

Course. Introduction to Machine Learning

Lecture 4. Factor Analysis

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Introduction to Machine Learning

Unsupervised
Learning

Supervised
Learning

Decision
Learning
Theory

Cluster
Analysis

**Factor
Analysi-
sis**

Visualization

Non Linear Decision

Linear
Decision

Basic
concepts of
Decision
Learning
Theory

K-Means ,
EM

**PCA
ICA**

Self
Organized
Maps (SOM) ,
Multi-
Dimensional
Scaling

Lazy
Learning
(K-NN, IBL,
CBR)

Overfitting,
model
selection and
feature
selection

Kernel
Learning

Ensemble
Learning
(NN, Trees,
Adaboost)

Perceptron,
SVM

Bias/Variance,
VC dimension,
Practical
advice of how
to use
learning
algorithms

1. Factor Analysis

1. Introduction to factor analysis
2. What is factor analysis?
3. General steps to factor analysis
4. Problems with factor analysis
5. Types of factor analysis
6. Summary of factor analysis
7. Applications of factor analysis
8. Principal Component Analysis (PCA)
9. Independent Component Analysis (ICA)

Introduction to factor analysis



Karl Pearson, 1857-1936

Inventor of the correlation coefficient, the chi-square test and Principal Components Analysis (1901)

Introduction to Factor Analysis

*“Perhaps the most widely used (and misused) multivariate [technique] is factor analysis. Few statisticians are neutral about this technique. Proponents feel that factor analysis is the greatest invention since the double bed, while its detractors feel it is a useless procedure that can be used to support nearly any desired interpretation of the data. **The truth, as is usually the case, lies somewhere in between.** Used properly, factor analysis can yield much useful information; when applied blindly, without regard for its limitations, it is about as useful and informative as Tarot cards. In particular, factor analysis can be used to explore the data for patterns, confirm our hypotheses, or reduce the many variables to a more manageable number.*

-- Norman Streiner, PDQ Statistics

GOAL: To summarize patterns of correlations among observed variables

- Factor analysis is a method for investigating whether a number of variables of interest are **linearly related** to a smaller number of unobservable factors
- Patterns of **correlations** are identified and either used as descriptive (PCA) or as indicative of underlying theory (FA)
- Process of providing an operational definition for **latent construct** (through a regression equation)

- FA (factor analysis) and PCA (principal components analysis) are statistical methods of **data reduction**
 - Take many **variables** and explain them with a few “factors” or “components”
 - **Correlated variables** are grouped together and separated from other variables with low or no correlation
 - Factors are formed that are relatively **independent** of one another.
 - Two types of “variables”:
 - **Latent variables**: not directly observed but inferred
 - **Observable variables**: observed and directly measured

- **Step 1:** Selecting and measuring a set of variables in a given domain
- **Step 2:** Data screening in order to prepare the correlation matrix to perform PCA or FA
- **Step 3:** Factor Extraction
- **Step 4:** Factor Rotation to increase interpretability
- **Step 5:** Interpretation
- **Further Steps:** Validation and Reliability of the measures

What is a “Good Factor”?

- A good factor:
 - Makes sense
 - Will be easy to interpret
 - Simple structure
 - Lacks complex loadings

1. Unlike many of the analyses so far there is no statistical criterion to compare the linear combination to
 - In MANOVA, they create linear combinations that maximally differentiate groups
 - In Canonical correlation one linear combination is used to correlate with another
2. After extraction, there is an infinite number of rotations available
3. FA is frequently used to “save” poorly conceived research

- Exploratory FA
 - Summarizing data by grouping correlated variables
 - Investigating sets of measured variables related to theoretical constructs
 - Usually done near the onset of research
 - This is the type of FA and PCA we are talking about in this course

- Confirmatory FA
 - More advanced technique
 - When factor structure is known or at least theorized
 - Testing generalization of factor structure to new data, etc.

- Factor Analysis enables us to **reduce the complexity of data**
- The exercise of data reduction makes it possible to identify the **latent variables** which exist underneath a set of variables which are actually observed
- These latent variables can then be used for further analysis to make sense of relationships in the data

1. Identification of Underlying Factors:

- clusters variables into homogeneous sets
- creates new variables (i.e. factors)
- allows us to gain insight to categories

2. Screening of Variables:

- identifies groupings to allow us to select one variable to represent many
- useful in regression (recall collinearity)

3. Summary:

- Allows us to describe many variables using a few factors

4. Clustering of objects:

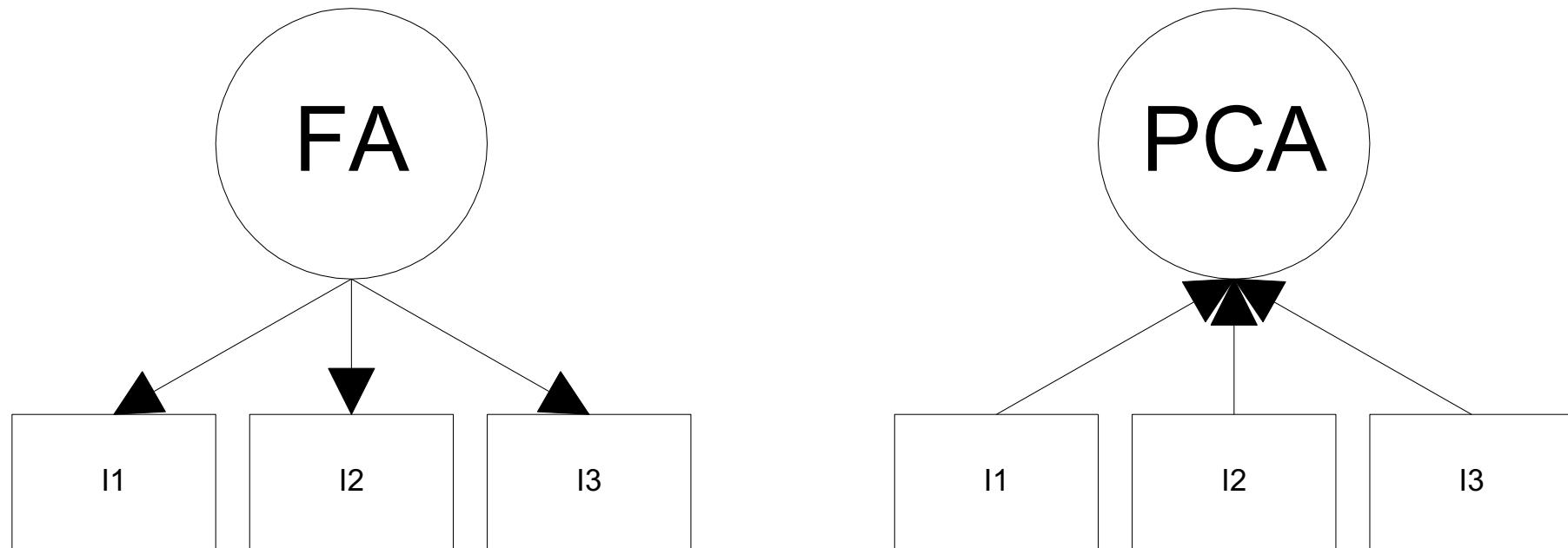
- Helps us to put objects (people) into categories depending on their factor scores



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Principal Components Analysis (PCA)

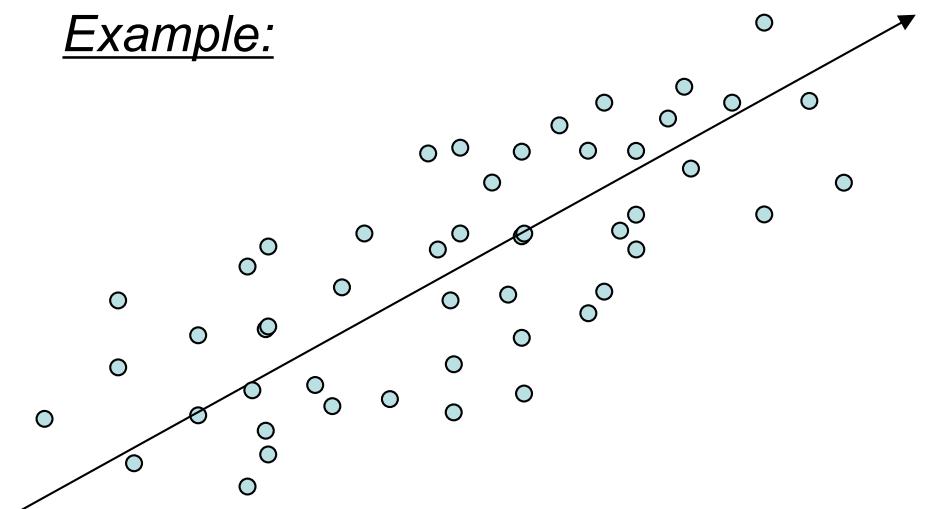


*FA is used to understand what constructs underlie the data.
PCA is used to reduce the data into a smaller number of components.*

- FA produces **factors**; PCA produces **components**
- **Factors** cause variables; **components** are aggregates of the variables
- **FA** analyzes only the variance shared among the variables (common variance without error or unique variance); **PCA** analyzes all of the variance
- **FA**: “What are the underlying processes that could produce these correlations?”
- **PCA**: Just summarize empirical associations, very data driven

- Imagine that we have many examples of the same pattern.
- Data appear clouded, unclear and even redundant.

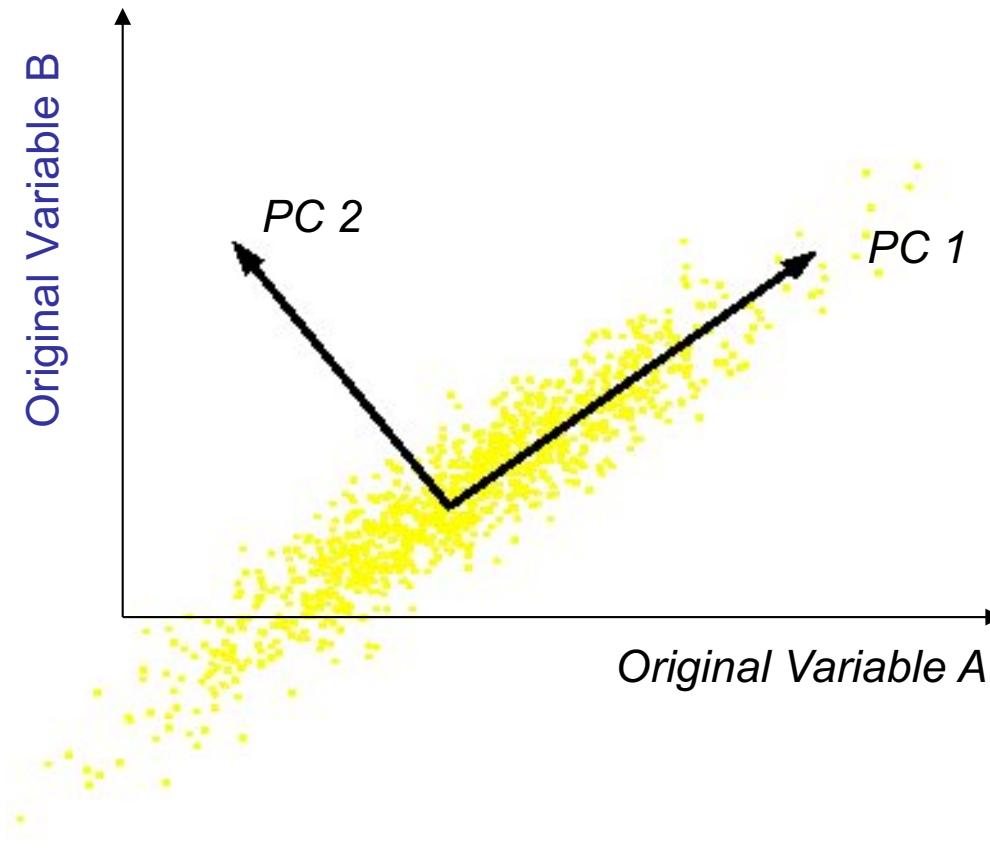
Example:



Goal of PCA: find new representation (basis) to filter the noise and reveal hidden dynamics.

