PS3_Babaei

September 30, 2019

[1]:
```
# Saharnaz Babaei Balderlou
# Problem Set 3
#-------------------------------------------------------------------------------
# Import required packages
#-------------------------------------------------------------------------------
import pandas as pd # will be used to read .dta file by .read_stata()
import numpy as np
import matplotlib.pyplot as plt # will be used to see the obvious relationship␣
 ↪of desired variables in a scatterplot
#%matplotlib inline
import math # to use in some equations
from scipy.optimize import minimize # for optimization of Likelihood function␣
 ↪(MLE method)
import scipy.optimize as opt
import statsmodels.api as statmod
import scipy.stats as stats
```

[2]:
```
#-------------------------------------------------------------------------------
# Read data and prepare to utilize (Part 1 & 2)
#-------------------------------------------------------------------------------
df1 = pd.read_stata('PS3_data.dta')
# My note for later: https://www.shanelynn.ie/
 ↪using-pandas-dataframe-creating-editing-viewing-data-in-python/
```

[3]:
```
print("Dataframe: ")
df1.head()
```

```
Dataframe:
```

[3]:
```
   id68  year  intid relhh  hannhrs  wannhrs  hlabinc  wlabinc  nochild  \
0     1  1967      1  Head   1200.0   2000.0      NaN      NaN        0
1     2  1967      2  Head      0.0      0.0      NaN      NaN        0
2     3  1967      3  Head      0.0      0.0      NaN      NaN        0
3     4  1967      4  Head   1560.0      0.0      NaN      NaN        6
4     5  1967      5  Head   2500.0   2000.0      NaN      NaN        3

   wrace  ...  redpregovinc  hsex  wsex   age  wage  hpersno  wpersno  hyrsed  \
0    NaN  ...        5614.0   1.0   2.0  52.0  46.0      1.0      2.0     8.0
```

```
1     NaN   ...          0.0    1.0    2.0   56.0   57.0        1.0        2.0       3.0
2     NaN   ...          0.0    1.0    2.0   77.0   64.0        1.0        2.0       NaN
3     1.0   ...       3280.0    1.0    2.0   45.0   44.0        1.0        2.0       8.0
4     1.0   ...       7900.0    1.0    2.0   24.0   22.0        1.0        2.0      10.0

   wyrsed  pce
0     8.0  0.0
1     3.0  0.0
2     3.0  0.0
3     5.0  0.0
4     9.0  0.0

[5 rows x 52 columns]
```

[4]:
```
'''
hlabinc = annual labor income of the head
hannhrs = annual hours of the head
hsex = gender of the head (1 = Male, 2 = Female)
hrace = race of the head (1 = white, 2 = Black, 3 = Native American, 4 = Asian/
↪Pacific Islander, 5 = Hispanic, 6,7 = Other)
age = age of the head
hyrsed = years of education of the head
'''
print("Data Statistics:")
df1.describe()
```

