Zusammenfassung
Halbleiter-Schaltungstechnik / Electronic Circuits

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1 diode.

1.1 diode as capacitor

A diode contains a pn-junction, which grows when a reverse voltage is applied. It thereby forms a capacitor, of which the capacity can be altered by a voltage V_R.
Built in potential: $V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2}$
Reverse voltage: V_R

$$C_j = \frac{C_{j0}}{\sqrt{1 - \frac{V_R}{V_0}}}$$
$$C_{j0} = \sqrt{\frac{\epsilon_{si} q}{2} \cdot \frac{N_A N_D}{N_A + N_D} \cdot \frac{1}{V_0}}$$

1.2 *almost* ideal diode (with voltage drop)

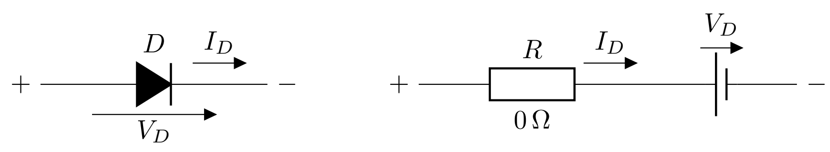


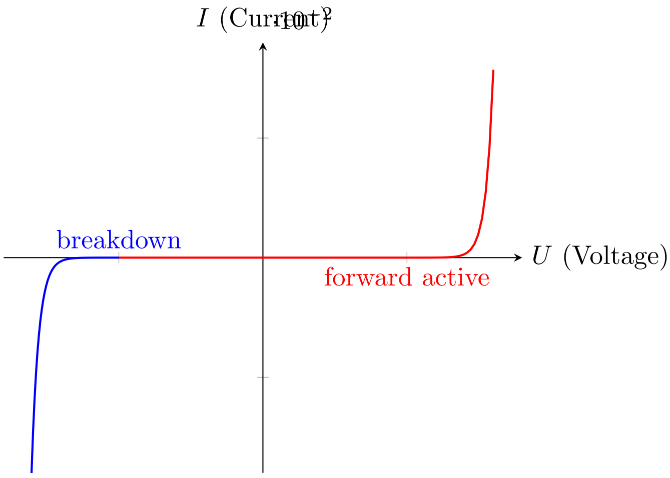
Figure 1: Ideal diode in ON-region: equivalent circuit

$$I_D = \begin{cases} I & V_D > 0 \\ 0 & V_D \leq 0 \end{cases}$$
$$R_D = \begin{cases} 0 & V_D > 0 \\ \infty & V_D \leq 0 \end{cases}$$

A Diode with no voltage drop is called a *ideal* diode.

1.3 breakdown physics

A to high reverse voltage will cause a diode to go into "breakdown". This means that the diode won't block the reverse current anymore but will allow current to flow pretty unhindered.



There are two kinds of breakdown phenomena:

- Zener Breakdown: All the electrons enter breakdown at the same time.
- Avalache Breakdown: A loose electron accelerates and kicks other electrons out of the crystal structure.

1.3.1 zener diode

This is a zener Diode:

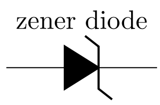


Figure 2: a zener diode

1.4 example 3.29 b

Plot the current through R_1 and D_1 as a function of V_{in}.
Assume a constant-voltage diode model and $V_B = 2\text{V}$

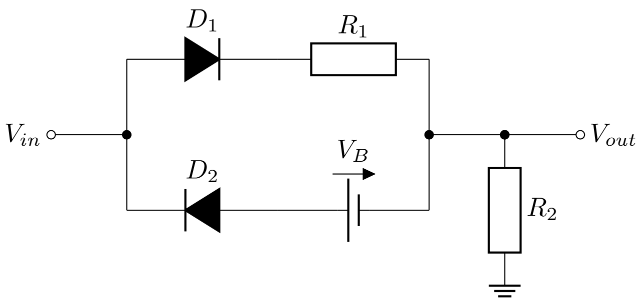


Figure 3: Given Schematic

First, we replace the diodes with their constant-voltage model.

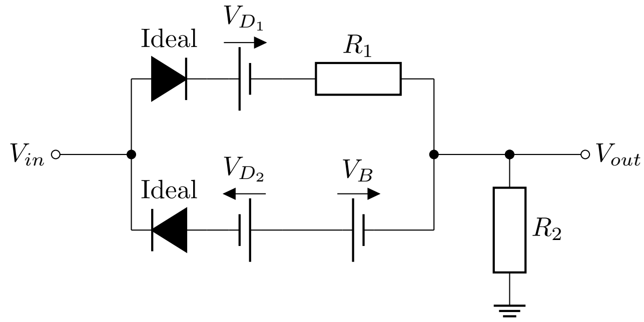


Figure 4: Schematic with Ideal diode model

From here, we can see that there are 2 possible scenarios for the current:

- $V_{in} << V_{out}$: Current goes through R_2 and D_2. The slope is $\frac{V_{in}}{V_{out}} = 1$, with an offset of $-V_B + V_{D_2}$
- $V_{in} >> V_{out}$: Current flows through D_1, R_1 and R_2. The slope of the curve is $\frac{R_2}{R_1+R_2}$ (by the voltage divider rule). The offset of the curve is $-V_{D_1}$.

To summarize:

- $V_{out} = V_{in} - V_B + V_{D_2} \quad V_{in} << V_{out}$
- $V_{out} = (V_{in} - V_{D_1}) \cdot \frac{R_2}{R_1+R_2} \quad V_{in} >> V_{out}$

Note: one may ask why the offset in the second equation is multiplied with the scaling factor $\frac{R_2}{R_1+R_2}$. The short answer to this is that the diode is *in* the voltage divider and therefore influenced by it, but to check this one can connect V_{in} to GND.