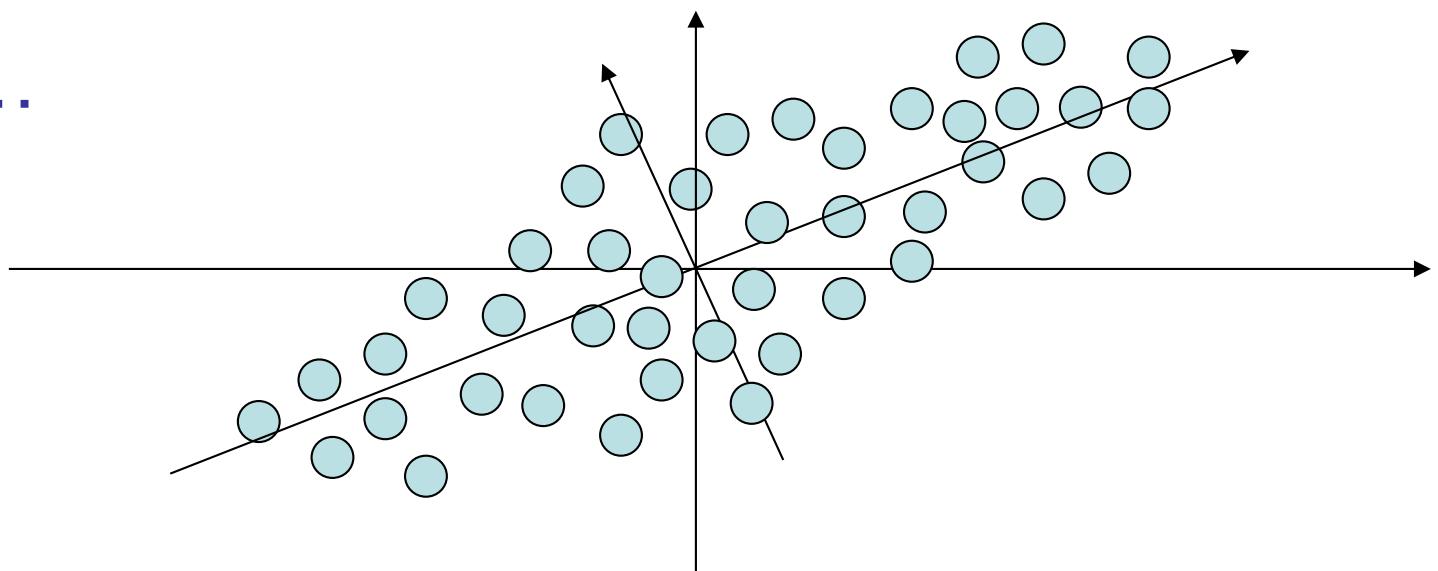
- It can be seen as the most common form of factor analysis
- The new variables/dimensions
 - Are linear combinations of the original ones
 - Are uncorrelated with one another
 - Orthogonal in original dimension space
 - Capture as much of the original variance in the data as possible
 - Are called Principal Components

What PCA does?



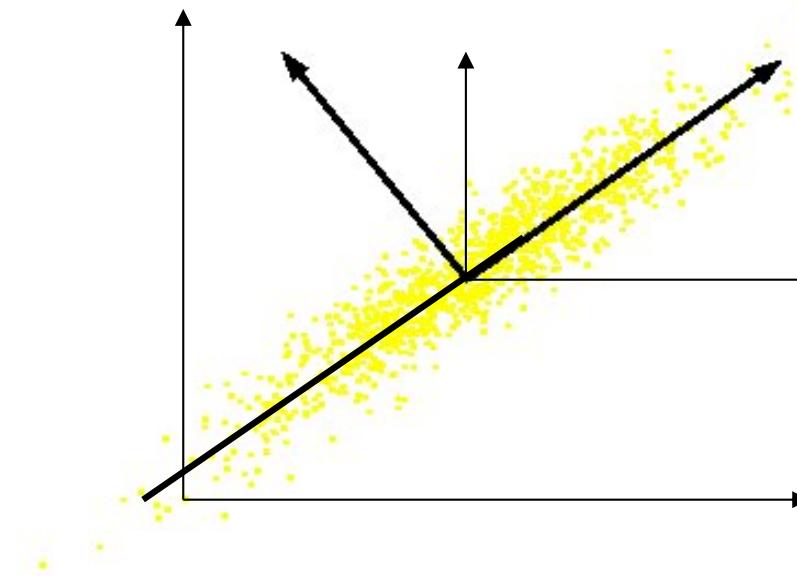
- *Orthogonal directions of greatest variance in data*
- *Projections along PC1 discriminate the data most along any one axis*

- **First principal component** is the direction of greatest variability (covariance) in the data
- **Second** is the next orthogonal (uncorrelated) direction of greatest variability
 - So first remove all the variability along the first component, and then find the next direction of greatest variability
- And so on ...



Computing the Components

- Data points are vectors in a multidimensional space
- Projection of vector \mathbf{x} onto an axis (dimension) \mathbf{u} is $\mathbf{u} \cdot \mathbf{x}$
- Direction of greatest variability is that in which the average square of the projection is greatest
 - i.e. \mathbf{u} such that $E((\mathbf{u} \cdot \mathbf{x})^2)$ over all \mathbf{x} is maximized
 - (we subtract the mean along each dimension, and center the original axis system at the centroid of all data points, for simplicity)
 - This direction of \mathbf{u} is the direction of the first Principal Component



- Principle
 - Linear projection method to reduce the number of parameters
 - Transfer a set of **correlated variables** into a new set of **uncorrelated variables**
 - Map the data into a space of lower dimensionality
 - Form of unsupervised learning
- Properties
 - It can be viewed as a rotation of the existing axes to new positions in the space defined by original variables
 - New axes are orthogonal and represent the directions with maximum variability

- Objects are represented as a cloud of n points in a multidimensional space with an axis for each of the p variables
 - First, the centroid of the points is defined by the mean of each variable
 - Second, the variance of each variable is the average squared deviation of its n values around the mean of that variable.

$$V_i = \frac{1}{n-1} \sum_{m=1}^n (X_{im} - \bar{X}_i)^2$$

- Suppose:
 - attributes are A_1 and A_2 ,
 - we have n training examples. x 's denote values of A_1 and y 's denote values of A_2 over the training examples.
- Variance of an attribute:

$$\text{var}(A_1) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{(n - 1)}$$

- Covariance of two attributes:

$$\text{cov}(A_1, A_2) = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{(n-1)}$$

- A **positive** covariance means both dimensions increase together.
- A **negative** covariance means as one dimension increases, the other decreases.
- A **Zero** covariance means dimensions are independent of each other.

- Covariance matrix

- Suppose we have n attributes, A_1, \dots, A_n .

- Covariance matrix:

$$C^{n \times n} = (c_{i,j}), \text{ where } c_{i,j} = \text{cov}(A_i, A_j)$$

$$\begin{pmatrix} \text{cov}(H,H) & \text{cov}(H,M) \\ \text{cov}(M,H) & \text{cov}(M,M) \end{pmatrix}$$

	Hours(H)	Mark(M)
Data	9	39
	15	56
	25	93
	14	61
	10	50
	18	75
	0	32
	16	85
	5	42
	19	70
	16	66
	20	80
Totals	167	749
Averages	13.92	62.42

Covariance matrix

$$\begin{pmatrix} \text{cov}(H,H) & \text{cov}(H,M) \\ \text{cov}(M,H) & \text{cov}(M,M) \end{pmatrix}$$

H	M	$(H_i - \bar{H})$	$(M_i - \bar{M})$	$(H_i - \bar{H})(M_i - \bar{M})$
9	39	-4.92	-23.42	115.23
15	56	1.08	-6.42	-6.93
25	93	11.08	30.58	338.83
14	61	0.08	-1.42	-0.11
10	50	-3.92	-12.42	48.69
18	75	4.08	12.58	51.33
0	32	-13.92	-30.42	423.45
16	85	2.08	22.58	46.97
5	42	-8.92	-20.42	182.15
19	70	5.08	7.58	38.51
16	66	2.08	3.58	7.45
20	80	6.08	17.58	106.89
Total				1149.89
Average				104.54

Covariance matrix



- Let M be an $n \times n$ matrix.
 - v is an *eigenvector* of M if $M \times v = \lambda v$
 - λ is called the *eigenvalue* associated with v
- For any eigenvector v of M and scalar a ,

$$M \times a v = \lambda a v$$

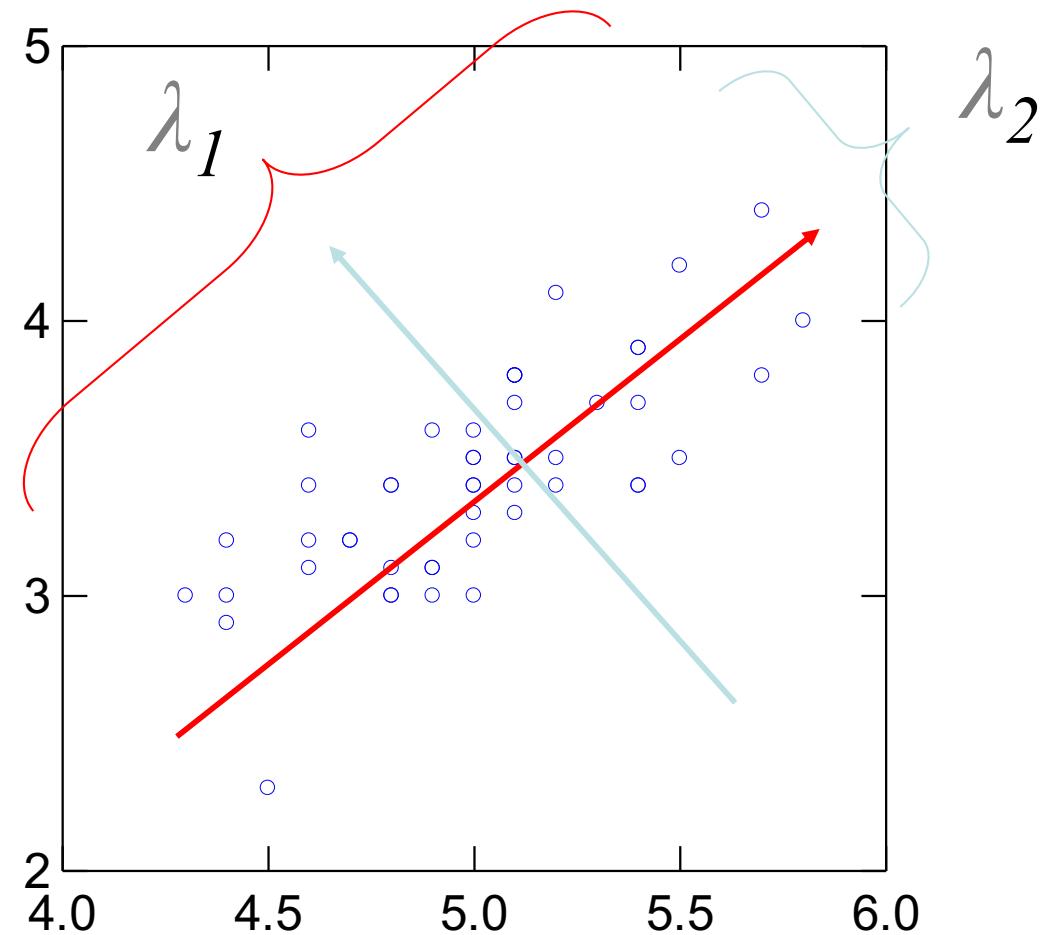
- Thus you can always choose eigenvectors of length 1:

$$\sqrt{v_1^2 + \dots + v_n^2} = 1$$

- If M has any eigenvectors, it has n of them, and they are orthogonal to one another.
- Thus eigenvectors can be used as a new basis for a n -dimensional vector space.



PCA Eigenvalues





0. Get some data
1. Subtract the mean from data
2. Calculate the covariance matrix
3. Calculate the eigenvectors and eigenvalues of the covariance matrix
4. Choose components and construct a new feature vector
5. Derive the new data set
6. Reconstruct the old data back

1. Given original data set $S = \{\mathbf{x}^1, \dots, \mathbf{x}^k\}$,
 produce new set by subtracting the mean
 of attribute A_i from each x_i .

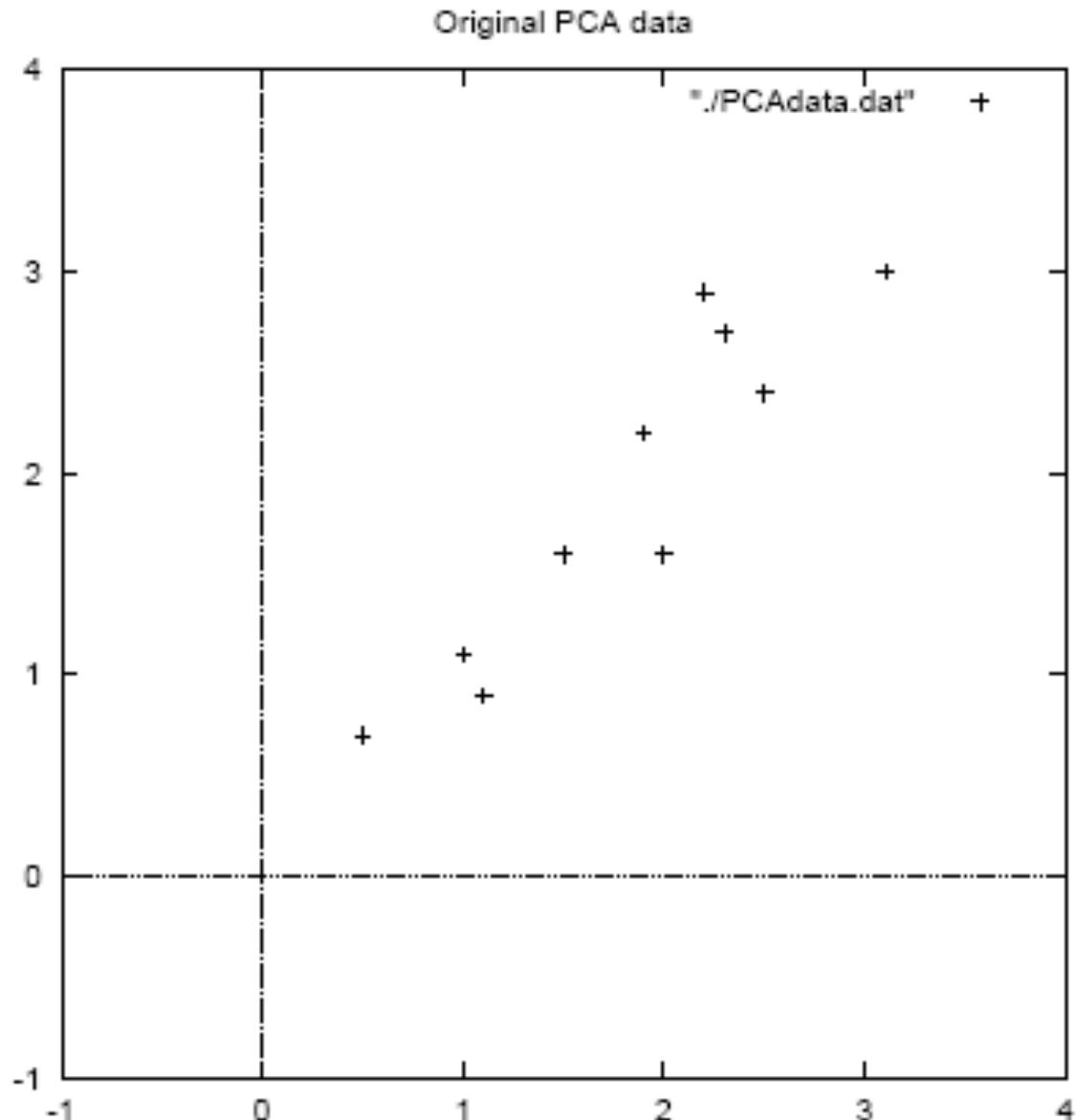
	x	y
	2.5	2.4
	0.5	0.7
	2.2	2.9
	1.9	2.2
Data =	3.1	3.0
	2.3	2.7
	2	1.6
	1	1.1
	1.5	1.6
	1.1	0.9

Mean: 1.81 1.91

	x	y
	.69	.49
	-1.31	-1.21
	.39	.99
	.09	.29
DataAdjust =	1.29	1.09
	.49	.79
	.19	-.31
	-.81	-.81
	-.31	-.31
	-.71	-1.01

Mean: 0 0

PCA example: original data set



2. Calculate the covariance matrix:

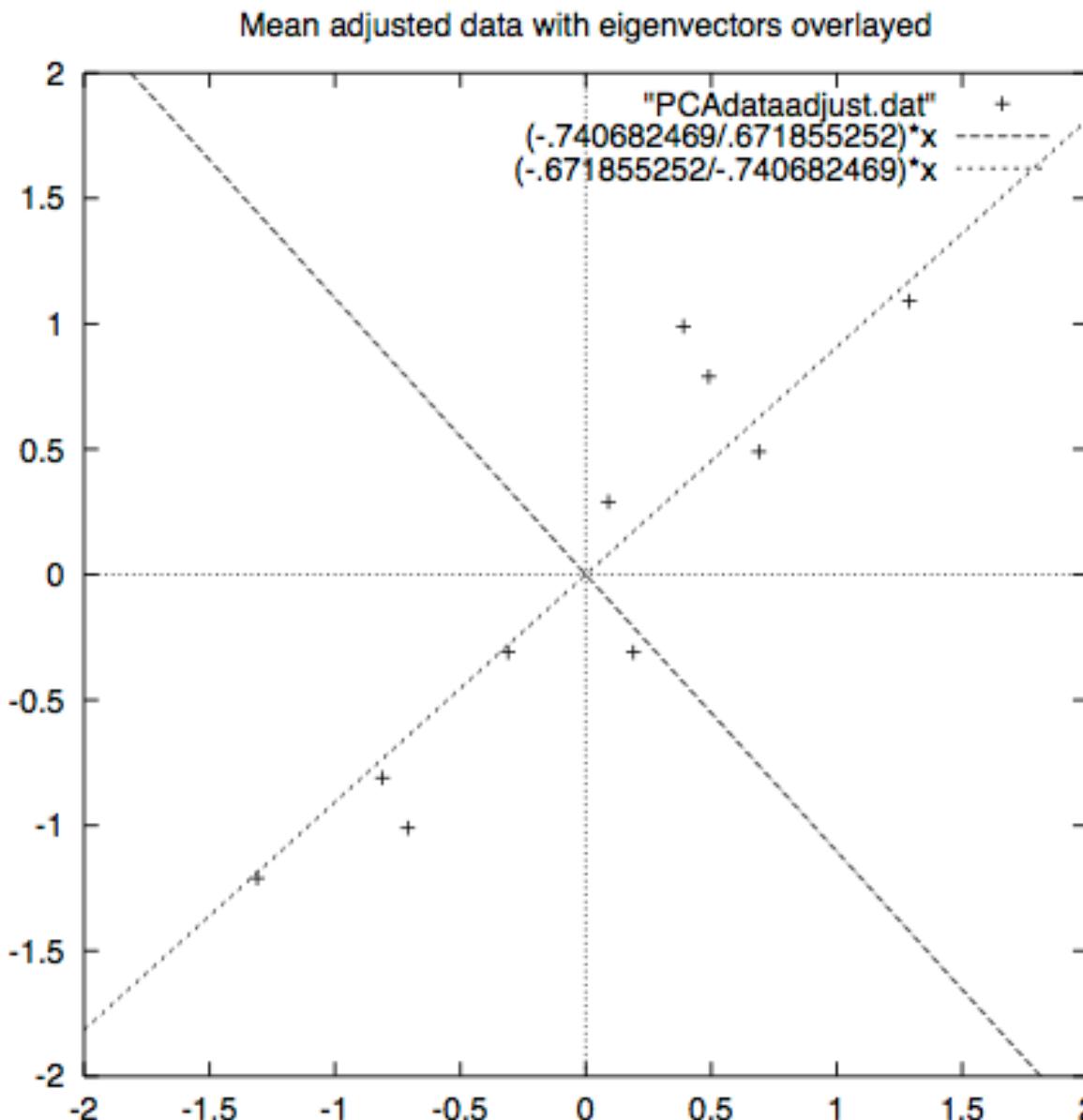
$$cov = \begin{pmatrix} & x & y \\ x & .616555556 & .615444444 \\ y & .615444444 & .716555556 \end{pmatrix}$$

3. Calculate the (unit) eigenvectors and eigenvalues of the covariance matrix:

$$eigenvalues = \begin{pmatrix} .0490833989 \\ 1.28402771 \end{pmatrix}$$

$$eigenvectors = \begin{pmatrix} -.735178656 & -.677873399 \\ .677873399 & -.735178656 \end{pmatrix}$$

PCA example



*Eigenvector with
largest
eigenvalue traces
linear pattern in data*

Order eigenvectors by eigenvalue, highest to lowest.

$$\mathbf{v}_1 = \begin{pmatrix} -.677873399 \\ -.735178956 \end{pmatrix} \quad \lambda = 1.28402771$$

$$\mathbf{v}_2 = \begin{pmatrix} -.735178956 \\ .677873399 \end{pmatrix} \quad \lambda = .0490833989$$

In general, you get n components.

