Household Savings and Monetary Policy under Individual and Aggregate Stochastic Volatility

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Motivation

- Four major developments in macroeconomics:
  – heterogeneity in income or productivity and assets with differing liquidity (machine, liquid bonds)
  – aggregate (and idiosyncratic) uncertainty done right
  – global solutions
  – redistribution
- These are usually done in isolation.

This paper: We do all four in one framework

HANK Model

- **Households**
  - Two assets: bonds (liquid) and machines (illiquid)
  - Two borrowing constraints
  - Idiosyncratic shocks to productivity level and variance
  - Sticky wages
- **Firms**
  - CRS with machines and labor
  - Aggregate shocks to TFP level and variance
- **Government**
  - Fiscal policy (progressive income taxation as in Heathcote, Storesletten, and Violante 2017)
  - Monetary policy (Taylor rule with ZLB)

Levels and Uncertainty Shocks

- Household productivity

$$
\begin{aligned}
\text{risk:} \quad \eta_{\ell,t}(j) &= \rho^{\ell}\eta_{\ell,t-1}(j) + \\
&\quad \exp\left(\sigma_{\ell,t-1} - \frac{\sigma_{\sigma_\ell}}{\sqrt{1-(\rho^{\sigma_\ell})^2}}\right)\bar{\sigma}_\ell \varepsilon_{\ell,t}(j) \\
\text{uncertainty:} \quad \sigma_{\ell,t} &= \rho^{\sigma_l}\sigma_{\ell,t-1} + \sigma_{\sigma_\ell}\varepsilon_{\sigma_\ell,t}
\end{aligned}
$$

- Aggregate TFP

$$
\begin{aligned}
\text{risk:} \quad \eta_{\theta,t} &= \rho^{\theta}\eta_{\theta,t-1} + \exp\left(\sigma_{\theta,t-1} - \frac{\sigma_{\sigma_\theta}}{\sqrt{1-(\rho^{\sigma_\theta})^2}}\right)\bar{\sigma}_\theta \varepsilon_{\theta,t} \\
\text{uncertainty:} \quad \sigma_{\theta,t} &= \rho^{\sigma_\theta}\sigma_{\theta,t-1} + \sigma_{\sigma_\theta}\varepsilon_{\sigma_\theta,t}
\end{aligned}
$$

where $\varepsilon_{\ell,t}$, $\varepsilon_{\sigma_\ell,t}$, $\varepsilon_{\theta,t}$, $\varepsilon_{\sigma_\theta,t} \sim \mathcal{N}(0,1)$

Relation to Literature about Uncertainty

1. *Aggregate uncertainty in RA models.*
    - Different sources of uncertainty: *TFP* (Fernández-Villaverdeet al. 2015); *monetary and fiscal policy* (Born and Pfeifer 2014), *fiscal instruments* (Fernández-Villaverde et al. 2015); *political factors* (Kelly, Pastor and Veronesi 2016); *preference shocks* (Basu and Bundick 2017)
2. *Idiosyncratic uncertainty on the production side.*
    - Assume representative household – uncertainty does not affect households of different income and wealth levels.
    - E.g., Bloom et al. (2018), Bahmann and Bayer (2013, 2014).
3. *Stochastic volatility in HA models.*
    - Bayer et al. (2019) and Schabb (2020).

Proper Solution Methods for Models with Uncertainty

- *Fernandez-Villaverde (2016):*
  – Perturbation solutions must be at least of order three
  $\Longrightarrow$ Volatility of shocks nontrivially enters decision rules
- *Groot (2020)*:
  – Even third-order perturbation methods may not be sufficient.
  "The answer is often no. A key parameter - the standard deviation of stochastic volatility innovations - does not appear in the coefficients of the decision rules of endogenous variables until a fourth- or sixth-order perturbation approximation (depending on the functional form of the stochastic volatility process)."

$$\Downarrow$$

Need global solutions to capture effects of volatility on decision rules

Relation to HANK Literature

- *Aggregate MIT risk shocks + No idiosyncratic or aggregate uncertainty shocks*
  – Kaplan, Moll and Violante (2018), Alves, Kaplan, Moll and Violante (2020)
- *Aggregate MIT risk shocks + Idiosyncratic uncertainty shocks + No aggregate uncertainty shocks*
  – Bayer, Luetticke, Pham-Dao and Tjaden (2019)
- *Aggregate risk shocks + Idiosyncratic MIT uncertainty shocks + No aggregate uncertainty shocks*
  – Schabb (2020)

