Algebra is $Cl_{1,4}(\mathbb{R})$
The five dimensions are (t, x, y, z, w).
It defines two sets of quaternions with one imaginary unit.
Signature is (+ - - - -)
CoQuaternions $(1, \mathbf{i}, \mathbf{j}, \mathbf{k})$ http://en.wikipedia.org/wiki/Split-quaternion

$$\mathbf{i} = \gamma_t$$

$$\mathbf{j} = \gamma_t\gamma_x\gamma_y\gamma_z$$

$$\mathbf{k} = \gamma_x\gamma_y\gamma_z$$

$$\mathbf{i}^2 = 1$$

$$\mathbf{j}^2 = -1$$

$$\mathbf{k}^2 = 1$$

$$\mathbf{ij} = \gamma_x\gamma_y\gamma_z$$

$$\mathbf{ji} = -\gamma_x\gamma_y\gamma_z$$

$$\mathbf{jk} = \gamma_t$$

$$\mathbf{kj} = -\gamma_t$$

$$\mathbf{ki} = -\gamma_t\gamma_x\gamma_y\gamma_z$$

$$\mathbf{ik} = \gamma_t\gamma_x\gamma_y\gamma_z$$

$$\mathbf{ijk} = 1$$

Quaternions $(1, \boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k})$ http://en.wikipedia.org/wiki/Quaternion

$$\boldsymbol{i} = \gamma_y\gamma_z$$

$$\boldsymbol{j} = -\gamma_x\gamma_z$$

$$\boldsymbol{k} = \gamma_x\gamma_y$$

$$\boldsymbol{i}^2 = -1$$

$$\boldsymbol{j}^2 = -1$$

$$\boldsymbol{k}^2 = -1$$

$$\boldsymbol{ij} = \gamma_x\gamma_y$$

$$\boldsymbol{ji} = -\gamma_x\gamma_y$$

$$\boldsymbol{jk} = \gamma_y\gamma_z$$

$$\boldsymbol{kj} = -\gamma_y\gamma_z$$

$$\boldsymbol{ki} = -\gamma_x\gamma_z$$

$$\boldsymbol{ik} = \gamma_x\gamma_z$$

$$\boldsymbol{ijk} = -1$$

Imaginary unit i

$$i = \gamma_w$$

$$i^2 = -1$$

Gradient definition

$$\nabla = (\gamma_t \frac{\partial}{\partial t} + \gamma_x \frac{\partial}{\partial x} + \gamma_y \frac{\partial}{\partial y} + \gamma_z \frac{\partial}{\partial z} + \gamma_w \frac{\partial}{\partial w})$$

http://www.mrao.cam.ac.uk/ clifford/ptIIIcourse/course99/handouts/hout07.ps.gz
Wave : K is a constant and f is a function of (t, x, y, z, w)

$$\psi = Ke^f$$

First derivative

$$\nabla\psi = \nabla(Ke^f)$$

$$\nabla\psi = (\nabla K)e^f + K\nabla(e^f)$$

$$\nabla K = 0$$

$$\nabla\psi = K(\nabla f)e^f$$

The following symbols are defined :
$E \in \mathbb{R}$
$m \in \mathbb{R}$
$\mathbf{p}$ is defined with $p_x, p_y, p_z \in \mathbb{R}$

$$\mathbf{p} = p_x \boldsymbol{i} + p_y \boldsymbol{j} + p_z \boldsymbol{k}$$

$$\mathbf{p} = p_z \gamma_x \gamma_y - p_y \gamma_x \gamma_z + p_x \gamma_y \gamma_z$$

Exponential function f

$$f = -mw + (p_y y + p_x x + p_z z - Et)\,\gamma_w$$

Gradient for f

$$\nabla f = m\gamma_w - E\gamma_t\gamma_w - p_x\gamma_x\gamma_w - p_y\gamma_y\gamma_w - p_z\gamma_z\gamma_w$$

Square of the gradient

$$\nabla f^2 = -m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2$$

Dirac

$$http : //en.wikipedia.org/wiki/Dirac\_equation$$

$$0 = (\gamma_0 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} + im)\psi$$

With the above gradients, identify the Dirac algebra aka gamma matrices

$$\gamma_0 = \gamma_t \gamma_w$$
$$-i\mathbf{i} = \gamma_t \gamma_w$$
$$\gamma_0^2 = 1$$
$$\gamma_1 = -\gamma_x \gamma_w$$
$$-i\boldsymbol{i}\mathbf{k} = -\gamma_x \gamma_w$$
$$\gamma_1^2 = -1$$
$$\gamma_2 = -\gamma_y \gamma_w$$
$$-i\boldsymbol{j}\mathbf{k} = -\gamma_y \gamma_w$$
$$\gamma_2^2 = -1$$
$$\gamma_3 = -\gamma_z \gamma_w$$
$$-i\boldsymbol{k}\mathbf{k} = -\gamma_z \gamma_w$$
$$\gamma_3^2 = -1$$
$$\gamma_5 = -\gamma_t \gamma_x \gamma_y \gamma_z \gamma_w$$
$$-i\mathbf{j} = -\gamma_t \gamma_x \gamma_y \gamma_z \gamma_w$$
$$\gamma_5^2 = 1$$

