a % of the Total Amount invested in Long Bonds
short_l += lpSum([cf_df['px_bid'][i] * short_q[i] for i in dec_var_names]) <= lpSum([cf_df['px_ask'][i] * long_q[i] for i in dec_var_names])*short_limit

#Liabilities Constraints
for i in range(0,len(cf_matrix[0])):
    short_l += lpSum([cf_df[i][j]*long_q[j] - cf_df[i][j]*short_q[j] for j in dec_var_names]) + excess_cfs[i] - excess_cfs[i+1] == liabilities[i]

short_l.solve()
```

[57]: 1

7.2.3 Portfolio Allocation

[58]:
```python
'''
Print Solution to Short portfolio with Shorting Limit
'''
bonds_short_l = pd.DataFrame(
    [v.varValue for v in short_l.variables() if v.varValue != 0],
    index=[str(v.name[:-8].replace('_', ' ')+v.name[-8:].replace('_', '/')) for v in short_l.variables() if v.varValue != 0],
    columns=['Quantity']
    )

md('''

<center> Portfolio Value of ${:.2f} MM  </center> <br>

<center>

{}

'''.format(value(short_l.objective), bonds_short_l[bonds_short_l['Quantity']!=0].to_markdown(colalign = ("right",)))
)
```

[58]: Portfolio Value of $115.62 MM

	Quantity
ExcessCf/3	-1.38988e-09
Long T 0.625 05/15/30	0.0697819
Long T 0.75 05/31/26	0.107674
Long T 1.25 05/31/28	0.0637436
Long T 1.5 11/30/24	0.0714316
Long T 2.0 05/31/24	0.0509223
Long T 2.875 04/30/29	0.0664559
Long T 5.5 08/15/28	0.084142
Long T 6.125 08/15/29	0.0774112
Long T 6.25 08/15/23	1.21302
Long T 6.375 08/15/27	0.0423924
Long T 6.625 02/15/27	0.0507125
Long T 6.75 08/15/26	0.0780774
Long T 6.875 08/15/25	0.0750923
Long T 7.125 02/15/23	0.0325975
Long T 7.25 08/15/22	0.0507575
Long T 7.625 02/15/25	0.0819673
Short T 0.125 06/30/23	1.18926

7.2.4 Discussion

8 Immunization, Short selling, and Dedication

Consider proposing a new portfolio of bonds using any additional consideration or change to the model that you see fit. For example, can you do something to make your portfolio of bonds immune to nonparallel changes in the term structure. Is there a better way to combine the techniques you used before. Explain clearly what you do and your results.

8.1 Mathematical Formulaation

[59]:
```
'''
Immunization With Shorting Available (Limited)
'''

long_q = LpVariable.dicts('Long',dec_var_names,lowBound=0)
short_q = LpVariable.dicts('Short',dec_var_names,lowBound=0)


imm_s = LpProblem('Immunized_Short', LpMinimize)

imm_s += lpSum([cf_df['px_bid'][i] * short_q[i] for i in dec_var_names]) <=␣
↪lpSum([cf_df['px_ask'][i] * long_q[i] for i in dec_var_names])*short_limit
```

```
imm_s += lpSum([cf_df['px_ask'][i] * long_q[i] - cf_df['px_bid'][i] *␣
 ↪short_q[i] for i in dec_var_names])
imm_s += lpSum([immunization_df['npv'].loc[i] * long_q[i] -␣
 ↪immunization_df['npv'].loc[i] * short_q[i] for i in dec_var_names]) == npv
imm_s += lpSum([immunization_df['duration'].loc[i] * long_q[i] -␣
 ↪immunization_df['duration'].loc[i] * short_q[i] for i in dec_var_names]) ==␣
 ↪dur
imm_s += lpSum([immunization_df['convexity'].loc[i] * long_q[i] -␣
 ↪immunization_df['convexity'].loc[i] * short_q[i] for i in dec_var_names]) ==␣
 ↪con

imm_s.solve()
```

[59]: 1

8.2 Portfolio Allocation

[60]:
```
'''
Print Solution to Short Immunized portfolio with Limits (Same Limit as Last␣
 ↪Prob)
'''
imms_bonds = pd.DataFrame(
    [v.varValue for v in imm_s.variables() if v.varValue != 0],
    index=[str(v.name[:-8].replace('_', ' ')+v.name[-8:].replace('_', '/')) for␣
 ↪v in imm_s.variables() if v.varValue != 0],
    columns=['Quantity']
    )

md('''

<center> Portfolio Value of ${:.2f} MM  </center> <br>

<center>

{}

'''.format(value(imm_s.objective), imms_bonds[imms_bonds['Quantity']!=0].
 ↪to_markdown(colalign = ("right",)))
)
```

[60]: Portfolio Value of $113.44 MM

	Quantity
Long T 2.75 06/30/25	0.378032
Long T 6.625 02/15/27	1.27248
Long T 7.25 08/15/22	0.41118

	Quantity
Short T 0.75 11/15/24	1.19349

8.3 Disscussion