This paper: the first HANK model with both

- aggregate uncertainty shocks
- aggregate risk shocks

Relation to HANK Computational Literature

- Based on **Reiter (2009)**:
  – *Idea:* local (perturbation) solutions at the aggregate level + Global solutions at the individual level
  – *Papers:* Ahn et al. (2018), Boppart et al. (2018), Bayer and Luetticke (2019), and Auclert et al. (2020).
  $\Rightarrow$ No TFP dynamics over time.
- Thus, **approaches in the HANK literature**
  – MIT aggregate shocks
  – Low-order perturbation
- **We address these problems with AI and deep learning**
  – Aggregate shocks in the solution procedure
  – Global nonlinear solutions

Deep Learning Analysis of Maliar, Maliar, Winant (2019)

1. **HANK model**: $\begin{cases} E_\epsilon \left[f_1 \left(X(s), \epsilon \right) \right] = 0 \\ \dots \\ E_\epsilon \left[f_n \left(X(s), \epsilon \right) \right] = 0 \end{cases}$

   s = state, $X(s)$ = decision function, ϵ = innovations.
2. Parameterize $X(s) \simeq \varphi(s;\theta)$ with a **deep neural network**.
3. Construct **objective function** $\Xi^n(\theta)$ for DL training

$$\min_\theta \left(E_\epsilon \left[f_1 \left(\varphi(s;\theta), \epsilon \right) \right] \right)^2 + \dots + \left(E_\epsilon \left[f_n \left(\varphi(s;\theta), \epsilon \right) \right] \right)^2 \to 0$$

4. **All-in-one expectation** operator is a critical novelty:

$$\left(E_\epsilon \left[f_j \left(\varphi(s;\theta), \epsilon \right) \right] \right)^2 = E_{(\epsilon_1,\epsilon_2)} \left[f_j \left(\varphi(s;\theta), \epsilon_1 \right) \cdot f_j \left(\varphi(s;\theta), \epsilon_2 \right) \right]$$

   with ϵ_1, ϵ_2 = two independent draws.
4. **Stochastic gradient descent** for training (random grids)
5. Google **TensorFlow** platform – software that leads to ground-breaking applications (image, speech recognition, etc).

Krusell and Smith (1998) versus the Present Paper

- **Krusell and Smith** (1998) use a reduced state space:
  X_i (*variables of agent* i, *aggregate moments*)
  $\Rightarrow$ few state variables
- **The present paper** uses the true state space:
  X_i (*variables of all agents*, *distributions*)
  $\Rightarrow$ hundreds of state variables

How do we deal with such a large state space?

1. Neural network automatically performs the model reduction
   – it learns to summarize information from many inputs into a smaller set of hidden layers.
2. Neural network deals with ill conditioning
   – it learns to ignore collinear and redundant variables.

Household Problem

Agent j solves

$$\max_{\{c_t, i_t, b_t, k_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t(j)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{(h_t(j))^{1+\vartheta} - 1}{1+\vartheta} \right\}$$

$$\text{s.t. } c_t(j) + i_t(j) + b_t(j) + \Psi(i_t(j), k_{t-1}(j)) =$$

$$\frac{R_{t-1}}{\pi_t} b_{t-1}(j) + w_t h_t(j) \exp\left(\eta_{\ell,t}(j) - \frac{\bar{\sigma_\ell}}{\sqrt{1-(\rho^\ell)^2}} \right) - \tau_t(j)$$

$$k_t(j) = \left[1 - d + r_t^k\right] k_{t-1}(j) + i_t(j)$$

$$k_t(j) \geq 0, \; b_t(j) \geq \overline{b}$$

$\Psi(\cdot,\cdot)$: adjustment costs on machines; $\tau_t(j)$: taxes;

$h_t(j) = \int^1 h_t(j,m)\, dm$: hours worked across $m \in [0,1]$ sectors

Adjustment-Cost and Tax Functions

- Adjustment costs on machines (Kaplan et al. 2018)

$$\Psi\left(i_t\left(j\right), k_{t-1}\left(j\right)\right) =$$

$$\Gamma_1 \left|i_t\left(j\right)\right| + \frac{\Gamma_2}{\Gamma_3}\left(\frac{\left|i_t\left(j\right)\right|}{\left[1-d+r_t^k\right] k_{t-1}\left(j\right)+\varepsilon} - \xi\right)^{\Gamma_3} \times \left(\left[1-d+r_t^k\right] k_{t-1}\left(j\right)+\varepsilon\right)$$

- Tax function

$$\tau_t\left(j\right) = y_t\left(j\right) - \tau_1 y_t\left(j\right)^{1-\tau_2}$$

$$y_t\left(j\right) \equiv w_t h_t\left(j\right) \exp\left(\eta_{\ell,t}\left(j\right) - \frac{\bar{\sigma_\ell}}{\sqrt{1-\left(\rho^\ell\right)^2}}\right).$$

