

Inequality and the Zero Lower Bound

Jesús Fernández-Villaverde

University of Pennsylvania, NBER, CEPR

Joël Marbet

CEMFI

Galo Nuño

Banco de España and BIS

Omar Rachedi

ESADE Business School

August 24, 2023

Outline

- 1 Introduction
- 2 Model
- 3 Solution Approach
- 4 Results: Aggregate dynamics and IRFs
- 5 Inflation target and real interest rates
- 6 Conclusion

Motivation

- Secular decline in global real rates over the past 30 years.
Fiorentini et al. (2019), Del Negro et al. (2019).
- Decline made acuter by the 2007-2008 financial crisis and the COVID-19 shock.
- As a result, the **zero lower bound (ZLB)** on nominal rates has become a pervasive feature of advanced economies.
- Likely to return once current inflationary pressures abate.
- Traditional analysis of the macro effects of the ZLB rely on representative agent models.
Eggertsson and Woodford (2003); Christiano, Eichenbaum, and Rebelo (2011); Fernández-Villaverde et al., (2015).
- We argue that the effects of the ZLB on both aggregate dynamics and the stance of monetary policy depend on **household inequality**.

What do we do?

- Heterogeneous-agent new Keynesian ([HANK](#)) model with aggregate shocks and the [ZLB](#).
 - ▶ Fully non-linear solution: neural networks approximate the aggregate laws of motion
[Fernández-Villaverde, Hurtado, and Nuño \(2023\)](#).
- The presence of the ZLB implies three channels that alter the room for maneuver of the central bank by reducing the level of the interest rates:
 1. [Deflationary bias](#) (operative also in representative-agent model):
 - ▶ Agents expect that inflation cannot be stabilized at the ZLB → nominal rates decrease out of the ZLB.
 2. [Precautionary savings due to idiosyncratic risk](#):
 - ▶ Agents insure against the realizations of the idiosyncratic shocks to labor earnings.
 3. [Precautionary savings due to aggregate risk](#):
 - ▶ ZLB recessions are relatively larger and weigh more on wealth-poor households.
 - ▶ Agents insure against the occurrence of ZLB events.

The long-run Fisher equation

- In this setting, **monetary policy is non-neutral in the long run**:
 - ▶ A reduction in the inflation target leads to a drop in the real interest rate.
 - ▶ The model features a **long-run Fisher equation** that equals

$$i(\tilde{\pi}) = r(\tilde{\pi}) + \pi(\tilde{\pi}), \quad \text{where } dr/d\tilde{\pi} > 0.$$

- ▶ Households' inequality amplifies the degree of non-neutrality.
- The combination of changes in trend inflation and households' inequality explains at least 21% of the drop in real rates over the recent decades:
 - ▶ We consider a drop in trend inflation from 4% to 1.7% and an increase in the Gini index of wealth matching its variation in the 2000s.
 - ▶ The real rate drops by 46 bps in our HANK economy and 14 bps in the RANK model.
 - ▶ In the data, the real rate drops by around 150 bps from the late 1980s on.

Related literature

- Representative agent models.
 - ▶ **Monetary policy and low rates:** Blanchard, Dell'Ariccia, and Mauro (2010); Andrade *et al.* (2019).
 - ▶ **ZLB:** Eggertsson and Woodford (2003); Christiano, Eichenbaum, and Rebelo (2011); Fernández-Villaverde *et al.* (2015).
 - ▶ **Deflationary bias:** Adam and Billi (2007), Nakov (2008), Hills, Nakata, and Schmidt (2019); Bianchi, Melosi, and Rottner (2020).
- Heterogeneous agent models.
 - ▶ **Nominal Rigidities:** McKay, Nakamura, and Steinsson (2016); Kaplan, Moll, and Violante (2018); Luetticke (2019).
 - ▶ **Methodology:** Krusell and Smith (1998); Boppart, Krusell, and Mitman (2018); Auclert *et al.* (2019); Fernández-Villaverde, Hurtado, and Nuño (2020).
- Closest papers.
 - ▶ **HANK with Permanently-binding ZLB:** McKay and Reis (2016); Auclert and Rognlie (2020).