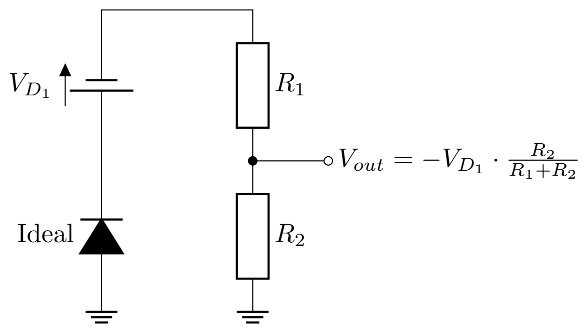


Figure 5: Find the offset
The diode has no effect on the voltages.

Now we need the exact point where the two lines cross to find the transition point. We will from now on assume that both diodes are equivalent and $V_{D_1} = V_{D_2} = V_D$.

$$V_{in} - V_B + V_{D_2} = (V_{in} - V_{D_1}) \cdot \frac{R_2}{R_1 + R_2}$$

$$V_{in} - V_B + V_{D_2} = V_{in} \cdot \frac{R_2}{R_1 + R_2} - V_{D_1} \cdot \frac{R_2}{R_1 + R_2}$$

$$V_{in} = V_{in} \cdot \frac{R_2}{R_1 + R_2} + V_B - \frac{R_2}{R_1 + R_2} \cdot V_D - V_D \qquad \text{move all } V_{in} \text{ to the left}$$

$$V_{in} \cdot \left(1 - \frac{R_2}{R_1 + R_2}\right) = V_B - \left(1 + \frac{R_2}{R_1 + R_2} \cdot\right) V_D \qquad \text{Divide}$$

$$V_{in} = \frac{V_B}{1 - \frac{R_2}{R_1+R_2}} - \frac{(1 + \frac{R_2}{R_1+R_2} \cdot)}{1 - \frac{R_2}{R_1+R_2}} V_D = \frac{V_B}{1 - \frac{R_2}{R_1+R_2}} \frac{R_1 + R_2}{R_1 + R_2} - \frac{(1 + \frac{R_2}{R_1+R_2} \cdot)}{1 - \frac{R_2}{R_1+R_2}} \frac{R_1 + R_2}{R_1 + R_2} V_D$$

$$V_{in} = V_B \frac{(R_1 + R_2)}{R_1 + R_2 - R_2} - V_D \frac{R_1 + R_2 + R_2}{R_1 + R_2 - R_2}$$

$$V_{in} = V_B \frac{R_1 + R_2}{R_1} - V_D \frac{R_1 + 2R_2}{R_1}$$

$$V_{in} = V_B \frac{R_1 + R_2}{R_1} - V_D - V_D \frac{2R_2}{R_1} + V_D - V_D$$

$$V_{in} = \frac{R_1 + R_2}{R_1} (V_B - 2V_D) + V_D$$

Thereby the V_{in} - V_{out} relationship is:

$$V_{out} = \begin{cases} V_{in} - V_B + V_D & V_{in} \leq \frac{R_1+R_2}{R_1}(V_B - 2V_D) + V_D \\ \frac{R_2}{R_1+R_2}(V_{in} - V_D) & V_{in} > \frac{R_1+R_2}{R_1}(V_B - 2V_D) + V_D \end{cases}$$

Plotting the function is left as an exercise to the reader, as it is trivial :P
Jokes aside, I don't really have time for this one.

1.5 forward active mode

Resistors link voltage and current as a linear function $V = f(I) = I \cdot R$ and $I = f(V)^{-1} = V/R$. That function isn't linear anymore in semiconductors.
Diode equation:

$$I_D = I_S \cdot \left(e^{\frac{V_D}{V_T}} - 1\right) \approx I_S \cdot e^{\frac{V_D}{V_T}}$$

$$V_D = V_T \cdot \ln\left(\frac{I_D}{I_S}\right)$$

1.6 small signal model

first, calculate operating point!
After that, we calculate the derivative of the I-V function to get a linear approximation. As long as we stay close to the original operation point (that's why it's called *small* signal model), our approximation is relatively good and we can model the diode as normal (linear) resistor r_D. Don't use a offsets voltage, that's wrong.

$$G \quad \hat{=} \quad [\text{conductance}] = \frac{1}{r_D} = \frac{dI_D}{dV_D} = \frac{1}{V_T} \cdot \left(I_S \cdot e^{\frac{V_D}{V_T}}\right) = \frac{I_D}{V_T}$$

$$r_D = G^{-1} = \frac{V_T}{I_D}$$

1.7 example 2.29

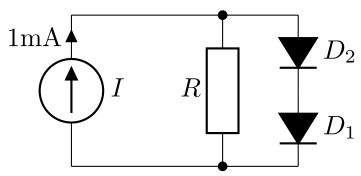


Figure 6: Problem 2.29

The current source supplies 1mA.
We want to find the resistor value R, such that $I_R = 0.5\text{mA}$.
Assume that both diodes are equivalent, $I_S = 10^{-16}\text{A}$.

$$
\begin{aligned}
\text{by using Kirchhoff's law: } I &= I_R + I_{D_1,D_2} \\
\text{and: } I_{D_1} &= I_{D_2} \\
\rightarrow I_{D_1,D_2} &= I - I_R = 1\text{mA} - 0.5\text{mA} = 0.5\text{mA} \\
V_R &= V_{D_1} + V_{D_2} \\
\text{U-I function of a diode: } V_{D_1} + V_{D_2} &= 2 \cdot V_T \cdot \ln\left(\frac{I_D}{I_S}\right) = 2 \cdot V_T \cdot \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right) \\
R &= \frac{V_R}{I_R} = \frac{2 \cdot V_T \cdot \ln\left(\frac{0.5\text{mA}}{10^{-16}\text{A}}\right)}{0.5\text{mA}} = 2.87\text{kOhm}
\end{aligned}
$$

2 rectifier.