Labor Union

$$\max_{W_t(m)} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t(j)^{1-\gamma} - 1}{1-\gamma} - \psi \frac{(h_t(j))^{1+\vartheta} - 1}{1+\vartheta} \right\}$$

$$- \frac{\mu_w}{1-\mu_w} \frac{1}{2\kappa_w} \left[\log \left(\frac{W_t(m)}{W_{t-1}(m)} \frac{1}{\pi^*} \right) \right]^2 H_t$$

$$\text{s.t. } H_t(m) = \left(\frac{W_t(m)}{W_t} \right)^{\mu_w - 1} H_t$$

$$h_t(j) = \int \left(\frac{W_t(m)}{W_t} \right)^{\mu_w - 1} H_t dm$$

Labor Union (Continued)

- Wage Phillips curve

$$\log\left(\frac{\pi_t}{\pi^*}\right) = \kappa_w\left(\psi H_t^{1+\vartheta} - \mu_w\left(1-\tau_2\right) Z_t \widetilde{\Lambda}_t\right) + \beta E_t \log\left(\frac{\pi_{t+1}}{\pi^*}\right)$$

- Z_t: the average after-tax income

$$Z_t \equiv \tau_1 \left(w_t H_t\right)^{(1-\tau_2)} \int_0^1 (\exp\left(\eta_{\ell,t}(j) - \frac{\bar{\sigma}_\ell}{\sqrt{1-(\rho^\ell)^2}}\right))^{(1-\tau_2)} dj$$

- $\widetilde{\Lambda}_t$: weighted average of the individual marginal utilities

Firms

- Production function

$$Y_t = \overline{A} \exp \left(\eta_{\theta,t} - \frac{\sigma_\theta}{\sqrt{1 - \left(\rho^\theta\right)^2}} \right) K_{t-1}^{\alpha} H_t^{1-\alpha}$$

Central Bank

- Taylor rule subject to ZLB

$$R_t \equiv \max\left\{1.0,\right.$$

$$\left. R_* \left(\frac{R_{t-1}}{R_*}\right)^{\mu} \left[\left(\frac{\pi_t}{\pi_*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_*}\right)^{\phi_y}\right]^{1-\mu} \exp\left(\eta_{R,t} - \frac{\sigma_R}{\sqrt{1-\left(\rho^R\right)^2}}\right)\right\}$$

- Monetary policy shock

$$\eta_{R,t} = \rho^R \eta_{R,t-1} + \sigma_R \varepsilon_{R,t}, \qquad \varepsilon_{R,t} \sim \mathcal{N}(0,1)$$

Market Cleaning

- Market clearing

$$\int_0^1 b_t(j)\,dj = 0$$

$$C_t + K_t - (1-d)\,K_{t-1} + \int_0^1 \mathrm{AC}(i)\,di = Y_t$$

- $\int_0^1 \mathrm{AC}(i)\,di = \int_0^1 \Psi\left(i_t(j), k_{t-1}(j)\right) dj$: aggregate cost of adjustment; $C_t = \int_0^1 c_t(j)\,dj$; $K_{t-1} = \int_0^1 k_{t-1}(j)\,dj$

Calibration

Parameter	*Description*	*Target/Source*
Household		
$\gamma = 2.0$	Risk aversion	standard
$\beta = .975$	Discount factor	standard
$d = 0.0135$	Depreciation rate	standard
$\Gamma_2 = 1.1686$	Illiquid asset adjustment cost	
$\Gamma_3 = 2.0$	Illiquid asset adjustment cost	
$\xi = 0.0$	Illiquid asset adjustment cost	
$\varepsilon = 0.25$	Illiquid asset adjustment cost	
$\overline{b} = -0.1$	Liquid asset borrowing constraint	75% of people have liquid assets Kaplan, Violante, and Weidner 2014
$\tau_1 = 0.8$	Tax function parameter	Heathcote, Storesletten, and Violante 2017
$\tau_2 = 0.181$	Tax function parameter	Heathcote, Storesletten, and Violante 2017
Labor Union		
$\vartheta = 1.0$	Labor supply elasticity	standard
$\psi = 0.8796$	Disutility of labor shift	$H = 1$ in model without agg risk
$\mu_w = 1.1$	Elasticity of substitution among goods	profits share of 10%
$\kappa_w = 0.15$	Slope of wage Phillips curve	Auclert et al. 2021
Firm		
$\alpha = 0.325$	Capital share	standard
$\overline{A} = 0.4735$	Constant in production function	$Y = 1$ in model without agg risk

Calibration (Continued)

