Algebra is $Cl_{1,4}(\mathbb{R})$
The five dimensions are (t, x, y, z, w).
It defines two sets of quaternions with one imaginary unit.
Signature is (+ - - - -)
CoQuaternions $(1, \mathbf{i}, \mathbf{j}, \mathbf{k})$ http://en.wikipedia.org/wiki/Split-quaternion

$$\mathbf{i} = \gamma_t$$

$$\mathbf{j} = \gamma_t\gamma_x\gamma_y\gamma_z$$

$$\mathbf{k} = \gamma_x\gamma_y\gamma_z$$

$$\mathbf{i}^2 = 1$$

$$\mathbf{j}^2 = -1$$

$$\mathbf{k}^2 = 1$$

$$\mathbf{ij} = \gamma_x\gamma_y\gamma_z$$

$$\mathbf{ji} = -\gamma_x\gamma_y\gamma_z$$

$$\mathbf{jk} = \gamma_t$$

$$\mathbf{kj} = -\gamma_t$$

$$\mathbf{ki} = -\gamma_t\gamma_x\gamma_y\gamma_z$$

$$\mathbf{ik} = \gamma_t\gamma_x\gamma_y\gamma_z$$

$$\mathbf{ijk} = 1$$

Quaternions $(1, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ http://en.wikipedia.org/wiki/Quaternion

$$\boldsymbol{i} = \gamma_y\gamma_z$$

$$\boldsymbol{j} = -\gamma_x\gamma_z$$

$$\boldsymbol{k} = \gamma_x\gamma_y$$

$$\boldsymbol{i}^2 = -1$$

$$\boldsymbol{j}^2 = -1$$

$$\boldsymbol{k}^2 = -1$$

$$\boldsymbol{ij} = \gamma_x\gamma_y$$

$$\boldsymbol{ji} = -\gamma_x\gamma_y$$

$$\boldsymbol{jk} = \gamma_y\gamma_z$$

$$\boldsymbol{kj} = -\gamma_y\gamma_z$$

$$\boldsymbol{ki} = -\gamma_x\gamma_z$$

$$\boldsymbol{ik} = \gamma_x\gamma_z$$

$$\boldsymbol{ijk} = -1$$

Imaginary unit i

$$i = \gamma_w$$
$$i^2 = -1$$

Gradient definition

$$\nabla = (\gamma_t \frac{\partial}{\partial t} + \gamma_x \frac{\partial}{\partial x} + \gamma_y \frac{\partial}{\partial y} + \gamma_z \frac{\partial}{\partial z} + \gamma_w \frac{\partial}{\partial w})$$

http://www.mrao.cam.ac.uk/ clifford/ptIIIcourse/course99/handouts/hout07.ps.gz
Wave : K is a constant and f is a function of (t, x, y, z, w)

$$\psi = Ke^f$$

Left and right derivatives
http://geocalc.clas.asu.edu/pdf/NFMPchapt1.pdf see chapter 1-3 Differentiation by vectors

$$\nabla\psi = \nabla(Ke^f)$$
$$\nabla\psi = (\nabla K)e^f + K\nabla(e^f)$$
$$\nabla K = 0$$
$$\nabla\psi_L = K(\nabla f)e^f$$
$$\nabla\psi_R = (\nabla f)Ke^f$$

The following symbols are defined :
$E \in \mathbb{R}$
$m \in \mathbb{R}$
$\mathbf{p}$ is defined with $p_x, p_y, p_z \in \mathbb{R}$

$$\mathbf{p} = p_x\boldsymbol{i} + p_y\boldsymbol{j} + p_z\boldsymbol{k}$$
$$\mathbf{p} = p_z\gamma_x\gamma_y - p_y\gamma_x\gamma_z + p_x\gamma_y\gamma_z$$

Exponential function f

$$f_1 = -mw + (p_y y + p_x x + p_z z - Et)\,\gamma_w$$
$$f_2 = mw + (-p_z z - p_y y - p_x x - Et)\,\gamma_w$$
$$f_3 = mw + (p_y y + p_x x + p_z z - Et)\,\gamma_w$$
$$f_4 = -mw + (-p_z z - p_y y - p_x x - Et)\,\gamma_w$$

Gradient for f

$$\nabla f_1 = m\gamma_w - E\gamma_t\gamma_w - p_x\gamma_x\gamma_w - p_y\gamma_y\gamma_w - p_z\gamma_z\gamma_w$$
$$\nabla f_2 = -m\gamma_w - E\gamma_t\gamma_w + p_x\gamma_x\gamma_w + p_y\gamma_y\gamma_w + p_z\gamma_z\gamma_w$$

$$\nabla f_3 = -m\gamma_w - E\gamma_t\gamma_w - p_x\gamma_x\gamma_w - p_y\gamma_y\gamma_w - p_z\gamma_z\gamma_w$$
$$\nabla f_4 = m\gamma_w - E\gamma_t\gamma_w + p_x\gamma_x\gamma_w + p_y\gamma_y\gamma_w + p_z\gamma_z\gamma_w$$

Square of the gradient

$$\nabla f_1^2 = -m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2$$
$$\nabla f_2^2 = -m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2$$
$$\nabla f_3^2 = -m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2$$
$$\nabla f_4^2 = -m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2$$

Dirac

$$http://en.wikipedia.org/wiki/Dirac\_equation$$
$$0 = (\gamma_0\frac{\partial}{\partial t} + \gamma_1\frac{\partial}{\partial x} + \gamma_2\frac{\partial}{\partial y} + \gamma_3\frac{\partial}{\partial z} + im)\psi$$

With the above gradients, identify the Dirac algebra aka gamma matrices

$$\gamma_0 = \gamma_t\gamma_w$$
$$-i\mathbf{i} = \gamma_t\gamma_w$$
$$\gamma_0^2 = 1$$
$$\gamma_1 = -\gamma_x\gamma_w$$
$$-i\boldsymbol{i}\mathbf{k} = -\gamma_x\gamma_w$$
$$\gamma_1^2 = -1$$
$$\gamma_2 = -\gamma_y\gamma_w$$
$$-i\boldsymbol{j}\mathbf{k} = -\gamma_y\gamma_w$$
$$\gamma_2^2 = -1$$
$$\gamma_3 = -\gamma_z\gamma_w$$
$$-i\boldsymbol{k}\mathbf{k} = -\gamma_z\gamma_w$$
$$\gamma_3^2 = -1$$
$$\gamma_5 = -\gamma_t\gamma_x\gamma_y\gamma_z\gamma_w$$
$$-i\mathbf{j} = -\gamma_t\gamma_x\gamma_y\gamma_z\gamma_w$$
$$\gamma_5^2 = 1$$

Substitution in Dirac equation

$$0 = (-i\mathbf{i}\frac{\partial}{\partial t} - i\boldsymbol{i}\mathbf{k}\frac{\partial}{\partial x} - i\boldsymbol{j}\mathbf{k}\frac{\partial}{\partial y} - i\boldsymbol{k}\mathbf{k}\frac{\partial}{\partial z} + im)\psi$$

