Monte Carlo Methods

DEPARTMENT OF COMPUTER SCIENCE
ST. FRANCIS XAVIER UNIVERSITY

CSCI-531 - Reinforcement Learning

Fall 2025

From Model-Based to Model-Free Learning

Dynamic Programming Limitations

- Requires **complete knowledge** of the environment

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- Average returns to estimate value functions
- No need for transition probabilities

Real-World Motivation

Think About...

How would you learn to play a new video game?

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Key Insight

Most real problems are more like the video game scenario!

The Monte Carlo Approach

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Value = Expected Return

If we can collect many returns from a state, we can **average** them to estimate the true value!

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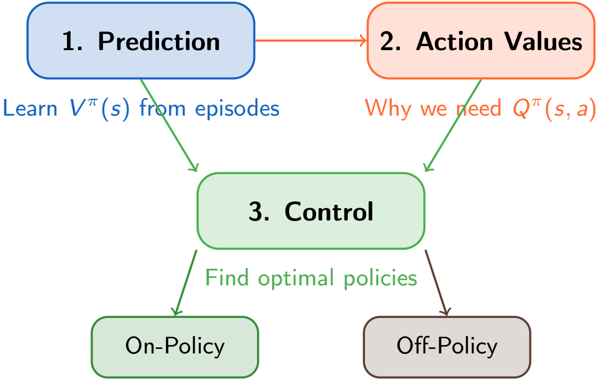
Important Note

Model still needed for interaction, but not for learning!

Learning Roadmap: Our Journey Through Monte Carlo

Where We're Going



Monte Carlo Prediction: The Basic Idea

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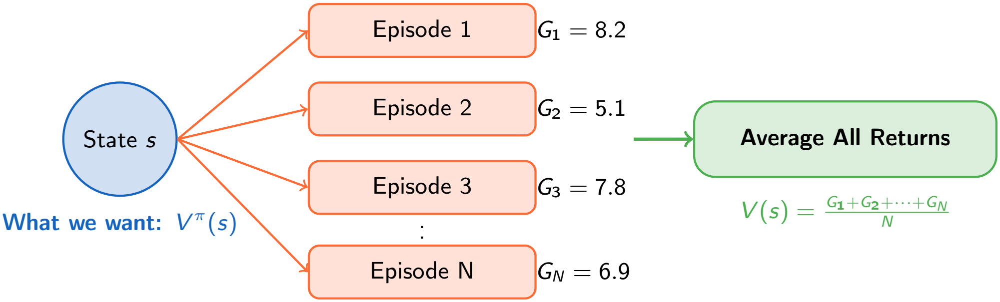
Mathematical Foundation

$$v_\pi(s) = \mathbb{E}_\pi[G_t | S_t = s]$$

$$V(s) \approx \frac{1}{N} \sum_{i=1}^{N} G_i$$

where G_i are returns from N visits to state s.

Episode Collection: The Visual Process



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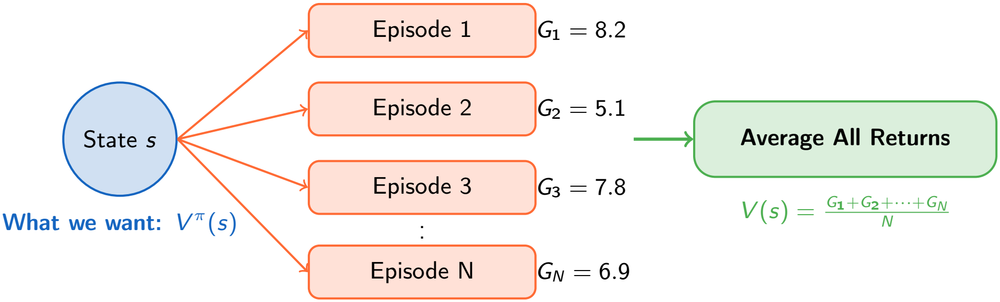

Key Insight

More episodes $\Rightarrow$ Better estimate of true value!