Parameter	*Description*	*Target/Source*
	Monetary Policy	
$\mu = 0.0$	Nominal rate persistence	
$R_* = 1.0175$	Long run nominal rate	
$Y_* = 1$	Long run output	
$\pi_* = 1.005$	Inflation target	
$\phi_\pi = 1.5$	MP response to inflation	
$\phi_y = \frac{.25}{4}$	MP response to output	
	Exogenous Variables	
$\rho^\ell = 0.966$	Persistence of idiosyncratic shocks	Auclert et al. 2021
$\sigma_\ell^* = 0.2379$	Standard deviation of idios.-level shocks (in the absence of uncertainty shocks)	Auclert et al. 2021
$\rho^{\sigma_\ell} = 0.84$	Persistence of idios.-volatility shocks	Based on Bayer et al. (2019)
$\sigma_{\sigma_\ell} = 0.02$	Standard deviation of idios.-volatility shocks	Based on Bayer et al. (2019)
$\rho^\theta = 0.9$	Persistence of TFP-level shocks	standard
$\sigma_\theta^* = 0.016$	Standard deviation of TFP-level shocks (in the absence of uncertainty shocks)	standard
$\rho^{\sigma_\theta} = 0.73$	Persistence of TFP-volatility shocks	Based on Fernandez-Villaverde et al. (2015)
$\sigma_{\sigma_\theta} = 0.04$	Standard deviation of TFP-volatility shocks	Based on Fernandez-Villaverde et al. (2015)
$\rho^R = 0.5$	Persistence of monetary-policy shocks	standard
$\sigma_R = 0.01$	Standard deviation of monetary-policy shocks	standard

DL Algorithm of Maliar, Maliar, Winant (2021)

Steps

1. Initialize the algorithm
   - construct a loss function $\Xi^n(\theta)$ to be minimized
   - define a topology of neural network parameterizing unknown decision functions $\varphi(\cdot, \theta)$
   - fix initial vector of coefficients θ
2. Train the machine, i.e., find θ that minimizes $\Xi^n(\theta)$
   i) simulate the model to produce data using $\varphi(\cdot, \theta)$
   ii) construct the gradient of the loss function, $\nabla\Xi^n(\theta)$, using backpropagation
   iii) update the coefficients $\theta' = \theta - \lambda\nabla\Xi^n(\theta)$ and go to step 2i)
   End Step 2 if the convergence criterion is satisfied
   $\left\|\theta' - \theta\right\| \leq \varepsilon$
3. Assess the accuracy of the constructed approximation $\varphi(\cdot, \theta)$ on a new sample

Key Features of the DL Algorithm for Heterogeneous Models

1. Represent the model as a DL objective
2. Use an all-in-one expectation operator
3. Work with the entire state space in Krusell-Smith type of model

1 Representing the Model as a DL Objective

Consumption-saving problem

$$\max_{\{c_t, w_{t+1}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$\text{s.t. } w_{t+1} = r(w_t - c_t) + e^{z_t},$$

$$c_t \leq w_t,$$

$$z_{j,t+1} = \rho_j z_{j,t} + \sigma_j \epsilon_{j,t} \text{ and } \epsilon_{j,t} \sim \mathcal{N}(0,1)$$

c_t, w_t: consumption & the beginning-of-period cash-on-hand; z_t : exogenous shock

- **Euler equation and Kuhn-Tucker conditions**: Use a Fischer-Burmeister (FB) function

$$\Psi^{FB}(a,b) = a + b - \sqrt{a^2 + b^2} = 0;$$

where $a = w - c$ and $b = u'(c) - \beta r E_\epsilon [u'(c')]$. It leads to the solution $a \geq 0$, $b \geq 0$ and $ab = 0$ but is differentiable

1 Representing the Model as a DL Objective

- **Euler-equation residuals:** select a consumption decision rule $c(\cdot;\theta)$, draw a random state $s \equiv (z, w)$ and define residuals in $\Psi^{FB}(a, b)$

$$E_{(z,w)}\left[\Psi^{FB}\left(w - c, u'(c) - \beta r E_{\epsilon}\left[u'(c')\right]\right)\right]^2$$

Not all-in-one-expectation operator yet!

- Re-write

$$\min_{\theta} E_{(z,w)}\left\{\left[\Psi^{FB}\left(w - c, u'(c) - \mu\right)\right]^2 + v\left[\beta r E_{\epsilon}\left[u'(c')\right] - \mu\right]^2\right\}$$

μ: expectation function; v: exogenous weight

2. All-in-one-expectation Operator

Combines integration with respect to z, w and ϵ in one place

$$\min_{\theta} E_{(z,w,\epsilon_1,\epsilon_2)} \left\{ \left[\Psi^{FB} \left(w - c, u'(c) - \mu \right) \right]^2 + v \left[\beta r \left[u' \left(c' \right) \Big|_{\epsilon=\epsilon_1} \right] - \mu \right] \left[\beta r \left[u' \left(c' \right) \Big|_{\epsilon=\epsilon_2} \right] - \mu \right] \right\}$$

