Examples of Events in a Dice Roll

Here are examples of events in a single dice roll:

1. Not independent and not disjoint

- Event A: The rolled number is even (i.e., 2, 4, or 6).
- Event B: The rolled number is greater than 3 (i.e., 4, 5, or 6).

These events are **not independent** because the occurrence of A does change the probability of B and vice versa. However, they are **not disjoint** because the numbers 4 and 6 are both even and greater than 3 (so $A \cap B = \{4, 6\} \neq \emptyset$).

$\mathbb{P}(A|B) = \frac{2}{3} \neq \mathbb{P}(A) = \frac{1}{2}$

$\mathbb{P}(B|A) = \frac{2}{3} \neq \mathbb{P}(B) = \frac{1}{2}$

2. Not independent, but disjoint

- Event C: The rolled number is 2.
- Event D: The rolled number is 5.

These events are **disjoint** because they are mutually exclusive (if the number 2 is rolled, it cannot simultaneously be 5, so $C \cap D = \emptyset$). However, they are **not independent** because the occurrence of C makes the probability of D zero and vice versa.

3. Independent and not disjoint

- Event E: The rolled number is even (i.e., 2, 4, or 6).
- Event F: The rolled number is a multiple of 3 (i.e., 3 or 6).

These events are **independent** because the occurrence of E does not affect the probability of F and vice versa:

$$\mathbb{P}(E) = \frac{3}{6} = 0.5, \quad \mathbb{P}(F) = \frac{2}{6} = \frac{1}{3}$$

$$\mathbb{P}(E \cap F) = \frac{1}{6} = \mathbb{P}(E) \cdot \mathbb{P}(F)$$

However, they are **not disjoint** because the number 6 is both even and a multiple of 3 (so $E \cap F = \{6\} \neq \emptyset$).