Outline

- 1 Introduction
- 2 Model**
- 3 Solution Approach
- 4 Results: Aggregate dynamics and IRFs
- 5 Inflation target and real interest rates
- 6 Conclusion

Setup

- Discrete-time, infinite horizon, sticky-price economy.
- Heterogeneous households:
 - ▶ Ex-ante identical and face idiosyncratic productivity shocks.
 - ▶ Choose consumption, bond holdings, and labor supply.
 - ▶ Bond holdings limited by a borrowing constraint.
- Firms:
 - ▶ Final-good producer (perfectly competitive; CES aggregator).
 - ▶ Intermediate-good producers (monopolistic competition).
 - ▶ Nominal rigidity: Rotemberg price adjustment costs.
- Preference shocks as source of aggregate uncertainty.
Christiano, Eichenbaum, and Rebelo (2011).

Households

- Households:

$$\begin{aligned} & \max_{\{c_{i,t}, b_{i,t}, h_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \xi_t \frac{1}{1-\sigma} \left(c_{i,t} - \chi \frac{h_{i,t}^{1+\nu}}{1+\nu} \right)^{1-\sigma} \\ \text{s.t. } & c_{i,t} + b_{i,t} = \tau (w_t s_{i,t} h_{i,t})^{1-\gamma} + \frac{R_{t-1}}{\pi_t} b_{i,t-1} + \Pi_t s_{i,t}, \\ & b_{i,t} \geq \underline{b} \end{aligned}$$

- Aggregate preference shock ξ_t follows AR(1) process.
- Idiosyncratic productivity shock $s_{i,t}$ follows a Markov chain.
- Progressive labor-income taxation (i.e., flat tax if $\gamma = 0$).
- Bond holdings limited by the borrowing constraint \underline{b} .
- Firm profits Π_t are re-distributed according to households' idiosyncratic productivity.

Firms

- **Final-good producer** assembles intermediate goods with $Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}$.
- CRS technology of **intermediate-good producers** is $y_{j,t} = l_{j,t}$.
- Intermediate-good producers choose prices $\{p_{j,t}\}_{t \geq 0}$ to maximize:

$$\mathbb{E}_t \sum_{k=t}^{\infty} \beta^k \left[\underbrace{\left(\frac{p_{j,k}}{P_k} - mg \text{ cost}_k \right) \left(\frac{p_{j,k}}{P_k} \right)^{-\varepsilon} Y_k}_{\text{Profits net of adjustment cost}} - \underbrace{\frac{\theta}{2} \left(\log \left(\frac{p_{j,k}}{p_{j,k-1} \times \tilde{\pi}} \right) \right)^2 Y_k}_{\text{Rotemberg adjustment cost}} \right],$$

where $\tilde{\pi}$ is the inflation target and P_t is the aggregate price level.

- Solving this problem yields the **New Keynesian Philips curve**:

$$\log \left(\frac{\pi_t}{\tilde{\pi}} \right) = \beta \mathbb{E}_t \left[\log \left(\frac{\pi_{t+1}}{\tilde{\pi}} \right) \frac{Y_{t+1}}{Y_t} \right] + \frac{\varepsilon}{\theta} \left(mg \text{ cost}_k - \frac{\varepsilon - 1}{\varepsilon} \right).$$

Monetary and fiscal authority

- The **monetary authority** follows a Taylor rule subject to the ZLB constraint:

$$R_t = \max \left\{ 1, \tilde{R} \left(\frac{\pi_t}{\tilde{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\tilde{Y}} \right)^{\phi_y} \right\},$$

where \tilde{R} is the DSS nominal rate, and \tilde{Y} is DSS output.

