

# Temporal-Difference Learning



DEPARTMENT OF COMPUTER SCIENCE  
ST. FRANCIS XAVIER UNIVERSITY

CSCI-531 - Reinforcement Learning

Fall 2025

# The Best of Both Worlds

What is Temporal-Difference Learning?

- ▶ Combines **Dynamic Programming** and **Monte Carlo** methods

# The Best of Both Worlds

What is Temporal-Difference Learning?

- ▶ Combines **Dynamic Programming** and **Monte Carlo** methods
- ▶ **Model-free** like Monte Carlo

# The Best of Both Worlds

What is Temporal-Difference Learning?

- ▶ Combines **Dynamic Programming** and **Monte Carlo** methods
- ▶ **Model-free** like Monte Carlo
- ▶ **Bootstraps** like Dynamic Programming

# The Best of Both Worlds

## What is Temporal-Difference Learning?

- ▶ Combines **Dynamic Programming** and **Monte Carlo** methods
- ▶ **Model-free** like Monte Carlo
- ▶ **Bootstraps** like Dynamic Programming

## Key Insight

- ▶ Updates estimates based on **other estimates**

# The Best of Both Worlds

## What is Temporal-Difference Learning?

- ▶ Combines **Dynamic Programming** and **Monte Carlo** methods
- ▶ **Model-free** like Monte Carlo
- ▶ **Bootstraps** like Dynamic Programming

## Key Insight

- ▶ Updates estimates based on **other estimates**
- ▶ No need to wait until episode completion

# The Best of Both Worlds

## What is Temporal-Difference Learning?

- ▶ Combines **Dynamic Programming** and **Monte Carlo** methods
- ▶ **Model-free** like Monte Carlo
- ▶ **Bootstraps** like Dynamic Programming

## Key Insight

- ▶ Updates estimates based on **other estimates**
- ▶ No need to wait until episode completion
- ▶ Updates at **every time step**

# Method Comparison

Method	Model-Free	Bootstraps	Online
Dynamic Programming	No	Yes	Yes
Monte Carlo	Yes	No	No
TD Learning	Yes	Yes	Yes

# Method Comparison

Method	Model-Free	Bootstraps	Online
Dynamic Programming	No	Yes	Yes
Monte Carlo	Yes	No	No
<b>TD Learning</b>	Yes	Yes	Yes

## Why This Matters

- ▶ Can learn from incomplete episodes

# Method Comparison

Method	Model-Free	Bootstraps	Online
Dynamic Programming	No	Yes	Yes
Monte Carlo	Yes	No	No
<b>TD Learning</b>	Yes	Yes	Yes

## Why This Matters

- ▶ Can learn from **incomplete episodes**
- ▶ Faster convergence than Monte Carlo

# Method Comparison

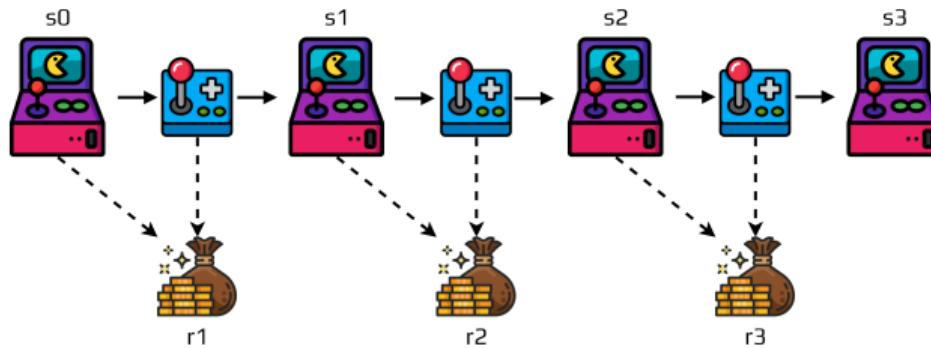
Method	Model-Free	Bootstraps	Online
Dynamic Programming	No	Yes	Yes
Monte Carlo	Yes	No	No
TD Learning	Yes	Yes	Yes

## Why This Matters

- ▶ Can learn from **incomplete episodes**
- ▶ Faster convergence than Monte Carlo
- ▶ More practical for **online learning**

# Activity: Monte Carlo Limitation

Recall: Monte Carlo Update



$$V(s_0) = V(s_0) + \alpha [G - V(s_0)]$$

Question

What is the main disadvantage of this approach?

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**
- ▶ Cannot learn from **partial episodes**

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**
- ▶ Cannot learn from **partial episodes**
- ▶ Updates are **delayed**

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**
- ▶ Cannot learn from **partial episodes**
- ▶ Updates are **delayed**
- ▶ Not suitable for **continuing tasks**

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**
- ▶ Cannot learn from **partial episodes**
- ▶ Updates are **delayed**
- ▶ Not suitable for **continuing tasks**

Impact

- ▶ Slow learning in long episodes

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**
- ▶ Cannot learn from **partial episodes**
- ▶ Updates are **delayed**
- ▶ Not suitable for **continuing tasks**

Impact

- ▶ Slow learning in long episodes
- ▶ Cannot handle infinite episodes

# Monte Carlo Limitations

Answer: Key Disadvantages

- ▶ Must wait until **episode completion**
- ▶ Cannot learn from **partial episodes**
- ▶ Updates are **delayed**
- ▶ Not suitable for **continuing tasks**

Impact

- ▶ Slow learning in long episodes
- ▶ Cannot handle infinite episodes
- ▶ Memory grows with episode length

# Building the TD Update: What Do We Want?

Recall: Value Function Definition

$$V(s) = \mathbb{E}[G_t | S_t = s] = \mathbb{E}[R_{t+1} + \gamma G_{t+1} | S_t = s] \quad (1)$$

# Building the TD Update: What Do We Want?

Recall: Value Function Definition

$$V(s) = \mathbb{E}[G_t | S_t = s] = \mathbb{E}[R_{t+1} + \gamma G_{t+1} | S_t = s] \quad (1)$$

Breaking Down the Return

$$V(s) = \mathbb{E}[R_{t+1} + \gamma G_{t+1} | S_t = s] \quad (2)$$

$$= \mathbb{E}[R_{t+1} | S_t = s] + \gamma \mathbb{E}[G_{t+1} | S_t = s] \quad (3)$$

$$= \mathbb{E}[R_{t+1} | S_t = s] + \gamma \mathbb{E}[V(S_{t+1}) | S_t = s] \quad (4)$$

# Building the TD Update: What Do We Want?

Recall: Value Function Definition

$$V(s) = \mathbb{E}[G_t | S_t = s] = \mathbb{E}[R_{t+1} + \gamma G_{t+1} | S_t = s] \quad (1)$$

Breaking Down the Return

$$V(s) = \mathbb{E}[R_{t+1} + \gamma G_{t+1} | S_t = s] \quad (2)$$

$$= \mathbb{E}[R_{t+1} | S_t = s] + \gamma \mathbb{E}[G_{t+1} | S_t = s] \quad (3)$$

$$= \mathbb{E}[R_{t+1} | S_t = s] + \gamma \mathbb{E}[V(S_{t+1}) | S_t = s] \quad (4)$$

The Key Insight

The future return  $G_{t+1}$  starting from  $S_{t+1}$  is just  $V(S_{t+1})$ !