To reduce dimensionality to p , ignore $n-p$ components at the bottom of the list.

4. Construct new feature vector.

Feature vector = ($\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_p$)

$$\textit{FeatureVector1} = \begin{pmatrix} -.677873399 & -.735178956 \\ -.735178956 & .677873399 \end{pmatrix}$$

or reduced dimension feature vector :

$$\textit{FeatureVector2} = \begin{pmatrix} -.677873399 \\ -.735178956 \end{pmatrix}$$

5. Derive the new data set.

TransformedData = RowFeatureVector × RowDataAdjust

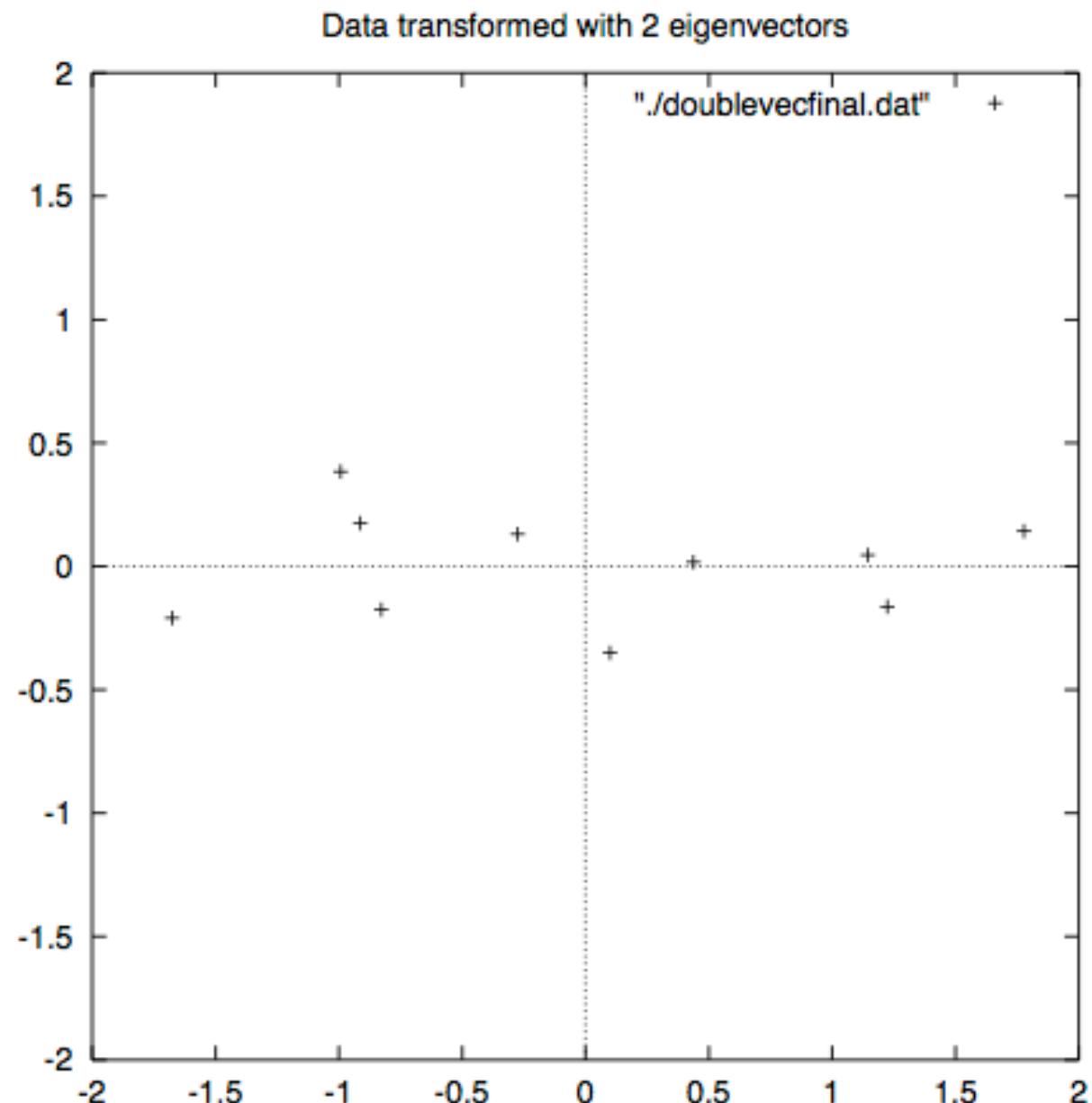
$$\text{RowFeatureVector1} = \begin{pmatrix} -.677873399 & -.735178956 \\ -.735178956 & .677873399 \end{pmatrix}$$

$$\text{RowFeatureVector2} = (-.677873399 \quad -.735178956)$$

$$\text{RowDataAdjust} = \begin{pmatrix} .69 & -1.31 & .39 & .09 & 1.29 & .49 & .19 & -.81 & -.31 & -.71 \\ .49 & -1.21 & .99 & .29 & 1.09 & .79 & -.31 & -.81 & -.31 & -1.01 \end{pmatrix}$$

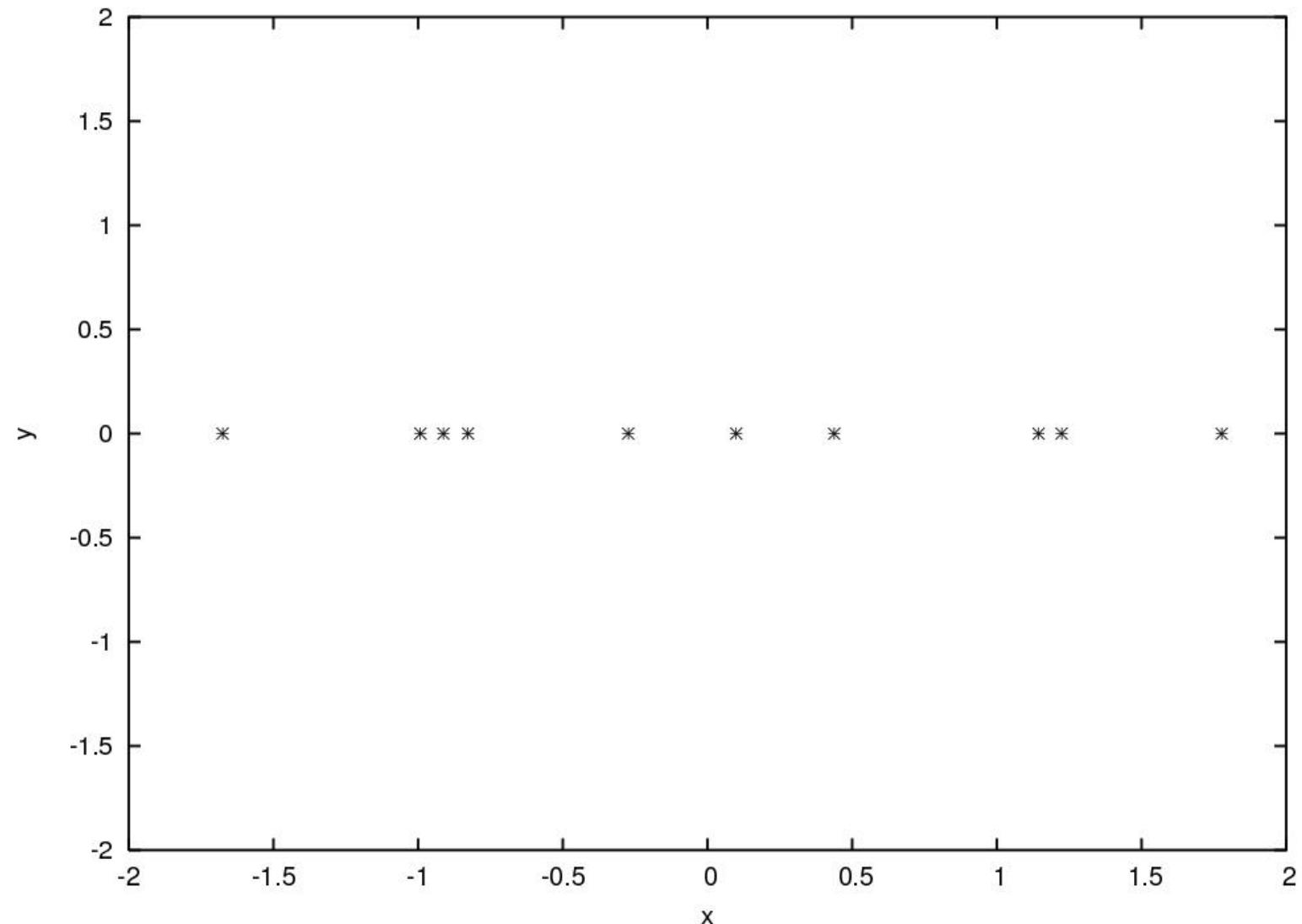
Transformed data

x	y
-.827970186	-.175115307
1.77758033	.142857227
-.992197494	.384374989
-.274210416	.130417207
-1.67580142	-.209498461
-.912949103	.175282444
.0991094375	-.349824698
1.14457216	.0464172582
.438046137	.0177646297
1.22382056	-.162675287



Transformed data with a Single eigenvector

x
-.827970186
1.77758033
-.992197494
-.274210416
-1.67580142
-.912949103
.0991094375
1.14457216
.438046137
1.22382056