Factorization and multiplication

$$0 = \mathbf{j}i(\mathbf{k}\frac{\partial}{\partial t} + \boldsymbol{i}\mathbf{i}\frac{\partial}{\partial x} + \boldsymbol{j}\mathbf{i}\frac{\partial}{\partial y} + \boldsymbol{k}\mathbf{i}\frac{\partial}{\partial z} - \mathbf{j}m)\psi$$

Simple Constants K (exactly the gradient)

$$K_1 = \mathbf{j}i(\mathbf{k}E - \mathbf{j}m + \mathbf{ip})$$

$$K_2 = \mathbf{j}i(\mathbf{k}E + \mathbf{j}m - \mathbf{ip})$$

$$K_3 = \mathbf{j}i(\mathbf{k}E + \mathbf{j}m + \mathbf{ip})$$

$$K_4 = \mathbf{j}i(\mathbf{k}E - \mathbf{j}m - \mathbf{ip})$$

Mixed Constants A (built from simple constants and the coquaternions)

L_1

$$\begin{pmatrix} L_1^1 \\ L_1^2 \\ L_1^3 \\ L_1^4 \end{pmatrix} = \begin{pmatrix} +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} \\ +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} \\ -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} \\ -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} \end{pmatrix}$$

L_2

$$\begin{pmatrix} L_2^1 \\ L_2^2 \\ L_2^3 \\ L_2^4 \end{pmatrix} = \begin{pmatrix} +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} \\ +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} \\ -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} \\ -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} \end{pmatrix}$$

L_3

$$\begin{pmatrix} L_3^1 \\ L_3^2 \\ L_3^3 \\ L_3^4 \end{pmatrix} = \begin{pmatrix} +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} \\ +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} \\ -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} \\ -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} \end{pmatrix}$$

L_4

$$\begin{pmatrix} L_4^1 \\ L_4^2 \\ L_4^3 \\ L_4^4 \end{pmatrix} = \begin{pmatrix} +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} \\ +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} \\ -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} \\ -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} \end{pmatrix}$$

R_1

$$\begin{pmatrix} R_1^1 \\ R_1^2 \\ R_1^3 \\ R_1^4 \end{pmatrix} = \begin{pmatrix} +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 \\ +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 \\ -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 \\ -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 \end{pmatrix}$$

R_2

$$\begin{pmatrix} R_2^1 \\ R_2^2 \\ R_2^3 \\ R_2^4 \end{pmatrix} = \begin{pmatrix} +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 \\ +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 \\ -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 \\ -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 \end{pmatrix}$$

R_3

$$\begin{pmatrix} R_3^1 \\ R_3^2 \\ R_3^3 \\ R_3^4 \end{pmatrix} = \begin{pmatrix} +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 \\ +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 \\ -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 \\ -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 \end{pmatrix}$$

R_4

$$\begin{pmatrix} R_4^1 \\ R_4^2 \\ R_4^3 \\ R_4^4 \end{pmatrix} = \begin{pmatrix} +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 \\ +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 \\ -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 \\ -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 \end{pmatrix}$$

$L_i^j K_i = 0$
$K_i R_i^j = 0$
$L_i^j R_i^j = 0$

Details about full solutions (built from mixed constants and the imaginary unit)
ψ_{L1}

$$\psi^1_{L1} \equiv (L^1_1 + iL^1_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^1_{L1}K_1 = 0$$
$$\psi^2_{L1} \equiv (L^1_1 - iL^1_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^2_{L1}K_1 = 0$$
$$\psi^3_{L1} \equiv (L^1_1 + iL^2_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^3_{L1}K_1 = 0$$
$$\psi^4_{L1} \equiv (L^1_1 - iL^2_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^4_{L1}K_1 = 0$$
$$\psi^5_{L1} \equiv (L^1_1 + iL^3_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^5_{L1}K_1 = 0$$
$$\psi^6_{L1} \equiv (L^1_1 - iL^3_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^6_{L1}K_1 = 0$$
$$\psi^7_{L1} \equiv (L^1_1 + iL^4_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^7_{L1}K_1 = 0$$
$$\psi^8_{L1} \equiv (L^1_1 - iL^4_1) = +K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^8_{L1}K_1 = 0$$
$$\psi^9_{L1} \equiv (L^2_1 + iL^1_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^9_{L1}K_1 = 0$$
$$\psi^{10}_{L1} \equiv (L^2_1 - iL^1_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{10}_{L1}K_1 = 0$$
$$\psi^{11}_{L1} \equiv (L^2_1 + iL^2_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{11}_{L1}K_1 = 0$$
$$\psi^{12}_{L1} \equiv (L^2_1 - iL^2_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{12}_{L1}K_1 = 0$$
$$\psi^{13}_{L1} \equiv (L^2_1 + iL^3_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{13}_{L1}K_1 = 0$$
$$\psi^{14}_{L1} \equiv (L^2_1 - iL^3_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi^{14}_{L1}K_1 = 0$$
$$\psi^{15}_{L1} \equiv (L^2_1 + iL^4_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{15}_{L1}K_1 = 0$$
$$\psi^{16}_{L1} \equiv (L^2_1 - iL^4_1) = +K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{16}_{L1}K_1 = 0$$
$$\psi^{17}_{L1} \equiv (L^3_1 + iL^1_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{17}_{L1}K_1 = 0$$
$$\psi^{18}_{L1} \equiv (L^3_1 - iL^1_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{18}_{L1}K_1 = 0$$
$$\psi^{19}_{L1} \equiv (L^3_1 + iL^2_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{19}_{L1}K_1 = 0$$
$$\psi^{20}_{L1} \equiv (L^3_1 - iL^2_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{20}_{L1}K_1 = 0$$
$$\psi^{21}_{L1} \equiv (L^3_1 + iL^3_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{21}_{L1}K_1 = 0$$
$$\psi^{22}_{L1} \equiv (L^3_1 - iL^3_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{22}_{L1}K_1 = 0$$
$$\psi^{23}_{L1} \equiv (L^3_1 + iL^4_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{23}_{L1}K_1 = 0$$
$$\psi^{24}_{L1} \equiv (L^3_1 - iL^4_1) = -K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{24}_{L1}K_1 = 0$$
$$\psi^{25}_{L1} \equiv (L^4_1 + iL^1_1) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{25}_{L1}K_1 = 0$$
$$\psi^{26}_{L1} \equiv (L^4_1 - iL^1_1) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{26}_{L1}K_1 = 0$$
$$\psi^{27}_{L1} \equiv (L^4_1 + iL^2_1) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{27}_{L1}K_1 = 0$$
$$\psi^{28}_{L1} \equiv (L^4_1 - iL^2_1) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(+K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{28}_{L1}K_1 = 0$$
$$\psi^{29}_{L1} \equiv (L^4_1 + iL^3_1) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi^{29}_{L1}K_1 = 0$$

$$\psi_{L1}^{30} \equiv (L_1^4 - iL_1^3) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 + K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi_{L1}^{30}K_1 = 0$$
$$\psi_{L1}^{31} \equiv (L_1^4 + iL_1^4) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi_{L1}^{31}K_1 = 0$$
$$\psi_{L1}^{32} \equiv (L_1^4 - iL_1^4) = -K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(-K_1 - K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$
$$\psi_{L1}^{32}K_1 = 0$$