# From Theory to Practice: The Approximation

## What We Have (Theory)

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s] \quad (5)$$

# From Theory to Practice: The Approximation

## What We Have (Theory)

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s] \quad (5)$$

## What We Can Observe (Practice)

- We see actual reward:  $r_{t+1}$  (sample of  $R_{t+1}$ )

# From Theory to Practice: The Approximation

## What We Have (Theory)

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s] \quad (5)$$

## What We Can Observe (Practice)

- ▶ We see actual reward:  $r_{t+1}$  (sample of  $R_{t+1}$ )
- ▶ We see next state:  $s_{t+1}$  (sample of  $S_{t+1}$ )

# From Theory to Practice: The Approximation

## What We Have (Theory)

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s] \quad (5)$$

## What We Can Observe (Practice)

- ▶ We see actual reward:  $r_{t+1}$  (sample of  $R_{t+1}$ )
- ▶ We see next state:  $s_{t+1}$  (sample of  $S_{t+1}$ )
- ▶ We have current estimate:  $V(s_{t+1})$

# From Theory to Practice: The Approximation

## What We Have (Theory)

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s] \quad (5)$$

## What We Can Observe (Practice)

- ▶ We see actual reward:  $r_{t+1}$  (sample of  $R_{t+1}$ )
- ▶ We see next state:  $s_{t+1}$  (sample of  $S_{t+1}$ )
- ▶ We have current estimate:  $V(s_{t+1})$

## The Approximation

$r_{t+1} + \gamma V(s_{t+1})$  is our estimate of  $V(s_t)$  (6)

# From Theory to Practice: The Approximation

## What We Have (Theory)

$$V(s) = \mathbb{E}[R_{t+1} + \gamma V(S_{t+1}) | S_t = s] \quad (5)$$

## What We Can Observe (Practice)

- ▶ We see actual reward:  $r_{t+1}$  (sample of  $R_{t+1}$ )
- ▶ We see next state:  $s_{t+1}$  (sample of  $S_{t+1}$ )
- ▶ We have current estimate:  $V(s_{t+1})$

## The Approximation

$r_{t+1} + \gamma V(s_{t+1})$  is our estimate of  $V(s_t)$  (6)

# Resolving the Conflict: Learning from Error

Two Estimates of  $V(s_t)$

- ▶ **Old estimate:**  $V(s_t)$  (what we thought before)

# Resolving the Conflict: Learning from Error

Two Estimates of  $V(s_t)$

- ▶ **Old estimate:**  $V(s_t)$  (what we thought before)
- ▶ **New evidence:**  $r_{t+1} + \gamma V(s_{t+1})$  (what we observe)

# Resolving the Conflict: Learning from Error

Two Estimates of  $V(s_t)$

- ▶ **Old estimate:**  $V(s_t)$  (what we thought before)
- ▶ **New evidence:**  $r_{t+1} + \gamma V(s_{t+1})$  (what we observe)

The Error

$$\delta_t = \underbrace{r_{t+1} + \gamma V(s_{t+1})}_{\text{new evidence}} - \underbrace{V(s_t)}_{\text{old estimate}} \quad (7)$$

# Resolving the Conflict: Learning from Error

Two Estimates of  $V(s_t)$

- ▶ **Old estimate:**  $V(s_t)$  (what we thought before)
- ▶ **New evidence:**  $r_{t+1} + \gamma V(s_{t+1})$  (what we observe)

The Error

$$\delta_t = \underbrace{r_{t+1} + \gamma V(s_{t+1})}_{\text{new evidence}} - \underbrace{V(s_t)}_{\text{old estimate}} \quad (7)$$

Learning Rule

Move our estimate towards the new evidence:

$$V(s_t)^{\text{new}} = V(s_t)^{\text{old}} + \alpha \times \delta_t \quad (8)$$

# The Complete TD Update Rule

Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

# The Complete TD Update Rule

## Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

## Components Explained

- ▶  $\alpha \in [0, 1]$ : **learning rate** (how much to change)

# The Complete TD Update Rule

## Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

## Components Explained

- ▶  $\alpha \in [0, 1]$ : **learning rate** (how much to change)
- ▶  $r_{t+1} + \gamma V(s_{t+1})$ : **TD target** (new evidence)

# The Complete TD Update Rule

## Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

## Components Explained

- ▶  $\alpha \in [0, 1]$ : **learning rate** (how much to change)
- ▶  $r_{t+1} + \gamma V(s_{t+1})$ : **TD target** (new evidence)
- ▶  $r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$ : **TD error** (how wrong we were)

# The Complete TD Update Rule

## Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

## Components Explained

- ▶  $\alpha \in [0, 1]$ : **learning rate** (how much to change)
- ▶  $r_{t+1} + \gamma V(s_{t+1})$ : **TD target** (new evidence)
- ▶  $r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$ : **TD error** (how wrong we were)

## Intuition

- ▶ If error is **positive**: our estimate was too low → increase it

# The Complete TD Update Rule

## Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

## Components Explained

- ▶  $\alpha \in [0, 1]$ : **learning rate** (how much to change)
- ▶  $r_{t+1} + \gamma V(s_{t+1})$ : **TD target** (new evidence)
- ▶  $r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$ : **TD error** (how wrong we were)

## Intuition

- ▶ If error is **positive**: our estimate was too low → increase it
- ▶ If error is **negative**: our estimate was too high → decrease it

# The Complete TD Update Rule

## Putting It All Together

$$V(s_t) \leftarrow V(s_t) + \alpha [r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \quad (9)$$

## Components Explained

- ▶  $\alpha \in [0, 1]$ : **learning rate** (how much to change)
- ▶  $r_{t+1} + \gamma V(s_{t+1})$ : **TD target** (new evidence)
- ▶  $r_{t+1} + \gamma V(s_{t+1}) - V(s_t)$ : **TD error** (how wrong we were)

## Intuition

- ▶ If error is **positive**: our estimate was too low → increase it
- ▶ If error is **negative**: our estimate was too high → decrease it
- ▶ If error is **zero**: our estimate was perfect → no change

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates
- ▶ Like improving a recipe by tasting and adjusting as you cook

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates
- ▶ Like improving a recipe by tasting and adjusting as you cook
- ▶ Don't wait for the final dish (complete episode)

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates
- ▶ Like improving a recipe by tasting and adjusting as you cook
- ▶ Don't wait for the final dish (complete episode)

## Mathematical Justification

- ▶ Each update moves us **closer** to the true value

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates
- ▶ Like improving a recipe by tasting and adjusting as you cook
- ▶ Don't wait for the final dish (complete episode)

## Mathematical Justification

- ▶ Each update moves us **closer** to the true value
- ▶ Estimates **propagate** through the state space

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates
- ▶ Like improving a recipe by tasting and adjusting as you cook
- ▶ Don't wait for the final dish (complete episode)

## Mathematical Justification

- ▶ Each update moves us **closer** to the true value
- ▶ Estimates **propagate** through the state space
- ▶ Converges to optimal values under mild conditions

# Why This Works: The Bootstrap Principle

## Bootstrap Analogy

"Pulling yourself up by your bootstraps"

- ▶ Use current estimates to improve current estimates
- ▶ Like improving a recipe by tasting and adjusting as you cook
- ▶ Don't wait for the final dish (complete episode)

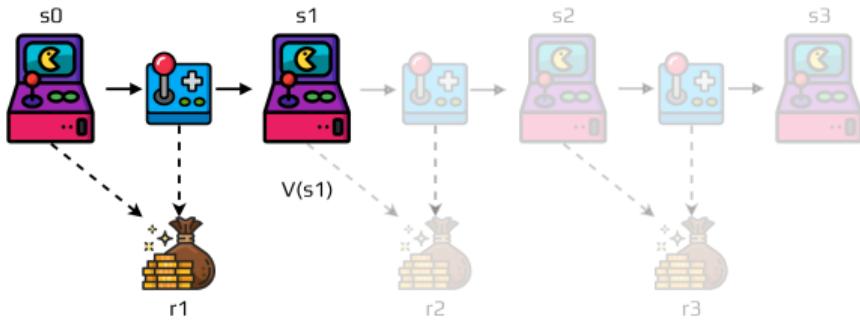
## Mathematical Justification

- ▶ Each update moves us **closer** to the true value
- ▶ Estimates **propagate** through the state space
- ▶ Converges to optimal values under mild conditions

