

Algebra is $Cl_{1,4}(\mathbb{R})$

The five dimensions are (t, x, y, z, w) .

It defines two sets of quaternions with one imaginary unit.

Signature is $(+ - - -)$

CoQuaternions $(1, \mathbf{i}, \mathbf{j}, \mathbf{k})$ <http://en.wikipedia.org/wiki/Split-quaternion>

$$\mathbf{i} = \gamma_t$$

$$\mathbf{j} = \gamma_t \gamma_x \gamma_y \gamma_z$$

$$\mathbf{k} = \gamma_x \gamma_y \gamma_z$$

$$\mathbf{i}^2 = 1$$

$$\mathbf{j}^2 = -1$$

$$\mathbf{k}^2 = 1$$

$$\mathbf{ij} = \gamma_x \gamma_y \gamma_z$$

$$\mathbf{ji} = -\gamma_x \gamma_y \gamma_z$$

$$\mathbf{jk} = \gamma_t$$

$$\mathbf{kj} = -\gamma_t$$

$$\mathbf{ki} = -\gamma_t \gamma_x \gamma_y \gamma_z$$

$$\mathbf{ik} = \gamma_t \gamma_x \gamma_y \gamma_z$$

$$\mathbf{ijk} = 1$$

Quaternions $(1, \mathbf{i}, \mathbf{j}, \mathbf{k})$ <http://en.wikipedia.org/wiki/Quaternion>

$$\mathbf{i} = \gamma_y \gamma_z$$

$$\mathbf{j} = -\gamma_x \gamma_z$$

$$\mathbf{k} = \gamma_x \gamma_y$$

$$\mathbf{i}^2 = -1$$

$$\mathbf{j}^2 = -1$$

$$\mathbf{k}^2 = -1$$

$$\mathbf{ij} = \gamma_x \gamma_y$$

$$\mathbf{ji} = -\gamma_x \gamma_y$$

$$\mathbf{jk} = \gamma_y \gamma_z$$

$$\mathbf{kj} = -\gamma_y \gamma_z$$

$$\mathbf{ki} = -\gamma_x \gamma_z$$

$$\mathbf{ik} = \gamma_x \gamma_z$$

$$\mathbf{ijk} = -1$$

Imaginary unit i

$$i = \gamma_w$$

$$i^2 = -1$$

Gradient definition

$$\nabla = (\gamma_t \frac{\partial}{\partial t} + \gamma_x \frac{\partial}{\partial x} + \gamma_y \frac{\partial}{\partial y} + \gamma_z \frac{\partial}{\partial z} + \gamma_w \frac{\partial}{\partial w})$$

<http://www.mrao.cam.ac.uk/~clifford/ptIIIcourse/course99/handouts/hout07.ps.gz>

Wave : K is a constant and f is a function of (t, x, y, z, w)

$$\psi = Ke^f$$

Left and right derivatives

<http://geocalc.clas.asu.edu/pdf/NFMPchapt1.pdf> see chapter 1-3 Differentiation by vectors

$$\nabla \psi = \nabla (Ke^f)$$

$$\nabla \psi = (\nabla K)e^f + K\nabla(e^f)$$

$$\nabla K = 0$$

$$\nabla \psi_L = K(\nabla f)e^f$$

$$\nabla \psi_R = (\nabla f)Ke^f$$

The following symbols are defined :

$$E \in \mathbb{R}$$

$$m \in \mathbb{R}$$

\mathbf{p} is defined with $p_x, p_y, p_z \in \mathbb{R}$

$$\mathbf{p} = p_x \mathbf{i} + p_y \mathbf{j} + p_z \mathbf{k}$$

$$\mathbf{p} = p_z \gamma_x \gamma_y - p_y \gamma_x \gamma_z + p_x \gamma_y \gamma_z$$

Exponential function f

$$f_1 = -mw + (p_y y + p_x x + p_z z - Et) \gamma_w$$

$$f_2 = mw + (-p_z z - p_y y - p_x x - Et) \gamma_w$$

$$f_3 = mw + (p_y y + p_x x + p_z z - Et) \gamma_w$$

$$f_4 = -mw + (-p_z z - p_y y - p_x x - Et) \gamma_w$$

Gradient for f

$$\nabla f_1 = m\gamma_w - E\gamma_t\gamma_w - p_x\gamma_x\gamma_w - p_y\gamma_y\gamma_w - p_z\gamma_z\gamma_w$$

$$\nabla f_2 = -m\gamma_w - E\gamma_t\gamma_w + p_x\gamma_x\gamma_w + p_y\gamma_y\gamma_w + p_z\gamma_z\gamma_w$$

$$\nabla f_3 = -m\gamma_w - E\gamma_t\gamma_w - p_x\gamma_x\gamma_w - p_y\gamma_y\gamma_w - p_z\gamma_z\gamma_w$$

$$\nabla f_4 = m\gamma_w - E\gamma_t\gamma_w + p_x\gamma_x\gamma_w + p_y\gamma_y\gamma_w + p_z\gamma_z\gamma_w$$

Square of the gradient

$$\nabla f_1^2 = -m^2 - p_z^2 + E^2 - p_y^2 - p_x^2$$

$$\nabla f_2^2 = -m^2 - p_z^2 + E^2 - p_y^2 - p_x^2$$

$$\nabla f_3^2 = -m^2 - p_z^2 + E^2 - p_y^2 - p_x^2$$

$$\nabla f_4^2 = -m^2 - p_z^2 + E^2 - p_y^2 - p_x^2$$

Dirac

http://en.wikipedia.org/wiki/Dirac_equation

$$0 = (\gamma_0 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} + im)\psi$$

With the above gradients, identify the Dirac algebra aka gamma matrices

$$\gamma_0 = \gamma_t\gamma_w$$

$$-i\mathbf{i} = \gamma_t\gamma_w$$

$$\gamma_0^2 = 1$$

$$\gamma_1 = -\gamma_x\gamma_w$$

$$-i\mathbf{i}\mathbf{k} = -\gamma_x\gamma_w$$

$$\gamma_1^2 = -1$$

$$\gamma_2 = -\gamma_y\gamma_w$$

$$-i\mathbf{j}\mathbf{k} = -\gamma_y\gamma_w$$

$$\gamma_2^2 = -1$$

$$\gamma_3 = -\gamma_z\gamma_w$$

$$-i\mathbf{k}\mathbf{k} = -\gamma_z\gamma_w$$

$$\gamma_3^2 = -1$$

$$\gamma_5 = -\gamma_t\gamma_x\gamma_y\gamma_z\gamma_w$$

$$-i\mathbf{j} = -\gamma_t\gamma_x\gamma_y\gamma_z\gamma_w$$

$$\gamma_5^2 = 1$$

Substitution in Dirac equation

$$0 = (-i\mathbf{i}\frac{\partial}{\partial t} - i\mathbf{i}\mathbf{k}\frac{\partial}{\partial x} - i\mathbf{j}\mathbf{k}\frac{\partial}{\partial y} - i\mathbf{k}\mathbf{k}\frac{\partial}{\partial z} + im)\psi$$

Factorization and multiplication

$$0 = \mathbf{j}i(\mathbf{k}\frac{\partial}{\partial t} + \mathbf{i}\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\mathbf{i}\frac{\partial}{\partial y} + \mathbf{k}\mathbf{i}\frac{\partial}{\partial z} - \mathbf{j}m)\psi$$

Simple Constants K (exactly the gradient)

$$K_1 = \mathbf{j}i(\mathbf{k}E - \mathbf{j}m + \mathbf{i}\mathbf{p})$$

$$K_2 = \mathbf{j}i(\mathbf{k}E + \mathbf{j}m - \mathbf{i}\mathbf{p})$$

$$K_3 = \mathbf{j}i(\mathbf{k}E + \mathbf{j}m + \mathbf{i}\mathbf{p})$$

$$K_4 = \mathbf{j}i(\mathbf{k}E - \mathbf{j}m - \mathbf{i}\mathbf{p})$$

Mixed Constants A (built from simple constants and the coquaternions)

L_1

$$\begin{pmatrix} L_1^1 \\ L_1^2 \\ L_1^3 \\ L_1^4 \end{pmatrix} \equiv \begin{pmatrix} K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} \\ K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} \\ K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} \\ K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} \end{pmatrix} - \begin{pmatrix} (1 - \mathbf{i} + \mathbf{j} - \mathbf{k})K_1 \\ (1 - \mathbf{i} - \mathbf{j} + \mathbf{k})K_1 \\ (1 + \mathbf{i} - \mathbf{j} - \mathbf{k})K_1 \\ (1 + \mathbf{i} + \mathbf{j} + \mathbf{k})K_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

L_2

$$\begin{pmatrix} L_2^1 \\ L_2^2 \\ L_2^3 \\ L_2^4 \end{pmatrix} \equiv \begin{pmatrix} K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} \\ K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} \\ K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} \\ K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} \end{pmatrix} - \begin{pmatrix} (1 - \mathbf{i} + \mathbf{j} - \mathbf{k})K_2 \\ (1 - \mathbf{i} - \mathbf{j} + \mathbf{k})K_2 \\ (1 + \mathbf{i} - \mathbf{j} - \mathbf{k})K_2 \\ (1 + \mathbf{i} + \mathbf{j} + \mathbf{k})K_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