6. Reconstruct the old data back

- We did:

$$\text{TransformedData} = \text{RowFeatureVector} \times \text{RowDataAdjust}$$

- so we can do

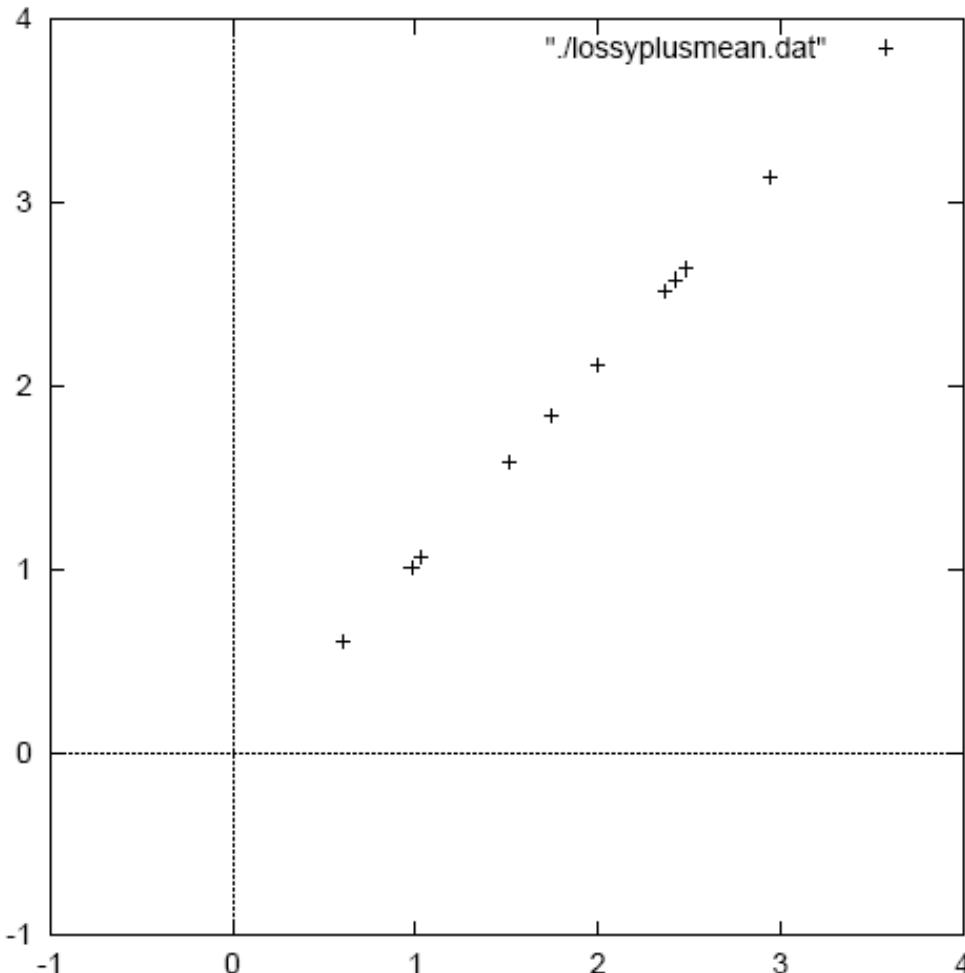
$$\text{RowDataAdjust} = \text{RowFeatureVector}^{-1} \times \text{TransformedData}$$
$$= \text{RowFeatureVector}^T \times \text{TransformedData}$$

- and

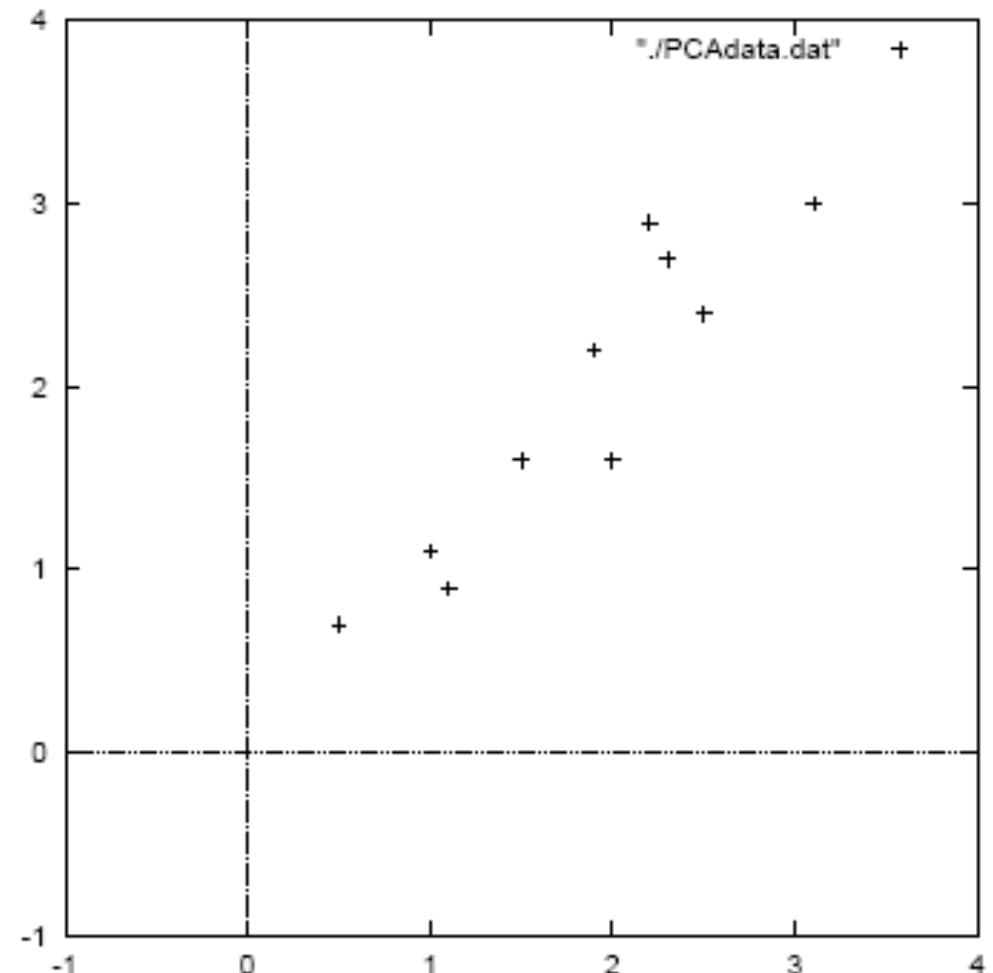
$$\text{RowDataOriginal} = \text{RowDataAdjust} + \text{OriginalMean}$$

PCA example

Original data restored using only a single eigenvector



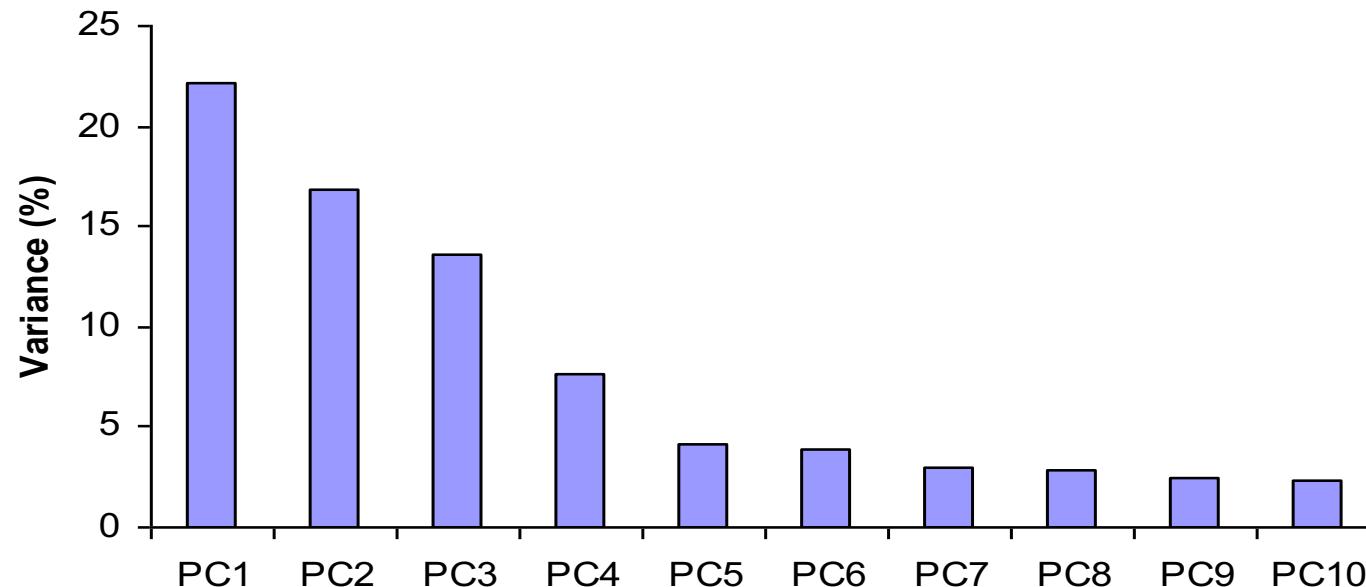
Original PCA data



- For n original dimensions, correlation matrix is $n \times n$, and has up to n eigenvectors. So n PCs.
- As a rule of thumb, it is worth the selection of those eigenvalues > 1
- Where does dimensionality reduction come from?

Dimensionality Reduction

Can ignore the components of lesser significance.



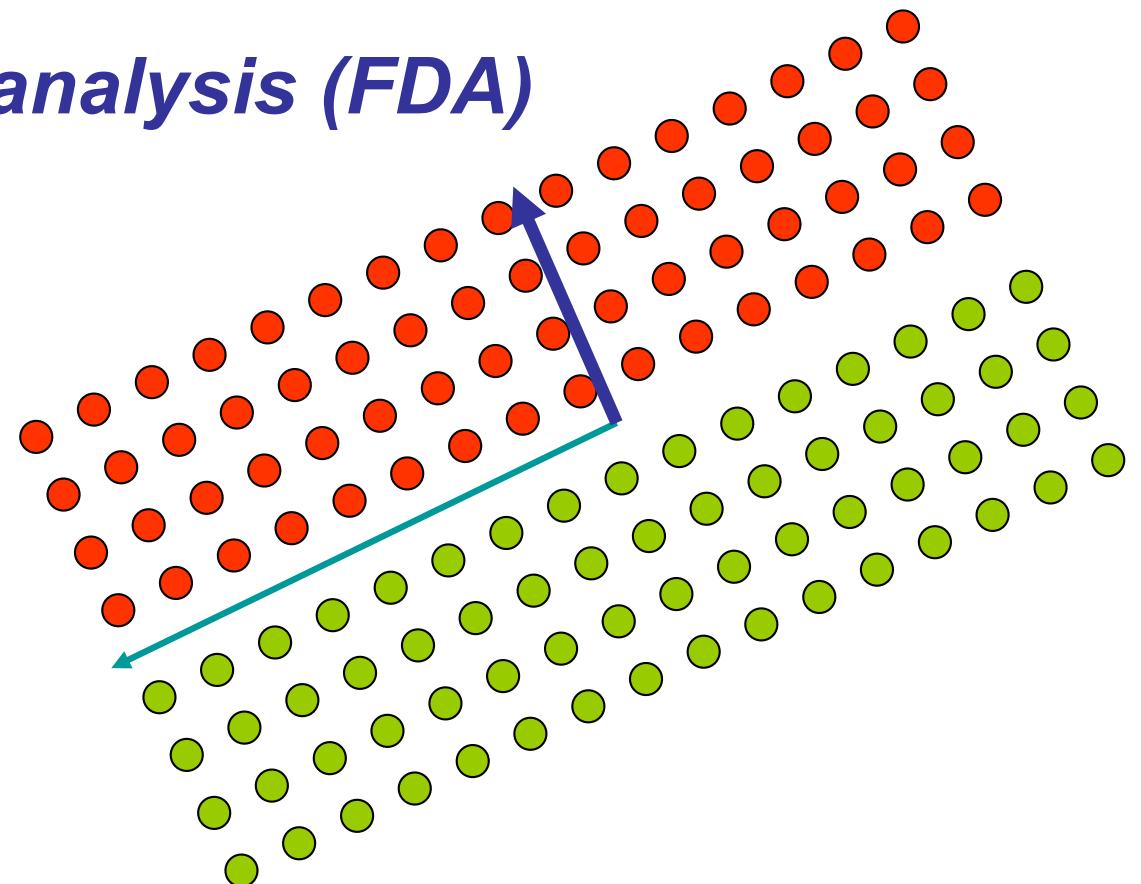
You do *lose some information*, but if the eigenvalues are small, you don't lose much

- n dimensions in original data and calculate n eigenvectors and eigenvalues
- choose only the first p eigenvectors, based on their eigenvalues and final data set has only p dimensions

- M. Turk, A. Pentland, "Eigenfaces for Recognition", *Journal of Cognitive Neuroscience*, vol. 3, no. 1, pp. 71-86, 1991.
- *Principal Component Analysis for clustering gene expression data*. Yeung and Ruzzo. 2001.
- *Probabilistic Disease Classification of Expression-Dependent Proteomic Data from Mass Spectrometry of Human Serum*. Lilien. 2003.