2.1 half bridge rectifier

Input: a sinusodial voltage V_{in} with frequency f_{in}.

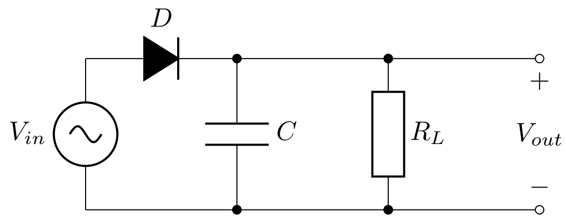


Figure 7: half bridge rectifier

We assume the voltage in the capacitor decreases linearly with time and current (In reality it decreases exponentially) ($I_{R_L} = \frac{V_C}{R_L}$). The voltage ripple ($V_R$) is the max difference of V_{in} and V_{out}.

$$
\begin{aligned}
V_{out}(t) &\approx (V_{in} - V_{D_{on}}) \cdot \left(1 - \frac{t}{R_L C}\right) \\
V_{out}(t=0) - V_{out}(t=T) = V_R &\approx \frac{V_{in} - V_{D_{on}}}{R_L C \cdot f_{in}}
\end{aligned}
$$

2.2 full bridge rectifier

In a full bridge rectifier, the input voltage has to pass trough 2 diodes, thereby doubling the voltage drop. The discharge time gets halved and therefore the voltage ripple too.

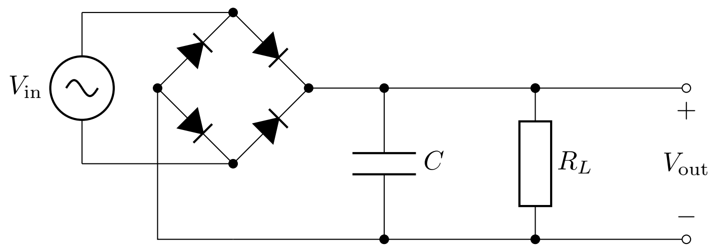


Figure 8: full bridge rectifier

$$V_R \approx \frac{1}{2} \frac{(V_{\text{in}} - 2 \cdot V_{D_{on}})}{R_L C \cdot f_{in}}$$

3 transistor.

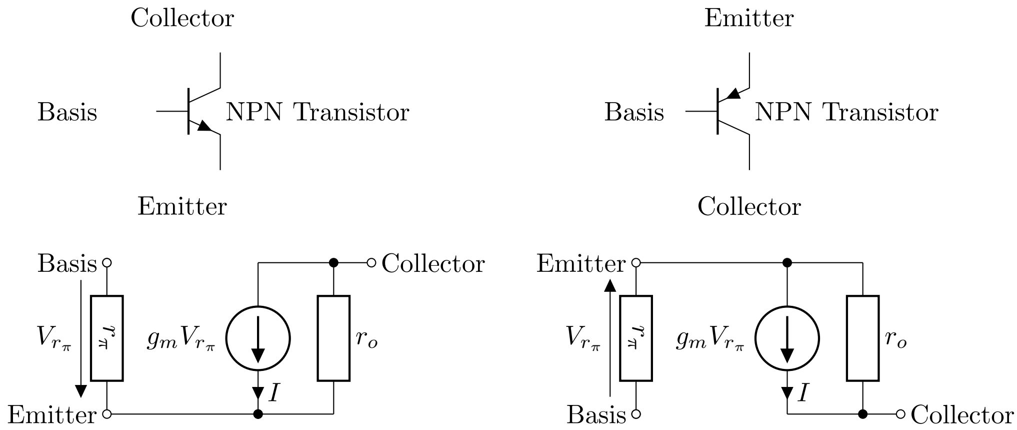


Figure 9: transistors and their small-signal equivalents

3.1 forward active region

$V_{CE} > V_{BE} > 0$
A transistor will behave as a voltage dependent current source (or a linear current amplifier, but it's easier to adjust voltage then current). The current I_C is exponentially linked to the voltage between base and emitter V_{BE}, while the current I_B and I_C are linearly linked.

$$I_C = I_S \cdot e^{\frac{V_{BE}}{V_T}}$$
$$I_C = \beta \cdot I_B$$
$$I_E = I_C + I_B = (1+\beta) \cdot I_B = \left(\frac{1}{\beta} + 1\right) \cdot I_C.$$
$$V_{BE} = V_T \cdot \ln\left(\frac{I_C}{I_S}\right)$$

3.1.1 small signal model

V_{CE} is constant.
We take the derivative of the voltage-current function $I_C = f(V_{BE})$ to get a linear approximation of the transconductance (current per voltage).

$$g_m \;\widehat{=}\; [\text{transconductance}] = \frac{dI_C}{dV_{BE}} = \frac{1}{V_T} \cdot \left(I_S \cdot e^{\frac{V_{BE}}{V_T}}\right) = \frac{I_C}{V_T} \tag{1}$$

$$\Delta I_C = g_m \cdot \Delta V_{BE}$$
$$\Delta I_B = \frac{g_m}{\beta} \cdot \Delta V_{BE}$$

We now have a approximation $I_C = g_m \cdot V_{BE}$, and with that, we can model the Basis current I_B as a linear function of V_{BE}.
We know about the linear dependence $I_C = \beta \cdot I_B$.

$$\rightarrow I_B = \frac{\beta}{I_C} = \frac{\beta}{V_{BE} \cdot g_m}$$

We can find a input resistor r_π to model this behavior in a circuit.

$$r_\pi = \frac{V_{BE}}{I_B} = \frac{\beta}{g_m} \tag{2}$$

3.2 eary effect

In all previous examples, I_C was independent of V_{CE}, but this isn't true in reality. $\rightarrow I_C = f(V_{BE}, V_{CE})$
The early voltage is V_A. This is a kind of scaling factor for the V_{CE} influence.