L_3

$$\begin{pmatrix} L_3^1 \\ L_3^2 \\ L_3^3 \\ L_3^4 \end{pmatrix} \equiv \begin{pmatrix} K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} \\ K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} \\ K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} \\ K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} \end{pmatrix} - \begin{pmatrix} (1 - \mathbf{i} + \mathbf{j} - \mathbf{k})K_3 \\ (1 - \mathbf{i} - \mathbf{j} + \mathbf{k})K_3 \\ (1 + \mathbf{i} - \mathbf{j} - \mathbf{k})K_3 \\ (1 + \mathbf{i} + \mathbf{j} + \mathbf{k})K_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

L_4

$$\begin{pmatrix} L_4^1 \\ L_4^2 \\ L_4^3 \\ L_4^4 \end{pmatrix} \equiv \begin{pmatrix} K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} \\ K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} \\ K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} \\ K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} \end{pmatrix} - \begin{pmatrix} (1 - \mathbf{i} + \mathbf{j} - \mathbf{k})K_4 \\ (1 - \mathbf{i} - \mathbf{j} + \mathbf{k})K_4 \\ (1 + \mathbf{i} - \mathbf{j} - \mathbf{k})K_4 \\ (1 + \mathbf{i} + \mathbf{j} + \mathbf{k})K_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

R_1

$$\begin{pmatrix} R_1^1 \\ R_1^2 \\ R_1^3 \\ R_1^4 \end{pmatrix} \equiv \begin{pmatrix} K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 \\ K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 \\ K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 \\ K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 \end{pmatrix} - \begin{pmatrix} K_1(1 - \mathbf{i} + \mathbf{j} - \mathbf{k}) \\ K_1(1 - \mathbf{i} - \mathbf{j} + \mathbf{k}) \\ K_1(1 + \mathbf{i} - \mathbf{j} - \mathbf{k}) \\ K_1(1 + \mathbf{i} + \mathbf{j} + \mathbf{k}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

R_2

$$\begin{pmatrix} R_2^1 \\ R_2^2 \\ R_2^3 \\ R_2^4 \end{pmatrix} \equiv \begin{pmatrix} K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 \\ K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 \\ K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 \\ K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 \end{pmatrix} - \begin{pmatrix} K_2(1 - \mathbf{i} + \mathbf{j} - \mathbf{k}) \\ K_2(1 - \mathbf{i} - \mathbf{j} + \mathbf{k}) \\ K_2(1 + \mathbf{i} - \mathbf{j} - \mathbf{k}) \\ K_2(1 + \mathbf{i} + \mathbf{j} + \mathbf{k}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

R_3

$$\begin{pmatrix} R_3^1 \\ R_3^2 \\ R_3^3 \\ R_3^4 \end{pmatrix} \equiv \begin{pmatrix} K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 \\ K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 \\ K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 \\ K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 \end{pmatrix} - \begin{pmatrix} K_3(1 - \mathbf{i} + \mathbf{j} - \mathbf{k}) \\ K_3(1 - \mathbf{i} - \mathbf{j} + \mathbf{k}) \\ K_3(1 + \mathbf{i} - \mathbf{j} - \mathbf{k}) \\ K_3(1 + \mathbf{i} + \mathbf{j} + \mathbf{k}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$R_4$$

$$\begin{pmatrix} R_4^1 \\ R_4^2 \\ R_4^3 \\ R_4^4 \end{pmatrix} \equiv \begin{pmatrix} K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 \\ K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 \\ K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 \\ K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 \end{pmatrix} - \begin{pmatrix} K_4(1 - \mathbf{i} + \mathbf{j} - \mathbf{k}) \\ K_4(1 - \mathbf{i} - \mathbf{j} + \mathbf{k}) \\ K_4(1 + \mathbf{i} - \mathbf{j} - \mathbf{k}) \\ K_4(1 + \mathbf{i} + \mathbf{j} + \mathbf{k}) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$L_i^j K_i = 0$$

$$K_i R_i^j = 0$$

$$L_i^j R_i^j = 0$$

Details about full solutions (built from mixed constants and the imaginary unit)

ψ_{L1}

$$\psi_{L1}^1 \equiv (L_1^1 + iL_1^1) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^1 K_1 = 0$$

$$\psi_{L1}^2 \equiv (L_1^1 - iL_1^1) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^2 K_1 = 0$$

$$\psi_{L1}^3 \equiv (L_1^1 + iL_1^2) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^3 K_1 = 0$$

$$\psi_{L1}^4 \equiv (L_1^1 - iL_1^2) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^4 K_1 = 0$$

$$\psi_{L1}^5 \equiv (L_1^1 + iL_1^3) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^5 K_1 = 0$$

$$\psi_{L1}^6 \equiv (L_1^1 - iL_1^3) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^6 K_1 = 0$$

$$\psi_{L1}^7 \equiv (L_1^1 + iL_1^4) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^7 K_1 = 0$$

$$\psi_{L1}^8 \equiv (L_1^1 - iL_1^4) = K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^8 K_1 = 0$$

$$\psi_{L1}^9 \equiv (L_1^2 + iL_1^1) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^9 K_1 = 0$$

$$\psi_{L1}^{10} \equiv (L_1^2 - iL_1^1) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^{10} K_1 = 0$$

$$\psi_{L1}^{11} \equiv (L_1^2 + iL_1^2) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^{11} K_1 = 0$$

$$\psi_{L1}^{12} \equiv (L_1^2 - iL_1^2) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^{12} K_1 = 0$$

$$\psi_{L1}^{13} \equiv (L_1^2 + iL_1^3) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^{13} K_1 = 0$$

$$\psi_{L1}^{14} \equiv (L_1^2 - iL_1^3) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k})$$

$$\begin{aligned}
\psi_{L1}^{14} K_1 &= 0 \\
\psi_{L1}^{15} &\equiv (L_1^2 + iL_1^4) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{15} K_1 &= 0 \\
\psi_{L1}^{16} &\equiv (L_1^2 - iL_1^4) = K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{16} K_1 &= 0 \\
\psi_{L1}^{17} &\equiv (L_1^3 + iL_1^1) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{17} K_1 &= 0 \\
\psi_{L1}^{18} &\equiv (L_1^3 - iL_1^1) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{18} K_1 &= 0 \\
\psi_{L1}^{19} &\equiv (L_1^3 + iL_1^2) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{19} K_1 &= 0 \\
\psi_{L1}^{20} &\equiv (L_1^3 - iL_1^2) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{20} K_1 &= 0 \\
\psi_{L1}^{21} &\equiv (L_1^3 + iL_1^3) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{21} K_1 &= 0 \\
\psi_{L1}^{22} &\equiv (L_1^3 - iL_1^3) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{22} K_1 &= 0 \\
\psi_{L1}^{23} &\equiv (L_1^3 + iL_1^4) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{23} K_1 &= 0 \\
\psi_{L1}^{24} &\equiv (L_1^3 - iL_1^4) = K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{24} K_1 &= 0 \\
\psi_{L1}^{25} &\equiv (L_1^4 + iL_1^1) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{25} K_1 &= 0 \\
\psi_{L1}^{26} &\equiv (L_1^4 - iL_1^1) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} - K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{26} K_1 &= 0 \\
\psi_{L1}^{27} &\equiv (L_1^4 + iL_1^2) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{27} K_1 &= 0 \\
\psi_{L1}^{28} &\equiv (L_1^4 - iL_1^2) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 + K_4\mathbf{i} + K_3\mathbf{j} + K_2\mathbf{k}) \\
\psi_{L1}^{28} K_1 &= 0 \\
\psi_{L1}^{29} &\equiv (L_1^4 + iL_1^3) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k}) \\
\psi_{L1}^{29} K_1 &= 0
\end{aligned}$$

$$\psi_{L1}^{30} \equiv (L_1^4 - iL_1^3) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} + K_3\mathbf{j} - K_2\mathbf{k})$$

$$\psi_{L1}^{30}K_1 = 0$$

$$\psi_{L1}^{31} \equiv (L_1^4 + iL_1^4) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} + i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^{31}K_1 = 0$$

$$\psi_{L1}^{32} \equiv (L_1^4 - iL_1^4) = K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k} - i(K_1 - K_4\mathbf{i} - K_3\mathbf{j} + K_2\mathbf{k})$$

$$\psi_{L1}^{32}K_1 = 0$$

$$\psi_{L2}$$

$$\psi_{L2}^1 \equiv (L_2^1 + iL_2^1) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^1K_2 = 0$$

$$\psi_{L2}^2 \equiv (L_2^1 - iL_2^1) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^2K_2 = 0$$

$$\psi_{L2}^3 \equiv (L_2^1 + iL_2^2) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^3K_2 = 0$$