ψ_{L2}

$$\psi_{L2}^{1} \equiv (L_2^1 + iL_2^1) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{1}K_2 = 0$$
$$\psi_{L2}^{2} \equiv (L_2^1 - iL_2^1) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{2}K_2 = 0$$
$$\psi_{L2}^{3} \equiv (L_2^1 + iL_2^2) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{3}K_2 = 0$$
$$\psi_{L2}^{4} \equiv (L_2^1 - iL_2^2) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{4}K_2 = 0$$
$$\psi_{L2}^{5} \equiv (L_2^1 + iL_2^3) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{5}K_2 = 0$$
$$\psi_{L2}^{6} \equiv (L_2^1 - iL_2^3) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{6}K_2 = 0$$
$$\psi_{L2}^{7} \equiv (L_2^1 + iL_2^4) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{7}K_2 = 0$$
$$\psi_{L2}^{8} \equiv (L_2^1 - iL_2^4) = +K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{8}K_2 = 0$$
$$\psi_{L2}^{9} \equiv (L_2^2 + iL_2^1) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{9}K_2 = 0$$
$$\psi_{L2}^{10} \equiv (L_2^2 - iL_2^1) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{10}K_2 = 0$$
$$\psi_{L2}^{11} \equiv (L_2^2 + iL_2^2) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{11}K_2 = 0$$
$$\psi_{L2}^{12} \equiv (L_2^2 - iL_2^2) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{12}K_2 = 0$$

$\psi_{L2}^{13} \equiv (L_2^2 + iL_2^3) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{13} K_2 = 0$

$\psi_{L2}^{14} \equiv (L_2^2 - iL_2^3) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{14} K_2 = 0$

$\psi_{L2}^{15} \equiv (L_2^2 + iL_2^4) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{15} K_2 = 0$

$\psi_{L2}^{16} \equiv (L_2^2 - iL_2^4) = +K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{16} K_2 = 0$

$\psi_{L2}^{17} \equiv (L_2^3 + iL_2^1) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{17} K_2 = 0$

$\psi_{L2}^{18} \equiv (L_2^3 - iL_2^1) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{18} K_2 = 0$

$\psi_{L2}^{19} \equiv (L_2^3 + iL_2^2) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{19} K_2 = 0$

$\psi_{L2}^{20} \equiv (L_2^3 - iL_2^2) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{20} K_2 = 0$

$\psi_{L2}^{21} \equiv (L_2^3 + iL_2^3) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{21} K_2 = 0$

$\psi_{L2}^{22} \equiv (L_2^3 - iL_2^3) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{22} K_2 = 0$

$\psi_{L2}^{23} \equiv (L_2^3 + iL_2^4) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{23} K_2 = 0$

$\psi_{L2}^{24} \equiv (L_2^3 - iL_2^4) = -K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{24} K_2 = 0$

$\psi_{L2}^{25} \equiv (L_2^4 + iL_2^1) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{25} K_2 = 0$

$\psi_{L2}^{26} \equiv (L_2^4 - iL_2^1) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{26} K_2 = 0$

$\psi_{L2}^{27} \equiv (L_2^4 + iL_2^2) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$\psi_{L2}^{27} K_2 = 0$

$\psi_{L2}^{28} \equiv (L_2^4 - iL_2^2) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(+K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$

$$\psi_{L2}^{28} K_2 = 0$$
$$\psi_{L2}^{29} \equiv (L_2^4 + iL_2^3) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{29} K_2 = 0$$
$$\psi_{L2}^{30} \equiv (L_2^4 - iL_2^3) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 + K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{30} K_2 = 0$$
$$\psi_{L2}^{31} \equiv (L_2^4 + iL_2^4) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{31} K_2 = 0$$
$$\psi_{L2}^{32} \equiv (L_2^4 - iL_2^4) = -K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(-K_2 - K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$
$$\psi_{L2}^{32} K_2 = 0$$

ψ_{L3}

$$\psi_{L3}^{1} \equiv (L_3^1 + iL_3^1) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{1} K_3 = 0$$
$$\psi_{L3}^{2} \equiv (L_3^1 - iL_3^1) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{2} K_3 = 0$$
$$\psi_{L3}^{3} \equiv (L_3^1 + iL_3^2) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{3} K_3 = 0$$
$$\psi_{L3}^{4} \equiv (L_3^1 - iL_3^2) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{4} K_3 = 0$$
$$\psi_{L3}^{5} \equiv (L_3^1 + iL_3^3) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{5} K_3 = 0$$
$$\psi_{L3}^{6} \equiv (L_3^1 - iL_3^3) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{6} K_3 = 0$$
$$\psi_{L3}^{7} \equiv (L_3^1 + iL_3^4) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{7} K_3 = 0$$
$$\psi_{L3}^{8} \equiv (L_3^1 - iL_3^4) = +K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{8} K_3 = 0$$
$$\psi_{L3}^{9} \equiv (L_3^2 + iL_3^1) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{9} K_3 = 0$$
$$\psi_{L3}^{10} \equiv (L_3^2 - iL_3^1) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi_{L3}^{10} K_3 = 0$$
$$\psi_{L3}^{11} \equiv (L_3^2 + iL_3^2) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$

$\psi_{L3}^{11} K_3 = 0$

$\psi_{L3}^{12} \equiv (L_3^2 - iL_3^2) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{12} K_3 = 0$

$\psi_{L3}^{13} \equiv (L_3^2 + iL_3^3) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{13} K_3 = 0$

$\psi_{L3}^{14} \equiv (L_3^2 - iL_3^3) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{14} K_3 = 0$

$\psi_{L3}^{15} \equiv (L_3^2 + iL_3^4) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{15} K_3 = 0$

$\psi_{L3}^{16} \equiv (L_3^2 - iL_3^4) = +K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{16} K_3 = 0$

$\psi_{L3}^{17} \equiv (L_3^3 + iL_3^1) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{17} K_3 = 0$

$\psi_{L3}^{18} \equiv (L_3^3 - iL_3^1) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{18} K_3 = 0$

$\psi_{L3}^{19} \equiv (L_3^3 + iL_3^2) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{19} K_3 = 0$

$\psi_{L3}^{20} \equiv (L_3^3 - iL_3^2) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{20} K_3 = 0$

$\psi_{L3}^{21} \equiv (L_3^3 + iL_3^3) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{21} K_3 = 0$

$\psi_{L3}^{22} \equiv (L_3^3 - iL_3^3) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{22} K_3 = 0$

$\psi_{L3}^{23} \equiv (L_3^3 + iL_3^4) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{23} K_3 = 0$

$\psi_{L3}^{24} \equiv (L_3^3 - iL_3^4) = -K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{24} K_3 = 0$

$\psi_{L3}^{25} \equiv (L_3^4 + iL_3^1) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{25} K_3 = 0$

$\psi_{L3}^{26} \equiv (L_3^4 - iL_3^1) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$