- The **fiscal authority** raises progressive labor income taxes to finance a fixed amount of outstanding debt \tilde{B} .
- The government budget constraint:

$$\int_0^1 \left[w_t s_{i,t} h_{i,t} - \tau_t (w_t s_{i,t} h_{i,t})^{1-\gamma} \right] di = (r_t - 1) \tilde{B}.$$

Closing the model

- Labor market clears:

$$\int_0^1 l_{j,t} dj = \int_0^1 s_{it, h_{i,t}} di.$$

- Bond market clears:

$$\tilde{B} = \int_0^1 b_{i,t} di.$$

- Resource constraint:

$$Y_t = \int_0^1 l_{j,t} dj = \int_0^1 c_{it} di .$$

Calibration

- Inflation target is 2% (annualized). Time discount factor implies a 1% real rate in the DSS.
- Volatility of demand shock reproduces a 10% ZLB frequency.
Coibion *et al.* (2016).
- Idiosyncratic risk calibrated to match:
 - ▶ 30% share of borrowers
Kaplan, Violante, and Weidner (2014).
 - ▶ 10% average marginal propensity to consume
Johnson, Parker, and Souleles (2006); Parker *et al.* (2013).
- Aggregate liquid wealth (i.e., government debt) equals 25% of annual GDP
McKay, Nakamura, and Steinsson (2016); Kaplan, Moll, and Violante (2018).
- Tax progressivity is $\gamma = 0.18$
Heathcote, Storesletten, and Violante (2017).
- Rotemberg costs equivalent to a Calvo parameter that yields a 1-year price duration.

More on calibration, I

Parameter		Value	Target/Source
Panel A. Aggregate Risk			
ρ_ξ	AR coefficient of process for ξ	0.6	Bianchi, Melosi, and Rottner (2020)
ω_ξ	Standard deviation of ξ shock	0.0105	10% ZLB frequency
Panel B. Idiosyncratic Risk			
ρ_s	AR coefficient of process for s_t	0.8	10% Average MPC
ω_s	Standard deviation of s_t shock	0.05	30% Borrowers
\underline{b}	Borrowing limit	-0.29	Monthly average labor income
Panel C. Preferences			
β	Discount factor	0.997	1% real interest rate in the DSS
σ	Risk aversion	1	Standard value
$1/\nu$	Frisch elasticity of labor supply	1	Standard value
χ	Disutility of labor	0.71	Labor supply equals 1 in the DSS

More on calibration, II

Parameter		Value	Target/Source
Panel D. Production			
ε	Demand elasticity	7.67	15% price markup
θ	Rotemberg price adjustment cost	79.41	Equivalent to 0.75 Calvo parameter
Panel E. Monetary Authority			
$\tilde{\pi}$	Inflation Target	$\exp(0.02/4)$	2% Annual inflation target
ϕ_{π}	Coeff. on inflation	2.5	Standard value
ϕ_y	Coeff. on deviations from DSS output	0.1	Standard value
Panel F. Fiscal Authority			
B	Government debt	0.25	Liquid assets = 25% annual GDP
γ	Tax prog,	0.18	Heathcote, Storesletten, and Violante (2017)

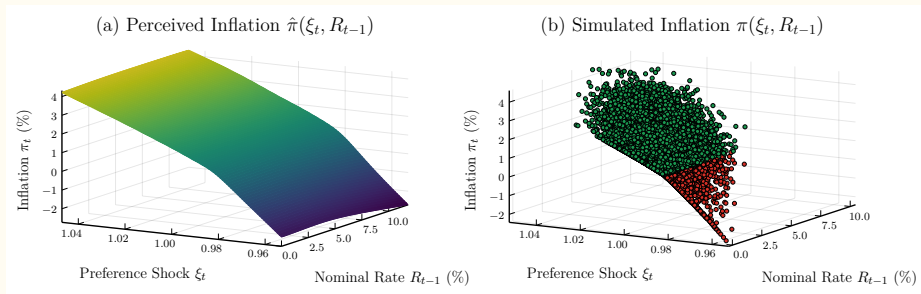
Outline

- 1 Introduction
- 2 Model
- 3 Solution Approach**
- 4 Results: Aggregate dynamics and IRFs
- 5 Inflation target and real interest rates
- 6 Conclusion

Solution approach

- Agents form expectations keeping track of how the **distribution** of bonds evolves:
 - ▶ Computationally intractable.
 - ▶ Possible solution: **Bounded rationality** as in **Krusell and Smith (1998)**.
 - ▶ However, this approach hinges on **log-linear law of motion**.
- We use **neural networks** to determine the **fully non-linear laws of motion**:
 - ▶ In our case, agents predict inflation, $\log \pi_t$, and a term related to inflation expectations, $\log \left(\frac{\pi_{t+1}}{\tilde{\pi}} \right) \frac{Y_{t+1}}{Y_t}$, from the NK Philips curve.
 - ▶ ZLB introduces **non-linearities** into the aggregate law of motion.
 - ▶ Neural Network is able to capture this non-linearity.