$$\psi_{L2}^4 \equiv (L_2^1 - iL_2^2) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^4K_2 = 0$$

$$\psi_{L2}^5 \equiv (L_2^1 + iL_2^3) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^5K_2 = 0$$

$$\psi_{L2}^6 \equiv (L_2^1 - iL_2^3) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^6K_2 = 0$$

$$\psi_{L2}^7 \equiv (L_2^1 + iL_2^4) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^7K_2 = 0$$

$$\psi_{L2}^8 \equiv (L_2^1 - iL_2^4) = K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^8K_2 = 0$$

$$\psi_{L2}^9 \equiv (L_2^2 + iL_2^1) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^9K_2 = 0$$

$$\psi_{L2}^{10} \equiv (L_2^2 - iL_2^1) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^{10}K_2 = 0$$

$$\psi_{L2}^{11} \equiv (L_2^2 + iL_2^2) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^{11}K_2 = 0$$

$$\psi_{L2}^{12} \equiv (L_2^2 - iL_2^2) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^{12}K_2 = 0$$

$$\psi_{L_2}^{13} \equiv (L_2^2 + iL_2^3) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{13}K_2 = 0$$

$$\psi_{L_2}^{14} \equiv (L_2^2 - iL_2^3) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{14}K_2 = 0$$

$$\psi_{L_2}^{15} \equiv (L_2^2 + iL_2^4) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{15}K_2 = 0$$

$$\psi_{L_2}^{16} \equiv (L_2^2 - iL_2^4) = K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{16}K_2 = 0$$

$$\psi_{L_2}^{17} \equiv (L_2^3 + iL_2^1) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{17}K_2 = 0$$

$$\psi_{L_2}^{18} \equiv (L_2^3 - iL_2^1) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{18}K_2 = 0$$

$$\psi_{L_2}^{19} \equiv (L_2^3 + iL_2^2) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{19}K_2 = 0$$

$$\psi_{L_2}^{20} \equiv (L_2^3 - iL_2^2) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{20}K_2 = 0$$

$$\psi_{L_2}^{21} \equiv (L_2^3 + iL_2^3) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{21}K_2 = 0$$

$$\psi_{L_2}^{22} \equiv (L_2^3 - iL_2^3) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{22}K_2 = 0$$

$$\psi_{L_2}^{23} \equiv (L_2^3 + iL_2^4) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{23}K_2 = 0$$

$$\psi_{L_2}^{24} \equiv (L_2^3 - iL_2^4) = K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{24}K_2 = 0$$

$$\psi_{L_2}^{25} \equiv (L_2^4 + iL_2^1) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{25}K_2 = 0$$

$$\psi_{L_2}^{26} \equiv (L_2^4 - iL_2^1) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} - K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L_2}^{26}K_2 = 0$$

$$\psi_{L_2}^{27} \equiv (L_2^4 + iL_2^2) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L_2}^{27}K_2 = 0$$

$$\psi_{L_2}^{28} \equiv (L_2^4 - iL_2^2) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 + K_3\mathbf{i} + K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^{28} K_2 = 0$$

$$\psi_{L2}^{29} \equiv (L_2^4 + iL_2^3) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^{29} K_2 = 0$$

$$\psi_{L2}^{30} \equiv (L_2^4 - iL_2^3) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} + K_4\mathbf{j} - K_1\mathbf{k})$$

$$\psi_{L2}^{30} K_2 = 0$$

$$\psi_{L2}^{31} \equiv (L_2^4 + iL_2^4) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} + i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^{31} K_2 = 0$$

$$\psi_{L2}^{32} \equiv (L_2^4 - iL_2^4) = K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k} - i(K_2 - K_3\mathbf{i} - K_4\mathbf{j} + K_1\mathbf{k})$$

$$\psi_{L2}^{32} K_2 = 0$$

$$\psi_{L3}$$

$$\psi_{L3}^1 \equiv (L_3^1 + iL_3^1) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^1 K_3 = 0$$

$$\psi_{L3}^2 \equiv (L_3^1 - iL_3^1) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^2 K_3 = 0$$

$$\psi_{L3}^3 \equiv (L_3^1 + iL_3^2) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^3 K_3 = 0$$

$$\psi_{L3}^4 \equiv (L_3^1 - iL_3^2) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^4 K_3 = 0$$

$$\psi_{L3}^5 \equiv (L_3^1 + iL_3^3) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^5 K_3 = 0$$

$$\psi_{L3}^6 \equiv (L_3^1 - iL_3^3) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^6 K_3 = 0$$

$$\psi_{L3}^7 \equiv (L_3^1 + iL_3^4) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^7 K_3 = 0$$

$$\psi_{L3}^8 \equiv (L_3^1 - iL_3^4) = K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^8 K_3 = 0$$

$$\psi_{L3}^9 \equiv (L_3^2 + iL_3^1) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^9 K_3 = 0$$

$$\psi_{L3}^{10} \equiv (L_3^2 - iL_3^1) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^{10} K_3 = 0$$

$$\psi_{L3}^{11} \equiv (L_3^2 + iL_3^2) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$

$$\begin{aligned}
\psi_{L3}^{11}K_3 &= 0 \\
\psi_{L3}^{12} &\equiv (L_3^2 - iL_3^2) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{12}K_3 &= 0 \\
\psi_{L3}^{13} &\equiv (L_3^2 + iL_3^3) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{13}K_3 &= 0 \\
\psi_{L3}^{14} &\equiv (L_3^2 - iL_3^3) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{14}K_3 &= 0 \\
\psi_{L3}^{15} &\equiv (L_3^2 + iL_3^4) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{15}K_3 &= 0 \\
\psi_{L3}^{16} &\equiv (L_3^2 - iL_3^4) = K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{16}K_3 &= 0 \\
\psi_{L3}^{17} &\equiv (L_3^3 + iL_3^1) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{17}K_3 &= 0 \\
\psi_{L3}^{18} &\equiv (L_3^3 - iL_3^1) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{18}K_3 &= 0 \\
\psi_{L3}^{19} &\equiv (L_3^3 + iL_3^2) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{19}K_3 &= 0 \\
\psi_{L3}^{20} &\equiv (L_3^3 - iL_3^2) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{20}K_3 &= 0 \\
\psi_{L3}^{21} &\equiv (L_3^3 + iL_3^3) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{21}K_3 &= 0 \\
\psi_{L3}^{22} &\equiv (L_3^3 - iL_3^3) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{22}K_3 &= 0 \\
\psi_{L3}^{23} &\equiv (L_3^3 + iL_3^4) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{23}K_3 &= 0 \\
\psi_{L3}^{24} &\equiv (L_3^3 - iL_3^4) = K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k}) \\
\psi_{L3}^{24}K_3 &= 0 \\
\psi_{L3}^{25} &\equiv (L_3^4 + iL_3^1) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{25}K_3 &= 0 \\
\psi_{L3}^{26} &\equiv (L_3^4 - iL_3^1) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} - K_1\mathbf{j} - K_4\mathbf{k}) \\
\psi_{L3}^{26}K_3 &= 0
\end{aligned}$$

$$\psi_{L3}^{27} \equiv (L_3^4 + iL_3^2) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^{27}K_3 = 0$$

$$\psi_{L3}^{28} \equiv (L_3^4 - iL_3^2) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 + K_2\mathbf{i} + K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^{28}K_3 = 0$$

$$\psi_{L3}^{29} \equiv (L_3^4 + iL_3^3) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^{29}K_3 = 0$$

$$\psi_{L3}^{30} \equiv (L_3^4 - iL_3^3) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} + K_1\mathbf{j} - K_4\mathbf{k})$$

$$\psi_{L3}^{30}K_3 = 0$$

$$\psi_{L3}^{31} \equiv (L_3^4 + iL_3^4) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} + i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^{31}K_3 = 0$$

$$\psi_{L3}^{32} \equiv (L_3^4 - iL_3^4) = K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k} - i(K_3 - K_2\mathbf{i} - K_1\mathbf{j} + K_4\mathbf{k})$$

$$\psi_{L3}^{32}K_3 = 0$$

$$\psi_{L4}$$

$$\psi_{L4}^1 \equiv (L_4^1 + iL_4^1) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L4}^1K_4 = 0$$

$$\psi_{L4}^2 \equiv (L_4^1 - iL_4^1) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L4}^2K_4 = 0$$

$$\psi_{L4}^3 \equiv (L_4^1 + iL_4^2) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L4}^3K_4 = 0$$

$$\psi_{L4}^4 \equiv (L_4^1 - iL_4^2) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L4}^4K_4 = 0$$

$$\psi_{L4}^5 \equiv (L_4^1 + iL_4^3) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L4}^5K_4 = 0$$

$$\psi_{L4}^6 \equiv (L_4^1 - iL_4^3) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L4}^6K_4 = 0$$

$$\psi_{L4}^7 \equiv (L_4^1 + iL_4^4) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L4}^7K_4 = 0$$