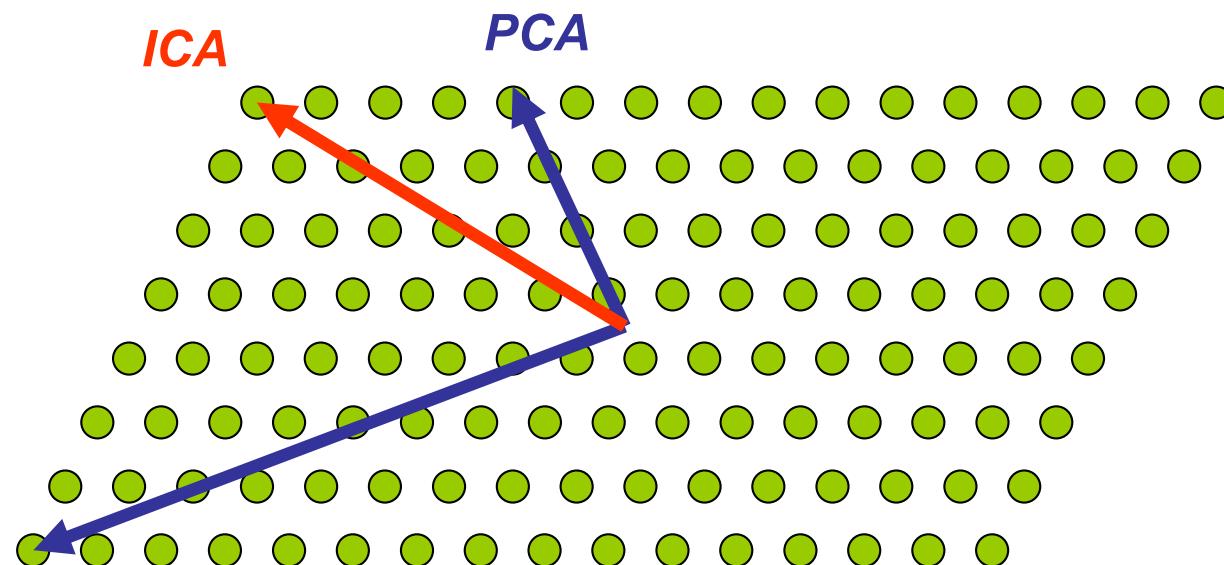
Are the maximal variance dimensions the relevant dimensions for preservation?

- **Relevant Component Analysis (RCA)**
- **Fisher Discriminant analysis (FDA)**

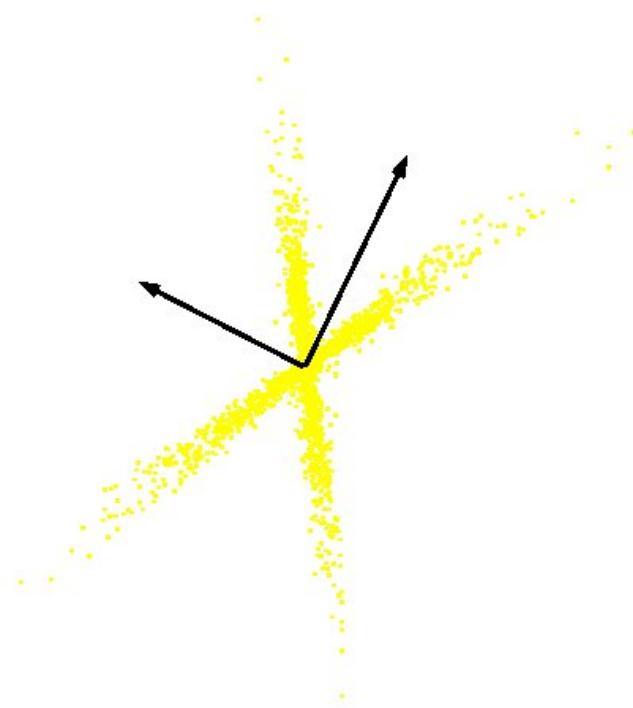


Should the goal be finding independent rather than pair-wise uncorrelated dimensions

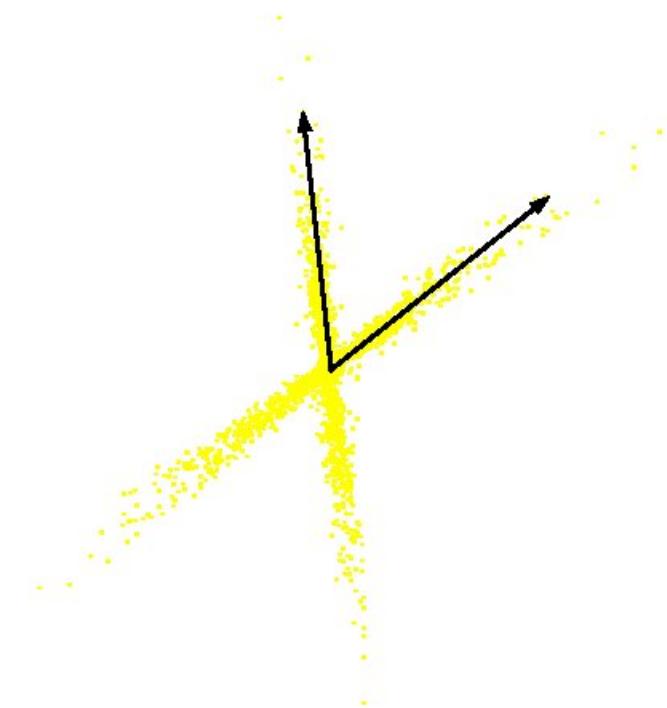
- **Independent Component Analysis (ICA)**



Independent Component Analysis (ICA)



PCA
(orthogonal coordinate)



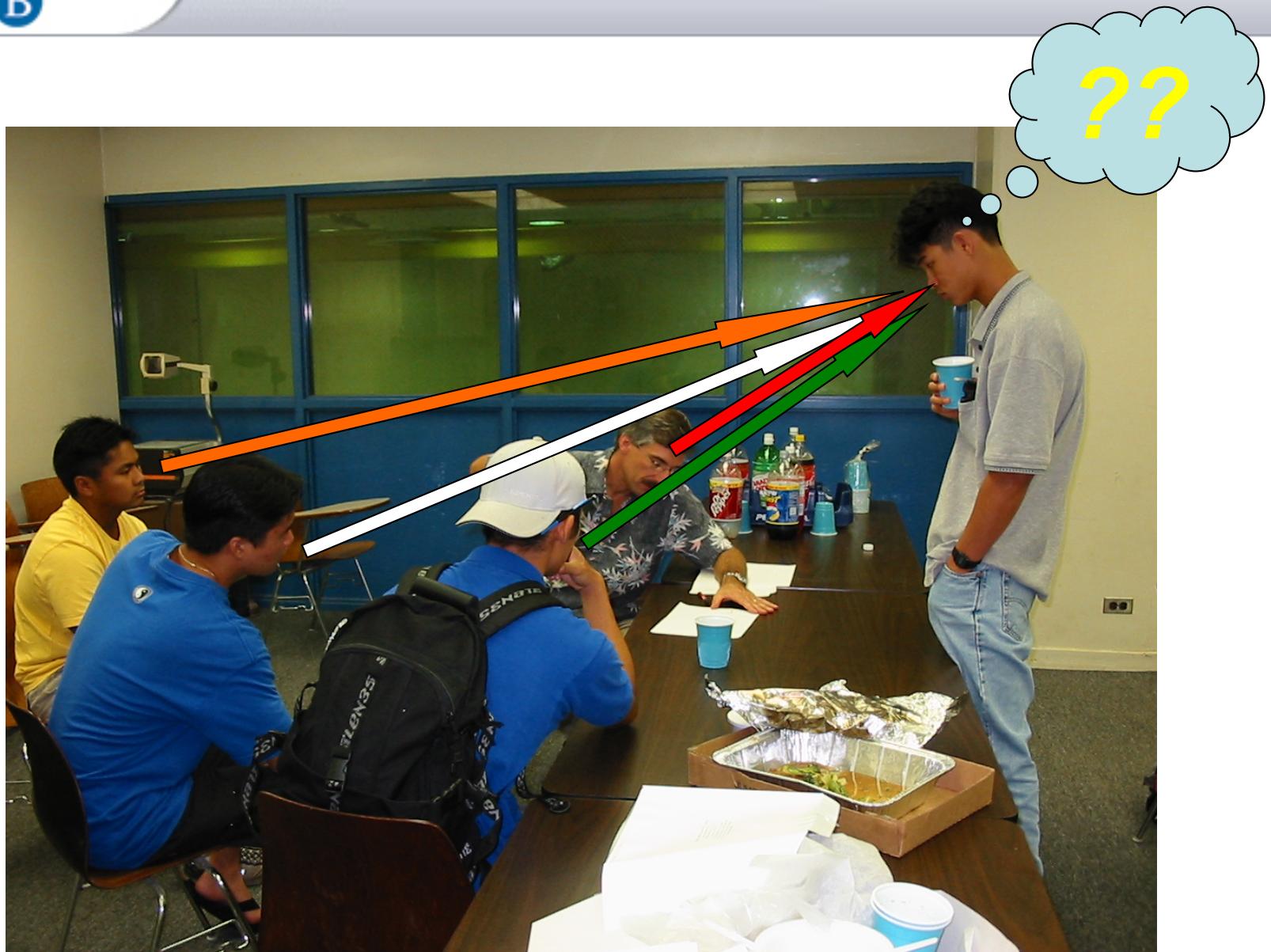
ICA
(non-orthogonal coordinate)



- PCA
 - Focus on uncorrelated and Gaussian components
 - Second-order statistics
 - Orthogonal transformation
- ICA
 - Focus on independent and non-Gaussian components
 - Higher-order statistics
 - Non-orthogonal transformation

- Cocktail Party Problem
 - Suppose you are in a crowded room with many people. How do you understand what any one person is saying?
- Separation of Independent Signals
 - Similar to Blind Source Separation
 - Little knowledge of the signals
 - Access to mixed signals only

Cocktail Party Problem



- ICA Separation Algorithm
 - Separation of Speech Signals
 - Humans can separate multiple signals with only two ears/sensors
 - ICA needs as many ears/sensors as message signals
 - Here we assume he has four ears!