$$I_C = \left(I_S \cdot e^{\frac{V_{BE}}{V_T}}\right) \cdot \left(1 + \frac{V_{CE}}{V_A}\right)$$

3.2.1 small signal model

We take the derivative of I_C in respect to V_{CE} to approximate a linear dependence, which we model as a resistor r_o.

$$g_o \quad \hat{=} \quad [\text{conductance}] = \frac{dI_C}{dV_{CE}} = \frac{dI_C}{dV_{CE}}\left(\frac{V_{CE}\cdot I_C}{V_A} + I_C\right) = \frac{I_C}{V_A}$$

$$r_o = \frac{dV_{CE}}{dI_C} = \frac{V_A}{I_C}$$

3.3 example 4.55

Consider this circuit, where $I_{S_1} = 3I_{S_2} = 5\cdot 10^{-16}A$, $\beta_1 = 100$, $\beta_2 = 50$, $V_A = \infty$, and $R_C = 500\Omega$.

a. What is the maximum allowable value of V_{in}, such that the forward collector-basis bias of Q_2 does not exceed $0.2V$?

b. Construct the small-signal equivalent circuit.

$V_A = \infty \rightarrow$ that means we can neglect the early effect.
You may be tempted to draw a voltage arrow for the Q_2 transostor from collector to base, but this is actually wrong. Q_2 is a NPN transistor. That means that the collector is made out of N-type material, and the base is made out of P-type material. So the PN-junction actually goes from base to collector and if we want to forward bias it by $\max 0.2V$, then the **base is at a higher potential than the collector**.

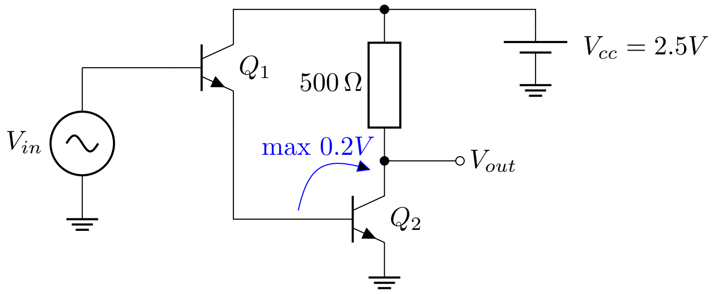


Figure 10: given circuit

The first step is to calculate this circuit is to use Kirchhoffs law to set up the equation. It is also important to remember that in a transistor, the base current and the collector current combine to form the emitter current.

$$I_E = I_B + \underbrace{I_C}_{=\beta\cdot I_B} = (1+\beta)\cdot I_B = \left(1+\frac{1}{\beta}\right)\cdot I_C$$

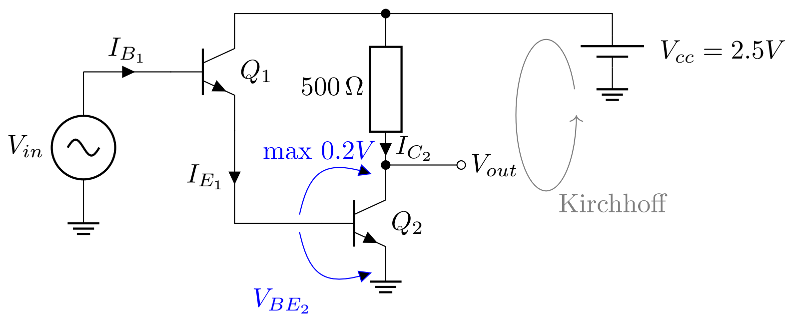


Figure 11: use Kirchhoffs law

$$
\begin{aligned}
V_{cc} + \underbrace{V_{BC_2}}_{=0.2V} &= V_{R_C} + V_{BE_2} \quad \text{using Kirchhoffs law} \\
V_{BE_2} &= V_{cc} + 0.2V - I_{C_2} R_C \\
V_T \log\left(\frac{I_{C_2}}{I_{S_2}}\right) &= V_{cc} + 0.2V - I_{C_2} R_C \quad \rightarrow \text{use calculator to solve for} I_{C_2}! \\
\rightarrow\text{calculator: } I_{C_2} &= 3.80\text{mA} \\
V_{BE_2} = V_T \log\left(\frac{I_{C_2}}{I_{S_2}}\right) &= 799.7\text{mV} \quad \rightarrow \text{we will need this later} \\
I_{B_2} = \frac{I_{C_2}}{\beta_2} &= I_{E_1} = I_{C_1}\left(1 + \frac{1}{\beta_1}\right) \\
I_{C_1} = I_{C_2} \cdot \frac{1}{\left(1+\frac{1}{\beta_1}\right)\cdot \beta_2} &= 75.2\mu\text{A} \\
V_{BE_1} = V_T \log\left(\frac{I_{C_1}}{I_{S_1}}\right) &= 669.2\text{mV} \\
V_{in} = V_{BE_1} + V_{BE_2} &= 1.469\text{mV}
\end{aligned}
$$

Therefore, $V_{in} \leq 1.469\text{mV}$

Now, lets construct the small signal equivalent model. Note: $V_A = \infty$, therefore we can ignore all r_o resistors. We cancel all static voltage sources and replace all transistor with their small signal equivalent circuit.