μ: expectation function; v: exogenous weight

3. Working with the Entire State Space

Krusell-Smith (1998) model

- A set of heterogeneous agents $i = 1, ..., \ell$

$$\max_{\left\{c_t^i, k_{t+1}^i\right\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^i\right)\right]$$

$$\text{s.t. } c_t^i + k_{t+1}^i = R_t k_t^i + W_t z_t^i$$

$$z_{t+1}^i = \rho_z z_t^i + \sigma_z \epsilon_t^i \text{ with } \epsilon_t^i \sim \mathcal{N}(0,1)$$

$$k_{t+1}^i \geq 0, \ \left(k_0^i, z_0^i\right) \text{ is given}$$

c_t^i, k_t^i, z_t^i: consumption, capital, labor productivity

- Cobb-Douglas production function $z_t k_t^{\alpha}$ with $k_t = \sum k_t^i =$ aggregate capital, $z_t =$ aggregate productivity shock

$$R_t = 1 - d + z_t \alpha k_t^{\alpha-1} \text{ and } W_t = z_t (1-\alpha) k_t^{\alpha-1}$$

$$z_{t+1} = \rho z_t + \sigma \epsilon_t \text{ with } \epsilon_t \sim \mathcal{N}(0,1)$$

- State space: $\left(\left\{k_t^i, z_t^i\right\}_{i=1}^{\ell}, z_t\right)$.

All-in-one-expectation

- **Objective function**

$$\min E_{(K,Z,z,\Sigma_1,\Sigma_2,\epsilon_1,,\epsilon_2)} \left\{ \left[\Psi^{FB} \left(w^i - c^i, u'(c^i) - \mu^i \right) \right]^2 \right.$$
$$\left. + v \left[\beta \left[u' \left(c^{i'} \right) \Big|_{\Sigma=\Sigma_1, \epsilon=\epsilon_1} \right] - \mu^i \right] \left[\beta \left[u' \left(c^{i'} \right) \Big|_{\Sigma=\Sigma_2, \epsilon=\epsilon_2} \right] - \mu^i \right] \right\},$$

  - $K = \left(k^1, ..., k^\ell \right)$, $Z = \left(z^1, ..., z^\ell \right)$
  - $\Sigma_1 = \left(\epsilon_1^1, ..., \epsilon_1^\ell \right)$, $\Sigma_2 = \left(\epsilon_2^1, ..., \epsilon_2^\ell \right)$
  - $\epsilon_{1,}$, ϵ_2: two uncorrelated random draws for the aggregate productivity shocks

DL Solution Algorithm

- Krusell and Smith (1998): replace distributions with a finite set of moments $m_t \Rightarrow$ approximate state space by $\{k_{it}, z_{it}, z_t, m_t\}$.
- We work directly with the actual state space: $K = \left(k^1, ..., k^\ell\right)$, $Z = \left(z^1, ..., z^\ell\right)$
  Procedure:

(i) draw initial state z_0 & $\{K_0, Z_0\} = \left\{k_0^i, z_0^i\right\}_{i=1}^{\ell}$;

(ii) compute aggregate capital k_0 and prices R_0, W_0;

(iii) train neural network for ℓ agents;

(iv) compute next period distribution z_1 & $\{K_1, Z_1\} = \left\{k_1^i, z_1^i\right\}_{i=1}^{\ell}$.

Proceed iteratively until convergence.

Solution Algorithm for HANK

- Use algorithm of Maliar, Maliar and Winant (2021)
- 13 agg. variables $\left\{\begin{array}{c} C_t, H_t, K_t, I_t, Y_t, \pi_t, w_t, r_t^k, \\ R_{t-1}, \eta_{R,t}, \eta_{\theta,t}, \sigma_{\theta,t}, \sigma_{\ell,t} \end{array}\right\}$
- 8 individual variables $\left\{\begin{array}{c} c_t(j), k_t(j), i_t(j), b_t(j), \\ q_t(j), \eta_{\ell,t}(j), \upsilon_t(j), \varphi_t(j) \end{array}\right\}$,

  $\upsilon_t(j), \varphi_t(j) =$ Lagrange multipliers; $q_t(j) =$ value of an additional unit of illiquid assets
- 5 aggregate state variables:

$$\underbrace{\{R_{t-1}\}}_{endogenous} \underbrace{\{\eta_{R,t}, \eta_{\theta,t}, \sigma_{\theta,t}, \sigma_{\ell,t}\}}_{exogenous}$$

- 3 individual state variables:

$$\underbrace{\{k_{t-1}(j), b_{t-1}(j)\}}_{endogenous} \underbrace{\{\eta_{\ell,t}(j)\}}_{exogenous}$$