Recovered Messages

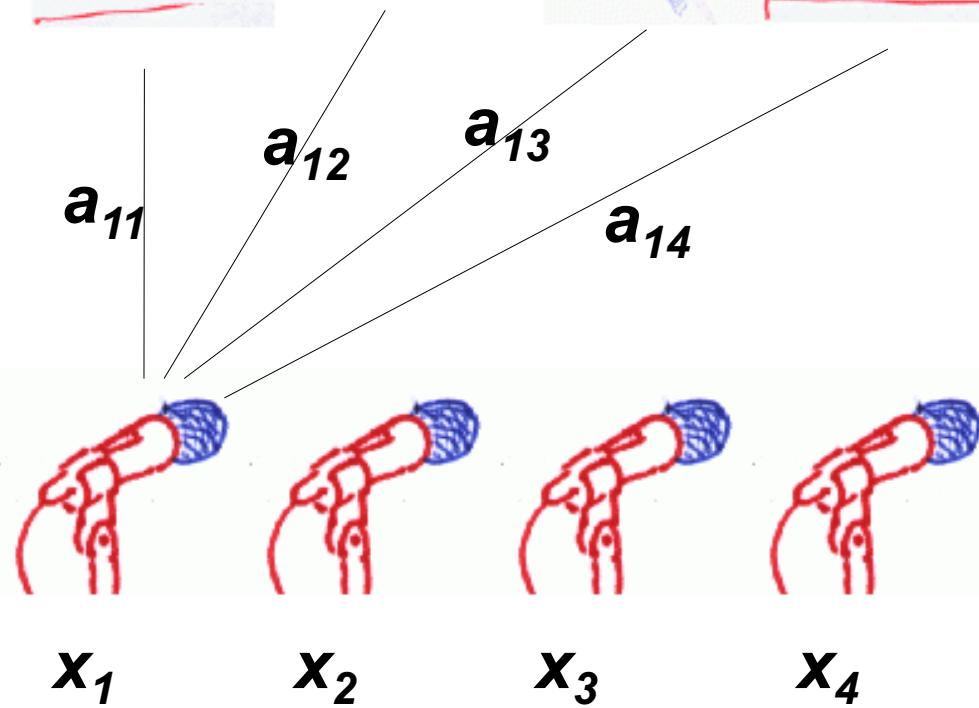
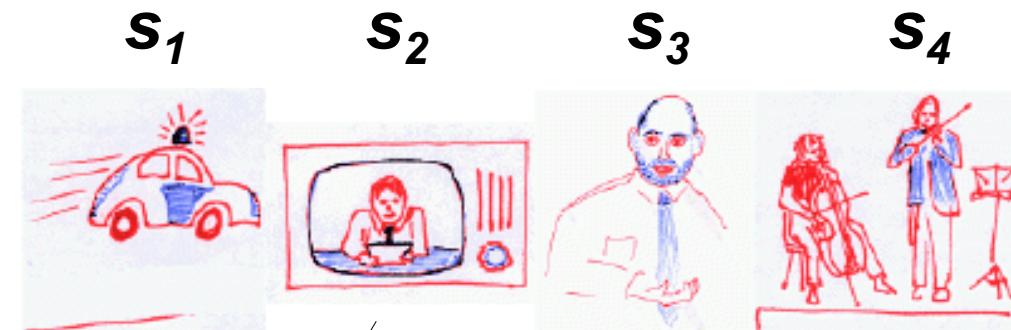


What is ICA?

“Independent component analysis (ICA) is a method for finding **underlying factors** or components from multivariate (multi-dimensional) statistical data. What distinguishes ICA from other methods is that it looks for components that are **both *statistically independent*, and *nonGaussian*.**”

A.Hyvarinen, A.Karhunen, E.Oja
‘Independent Component Analysis’

The ICA model



$$x_i(t) = a_{i1} * s_1(t) + a_{i2} * s_2(t) + a_{i3} * s_3(t) + a_{i4} * s_4(t)$$

Here, $i=1:4$.

In vector-matrix notation, and dropping index t , this is

$$\mathbf{x} = \mathbf{A} * \mathbf{s}$$

- ICA is a statistical method, the goal of which is to decompose given multivariate data into a linear sum of statistically independent components.
- For example, given two-dimensional vector , $\mathbf{x} = [x_1 \ x_2]^T$, ICA aims at finding the following decomposition

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} s_1 + \begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} s_2$$

$$\mathbf{x} = \mathbf{a}_1 s_1 + \mathbf{a}_2 s_2$$

where $\mathbf{a}_1, \mathbf{a}_2$ are basis vectors and s_1, s_2 are basis coefficients.

Constraint: Basis coefficients s_1 and s_2 are statistically independent.

- Mixed Signals in Matrix Notation

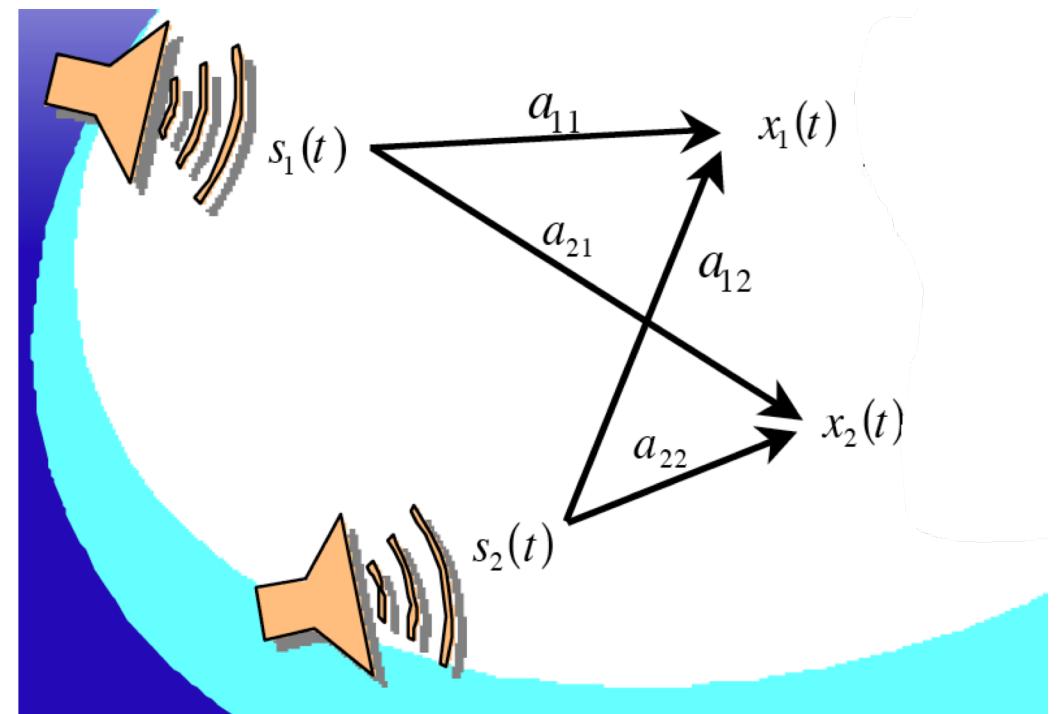
$$\mathbf{X} = \mathbf{A} \times \mathbf{S} = \sum_{i=1}^n \mathbf{a}_i \bullet s_i$$

- **Variable Definitions**

\mathbf{x}_j ~ Multiplexed Signal

\mathbf{A} ~ Multiplexing Matrix

s_i ~ i^{th} Independent Signal



- Signal Separation

$$\begin{aligned}\hat{\mathbf{s}} &= \mathbf{W} \times \mathbf{x} = \mathbf{W} \times (\mathbf{A} \times \mathbf{s}) \\ &= (\mathbf{W} \times \mathbf{A}) \times \mathbf{s} = \mathbf{I} \times \mathbf{s} = \mathbf{s}\end{aligned}$$

- Find \mathbf{W} using the ICA Algorithm

Goal will be to find a matrix W such that the entries of $\hat{\mathbf{s}} = \mathbf{W}\mathbf{x}$ are as independent as possible.

- Central Limit Theorem
 - If two random (non-Gaussian) signals are added, the resulting signal will be more Gaussian than the original two random signals
- ICA Separation Concept
 - Central Limit Theorem (in Reverse)
 - Maximizing Non-Gaussianity
 - Results in separating the two signals

- **Probability Density Definition**

$$p(y_1, y_2) = p_1(y_1)p_2(y_2)$$

p_1, p_2 are pdf's

- **Expected Value Definition**

$$E\{h_1(y_1) * h_2(y_2)\} = E\{h_1(y_1)\} * E\{h_2(y_2)\}$$

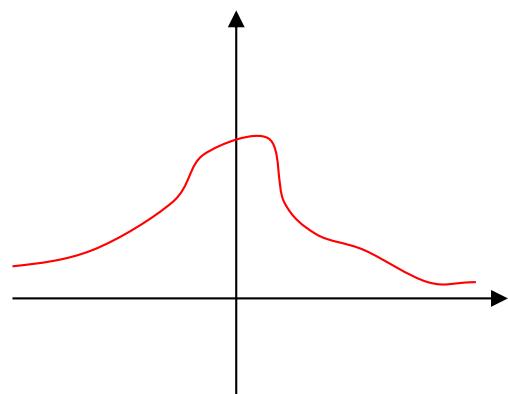
h_1, h_2 are functions

What is independent component?

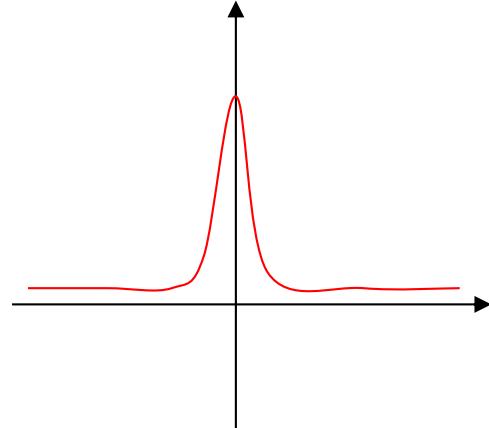
- If one variable can not be estimated from other variables, **it is independent.**
- By **Central Limit Theorem**, a sum of two independent random variables is more gaussian than original variables → distribution of independent components are nongaussian
- To estimate ICs, **y** should have nongaussian distribution, i.e. we should **maximize nongaussianity.**

What is nongaussianity?

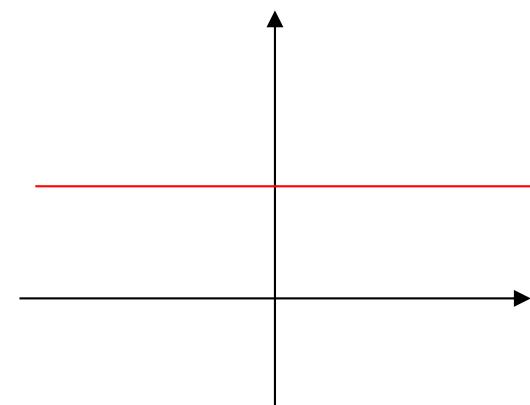
- Supergaussian
- Subgaussian
- Low entropy



Gaussian



Supergaussian



Subgaussian

- Property of Gaussian signals
 - Addition of two independent Gaussian random variables is another single Gaussian random variable.
 - Information Lost!
- Measuring nongaussianity by Kurtosis Function
 - Kurtosis : 4th order cumulant of random variable
$$kurt(y) = E\{y^4\} - 3(E\{y^2\})^2$$
 - Special case $Kurt(y) = 0$

How to measure nongaussianity?