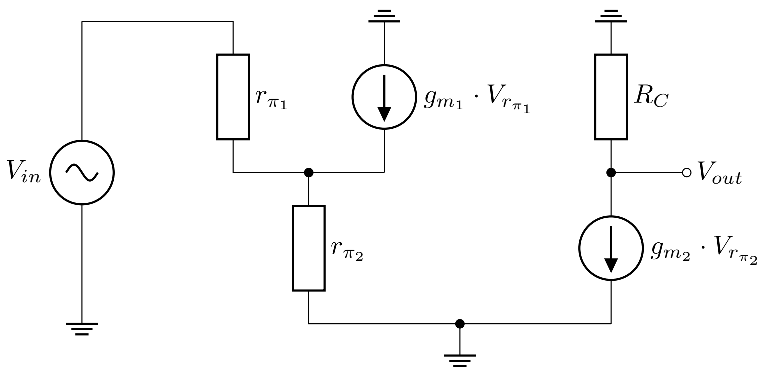


Figure 12: small signal equivalent circuit

Now we can calculate the all the values using the formulas (1) and (2).

$$
\begin{aligned}
g_{m_1} &= \frac{I_{C_1}}{V_T} = 2.895\,\text{mS} \\
g_{m_2} &= \frac{I_{C_2}}{V_T} = 146.2\,\text{mS} \\
r_{\pi_1} &= \frac{\beta_1}{g_{m_1}} = 34.55\,\text{k}\Omega \\
r_{\pi_2} &= \frac{\beta_2}{g_{m_2}} = 342.1\,\Omega
\end{aligned}
$$

3.4 example 5.47 c

Compute the voltage Gain and I/O impedances.
$V_A = \infty$

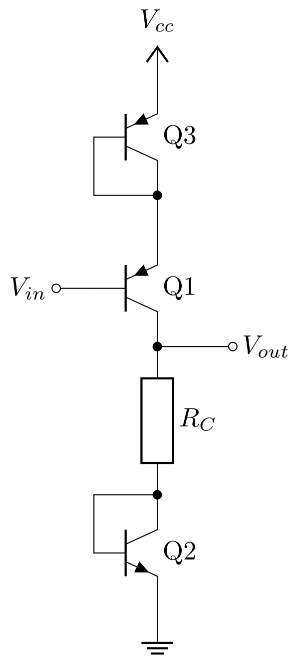


Figure 13: Biploar Amplifer

First, lets transform this circuit to the small signal equivalent.

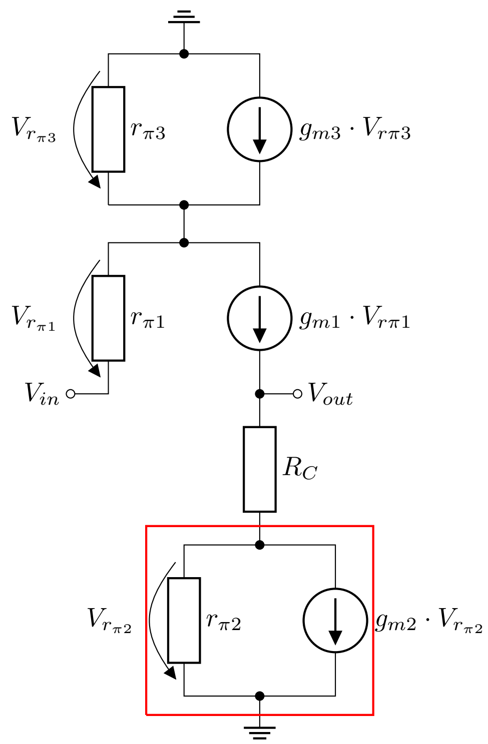


Figure 14: Small signal model of bipolar amplifier

You might notice the parallel current source and resistor in the red box. We will replace it with a equivalent resistor. But what is its equivalent resistance?

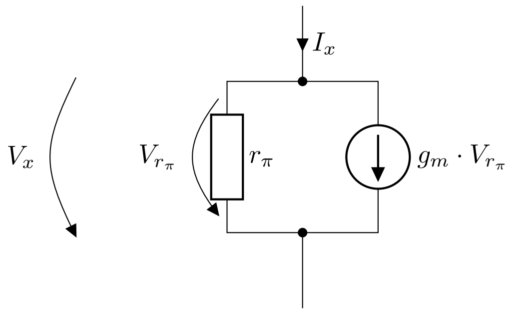


Figure 15: Resistance of a resistor // current source

To calculate the resistance we supply a test current I_x and check the text voltage V_x.

$$R_x = \frac{V_x}{I_x}$$
$$V_x = V_{r_\pi}$$
$$I_x = \frac{V_x}{r_\pi} + g_m \cdot V_x$$
$$R_x = \frac{V_x}{\frac{V_x}{r_\pi} + g_m \cdot V_x} = \frac{1}{\frac{1}{r_\pi} + g_m} = r_\pi /\!/ \frac{1}{g_m}$$

Now we can see a more simplified circuit!

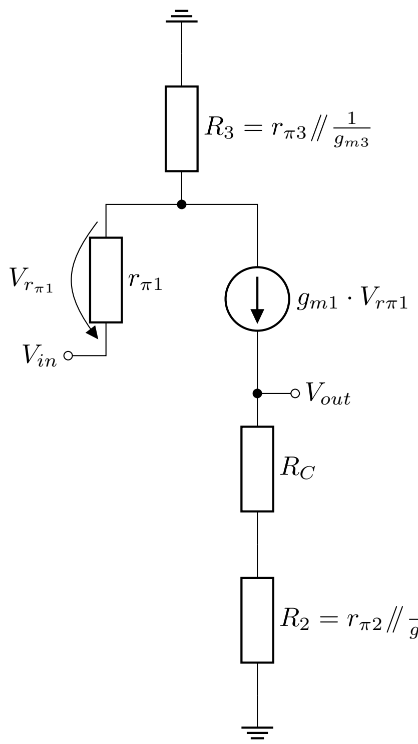


Figure 16: Simplified small signal model of bipolar amplifier

Now we can easely see that $R_{out} = \underline{\underline{R_C + r_{\pi 2} /\!/ \frac{1}{g_{m2}}}}$. To calculate the amplification A_v, it is easier to first find R_{in}.
To find R_{in}, we supply a test current and measure the test voltage.