$\psi_{L3}^{26} K_3 = 0$

$$\psi^{27}_{L3} \equiv (L^4_3 + iL^2_3) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi^{27}_{L3}K_3 = 0$$
$$\psi^{28}_{L3} \equiv (L^4_3 - iL^2_3) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(+K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi^{28}_{L3}K_3 = 0$$
$$\psi^{29}_{L3} \equiv (L^4_3 + iL^3_3) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi^{29}_{L3}K_3 = 0$$
$$\psi^{30}_{L3} \equiv (L^4_3 - iL^3_3) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 + K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi^{30}_{L3}K_3 = 0$$
$$\psi^{31}_{L3} \equiv (L^4_3 + iL^4_3) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi^{31}_{L3}K_3 = 0$$
$$\psi^{32}_{L3} \equiv (L^4_3 - iL^4_3) = -K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(-K_3 - K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$
$$\psi^{32}_{L3}K_3 = 0$$

ψ_{L4}

$$\psi^{1}_{L4} \equiv (L^1_4 + iL^1_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{1}_{L4}K_4 = 0$$
$$\psi^{2}_{L4} \equiv (L^1_4 - iL^1_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{2}_{L4}K_4 = 0$$
$$\psi^{3}_{L4} \equiv (L^1_4 + iL^2_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{3}_{L4}K_4 = 0$$
$$\psi^{4}_{L4} \equiv (L^1_4 - iL^2_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{4}_{L4}K_4 = 0$$
$$\psi^{5}_{L4} \equiv (L^1_4 + iL^3_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{5}_{L4}K_4 = 0$$
$$\psi^{6}_{L4} \equiv (L^1_4 - iL^3_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{6}_{L4}K_4 = 0$$
$$\psi^{7}_{L4} \equiv (L^1_4 + iL^4_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{7}_{L4}K_4 = 0$$
$$\psi^{8}_{L4} \equiv (L^1_4 - iL^4_4) = +K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{8}_{L4}K_4 = 0$$
$$\psi^{9}_{L4} \equiv (L^2_4 + iL^1_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{9}_{L4}K_4 = 0$$

$$\psi^{10}_{L4} \equiv (L^2_4 - iL^1_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{10}_{L4} K_4 = 0$$
$$\psi^{11}_{L4} \equiv (L^2_4 + iL^2_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{11}_{L4} K_4 = 0$$
$$\psi^{12}_{L4} \equiv (L^2_4 - iL^2_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{12}_{L4} K_4 = 0$$
$$\psi^{13}_{L4} \equiv (L^2_4 + iL^3_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{13}_{L4} K_4 = 0$$
$$\psi^{14}_{L4} \equiv (L^2_4 - iL^3_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{14}_{L4} K_4 = 0$$
$$\psi^{15}_{L4} \equiv (L^2_4 + iL^4_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{15}_{L4} K_4 = 0$$
$$\psi^{16}_{L4} \equiv (L^2_4 - iL^4_4) = +K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{16}_{L4} K_4 = 0$$
$$\psi^{17}_{L4} \equiv (L^3_4 + iL^1_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{17}_{L4} K_4 = 0$$
$$\psi^{18}_{L4} \equiv (L^3_4 - iL^1_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{18}_{L4} K_4 = 0$$
$$\psi^{19}_{L4} \equiv (L^3_4 + iL^2_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{19}_{L4} K_4 = 0$$
$$\psi^{20}_{L4} \equiv (L^3_4 - iL^2_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{20}_{L4} K_4 = 0$$
$$\psi^{21}_{L4} \equiv (L^3_4 + iL^3_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{21}_{L4} K_4 = 0$$
$$\psi^{22}_{L4} \equiv (L^3_4 - iL^3_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{22}_{L4} K_4 = 0$$
$$\psi^{23}_{L4} \equiv (L^3_4 + iL^4_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{23}_{L4} K_4 = 0$$
$$\psi^{24}_{L4} \equiv (L^3_4 - iL^4_4) = -K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{24}_{L4} K_4 = 0$$
$$\psi^{25}_{L4} \equiv (L^4_4 + iL^1_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi^{25}_{L4}K_4 = 0$$
$$\psi^{26}_{L4} \equiv (L^4_4 - iL^1_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{26}_{L4}K_4 = 0$$
$$\psi^{27}_{L4} \equiv (L^4_4 + iL^2_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{27}_{L4}K_4 = 0$$
$$\psi^{28}_{L4} \equiv (L^4_4 - iL^2_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(+K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{28}_{L4}K_4 = 0$$
$$\psi^{29}_{L4} \equiv (L^4_4 + iL^3_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{29}_{L4}K_4 = 0$$
$$\psi^{30}_{L4} \equiv (L^4_4 - iL^3_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 + K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{30}_{L4}K_4 = 0$$
$$\psi^{31}_{L4} \equiv (L^4_4 + iL^4_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{31}_{L4}K_4 = 0$$
$$\psi^{32}_{L4} \equiv (L^4_4 - iL^4_4) = -K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(-K_4 - K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$
$$\psi^{32}_{L4}K_4 = 0$$

ψ_{R1}

$$\psi^{1}_{R1} \equiv (R^1_1 + R^1_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{1}_{R1} = 0$$
$$\psi^{2}_{R1} \equiv (R^1_1 - R^1_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{2}_{R1} = 0$$
$$\psi^{3}_{R1} \equiv (R^1_1 + R^2_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{3}_{R1} = 0$$
$$\psi^{4}_{R1} \equiv (R^1_1 - R^2_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{4}_{R1} = 0$$
$$\psi^{5}_{R1} \equiv (R^1_1 + R^3_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{5}_{R1} = 0$$
$$\psi^{6}_{R1} \equiv (R^1_1 - R^3_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{6}_{R1} = 0$$
$$\psi^{7}_{R1} \equiv (R^1_1 + R^4_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{7}_{R1} = 0$$
$$\psi^{8}_{R1} \equiv (R^1_1 - R^4_1 i) = +K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{8}_{R1} = 0$$
$$\psi^{9}_{R1} \equiv (R^2_1 + R^1_1 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{9}_{R1} = 0$$
$$\psi^{10}_{R1} \equiv (R^2_1 - R^1_1 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{10}_{R1} = 0$$
$$\psi^{11}_{R1} \equiv (R^2_1 + R^2_1 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{11}_{R1} = 0$$
$$\psi^{12}_{R1} \equiv (R^2_1 - R^2_1 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{12}_{R1} = 0$$
$$\psi^{13}_{R1} \equiv (R^2_1 + R^3_1 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi^{13}_{R1} = 0$$
$$\psi^{14}_{R1} \equiv (R^2_1 - R^3_1 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1\psi_{R1}^{14} = 0$$
$$\psi_{R1}^{15} \equiv (R_1^2 + R_1^4 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{15} = 0$$
$$\psi_{R1}^{16} \equiv (R_1^2 - R_1^4 i) = +K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{16} = 0$$
$$\psi_{R1}^{17} \equiv (R_1^3 + R_1^1 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{17} = 0$$
$$\psi_{R1}^{18} \equiv (R_1^3 - R_1^1 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{18} = 0$$
$$\psi_{R1}^{19} \equiv (R_1^3 + R_1^2 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{19} = 0$$
$$\psi_{R1}^{20} \equiv (R_1^3 - R_1^2 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{20} = 0$$
$$\psi_{R1}^{21} \equiv (R_1^3 + R_1^3 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{21} = 0$$
$$\psi_{R1}^{22} \equiv (R_1^3 - R_1^3 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{22} = 0$$
$$\psi_{R1}^{23} \equiv (R_1^3 + R_1^4 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{23} = 0$$
$$\psi_{R1}^{24} \equiv (R_1^3 - R_1^4 i) = -K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{24} = 0$$
$$\psi_{R1}^{25} \equiv (R_1^4 + R_1^1 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{25} = 0$$
$$\psi_{R1}^{26} \equiv (R_1^4 - R_1^1 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{26} = 0$$
$$\psi_{R1}^{27} \equiv (R_1^4 + R_1^2 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{27} = 0$$
$$\psi_{R1}^{28} \equiv (R_1^4 - R_1^2 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (+K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{28} = 0$$
$$\psi_{R1}^{29} \equiv (R_1^4 + R_1^3 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{29} = 0$$

$$\psi_{R1}^{30} \equiv (R_1^4 - R_1^3 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{30} = 0$$
$$\psi_{R1}^{31} \equiv (R_1^4 + R_1^4 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{31} = 0$$
$$\psi_{R1}^{32} \equiv (R_1^4 - R_1^4 i) = -K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (-K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$
$$K_1\psi_{R1}^{32} = 0$$

