ALGEBRA

Algebra playground is Clifford $Cl_{1,4}(\mathbb{R})$, hence signature is $(+----)$. The five dimensions are (t,x,y,z,w). It defines two sets of quaternions with one imaginary unit.
Associative Hyperbolic Quaternions $(1,\mathbf{i},\mathbf{j},\mathbf{k})$

$$\mathbf{i}=\gamma_t$$
$$\mathbf{j}=\gamma_t\wedge\gamma_x\wedge\gamma_y\wedge\gamma_z\wedge\gamma_w$$
$$\mathbf{k}=\gamma_x\wedge\gamma_y\wedge\gamma_z\wedge\gamma_w$$
$$\mathbf{i}^2=1$$
$$\mathbf{j}^2=1$$
$$\mathbf{k}^2=1$$
$$\mathbf{ij}=\gamma_x\wedge\gamma_y\wedge\gamma_z\wedge\gamma_w$$
$$\mathbf{ji}=\gamma_x\wedge\gamma_y\wedge\gamma_z\wedge\gamma_w$$
$$\mathbf{jk}=\gamma_t$$
$$\mathbf{kj}=\gamma_t$$
$$\mathbf{ki}=\gamma_t\wedge\gamma_x\wedge\gamma_y\wedge\gamma_z\wedge\gamma_w$$
$$\mathbf{ik}=\gamma_t\wedge\gamma_x\wedge\gamma_y\wedge\gamma_z\wedge\gamma_w$$
$$\mathbf{ijk}=1$$

Quaternions $(1,\boldsymbol{i},\boldsymbol{j},\boldsymbol{k})$ http://en.wikipedia.org/wiki/Quaternion

$$\boldsymbol{i}=\gamma_y\wedge\gamma_z$$
$$\boldsymbol{j}=-1\gamma_x\wedge\gamma_z$$
$$\boldsymbol{k}=\gamma_x\wedge\gamma_y$$
$$\boldsymbol{i}^2=-1$$
$$\boldsymbol{j}^2=-1$$
$$\boldsymbol{k}^2=-1$$
$$\boldsymbol{ij}=\gamma_x\wedge\gamma_y$$
$$\boldsymbol{ji}=-1\gamma_x\wedge\gamma_y$$
$$\boldsymbol{jk}=\gamma_y\wedge\gamma_z$$
$$\boldsymbol{kj}=-1\gamma_y\wedge\gamma_z$$
$$\boldsymbol{ki}=-1\gamma_x\wedge\gamma_z$$
$$\boldsymbol{ik}=\gamma_x\wedge\gamma_z$$
$$\boldsymbol{ijk}=-1$$

Imaginary unit i

$$i=\gamma_w$$
$$i^2=-1$$

Gradient definition

$$\nabla=(\gamma_t\frac{\partial}{\partial t}+\gamma_x\frac{\partial}{\partial x}+\gamma_y\frac{\partial}{\partial y}+\gamma_z\frac{\partial}{\partial z}+\gamma_w\frac{\partial}{\partial w})$$

http://www.mrao.cam.ac.uk/ clifford/ptIIIcourse/course99/handouts/hout07.ps.gz
Idempotents in $Cl_{1,4}(\mathbb{R})$

$$U_1:\frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1-\mathbf{i}i+i-\mathbf{k})\Rightarrow U_1U_1-U_1=0$$

$$U_2 : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1-\mathbf{i}i+i-\mathbf{k}) \Rightarrow U_2U_2 - U_2 = 0$$

$$U_3 : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1-\mathbf{i}i-i+\mathbf{k}) \Rightarrow U_3U_3 - U_3 = 0$$

$$U_4 : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1-\mathbf{i}i-i+\mathbf{k}) \Rightarrow U_4U_4 - U_4 = 0$$

$$U_5 : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1+\mathbf{i}i-i-\mathbf{k}) \Rightarrow U_5U_5 - U_5 = 0$$

$$U_6 : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1+\mathbf{i}i-i-\mathbf{k}) \Rightarrow U_6U_6 - U_6 = 0$$

$$U_7 : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1+\mathbf{i}i+i+\mathbf{k}) \Rightarrow U_7U_7 - U_7 = 0$$

$$U_8 : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1+\mathbf{i}i+i+\mathbf{k}) \Rightarrow U_8U_8 - U_8 = 0$$

$$U_9 : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1+\mathbf{i}i-i+\mathbf{k}) \Rightarrow U_9U_9 - U_9 = 0$$

$$U_{10} : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1+\mathbf{i}i-i+\mathbf{k}) \Rightarrow U_{10}U_{10} - U_{10} = 0$$

$$U_{11} : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1+\mathbf{i}i+i-\mathbf{k}) \Rightarrow U_{11}U_{11} - U_{11} = 0$$

$$U_{12} : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1+\mathbf{i}i+i-\mathbf{k}) \Rightarrow U_{12}U_{12} - U_{12} = 0$$

$$U_{13} : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1-\mathbf{i}i+i+\mathbf{k}) \Rightarrow U_{13}U_{13} - U_{13} = 0$$

$$U_{14} : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1-\mathbf{i}i+i+\mathbf{k}) \Rightarrow U_{14}U_{14} - U_{14} = 0$$

$$U_{15} : \frac{1}{2}(1+\mathbf{j})\frac{1}{2}(1-\mathbf{i}i-i-\mathbf{k}) \Rightarrow U_{15}U_{15} - U_{15} = 0$$

$$U_{16} : \frac{1}{2}(1-\mathbf{j})\frac{1}{2}(1-\mathbf{i}i-i-\mathbf{k}) \Rightarrow U_{16}U_{16} - U_{16} = 0$$

PHYSICS
The following symbols are defined :
Energy $E \in \mathbb{R}$
Mass $m \in \mathbb{R}$
Momentum $\mathbf{p}$ is defined with $p_x, p_y, p_z \in \mathbb{R}$

$$\mathbf{p} = p_x\boldsymbol{i} + p_y\boldsymbol{j} + p_z\boldsymbol{k}$$

$$\mathbf{p} = p_z\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - p_y\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + p_x\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$$

Electric field $\mathbf{E}$ is defined with $E_x, E_y, E_z \in \mathbb{R}$

$$\mathbf{E} = E_x\boldsymbol{i} + E_y\boldsymbol{j} + E_z\boldsymbol{k}$$

$$\mathbf{E} = E_z\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - E_y\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + E_x\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$$

Magnetic field $\mathbf{B}$ is defined with $B_x, B_y, B_z \in \mathbb{R}$

$$\mathbf{B} = B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k}$$

$$\mathbf{B} = B_z\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - B_y\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + B_x\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$$

Wavefunction : K is a constant and f is a function of (t, x, y, z, w)

$$\psi = Ke^f$$

First derivative

$$\nabla\psi = \nabla(Ke^f)$$

$$\nabla\psi = (\nabla K)e^f + K\nabla(e^f)$$

$$\nabla K = 0$$

$$\nabla\psi_L = (\nabla f)Ke^f$$

$$\nabla\psi_R = Ke^f(\nabla f)$$

$$\nabla\psi_R = K(\nabla f)e^f$$

$$K(\nabla f) = (\nabla f)K = 0$$

Find solutions that fullfills left and right multiplication
f is the exponential function

$$f = -mw + (E_x x + E_y y + E_z z)\,\boldsymbol{\gamma_t} - B_y z\boldsymbol{\gamma_x} - B_z x\boldsymbol{\gamma_y} - B_x y\boldsymbol{\gamma_z} + (-Et + p_x x + p_y y + p_z z)\,\boldsymbol{\gamma_w}$$

DIRAC
http://en.wikipedia.org/wiki/Dirac_equation

$$0 = (\gamma_0 \frac{\partial}{\partial t} + \gamma_1 \frac{\partial}{\partial x} + \gamma_2 \frac{\partial}{\partial y} + \gamma_3 \frac{\partial}{\partial z} + im)\psi$$

Define the Dirac algebra (aka gamma matrices) with the two sets of quaternions

$$\gamma_0 = -1\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w}$$
$$-\mathbf{i}i = -1\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w}$$
$$\gamma_0^2 = 1$$
$$\gamma_1 = \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w}$$
$$-\boldsymbol{i}\mathbf{k} = \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w}$$
$$\gamma_1^2 = -1$$
$$\gamma_2 = \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w}$$
$$-\boldsymbol{j}\mathbf{k} = \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w}$$
$$\gamma_2^2 = -1$$
$$\gamma_3 = \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$
$$-\boldsymbol{k}\mathbf{k} = \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$
$$\gamma_3^2 = -1$$
$$\gamma_5 = -1\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$
$$-\mathbf{j} = -1\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$
$$\gamma_5^2 = 1$$

Substitution in Dirac equation

$$-i\frac{\partial}{\partial w}\psi = (-\mathbf{i}i\frac{\partial}{\partial t} - \boldsymbol{i}\mathbf{k}\frac{\partial}{\partial x} - \boldsymbol{j}\mathbf{k}\frac{\partial}{\partial y} - \boldsymbol{k}\mathbf{k}\frac{\partial}{\partial z})\psi$$

Exponential function with energy and momentum only, electric and magnetic fields are null f

$$f = -mw + (-Et + p_x x + p_y y + p_z z)\,\boldsymbol{\gamma_w}$$

Gradient for f

$$\nabla f = m\boldsymbol{\gamma_w} - E\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w} - p_x\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} - p_y\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} - p_z\boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$

Square of the gradient is a Lorentz invariant

$$\nabla f^2 = E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2$$

Particular solutions K

$$K_1 = (-\mathbf{i}iE + im + \mathbf{kp})$$
$$K_2 = -(-\mathbf{i}iE + im - \mathbf{kp})\mathbf{k}$$
$$K_3 = (-\mathbf{i}iE + im + \mathbf{kp})\mathbf{j}$$
$$K_4 = (-\mathbf{i}iE - im - \mathbf{kp})\mathbf{i}i$$
$$K_5 = (-\mathbf{i}iE - im + \mathbf{kp})\mathbf{j}i$$
$$K_6 = -(-\mathbf{i}iE - im + \mathbf{kp})i$$
$$K_7 = (-\mathbf{i}iE + im - \mathbf{kp})\mathbf{i}$$
$$K_8 = -(-\mathbf{i}iE - im - \mathbf{kp})\mathbf{k}i$$

Check first derivative is null $K\nabla f$

$$K_1\nabla f = E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2$$
$$K_2\nabla f = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$K_3\nabla f = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$K_4\nabla f = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_w}$$
$$K_5\nabla f = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}$$
$$K_6\nabla f = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_w}$$
$$K_7\nabla f = \left(-E^2 + m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_t}$$
$$K_8\nabla f = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}$$

Check first derivative is null $\nabla f K$

$$\nabla f K_1 = E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2$$
$$\nabla f K_2 = 2mp_z\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} - 2mp_y\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} + 2mp_x\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} - 2Ep_z\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} + 2Ep_y\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} - 2Ep_x\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} + \left(-E^2 + m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$\nabla f K_3 = \left(E^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$\nabla f K_4 = -2Em\boldsymbol{\gamma_w} + \left(E^2 + m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_w} + 2Ep_x\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_w} + 2Ep_y\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} + 2Ep_z\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$\nabla f K_5 = 2mp_z\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y} - 2mp_y\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z} + 2mp_x\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} + 2Em\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} + \left(-E^2 - m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}$$
$$\nabla f K_6 = \left(-E^2 - m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_w} + 2Em\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_w} + 2mp_x\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_w} + 2mp_y\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} + 2mp_z\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$\nabla f K_7 = \left(E^2 - m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_t} + 2Ep_x\boldsymbol{\gamma_x} + 2Ep_y\boldsymbol{\gamma_y} + 2Ep_z\boldsymbol{\gamma_z} - 2mp_x\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x} - 2mp_y\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y} - 2mp_z\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_z}$$
$$\nabla f K_8 = 2Ep_z\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y} - 2Ep_y\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z} + 2Ep_x\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} + \left(E^2 + m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} - 2Em\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}$$

Right multiplication, left multiplication

$$U_1 = \frac{1}{4} - \frac{1}{4}\boldsymbol{\gamma_t} + \frac{1}{4}\boldsymbol{\gamma_w} - \frac{1}{4}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_w} + \frac{1}{4}\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} - \frac{1}{4}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} - \frac{1}{4}\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} + \frac{1}{4}\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}$$
$$\mathbf{j}U_1 - U_1 = 0$$
$$U_1\mathbf{j} - U_1 = 0$$
$$K_3 - K_1\mathbf{j} = 0$$
$$K_3 - \mathbf{j}K_1 = 0$$

$$\begin{aligned}
U_1K_1\nabla f = &\left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t} + \left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_w}\\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_w}\\
&+ \left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} + \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}\\
&+ \left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}
\end{aligned}$$

$$
\begin{aligned}
\nabla f K_1 U_1 = &\left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right) \\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_t} + \left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_w} \\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w} \\
&+ \left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} + \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} \\
&+ \left(\frac{E^2}{4} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}
\end{aligned}
$$

$$
\begin{aligned}
U_1 K_1 + K_1 U_1 = &\left(\frac{E}{2} - \frac{m}{2}\right) \\
&+ \frac{m}{2}\boldsymbol{\gamma_w} \\
&- \frac{E}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w} - \frac{p_z}{2}\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} + \frac{p_y}{2}\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} - \frac{p_x}{2}\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} - \frac{p_x}{2}\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} - \frac{p_y}{2}\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} - \frac{p_z}{2}\boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} \\
&+ \frac{p_z}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - \frac{p_y}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + \frac{p_x}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} + \frac{p_x}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} + \frac{p_y}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} + \frac{p_z}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + \frac{E}{2}\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
&- \frac{m}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \\
&+ \left(\frac{E}{2} - \frac{m}{2}\right) \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}
\end{aligned}
$$

16 idempotents, 16 solutions ?

$$
\psi = U_1 K_1 e^f
$$

Dirac observables handout 11 chapter 3.1

$$\psi^\dagger = -e^{-f} K_1 U_1$$
$$\rho = \psi^\dagger \psi = -e^{-f} K_1 U_1 U_1 K_1 e^f$$
$$\rho = \psi^\dagger \psi = -e^{-f} K_1 U_1 K_1 e^f$$

this is the definition of a rotation :

$$e^{-f} e^f = 1$$

$$-K_1 U_1 K_1 = \Phi$$

$$\begin{aligned}
\Phi = &\left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right) \\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t} - \frac{Ep_x}{2}\boldsymbol{\gamma_x} - \frac{Ep_y}{2}\boldsymbol{\gamma_y} - \frac{Ep_z}{2}\boldsymbol{\gamma_z} + \left(\frac{E^2}{4} - \frac{Em}{2} + \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_w} \\
&+ \frac{mp_x}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} + \frac{mp_y}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} + \frac{mp_z}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} + \left(\frac{E^2}{4} - \frac{Em}{2} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w} + \frac{p_x}{2}(E-m)\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} + \frac{p_y}{2}(E-m)\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} + \frac{p_z}{2}(E-m)\boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} \\
&+ \frac{p_z}{2}(-E+m)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} + \frac{p_y}{2}(E-m)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + \frac{p_x}{2}(-E+m)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} + \left(-\frac{E^2}{4} + \frac{Em}{2} - \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} + \frac{mp_z}{2}\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} - \frac{mp_y}{2}\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + \frac{mp_x}{2}\boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \\
&+ \left(-\frac{E^2}{4} + \frac{Em}{2} - \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} - \frac{Ep_z}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} + \frac{Ep_y}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} - \frac{Ep_x}{2}\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + \left(-\frac{E^2}{4} + \frac{m^2}{4} - \frac{1}{4}(p_x)^2 - \frac{1}{4}(p_y)^2 - \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \\
&+ \left(-\frac{E^2}{4} + \frac{m^2}{4} + \frac{1}{4}(p_x)^2 + \frac{1}{4}(p_y)^2 + \frac{1}{4}(p_z)^2\right)\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}
\end{aligned}$$

MAXWELL
Exponential function with electric and magnetic fields only f

$$f = (E_x x + E_y y + E_z z)\,\boldsymbol{\gamma_t} - B_y z \boldsymbol{\gamma_x} - B_z x \boldsymbol{\gamma_y} - B_x y \boldsymbol{\gamma_z}$$

Gradient for f

$$\nabla f = E_x \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} + E_y \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} + E_z \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} + B_z \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - B_y \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + B_x \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$$

Square of the gradient is a Lorentz invariant

$$\nabla f^2 = \left(-(B_x)^2 - (B_y)^2 - (B_z)^2 + (E_x)^2 + (E_y)^2 + (E_z)^2\right) + (2B_x E_x + 2B_y E_y + 2B_z E_z)\,\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$$

DIRAC MAXWELL

∇f is the electromagnetic field F with a Dirac component

$$f = -mw + (E_x x + E_y y + E_z z)\,\boldsymbol{\gamma_t} - B_y z \boldsymbol{\gamma_x} - B_z x \boldsymbol{\gamma_y} - B_x y \boldsymbol{\gamma_z} + (-Et + p_x x + p_y y + p_z z)\,\boldsymbol{\gamma_w}$$

$$F_{DiracMaxwell} : \nabla f = m\boldsymbol{\gamma_w} + E_x \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} + E_y \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} + E_z \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} - E\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w} + B_z \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - B_y \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} - p_x \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} + B_x \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} - p_y \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} - p_z \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$

$$F_{Maxwell} = -i(i\mathbf{B} - \mathbf{jE})$$

$$F_{Maxwell} = E_x \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} + E_y \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} + E_z \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} + B_z \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} - B_y \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} + B_x \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z}$$

$$F_{Dirac} = (\mathbf{kp} - \mathbf{i}iE)$$

$$F_{Dirac} = m\boldsymbol{\gamma_w} - E\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_w} - p_x \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} - p_y \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} - p_z \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w}$$

Square of the gradient is a Lorentz invariant

$$\begin{aligned}
\nabla f^2 = &\left(-(B_x)^2 - (B_y)^2 - (B_z)^2 + E^2 + (E_x)^2 + (E_y)^2 + (E_z)^2 - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right) \\
&+ 2E_x m \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_w} + 2E_y m \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} + 2E_z m \boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + 2B_z m \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} - 2B_y m \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + 2B_x m \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} \\
&+ (2B_x E_x + 2B_y E_y + 2B_z E_z)\,\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} + (-2B_z E - 2E_x p_y + 2E_y p_x)\,\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_w} + (2B_y E - 2E_x p_z + 2E_z p_x)\,\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_x} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + (-2B_x E - 2E_y p_z + 2E_z p_y)\,\boldsymbol{\gamma_t} \wedge \boldsymbol{\gamma_y} \wedge \boldsymbol{\gamma_z} \wedge \boldsymbol{\gamma_w} + (-2B_x p_x - 2
\end{aligned}$$

DIRAC COUPLING

$$-(im + \mathbf{i}iq\phi + \mathbf{k}q\mathbf{A})\psi = (-\mathbf{i}i\frac{\partial}{\partial t} - \boldsymbol{i}\mathbf{k}\frac{\partial}{\partial x} - \boldsymbol{j}\mathbf{k}\frac{\partial}{\partial y} - \boldsymbol{k}\mathbf{k}\frac{\partial}{\partial z})\psi$$

Exponential function with energy and momentum only, and a potential that depends on w
Electric and magnetic field are null

$$f = \frac{qw}{4}\left(-E^2 + m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right) + \frac{qw}{4}\left(-E^2 + m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_t} + \frac{qw}{2}\left(2A_x - mp_x\right)\boldsymbol{\gamma_x} + \frac{qw}{2}\left(2A_y - mp_y\right)\boldsymbol{\gamma_y} + \frac{qw}{2}\left(2A_z - mp_z\right)\boldsymbol{\gamma_z} + \left(-\frac{qw}{4}E^2 + \frac{Em}{2}qw - Et - \frac{qw}{4}m^2 - \frac{qw}{4}(p_x)^2 + p_x x - \frac{qw}{4}(p_y)^2 + p_y$$

Gradient for f

$$\nabla f = \frac{q}{4}\left(-E^2 + 2Em - m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right) + \frac{q}{4}\left(E^2 - 2Em + m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right)\boldsymbol{\gamma_t} + \frac{q}{4}\left(E^2 - m^2 + (p_x)^2 + (p_y)^2 + (p_z)^2\right)\boldsymbol{\gamma_w} + \frac{p_x q}{2}(E - m)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x} + \frac{p_y q}{2}(E - m)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y} + \frac{p_z q}{2}(E - m)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_z} + \left(-\frac{E^2 q}{4} - E$$

Full exponential function

$$f = \frac{qw}{4}\left(-E^2 + m^2 - (p_x)^2 - (p_y)^2 - (p_z)^2\right) + \left(-\frac{qw}{4}E^2 + E_x x + E_y y + E_z z + \frac{qw}{4}m^2 + \frac{qw}{4}(p_x)^2 + \frac{qw}{4}(p_y)^2 + \frac{qw}{4}(p_z)^2\right)\boldsymbol{\gamma_t} + \left(A_x qw - B_y z - \frac{mp_x}{2}qw\right)\boldsymbol{\gamma_x} + \left(A_y qw - B_z x - \frac{mp_y}{2}qw\right)\boldsymbol{\gamma_y} + \left(A_z qw - B_x y - \frac{mp_z}{2}qw\right)\boldsymbol{\gamma_z} + \left(-\frac{qw}{4}\right.$$

Gradient for f

$$
\begin{aligned}
\nabla f =& \frac{q}{4}\left(-E^2+2Em-m^2-(p_x)^2-(p_y)^2-(p_z)^2\right) \\
&+\frac{q}{4}\left(E^2-2Em+m^2-(p_x)^2-(p_y)^2-(p_z)^2\right)\boldsymbol{\gamma_t} \\
&+\frac{q}{4}\left(E^2-m^2+(p_x)^2+(p_y)^2+(p_z)^2\right)\boldsymbol{\gamma_w} \\
&+\left(\frac{Ep_x}{2}q+E_x-\frac{mp_x}{2}q\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x} \\
&+\left(\frac{Ep_y}{2}q+E_y-\frac{mp_y}{2}q\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y} \\
&+\left(\frac{Ep_z}{2}q+E_z-\frac{mp_z}{2}q\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_z} \\
&+\left(-\frac{E^2q}{4}-E+\frac{m^2q}{4}+\frac{q}{4}(p_x)^2+\frac{q}{4}(p_y)^2+\frac{q}{4}(p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_w} \\
&+\left(B_z+\frac{Ep_z}{2}q\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y} \\
&+\left(-B_y-\frac{Ep_y}{2}q\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z} \\
&+\left(A_xq-\frac{mp_x}{2}q-p_x\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_w} \\
&+\left(B_x+\frac{Ep_x}{2}q\right)\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} \\
&+\left(A_yq-\frac{mp_y}{2}q-p_y\right)\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} \\
&+\left(A_zq-\frac{mp_z}{2}q-p_z\right)\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} \\
&+\frac{mp_z}{2}q\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y} \\
&-\frac{mp_y}{2}q\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z} \\
&-\frac{Ep_x}{2}q\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_w} \\
&+\frac{mp_x}{2}q\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} \\
&-\frac{Ep_y}{2}q\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} \\
&-\frac{Ep_z}{2}q\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} \\
&+\frac{q}{4}\left(E^2-m^2-(p_x)^2-(p_y)^2-(p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} \\
&+\frac{p_zq}{2}\left(E-m\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_w} \\
&+\frac{p_yq}{2}\left(-E+m\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} \\
&+\frac{p_xq}{2}\left(E-m\right)\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} \\
&+\frac{q}{4}\left(-E^2+m^2-(p_x)^2-(p_y)^2-(p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z} \\
&+\frac{q}{4}\left(E^2-2Em+m^2-(p_x)^2-(p_y)^2-(p_z)^2\right)\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w} \\
&+\frac{q}{4}\left(-E^2+2Em-m^2-(p_x)^2-(p_y)^2-(p_z)^2\right)\boldsymbol{\gamma_t}\wedge\boldsymbol{\gamma_x}\wedge\boldsymbol{\gamma_y}\wedge\boldsymbol{\gamma_z}\wedge\boldsymbol{\gamma_w}
\end{aligned}
$$