Substitution in Dirac equation

$$0 = (-i\mathbf{i}\frac{\partial}{\partial t} - i\boldsymbol{i}\mathbf{k}\frac{\partial}{\partial x} - i\boldsymbol{j}\mathbf{k}\frac{\partial}{\partial y} - i\boldsymbol{k}\mathbf{k}\frac{\partial}{\partial z} + im)\psi$$

Factorization and multiplication

$$0 = \mathbf{j}i(\mathbf{k}\frac{\partial}{\partial t} + \boldsymbol{i}\mathbf{i}\frac{\partial}{\partial x} + \boldsymbol{j}\mathbf{i}\frac{\partial}{\partial y} + \boldsymbol{k}\mathbf{i}\frac{\partial}{\partial z} - \mathbf{j}m)\psi$$

Particular solutions K

$$K_1 = \mathbf{j}i(\mathbf{k}E - \mathbf{j}m + \mathbf{ip})$$
$$K_2 = \mathbf{j}i(\mathbf{k}E + \mathbf{j}m - \mathbf{ip})\mathbf{k}$$
$$K_3 = -\mathbf{j}i(\mathbf{k}E + \mathbf{j}m + \mathbf{ip})\mathbf{j}$$
$$K_4 = -\mathbf{j}i(\mathbf{k}E - \mathbf{j}m - \mathbf{ip})\mathbf{i}$$
$$K_5 = \mathbf{j}i(\mathbf{k}E - \mathbf{j}m + \mathbf{ip})\mathbf{j}i$$
$$K_6 = -\mathbf{j}i(\mathbf{k}E + \mathbf{j}m + \mathbf{ip})i$$
$$K_7 = \mathbf{j}i(\mathbf{k}E + \mathbf{j}m - \mathbf{ip})\mathbf{i}i$$
$$K_8 = -\mathbf{j}i(\mathbf{k}E - \mathbf{j}m - \mathbf{ip})\mathbf{k}i$$

Check first derivative is null $K\nabla f$

$$K_1\nabla f = -m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2$$
$$K_2\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_x\gamma_y\gamma_z$$
$$K_3\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_t\gamma_x\gamma_y\gamma_z$$
$$K_4\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_t$$
$$K_5\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_t\gamma_x\gamma_y\gamma_z\gamma_w$$
$$K_6\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_w$$
$$K_7\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_t\gamma_w$$
$$K_8\nabla f = \left(-m^2 - {p_z}^2 + E^2 - {p_y}^2 - {p_x}^2\right)\gamma_x\gamma_y\gamma_z\gamma_w$$

8 previous solutions can be combined linearly together and factorized, this gives idempotents !
Some idempotents in $Cl_{1,4}(\mathbb{R})$

$$Id_1 = \frac{1}{2}(1 + \mathbf{j}i)\frac{1}{2}(1 - \mathbf{i} + \mathbf{j} - \mathbf{k})$$
$$Id_1 Id_1 - Id_1 = 0$$
$$Id_2 = \frac{1}{2}(1 - \mathbf{j}i)\frac{1}{2}(1 - \mathbf{i} + \mathbf{j} - \mathbf{k})$$
$$Id_2 Id_2 - Id_2 = 0$$

$$Id_3 = \frac{1}{2}(1 + \mathbf{j}i)\frac{1}{2}(1 - \mathbf{i} - \mathbf{j} + \mathbf{k})$$
$$Id_3 Id_3 - Id_3 = 0$$
$$Id_4 = \frac{1}{2}(1 - \mathbf{j}i)\frac{1}{2}(1 - \mathbf{i} - \mathbf{j} + \mathbf{k})$$
$$Id_4 Id_4 - Id_4 = 0$$
$$Id_5 = \frac{1}{2}(1 + \mathbf{j}i)\frac{1}{2}(1 + \mathbf{i} - \mathbf{j} - \mathbf{k})$$
$$Id_5 Id_5 - Id_5 = 0$$
$$Id_6 = \frac{1}{2}(1 - \mathbf{j}i)\frac{1}{2}(1 + \mathbf{i} - \mathbf{j} - \mathbf{k})$$
$$Id_6 Id_6 - Id_6 = 0$$
$$Id_7 = \frac{1}{2}(1 + \mathbf{j}i)\frac{1}{2}(1 + \mathbf{i} + \mathbf{j} + \mathbf{k})$$
$$Id_7 Id_7 - Id_7 = 0$$
$$Id_8 = \frac{1}{2}(1 - \mathbf{j}i)\frac{1}{2}(1 + \mathbf{i} + \mathbf{j} + \mathbf{k})$$
$$Id_8 Id_8 - Id_8 = 0$$