## Key Insight

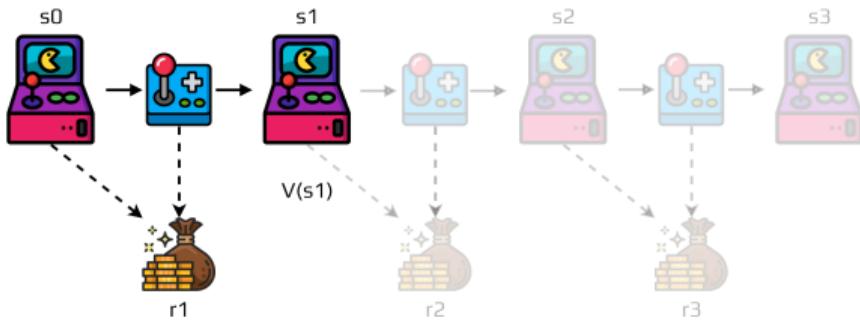
We don't need perfect information - just better information than what we had!

## Visual Comparison: MC vs TD



$$V(s_0) = V(s_0) + \alpha [r_1 + \gamma V(s_1) - V(s_0)]$$

## Visual Comparison: MC vs TD

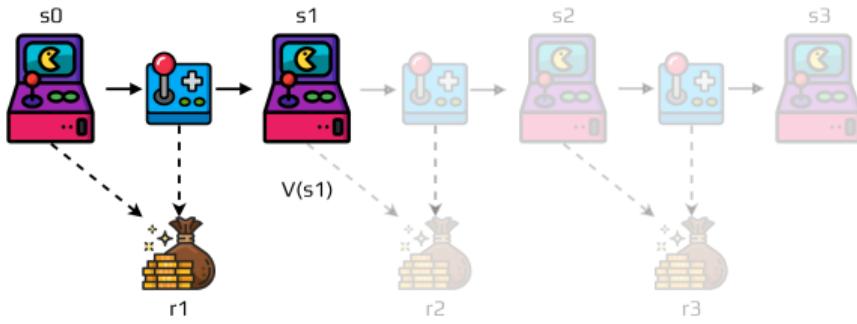


$$V(s_0) = V(s_0) + \alpha [r_1 + \gamma V(s_1) - V(s_0)]$$

## Key Differences

- **MC:** Uses actual return  $G_t$

## Visual Comparison: MC vs TD

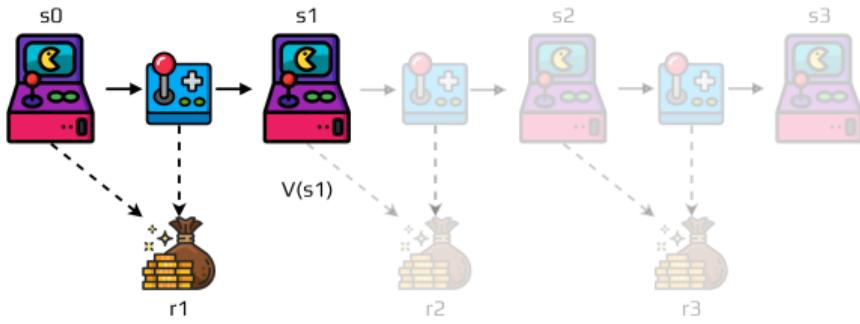


$$V(s_0) = V(s_0) + \alpha [r_1 + \gamma V(s_1) - V(s_0)]$$

## Key Differences

- ▶ **MC:** Uses actual return  $G_t$
- ▶ **TD:** Uses estimated return  $r_{t+1} + \gamma V(s_{t+1})$

## Visual Comparison: MC vs TD



## Key Differences

- ▶ **MC:** Uses actual return  $G_t$
- ▶ **TD:** Uses estimated return  $r_{t+1} + \gamma V(s_{t+1})$
- ▶ **TD:** Updates **immediately** after each step

# TD(0) for Policy Evaluation

---

## Algorithm TD(0) Algorithm

---

```
1: Inputs:  $\pi$  (policy to evaluate),  $N$  (episodes),  $\alpha \in [0, 1]$  (step size)
2: Initialize:  $V_\pi(s) \in \mathbb{R}$  arbitrarily,  $\forall s \in S$ , except  $V(\text{terminal}) = 0$ 
3: for  $\text{episode} = 1$  to  $N$  do
4:   Initialize  $s$ 
5:   while  $s \neq \text{terminal}$  do
6:      $a \leftarrow \pi(s)$ 
7:      $r, s' \leftarrow \text{Execute } a$ 
8:      $V(s) \leftarrow V(s) + \alpha[r + \gamma V(s') - V(s)]$ 
9:      $s \leftarrow s'$ 
10:  end while
11: end for
```

---

# Why "TD(0)"?

The "0" in TD(0)

The target uses the estimate **0 steps** into the future:

$$r_{t+1} + \gamma V(s_{t+1}) \quad (10)$$

# Why "TD(0)"?

The "0" in TD(0)

The target uses the estimate **0 steps** into the future:

$$r_{t+1} + \gamma V(s_{t+1}) \quad (10)$$

TD(n) Family

- ▶ **TD(0):** Uses 1-step return (immediate reward + next state value)

# Why "TD(0)"?

The "0" in TD(0)

The target uses the estimate **0 steps** into the future:

$$r_{t+1} + \gamma V(s_{t+1}) \quad (10)$$

TD(n) Family

- ▶ **TD(0)**: Uses 1-step return (immediate reward + next state value)
- ▶ **TD(1)**: Would use 2-step return

# Why "TD(0)"?

The "0" in TD(0)

The target uses the estimate **0 steps** into the future:

$$r_{t+1} + \gamma V(s_{t+1}) \quad (10)$$

TD(n) Family

- ▶ TD(0): Uses 1-step return (immediate reward + next state value)
- ▶ TD(1): Would use 2-step return
- ▶ TD(n): Uses n-step return

# Why "TD(0)"?

The "0" in TD(0)

The target uses the estimate **0 steps** into the future:

$$r_{t+1} + \gamma V(s_{t+1}) \quad (10)$$

TD(n) Family

- ▶ TD(0): Uses 1-step return (immediate reward + next state value)
- ▶ TD(1): Would use 2-step return
- ▶ TD(n): Uses n-step return
- ▶ TD( $\infty$ ): Equivalent to Monte Carlo (full episode)

# Why "TD(0)"?

## TD(n) Family

- ▶ **TD(0)**: Uses 1-step return (immediate reward + next state value)
- ▶ **TD(1)**: Would use 2-step return
- ▶ **TD(n)**: Uses n-step return
- ▶ **TD( $\infty$ )**: Equivalent to Monte Carlo (full episode)

# Why "TD(0)"?

## TD(n) Family

- ▶ TD(0): Uses 1-step return (immediate reward + next state value)
- ▶ TD(1): Would use 2-step return
- ▶ TD(n): Uses n-step return
- ▶ TD( $\infty$ ): Equivalent to Monte Carlo (full episode)

## Key Insight

**Higher n:** More like Monte Carlo (less bias, more variance)

**Lower n:** More like DP (more bias, less variance)

# Activity: TD vs Monte Carlo Differences

## Question

What major differences can you see between TD(0) and Monte Carlo?

# Activity: TD vs Monte Carlo Differences

## Question

What major differences can you see between TD(0) and Monte Carlo?