- Measuring nongaussianity by Kurtosis
 - Kurtosis : 4th order cumulant of random variable

$$kurt(z) = E\{z^4\} - 3(E\{z^2\})^2$$

- If $kurt(z)$ is zero, gaussian
- If $kurt(z)$ is positive, supergaussian
- If $kurt(z)$ is negative, subgaussian

- Maximization of $|kurt(z)|$ by gradient method

$$\frac{\partial |kurt(\mathbf{w}^T \mathbf{x})|}{\partial \mathbf{w}} = 4sign(kurt(\mathbf{w}^T \mathbf{x}))[E\{\mathbf{x}(\mathbf{w}^T \mathbf{x})^3\} - 3\mathbf{w} \|\mathbf{w}\|^2]$$

*Simply change
The norm of w*

$$\Delta \mathbf{w} \propto sign(kurt(\mathbf{w}^T \mathbf{x}))E\{\mathbf{x}(\mathbf{w}^T \mathbf{x})^3\}$$

$$\mathbf{w} \leftarrow \mathbf{w} / \|\mathbf{w}\|$$



$$\mathbf{w} \leftarrow E\{\mathbf{x}(\mathbf{w}^T \mathbf{x})^3\} - 3\mathbf{w}$$

Fast-fixed point algorithm

- Kurtosis : gauss=0 (sensitive to outliers)
- Entropy : gauss=largest $kurt(y) = E\{y^4\} - 3(E\{y^2\})^2$
- Negentropy : gauss = 0 $H(y) = - \int f(y) \log f(y) dy$
– (difficult to estimate)
- Approximations
$$\begin{cases} J(y) = H(y_{gauss}) - H(y) \\ J(y) = \frac{1}{12} E\{y^2\}^2 + \frac{1}{48} kurt(y)^2 \\ J(y) \approx [E\{G(y)\} - E\{G(v)\}]^2 \end{cases}$$
- where v is a standard gaussian random variable and :

$$G(y) = \frac{1}{a} \log \cosh(a.y)$$

$$G(y) = -\exp(-a.u^2 / 2)$$

Limitation #1: Scaling

- ICA maximizes independence between signals.

$$E\{\lambda_1 h_1(\hat{X}_1) * \lambda_2 h_2(\hat{X}_2)\} = \lambda_1 \lambda_2 E\{h_1(\hat{X}_1) * h_2(\hat{X}_2)\}$$

$$\xrightarrow{\quad} = \lambda_1 E\{h_1(\hat{X}_1)\} * \lambda_2 E\{h_2(\hat{X}_2)\}$$

Assuming the Separated Signals are Independent

Scaling Factor for Signal #1

Scaling Factor for Signal #2

- The mixing matrix and independent components are unknown.

$$\mathbf{x} = \mathbf{A} * \mathbf{s} = \mathbf{A} * \mathbf{I} * \mathbf{s} = \mathbf{A} * (\mathbf{P}^{-1} * \mathbf{P}) * \mathbf{s}$$

$$= (\mathbf{A} * \mathbf{P}^{-1}) * (\mathbf{P} * \mathbf{s}) = \mathbf{A}' * \mathbf{s}'$$

$$\hat{\mathbf{x}} = \mathbf{W}' * \mathbf{A}' * \mathbf{s}' = (\mathbf{P} * \mathbf{W}) * (\mathbf{A} * \mathbf{P}^{-1}) * \mathbf{s}'$$

$$= (\mathbf{P} * (\mathbf{W} * \mathbf{A}) * \mathbf{P}^{-1}) * \mathbf{s}' = (\mathbf{P} * (\mathbf{I} * \mathbf{P}^{-1})) * \mathbf{s}'$$

$$= (\mathbf{P} * \mathbf{P}^{-1}) * \mathbf{s}' = \mathbf{I} * \mathbf{s}' = \mathbf{s}' = \mathbf{P} * \mathbf{s}$$

- Sensor Requirement
 - The number of separated signals cannot be larger than the number of inputs.
 - Current research is being done to reduce this constraint.

- Blind source separation (BSS)
- Image denoising
- Medical signal processing – fMRI, ECG, EEG
- Modelling of the hippocampus and visual cortex
- Feature extraction, face recognition
- Compression, redundancy reduction
- Watermarking
- Clustering
- Time series analysis (stock market, microarray data)
- Topic extraction
- Econometrics: Finding hidden factors in financial data

```
import numpy as np
import matplotlib.pyplot as plt
from scipy import signal
from sklearn.decomposition import FastICA, PCA

# Generate sample data
np.random.seed(0)
n_samples = 2000
time = np.linspace(0, 8, n_samples)

s1 = np.sin(2 * time) # Signal 1 : sinusoidal signal
s2 = np.sign(np.sin(3 * time)) # Signal 2 : square signal
s3 = signal.sawtooth(2 * np.pi * time) # Signal 3: saw tooth signal

S = np.c_[s1, s2, s3]
S += 0.2 * np.random.normal(size=S.shape) # Add noise
```



```
S /= S.std(axis=0) # Standardize data  
# Mix data  
A = np.array([[1, 1, 1], [0.5, 2, 1.0], [1.5, 1.0, 2.0]]) # Mixing matrix  
X = np.dot(S, A.T) # Generate observations
```

Compute ICA

```
ica = FastICA(n_components=3)  
S_ = ica.fit_transform(X) # Reconstruct signals  
A_ = ica.mixing_ # Get estimated mixing matrix
```

We can `prove` that the ICA model applies by reverting the unmixing.
assert np.allclose(X, np.dot(S_, A_.T) + ica.mean_)

For comparison, compute PCA

```
pca = PCA(n_components=3)
```

```
H = pca.fit_transform(X) # Reconstruct signals based on orthogonal components
```

Plot results

```
plt.figure()
```

```
models = [X, S, S_, H]
```

```
names = ['Observations (mixed signal)', 'True Sources', 'ICA recovered signals',
         'PCA recovered signals']
```

```
colors = ['red', 'steelblue', 'orange']
```

```
for ii, (model, name) in enumerate(zip(models, names), 1):
```

```
    plt.subplot(4, 1, ii)
```

```
    plt.title(name)
```

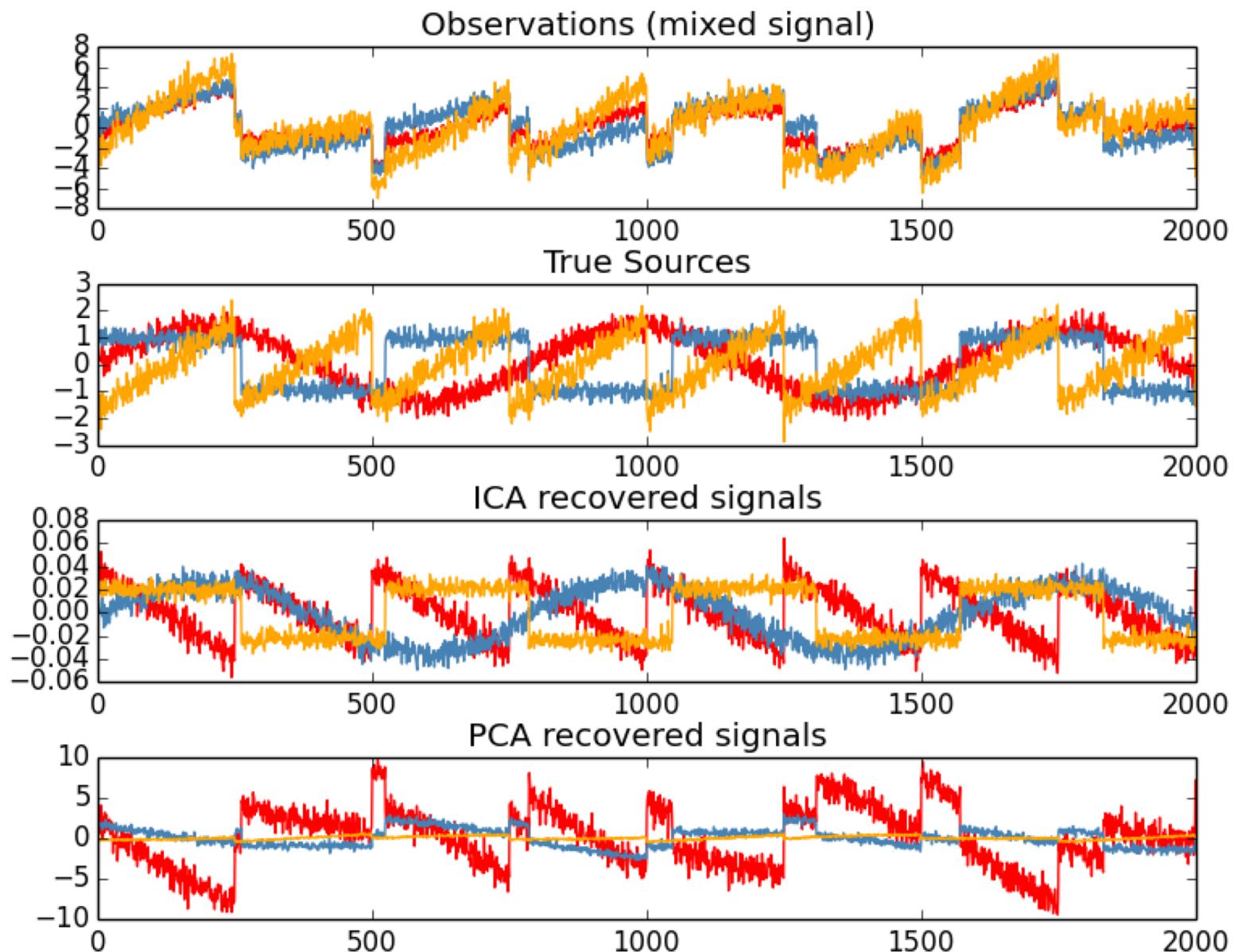
```
    for sig, color in zip(model.T, colors):
```

```
        plt.plot(sig, color=color)
```

```
plt.subplots_adjust(0.09, 0.04, 0.94, 0.94, 0.26, 0.46)
```

```
plt.show()
```

ICA in Python (Example)





- PCA : Proper to dimension reduction
- ICA : Proper to blind source separation or classification using ICs when class id of training data is not available

- R.O. Duda, P.E. Hart and D.G. Stork, "Pattern Classification". Wiley-Interscience Publication (2000). Chapter 3
- Lindsay I Smith. A tutorial on Principal Components Analysis, 2002.
- Christopher Bishop. Pattern Recognition and Machine Learning. Chapter 12.
- A. Hyvärinen, J. Karhunen and E. Oja, “Independent Component Analysis,”, John Willy & Sons, Inc.
- ICA demo step-by-step
 - <http://www.cis.hut.fi/projects/ica/icademo/>