Exercise 7 - Conditional Probability

The event space is:

$$\Omega = \{(R1_{\text{pos}}, R2_{\text{pos}}), (R1_{\text{pos}}, R2_{\text{neg}}), (R1_{\text{neg}}, R2_{\text{pos}}), (R1_{\text{neg}}, R2_{\text{neg}})\}$$

We are interested in the conditional probability:

$$P(\text{R1 finds an effect} \mid \text{R2 finds an effect})$$

This is given by:

$$P(R1_{\text{pos}} \mid R2_{\text{pos}}) = \frac{P(R1_{\text{pos}} \cap R2_{\text{pos}})}{P(R2_{\text{pos}})}$$

Step 1: Compute the probability of $R2_{\text{pos}}$ The probability that researcher 2 finds an effect, $P(R2_{\text{pos}})$, is the sum of the probabilities of the elementary events where $R2$ finds an effect:

$$P(R2_{\text{pos}}) = P(R1_{\text{pos}} \cap R2_{\text{pos}}) + P(R1_{\text{neg}} \cap R2_{\text{pos}})$$

Step 2: Compute the probability of $R1_{\text{pos}} \cap R2_{\text{pos}}$ This is the probability of both researchers finding an effect, $P(R1_{\text{pos}} \cap R2_{\text{pos}})$.

Step 3: Finally, the conditional probability is:

$$P(R1_{\text{pos}} \mid R2_{\text{pos}}) = \frac{P(R1_{\text{pos}} \cap R2_{\text{pos}})}{P(R2_{\text{pos}})} = \frac{0.04^2}{0.04 \cdot 0.04 + 0.96 \cdot 0.04} = 0.04$$

This shows that researcher 1 finding an effect is independent from researcher 2 finding an effect.