## Key Differences

1. **Update frequency:** Every step vs. end of episode

# Activity: TD vs Monte Carlo Differences

## Question

What major differences can you see between TD(0) and Monte Carlo?

## Key Differences

1. **Update frequency:** Every step vs. end of episode
2. **Target:** Estimate vs. actual return

# Activity: TD vs Monte Carlo Differences

## Question

What major differences can you see between TD(0) and Monte Carlo?

## Key Differences

1. **Update frequency:** Every step vs. end of episode
2. **Target:** Estimate vs. actual return
3. **Memory:** Constant vs. growing with episode length

# Activity: TD vs Monte Carlo Differences

## Question

What major differences can you see between TD(0) and Monte Carlo?

## Key Differences

1. **Update frequency:** Every step vs. end of episode
2. **Target:** Estimate vs. actual return
3. **Memory:** Constant vs. growing with episode length
4. **Convergence:** May converge faster

# Understanding TD Error

## TD Error Definition

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \quad (11)$$

# Understanding TD Error

## TD Error Definition

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \quad (11)$$

## What TD Error Represents

- ▶ **Prediction error** at time  $t$

# Understanding TD Error

## TD Error Definition

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \quad (11)$$

## What TD Error Represents

- ▶ **Prediction error** at time  $t$
- ▶ Difference between **current estimate** and **better estimate**

# Understanding TD Error

## TD Error Definition

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \quad (11)$$

## What TD Error Represents

- ▶ **Prediction error** at time  $t$
- ▶ Difference between **current estimate** and **better estimate**
- ▶ Available only at time  $t + 1$  (needs next state and reward)

# Understanding TD Error

## TD Error Definition

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \quad (11)$$

## What TD Error Represents

- ▶ **Prediction error** at time  $t$
- ▶ Difference between **current estimate** and **better estimate**
- ▶ Available only at time  $t + 1$  (needs next state and reward)

## Why TD Target is an Estimate

1. Expected value  $V(s')$  is **sampled**

# Understanding TD Error

## TD Error Definition

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \quad (11)$$

## What TD Error Represents

- ▶ **Prediction error** at time  $t$
- ▶ Difference between **current estimate** and **better estimate**
- ▶ Available only at time  $t + 1$  (needs next state and reward)

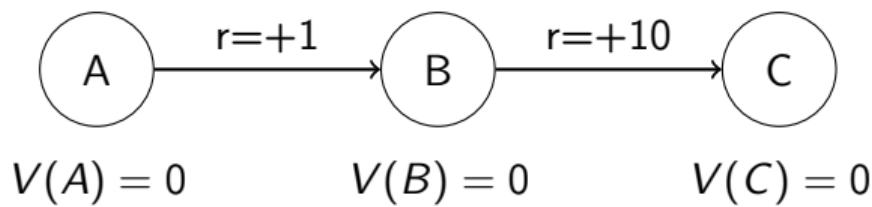
## Why TD Target is an Estimate

1. Expected value  $V(s')$  is **sampled**
2. Uses current **estimate**  $V(s')$ , not true value

# TD Learning with Numbers: A Simple Example

Setup: 3-State Chain

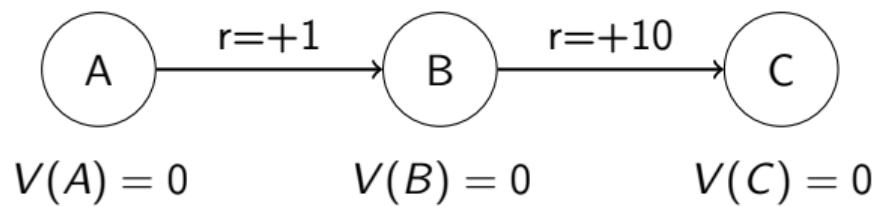
- ▶ States:  $A \rightarrow B \rightarrow C$  (terminal)



# TD Learning with Numbers: A Simple Example

## Setup: 3-State Chain

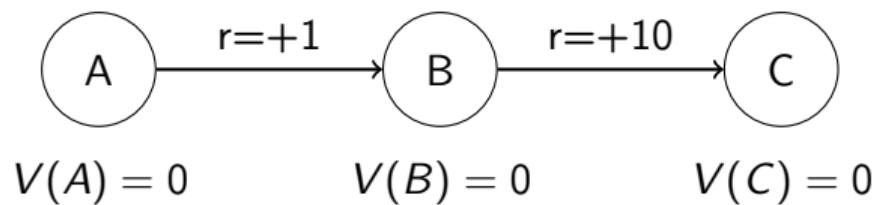
- ▶ States:  $A \rightarrow B \rightarrow C$  (terminal)
- ▶ Rewards:  $A \rightarrow B$  gives +1,  $B \rightarrow C$  gives +10



# TD Learning with Numbers: A Simple Example

## Setup: 3-State Chain

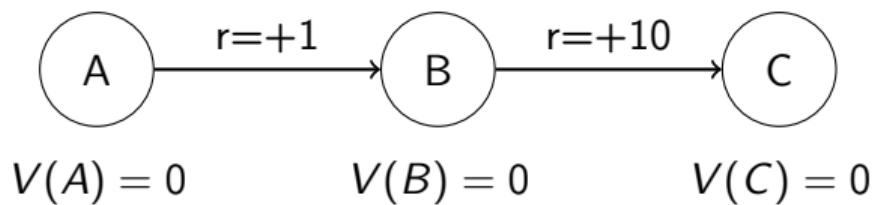
- ▶ States:  $A \rightarrow B \rightarrow C$  (terminal)
- ▶ Rewards:  $A \rightarrow B$  gives +1,  $B \rightarrow C$  gives +10
- ▶  $\gamma = 0.9$ ,  $\alpha = 0.5$



# TD Learning with Numbers: A Simple Example

## Setup: 3-State Chain

- ▶ States:  $A \rightarrow B \rightarrow C$  (terminal)
- ▶ Rewards:  $A \rightarrow B$  gives +1,  $B \rightarrow C$  gives +10
- ▶  $\gamma = 0.9$ ,  $\alpha = 0.5$
- ▶ Initial estimates:  $V(A) = 0$ ,  $V(B) = 0$ ,  $V(C) = 0$



# Step-by-Step TD Updates

Episode: A → B → C

Let's trace through one episode and see how values update:

# Step-by-Step TD Updates

Episode: A → B → C

Let's trace through one episode and see how values update:

Step 1: A → B (receive reward = +1)

TD Update for A:

$$\begin{aligned}V(A) &= V(A) + \alpha[r + \gamma V(B) - V(A)] \\&= 0 + 0.5[1 + 0.9 \times 0 - 0] \\&= 0 + 0.5 \times 1 = 0.5\end{aligned}$$

# Step-by-Step TD Updates

Episode: A → B → C

Let's trace through one episode and see how values update:

# Step-by-Step TD Updates

Episode: A → B → C

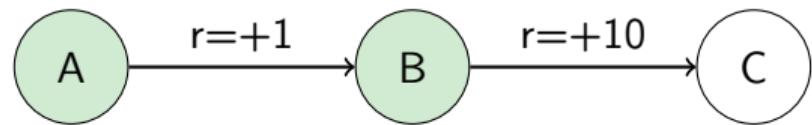
Let's trace through one episode and see how values update:

Step 2: B → C (receive reward = +10)

TD Update for B:

$$\begin{aligned}V(B) &= V(B) + \alpha[r + \gamma V(C) - V(B)] \\&= 0 + 0.5[10 + 0.9 \times 0 - 0] \\&= 0 + 0.5 \times 10 = 5.0\end{aligned}$$