$$\psi_{L4}^8 \equiv (L_4^1 - iL_4^4) = K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L4}^8K_4 = 0$$

$$\psi_{L4}^9 \equiv (L_4^2 + iL_4^1) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L4}^9K_4 = 0$$

$$\psi_{L_4}^{10} \equiv (L_4^2 - iL_4^1) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{10}K_4 = 0$$

$$\psi_{L_4}^{11} \equiv (L_4^2 + iL_4^2) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{11}K_4 = 0$$

$$\psi_{L_4}^{12} \equiv (L_4^2 - iL_4^2) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{12}K_4 = 0$$

$$\psi_{L_4}^{13} \equiv (L_4^2 + iL_4^3) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{13}K_4 = 0$$

$$\psi_{L_4}^{14} \equiv (L_4^2 - iL_4^3) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{14}K_4 = 0$$

$$\psi_{L_4}^{15} \equiv (L_4^2 + iL_4^4) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} + i(K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{15}K_4 = 0$$

$$\psi_{L_4}^{16} \equiv (L_4^2 - iL_4^4) = K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k} - i(K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{16}K_4 = 0$$

$$\psi_{L_4}^{17} \equiv (L_4^3 + iL_4^1) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{17}K_4 = 0$$

$$\psi_{L_4}^{18} \equiv (L_4^3 - iL_4^1) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{18}K_4 = 0$$

$$\psi_{L_4}^{19} \equiv (L_4^3 + iL_4^2) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{19}K_4 = 0$$

$$\psi_{L_4}^{20} \equiv (L_4^3 - iL_4^2) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 + K_1\mathbf{i} + K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{20}K_4 = 0$$

$$\psi_{L_4}^{21} \equiv (L_4^3 + iL_4^3) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{21}K_4 = 0$$

$$\psi_{L_4}^{22} \equiv (L_4^3 - iL_4^3) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{22}K_4 = 0$$

$$\psi_{L_4}^{23} \equiv (L_4^3 + iL_4^4) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} + i(K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{23}K_4 = 0$$

$$\psi_{L_4}^{24} \equiv (L_4^3 - iL_4^4) = K_4 - K_1\mathbf{i} + K_2\mathbf{j} - K_3\mathbf{k} - i(K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k})$$

$$\psi_{L_4}^{24}K_4 = 0$$

$$\psi_{L_4}^{25} \equiv (L_4^4 + iL_4^1) = K_4 - K_1\mathbf{i} - K_2\mathbf{j} + K_3\mathbf{k} + i(K_4 + K_1\mathbf{i} - K_2\mathbf{j} - K_3\mathbf{k})$$

$$\psi_{L_4}^{25} K_4 = 0$$

$$\psi_{L_4}^{26} \equiv (L_4^4 - iL_4^1) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} - i(K_4 + K_1 \mathbf{i} - K_2 \mathbf{j} - K_3 \mathbf{k})$$

$$\psi_{L_4}^{26} K_4 = 0$$

$$\psi_{L_4}^{27} \equiv (L_4^4 + iL_4^2) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} + i(K_4 + K_1 \mathbf{i} + K_2 \mathbf{j} + K_3 \mathbf{k})$$

$$\psi_{L_4}^{27} K_4 = 0$$

$$\psi_{L_4}^{28} \equiv (L_4^4 - iL_4^2) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} - i(K_4 + K_1 \mathbf{i} + K_2 \mathbf{j} + K_3 \mathbf{k})$$

$$\psi_{L_4}^{28} K_4 = 0$$

$$\psi_{L_4}^{29} \equiv (L_4^4 + iL_4^3) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} + i(K_4 - K_1 \mathbf{i} + K_2 \mathbf{j} - K_3 \mathbf{k})$$

$$\psi_{L_4}^{29} K_4 = 0$$

$$\psi_{L_4}^{30} \equiv (L_4^4 - iL_4^3) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} - i(K_4 - K_1 \mathbf{i} + K_2 \mathbf{j} - K_3 \mathbf{k})$$

$$\psi_{L_4}^{30} K_4 = 0$$

$$\psi_{L_4}^{31} \equiv (L_4^4 + iL_4^4) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} + i(K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k})$$

$$\psi_{L_4}^{31} K_4 = 0$$

$$\psi_{L_4}^{32} \equiv (L_4^4 - iL_4^4) = K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k} - i(K_4 - K_1 \mathbf{i} - K_2 \mathbf{j} + K_3 \mathbf{k})$$

$$\psi_{L_4}^{32} K_4 = 0$$

$$\psi_{R1}$$

$$\psi_{R1}^1 \equiv (R_1^1 + R_1^1 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^1 = 0$$

$$\psi_{R1}^2 \equiv (R_1^1 - R_1^1 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^2 = 0$$

$$\psi_{R1}^3 \equiv (R_1^1 + R_1^2 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^3 = 0$$

$$\psi_{R1}^4 \equiv (R_1^1 - R_1^2 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^4 = 0$$

$$\psi_{R1}^5 \equiv (R_1^1 + R_1^3 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^5 = 0$$

$$\psi_{R1}^6 \equiv (R_1^1 - R_1^3 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^6 = 0$$

$$\psi_{R1}^7 \equiv (R_1^1 + R_1^4 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^7 = 0$$

$$\psi_{R1}^8 \equiv (R_1^1 - R_1^4 i) = K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^8 = 0$$

$$\psi_{R1}^9 \equiv (R_1^2 + R_1^1 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^9 = 0$$

$$\psi_{R1}^{10} \equiv (R_1^2 - R_1^1 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^{10} = 0$$

$$\psi_{R1}^{11} \equiv (R_1^2 + R_1^2 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^{11} = 0$$

$$\psi_{R1}^{12} \equiv (R_1^2 - R_1^2 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^{12} = 0$$

$$\psi_{R1}^{13} \equiv (R_1^2 + R_1^3 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$K_1 \psi_{R1}^{13} = 0$$

$$\psi_{R1}^{14} \equiv (R_1^2 - R_1^3 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i$$

$$\begin{aligned}
K_1\psi_{R1}^{14} &= 0 \\
\psi_{R1}^{15} &\equiv (R_1^2 + R_1^4 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{15} &= 0 \\
\psi_{R1}^{16} &\equiv (R_1^2 - R_1^4 i) = K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{16} &= 0 \\
\psi_{R1}^{17} &\equiv (R_1^3 + R_1^1 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{17} &= 0 \\
\psi_{R1}^{18} &\equiv (R_1^3 - R_1^1 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{18} &= 0 \\
\psi_{R1}^{19} &\equiv (R_1^3 + R_1^2 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{19} &= 0 \\
\psi_{R1}^{20} &\equiv (R_1^3 - R_1^2 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{20} &= 0 \\
\psi_{R1}^{21} &\equiv (R_1^3 + R_1^3 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{21} &= 0 \\
\psi_{R1}^{22} &\equiv (R_1^3 - R_1^3 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{22} &= 0 \\
\psi_{R1}^{23} &\equiv (R_1^3 + R_1^4 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{23} &= 0 \\
\psi_{R1}^{24} &\equiv (R_1^3 - R_1^4 i) = K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{24} &= 0 \\
\psi_{R1}^{25} &\equiv (R_1^4 + R_1^1 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{25} &= 0 \\
\psi_{R1}^{26} &\equiv (R_1^4 - R_1^1 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 - \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{26} &= 0 \\
\psi_{R1}^{27} &\equiv (R_1^4 + R_1^2 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{27} &= 0 \\
\psi_{R1}^{28} &\equiv (R_1^4 - R_1^2 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 + \mathbf{i}K_4 + \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1\psi_{R1}^{28} &= 0 \\
\psi_{R1}^{29} &\equiv (R_1^4 + R_1^3 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1\psi_{R1}^{29} &= 0
\end{aligned}$$

$$\begin{aligned}
\psi_{R1}^{30} &\equiv (R_1^4 - R_1^3 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 + \mathbf{j}K_3 - \mathbf{k}K_2)i \\
K_1 \psi_{R1}^{30} &= 0 \\
\psi_{R1}^{31} &\equiv (R_1^4 + R_1^4 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 + (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1 \psi_{R1}^{31} &= 0 \\
\psi_{R1}^{32} &\equiv (R_1^4 - R_1^4 i) = K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2 - (K_1 - \mathbf{i}K_4 - \mathbf{j}K_3 + \mathbf{k}K_2)i \\
K_1 \psi_{R1}^{32} &= 0
\end{aligned}$$