ψ_{R2}

$$\psi_{R2}^{1} \equiv (R_2^1 + R_2^1 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{1} = 0$$
$$\psi_{R2}^{2} \equiv (R_2^1 - R_2^1 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{2} = 0$$
$$\psi_{R2}^{3} \equiv (R_2^1 + R_2^2 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{3} = 0$$
$$\psi_{R2}^{4} \equiv (R_2^1 - R_2^2 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{4} = 0$$
$$\psi_{R2}^{5} \equiv (R_2^1 + R_2^3 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{5} = 0$$
$$\psi_{R2}^{6} \equiv (R_2^1 - R_2^3 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{6} = 0$$
$$\psi_{R2}^{7} \equiv (R_2^1 + R_2^4 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{7} = 0$$
$$\psi_{R2}^{8} \equiv (R_2^1 - R_2^4 i) = +K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{8} = 0$$
$$\psi_{R2}^{9} \equiv (R_2^2 + R_2^1 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{9} = 0$$
$$\psi_{R2}^{10} \equiv (R_2^2 - R_2^1 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{10} = 0$$
$$\psi_{R2}^{11} \equiv (R_2^2 + R_2^2 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{11} = 0$$
$$\psi_{R2}^{12} \equiv (R_2^2 - R_2^2 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{12} = 0$$

$$\psi_{R2}^{13} \equiv (R_2^2 + R_2^3 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{13} = 0$$
$$\psi_{R2}^{14} \equiv (R_2^2 - R_2^3 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{14} = 0$$
$$\psi_{R2}^{15} \equiv (R_2^2 + R_2^4 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{15} = 0$$
$$\psi_{R2}^{16} \equiv (R_2^2 - R_2^4 i) = +K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{16} = 0$$
$$\psi_{R2}^{17} \equiv (R_2^3 + R_2^1 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{17} = 0$$
$$\psi_{R2}^{18} \equiv (R_2^3 - R_2^1 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{18} = 0$$
$$\psi_{R2}^{19} \equiv (R_2^3 + R_2^2 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{19} = 0$$
$$\psi_{R2}^{20} \equiv (R_2^3 - R_2^2 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{20} = 0$$
$$\psi_{R2}^{21} \equiv (R_2^3 + R_2^3 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{21} = 0$$
$$\psi_{R2}^{22} \equiv (R_2^3 - R_2^3 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{22} = 0$$
$$\psi_{R2}^{23} \equiv (R_2^3 + R_2^4 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{23} = 0$$
$$\psi_{R2}^{24} \equiv (R_2^3 - R_2^4 i) = -K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{24} = 0$$
$$\psi_{R2}^{25} \equiv (R_2^4 + R_2^1 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{25} = 0$$
$$\psi_{R2}^{26} \equiv (R_2^4 - R_2^1 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{26} = 0$$
$$\psi_{R2}^{27} \equiv (R_2^4 + R_2^2 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{27} = 0$$
$$\psi_{R2}^{28} \equiv (R_2^4 - R_2^2 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (+K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$

$$K_2\psi_{R2}^{28} = 0$$
$$\psi_{R2}^{29} \equiv (R_2^4 + R_2^3 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{29} = 0$$
$$\psi_{R2}^{30} \equiv (R_2^4 - R_2^3 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{30} = 0$$
$$\psi_{R2}^{31} \equiv (R_2^4 + R_2^4 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{31} = 0$$
$$\psi_{R2}^{32} \equiv (R_2^4 - R_2^4 i) = -K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (-K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i$$
$$K_2\psi_{R2}^{32} = 0$$

ψ_{R3}

$$\psi_{R3}^{1} \equiv (R_3^1 + R_3^1 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{1} = 0$$
$$\psi_{R3}^{2} \equiv (R_3^1 - R_3^1 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{2} = 0$$
$$\psi_{R3}^{3} \equiv (R_3^1 + R_3^2 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{3} = 0$$
$$\psi_{R3}^{4} \equiv (R_3^1 - R_3^2 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{4} = 0$$
$$\psi_{R3}^{5} \equiv (R_3^1 + R_3^3 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{5} = 0$$
$$\psi_{R3}^{6} \equiv (R_3^1 - R_3^3 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{6} = 0$$
$$\psi_{R3}^{7} \equiv (R_3^1 + R_3^4 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{7} = 0$$
$$\psi_{R3}^{8} \equiv (R_3^1 - R_3^4 i) = +K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{8} = 0$$
$$\psi_{R3}^{9} \equiv (R_3^2 + R_3^1 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{9} = 0$$
$$\psi_{R3}^{10} \equiv (R_3^2 - R_3^1 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{10} = 0$$
$$\psi_{R3}^{11} \equiv (R_3^2 + R_3^2 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$

$K_3\psi_{R3}^{11} = 0$

$\psi_{R3}^{12} \equiv (R_3^2 - R_3^2 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{12} = 0$

$\psi_{R3}^{13} \equiv (R_3^2 + R_3^3 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{13} = 0$

$\psi_{R3}^{14} \equiv (R_3^2 - R_3^3 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{14} = 0$

$\psi_{R3}^{15} \equiv (R_3^2 + R_3^4 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{15} = 0$

$\psi_{R3}^{16} \equiv (R_3^2 - R_3^4 i) = +K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{16} = 0$

$\psi_{R3}^{17} \equiv (R_3^3 + R_3^1 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{17} = 0$

$\psi_{R3}^{18} \equiv (R_3^3 - R_3^1 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{18} = 0$

$\psi_{R3}^{19} \equiv (R_3^3 + R_3^2 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{19} = 0$

$\psi_{R3}^{20} \equiv (R_3^3 - R_3^2 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{20} = 0$

$\psi_{R3}^{21} \equiv (R_3^3 + R_3^3 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{21} = 0$

$\psi_{R3}^{22} \equiv (R_3^3 - R_3^3 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{22} = 0$

$\psi_{R3}^{23} \equiv (R_3^3 + R_3^4 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{23} = 0$

$\psi_{R3}^{24} \equiv (R_3^3 - R_3^4 i) = -K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{24} = 0$

$\psi_{R3}^{25} \equiv (R_3^4 + R_3^1 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{25} = 0$