- $3J + 5$ dimensional state space, where $J =$ number of agents

Neural Networks

- 2 neural networks (NN) with 4 hidden layers each and 128 neurons in each layer.
- Leaky relu as activation function. ADAM optimization algorithm. Batch size of 10.
- **Outputs of NNs**:
    - 1st NN ($\mathcal{N}^{agg}$): aggregate variables $\{H_t, \pi_t\}$
    - 2d NN $\left(\mathcal{N}^{indiv}\right)$: individual variables $\left\{\xi_t^c(j), \xi_t^k(j), \upsilon_t(j), \varphi_t(j)\right\}$
        - $\xi_t^c(j)$: share of consumption out of income, net the borrowing limit
        - $\xi_t^k(j)$: share of capital out of income, net the borrowing limit and consumption
        - $\upsilon_t(j)$, $\varphi_t(j)$: multipliers
- Need to approximate six decision function each of which is of dimensionality $3J + 5$

Recovering Aggregate Variables

- Use weights of NNs to compute **aggregate** variables

$$\mathcal{N}^{agg}(\Sigma) \rightarrow (H_t, \pi_t)$$
$$k(j) \rightarrow K_t$$
$$(H_t, K_t, \eta_{\theta,t}) \rightarrow \left(w_t, r_t^k, Y_t\right)$$
$$\left(\pi_t, Y_t, R_{t-1}, \eta_{R,t}\right) \rightarrow R_t$$

Recovering Individual Variables

- NN for individuals

$$\mathcal{N}^{indiv}\left(\Sigma\right) \rightarrow \left(\xi_t^k\left(j\right), \xi_t^c\left(j\right), \upsilon_t\left(j\right), \varphi_t\left(j\right)\right) \tag{1}$$

- resources

$$\begin{aligned} M_t\left(j\right) \quad &\equiv \quad \frac{R_{t-1}}{\pi_t} b_{t-1}\left(j\right)\left[1-d+r_t^k\right] k_{t-1}\left(j\right) \\ &+\tau_1\left[w_t H_t \exp\left(\eta_{\ell,t}\left(j\right) - \frac{\bar{\sigma_\ell}}{\sqrt{1-\left(\rho^\ell\right)^2}}\right)\right]^{1-\tau_2} \end{aligned} \tag{2}$$

- consumption

$$c_t\left(j\right) = \xi_t^c\left(M_t\left(j\right) - \overline{b}\right)$$

Recovering Individual Variables (Continued)

- machines

$$k_t(j) = \max\left(\xi_t^k(j) \cdot \left[M_t(j) - \overline{b} - c_t(j)\right], 0.0\right)$$

- adjustment cost

$$(k_t(j), k_{t-1}(j)) \to i_t(j) \to (\Psi_t(j), q_t(j))$$

- bonds

$$b(j) = \max\left(\left[M_t(j) - \overline{b} - c_t(j) - k_t(j) - \Psi_t(j)\right], \overline{b}\right)$$

Model Generated Statistics

	Wealth Gini	Consumption Gini	Net Income Gini
Data	0.79		
Model	0.66	0.276	0.360
95% CI	(0.655,0.684)	(0.274,0.277)	(0.359,0.361)

Generalized Impulse Response

- Koop, Pesaran, and Potter (1996)
- Period 0:
  - "No innovation": ε_0
  - "Innovation": $\varepsilon_0 + 1$
- Period 1 to T:
  - "No innovation": $\{\varepsilon_t\}_{t=1}^{T}$
  - "Innovation": $\{\varepsilon_t\}_{t=1}^{T}$
- Relative response:

$$\frac{X^{innovation} - X^{no\_innovation}}{X^{no\_innovation}} \times 100$$

Generalized Impulse Response (Continued)

- TFP uncertainty

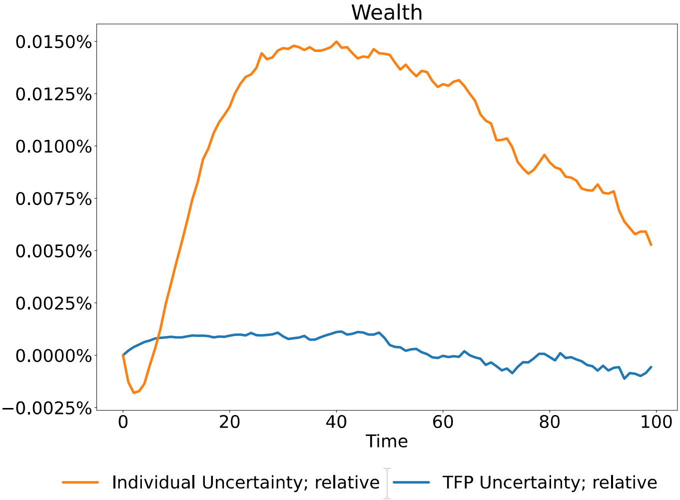
$$\sigma_{\theta,0} = \rho^{\sigma_\theta} \sigma_{\theta,-1} + \sigma_{\sigma_\theta} \left(\varepsilon_{\sigma_\theta,0} + 1 \right)$$