ψ_{R2}

$$\begin{aligned}
\psi_{R2}^1 &\equiv (R_2^1 + R_2^1 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^1 &= 0 \\
\psi_{R2}^2 &\equiv (R_2^1 - R_2^1 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^2 &= 0 \\
\psi_{R2}^3 &\equiv (R_2^1 + R_2^2 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^3 &= 0 \\
\psi_{R2}^4 &\equiv (R_2^1 - R_2^2 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^4 &= 0 \\
\psi_{R2}^5 &\equiv (R_2^1 + R_2^3 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^5 &= 0 \\
\psi_{R2}^6 &\equiv (R_2^1 - R_2^3 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^6 &= 0 \\
\psi_{R2}^7 &\equiv (R_2^1 + R_2^4 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^7 &= 0 \\
\psi_{R2}^8 &\equiv (R_2^1 - R_2^4 i) = K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^8 &= 0 \\
\psi_{R2}^9 &\equiv (R_2^2 + R_2^1 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^9 &= 0 \\
\psi_{R2}^{10} &\equiv (R_2^2 - R_2^1 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{10} &= 0 \\
\psi_{R2}^{11} &\equiv (R_2^2 + R_2^2 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{11} &= 0 \\
\psi_{R2}^{12} &\equiv (R_2^2 - R_2^2 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{12} &= 0
\end{aligned}$$

$$\begin{aligned}
\psi_{R2}^{13} &\equiv (R_2^2 + R_2^3 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{13} &= 0 \\
\psi_{R2}^{14} &\equiv (R_2^2 - R_2^3 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{14} &= 0 \\
\psi_{R2}^{15} &\equiv (R_2^2 + R_2^4 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{15} &= 0 \\
\psi_{R2}^{16} &\equiv (R_2^2 - R_2^4 i) = K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{16} &= 0 \\
\psi_{R2}^{17} &\equiv (R_2^3 + R_2^1 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{17} &= 0 \\
\psi_{R2}^{18} &\equiv (R_2^3 - R_2^1 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{18} &= 0 \\
\psi_{R2}^{19} &\equiv (R_2^3 + R_2^2 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{19} &= 0 \\
\psi_{R2}^{20} &\equiv (R_2^3 - R_2^2 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{20} &= 0 \\
\psi_{R2}^{21} &\equiv (R_2^3 + R_2^3 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{21} &= 0 \\
\psi_{R2}^{22} &\equiv (R_2^3 - R_2^3 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{22} &= 0 \\
\psi_{R2}^{23} &\equiv (R_2^3 + R_2^4 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{23} &= 0 \\
\psi_{R2}^{24} &\equiv (R_2^3 - R_2^4 i) = K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{24} &= 0 \\
\psi_{R2}^{25} &\equiv (R_2^4 + R_2^1 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{25} &= 0 \\
\psi_{R2}^{26} &\equiv (R_2^4 - R_2^1 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 - \mathbf{j}K_4 - \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{26} &= 0 \\
\psi_{R2}^{27} &\equiv (R_2^4 + R_2^2 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i \\
K_2 \psi_{R2}^{27} &= 0 \\
\psi_{R2}^{28} &\equiv (R_2^4 - R_2^2 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 + \mathbf{i}K_3 + \mathbf{j}K_4 + \mathbf{k}K_1)i
\end{aligned}$$

$$K_2\psi_{R2}^{28} = 0$$

$$\psi_{R2}^{29} \equiv (R_2^4 + R_2^3 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i$$

$$K_2\psi_{R2}^{29} = 0$$

$$\psi_{R2}^{30} \equiv (R_2^4 - R_2^3 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 + \mathbf{j}K_4 - \mathbf{k}K_1)i$$

$$K_2\psi_{R2}^{30} = 0$$

$$\psi_{R2}^{31} \equiv (R_2^4 + R_2^4 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 + (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$

$$K_2\psi_{R2}^{31} = 0$$

$$\psi_{R2}^{32} \equiv (R_2^4 - R_2^4 i) = K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1 - (K_2 - \mathbf{i}K_3 - \mathbf{j}K_4 + \mathbf{k}K_1)i$$

$$K_2\psi_{R2}^{32} = 0$$

$$\psi_{R3}$$

$$\psi_{R3}^1 \equiv (R_3^1 + R_3^1 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^1 = 0$$

$$\psi_{R3}^2 \equiv (R_3^1 - R_3^1 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^2 = 0$$

$$\psi_{R3}^3 \equiv (R_3^1 + R_3^2 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^3 = 0$$

$$\psi_{R3}^4 \equiv (R_3^1 - R_3^2 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^4 = 0$$

$$\psi_{R3}^5 \equiv (R_3^1 + R_3^3 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^5 = 0$$

$$\psi_{R3}^6 \equiv (R_3^1 - R_3^3 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^6 = 0$$

$$\psi_{R3}^7 \equiv (R_3^1 + R_3^4 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^7 = 0$$

$$\psi_{R3}^8 \equiv (R_3^1 - R_3^4 i) = K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^8 = 0$$

$$\psi_{R3}^9 \equiv (R_3^2 + R_3^1 i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^9 = 0$$

$$\psi_{R3}^{10} \equiv (R_3^2 - R_3^1 i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i$$

$$K_3\psi_{R3}^{10} = 0$$

$$\psi_{R3}^{11} \equiv (R_3^2 + R_3^2 i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i$$

$$\begin{aligned}
K_3\psi_{R3}^{11} &= 0 \\
\psi_{R3}^{12} &\equiv (R_3^2 - R_3^2i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{12} &= 0 \\
\psi_{R3}^{13} &\equiv (R_3^2 + R_3^3i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{13} &= 0 \\
\psi_{R3}^{14} &\equiv (R_3^2 - R_3^3i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{14} &= 0 \\
\psi_{R3}^{15} &\equiv (R_3^2 + R_3^4i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{15} &= 0 \\
\psi_{R3}^{16} &\equiv (R_3^2 - R_3^4i) = K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{16} &= 0 \\
\psi_{R3}^{17} &\equiv (R_3^3 + R_3^1i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{17} &= 0 \\
\psi_{R3}^{18} &\equiv (R_3^3 - R_3^1i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{18} &= 0 \\
\psi_{R3}^{19} &\equiv (R_3^3 + R_3^2i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{19} &= 0 \\
\psi_{R3}^{20} &\equiv (R_3^3 - R_3^2i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{20} &= 0 \\
\psi_{R3}^{21} &\equiv (R_3^3 + R_3^3i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{21} &= 0 \\
\psi_{R3}^{22} &\equiv (R_3^3 - R_3^3i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{22} &= 0 \\
\psi_{R3}^{23} &\equiv (R_3^3 + R_3^4i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{23} &= 0 \\
\psi_{R3}^{24} &\equiv (R_3^3 - R_3^4i) = K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3\psi_{R3}^{24} &= 0 \\
\psi_{R3}^{25} &\equiv (R_3^4 + R_3^1i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{25} &= 0 \\
\psi_{R3}^{26} &\equiv (R_3^4 - R_3^1i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 - \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3\psi_{R3}^{26} &= 0
\end{aligned}$$

$$\begin{aligned}
\psi_{R3}^{27} &\equiv (R_3^4 + R_3^2 i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3 \psi_{R3}^{27} &= 0 \\
\psi_{R3}^{28} &\equiv (R_3^4 - R_3^2 i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 + \mathbf{i}K_2 + \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3 \psi_{R3}^{28} &= 0 \\
\psi_{R3}^{29} &\equiv (R_3^4 + R_3^3 i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3 \psi_{R3}^{29} &= 0 \\
\psi_{R3}^{30} &\equiv (R_3^4 - R_3^3 i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 + \mathbf{j}K_1 - \mathbf{k}K_4)i \\
K_3 \psi_{R3}^{30} &= 0 \\
\psi_{R3}^{31} &\equiv (R_3^4 + R_3^4 i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 + (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3 \psi_{R3}^{31} &= 0 \\
\psi_{R3}^{32} &\equiv (R_3^4 - R_3^4 i) = K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4 - (K_3 - \mathbf{i}K_2 - \mathbf{j}K_1 + \mathbf{k}K_4)i \\
K_3 \psi_{R3}^{32} &= 0
\end{aligned}$$