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Convergence Guarantee

As $N \to \infty$, $V(s) \to v_\pi(s)$ by the **Law of Large Numbers**

Concrete Example: Blackjack

Consider playing Blackjack with a simple policy:

- Hit if hand value < 17
- Stand if hand value ≥ 17

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Sample Episodes from State "Hand = 16"

Episode	Outcome	Return
1	Bust	-1
2	Win	+1
3	Bust	-1
4	Bust	-1
5	Lose	-1

Concrete Example: Blackjack

Worked Example and Value Estimate

Worked episodes (hand = 16), assume $\gamma = 1$:

- Episode 1: Bust $\rightarrow R = -1 \Rightarrow G = -1$
- Episode 2: Win $\rightarrow R = +1 \Rightarrow G = +1$
- Episode 3: Bust $\rightarrow R = -1 \Rightarrow G = -1$
- Episode 4: Bust $\rightarrow R = -1 \Rightarrow G = -1$
- Episode 5: Lose $\rightarrow R = -1 \Rightarrow G = -1$

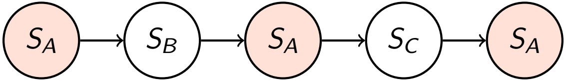
Averaging these returns:

$$V(\text{Hand} = 16) \approx \frac{-1 + 1 + (-1) + (-1) + (-1)}{5} = \textbf{-0.6}.$$

First-Visit vs Every-Visit Monte Carlo

A Key Decision

If we visit the same state multiple times in an episode, should we count all visits or just the first one?



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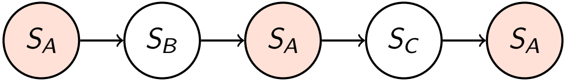

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- More data per episode

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First-Visit vs Every-Visit — Small numeric example

Episode

Consider the episode (with $\gamma = 1$):

$$S_A \xrightarrow{2} S_B \xrightarrow{1} S_A \xrightarrow{3} S_C \xrightarrow{0}$$

- First-Visit for S_A: use return from its first occurrence: $G_0 = 2 + 1 + 3 + 0 = 6$.
- Every-Visit for S_A: include returns from each visit: first visit $G_0 = 6$, second visit (at time 2) $G_2 = 3 + 0 = 3$.

Discussion: First-Visit yields one sample per episode (independent), Every-Visit yields multiple correlated samples but more data per episode.

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Different Perspectives

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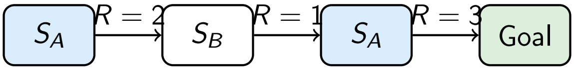
Practical Guidelines

- **Use First-Visit** for theoretical guarantees and textbook implementations
- **Use Every-Visit** for faster learning and limited computational budget

Interactive Example: Computing Returns

Activity

Given this episode with $\gamma = 0.9$:



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Calculate Returns

- From first visit to S_A: $G_0 = 2 + 0.9 \times 1 + 0.9^2 \times 3 = 4.33$

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Value Estimates

- First-Visit MC: $V(S_A) = 4.33$

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Value Estimates

- First-Visit MC: $V(S_A) = 4.33$
- Every-Visit MC: $V(S_A) = \frac{4.33+3}{2} = 3.67$

Monte Carlo Prediction Algorithm

Algorithm First-Visit Monte Carlo Prediction

Input: Policy π, number of episodes N
Initialize: $V(s) \in \mathbb{R}$ arbitrarily, $\forall s \in S$; $Returns(s) \leftarrow$ empty list, $\forall s \in S$
for $episode = 1$ to N **do**
    Generate episode using π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$
    for $t = T-1, T-2, \ldots, 0$ **do**
        $G \leftarrow \gamma G + R_{t+1}$
        if $S_t \notin \{S_0, S_1, \ldots, S_{t-1}\}$ **then**
            Append G to $Returns(S_t)$
            $V(S_t) \leftarrow \text{average}(Returns(S_t))$
        end if
    end for
end for

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Why do we go backwards through the episode?

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- Returns are calculated from **future rewards**
- Working backwards ensures we have all future rewards available
- More efficient than recalculating from scratch each time

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How many episodes do we need?