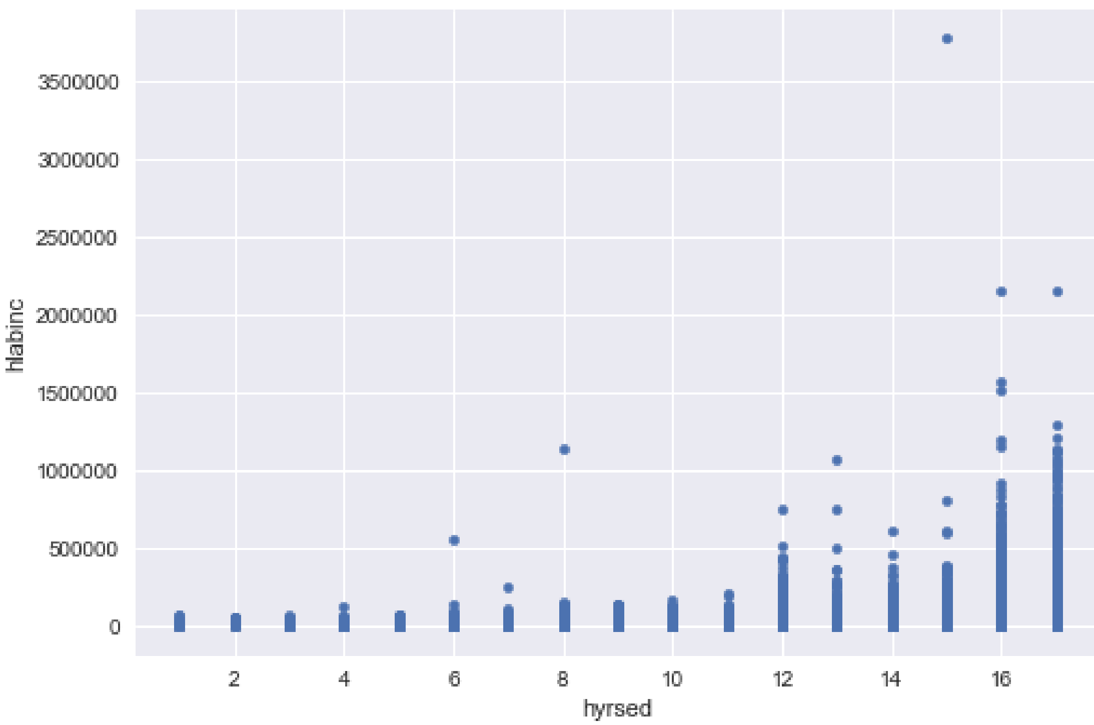
```
Data Statistics:
```

[4]:
```
                id68           year          intid        hannhrs  \
count  123786.000000  123786.000000  123786.000000  123786.000000
mean     1494.639475    1984.831273    3271.379429    1679.269897
std       838.901790       9.836212    2277.056058    1061.704712
min         1.000000    1967.000000       1.000000       0.000000
25%       772.000000    1977.000000    1444.000000     832.000000
50%      1517.000000    1985.000000    2984.000000    1976.000000
75%      2224.000000    1993.000000    4763.000000    2350.000000
max      2930.000000    2002.000000   16968.000000    7800.000000

             wannhrs       hlabinc        wlabinc        nochild  \
count  123786.000000  9.023300e+04   48496.000000  123786.000000
mean      633.026917  4.211505e+04   22026.289062       0.843771
std       878.422791  4.670424e+04   21336.107422       1.182829
min         0.000000  6.353981e-01       1.192780       0.000000
25%         0.000000  1.979858e+04    8016.247070       0.000000
50%         0.000000  3.460022e+04   18122.412109       0.000000
75%      1454.000000  5.267309e+04   30256.060547       2.000000
max      5840.000000  3.771521e+06  856942.062500      11.000000
```

```
               wrace          hrace  ...  redpregovinc           hsex     wsex  \
count  90603.000000  123656.000000  ...  1.237860e+05  123786.000000  80758.0
mean       1.098220       1.129731  ...  3.012258e+04       1.233072      2.0
std        0.356161       0.394627  ...  4.588795e+04       0.422940      0.0
min        1.000000       1.000000  ... -1.324040e+05       1.000000      2.0
25%        1.000000       1.000000  ...  7.700000e+03       1.000000      2.0
50%        1.000000       1.000000  ...  1.900000e+04       1.000000      2.0
75%        1.000000       1.000000  ...  3.910775e+04       1.000000      2.0
max        8.000000       8.000000  ...  3.660000e+06       2.000000      2.0

                  age          wage        hpersno       wpersno  \
count  123786.000000  80758.000000  123786.000000  80758.000000
mean       45.545547     41.390785      39.620201     55.346169
std        17.623671     14.786721      69.003265     77.864296
min        16.000000     13.000000       1.000000      1.000000
25%        31.000000     29.000000       1.000000      2.000000
50%        42.000000     39.000000       3.000000      3.000000
75%        58.000000     51.000000      22.000000    170.000000
max       102.000000     95.000000     227.000000    231.000000

               hyrsed        wyrsed            pce
count  122809.000000  80091.000000  123786.000000
mean       12.666091     12.720081       0.557690
std         2.917721      2.422607       0.265198
min         1.000000      1.000000       0.000000
25%        12.000000     12.000000       0.362158
50%        12.000000     12.000000       0.599887
75%        15.000000     14.000000       0.786908
max        17.000000     17.000000       0.928007

[8 rows x 51 columns]
```

[5]:
```python
print("Scatterplot between annual labor inome of the head and years of education of the head")
plt.style.use('seaborn')
df1.plot(x = 'hyrsed', y = 'hlabinc', kind = 'scatter')
plt.show()
plt.savefig('scat_inc_edu.png')
```

```
Scatterplot between annual labor inome of the head and years of education of the
head
```




```
<Figure size 576x396 with 0 Axes>
```

[6]:
```
#drop missing values
df1_subset = df1.dropna(how = 'any', subset = ['hlabinc', 'hannhrs', 'hsex', 'hrace', 'age', 'hyrsed'])
df1_subset.describe()
```

[6]:
```
              id68         year         intid      hannhrs      wannhrs  \
count  89688.000000  89688.000000  89688.000000  89688.000000  89688.000000
mean    1510.782992   1986.315338   3515.175007   2067.510254    763.457153
std      834.667982      8.791094   2314.034043    756.294312    913.115479
min        1.000000   1971.000000      1.000000      0.000000      0.000000
25%      783.000000   1979.000000   1673.000000   1800.000000      0.000000
50%     1542.000000   1986.000000   3321.000000   2064.000000     74.000000
75%     2240.000000   1993.000000   5058.000000   2453.000000   1700.000000
max     2930.000000   2002.000000  16968.000000   7800.000000   5840.000000

            hlabinc       wlabinc       nochild        wrace         hrace  \
count  8.968800e+04  45338.000000  89688.000000  73835.000000  89688.000000
mean   4.211382e+04  21914.417969      0.949737      1.096932      1.123695
std    4.675834e+04  20676.205078      1.168268      0.354777      0.390376
min    6.353981e-01      1.192780      0.000000      1.000000      1.000000
25%    1.976367e+04   8042.066406      0.000000      1.000000      1.000000
```

```
50%     3.460022e+04   18120.386719      0.000000      1.000000      1.000000
75%     5.267309e+04   30172.169922      2.000000      1.000000      1.000000
max     3.771521e+06  685266.750000     11.000000      8.000000      3.000000

       ...  redpregovinc          hsex     wsex           age          wage  \
count  ...  8.968800e+04  89688.000000  62705.0  89688.000000  62705.000000
mean   ...  3.774112e+04      1.179935      2.0     40.012321     38.289116
std    ...  4.853038e+04      0.384065      0.0     13.287443     12.255554
min    ... -9.359900e+04      1.000000      2.0     17.000000     14.000000
25%    ...  1.400000e+04      1.000000      2.0     29.000000     29.000000
50%    ...  2.640000e+04      1.000000      2.0     38.000000     36.000000
75%    ...  4.744325e+04      1.000000      2.0     49.000000     46.000000
max    ...  3.660000e+06      2.000000      2.0     95.000000     93.000000

            hpersno       wpersno        hyrsed        wyrsed           pce
count  89688.000000  62705.000000  89688.000000  62244.000000  89688.000000
mean      48.742619     65.553192     13.228726     13.029015      0.609756
std       73.709778     81.090965      2.526992      2.225578      0.208654
min        1.000000      1.000000      1.000000      1.000000      0.247121
25%        1.000000      2.000000     12.000000     12.000000      0.421747
50%        4.000000      4.000000     12.000000     12.000000      0.614522
75%      170.000000    170.000000     16.000000     15.000000      0.786908
max      227.000000    231.000000     17.000000     17.000000      0.928007

[8 rows x 51 columns]
```