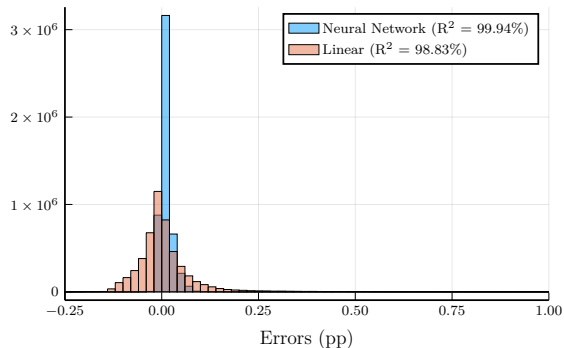
Non-linearity due to the ZLB



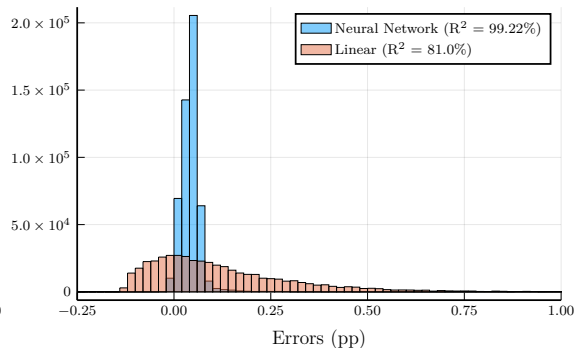
- Different inflation policies arise from bounded rationality assumption:
 - ▶ **Perceived inflation** → how agents nowcast inflation.
 - ▶ **Simulated inflation** → actual realization of inflation.
- ZLB introduces **non-linearities** into the inflation policies captured by the neural network.

Nowcast errors for inflation

(a) All Simulated Periods



(b) Only Periods with binding ZLB

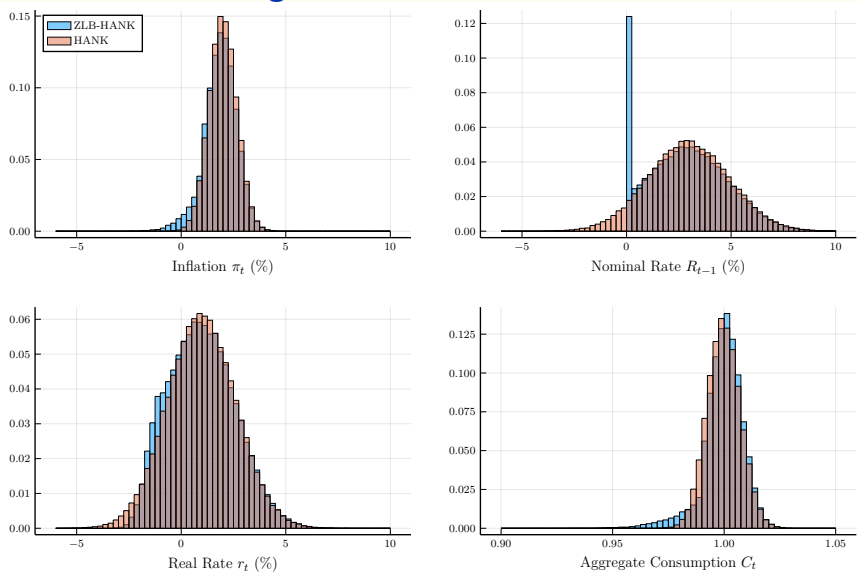


- Neural network improves upon the linear regression approach, especially at the ZLB.
- Results are very similar for forecasts of the inflation expectation term.