$$R_{in} = \frac{V_x}{I_x}$$
$$V_x = I_x \cdot r_{\pi 2} + I_x \cdot (1+\beta) \cdot R_3$$
$$R_{in} = \frac{I_x \cdot r_{\pi 2} + I_x \cdot (1+\beta) \cdot R_3}{I_x} = r_{\pi 2} + (1+\beta) R_3 \approx \underline{\underline{r_{\pi 2} + \beta(r_{\pi 3} /\!/ \frac{1}{g_{m3}})}}$$

Well, the amplification is $\frac{V_{out}}{V_{in}}$.

$$V_{out} = g_{m1} V_{r\pi 1} \cdot (R_C + R_2) \qquad \text{Just I} \times \text{R}$$

$$V_{r\pi 1} = V_{in} \cdot \underbrace{\frac{r_{\pi 1}}{R_{in}}}_{\text{Voltage divider}} = \frac{r_{\pi 1}}{r_{\pi 1} + \beta (r_{\pi 3} /\!/ \frac{1}{g_{m3}})}$$

$$V_{out} = V_{in} \cdot \frac{g_{m1} r_{\pi 1} \cdot (R_C + r_{\pi 2} /\!/ \frac{1}{g_{m2}})}{r_{\pi 1} + \beta (r_{\pi 3} /\!/ \frac{1}{g_{m3}})} \cdot \frac{\frac{1}{r_{\pi 1}}}{\frac{1}{r_{\pi 1}}} \qquad \text{notice that } g_m = \frac{\beta}{r_\pi}$$

$$\frac{V_{out}}{V_{in}} = \frac{g_{m1}(R_C + r_{\pi 2} /\!/ \frac{1}{g_{m2}})}{1 + g_{m1} \cdot (r_{\pi 3} /\!/ \frac{1}{g_{m3}})} = \underline{\underline{\frac{R_C + r_{\pi 2} /\!/ \frac{1}{g_{m2}}}{\frac{1}{g_{m1}} + r_{\pi 3} /\!/ \frac{1}{g_{m3}}}}}$$

3.5 saturation mode

3.5.1 soft saturation mode

$V_{BE} > V_{CE}$
$V_{BC} < 400$ mV

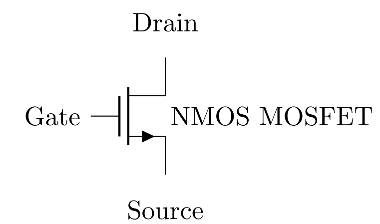
3.5.2 deep saturation

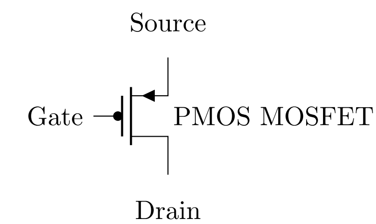
$V_{BE} > V_{CE}$
$V_{BC} > 400$ mV

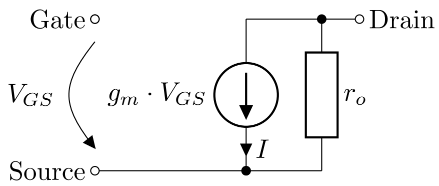
3.6 bipolar amplifiers

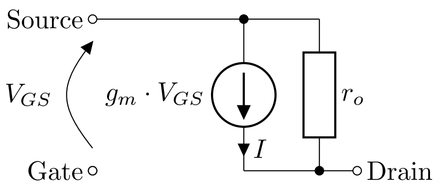
Any capacitors are a open circuit in DC-analysis, and a short circuit in AC-analysis!

4 MOSFET









4.1 cutoff region ($V_{GS} \leq V_{th}$)

The MOSFET device is off, no current flows.

4.2 triode/linear/ohmic region

$V_{DS} < (V_{GS} - V_{th})$
Deep saturation region: $V_{DS} \ll 2(V_{GS} - V_{th})$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot (2(V_{GS} - V_{th}) V_{DS} - V_{DS}^2)$$

$(V_{DS} = V_{GS} - V_{th}) \rightarrow$ The maximum achievable current is:

$$I_{D_{\max}} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \cdot (V_{GS} - V_{th})$$

4.2.1 small signal model

The MOSFET can be modelled as a voltage-dependent resistor, with resistance R_{on}.

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{th})}$$

4.3 saturation region

$V_{DS} > (V_{GS} - V_{th})$

4.3.1 channel-length modulation

λ is called the channel-length modulation coefficient. This effect is similar to the early effect in transistors. It artificially lowers the channel length in the MOSFET by "pinching off" the current I_D, when V_{DS} gets to big. λ is inversely proportional to L.

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2 \cdot (1 + \lambda \cdot V_{DS})$$

4.3.2 small signal model

The MOSFET can be modeled as a voltage-dependent current source (an increase in V_{DS} does not yield an increase in I_D) with transconductance g_m.

$$g_m = \frac{dI_D}{dV_{GS}} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th}) = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D} = \frac{2 \cdot I_D}{(V_{GS} - V_{th})}$$

Taking the channel-length modulation effect in consideration, we get:

$$r_o = \frac{dV_{DS}}{dI_D} = \frac{1}{\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{th})^2 \cdot \lambda} \approx \frac{1}{\lambda \cdot I_D}$$

4.3.3 Velocity Saturation

Normally, the electric field $\vec{E}$ accelerates electrons to a velocity v_x, but in reality, a certain velocity v_{max} can't be exceeded. ($\rightarrow$ We replace v_x with v_{max})

$$\begin{aligned} I &= \frac{v_x \cdot Q}{L} = \frac{v_{max} \cdot W \cdot L \cdot C_{ox} \cdot (V_{GS} - \frac{1}{2}V_{DS} - V_{th})}{L} \\ &= v_{max} \cdot W \cdot C_{ox} \cdot (V_{GS} - \frac{1}{2}V_{DS} - V_{th}) \end{aligned}$$