[7]:

```
# select male heads of HH whose age is between 25 & 60 (included!), and wage >
↪$7/hr
df2 = df1_subset[['id68', 'year', 'intid', 'hlabinc', 'hannhrs', 'hsex',
↪'hrace', 'age', 'hyrsed']]
df2['annhrs'] = df2['hannhrs'].where(df2['hannhrs']>0)
df2['hrwage'] = df2['hlabinc']/df2['annhrs'] # compute hourly wage
df2 = df2[(df2.hsex == 1.0) & (df2.age >= 25) & (df2.age <= 60) & (df2.hrwage >
↪7)] #Part 1
df2['ln_hrwage'] = np.log(df2['hrwage']) #log of wages
df2.describe()
```

```
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:3:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-
docs/stable/indexing.html#indexing-view-versus-copy
  This is separate from the ipykernel package so we can avoid doing imports
until
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:4:
SettingWithCopyWarning:
```

```
A value is trying to be set on a copy of a slice from a DataFrame.
Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-
docs/stable/indexing.html#indexing-view-versus-copy
  after removing the cwd from sys.path.
```

[7]:

```
               id68          year         intid       hlabinc       hannhrs  \
count  57062.000000  57062.000000  57062.000000  5.706200e+04  57062.000000
mean    1507.470558   1986.575672   3480.375311  5.282805e+04   2228.557617
std      828.361439      8.712311   2253.229068  5.236477e+04    620.054077
min        1.000000   1971.000000      1.000000  1.666980e+01      2.000000
25%      782.000000   1979.000000   1690.000000  3.037345e+04   1952.000000
50%     1542.000000   1987.000000   3296.000000  4.382858e+04   2160.000000
75%     2225.000000   1994.000000   5002.000000  6.138495e+04   2519.000000
max     2930.000000   2002.000000  16968.000000  3.771521e+06   5840.000000

          hsex         hrace           age        hyrsed        annhrs  \
count  57062.0  57062.000000  57062.000000  57062.000000  57062.000000
mean       1.0      1.101416     39.243629     13.529967   2228.557617
std        0.0      0.369015      9.578858      2.450013    620.054077
min        1.0      1.000000     25.000000      1.000000      2.000000
25%        1.0      1.000000     31.000000     12.000000   1952.000000
50%        1.0      1.000000     38.000000     13.000000   2160.000000
75%        1.0      1.000000     47.000000     16.000000   2519.000000
max        1.0      3.000000     60.000000     17.000000   5840.000000

             hrwage     ln_hrwage
count  57062.000000  57062.000000
mean      24.320921      3.010804
std       25.209909      0.544096
min        7.000252      1.945946
25%       13.950662      2.635527
50%       19.914677      2.991457
75%       27.790929      3.324710
max     1717.330322      7.448526
```

[8]:

```
# This is a note for me: https://stackoverflow.com/questions/11587782/
 ↪creating-dummy-variables-in-pandas-for-python
race_dummy = pd.get_dummies(df2['hrace']) # Defining race dummies (There is no␣
 ↪Hispanic individual in dataset)
df2['constant'] = 1
data = pd.concat([df2, race_dummy], axis = 1)
data.rename(columns = {1.0: 'White', 2.0: 'Black', 3.0: 'Others'}, inplace =␣
 ↪True) #final data "data"; ready to estimate model
data.head()
```

[8]:
```
        id68  year  intid      hlabinc  hannhrs  hsex  hrace   age  hyrsed  \
11161    402  1971      1  62928.707031   1523.0   1.0    1.0  51.0    12.0
11164    461  1971      4  22660.970703   2010.0   1.0    1.0  55.0     5.0
11166   1126  1971      8  29337.865234   2860.0   1.0    1.0  25.0    16.0
11173    284  1971     20  76885.437500   2400.0   1.0    1.0  39.0    16.0
11175     50  1971     29  31968.156250   3164.0   1.0    1.0  36.0    12.0

       annhrs     hrwage  ln_hrwage  constant  White  Black  Others
11161  1523.0  41.318916   3.721320         1      1      0       0
11164  2010.0  11.274115   2.422509         1      1      0       0
11166  2860.0  10.257995   2.328057         1      1      0       0
11173  2400.0  32.035599   3.466848         1      1      0       0
11175  3164.0  10.103716   2.312903         1      1      0       0
```

[9]:
```
data.dtypes
```

[9]:
```
id68           int16
year           int16
intid          int16
hlabinc      float32
hannhrs      float32
hsex         float32
hrace        float64
age          float32
hyrsed       float32
annhrs       float32
hrwage       float32
ln_hrwage    float32
constant       int64
White          uint8
Black          uint8
Others         uint8
dtype: object
```

[10]:
```
np.save("data", data)

# Separate years for estimation
data1971 = data[data['year'] == 1971]
np.save("data1971", data1971)
data1980 = data[data['year'] == 1980]
np.save("data1980", data1980)
data1990 = data[data['year'] == 1990]
np.save("data1990", data1990)
data2000 = data[data['year'] == 2000]
np.save("data2000", data2000)
```

[11]:
```
#--------------------------------------------------------------------------------
# ML estimation (Part 3)
#--------------------------------------------------------------------------------
```

```
'''
ln(w_it) = alpha + beta_1 * Educ_it + beta_2 * Age_it + beta_3 * Black_it +␣
↪beta_4 * Hispanic_it + beta_5 * OtherRace_it + epsilon_it
w_it = wage of individual i in survey year t
Educ_it = education in years
Age_it = age in years
Black_it, Hispanic_it, OtherRace_it = dummy variables for race = Black,␣
↪Hispanic, Not (belongs to {White, Black, Hispanic})
'''

# Define my objective function
def myLL(params, t):
    # Coeff.s
    beta0, beta1, beta2, beta3, beta4, sigma = params
    beta = np.array([beta0, beta1, beta2, beta3, beta4])
    n = len(t)
    # Independent variables matrix (No Hispanic)
    x0 = np.array(t['constant']).astype('float')
    x1 = np.array(t['hyrsed']).astype('float')
    x2 = np.array(t['age']).astype('float')
    x3 = np.array(t['Black']).astype('float')
    x4 = np.array(t['Others']).astype('float')
    X = np.column_stack((x0, x1, x2, x3, x4))
    # Dependent variable matrix
    y = np.array(t['ln_hrwage']).astype('float')
    y_bar = np.dot(X, beta)
    ll = (-(n/2)*np.log(2*np.pi) - (n/2)*np.log(sigma**2) - (1/
↪(2*sigma**2))*((y-y_bar).T @ (y - y_bar)))
    return (-ll)
```