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How many episodes do we need?

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- Start with 1000+ episodes for simple problems
- Monitor convergence of value estimates

From Prediction to Control: What's Missing?

What We've Learned So Far

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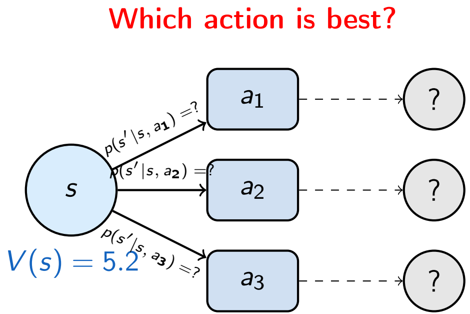
If we have perfect state values $V^*(s)$, can we immediately determine the optimal policy without knowing the model?



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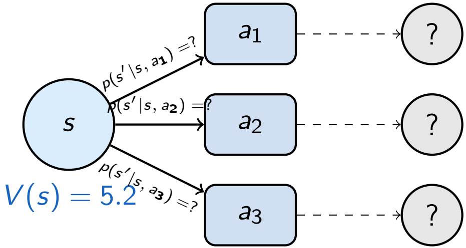

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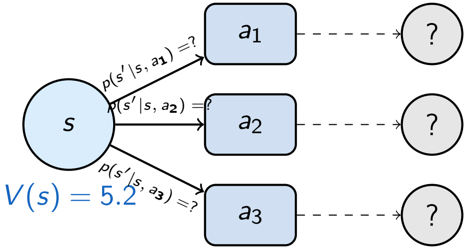

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- Not available in model-free settings!

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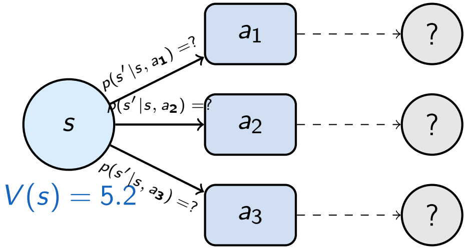

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You're playing a game and reach this state:

You have **perfect knowledge** that being here is worth +5.2 points on average

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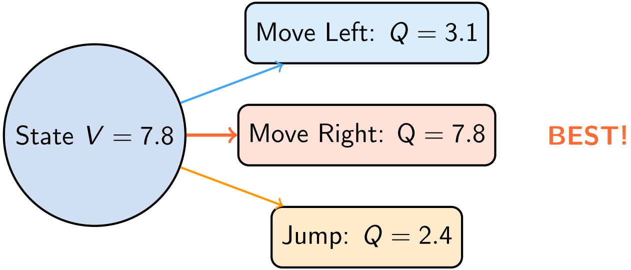
- State value doesn't tell you!

Action Values: The Missing Piece

What We Really Need



Action Values: The Missing Piece

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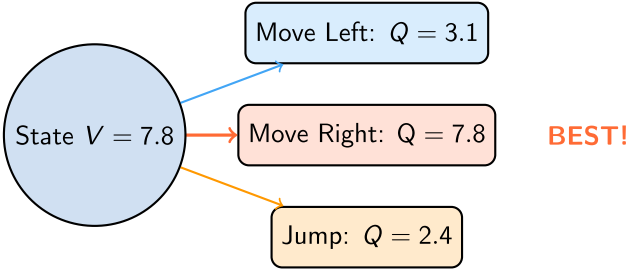

Now We Can Decide!

- **Move Right** has highest Q-value (7.8)

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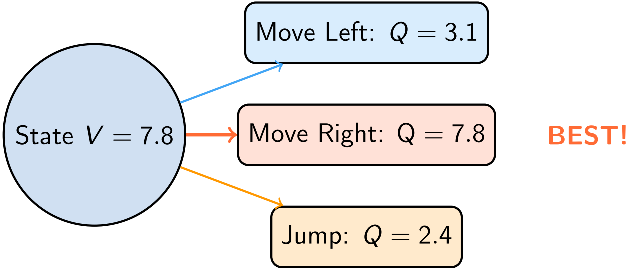

Now We Can Decide!