## After First Episode

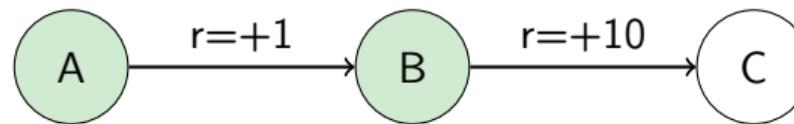


$$V(A) = 0.5$$

$$V(B) = 5.0$$

$$V(C) = 0$$

## After First Episode



$$V(A) = 0.5$$

$$V(B) = 5.0$$

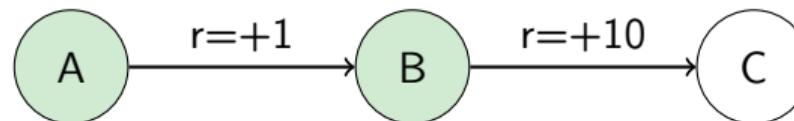
$$V(C) = 0$$

Second Episode:  $A \rightarrow B \rightarrow C$

Now A uses updated  $V(B)$ :

$$\begin{aligned} V(A) &\leftarrow 0.5 + 0.5[1 + 0.9 \times 5.0 - 0.5] \\ &\leftarrow 0.5 + 0.5[1 + 4.5 - 0.5] \\ &\leftarrow 0.5 + 0.5 \times 5.0 = 3.0 \end{aligned}$$

## After First Episode



$$V(A) = 0.5$$

$$V(B) = 5.0$$

$$V(C) = 0$$

Second Episode:  $A \rightarrow B \rightarrow C$

Now A uses updated  $V(B)$ :

$$\begin{aligned} V(A) &\leftarrow 0.5 + 0.5[1 + 0.9 \times 5.0 - 0.5] \\ &\leftarrow 0.5 + 0.5[1 + 4.5 - 0.5] \\ &\leftarrow 0.5 + 0.5 \times 5.0 = 3.0 \end{aligned}$$

Key Insight

TD uses updated estimates -  $V(A)$  learned from  $V(B)$ 's improvement!

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode

## TD Learning

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$

## TD Learning

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$

## TD Learning

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

- ▶ Updates at each step

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

- ▶ Updates at each step
- ▶ A uses estimate:  $1 + 0.9 \times 0 = 1$

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

- ▶ Updates at each step
- ▶ A uses estimate:  $1 + 0.9 \times 0 = 1$
- ▶ After step 1:  $V(A) = 0.5$

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

- ▶ Updates at each step
- ▶ A uses estimate:  $1 + 0.9 \times 0 = 1$
- ▶ After step 1:  $V(A) = 0.5$
- ▶ Learns progressively

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

- ▶ Updates at each step
- ▶ A uses estimate:  $1 + 0.9 \times 0 = 1$
- ▶ After step 1:  $V(A) = 0.5$
- ▶ Learns progressively

## Why This Matters

- ▶ TD starts learning **immediately**

# Compare with Monte Carlo

## Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

## TD Learning

- ▶ Updates at each step
- ▶ A uses estimate:  $1 + 0.9 \times 0 = 1$
- ▶ After step 1:  $V(A) = 0.5$
- ▶ Learns progressively

## Why This Matters

- ▶ TD starts learning **immediately**
- ▶ Values **propagate** through the chain

## Compare with Monte Carlo

### Monte Carlo

- ▶ Waits for complete episode
- ▶ A gets return:  $1 + 0.9 \times 10 = 10$
- ▶ After episode 1:  $V(A) = 5.0$
- ▶ No learning until end

### TD Learning

- ▶ Updates at each step
- ▶ A uses estimate:  $1 + 0.9 \times 0 = 1$
- ▶ After step 1:  $V(A) = 0.5$
- ▶ Learns progressively

### Why This Matters

- ▶ TD starts learning **immediately**
- ▶ Values **propagate** through the chain
- ▶ More robust to **incomplete episodes**

# Practical Example: Driving to Friend's House

## Scenario

- ▶ Predict travel time to friend's house

State	Elapsed Time	Predicted Total	Actual Total
Leaving office	0	30 min	43 min
Reach highway	5 min	35 min	43 min
Highway (no traffic)	20 min	40 min	43 min
Highway exit	30 min	43 min	43 min
Arrive at friend's	43 min	43 min	43 min

# Practical Example: Driving to Friend's House

## Scenario

- ▶ Predict travel time to friend's house
- ▶ Consider: time of day, weather, traffic, etc.

State	Elapsed Time	Predicted Total	Actual Total
Leaving office	0	30 min	43 min
Reach highway	5 min	35 min	43 min
Highway (no traffic)	20 min	40 min	43 min
Highway exit	30 min	43 min	43 min
Arrive at friend's	43 min	43 min	43 min

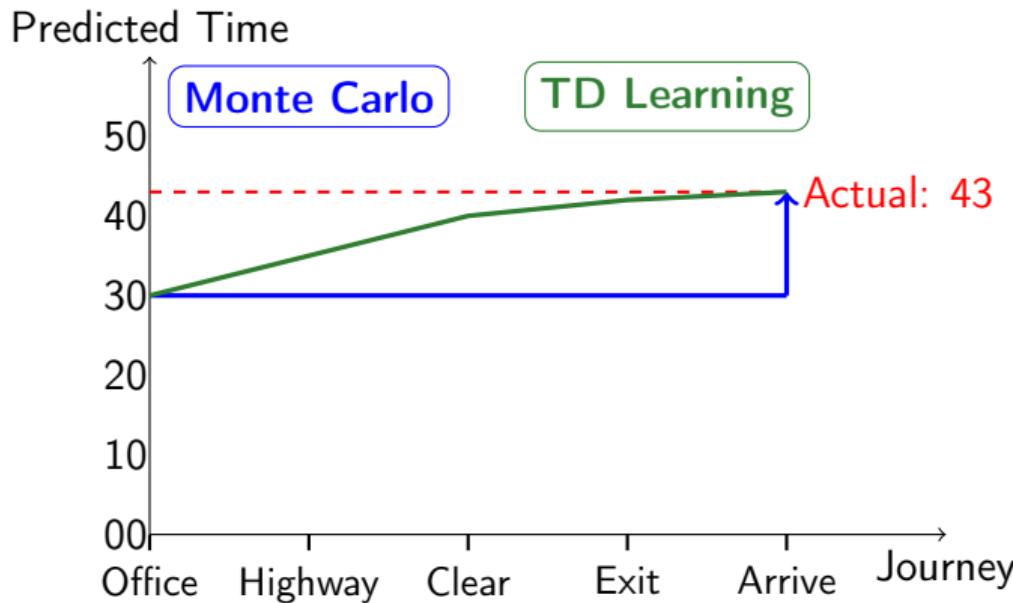
# Practical Example: Driving to Friend's House

## Scenario

- ▶ Predict travel time to friend's house
- ▶ Consider: time of day, weather, traffic, etc.
- ▶ Learn from experience over multiple trips

State	Elapsed Time	Predicted Total	Actual Total
Leaving office	0	30 min	43 min
Reach highway	5 min	35 min	43 min
Highway (no traffic)	20 min	40 min	43 min
Highway exit	30 min	43 min	43 min
Arrive at friend's	43 min	43 min	43 min

