Exercise 6 - Three Researchers

1. **What does the event space Ω look like?**

   The event space Ω for 3 researchers consists of all possible combinations of the researchers either finding a positive effect or not finding an effect. Each researcher can either find a positive effect (denoted by "pos") or not (denoted by "neg"). Thus, the event space Ω consists of $2^3 = 8$ elementary events.
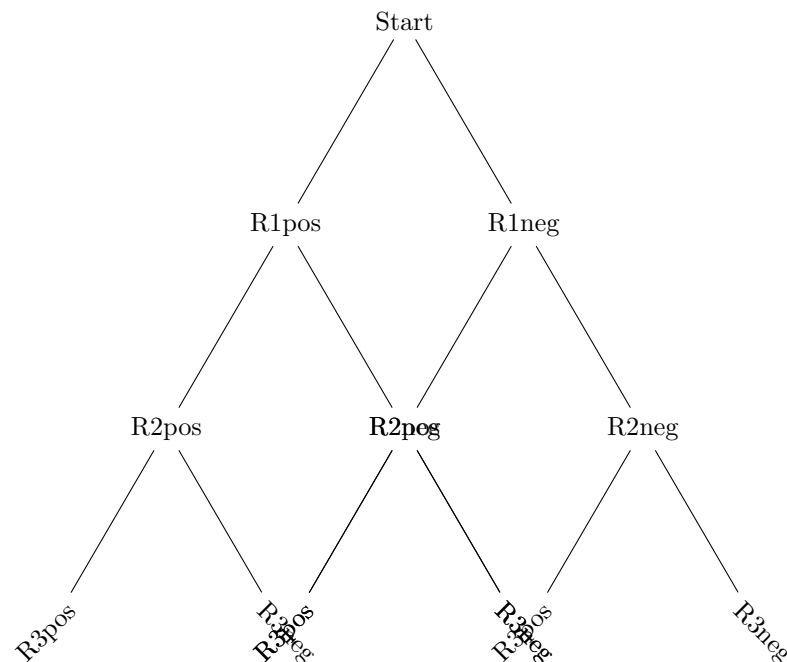
   $$\Omega = \{(R1_{\text{pos}}, R2_{\text{pos}}, R3_{\text{pos}}), (R1_{\text{pos}}, R2_{\text{pos}}, R3_{\text{neg}}), \ldots, (R1_{\text{neg}}, R2_{\text{neg}}, R3_{\text{neg}})\}$$

2. **Which elementary events are in the set of all possible outcomes of our 3-researcher experiment and how many are there?**

   The total number of elementary events is $2^3 = 8$. The set of all possible outcomes is:

   $$\{(R1_{\text{pos}}, R2_{\text{pos}}, R3_{\text{pos}}), (R1_{\text{pos}}, R2_{\text{pos}}, R3_{\text{neg}}), (R1_{\text{pos}}, R2_{\text{neg}}, R3_{\text{pos}}), (R1_{\text{pos}}, R2_{\text{neg}}, R3_{\text{neg}}),$$

   $$(R1_{\text{neg}}, R2_{\text{pos}}, R3_{\text{pos}}), (R1_{\text{neg}}, R2_{\text{pos}}, R3_{\text{neg}}), (R1_{\text{neg}}, R2_{\text{neg}}, R3_{\text{pos}}), (R1_{\text{neg}}, R2_{\text{neg}}, R3_{\text{neg}})\}$$

3. **Draw the corresponding binary tree for this experiment.**

4. **Which elementary events are in the following event: "Researcher 3 finds a positive effect"?**

   The event that "Researcher 3 finds a positive effect" consists of all outcomes where the third researcher's result is "pos". Thus, the event set is:

   $$\{(R1_{\text{pos}}, R2_{\text{pos}}, R3_{\text{pos}}), (R1_{\text{pos}}, R2_{\text{neg}}, R3_{\text{pos}}), (R1_{\text{neg}}, R2_{\text{pos}}, R3_{\text{pos}}), (R1_{\text{neg}}, R2_{\text{neg}}, R3_{\text{pos}})\}$$

5. **Are the events "only researcher 1 finds an effect" and "only researcher 3 finds an effect" disjoint and/or independent?**

   The event "only researcher 1 finds an effect" is represented by the elementary event:
   $$(R1_{\text{pos}}, R2_{\text{neg}}, R3_{\text{neg}})$$

   The event "only researcher 3 finds an effect" is represented by the elementary event:
   $$(R1_{\text{neg}}, R2_{\text{neg}}, R3_{\text{pos}})$$

   These two events are **disjoint** because they cannot happen simultaneously (i.e., one requires researcher 1 to find an effect while the other requires researcher 1 not to find an effect).

   Are they independent?

   $0 < 0.04 * 0.96^2 =$
   $\mathbb{P}(\text{only researcher 1 finds an effect}) \neq$
   $\mathbb{P}(\text{only researcher 1 finds an effect}|\text{only researcher 3 finds an effect}) = 0$
   since this is impossible, if only researcher 3 found an effect.