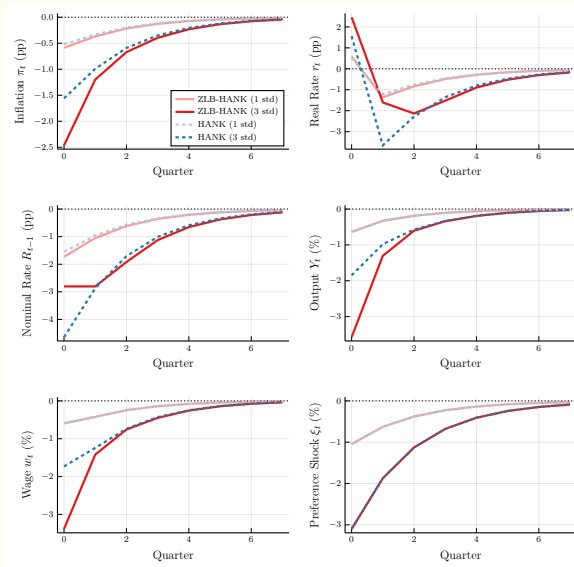
Outline

- 1 Introduction
- 2 Model
- 3 Solution Approach
- 4 Results: Aggregate dynamics and IRFs**
- 5 Inflation target and real interest rates
- 6 Conclusion