The resulting equation yields: (I don't know where the $-\frac{1}{2}V_{DS}$ went)

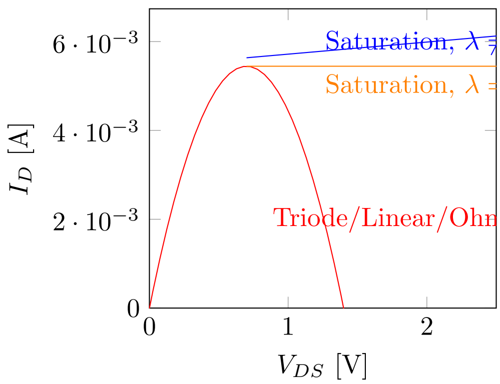
$$I_D = v_{max} \cdot W \cdot C_{ox} \cdot (V_{GS} - V_{th})$$

4.4 body effect

4.5 figure: different operation regions

This figure plots the different equations for I_D in the different operation regions.

Figure 17: I-V Plot for MOSFET in different operational regions

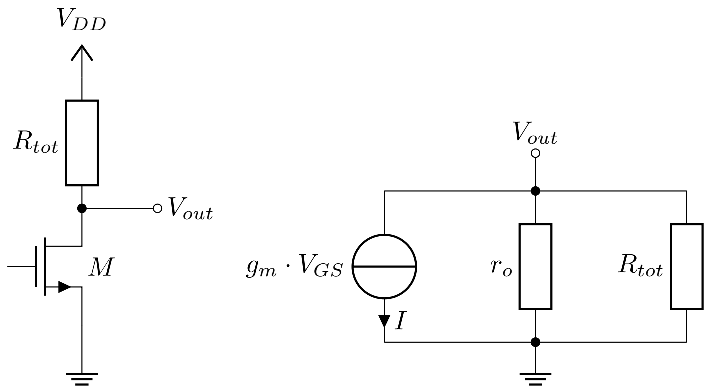


5 MOSFET: tricky situations.

5.1 The $R_{tot} /\!/ I_q$ technique

5.1.1 common source

A lot of MOSFET amplifier circuits can be transformed into the following form:



$$V_{out} = -(R_{tot} /\!/ r_o) \cdot g_m \cdot V_{GS}$$

5.1.2 common source & source follower

With degeration!
In a common source circuit, we search for V_{out} and in a source follower, we search for V_{R_S}.

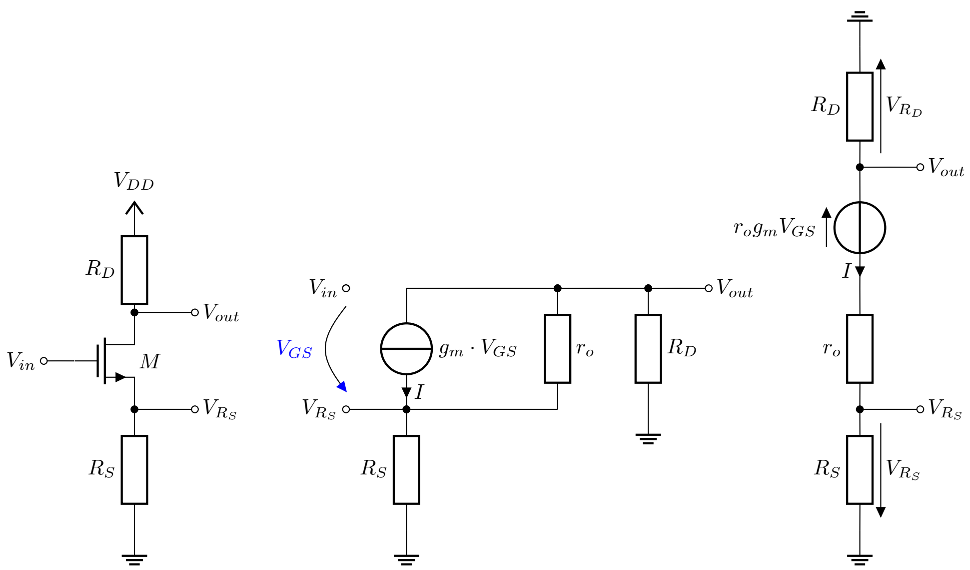


Figure 18: What is V_{out}?

We transform the current source // r_o into a equivalent Thevenin voltage.

$$
\begin{aligned}
V_{out} =& \underbrace{-g_m V_{GS} r_o}_{\text{Thevenin voltage}} \underbrace{\frac{R_D}{R_D + r_o + R_S}}_{\text{voltage divider}} = V_{R_D} \\
V_{GS} =& V_{in} - V_{R_S} \\
V_{R_S} =& \underbrace{g_m V_{GS} r_o}_{\text{Thevenin voltage}} \underbrace{\frac{R_S}{R_D + r_o + R_S}}_{\text{voltage divider}} \\
V_{in} =& V_{GS} \cdot (1 + \frac{g_m r_o R_S}{R_D + r_o + R_S}) && \text{solving for } V_{GS} \\
\rightarrow V_{GS} =& \frac{V_{in}}{1 + \frac{g_m r_o R_S}{R_D + r_o + R_S}} \\
V_{out} =& \frac{V_{in}}{1 + \frac{g_m r_o R_S}{R_D + r_o + R_S}} \frac{-g_m r_o R_D}{R_D + r_o + R_S} && \text{putting in } V_{GS} \\
V_{out} =& V_{in} \frac{-g_m r_o R_D}{R_D + r_o + R_S + g_m r_o R_S} \\
V_{R_S} =& \frac{V_{in}}{1 + \frac{g_m r_o R_S}{R_D + r_o + R_S}} \frac{g_m r_o R_S}{R_D + r_o + R_S} && \text{putting in } V_{GS} \\
V_{R_S} =& V_{in} \frac{g_m r_o R_S}{R_D + r_o + R_S + g_m r_o R_S} && \text{notice that } V_{R_S} < V_{in}
\end{aligned}
$$