ψ_{R4}

$$\begin{aligned}
\psi_{R4}^1 &\equiv (R_4^1 + R_4^1 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^1 &= 0 \\
\psi_{R4}^2 &\equiv (R_4^1 - R_4^1 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^2 &= 0 \\
\psi_{R4}^3 &\equiv (R_4^1 + R_4^2 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^3 &= 0 \\
\psi_{R4}^4 &\equiv (R_4^1 - R_4^2 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^4 &= 0 \\
\psi_{R4}^5 &\equiv (R_4^1 + R_4^3 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^5 &= 0 \\
\psi_{R4}^6 &\equiv (R_4^1 - R_4^3 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^6 &= 0 \\
\psi_{R4}^7 &\equiv (R_4^1 + R_4^4 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^7 &= 0 \\
\psi_{R4}^8 &\equiv (R_4^1 - R_4^4 i) = K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^8 &= 0 \\
\psi_{R4}^9 &\equiv (R_4^2 + R_4^1 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^9 &= 0
\end{aligned}$$

$$\begin{aligned}
\psi_{R4}^{10} &\equiv (R_4^2 - R_4^1 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{10} &= 0 \\
\psi_{R4}^{11} &\equiv (R_4^2 + R_4^2 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{11} &= 0 \\
\psi_{R4}^{12} &\equiv (R_4^2 - R_4^2 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{12} &= 0 \\
\psi_{R4}^{13} &\equiv (R_4^2 + R_4^3 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{13} &= 0 \\
\psi_{R4}^{14} &\equiv (R_4^2 - R_4^3 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{14} &= 0 \\
\psi_{R4}^{15} &\equiv (R_4^2 + R_4^4 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{15} &= 0 \\
\psi_{R4}^{16} &\equiv (R_4^2 - R_4^4 i) = K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{16} &= 0 \\
\psi_{R4}^{17} &\equiv (R_4^3 + R_4^1 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{17} &= 0 \\
\psi_{R4}^{18} &\equiv (R_4^3 - R_4^1 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{18} &= 0 \\
\psi_{R4}^{19} &\equiv (R_4^3 + R_4^2 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{19} &= 0 \\
\psi_{R4}^{20} &\equiv (R_4^3 - R_4^2 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{20} &= 0 \\
\psi_{R4}^{21} &\equiv (R_4^3 + R_4^3 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{21} &= 0 \\
\psi_{R4}^{22} &\equiv (R_4^3 - R_4^3 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{22} &= 0 \\
\psi_{R4}^{23} &\equiv (R_4^3 + R_4^4 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{23} &= 0 \\
\psi_{R4}^{24} &\equiv (R_4^3 - R_4^4 i) = K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4 \psi_{R4}^{24} &= 0 \\
\psi_{R4}^{25} &\equiv (R_4^4 + R_4^1 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i
\end{aligned}$$

$$\begin{aligned}
K_4\psi_{R4}^{25} &= 0 \\
\psi_{R4}^{26} &\equiv (R_4^4 - R_4^1 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 - \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4\psi_{R4}^{26} &= 0 \\
\psi_{R4}^{27} &\equiv (R_4^4 + R_4^2 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4\psi_{R4}^{27} &= 0 \\
\psi_{R4}^{28} &\equiv (R_4^4 - R_4^2 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 + \mathbf{i}K_1 + \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4\psi_{R4}^{28} &= 0 \\
\psi_{R4}^{29} &\equiv (R_4^4 + R_4^3 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4\psi_{R4}^{29} &= 0 \\
\psi_{R4}^{30} &\equiv (R_4^4 - R_4^3 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 + \mathbf{j}K_2 - \mathbf{k}K_3)i \\
K_4\psi_{R4}^{30} &= 0 \\
\psi_{R4}^{31} &\equiv (R_4^4 + R_4^4 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 + (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4\psi_{R4}^{31} &= 0 \\
\psi_{R4}^{32} &\equiv (R_4^4 - R_4^4 i) = K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3 - (K_4 - \mathbf{i}K_1 - \mathbf{j}K_2 + \mathbf{k}K_3)i \\
K_4\psi_{R4}^{32} &= 0
\end{aligned}$$

Symmetry with ψ_{L1} solutions

[1, 2, 3, 4, 9, 10, 11, 12]

$$\begin{aligned}
i\psi_{L1}^1 + \psi_{L1}^2 &= 0 \\
i\psi_{L1}^{11} + \psi_{L1}^{12} &= 0 \\
i\psi_{L1}^3 + \psi_{L1}^{10} &= 0 \\
i\psi_{L1}^9 + \psi_{L1}^4 &= 0
\end{aligned}$$

[1, 2, 11, 12]

$$\begin{aligned}
\mathbf{i}\psi_{L1}^1 - \psi_{L1}^2 &= -2E - 2m + (-2E - 2m)\gamma_t - 21p_x\gamma_x - 21p_y\gamma_y - 21p_z\gamma_z + (-2E - 2m)\gamma_w - 21p_x\gamma_t\gamma_x - 21p_y\gamma_t\gamma_y + 21p_z\gamma_x\gamma_y - 21p_z\gamma_t\gamma_z - 21p_y\gamma_x\gamma_z + 21p_x\gamma_y\gamma_z + (2m + \\
\mathbf{j}\psi_{L1}^1 + \psi_{L1}^{11} &= -2E - 21m\gamma_t + 21E\gamma_w - 21p_x\gamma_t\gamma_x - 21p_y\gamma_t\gamma_y - 21p_z\gamma_x\gamma_y - 21p_z\gamma_t\gamma_z + 21p_y\gamma_x\gamma_z - 21p_x\gamma_y\gamma_z - 21m\gamma_t\gamma_w + 21m\gamma_x\gamma_y\gamma_z + 21p_x\gamma_t\gamma_x\gamma_w + 21p_y\gamma_t\gamma_y\gamma_w + 21p_z\gamma_x\gamma_y\gamma_w \\
\mathbf{k}\psi_{L1}^1 - \psi_{L1}^{12} &= -2m - 21E\gamma_t - 21p_x\gamma_x - 21p_y\gamma_y - 21p_z\gamma_z - 21m\gamma_w + 21E\gamma_t\gamma_w + 21p_x\gamma_x\gamma_w + 21p_y\gamma_y\gamma_w + 21p_z\gamma_z\gamma_w - 21p_z\gamma_t\gamma_x\gamma_y + 21p_y\gamma_t\gamma_x\gamma_z - 21p_x\gamma_t\gamma_y\gamma_z - 21E\gamma_x\gamma_y
\end{aligned}$$

[3, 10]

$$\begin{aligned}
\mathbf{i}\psi_{L1}^3 - \psi_{L1}^3 &= (-2E - 2m)\gamma_w + (2m + 2E)\gamma_t\gamma_w + 21p_x\gamma_x\gamma_w + 21p_y\gamma_y\gamma_w + 21p_z\gamma_z\gamma_w - 21p_x\gamma_t\gamma_x\gamma_w - 21p_y\gamma_t\gamma_y\gamma_w + 21p_z\gamma_x\gamma_y\gamma_w - 21p_z\gamma_t\gamma_z\gamma_w - 21p_y\gamma_x\gamma_z\gamma_w + 21p_x\gamma_y\gamma_z\gamma_w \\
\mathbf{j}\psi_{L1}^3 + \psi_{L1}^{10} &= 2E + 21m\gamma_t + 21E\gamma_w + 21p_x\gamma_t\gamma_x + 21p_y\gamma_t\gamma_y - 21p_z\gamma_x\gamma_y + 21p_z\gamma_t\gamma_z + 21p_y\gamma_x\gamma_z - 21p_x\gamma_y\gamma_z - 21m\gamma_t\gamma_w + 21m\gamma_x\gamma_y\gamma_z + 21p_x\gamma_t\gamma_x\gamma_w + 21p_y\gamma_t\gamma_y\gamma_w + 21p_z\gamma_x\gamma_y\gamma_w \\
\mathbf{k}\psi_{L1}^3 - \psi_{L1}^{10} &= -2E - 21m\gamma_t - 21m\gamma_w - 21p_x\gamma_t\gamma_x - 21p_y\gamma_t\gamma_y + 21p_z\gamma_x\gamma_y - 21p_z\gamma_t\gamma_z - 21p_y\gamma_x\gamma_z + 21p_x\gamma_y\gamma_z + 21E\gamma_t\gamma_w + 21p_x\gamma_x\gamma_w + 21p_y\gamma_y\gamma_w + 21p_z\gamma_z\gamma_w - 21m\gamma_x\gamma_y\gamma_z
\end{aligned}$$

[4, 9]

$$\begin{aligned}\mathbf{i}\psi_{L1}^4 - \psi_{L1}^4 &= (-2E - 2m) \gamma_w + (2m + 2E) \gamma_t \gamma_w + 21p_x \gamma_x \gamma_w + 21p_y \gamma_y \gamma_w + 21p_z \gamma_z \gamma_w - 21p_x \gamma_t \gamma_x \gamma_w - 21p_y \gamma_t \gamma_y \gamma_w + 21p_z \gamma_x \gamma_y \gamma_w - 21p_z \gamma_t \gamma_z \gamma_w - 21p_y \gamma_x \gamma_z \gamma_w + 21p_x \gamma_t \gamma_z \gamma_w \\ \mathbf{j}\psi_{L1}^4 + \psi_{L1}^9 &= -2E - 21m \gamma_t + 21E \gamma_w - 21p_x \gamma_t \gamma_x - 21p_y \gamma_t \gamma_y + 21p_z \gamma_x \gamma_y - 21p_z \gamma_t \gamma_z - 21p_y \gamma_x \gamma_z + 21p_x \gamma_y \gamma_z - 21m \gamma_t \gamma_w - 21m \gamma_x \gamma_y \gamma_z + 21p_x \gamma_t \gamma_x \gamma_w + 21p_y \gamma_t \gamma_y \gamma_w + 21p_z \gamma_x \gamma_y \gamma_w \\ \mathbf{k}\psi_{L1}^4 - \psi_{L1}^9 &= 2E + 21m \gamma_t - 21m \gamma_w + 21p_x \gamma_t \gamma_x + 21p_y \gamma_t \gamma_y - 21p_z \gamma_x \gamma_y + 21p_z \gamma_t \gamma_z + 21p_y \gamma_x \gamma_z - 21p_x \gamma_y \gamma_z + 21E \gamma_t \gamma_w + 21p_x \gamma_x \gamma_w + 21p_y \gamma_y \gamma_w + 21p_z \gamma_z \gamma_w + 21m \gamma_x \gamma_y \gamma_z\end{aligned}$$