The macro of the ZLB: Ergodic distribution



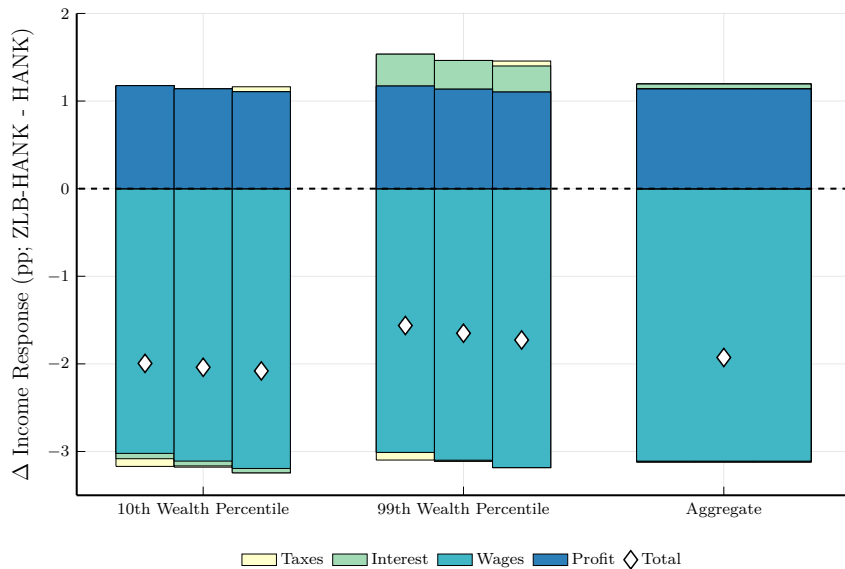
The macro of the ZLB: Aggregate IRFs



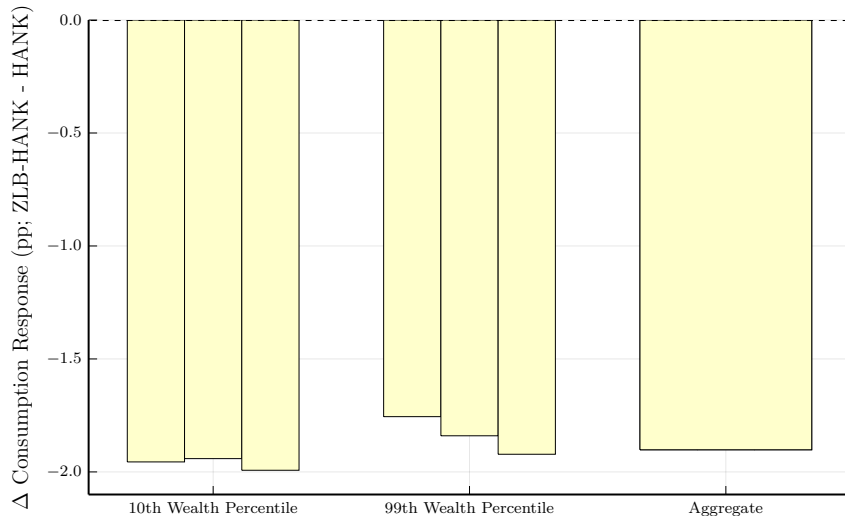
The macro of the ZLB: Taking stock

- ZLB skews the dynamics of the model to the left relative to a standard HANK:
 - ▶ These are the cases in which the ZLB. constrains the nominal rate
 - ▶ Sharp drop in aggregate consumption amidst a deflationary spiral.
 - ▶ All these dynamics are absent in the standard HANK model.
- IRFs to small demand shocks coincide in the model with and without ZLB:
- IRFs to large shocks do differ:
 - ▶ A large shock brings the nominal interest rate down to zero.
 - ▶ Much larger drop in both inflation and output.
- ZLB events are characterized by deflation and substantial household consumption losses.

The micro of the ZLB: Households' income IRFs



The micro of the ZLB: Households' consumption IRFs



The micro of the ZLB: Taking stock

- ZLB alters the **distributional effects** of a recessionary shocks.
- ZLB amplifies the drop in **total income**:
 - ▶ This holds for any realization of labor earnings and any position in wealth distribution.
 - ▶ ZLB makes wages drop more whereas interest payments relatively rise.
 - ▶ Larger drop in the total income of wealth-poor households.
- ZLB also amplifies the drop in **consumption**:
 - ▶ This drop is larger for wealth-poor households.
 - ▶ Consumption drop for wealth-poor individuals increases by 0.2 pp due to the ZLB.
- Burden of recessions tilted towards households at **lower end of the wealth distribution**.

Deterministic and stochastic steady states

- What is the difference between the deterministic and stochastic steady states?
 - ▶ **Deterministic Steady State (DSS):** Agents ignore aggregate risks ($\sigma_\xi = 0$).
 - ▶ **Stochastic Steady State (SSS):** Agents make their decisions taking into account aggregate risks ($\sigma_\xi > 0$) but no shock arrives along the equilibrium path.
 - ▶ Idiosyncratic shocks are taken into account by agents in both cases.
- In DSS households do not anticipate the effect of future aggregate shocks.
- Instead, in the SSS, households are aware of the existence of future aggregate shocks that may hit the economy.
- Why do we care about the SSS?

Comparison of DSS and SSS in ZLB-HANK, HANK, and ZLB-RANK

	ZLB-HANK		HANK		ZLB-RANK	
Variable	DSS	SSS	DSS	SSS	DSS	SSS
Inflation	2.0%	1.91%	2.0%	1.99%	2.0%	1.93%
Nominal Rate	3.0%	2.80%	3.0%	2.96%	3.22%	3.08%
Real Rate	1.0%	0.89%	1.0%	0.97%	1.22%	1.15%
(Shadow) ZLB Frequency	-	10.17%	-	(6.09%)	-	8.35%
(Shadow) ZLB Duration Quarters	-	1.65	-	(1.50)	-	1.60

Decomposition exercise

	Real Rate	Nominal Rate	Inflation
ZLB-RANK DSS	1.22%	3.22%	2.0%
ZLB-RANK SSS	1.15%	3.08%	1.93%
(i) Deflationary Bias	0.08pp	0.14pp	0.07pp
ZLB-RANK DSS	1.22%	3.22%	2.0%
ZLB-HANK DSS	1.0%	3.0%	2.0%
(ii) Precautionary Savings - Idiosyncratic Risk	0.22pp	0.22pp	0.0pp
ZLB-RANK DSS	1.22%	3.22%	2.0%
ZLB-HANK SSS	0.89%	2.8%	1.91%
(iii) Total	0.33pp	0.42pp	0.09pp
(iii)-(i)-(ii) Precautionary Savings - Aggregate Risk	0.03pp	0.05pp	0.02pp

The determinants of the differences between DSS and SSS Real Rates

- Deflationary bias reduces the level of real rate by 8 bps.
- Precautionary savings due to idiosyncratic risk reduce the level of real rate by 22 bps:
 - ▶ Although this result traces back to Aiyagari (1994), our setting grants it a novel perspective.
 - ▶ Precautionary savings reduce the room of maneuver for the central bank's policy rate.
 - ▶ In standard HANK literature, the precautionary savings are immaterial for aggregate dynamics because of the lack of the ZLB.
- Precautionary savings due to aggregate risk reduce the level of real rate by 3 bps.