5.1.3 finding R_{tot}

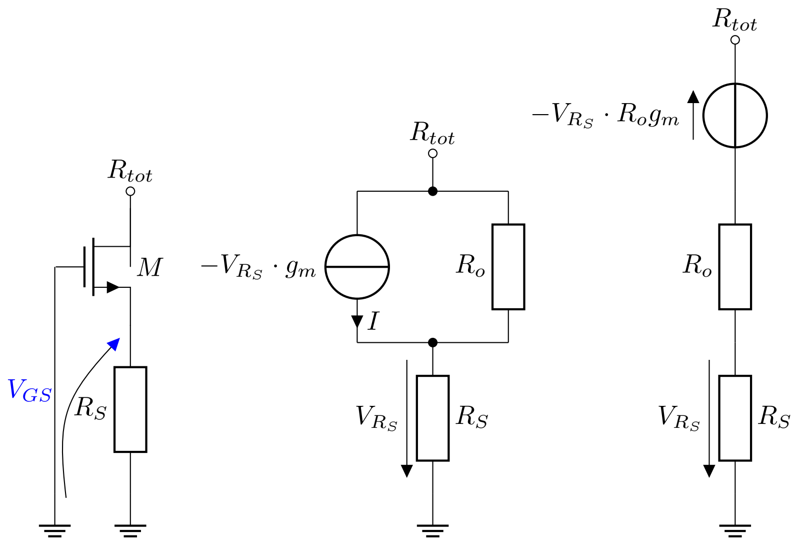


Figure 19: What is the resistance R_{tot}?
$V_{GS} = 0 - V_{R_S}$

The left Norton circuit (current source // resistor) can be replaced by a equivalent Thevenin circuit. The resulting total resistance can be calculated by a test-current I_X.

$$
\begin{aligned}
V_X &= - - V_{R_S} \cdot g_m R_o + I_X R_o + I_X R_S \\
&= I_X R_S \cdot g_m R_o + I_X R_o + I_X R_S \\
R_{tot} &= \frac{V_X}{I_X} = R_S + R_S g_m R_o + R_o
\end{aligned}
$$

6 OP-AMP's

Its very important to differentiate between a *open loop* or *closed loop* amplifier.

6.1 open loop.

Open loop amplifiers are basically useless!

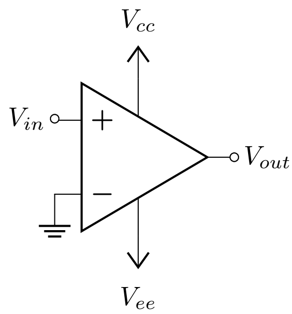


Figure 20: open loop op-amp

The op-amp amplifies any voltage difference of V_{in} and GND with ∞ amplification. However, this is not possible in reality, so V_{out} runs into the supply voltage.

$$V_{out} = \begin{cases} V_{cc} & V_{in} > 0 \\ 0 & V_{in} = 0 \\ V_{ee} & V_{in} < 0 \end{cases}$$

We don't always draw the supply voltages, but don't forget that V_{out} can never be greater than the supply voltage.

6.2 closed loop.

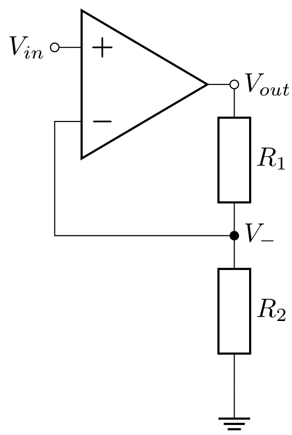


Figure 21: non-inverting closed loop op-amp

$$\begin{aligned}
V_{out} =& A_0 \cdot (V_{in} - V_-) \\
V_- =& V_{out} \cdot \frac{R_2}{R_1 + R_2} \\
V_{out} =& A_0 V_{in} - A_0 V_{out} \cdot \frac{R_2}{R_1 + R_2} \\
A_0 V_{in} =& V_{out} \cdot (1 + A_0 \cdot \frac{R_2}{R_1 + R_2}) \\
V_{out} =& \frac{A_0 V_{in}}{1 + A_0 \cdot \frac{R_2}{R_1 + R_2}} \cdot \frac{R_1 + R_2}{R_1 + R_2} \\
V_{out} =& V_{in} \cdot \frac{A_0 (R_1 + R_2)}{R_1 + R_2 + A_0 R_2} \\
\lim_{A_0 \to \infty} V_{out} =& V_{in} \frac{R_1 + R_2}{R_2} = V_{in} \cdot (1 + \frac{R_1}{R_2})
\end{aligned}$$

7 current mirrors.

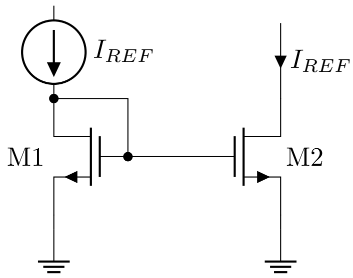


Figure 22: MOSFET current mirror

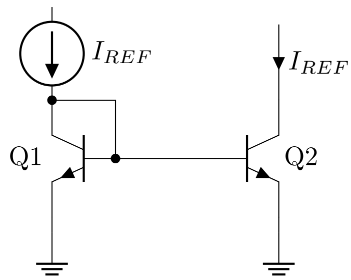


Figure 23: transistor current mirror

8 end