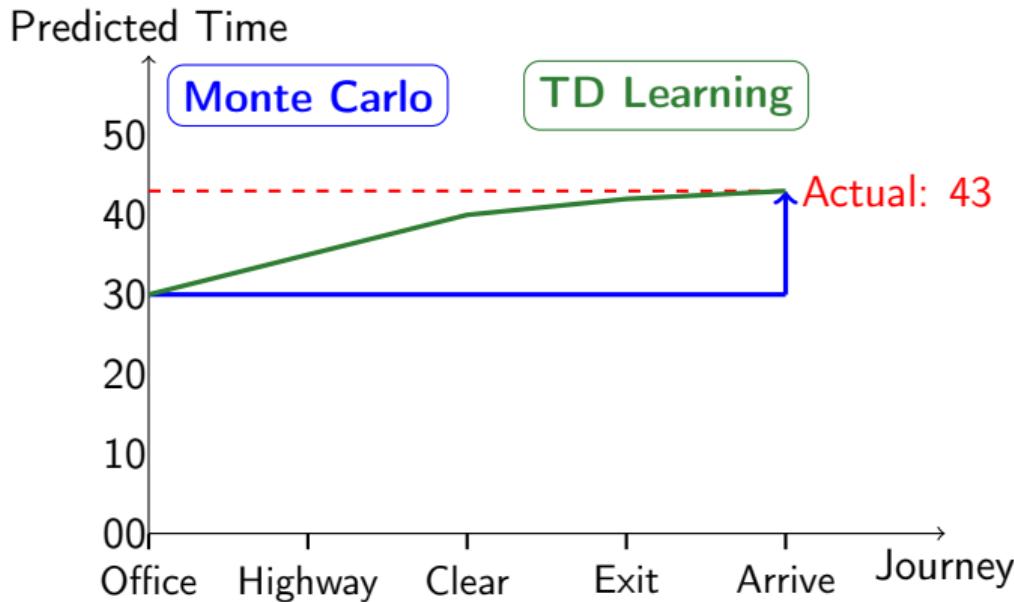
# MC vs TD Learning Comparison



## Observations

- TD: Updates prediction continuously during the trip

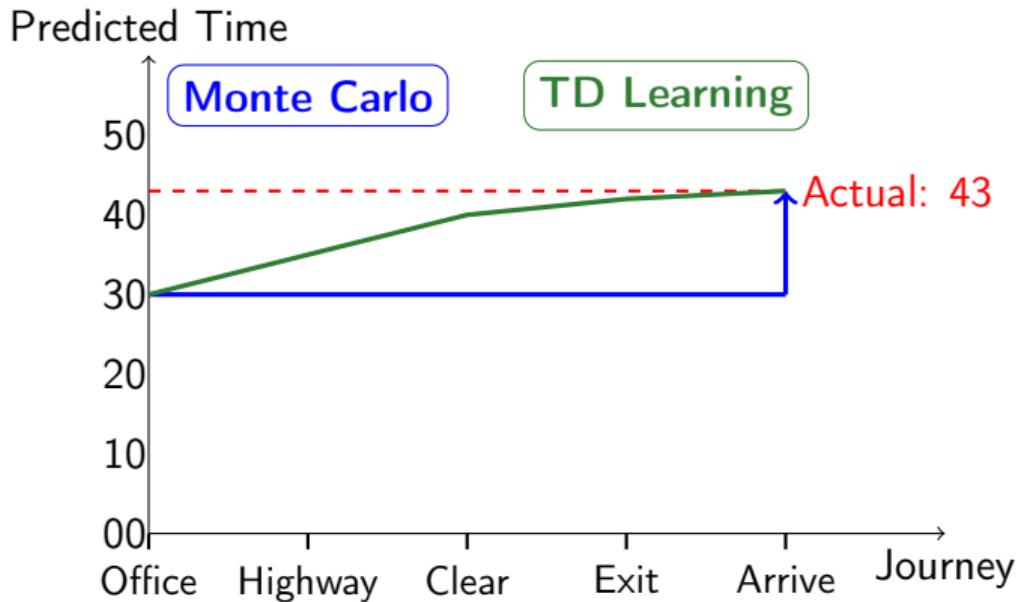
# MC vs TD Learning Comparison



## Observations

- ▶ **TD:** Updates prediction continuously during the trip
- ▶ **MC:** Only updates at the end of the trip

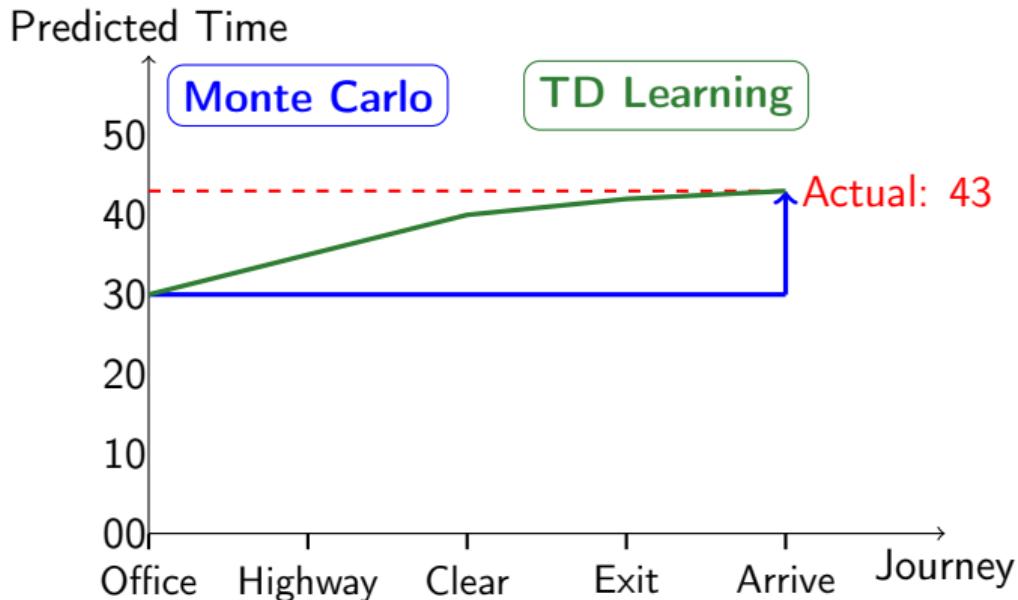
# MC vs TD Learning Comparison



## Observations

- ▶ **TD**: Updates prediction continuously during the trip
- ▶ **MC**: Only updates at the end of the trip
- ▶ TD can **adapt** to new information immediately

# MC vs TD Learning Comparison



## Observations

- **TD:** Updates prediction continuously during the trip
- **MC:** Only updates at the end of the trip
- TD can **adapt** to new information immediately
- MC waits for **complete** episode

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free:** No need for environment model

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free:** No need for environment model
- ▶ **Online learning:** Updates at every step

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free**: No need for environment model
- ▶ **Online learning**: Updates at every step
- ▶ **Fully incremental**: Constant memory requirements

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free**: No need for environment model
- ▶ **Online learning**: Updates at every step
- ▶ **Fully incremental**: Constant memory requirements
- ▶ **Bootstrapping**: Learns from estimates

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free**: No need for environment model
- ▶ **Online learning**: Updates at every step
- ▶ **Fully incremental**: Constant memory requirements
- ▶ **Bootstrapping**: Learns from estimates
- ▶ **Practical**: Works in continuing tasks

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free**: No need for environment model
- ▶ **Online learning**: Updates at every step
- ▶ **Fully incremental**: Constant memory requirements
- ▶ **Bootstrapping**: Learns from estimates
- ▶ **Practical**: Works in continuing tasks

## When to Use TD

- ▶ When episodes are very long or infinite

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free**: No need for environment model
- ▶ **Online learning**: Updates at every step
- ▶ **Fully incremental**: Constant memory requirements
- ▶ **Bootstrapping**: Learns from estimates
- ▶ **Practical**: Works in continuing tasks

## When to Use TD

- ▶ When episodes are very long or infinite
- ▶ When you need **real-time** learning

# Advantages of TD Learning

## Why TD Learning is Powerful

- ▶ **Model-free**: No need for environment model
- ▶ **Online learning**: Updates at every step
- ▶ **Fully incremental**: Constant memory requirements
- ▶ **Bootstrapping**: Learns from estimates
- ▶ **Practical**: Works in continuing tasks

## When to Use TD

- ▶ When episodes are very long or infinite
- ▶ When you need **real-time** learning
- ▶ When computational resources are limited

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions
- ▶ Essential for **policy improvement**

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions
- ▶ Essential for **policy improvement**
- ▶ Must balance **exploration** and **exploitation**

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions
- ▶ Essential for **policy improvement**
- ▶ Must balance **exploration** and **exploitation**

## Activity

Why do we need state-action pairs instead of just states?