[12]:
```
# MLE; 'Nelder-Mead'
nbeta = 5
beta = np.zeros(nbeta)
beta0 = 0.1
beta1 = 0.1
beta2 = 0.1
beta3 = 0.1
beta4 = 0.1
sigma = 0.1
beta = [beta0, beta1, beta2, beta3, beta4]
params = [beta0, beta1, beta2, beta3, beta4, sigma]

bounds = ((1e-10, None), (None,None), (None,None), (None,None), (None,None),␣
↪(None,None))

res_NM = opt.minimize(myLL, params, args=(data), method='Nelder-Mead',␣
↪bounds=bounds)
```

```
print("MLE coefficients: ", "Total dataset")
print("=============================")
print(res_NM)
print("_____________________________________")
res71_NM = opt.minimize(myLL, params, args=(data1971), method='Nelder-Mead',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 1971")
print("=============================")
print(res71_NM)
print("_____________________________________")
res80_NM = opt.minimize(myLL, params, args=(data1980), method='Nelder-Mead',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 1980")
print("=============================")
print(res80_NM)
print("_____________________________________")
res90_NM = opt.minimize(myLL, params, args=(data1990), method='Nelder-Mead',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 1990")
print("=============================")
print(res90_NM)
print("_____________________________________")
res2000_NM = opt.minimize(myLL, params, args=(data2000), method='Nelder-Mead',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 2000")
print("=============================")
print(res2000_NM)
print("_____________________________________")
'''
years = ["data1971.npy", "data1980.npy", "data1990.npy", "data2000.npy", "data.
 ↪npy"]
for t in years:
    res_MLE = opt.minimize(myLL, params, args=(t), method='Nelder-Mead',␣
 ↪bounds=bounds)
    print("MLE coefficients: ", t)
    print(res_MLE)
    print("_____________________________________")
#### I tried this loop so many times and attempted to troubleshoot but finally␣
 ↪I got this error: "string indices must be integers"
'''
```

```
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\_minimize.py:517:
RuntimeWarning: Method Nelder-Mead cannot handle constraints nor bounds.
  RuntimeWarning)

MLE coefficients:  Total dataset
=============================
```

```
 final_simplex: (array([[ 1.39309627,  0.0781139 ,  0.0145177 , -0.16239436,
0.01108259,
         0.48968236],
       [ 1.3930527 ,  0.0781193 ,  0.01451702, -0.1624352 ,  0.01106685,
         0.4896769 ],
       [ 1.39309914,  0.07811356,  0.01451801, -0.16244586,  0.0109965 ,
         0.48968791],
       [ 1.39307198,  0.07811841,  0.01451689, -0.16244504,  0.01111231,
         0.48968793],
       [ 1.39313048,  0.07811415,  0.01451716, -0.16247289,  0.01103884,
         0.48967662],
       [ 1.3930366 ,  0.07812007,  0.01451746, -0.16232213,  0.01103599,
         0.48968893],
       [ 1.39317897,  0.07811236,  0.01451641, -0.16240374,  0.01101757,
         0.48968747]]), array([40225.31420626, 40225.31421319, 40225.31422174,
40225.31422664,
       40225.31424347, 40225.3142488 , 40225.3142569 ]))
           fun: 40225.31420626389
       message: 'Optimization terminated successfully.'
          nfev: 1162
           nit: 749
        status: 0
       success: True
             x: array([ 1.39309627,  0.0781139 ,  0.0145177 , -0.16239436,
0.01108259,
        0.48968236])
-------------------------------------
MLE coefficients:  year == 1971
=============================
 final_simplex: (array([[ 1.47367052,  0.06973573,  0.01578288, -0.2340242 ,
-0.42911197,
         0.41157107],
       [ 1.47361046,  0.06974115,  0.01578231, -0.23397205, -0.42911159,
         0.4115673 ],
       [ 1.47358856,  0.06973973,  0.01578298, -0.23403964, -0.42908831,
         0.41157356],
       [ 1.4736112 ,  0.06973677,  0.01578324, -0.23399855, -0.42909645,
         0.41156275],
       [ 1.47371123,  0.06973561,  0.01578128, -0.23405239, -0.42911123,
         0.41156895],
       [ 1.47367534,  0.06973616,  0.01578253, -0.23405703, -0.42909264,
         0.41156107],
       [ 1.47371518,  0.06973268,  0.01578194, -0.23400247, -0.42914202,
         0.41158344]]), array([752.78870883, 752.78870947, 752.78871087,
752.78871124,
       752.78871183, 752.7887139 , 752.78871442]))
           fun: 752.7887088280063
       message: 'Optimization terminated successfully.'
```

```
          nfev: 786
           nit: 496
        status: 0
       success: True
             x: array([ 1.47367052,  0.06973573,  0.01578288, -0.2340242 ,
-0.42911197,
        0.41157107])
-------------------------------------
MLE coefficients:  year == 1980
=============================
 final_simplex: (array([[ 1.6132899 ,  0.0675381 ,  0.01269899, -0.10263128,
0.0136228 ,
         0.44926559],
       [ 1.61305248,  0.06756315,  0.01269762, -0.10267639,  0.01366701,
         0.44928481],
       [ 1.61340895,  0.06753724,  0.01269743, -0.10258163,  0.01316331,
         0.4492538 ],
       [ 1.61323268,  0.06753902,  0.01269958, -0.10288796,  0.01379478,
         0.44921281],
       [ 1.61374566,  0.06752382,  0.01269221, -0.10279164,  0.01359016,
         0.44925456],
       [ 1.61356332,  0.06752716,  0.01269722, -0.10289006,  0.01380762,
         0.44925926],
       [ 1.61340448,  0.06753553,  0.01269851, -0.10301816,  0.01315615,
         0.4492721 ]]), array([1148.39323838, 1148.39324439, 1148.39325561,
1148.39325705,
       1148.39326198, 1148.39326495, 1148.39326782]))
           fun: 1148.39323837555
       message: 'Maximum number of function evaluations has been exceeded.'
          nfev: 1201
           nit: 774
        status: 1
       success: False
             x: array([ 1.6132899 ,  0.0675381 ,  0.01269899, -0.10263128,
0.0136228 ,
        0.44926559])
-------------------------------------
MLE coefficients:  year == 1990
=============================
 final_simplex: (array([[ 0.66841963,  0.11863582,  0.01749401, -0.14739724,
-0.65477016,
         0.4893061 ],
       [ 0.66835602,  0.11864754,  0.0174916 , -0.14732871, -0.65471547,
         0.4893005 ],
       [ 0.66851798,  0.1186424 ,  0.01748935, -0.14746815, -0.65485297,
         0.48929941],
       [ 0.66837051,  0.11864686,  0.01749027, -0.14739821, -0.65472676,
         0.48930142],
```

```
       [ 0.66838604,  0.11864412,  0.01749209, -0.14745596, -0.65473298,
         0.48929077],
       [ 0.66840779,  0.11864285,  0.01749167, -0.14741798, -0.65476171,
         0.4893055 ],
       [ 0.66846521,  0.11864223,  0.01749013, -0.14735539, -0.65480545,
         0.48929722]]), array([1430.23330055, 1430.23330579, 1430.23330615,
1430.23330668,
       1430.23330737, 1430.23330866, 1430.23331002]))
           fun: 1430.233300554175
       message: 'Optimization terminated successfully.'
          nfev: 852
           nit: 547
        status: 0
       success: True
             x: array([ 0.66841963,  0.11863582,  0.01749401, -0.14739724,
-0.65477016,
        0.4893061 ])
-------------------------------------
MLE coefficients:  year == 2000
=============================
 final_simplex: (array([[ 1.16169112,  0.10915564,  0.01099325, -0.24608949,
-0.06074079,
         0.5395586 ],
       [ 1.16172663,  0.10915285,  0.01099324, -0.24602461, -0.06069615,
         0.53956308],
       [ 1.16176407,  0.10914997,  0.01099332, -0.2460459 , -0.06071587,
         0.53956483],
       [ 1.16165857,  0.10915336,  0.01099491, -0.24603163, -0.06075654,
         0.53955864],
       [ 1.16175584,  0.10915004,  0.01099372, -0.24609096, -0.06069359,
         0.53956002],
       [ 1.16161566,  0.10915489,  0.01099514, -0.24607217, -0.06067408,
         0.53955494],
       [ 1.1616349 ,  0.10915557,  0.01099461, -0.24605415, -0.06071328,
         0.53954611]]), array([2068.985145  , 2068.98514519, 2068.98514543,
2068.98514579,
       2068.98514608, 2068.98514614, 2068.98514625]))
           fun: 2068.985144996586
       message: 'Optimization terminated successfully.'
          nfev: 1143
           nit: 742
        status: 0
       success: True
             x: array([ 1.16169112,  0.10915564,  0.01099325, -0.24608949,
-0.06074079,
        0.5395586 ])
-------------------------------------
```