Outline

- 1 Introduction
- 2 Model
- 3 Solution Approach
- 4 Results: Aggregate dynamics and IRFs
- 5 Inflation target and real interest rates**
- 6 Conclusion

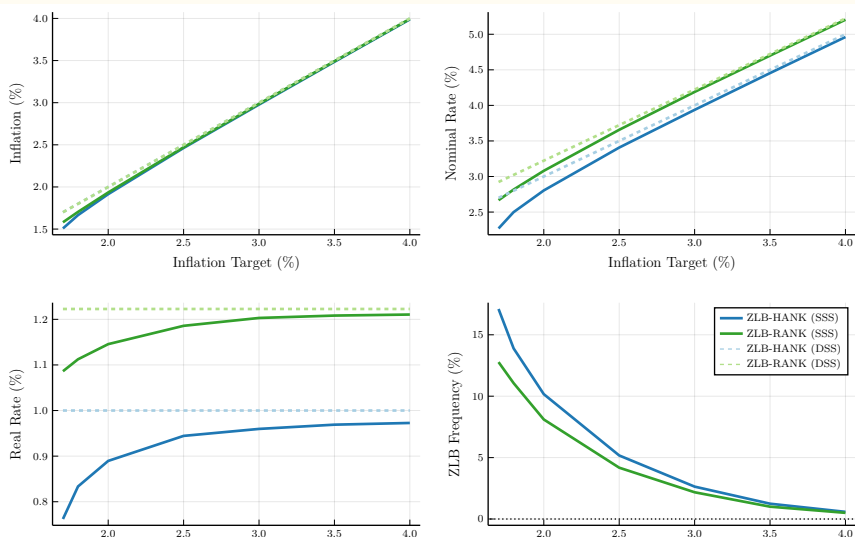
The role of the inflation target

- Changes in inflation target $\tilde{\pi}$ alter the ZLB frequency and households' expectations \rightarrow affect the level of real interest rates.
- Monetary policy is **not neutral**: the SSS real rate depends on central bank's inflation target.
- The model features a **long-run Fisher equation**:

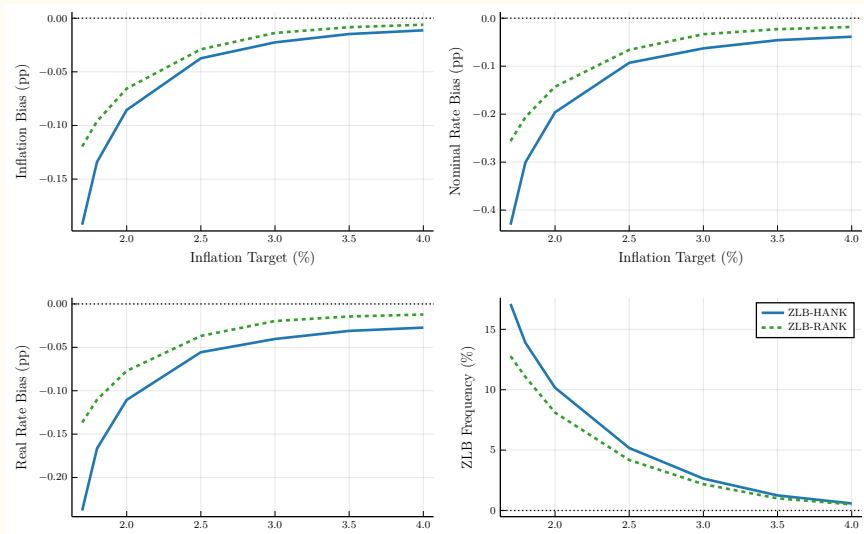
$$i(\tilde{\pi}) = r(\tilde{\pi}) + \pi(\tilde{\pi}), \quad \text{where } dr/d\tilde{\pi} > 0.$$

- To uncover this result, we compare the real interest rate level in different model economies, which uniquely differ in the inflation target $\tilde{\pi}$.