- **Move Right** has highest Q-value (7.8)
- **Simple policy**: $\pi(s) = \arg\max_a Q(s, a)$

Why This Changes Everything

The Key Insight

Action values give us direct decision-making power!

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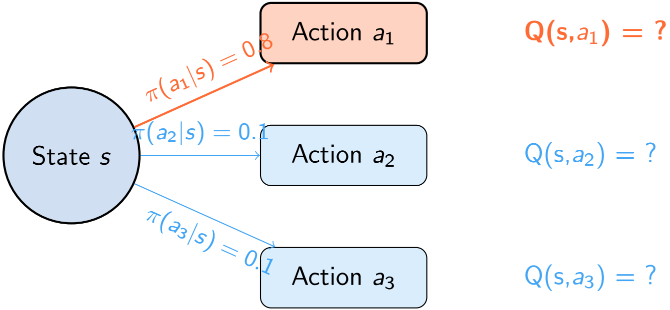
New Challenge

Exploration Problem: All actions must be visited to get good estimates!

The Exploration Problem



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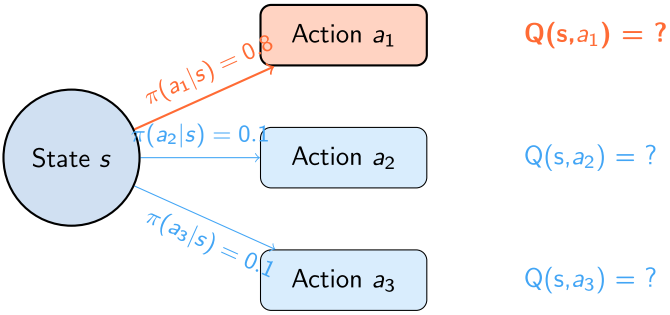

The Dilemma

- **Greedy action** gets lots of data → good estimate

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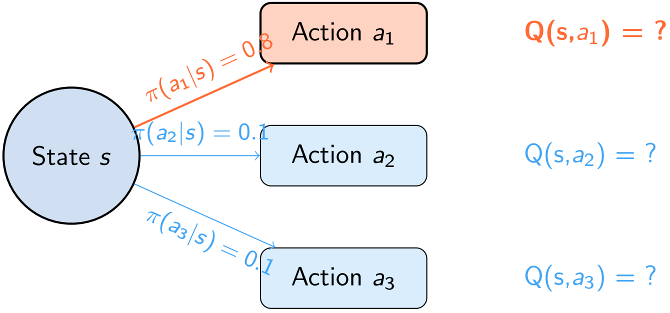

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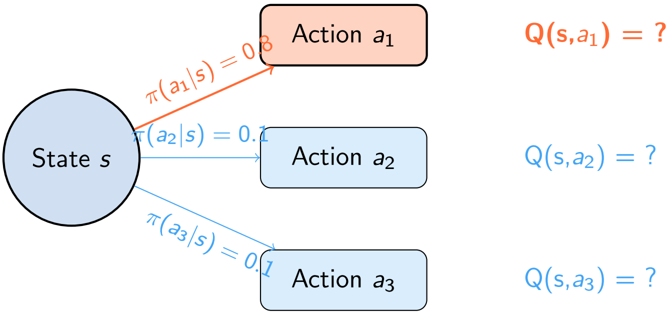

The Dilemma

- **Greedy action** gets lots of data $\rightarrow$ good estimate
- **Other actions** get little data $\rightarrow$ poor estimates
- How do we know if unexplored actions are actually better?