[12]:
```
'\nyears = ["data1971.npy", "data1980.npy", "data1990.npy", "data2000.npy",
"data.npy"]\nfor t in years:\n    res_MLE = opt.minimize(myLL, params, args=(t),
method=\'Nelder-Mead\', bounds=bounds)\n    print("MLE coefficients: ", t)\n
print(res_MLE)\n    print("_____________________________________")\n#### I tried
this loop so many times and attempted to troubleshoot but finally I got this
error: "string indices must be integers"\n'
```

[13]:
```python
# MLE; 'L-BFGS-B'
nbeta = 5
beta = np.zeros(nbeta)
beta0 = 0.1
beta1 = 0.1
beta2 = 0.1
beta3 = 0.1
beta4 = 0.1
sigma = 0.1
beta = [beta0, beta1, beta2, beta3, beta4]
params = [beta0, beta1, beta2, beta3, beta4, sigma]

bounds = ((1e-10, None), (None,None), (None,None), (None,None), (None,None),␣
 ↪(None,None))

res_B = opt.minimize(myLL, params, args=(data), method='L-BFGS-B',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "Total dataset")
print("=============================")
print(res_B)
print("_____________________________________")
res71_B = opt.minimize(myLL, params, args=(data1971), method='L-BFGS-B',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 1971")
print("=============================")
print(res71_B)
print("_____________________________________")
res80_B = opt.minimize(myLL, params, args=(data1980), method='L-BFGS-B',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 1980")
print("=============================")
print(res80_B)
print("_____________________________________")
res90_B = opt.minimize(myLL, params, args=(data1990), method='L-BFGS-B',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 1990")
print("=============================")
print(res90_B)
print("_____________________________________")
```

```
res2000_B = opt.minimize(myLL, params, args=(data2000), method='L-BFGS-B',␣
 ↪bounds=bounds)
print("MLE coefficients: ", "year == 2000")
print("=============================")
print(res2000_B)
print("_____________________________________")
```

```
MLE coefficients:  Total dataset
=============================
      fun: 40225.31422021307
 hess_inv: <6x6 LbfgsInvHessProduct with dtype=float64>
      jac: array([ 1.09284883, 11.08637662, 13.14037945,  0.50786184,
-0.14697434,
        2.48692231])
  message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
     nfev: 469
      nit: 57
   status: 0
  success: True
        x: array([ 1.39310199,  0.07811655,  0.01451693, -0.1623717 ,  0.0110262
,
        0.48968773])
_____________________________________
MLE coefficients:  year == 1971
=============================
      fun: 728.0628698968592
 hess_inv: <6x6 LbfgsInvHessProduct with dtype=float64>
      jac: array([-5.36374500e-02, -4.52814675e-01, -2.12465920e+00,
3.09682946e-02,
       -7.27595761e-04,  9.65997060e-02])
  message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
     nfev: 602
      nit: 76
   status: 0
  success: True
        x: array([ 1.55089935,  0.0668825 ,  0.01439164, -0.16381197,  0.0306993
,
        0.41010486])
_____________________________________
MLE coefficients:  year == 1980
=============================
      fun: 1148.3932188882504
 hess_inv: <6x6 LbfgsInvHessProduct with dtype=float64>
      jac: array([-0.00618456, -0.013506  , -0.2509978 ,  0.01757599,
-0.01077751,
       -0.02992238])
  message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
```

```
     nfev: 448
      nit: 52
   status: 0
  success: True
        x: array([ 1.61306755,  0.06755729,  0.01269855, -0.10270202,
0.01346053,
        0.44924099])
-------------------------------------
MLE coefficients:  year == 1990
=============================
      fun: 1393.8821505146818
 hess_inv: <6x6 LbfgsInvHessProduct with dtype=float64>
      jac: array([-1.40516931e-02, -2.04931894e-01, -5.94741323e-01,
5.91171556e-04,
       -2.95585778e-04, -1.26192390e-02])
  message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
     nfev: 511
      nit: 64
   status: 0
  success: True
        x: array([ 1.11859519,  0.0975577 ,  0.01346543, -0.17202183,
-0.05971275,
        0.4835987 ])
-------------------------------------
MLE coefficients:  year == 2000
=============================
      fun: 2068.985144417821
 hess_inv: <6x6 LbfgsInvHessProduct with dtype=float64>
      jac: array([ 0.00245564,  0.03237801,  0.03697096, -0.00118234,
0.00172804,
       -0.0125965 ])
  message: b'CONVERGENCE: REL_REDUCTION_OF_F_<=_FACTR*EPSMCH'
     nfev: 378
      nit: 47
   status: 0
  success: True
        x: array([ 1.16170054,  0.1091542 ,  0.01099335, -0.24604792,
-0.06072719,
        0.53955657])
-------------------------------------
```