[5, 6, 15, 16, 17, 18, 27, 28]

$$i\psi_{L1}^5 + \psi_{L1}^{18} = 0$$

$$i\psi_{L1}^{17} + \psi_{L1}^6 = 0$$

$$i\psi_{L1}^{15} + \psi_{L1}^{28} = 0$$

$$i\psi_{L1}^{27} + \psi_{L1}^{16} = 0$$

[7, 8, 13, 14, 19, 20, 25, 26]

$$i\psi_{L1}^7 + \psi_{L1}^{26} = 0$$

$$i\psi_{L1}^{25} + \psi_{L1}^8 = 0$$

$$i\psi_{L1}^{13} + \psi_{L1}^{20} = 0$$

$$i\psi_{L1}^{19} + \psi_{L1}^{14} = 0$$

[5, 6, 16, 15]

$$\begin{aligned}\mathbf{i}\psi_{L1}^5 - \psi_{L1}^6 &= (-2E - 2m) \gamma_w + (2m + 2E) \gamma_t \gamma_w + 21p_x \gamma_x \gamma_w + 21p_y \gamma_y \gamma_w + 21p_z \gamma_z \gamma_w - 21p_x \gamma_t \gamma_x \gamma_w - 21p_y \gamma_t \gamma_y \gamma_w + 21p_z \gamma_x \gamma_y \gamma_w - 21p_z \gamma_t \gamma_z \gamma_w - 21p_y \gamma_x \gamma_z \gamma_w + 21p_x \gamma_t \gamma_z \gamma_w \\ \mathbf{j}\psi_{L1}^5 + \psi_{L1}^{16} &= -2E - 21m \gamma_t + 21E \gamma_w - 21p_x \gamma_t \gamma_x - 21p_y \gamma_t \gamma_y + 21p_z \gamma_x \gamma_y - 21p_z \gamma_t \gamma_z - 21p_y \gamma_x \gamma_z + 21p_x \gamma_y \gamma_z - 21m \gamma_t \gamma_w - 21m \gamma_x \gamma_y \gamma_z + 21p_x \gamma_t \gamma_x \gamma_w + 21p_y \gamma_t \gamma_y \gamma_w + 21p_z \gamma_x \gamma_y \gamma_w \\ \mathbf{k}\psi_{L1}^5 - \psi_{L1}^{15} &= -2E - 21m \gamma_t - 21m \gamma_w - 21p_x \gamma_t \gamma_x - 21p_y \gamma_t \gamma_y + 21p_z \gamma_x \gamma_y - 21p_z \gamma_t \gamma_z - 21p_y \gamma_x \gamma_z + 21p_x \gamma_y \gamma_z + 21E \gamma_t \gamma_w + 21p_x \gamma_x \gamma_w + 21p_y \gamma_y \gamma_w + 21p_z \gamma_z \gamma_w - 21m \gamma_x \gamma_y \gamma_z\end{aligned}$$

[7, 13]

$$\begin{aligned}\mathbf{i}\psi_{L1}^7 - \psi_{L1}^7 &= 2m - 2E + (2E - 2m) \gamma_t + 21p_x \gamma_x + 21p_y \gamma_y + 21p_z \gamma_z + (-2E - 2m) \gamma_w - 21p_x \gamma_t \gamma_x - 21p_y \gamma_t \gamma_y + 21p_z \gamma_x \gamma_y - 21p_z \gamma_t \gamma_z - 21p_y \gamma_x \gamma_z + 21p_x \gamma_y \gamma_z + (2m + 2E) \gamma_w \\ \mathbf{j}\psi_{L1}^7 + \psi_{L1}^{13} &= 2E + 21m \gamma_t + 21E \gamma_w + 21p_x \gamma_t \gamma_x + 21p_y \gamma_t \gamma_y + 21p_z \gamma_x \gamma_y + 21p_z \gamma_t \gamma_z - 21p_y \gamma_x \gamma_z + 21p_x \gamma_y \gamma_z - 21m \gamma_t \gamma_w - 21m \gamma_x \gamma_y \gamma_z + 21p_x \gamma_t \gamma_x \gamma_w + 21p_y \gamma_t \gamma_y \gamma_w + 21p_z \gamma_x \gamma_y \gamma_w \\ \mathbf{k}\psi_{L1}^7 - \psi_{L1}^{13} &= 2m + 21E \gamma_t + 21p_x \gamma_x + 21p_y \gamma_y + 21p_z \gamma_z - 21m \gamma_w + 21E \gamma_t \gamma_w + 21p_x \gamma_x \gamma_w + 21p_y \gamma_y \gamma_w + 21p_z \gamma_z \gamma_w + 21p_z \gamma_t \gamma_x \gamma_y - 21p_y \gamma_t \gamma_x \gamma_z + 21p_x \gamma_t \gamma_y \gamma_z + 21E \gamma_x \gamma_y \gamma_z\end{aligned}$$

[8, 14]

$$\begin{aligned}\mathbf{i}\psi_{L1}^8 - \psi_{L1}^8 &= 2E - 2m + (2m - 2E) \gamma_t - 21p_x \gamma_x - 21p_y \gamma_y - 21p_z \gamma_z + (-2E - 2m) \gamma_w + 21p_x \gamma_t \gamma_x + 21p_y \gamma_t \gamma_y - 21p_z \gamma_x \gamma_y + 21p_z \gamma_t \gamma_z + 21p_y \gamma_x \gamma_z - 21p_x \gamma_y \gamma_z + (2m + 2E) \gamma_w \\ \mathbf{j}\psi_{L1}^8 + \psi_{L1}^{14} &= -2E - 21m \gamma_t + 21E \gamma_w - 21p_x \gamma_t \gamma_x - 21p_y \gamma_t \gamma_y - 21p_z \gamma_x \gamma_y - 21p_z \gamma_t \gamma_z + 21p_y \gamma_x \gamma_z - 21p_x \gamma_y \gamma_z - 21m \gamma_t \gamma_w + 21m \gamma_x \gamma_y \gamma_z + 21p_x \gamma_t \gamma_x \gamma_w + 21p_y \gamma_t \gamma_y \gamma_w + 21p_z \gamma_x \gamma_y \gamma_w \\ \mathbf{k}\psi_{L1}^8 - \psi_{L1}^{14} &= -2m - 21E \gamma_t - 21p_x \gamma_x - 21p_y \gamma_y - 21p_z \gamma_z - 21m \gamma_w + 21E \gamma_t \gamma_w + 21p_x \gamma_x \gamma_w + 21p_y \gamma_y \gamma_w + 21p_z \gamma_z \gamma_w - 21p_z \gamma_t \gamma_x \gamma_y + 21p_y \gamma_t \gamma_x \gamma_z - 21p_x \gamma_t \gamma_y \gamma_z - 21E \gamma_x \gamma_y \gamma_z\end{aligned}$$

[17, 18, 28, 27]

$$\mathbf{i}\psi_{L1}^{17} - \psi_{L1}^{18} = -2E - 2m + (-2E - 2m) \gamma_t - 21p_x \gamma_x - 21p_y \gamma_y - 21p_z \gamma_z - 21p_x \gamma_t \gamma_x - 21p_y \gamma_t \gamma_y + 21p_z \gamma_x \gamma_y - 21p_z \gamma_t \gamma_z - 21p_y \gamma_x \gamma_z + 21p_x \gamma_y \gamma_z + 21p_z \gamma_t \gamma_x \gamma_y - 21p_y \gamma_t \gamma_x \gamma_z$$

$$\begin{aligned}\mathbf{j}\psi_{L1}^{17}-\psi_{L1}^{28} &= -2E-21m\gamma_t+21E\gamma_w-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y-21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z+21p_y\gamma_x\gamma_z-21p_x\gamma_y\gamma_z-21m\gamma_t\gamma_w+21m\gamma_x\gamma_y\gamma_z+21p_x\gamma_t\gamma_x\gamma_w+21p_y\gamma_t\gamma_y\gamma_w-21p_z\gamma_t\gamma_z\gamma_w \\ \mathbf{k}\psi_{L1}^{17}+\psi_{L1}^{27} &= -2m-21E\gamma_t-21p_x\gamma_x-21p_y\gamma_y-21p_z\gamma_z-21E\gamma_w+21m\gamma_t\gamma_w-21p_z\gamma_t\gamma_x\gamma_y+21p_y\gamma_t\gamma_x\gamma_z-21p_x\gamma_t\gamma_y\gamma_z-21E\gamma_x\gamma_y\gamma_z-21p_x\gamma_t\gamma_x\gamma_w-21p_y\gamma_t\gamma_y\gamma_w+21p_z\gamma_t\gamma_z\gamma_w \\ [19, 25]\end{aligned}$$

