1 Exercise 2.68 p.96: Entanglement

Prove that $|\phi\rangle := \frac{|00\rangle+|11\rangle}{\sqrt{2}} \neq |a\rangle|b\rangle$ for all single qubit states $|a\rangle$ and $|b\rangle$.

$$
\begin{aligned}
|a\rangle|b\rangle &= (a_0|0\rangle + a_1|1\rangle) \otimes (b_0|0\rangle + b_1|1\rangle) = a_0b_0|00\rangle + a_1b_1|11\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle \\
&= |\phi\rangle \Rightarrow a_0b_1 = 0 \quad \text{or} \quad a_1b_0 = 0
\end{aligned}
$$

but if one of the coefficients is null, then a_ob_0 or a_1b_1 will be null as well, and we cannot obtain $|\phi\rangle$.

2 Exercise 4.5 p.175

$$
\begin{aligned}
(\hat{n}\cdot\vec{\sigma}) &= \begin{pmatrix} a \\ b \\ c \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = aX + bY + cZ \\
&= a\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + b\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} + c\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} c & (a-b) \\ (a+ib) & -c \end{pmatrix}
\end{aligned}
$$

$$
\begin{aligned}
(\hat{n}\cdot\vec{\sigma})^2 &= \begin{pmatrix} c & (a-b) \\ (a+ib) & -c \end{pmatrix} \begin{pmatrix} c & (a-b) \\ (a+ib) & -c \end{pmatrix} \\
&= \begin{pmatrix} c^2+a^2+b^2 & 0 \\ 0 & c^2+a^2+b^2 \end{pmatrix} = I
\end{aligned}
$$