[14]:
```
# MLE; 'SLSQP'
nbeta = 5
beta = np.zeros(nbeta)
beta0 = 1.20
beta1 = 0.11
beta2 = 0.01
```

```python
beta3 = 0.20
beta4 = 0.01
sigma = 0.50
beta = [beta0, beta1, beta2, beta3, beta4]
params = [beta0, beta1, beta2, beta3, beta4, sigma]

bounds = ((1e-10, None), (None,None), (None,None), (None,None), (None,None), (None,None))

res_S = opt.minimize(myLL, params, args=(data), method='SLSQP', bounds=bounds)
print("MLE coefficients: ", "Total dataset")
print("=============================")
print(res_S)
print("_____________________________________")
res71_S = opt.minimize(myLL, params, args=(data1971), method='SLSQP', bounds=bounds)
print("MLE coefficients: ", "year == 1971")
print("=============================")
print(res71_S)
print("_____________________________________")
res80_S = opt.minimize(myLL, params, args=(data1980), method='SLSQP', bounds=bounds)
print("MLE coefficients: ", "year == 1980")
print("=============================")
print(res80_S)
print("_____________________________________")
res90_S = opt.minimize(myLL, params, args=(data1990), method='SLSQP', bounds=bounds)
print("MLE coefficients: ", "year == 1990")
print("=============================")
print(res90_S)
print("_____________________________________")
res2000_S = opt.minimize(myLL, params, args=(data2000), method='SLSQP', bounds=bounds)
print("MLE coefficients: ", "year == 2000")
print("=============================")
print(res2000_S)
print("_____________________________________")
```

```
MLE coefficients:  Total dataset
=============================
     fun: 40225.314173424194
     jac: array([ 0.0078125 ,  0.08886719,  0.16503906, -0.00683594,  0.
,
        0.11083984])
 message: 'Optimization terminated successfully.'
    nfev: 140
```

```
     nit: 12
    njev: 12
  status: 0
 success: True
       x: array([ 1.39301236,  0.07811904,  0.01451826, -0.16240252,
0.01105784,
        0.48968273])
-------------------------------------
MLE coefficients:  year == 1971
=============================
     fun: 728.062867806588
     jac: array([-0.06733704, -0.85600281, -2.66024017,  0.01177979,
-0.00302124,
        0.07466125])
 message: 'Optimization terminated successfully.'
    nfev: 125
     nit: 11
    njev: 11
  status: 0
 success: True
       x: array([ 1.55097459,  0.06687866,  0.01439123, -0.1638947 ,
0.03068816,
        0.41009816])
-------------------------------------
MLE coefficients:  year == 1980
=============================
     fun: 1148.3932185943872
     jac: array([-0.0244751 , -0.30895996, -0.92105103,  0.01419067,
-0.00190735,
        0.05471802])
 message: 'Optimization terminated successfully.'
    nfev: 120
     nit: 11
    njev: 11
  status: 0
 success: True
       x: array([ 1.61308089,  0.06755625,  0.01269849, -0.10270663,
0.01350651,
        0.44924558])
-------------------------------------
MLE coefficients:  year == 1990
=============================
     fun: 1393.8821506055688
     jac: array([-0.02429199, -0.3656311 , -0.88607788, -0.00256348,  0.
,
        0.04708862])
 message: 'Optimization terminated successfully.'
    nfev: 117
```

```
     nit: 11
    njev: 11
  status: 0
 success: True
       x: array([ 1.11859812,  0.0975571 ,  0.01346555, -0.17202805, -0.0597097
,
        0.48360217])
-------------------------------------
MLE coefficients:  year == 2000
=============================
     fun: 2068.9851444238884
     jac: array([ 1.58691406e-02,  2.22412109e-01,  6.81030273e-01,
2.07519531e-03,
        4.27246094e-04, -1.78222656e-02])
 message: 'Optimization terminated successfully.'
    nfev: 115
     nit: 10
    njev: 10
  status: 0
 success: True
       x: array([ 1.16169916,  0.1091542 ,  0.01099342, -0.24604219,
-0.06073273,
        0.53955628])
-------------------------------------
```