- Individual uncertainty

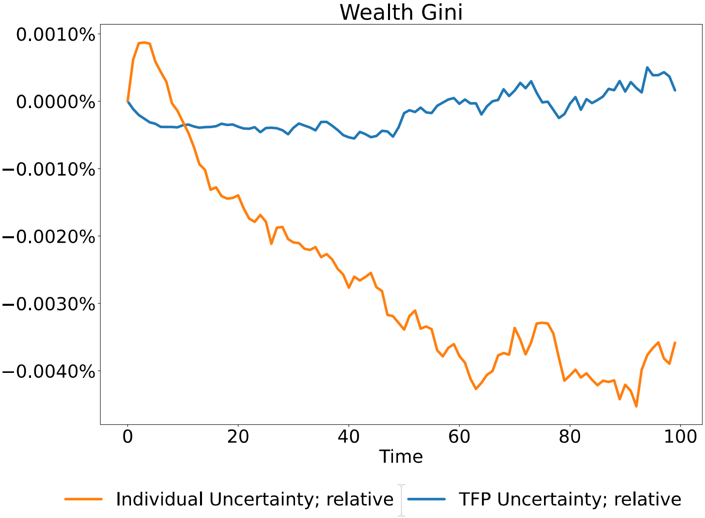
$$\sigma_{\ell,0} = \rho^{\sigma_\ell} \sigma_{\ell,-1} + \sigma_{\sigma_\ell} \left(\varepsilon_{\sigma_\ell,0} + 1 \right)$$

- 100 initial conditions
- 100 draws of innovations for each initial condition
- Time period: 1 quarter
- 200 agents

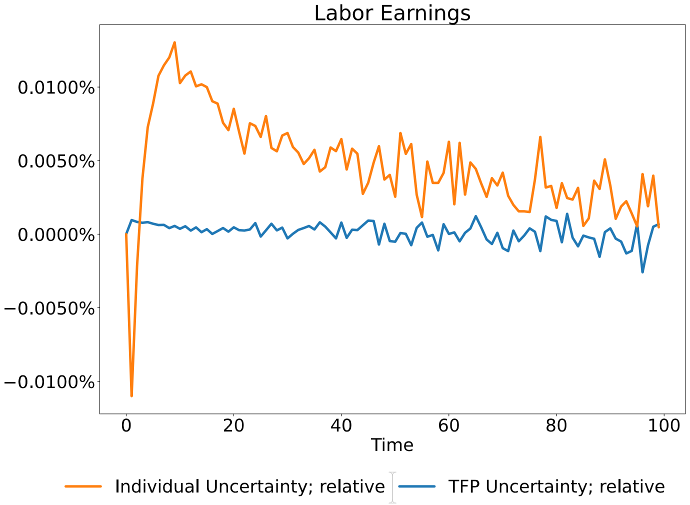
Wealth


Wealth
0.0150%
0.0125%
0.0100%
0.0075%
0.0050%
0.0025%
0.0000%
−0.0025%
0
20
40
60
80
100
Time
Individual Uncertainty; relative
TFP Uncertainty; relative

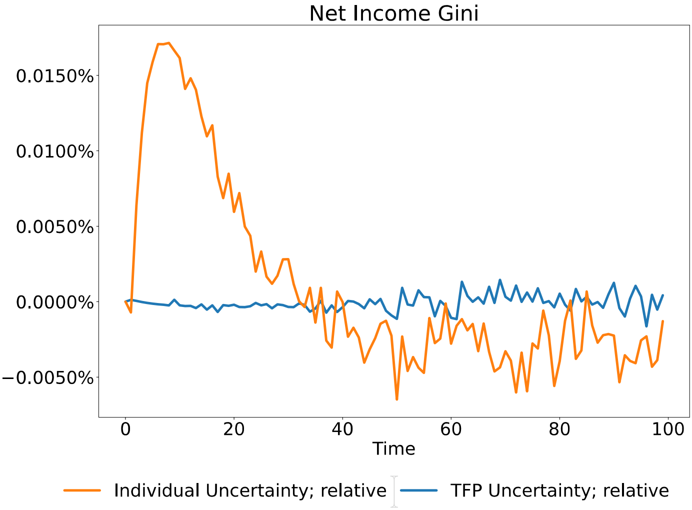
Wealth Gini


Wealth Gini
0.0010%
0.0000%
−0.0010%
−0.0020%
−0.0030%
−0.0040%
0
20
40
60
80
100
Time
Individual Uncertainty; relative
TFP Uncertainty; relative