$\psi_{R3}^{26} \equiv (R_3^4 - R_3^1 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$

$K_3\psi_{R3}^{26} = 0$

$$\psi_{R3}^{27} \equiv (R_3^4 + R_3^2 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{27} = 0$$
$$\psi_{R3}^{28} \equiv (R_3^4 - R_3^2 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (+K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{28} = 0$$
$$\psi_{R3}^{29} \equiv (R_3^4 + R_3^3 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{29} = 0$$
$$\psi_{R3}^{30} \equiv (R_3^4 - R_3^3 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{30} = 0$$
$$\psi_{R3}^{31} \equiv (R_3^4 + R_3^4 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{31} = 0$$
$$\psi_{R3}^{32} \equiv (R_3^4 - R_3^4 i) = -K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (-K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$
$$K_3\psi_{R3}^{32} = 0$$

ψ_{R4}

$$\psi_{R4}^{1} \equiv (R_4^1 + R_4^1 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{1} = 0$$
$$\psi_{R4}^{2} \equiv (R_4^1 - R_4^1 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{2} = 0$$
$$\psi_{R4}^{3} \equiv (R_4^1 + R_4^2 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{3} = 0$$
$$\psi_{R4}^{4} \equiv (R_4^1 - R_4^2 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{4} = 0$$
$$\psi_{R4}^{5} \equiv (R_4^1 + R_4^3 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{5} = 0$$
$$\psi_{R4}^{6} \equiv (R_4^1 - R_4^3 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{6} = 0$$
$$\psi_{R4}^{7} \equiv (R_4^1 + R_4^4 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{7} = 0$$
$$\psi_{R4}^{8} \equiv (R_4^1 - R_4^4 i) = +K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{8} = 0$$
$$\psi_{R4}^{9} \equiv (R_4^2 + R_4^1 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{9} = 0$$

$\psi^{10}_{R4} \equiv (R^2_4 - R^1_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{10}_{R4} = 0$

$\psi^{11}_{R4} \equiv (R^2_4 + R^2_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{11}_{R4} = 0$

$\psi^{12}_{R4} \equiv (R^2_4 - R^2_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{12}_{R4} = 0$

$\psi^{13}_{R4} \equiv (R^2_4 + R^3_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{13}_{R4} = 0$

$\psi^{14}_{R4} \equiv (R^2_4 - R^3_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{14}_{R4} = 0$

$\psi^{15}_{R4} \equiv (R^2_4 + R^4_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{15}_{R4} = 0$

$\psi^{16}_{R4} \equiv (R^2_4 - R^4_4 i) = +K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{16}_{R4} = 0$

$\psi^{17}_{R4} \equiv (R^3_4 + R^1_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{17}_{R4} = 0$

$\psi^{18}_{R4} \equiv (R^3_4 - R^1_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{18}_{R4} = 0$

$\psi^{19}_{R4} \equiv (R^3_4 + R^2_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{19}_{R4} = 0$

$\psi^{20}_{R4} \equiv (R^3_4 - R^2_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{20}_{R4} = 0$

$\psi^{21}_{R4} \equiv (R^3_4 + R^3_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{21}_{R4} = 0$

$\psi^{22}_{R4} \equiv (R^3_4 - R^3_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{22}_{R4} = 0$

$\psi^{23}_{R4} \equiv (R^3_4 + R^4_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{23}_{R4} = 0$

$\psi^{24}_{R4} \equiv (R^3_4 - R^4_4 i) = -K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$

$K_4\psi^{24}_{R4} = 0$

$\psi^{25}_{R4} \equiv (R^4_4 + R^1_4 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$

$$K_4\psi_{R4}^{25} = 0$$
$$\psi_{R4}^{26} \equiv (R_4^4 - R_4^1 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{26} = 0$$
$$\psi_{R4}^{27} \equiv (R_4^4 + R_4^2 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{27} = 0$$
$$\psi_{R4}^{28} \equiv (R_4^4 - R_4^2 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (+K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{28} = 0$$
$$\psi_{R4}^{29} \equiv (R_4^4 + R_4^3 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{29} = 0$$
$$\psi_{R4}^{30} \equiv (R_4^4 - R_4^3 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{30} = 0$$
$$\psi_{R4}^{31} \equiv (R_4^4 + R_4^4 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{31} = 0$$
$$\psi_{R4}^{32} \equiv (R_4^4 - R_4^4 i) = -K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (-K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i$$
$$K_4\psi_{R4}^{32} = 0$$

Symmetry with ψ_{L1} solutions
[1, 2, 3, 4, 9, 10, 11, 12]

$$i\psi_{L1}^{1} + \psi_{L1}^{2} = 0$$
$$i\psi_{L1}^{11} + \psi_{L1}^{12} = 0$$
$$i\psi_{L1}^{3} + \psi_{L1}^{10} = 0$$
$$i\psi_{L1}^{9} + \psi_{L1}^{4} = 0$$

[1, 2, 11, 12]

$$\mathbf{i}\psi_{L1}^{1} - \psi_{L1}^{2} = 0$$
$$\mathbf{j}\psi_{L1}^{1} + \psi_{L1}^{11} = 0$$
$$\mathbf{k}\psi_{L1}^{1} - \psi_{L1}^{12} = 0$$

[3, 10]

$$\mathbf{i}\psi_{L1}^{3} - \psi_{L1}^{3} = 0$$
$$\mathbf{j}\psi_{L1}^{3} + \psi_{L1}^{10} = 0$$
$$\mathbf{k}\psi_{L1}^{3} - \psi_{L1}^{10} = 0$$

[4, 9]

$$\mathbf{i}\psi_{L1}^{4} - \psi_{L1}^{4} = 0$$
$$\mathbf{j}\psi_{L1}^{4} + \psi_{L1}^{9} = 0$$
$$\mathbf{k}\psi_{L1}^{4} - \psi_{L1}^{9} = 0$$

[5, 6, 15, 16, 17, 18, 27, 28]

$$i\psi_{L1}^{5} + \psi_{L1}^{18} = 0$$
$$i\psi_{L1}^{17} + \psi_{L1}^{6} = 0$$
$$i\psi_{L1}^{15} + \psi_{L1}^{28} = 0$$
$$i\psi_{L1}^{27} + \psi_{L1}^{16} = 0$$

[7, 8, 13, 14, 19, 20, 25, 26]

$$i\psi_{L1}^{7} + \psi_{L1}^{26} = 0$$
$$i\psi_{L1}^{25} + \psi_{L1}^{8} = 0$$
$$i\psi_{L1}^{13} + \psi_{L1}^{20} = 0$$
$$i\psi_{L1}^{19} + \psi_{L1}^{14} = 0$$

[5, 6, 16, 15]

$$\mathbf{i}\psi_{L1}^{5} - \psi_{L1}^{6} = 0$$
$$\mathbf{j}\psi_{L1}^{5} + \psi_{L1}^{16} = 0$$
$$\mathbf{k}\psi_{L1}^{5} - \psi_{L1}^{15} = 0$$

[7, 13]

$$\mathbf{i}\psi_{L1}^{7} - \psi_{L1}^{7} = 0$$
$$\mathbf{j}\psi_{L1}^{7} + \psi_{1}^{13} = 0$$
$$\mathbf{k}\psi_{L1}^{7} - \psi_{L1}^{13} = 0$$