DSS/SSS in ZLB-RANK/ZLB-HANK as a function of the inflation target



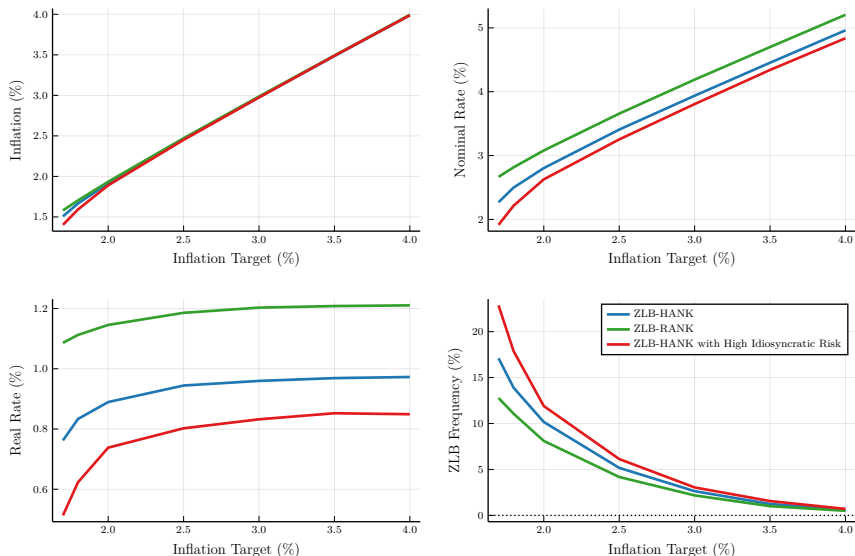
Differences between the SSS and DSS as a function of the inflation target



The monetary policy non-neutrality in the ZLB-HANK model

- When the inflation target is around 3%, the probability of ZLB events is low → not quantitatively relevant in shaping households' expectations.
- For targets below 3%, the non-linearity due to the ZLB kicks in → SSS and DSS levels diverge.
- When the target is 1.7%, the ZLB probability is as high as 20%, and the real rate is 0.75% → real rate is 25 bp lower than that associated with the 4% target.
- Non-neutrality is also present in RANK model.
- However, households' heterogeneity increases substantially the quantitative relevance of the long-run Fisher equation.

The interaction between the inflation target and wealth inequality



The role of wealth inequality

- A drop in the inflation target from 4% to 1.7% together with an increase in the Gini index of wealth of three p.p. reduces the level of the real rates by 46 bps.
- The drop in the inflation target is consistent with the reduction in inflation between the 1980s-1990s and 2000s-2010s.
- The increase in the Gini index of wealth is consistent with that measured by Kuhn and Ríos-Rull (2016) in the 2000s.
- In comparison, in the RANK model, the drop in the inflation target would reduce the real rate by just 14 bps.
- Overall, the interaction of these changes accounts for 21% of the overall 150 bps drop in the real rates over the last three decades.

Outline

- 1 Introduction
- 2 Model
- 3 Solution Approach
- 4 Results: Aggregate dynamics and IRFs
- 5 Inflation target and real interest rates
- 6 Conclusion**

Conclusion

- This paper introduces a HANK model that explicitly incorporates the non-linearity due to the **ZLB constraint**.
- We have solved the model with a novel **neural-network algorithm**.
- The model shows that the ZLB constraint alters the dynamics of both macroeconomic and individual variables.
- The burden of recessions is tilted towards wealth-poor individuals.
- We uncover the **non-neutrality of the central bank's inflation target**.
 - ▶ The model features a **long-run Fisher equation**.
 - ▶ Changes in the inflation target reduces the level of the real rate.
 - ▶ This channel is substantially amplified by **households' inequality** through changes in precautionary savings.