$$\begin{aligned}\mathbf{i}\psi_{L1}^{19}-\psi_{L1}^{19} &= 0 \\ \mathbf{j}\psi_{L1}^{19}-\psi_{L1}^{25} &= 2E+21m\gamma_t+21E\gamma_w+21p_x\gamma_t\gamma_x+21p_y\gamma_t\gamma_y-21p_z\gamma_x\gamma_y+21p_z\gamma_t\gamma_z+21p_y\gamma_x\gamma_z-21p_x\gamma_y\gamma_z-21m\gamma_t\gamma_w+21m\gamma_x\gamma_y\gamma_z+21p_x\gamma_t\gamma_x\gamma_w+21p_y\gamma_t\gamma_y\gamma_w-21p_z\gamma_t\gamma_z\gamma_w \\ \mathbf{k}\psi_{L1}^{19}+\psi_{L1}^{25} &= -2E-21m\gamma_t-21E\gamma_w-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y+21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z-21p_y\gamma_x\gamma_z+21p_x\gamma_y\gamma_z+21m\gamma_t\gamma_w-21m\gamma_x\gamma_y\gamma_z-21p_x\gamma_t\gamma_x\gamma_w-21p_y\gamma_t\gamma_y\gamma_w+21p_z\gamma_t\gamma_z\gamma_w \\ [20, 26]\end{aligned}$$

$$\begin{aligned}\mathbf{i}\psi_{L1}^{20}-\psi_{L1}^{20} &= 0 \\ \mathbf{j}\psi_{L1}^{20}-\psi_{L1}^{26} &= -2E-21m\gamma_t+21E\gamma_w-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y+21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z-21p_y\gamma_x\gamma_z+21p_x\gamma_y\gamma_z-21m\gamma_t\gamma_w-21m\gamma_x\gamma_y\gamma_z+21p_x\gamma_t\gamma_x\gamma_w+21p_y\gamma_t\gamma_y\gamma_w-21p_z\gamma_t\gamma_z\gamma_w \\ \mathbf{k}\psi_{L1}^{20}+\psi_{L1}^{26} &= 2E+21m\gamma_t-21E\gamma_w+21p_x\gamma_t\gamma_x+21p_y\gamma_t\gamma_y-21p_z\gamma_x\gamma_y+21p_z\gamma_t\gamma_z+21p_y\gamma_x\gamma_z-21p_x\gamma_y\gamma_z+21m\gamma_t\gamma_w+21m\gamma_x\gamma_y\gamma_z-21p_x\gamma_t\gamma_x\gamma_w-21p_y\gamma_t\gamma_y\gamma_w+21p_z\gamma_t\gamma_z\gamma_w \\ [21, 22, 23, 24, 29, 30, 31, 32]\end{aligned}$$

$$\begin{aligned}i\psi_{L1}^{21}+\psi_{L1}^{22} &= 0 \\ i\psi_{L1}^{31}+\psi_{L1}^{32} &= 0 \\ i\psi_{L1}^{23}+\psi_{L1}^{30} &= 0 \\ i\psi_{L1}^{29}+\psi_{L1}^{24} &= 0 \\ [21, 22, 31, 32]\end{aligned}$$

$$\begin{aligned}\mathbf{i}\psi_{L1}^{21}-\psi_{L1}^{22} &= 0 \\ \mathbf{j}\psi_{L1}^{21}-\psi_{L1}^{31} &= -2E-21m\gamma_t+21E\gamma_w-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y+21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z-21p_y\gamma_x\gamma_z+21p_x\gamma_y\gamma_z-21m\gamma_t\gamma_w-21m\gamma_x\gamma_y\gamma_z+21p_x\gamma_t\gamma_x\gamma_w+21p_y\gamma_t\gamma_y\gamma_w-21p_z\gamma_t\gamma_z\gamma_w \\ \mathbf{k}\psi_{L1}^{21}+\psi_{L1}^{32} &= -2E-21m\gamma_t-21E\gamma_w-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y+21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z-21p_y\gamma_x\gamma_z+21p_x\gamma_y\gamma_z+21m\gamma_t\gamma_w-21m\gamma_x\gamma_y\gamma_z-21p_x\gamma_t\gamma_x\gamma_w-21p_y\gamma_t\gamma_y\gamma_w+21p_z\gamma_t\gamma_z\gamma_w \\ [23, 30]\end{aligned}$$

$$\begin{aligned}\mathbf{i}\psi_{L1}^{23}-\psi_{L1}^{23} &= 2m-2E+(2E-2m)\gamma_t+21p_x\gamma_x+21p_y\gamma_y+21p_z\gamma_z-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y+21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z-21p_y\gamma_x\gamma_z+21p_x\gamma_y\gamma_z-21p_z\gamma_t\gamma_x\gamma_y+21p_y\gamma_t\gamma_x\gamma_z-21p_x\gamma_t\gamma_z\gamma_y \\ \mathbf{j}\psi_{L1}^{23}-\psi_{L1}^{30} &= 2E+21m\gamma_t+21E\gamma_w+21p_x\gamma_t\gamma_x+21p_y\gamma_t\gamma_y+21p_z\gamma_x\gamma_y+21p_z\gamma_t\gamma_z-21p_y\gamma_x\gamma_z+21p_x\gamma_y\gamma_z-21m\gamma_t\gamma_w-21m\gamma_x\gamma_y\gamma_z+21p_x\gamma_t\gamma_x\gamma_w+21p_y\gamma_t\gamma_y\gamma_w-21p_z\gamma_t\gamma_z\gamma_w \\ \mathbf{k}\psi_{L1}^{23}+\psi_{L1}^{30} &= 2m+21E\gamma_t+21p_x\gamma_x+21p_y\gamma_y+21p_z\gamma_z-21E\gamma_w+21m\gamma_t\gamma_w+21p_z\gamma_t\gamma_x\gamma_y-21p_y\gamma_t\gamma_x\gamma_z+21p_x\gamma_t\gamma_y\gamma_z+21E\gamma_x\gamma_y\gamma_z-21p_x\gamma_t\gamma_x\gamma_w-21p_y\gamma_t\gamma_y\gamma_w+21p_z\gamma_t\gamma_z\gamma_w \\ [24, 29]\end{aligned}$$

$$\begin{aligned}\mathbf{i}\psi_{L1}^{24}-\psi_{L1}^{24} &= 2E-2m+(2m-2E)\gamma_t-21p_x\gamma_x-21p_y\gamma_y-21p_z\gamma_z+21p_x\gamma_t\gamma_x+21p_y\gamma_t\gamma_y-21p_z\gamma_x\gamma_y+21p_z\gamma_t\gamma_z+21p_y\gamma_x\gamma_z-21p_x\gamma_y\gamma_z+21p_z\gamma_t\gamma_x\gamma_y-21p_y\gamma_t\gamma_x\gamma_z+21p_x\gamma_t\gamma_z\gamma_y \\ \mathbf{j}\psi_{L1}^{24}-\psi_{L1}^{29} &= -2E-21m\gamma_t+21E\gamma_w-21p_x\gamma_t\gamma_x-21p_y\gamma_t\gamma_y-21p_z\gamma_x\gamma_y-21p_z\gamma_t\gamma_z+21p_y\gamma_x\gamma_z-21p_x\gamma_y\gamma_z-21m\gamma_t\gamma_w+21m\gamma_x\gamma_y\gamma_z+21p_x\gamma_t\gamma_x\gamma_w+21p_y\gamma_t\gamma_y\gamma_w-21p_z\gamma_t\gamma_z\gamma_w \\ \mathbf{k}\psi_{L1}^{24}+\psi_{L1}^{29} &= -2m-21E\gamma_t-21p_x\gamma_x-21p_y\gamma_y-21p_z\gamma_z-21E\gamma_w+21m\gamma_t\gamma_w-21p_z\gamma_t\gamma_x\gamma_y+21p_y\gamma_t\gamma_x\gamma_z-21p_x\gamma_t\gamma_y\gamma_z-21E\gamma_x\gamma_y\gamma_z-21p_x\gamma_t\gamma_x\gamma_w-21p_y\gamma_t\gamma_y\gamma_w+21p_z\gamma_t\gamma_z\gamma_w \\ \text{Rotations } \psi_{L1}\end{aligned}$$

$$\mathbf{i}\psi_{L1}^1\mathbf{i}-\psi_{L4}^{22} = -2m-21E\gamma_t+21p_x\gamma_x+21p_y\gamma_y+21p_z\gamma_z-21m\gamma_w+21E\gamma_t\gamma_w-21p_x\gamma_x\gamma_w-21p_y\gamma_y\gamma_w-21p_z\gamma_z\gamma_w+21p_z\gamma_t\gamma_x\gamma_y-21p_y\gamma_t\gamma_x\gamma_z+21p_x\gamma_t\gamma_y\gamma_z-21E\gamma_x\gamma_y\gamma_z$$

[illegible]

[illegible]

[illegible]