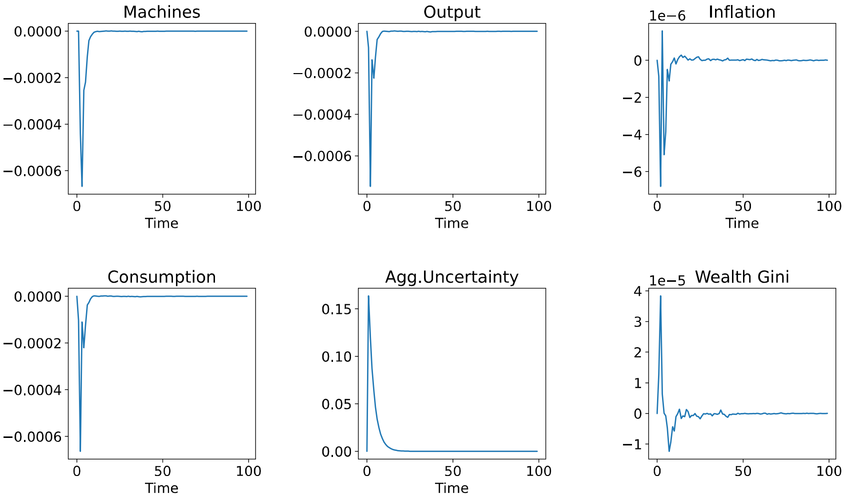
Average Labor Earnings



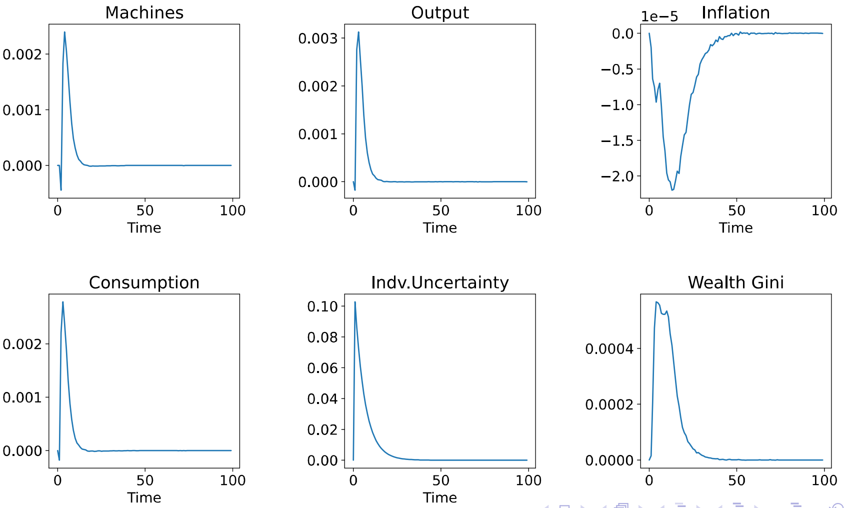
Net Income Gini



Aggregate Impulse Responses to TFP Uncertainty



Aggregate Impulse Responses to Individual Uncertainty

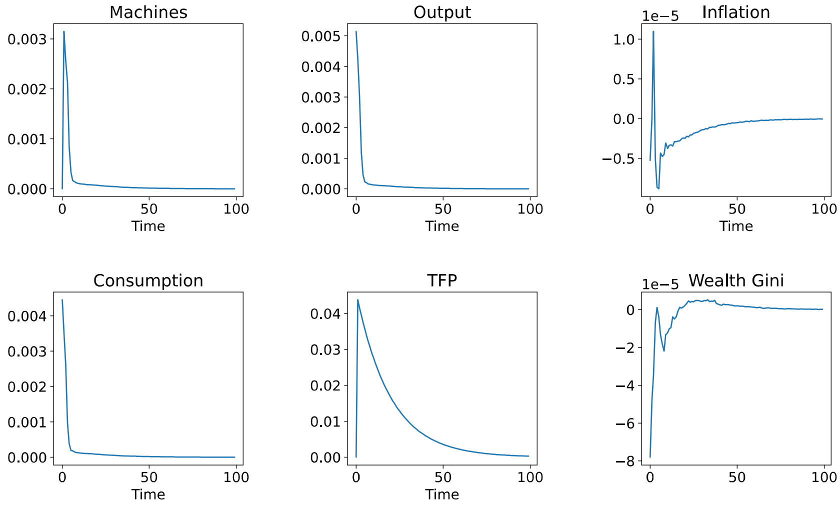


Conclusion

- Response to individual uncertainty shocks is much larger than the response to TFP uncertainty shocks
- Individual uncertainty shocks lead to a persistent increase in wealth following an initial reduction.
- Individual uncertainty shocks increase income inequality.
- **Future versions**:
  – Assume correlation between individual and aggregate uncertainty.
  – Compare to Azinovic, Luca and Scheidegger (2021): Young's (2010) deterministic simulation with Markov chains.
  – Analyze the effects of FP and MP on inequality

Thank you!

Aggregate Impulse Responses to TFP Risk Shock



Aggregate Impulse Responses to Monetary Policy Shock