[8, 14]

$$\mathbf{i}\psi_{L1}^{8} - \psi_{L1}^{8} = 0$$
$$\mathbf{j}\psi_{L1}^{8} + \psi_{1}^{14} = 0$$
$$\mathbf{k}\psi_{L1}^{8} - \psi_{L1}^{14} = 0$$

[17, 18, 28, 27]

$$\mathbf{i}\psi_{L1}^{17} - \psi_{L1}^{18} = 0$$

$$\mathbf{j}\psi_{L1}^{17} - \psi_{L1}^{28} = 0$$
$$\mathbf{k}\psi_{L1}^{17} + \psi_{L1}^{27} = 0$$

[19, 25]

$$\mathbf{i}\psi_{L1}^{19} - \psi_{L1}^{19} = 0$$
$$\mathbf{j}\psi_{L1}^{19} - \psi_{L1}^{25} = 0$$
$$\mathbf{k}\psi_{L1}^{19} + \psi_{L1}^{25} = 0$$

[20, 26]

$$\mathbf{i}\psi_{L1}^{20} - \psi_{L1}^{20} = 0$$
$$\mathbf{j}\psi_{L1}^{20} - \psi_{L1}^{26} = 0$$
$$\mathbf{k}\psi_{L1}^{20} + \psi_{L1}^{26} = 0$$

[21, 22, 23, 24, 29, 30, 31, 32]

$$i\psi_{L1}^{21} + \psi_{L1}^{22} = 0$$
$$i\psi_{L1}^{31} + \psi_{L1}^{32} = 0$$
$$i\psi_{L1}^{23} + \psi_{L1}^{30} = 0$$
$$i\psi_{L1}^{29} + \psi_{L1}^{24} = 0$$

[21, 22, 31, 32]

$$\mathbf{i}\psi_{L1}^{21} - \psi_{L1}^{22} = 0$$
$$\mathbf{j}\psi_{L1}^{21} - \psi_{L1}^{31} = 0$$
$$\mathbf{k}\psi_{L1}^{21} + \psi_{L1}^{32} = 0$$

[23, 30]

$$\mathbf{i}\psi_{L1}^{23} - \psi_{L1}^{23} = 0$$
$$\mathbf{j}\psi_{L1}^{23} - \psi_{L1}^{30} = 0$$
$$\mathbf{k}\psi_{L1}^{23} + \psi_{L1}^{30} = 0$$

[24, 29]

$$\mathbf{i}\psi_{L1}^{24} - \psi_{L1}^{24} = 0$$
$$\mathbf{j}\psi_{L1}^{24} - \psi_{L1}^{29} = 0$$
$$\mathbf{k}\psi_{L1}^{24} + \psi_{L1}^{29} = 0$$

Rotations ψ_{L1}

$$\mathbf{i}\psi_{L1}^{1}\mathbf{i} - \psi_{L4}^{22} = 0$$

$$\mathbf{j}\psi_{L1}^{1}\mathbf{j}+\psi_{L3}^{31}=0$$
$$\mathbf{k}\psi_{L1}^{1}\mathbf{k}-\psi_{L2}^{12}=0$$
$$\mathbf{i}\psi_{L1}^{2}\mathbf{i}-\psi_{L4}^{21}=0$$
$$\mathbf{j}\psi_{L1}^{2}\mathbf{j}+\psi_{L3}^{32}=0$$
$$\mathbf{k}\psi_{L1}^{2}\mathbf{k}-\psi_{L2}^{11}=0$$
$$\mathbf{i}\psi_{L1}^{3}\mathbf{i}-\psi_{L4}^{24}=0$$
$$\mathbf{j}\psi_{L1}^{3}\mathbf{j}+\psi_{L3}^{29}=0$$
$$\mathbf{k}\psi_{L1}^{3}\mathbf{k}-\psi_{L2}^{10}=0$$
$$\mathbf{i}\psi_{L1}^{4}\mathbf{i}-\psi_{L4}^{23}=0$$
$$\mathbf{j}\psi_{L1}^{4}\mathbf{j}+\psi_{L3}^{30}=0$$
$$\mathbf{k}\psi_{L1}^{4}\mathbf{k}-\psi_{L2}^{9}=0$$
$$\mathbf{i}\psi_{L1}^{5}\mathbf{i}-\psi_{L4}^{18}=0$$
$$\mathbf{j}\psi_{L1}^{5}\mathbf{j}+\psi_{L3}^{27}=0$$
$$\mathbf{k}\psi_{L1}^{5}\mathbf{k}-\psi_{L2}^{16}=0$$
$$\mathbf{i}\psi_{L1}^{6}\mathbf{i}-\psi_{L4}^{17}=0$$
$$\mathbf{j}\psi_{L1}^{6}\mathbf{j}+\psi_{L3}^{28}=0$$
$$\mathbf{k}\psi_{L1}^{6}\mathbf{k}-\psi_{L2}^{15}=0$$
$$\mathbf{i}\psi_{L1}^{7}\mathbf{i}-\psi_{L4}^{20}=0$$
$$\mathbf{j}\psi_{L1}^{7}\mathbf{j}+\psi_{L3}^{25}=0$$
$$\mathbf{k}\psi_{L1}^{7}\mathbf{k}-\psi_{L2}^{14}=0$$
$$\mathbf{i}\psi_{L1}^{8}\mathbf{i}-\psi_{L4}^{19}=0$$
$$\mathbf{j}\psi_{L1}^{8}\mathbf{j}+\psi_{L3}^{26}=0$$
$$\mathbf{k}\psi_{L1}^{8}\mathbf{k}-\psi_{L2}^{13}=0$$
$$\mathbf{i}\psi_{L1}^{9}\mathbf{i}-\psi_{L4}^{30}=0$$
$$\mathbf{j}\psi_{L1}^{9}\mathbf{j}+\psi_{L3}^{23}=0$$
$$\mathbf{k}\psi_{L1}^{9}\mathbf{k}-\psi_{L2}^{4}=0$$
$$\mathbf{i}\psi_{L1}^{10}\mathbf{i}-\psi_{L4}^{29}=0$$
$$\mathbf{j}\psi_{L1}^{10}\mathbf{j}+\psi_{L3}^{24}=0$$
$$\mathbf{k}\psi_{L1}^{10}\mathbf{k}-\psi_{L2}^{3}=0$$
$$\mathbf{i}\psi_{L1}^{11}\mathbf{i}-\psi_{L4}^{32}=0$$
$$\mathbf{j}\psi_{L1}^{11}\mathbf{j}+\psi_{L3}^{21}=0$$