[15]:
```python
# Comparing the MLE coefficients for 3 different methods:
print("All data; Nelder-Mead: ", res_NM.x)
print("1971; Nelder-Mead: ", res71_NM.x)
print("1980; Nelder-Mead: ", res80_NM.x)
print("1990; Nelder-Mead: ", res90_NM.x)
print("2000; Nelder-Mead: ", res2000_NM.x)
print("____________________________________________")
print("All data; L-BFGS-B: ", res_B.x)
print("1971; L-BFGS-B: ", res71_B.x)
print("1980; L-BFGS-B: ", res80_B.x)
print("1990; L-BFGS-B: ", res90_B.x)
print("2000; L-BFGS-B: ", res2000_B.x)
print("____________________________________________")
print("All data; SLSQP: ", res_S.x)
print("1971; L-BFGS-B: ", res71_S.x)
print("1980; L-BFGS-B: ", res80_S.x)
print("1990; L-BFGS-B: ", res90_S.x)
print("2000; L-BFGS-B: ", res2000_S.x)
```

```
All data; Nelder-Mead:  [ 1.39309627  0.0781139   0.0145177  -0.16239436
0.01108259  0.48968236]
1971; Nelder-Mead:  [ 1.47367052  0.06973573  0.01578288 -0.2340242  -0.42911197
```

```
0.41157107]
1980; Nelder-Mead:  [ 1.6132899   0.0675381   0.01269899 -0.10263128  0.0136228
0.44926559]
1990; Nelder-Mead:  [ 0.66841963  0.11863582  0.01749401 -0.14739724 -0.65477016
0.4893061 ]
2000; Nelder-Mead:  [ 1.16169112  0.10915564  0.01099325 -0.24608949 -0.06074079
0.5395586 ]
--------------------------------------------
All data; L-BFGS-B:  [ 1.39310199  0.07811655  0.01451693 -0.1623717   0.0110262
0.48968773]
1971; L-BFGS-B:  [ 1.55089935  0.0668825   0.01439164 -0.16381197  0.0306993
0.41010486]
1980; L-BFGS-B:  [ 1.61306755  0.06755729  0.01269855 -0.10270202  0.01346053
0.44924099]
1990; L-BFGS-B:  [ 1.11859519  0.0975577   0.01346543 -0.17202183 -0.05971275
0.4835987 ]
2000; L-BFGS-B:  [ 1.16170054  0.1091542   0.01099335 -0.24604792 -0.06072719
0.53955657]
--------------------------------------------
All data; SLSQP:  [ 1.39301236  0.07811904  0.01451826 -0.16240252  0.01105784
0.48968273]
1971; L-BFGS-B:  [ 1.55097459  0.06687866  0.01439123 -0.1638947   0.03068816
0.41009816]
1980; L-BFGS-B:  [ 1.61308089  0.06755625  0.01269849 -0.10270663  0.01350651
0.44924558]
1990; L-BFGS-B:  [ 1.11859812  0.0975571   0.01346555 -0.17202805 -0.0597097
0.48360217]
2000; L-BFGS-B:  [ 1.16169916  0.1091542   0.01099342 -0.24604219 -0.06073273
0.53955628]
```