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions
- ▶ Essential for **policy improvement**
- ▶ Must balance **exploration** and **exploitation**

## Activity

**Why do we need state-action pairs instead of just states?**

## Answer

- ▶ Need to know **value of each action** in each state

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions
- ▶ Essential for **policy improvement**
- ▶ Must balance **exploration** and **exploitation**

## Activity

Why do we need state-action pairs instead of just states?

## Answer

- ▶ Need to know **value of each action** in each state
- ▶ Essential for **policy improvement**

# Moving to Control: Action-Value Functions

## Why Action-Value Functions?

- ▶ Need to estimate  $q_{\pi}(s, a)$  for all states and actions
- ▶ Essential for **policy improvement**
- ▶ Must balance **exploration** and **exploitation**

## Activity

Why do we need state-action pairs instead of just states?

## Answer

- ▶ Need to know **value of each action** in each state
- ▶ Essential for **policy improvement**
- ▶ Cannot improve policy with only state values

# SARSA Update Rule

From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

# SARSA Update Rule

From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

# SARSA Update Rule

From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

# SARSA Update Rule

From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

- ▶ State  $s_t$

# SARSA Update Rule

From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

- ▶ State  $s_t$
- ▶ Action  $a_t$

# SARSA Update Rule

## From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

## SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

- ▶ State  $s_t$
- ▶ Action  $a_t$
- ▶ Reward  $r_{t+1}$

# SARSA Update Rule

## From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

## SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

- ▶ State  $s_t$
- ▶ Action  $a_t$
- ▶ Reward  $r_{t+1}$
- ▶ State  $s_{t+1}$

# SARSA Update Rule

## From State Values to Action Values

Replace  $V(s)$  with  $Q(s, a)$  in the TD update:

## SARSA Update

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] \quad (12)$$

Uses every element:  $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$

- ▶ State  $s_t$
- ▶ Action  $a_t$
- ▶ Reward  $r_{t+1}$
- ▶ State  $s_{t+1}$
- ▶ Action  $a_{t+1}$

# SARSA Algorithm

---

## Algorithm SARSA

---

```
1: Inputs:  $N$  (episodes),  $\alpha \in [0, 1]$  (step size)
2: Initialize:  $Q(s, a) \in \mathbb{R}$  arbitrarily,  $\forall s \in S, a \in A$ , except  $Q(\text{terminal}, \cdot) = 0$ 
3: for  $\text{episode} = 1$  to  $N$  do
4:   Initialize  $s$ 
5:    $a \leftarrow$  action from  $Q(s, \cdot)$  using  $\epsilon$ -greedy
6:   while  $s \neq \text{terminal}$  do
7:      $r, s' \leftarrow$  Execute  $a$ 
8:      $a' \leftarrow$  action from  $Q(s', \cdot)$  using  $\epsilon$ -greedy
9:      $Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma Q(s', a') - Q(s, a)]$ 
10:     $s \leftarrow s', a \leftarrow a'$ 
11:   end while
12: end for
```

---

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows
- ▶ Both behavior and target policy use  $\epsilon$ -greedy

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows
- ▶ Both behavior and target policy use  $\epsilon$ -greedy
- ▶ Updates  $Q(s, a)$  for the action actually taken

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows
- ▶ Both behavior and target policy use  $\epsilon$ -greedy
- ▶ Updates  $Q(s, a)$  for the action actually taken

## Action Selection Strategy

- ▶ Typically uses  **$\epsilon$ -greedy** policy

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows
- ▶ Both behavior and target policy use  $\epsilon$ -greedy
- ▶ Updates  $Q(s, a)$  for the action actually taken

## Action Selection Strategy

- ▶ Typically uses  **$\epsilon$ -greedy** policy
- ▶ Balances exploration and exploitation

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows
- ▶ Both behavior and target policy use  $\epsilon$ -greedy
- ▶ Updates  $Q(s, a)$  for the action actually taken

## Action Selection Strategy

- ▶ Typically uses  **$\epsilon$ -greedy** policy
- ▶ Balances exploration and exploitation
- ▶ Other strategies can be used (softmax, UCB, etc.)

# SARSA Key Properties

## On-Policy Learning

- ▶ Learns about the **same policy** it follows
- ▶ Both behavior and target policy use  $\epsilon$ -greedy
- ▶ Updates  $Q(s, a)$  for the action actually taken

## Action Selection Strategy

- ▶ Typically uses  **$\epsilon$ -greedy** policy
- ▶ Balances exploration and exploitation
- ▶ Other strategies can be used (softmax, UCB, etc.)

## Important Note

**No explicit policy is stored!** The policy is **implicit** in the Q-values.

# Q-Learning: A Classic Algorithm

## Historical Note

- ▶ Introduced in **1989** by Chris Watkins

# Q-Learning: A Classic Algorithm

## Historical Note

- ▶ Introduced in **1989** by Chris Watkins
- ▶ One of the most important RL algorithms

# Q-Learning: A Classic Algorithm

## Historical Note

- ▶ Introduced in **1989** by Chris Watkins
- ▶ One of the most important RL algorithms
- ▶ Foundation for many modern methods

# Q-Learning: A Classic Algorithm

## Historical Note

- ▶ Introduced in **1989** by Chris Watkins
- ▶ One of the most important RL algorithms
- ▶ Foundation for many modern methods

## Q-Learning Update Rule

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[ r_{t+1} + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right] \quad (13)$$

# Q-Learning: A Classic Algorithm

## Historical Note

- ▶ Introduced in **1989** by Chris Watkins
- ▶ One of the most important RL algorithms
- ▶ Foundation for many modern methods

## Q-Learning Update Rule

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[ r_{t+1} + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right] \quad (13)$$

## Key Difference from SARSA

- ▶ Uses  $\max_{a'} Q(s_{t+1}, a')$  instead of  $Q(s_{t+1}, a_{t+1})$

# Q-Learning: A Classic Algorithm

## Historical Note

- ▶ Introduced in **1989** by Chris Watkins
- ▶ One of the most important RL algorithms
- ▶ Foundation for many modern methods

## Q-Learning Update Rule

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left[ r_{t+1} + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t) \right] \quad (13)$$

## Key Difference from SARSA

- ▶ Uses  $\max_{a'} Q(s_{t+1}, a')$  instead of  $Q(s_{t+1}, a_{t+1})$
- ▶ Approximates  $q^*$  (optimal action-value function)

# Q-Learning Algorithm

---

## Algorithm Q-Learning

---

```
1: Inputs:  $N$  (episodes),  $\alpha \in [0, 1]$  (step size)
2: Initialize:  $Q(s, a) \in \mathbb{R}$  arbitrarily,  $\forall s \in S, a \in A$ , except  $Q(\text{terminal}, \cdot) = 0$ 
3: for  $\text{episode} = 1$  to  $N$  do
4:   Initialize  $s$ 
5:   while  $s \neq \text{terminal}$  do
6:      $a \leftarrow$  action from  $Q(s, \cdot)$  using  $\epsilon$ -greedy
7:      $r, s' \leftarrow$  Execute  $a$ 
8:      $Q(s, a) \leftarrow Q(s, a) + \alpha[r + \gamma \max_{a'} Q(s', a') - Q(s, a)]$ 
9:      $s \leftarrow s'$ 
10:    end while
11:  end for
```

---

# Activity: Spot the Differences

Question

What is the difference between SARSA and Q-Learning algorithms?