Solutions to the Exploration Problem

Option 1: Exploring Starts

- Every episode starts with random state-action pair

Solutions to the Exploration Problem

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- Maintain **non-zero probability** for all actions

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- Example: ϵ**-greedy policies**

On-Policy Monte Carlo Control

The Approach

- Learn the value of the same policy we're following

On-Policy Monte Carlo Control

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ϵ-Greedy Policy

$$\pi(a|s) = \begin{cases} 1 - \epsilon + \frac{\epsilon}{|A|} & \text{if } a = \arg\max_a Q(s,a) \\ \frac{\epsilon}{|A|} & \text{otherwise} \end{cases}$$

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Key Properties

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Key Properties

- **Soft policy:** All actions have non-zero probability
- **Improvement guarantee:** Any ϵ-greedy policy w.r.t. Q_π is better than π

ϵ-Greedy Policy Example

Interactive Example

State s has 3 actions with $\epsilon = 0.2$:

$$Q(s, a_1) = 5, \; Q(s, a_2) = 3, \; Q(s, a_3) = 1$$

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What happens as we learn and Q values change?

Algorithm On-Policy First-Visit MC Control

Initialize: $Q(s, a) \in \mathbb{R}$ arbitrarily, $\forall s \in S, a \in A$
Initialize: $\pi \leftarrow$ arbitrary ϵ-soft policy; $Returns(s, a) \leftarrow$ empty list, $\forall s \in S, a \in A$
for each episode **do**
  Generate episode using π: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
  $G \leftarrow 0$
  for $t = T-1, T-2, \ldots, 0$ **do**
    $G \leftarrow \gamma G + R_{t+1}$
    if $(S_t, A_t) \notin \{(S_0, A_0), \ldots, (S_{t-1}, A_{t-1})\}$ **then**
      Append G to $Returns(S_t, A_t)$
      $Q(S_t, A_t) \leftarrow$ average$(Returns(S_t, A_t))$
      $A^* \leftarrow \arg\max_a Q(S_t, a)$
      for all $a \in A$ **do**

$$\pi(a|S_t) \leftarrow \begin{cases} 1 - \epsilon + \frac{\epsilon}{|A|} & \text{if } a = A^* \\ \frac{\epsilon}{|A|} & \text{otherwise} \end{cases}$$

      end for

Concrete Example Part 1: Q-Value Update

Current Situation

State: Hand = 16, Actions: Hit or Stand

Action	**Current Q-value**	**Returns History**
Hit	$Q(16, \text{Hit}) = 0.2$	$[0.1, 0.3]$
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Episode Update

- Chose Hit, got return $G = 0.7$

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- $Q(16, \text{Hit}) = \frac{0.1+0.3+0.7}{3} = 0.37$
- **Result:** Hit improved from 0.2 to 0.37, Stand stays 0.5

Concrete Example Part 2: Policy Update

Current Q-Values

Action	Q-value
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Policy Update with $\epsilon = 0.1$:

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- $\pi(\text{Stand}|16) = 1 - 0.1 + \frac{0.1}{2} = 0.95$

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Off-Policy Monte Carlo: Motivation

Limitation of On-Policy Methods

What's the best policy on-policy MC can find?

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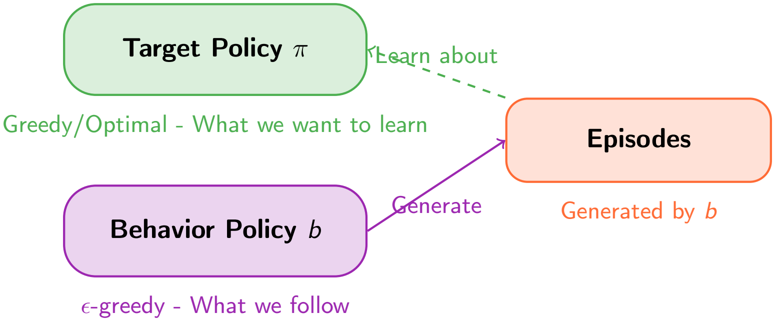
  - Behavior Policy b: The policy we follow for exploration
- Learn about π using data generated by b

Off-Policy Concept Visualization



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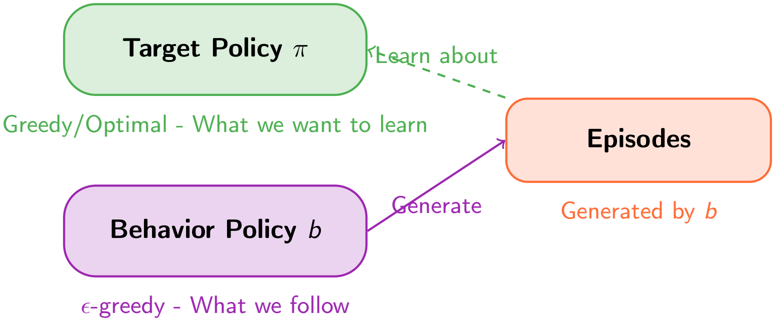

Coverage Assumption

$\pi(a|s) > 0 \Rightarrow b(a|s) > 0$ for all s, a

Why is this assumption crucial?

Simple Analogy: YouTube Gaming Videos

Imagine You Want to Know...

How good are speedrunners at this game?

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- Casual player videos are **over-represented** for safe moves

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- Casual player videos are **over-represented** for safe moves
- Weight risky moves by $\frac{0.80}{0.20} = 4.0$
- This "up-weights" the rare risky gameplay we want to learn about

From Gaming to Reinforcement Learning

The Connection

YouTube Gaming	**RL Off-Policy**
Casual players	Behavior policy b
Speedrunners	Target policy π
Strategy choices	Episode actions
Game scores	Episode returns
Weight $= 0.80/0.20$	Weight $= \pi(a\|s)/b(a\|s)$

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Key Insight

When gameplay comes from the "wrong" player type, we reweight it to match the strategy we want to learn!

Importance Sampling Intuition

The Problem

We have episodes from policy b, but want to learn about policy π. How do we account for the difference?

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The Solution: Importance Sampling

Weight each episode by the ratio of its probability under π vs b

Importance Sampling Mathematics

Importance Sampling Ratio

$$
\begin{aligned}
\rho_{t:T-1} &= \frac{\text{Prob of trajectory under } \pi}{\text{Prob of trajectory under } b} \\
&= \frac{\prod_{k=t}^{T-1} \pi(A_k|S_k)p(S_{k+1}|S_k, A_k)}{\prod_{k=t}^{T-1} b(A_k|S_k)p(S_{k+1}|S_k, A_k)} \\
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\end{aligned}
$$

Key Insight

Transition probabilities cancel out. Only depends on policies.

Importance Sampling: Numerical Example

Episode Trajectory

Episode: $S_0 \xrightarrow{A_0} S_1 \xrightarrow{A_1} S_2$

Importance Sampling: Numerical Example

Episode Trajectory

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Policy Probabilities

- At S_0: $\pi(A_0|S_0) = 0.1$, $b(A_0|S_0) = 0.3$

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Policy Probabilities

- At S_0: $\pi(A_0|S_0) = 0.1$, $b(A_0|S_0) = 0.3$
- At S_1: $\pi(A_1|S_1) = 0.8$, $b(A_1|S_1) = 0.4$

Importance Sampling: Numerical Example

Episode Trajectory

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Policy Probabilities

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- At S_1: $\pi(A_1|S_1) = 0.8$, $b(A_1|S_1) = 0.4$

Importance Sampling Ratio

$$\begin{aligned}\rho &= \frac{\pi(A_0|S_0)}{b(A_0|S_0)} \times \frac{\pi(A_1|S_1)}{b(A_1|S_1)} \\ &= \frac{0.1}{0.3} \times \frac{0.8}{0.4} = \frac{1}{3} \times 2 = \frac{2}{3}\end{aligned}$$

Algorithm Off-Policy Monte Carlo Prediction

Initialize: $Q(s, a) \in \mathbb{R}$ arbitrarily, $\forall s \in S, a \in A$
Initialize: $C(s, a) \leftarrow 0$, $\forall s \in S, a \in A$
for each episode **do**
    $b \leftarrow$ any policy with coverage of π
    Generate episode using b: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
    $G \leftarrow 0$, $W \leftarrow 1$
    for $t = T-1, T-2, \ldots, 0$ and $W \neq 0$ **do**
        $G \leftarrow \gamma G + R_{t+1}$
        $C(S_t, A_t) \leftarrow C(S_t, A_t) + W$
        $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{W}{C(S_t, A_t)}[G - Q(S_t, A_t)]$
        $W \leftarrow W \cdot \frac{\pi(A_t|S_t)}{b(A_t|S_t)}$
    end for
end for

Weighted Importance Sampling: Key Features

Implementation Details

- Uses **incremental implementation** with weighted average

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Why This Matters

- **Efficiency**: No need to store all returns
- **Robustness**: Handles impossible trajectories gracefully
- **Convergence**: Weighted averaging often has better properties

Algorithm Off-Policy Monte Carlo Control

Initialize: $\pi(s) \leftarrow \arg\max_a Q(s, a)$ (greedy policy)
for each episode **do**
  $b \leftarrow$ any policy with coverage of π
  Generate episode using b: $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$
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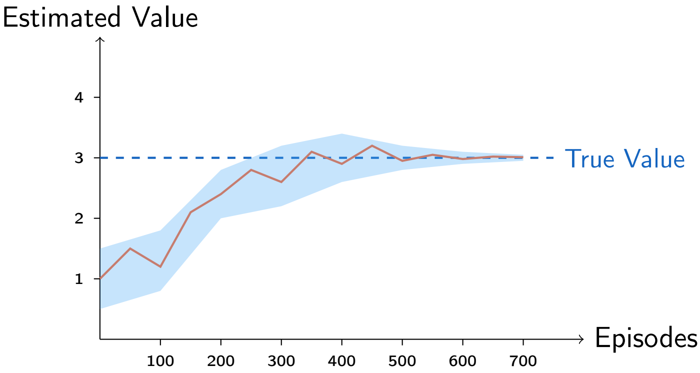
    $\pi(S_t) \leftarrow \arg\max_a Q(S_t, a)$
    if $A_t \neq \pi(S_t)$ **then**
      break (exit inner loop)
    end if
    $W \leftarrow W \cdot \frac{1}{b(A_t|S_t)}$
  end for

Learning Progress: How Monte Carlo Converges

What Does Learning Look Like?

Let's visualize how our value estimates improve over time



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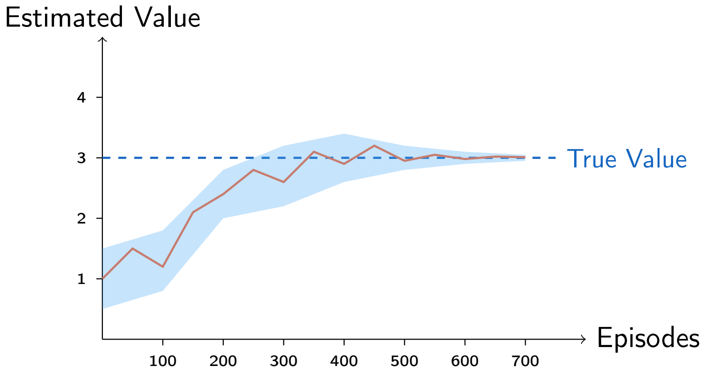

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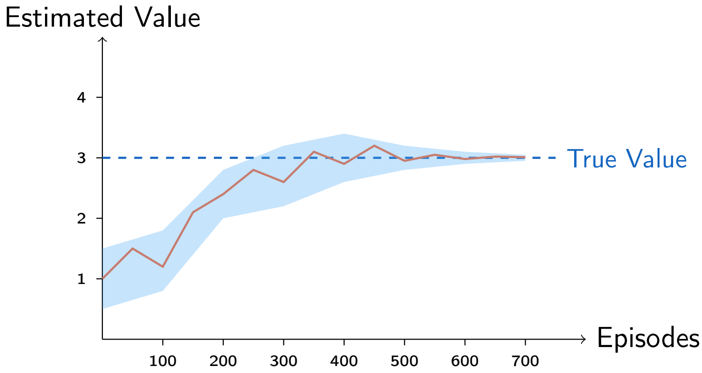

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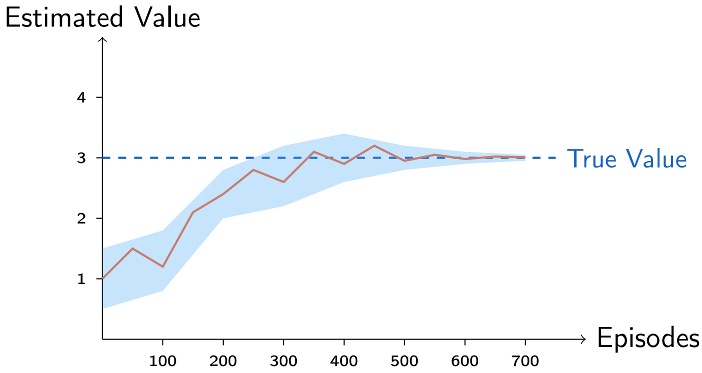

Key Observations

Practical Learning Considerations

What Affects Convergence Speed?

Faster Convergence:

Slower Convergence:

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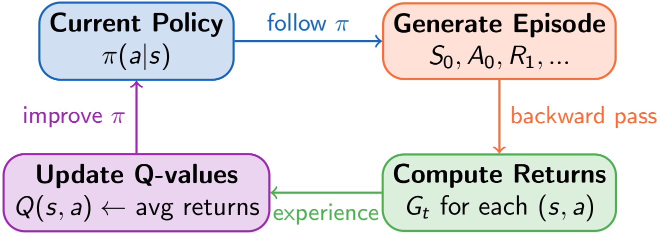
- Stochastic environments

Monte Carlo Control: The Complete Loop

The Big Picture

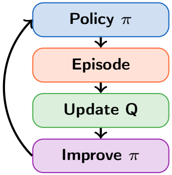
How do all the pieces fit together?



Repeat until convergence

On-Policy vs Off-Policy Control Flow

On-Policy Monte Carlo



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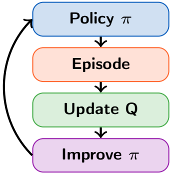

- Same policy for **acting** and **learning**

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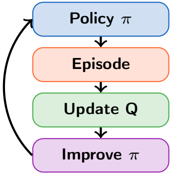

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- Simpler, more stable

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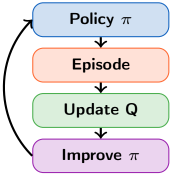

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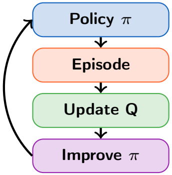

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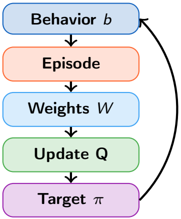

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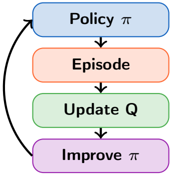

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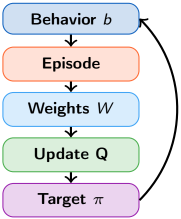

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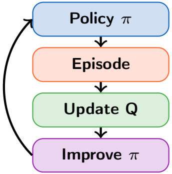

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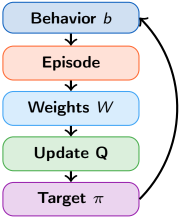

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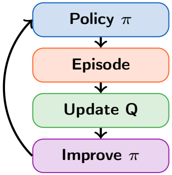

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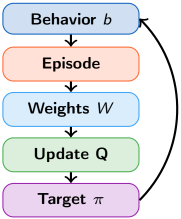

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- Better exploration potential

On-Policy vs Off-Policy: Trade-offs

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Advantages:

- Simpler to understand
- Lower variance
- Faster convergence
- Easier to implement

Disadvantages:

- Limited to ϵ-soft policies
- Cannot learn deterministic optimal policy

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- More general and powerful
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Discussion

When would you choose each approach?

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Implementation Tips

What should you consider when implementing Monte Carlo methods?

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- Start with $\epsilon = 0.1$ (10% exploration)

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- Consider computational budget

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- Need sufficient exploration

Key Takeaways

What We've Learned

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- What if episodes are very long or infinite?
- Next: **Temporal Difference Learning** - combine MC and DP benefits