$$\mathbf{k}\psi_{L1}^{11}\mathbf{k} - \psi_{L2}^{2} = 0$$
$$\mathbf{i}\psi_{L1}^{12}\mathbf{i} - \psi_{L4}^{31} = 0$$
$$\mathbf{j}\psi_{L1}^{12}\mathbf{j} + \psi_{L3}^{22} = 0$$
$$\mathbf{k}\psi_{L1}^{12}\mathbf{k} - \psi_{L2}^{1} = 0$$
$$\mathbf{i}\psi_{L1}^{13}\mathbf{i} - \psi_{L4}^{26} = 0$$
$$\mathbf{j}\psi_{L1}^{13}\mathbf{j} + \psi_{L3}^{19} = 0$$
$$\mathbf{k}\psi_{L1}^{13}\mathbf{k} - \psi_{L2}^{8} = 0$$
$$\mathbf{i}\psi_{L1}^{14}\mathbf{i} - \psi_{L4}^{25} = 0$$
$$\mathbf{j}\psi_{L1}^{14}\mathbf{j} + \psi_{L3}^{20} = 0$$
$$\mathbf{k}\psi_{L1}^{14}\mathbf{k} - \psi_{L2}^{7} = 0$$
$$\mathbf{i}\psi_{L1}^{15}\mathbf{i} - \psi_{L4}^{28} = 0$$
$$\mathbf{j}\psi_{L1}^{15}\mathbf{j} + \psi_{L3}^{17} = 0$$
$$\mathbf{k}\psi_{L1}^{15}\mathbf{k} - \psi_{L2}^{6} = 0$$
$$\mathbf{i}\psi_{L1}^{16}\mathbf{i} - \psi_{L4}^{27} = 0$$
$$\mathbf{j}\psi_{L1}^{16}\mathbf{j} + \psi_{L3}^{18} = 0$$
$$\mathbf{k}\psi_{L1}^{16}\mathbf{k} - \psi_{L2}^{5} = 0$$
$$\mathbf{i}\psi_{L1}^{17}\mathbf{i} - \psi_{L4}^{6} = 0$$
$$\mathbf{j}\psi_{L1}^{17}\mathbf{j} + \psi_{L3}^{15} = 0$$
$$\mathbf{k}\psi_{L1}^{17}\mathbf{k} - \psi_{L2}^{28} = 0$$
$$\mathbf{i}\psi_{L1}^{18}\mathbf{i} - \psi_{L4}^{5} = 0$$
$$\mathbf{j}\psi_{L1}^{18}\mathbf{j} + \psi_{L3}^{16} = 0$$
$$\mathbf{k}\psi_{L1}^{18}\mathbf{k} - \psi_{L2}^{27} = 0$$
$$\mathbf{i}\psi_{L1}^{19}\mathbf{i} - \psi_{L4}^{8} = 0$$
$$\mathbf{j}\psi_{L1}^{19}\mathbf{j} + \psi_{L3}^{13} = 0$$
$$\mathbf{k}\psi_{L1}^{19}\mathbf{k} - \psi_{L2}^{26} = 0$$
$$\mathbf{i}\psi_{L1}^{20}\mathbf{i} - \psi_{L4}^{7} = 0$$
$$\mathbf{j}\psi_{L1}^{20}\mathbf{j} + \psi_{L3}^{14} = 0$$
$$\mathbf{k}\psi_{L1}^{20}\mathbf{k} - \psi_{L2}^{25} = 0$$
$$\mathbf{i}\psi_{L1}^{21}\mathbf{i} - \psi_{L4}^{2} = 0$$
$$\mathbf{j}\psi_{L1}^{21}\mathbf{j} + \psi_{L3}^{11} = 0$$
$$\mathbf{k}\psi_{L1}^{21}\mathbf{k} - \psi_{L2}^{32} = 0$$

$$\mathbf{i}\psi_{L1}^{22}\mathbf{i} - \psi_{L4}^{1} = 0$$
$$\mathbf{j}\psi_{L1}^{22}\mathbf{j} + \psi_{L3}^{12} = 0$$
$$\mathbf{k}\psi_{L1}^{22}\mathbf{k} - \psi_{L2}^{31} = 0$$
$$\mathbf{i}\psi_{L1}^{23}\mathbf{i} - \psi_{L4}^{4} = 0$$
$$\mathbf{j}\psi_{L1}^{23}\mathbf{j} + \psi_{L3}^{9} = 0$$
$$\mathbf{k}\psi_{L1}^{23}\mathbf{k} - \psi_{L2}^{30} = 0$$
$$\mathbf{i}\psi_{L1}^{24}\mathbf{i} - \psi_{L4}^{3} = 0$$
$$\mathbf{j}\psi_{L1}^{24}\mathbf{j} + \psi_{L3}^{10} = 0$$
$$\mathbf{k}\psi_{L1}^{24}\mathbf{k} - \psi_{L2}^{29} = 0$$
$$\mathbf{i}\psi_{L1}^{25}\mathbf{i} - \psi_{L4}^{14} = 0$$
$$\mathbf{j}\psi_{L1}^{25}\mathbf{j} + \psi_{L3}^{7} = 0$$
$$\mathbf{k}\psi_{L1}^{25}\mathbf{k} - \psi_{L2}^{20} = 0$$
$$\mathbf{i}\psi_{L1}^{26}\mathbf{i} - \psi_{L4}^{13} = 0$$
$$\mathbf{j}\psi_{L1}^{26}\mathbf{j} + \psi_{L3}^{8} = 0$$
$$\mathbf{k}\psi_{L1}^{26}\mathbf{k} - \psi_{L2}^{19} = 0$$
$$\mathbf{i}\psi_{L1}^{27}\mathbf{i} - \psi_{L4}^{16} = 0$$
$$\mathbf{j}\psi_{L1}^{27}\mathbf{j} + \psi_{L3}^{5} = 0$$
$$\mathbf{k}\psi_{L1}^{27}\mathbf{k} - \psi_{L2}^{18} = 0$$
$$\mathbf{i}\psi_{L1}^{28}\mathbf{i} - \psi_{L4}^{15} = 0$$
$$\mathbf{j}\psi_{L1}^{28}\mathbf{j} + \psi_{L3}^{6} = 0$$
$$\mathbf{k}\psi_{L1}^{28}\mathbf{k} - \psi_{L2}^{17} = 0$$
$$\mathbf{i}\psi_{L1}^{29}\mathbf{i} - \psi_{L4}^{10} = 0$$
$$\mathbf{j}\psi_{L1}^{29}\mathbf{j} + \psi_{L3}^{3} = 0$$
$$\mathbf{k}\psi_{L1}^{29}\mathbf{k} - \psi_{L2}^{24} = 0$$
$$\mathbf{i}\psi_{L1}^{30}\mathbf{i} - \psi_{L4}^{9} = 0$$
$$\mathbf{j}\psi_{L1}^{30}\mathbf{j} + \psi_{L3}^{4} = 0$$
$$\mathbf{k}\psi_{L1}^{30}\mathbf{k} - \psi_{L2}^{23} = 0$$
$$\mathbf{i}\psi_{L1}^{31}\mathbf{i} - \psi_{L4}^{12} = 0$$
$$\mathbf{j}\psi_{L1}^{31}\mathbf{j} + \psi_{L3}^{1} = 0$$
$$\mathbf{k}\psi_{L1}^{31}\mathbf{k} - \psi_{L2}^{22} = 0$$
$$\mathbf{i}\psi_{L1}^{32}\mathbf{i} - \psi_{L4}^{11} = 0$$
$$\mathbf{j}\psi_{L1}^{32}\mathbf{j} + \psi_{L3}^{2} = 0$$
$$\mathbf{k}\psi_{L1}^{32}\mathbf{k} - \psi_{L2}^{21} = 0$$