[16]:
```python
# OLS

#https://lectures.quantecon.org/py/ols.html
# to estimate each year separately
res_OLS = statmod.OLS(endog=data['ln_hrwage'], exog=data[['constant', 'hyrsed',␣
↪'age', 'Black', 'Others']]).fit()
res71_OLS = statmod.OLS(endog=data1971['ln_hrwage'], exog=data1971[['constant',␣
↪'hyrsed', 'age', 'Black', 'Others']]).fit()
res80_OLS = statmod.OLS(endog=data1980['ln_hrwage'], exog=data1980[['constant',␣
↪'hyrsed', 'age', 'Black', 'Others']]).fit()
res90_OLS = statmod.OLS(endog=data1990['ln_hrwage'], exog=data1990[['constant',␣
↪'hyrsed', 'age', 'Black', 'Others']]).fit()
res2000_OLS = statmod.OLS(endog=data2000['ln_hrwage'],␣
↪exog=data2000[['constant', 'hyrsed', 'age', 'Black', 'Others']]).fit()

print('OLS; Full Sample')
print('===================')
```

```
print(res_OLS.summary())
print('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
print('OLS; year == 1971')
print('===================')
print(res71_OLS.summary())
print('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
print('OLS; year == 1980')
print('===================')
print(res80_OLS.summary())
print('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
print('OLS; year == 1990')
print('===================')
print(res90_OLS.summary())
print('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
print('OLS; year == 2000')
print('===================')
print(res2000_OLS.summary())
print('~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~')
'''
years = ['data1971.npy', 'data1980.npy', 'data1990.npy', 'data2000.npy', 'data.
 ↪npy']
for t in years:
    print('OLS for', t)
    print('===================')
    print(statmod.OLS(endog=t['ln_hrwage'], exog=t[['constant', 'hyrsed',␣
 ↪'age', 'Black', 'Others']]).fit().summary())
    ␣
 ↪print('_____________________________________________________________________________
'''
```

```
OLS; Full Sample
===================
                            OLS Regression Results
==============================================================================
Dep. Variable:              ln_hrwage   R-squared:                       0.190
Model:                            OLS   Adj. R-squared:                  0.190
Method:                 Least Squares   F-statistic:                     3346.
Date:                Mon, 30 Sep 2019   Prob (F-statistic):               0.00
Time:                        21:39:38   Log-Likelihood:                -40225.
No. Observations:               57062   AIC:                         8.046e+04
Df Residuals:                   57057   BIC:                         8.051e+04
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
constant       1.3930      0.015     94.165      0.000       1.364       1.422
```

```
hyrsed          0.0781      0.001     92.558      0.000       0.076       0.080
age             0.0145      0.000     67.707      0.000       0.014       0.015
Black          -0.1624      0.009    -18.129      0.000      -0.180      -0.145
Others          0.0111      0.014      0.800      0.424      -0.016       0.038
==============================================================================
Omnibus:                     5824.662   Durbin-Watson:                   1.937
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            16025.805
Skew:                           0.572   Prob(JB):                         0.00
Kurtosis:                       5.330   Cond. No.                         310.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
OLS; year == 1971
===================
                            OLS Regression Results
==============================================================================
Dep. Variable:             ln_hrwage   R-squared:                       0.244
Model:                           OLS   Adj. R-squared:                  0.241
Method:                Least Squares   F-statistic:                     110.7
Date:               Mon, 30 Sep 2019   Prob (F-statistic):           7.42e-82
Time:                       21:39:38   Log-Likelihood:                -728.06
No. Observations:               1380   AIC:                             1466.
Df Residuals:                   1375   BIC:                             1492.
Df Model:                          4
Covariance Type:           nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
constant        1.5510      0.073     21.382      0.000       1.409       1.693
hyrsed          0.0669      0.004     17.814      0.000       0.060       0.074
age             0.0144      0.001     12.902      0.000       0.012       0.017
Black          -0.1639      0.045     -3.638      0.000      -0.252      -0.076
Others          0.0307      0.069      0.447      0.655      -0.104       0.165
==============================================================================
Omnibus:                       96.384   Durbin-Watson:                   1.819
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              217.837
Skew:                           0.424   Prob(JB):                     4.98e-48
Kurtosis:                       4.752   Cond. No.                         291.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
OLS; year == 1980
```

```
===================
                            OLS Regression Results
==============================================================================
Dep. Variable:              ln_hrwage   R-squared:                       0.169
Model:                            OLS   Adj. R-squared:                  0.167
Method:                 Least Squares   F-statistic:                     94.08
Date:                Mon, 30 Sep 2019   Prob (F-statistic):           6.41e-73
Time:                        21:39:38   Log-Likelihood:                -1148.4
No. Observations:                1856   AIC:                             2307.
Df Residuals:                    1851   BIC:                             2334.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
constant       1.6131      0.075     21.590      0.000       1.467       1.760
hyrsed         0.0676      0.004     15.860      0.000       0.059       0.076
age            0.0127      0.001     12.281      0.000       0.011       0.015
Black         -0.1027      0.044     -2.355      0.019      -0.188      -0.017
Others         0.0135      0.071      0.190      0.849      -0.126       0.153
==============================================================================
Omnibus:                      109.135   Durbin-Watson:                   1.958
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              266.486
Skew:                           0.337   Prob(JB):                     1.36e-58
Kurtosis:                       4.730   Cond. No.                         302.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
OLS; year == 1990
===================
                            OLS Regression Results
==============================================================================
Dep. Variable:              ln_hrwage   R-squared:                       0.217
Model:                            OLS   Adj. R-squared:                  0.216
Method:                 Least Squares   F-statistic:                     139.3
Date:                Mon, 30 Sep 2019   Prob (F-statistic):          3.67e-105
Time:                        21:39:38   Log-Likelihood:                -1393.9
No. Observations:                2013   AIC:                             2798.
Df Residuals:                    2008   BIC:                             2826.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
constant       1.1186      0.084     13.312      0.000       0.954       1.283
```

```
hyrsed          0.0976      0.005     19.991      0.000       0.088       0.107
age             0.0135      0.001     10.785      0.000       0.011       0.016
Black          -0.1720      0.048     -3.601      0.000      -0.266      -0.078
Others         -0.0597      0.089     -0.670      0.503      -0.234       0.115
==============================================================================
Omnibus:                      111.834   Durbin-Watson:                   2.006
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              253.156
Skew:                           0.342   Prob(JB):                     1.07e-55
Kurtosis:                       4.597   Cond. No.                         349.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
OLS; year == 2000
===================
                            OLS Regression Results
==============================================================================
Dep. Variable:              ln_hrwage   R-squared:                       0.207
Model:                            OLS   Adj. R-squared:                  0.206
Method:                 Least Squares   F-statistic:                     168.3
Date:                Mon, 30 Sep 2019   Prob (F-statistic):         3.93e-128
Time:                        21:39:38   Log-Likelihood:                -2069.0
No. Observations:                2580   AIC:                             4148.
Df Residuals:                    2575   BIC:                             4177.
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
constant        1.1617      0.081     14.419      0.000       1.004       1.320
hyrsed          0.1092      0.005     21.168      0.000       0.099       0.119
age             0.0110      0.001      9.585      0.000       0.009       0.013
Black          -0.2460      0.048     -5.145      0.000      -0.340      -0.152
Others         -0.0607      0.060     -1.018      0.309      -0.178       0.056
==============================================================================
Omnibus:                      305.903   Durbin-Watson:                   1.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              708.951
Skew:                           0.696   Prob(JB):                    1.13e-154
Kurtosis:                       5.158   Cond. No.                         339.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
```

[16]:
```
"\nyears = ['data1971.npy', 'data1980.npy', 'data1990.npy', 'data2000.npy',
'data.npy']\nfor t in years:\n    print('OLS for', t)\n
print('===================')\n    print(statmod.OLS(endog=t['ln_hrwage'],
exog=t[['constant', 'hyrsed', 'age', 'Black', 'Others']]).fit().summary())\n
print('_____________________________________________________________________
______________________')\n"
```

Interpretation (Part 4)

To answer this question that how the returns to education change over time in these data, we can note that it is increasing. The results for OLS estimation in the full sample indicate that 1 unit increase in the years of education of males will result in a 7.8 % increase in their wages. Using data for 1971, we can see that this increase in the wage is as much as 6.7% as a result of each year of increased education for the male heads. We can also see that the percentage of increase in the wages is 6.8%, 9.8%, & 11% for 1980, 1990, and 2000, respectively, after one more year of education.

In addition, I estimated the same linear model using the maximum likelihood model and the estimated coefficients for the education are 7.8, 6.97, 6.7, 11.8, 10.9 percent for the full sample, 1971 sample of the data, 1980, 1990 and 2000 samples, respectively, which supports the estimation of the model using OLS both in magnitude and signature. MLE method can be estimated using several optimization methods for the likelihood function. After using Nelder-Mead method and gaining aforementioned estimations, we used bounded L-BFGS-B and SLSQP methods. The former estimates the coefficients of the education as 7.8, 6.7, 6.7, 9.7, and 10.9 % for the full dataset, 1971, 1980, 1990, 2000 datasets respectively which is againg very close to the estimations of the model using OLS methodology. But the latter estimated far away coefficients for the provided samples of the data with initial values of 0.1 for all coefficients and tolerance level of 1e-15. However, after changing the initial values of the coefficients in the optimization method, I received the same estimations as the ones in OLS and other MLE optimization methods.

All in all, education and wages are positively correlated and increasing the years of education will lead to higher wages.

[ ]: