Let $\alpha = 1/\sqrt{2}$

1 Eig(X)

$$|I - \lambda X| = \left| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right| = \begin{vmatrix} 1 & -\lambda \\ -\lambda & 1 \end{vmatrix} = 1 - \lambda^2 \Rightarrow \lambda = \pm 1$$

$$Xv = 1 \cdot v \Leftrightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} b = a \\ a = b \end{cases} \rightarrow v = \alpha \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Xv = 1 \cdot v \Leftrightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = - \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} b = -a \\ a = -b \end{cases} \rightarrow v = \alpha \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

2 Eig(Y)

$$|I - \lambda Y| = \left| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right| = \begin{vmatrix} 1 & \lambda i \\ -\lambda i & 1 \end{vmatrix} = 1 - \lambda^2 \Rightarrow \lambda = \pm 1$$

$$Yv = 1 \cdot v \Leftrightarrow \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} -ib = a \\ ia = b \end{cases} \rightarrow v = \alpha \begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$Yv = 1 \cdot v \Leftrightarrow \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = - \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} -ib = -a \\ ia = -b \end{cases} \rightarrow v = \alpha \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

3 Eig(Z)

$$|I - \lambda Z| = \left| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right| = \begin{vmatrix} 1 - \lambda & 0 \\ 0 & 1 + \lambda \end{vmatrix} \Rightarrow \lambda = \pm 1$$

$$Zv = 1 \cdot v \Leftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} a = a \\ -b = b \end{cases} \rightarrow v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$Zv = 1 \cdot v \Leftrightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = - \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} a = -a \\ -b = -b \end{cases} \rightarrow v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

4 Eig(H)

$$|I - \lambda H| = \left| \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \lambda\alpha \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \right| = \begin{vmatrix} 1 - \alpha\lambda & -\alpha\lambda \\ -\alpha\lambda & 1 + \alpha\lambda \end{vmatrix}$$
$$= 1 - (\alpha\lambda)^2 - (\alpha\lambda)^2 = 1 - \lambda^2 \Rightarrow \lambda = \pm 1$$

$$Hv = 1 \cdot v \Leftrightarrow \alpha \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} \alpha(a + b) = a \\ \alpha(a - b) = b \end{cases}$$
$$\rightarrow v = \begin{pmatrix} 1 \\ \sqrt{2} - 1 \end{pmatrix}$$

$$Hv = 1 \cdot v \Leftrightarrow \alpha \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = - \begin{pmatrix} a \\ b \end{pmatrix} \Leftrightarrow \begin{cases} \alpha(a + b) = -a \\ \alpha(a - b) = -b \end{cases}$$
$$\rightarrow v = \begin{pmatrix} 1 \\ \sqrt{2} + 1 \end{pmatrix}$$