# Activity: Spot the Differences

## Question

What is the difference between SARSA and Q-Learning algorithms?

## Key Differences

### 1. Policy Type:

- ▶ SARSA: **On-policy** (learns about policy it follows)
- ▶ Q-Learning: **Off-policy** (learns about optimal policy)

# Activity: Spot the Differences

## Question

What is the difference between SARSA and Q-Learning algorithms?

## Key Differences

### 1. Policy Type:

- ▶ SARSA: **On-policy** (learns about policy it follows)
- ▶ Q-Learning: **Off-policy** (learns about optimal policy)

### 2. Update Target:

- ▶ SARSA:  $Q(s_{t+1}, a_{t+1})$  (actual next action)
- ▶ Q-Learning:  $\max_{a'} Q(s_{t+1}, a')$  (best possible action)

# Activity: Spot the Differences

## Question

What is the difference between SARSA and Q-Learning algorithms?

## Key Differences

### 1. Policy Type:

- ▶ SARSA: **On-policy** (learns about policy it follows)
- ▶ Q-Learning: **Off-policy** (learns about optimal policy)

### 2. Update Target:

- ▶ SARSA:  $Q(s_{t+1}, a_{t+1})$  (actual next action)
- ▶ Q-Learning:  $\max_{a'} Q(s_{t+1}, a')$  (best possible action)

### 3. Next Action Selection:

- ▶ SARSA: Needs  $a_{t+1}$  before update
- ▶ Q-Learning: No need for actual next action

# On-Policy vs Off-Policy Behavior

SARSA (On-Policy)

Q-Learning (Off-Policy)

- ▶ **Conservative**

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed

## Q-Learning (Off-Policy)

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning

## Q-Learning (Off-Policy)

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

- ▶ **Optimistic**

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

- ▶ **Optimistic**
- ▶ Learns the value of the **optimal policy**

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

- ▶ **Optimistic**
- ▶ Learns the value of the **optimal policy**
- ▶ Ignores exploration in learning

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

- ▶ **Optimistic**
- ▶ Learns the value of the **optimal policy**
- ▶ Ignores exploration in learning
- ▶ Faster convergence to optimal

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

- ▶ **Optimistic**
- ▶ Learns the value of the **optimal policy**
- ▶ Ignores exploration in learning
- ▶ Faster convergence to optimal

## Practical Implications

- ▶ **SARSA:** Better for **risky** environments (cliff walking)

# On-Policy vs Off-Policy Behavior

## SARSA (On-Policy)

- ▶ **Conservative**
- ▶ Learns the value of the **actual policy** followed
- ▶ Considers exploration in learning
- ▶ Safer in dangerous environments

## Q-Learning (Off-Policy)

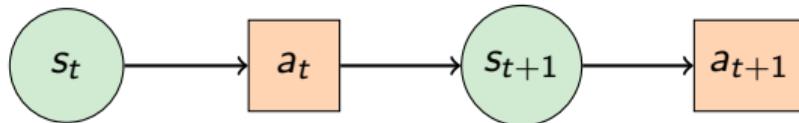
- ▶ **Optimistic**
- ▶ Learns the value of the **optimal policy**
- ▶ Ignores exploration in learning
- ▶ Faster convergence to optimal

## Practical Implications

- ▶ **SARSA:** Better for **risky** environments (cliff walking)
- ▶ **Q-Learning:** Better for finding **optimal** policies

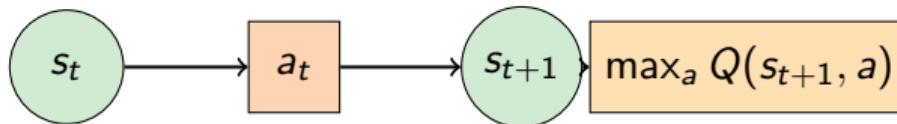
# Visual Comparison: Update Mechanisms

## SARSA



Uses actual  $a_{t+1}$

## Q-Learning



Uses best possible action

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free:** No environment model required

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free:** No environment model required
- ▶ **Online:** Learn during interaction

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free**: No environment model required
- ▶ **Online**: Learn during interaction
- ▶ **Incremental**: Constant memory and computation

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free**: No environment model required
- ▶ **Online**: Learn during interaction
- ▶ **Incremental**: Constant memory and computation
- ▶ **Bootstrapping**: Faster than Monte Carlo

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free**: No environment model required
- ▶ **Online**: Learn during interaction
- ▶ **Incremental**: Constant memory and computation
- ▶ **Bootstrapping**: Faster than Monte Carlo

## Practical Benefits

- ▶ Works with **continuing tasks**

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free**: No environment model required
- ▶ **Online**: Learn during interaction
- ▶ **Incremental**: Constant memory and computation
- ▶ **Bootstrapping**: Faster than Monte Carlo

## Practical Benefits

- ▶ Works with **continuing tasks**
- ▶ Suitable for **real-time** applications

# Why TD Learning Matters

## Core Advantages

- ▶ **Model-free**: No environment model required
- ▶ **Online**: Learn during interaction
- ▶ **Incremental**: Constant memory and computation
- ▶ **Bootstrapping**: Faster than Monte Carlo

## Practical Benefits

- ▶ Works with **continuing tasks**
- ▶ Suitable for **real-time** applications
- ▶ Foundation for **modern deep RL**

# Method Comparison Summary

Method	Model-Free	Online	Policy Type	Convergence
Monte Carlo	Yes	No	On/Off	Guaranteed
TD(0)	Yes	Yes	On	Guaranteed
SARSA	Yes	Yes	On	Guaranteed
Q-Learning	Yes	Yes	Off	Guaranteed

# Method Comparison Summary

Method	Model-Free	Online	Policy Type	Convergence
Monte Carlo	Yes	No	On/Off	Guaranteed
TD(0)	Yes	Yes	On	Guaranteed
SARSA	Yes	Yes	On	Guaranteed
Q-Learning	Yes	Yes	Off	Guaranteed

## When to Use Each

- ▶ **SARSA:** Safe exploration, learns about followed policy

# Method Comparison Summary

Method	Model-Free	Online	Policy Type	Convergence
Monte Carlo	Yes	No	On/Off	Guaranteed
TD(0)	Yes	Yes	On	Guaranteed
SARSA	Yes	Yes	On	Guaranteed
Q-Learning	Yes	Yes	Off	Guaranteed

## When to Use Each

- ▶ **SARSA:** Safe exploration, learns about followed policy
- ▶ **Q-Learning:** Optimal policies, faster convergence

# Method Comparison Summary

Method	Model-Free	Online	Policy Type	Convergence
Monte Carlo	Yes	No	On/Off	Guaranteed
TD(0)	Yes	Yes	On	Guaranteed
SARSA	Yes	Yes	On	Guaranteed
Q-Learning	Yes	Yes	Off	Guaranteed

## When to Use Each

- ▶ **SARSA:** Safe exploration, learns about followed policy
- ▶ **Q-Learning:** Optimal policies, faster convergence
- ▶ Both: Foundation for function approximation methods

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

## Algorithm Choice

- ▶ Use **SARSA** when safety matters

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

## Algorithm Choice

- ▶ Use **SARSA** when safety matters
- ▶ Use **Q-Learning** for optimal performance

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

## Algorithm Choice

- ▶ Use **SARSA** when safety matters
- ▶ Use **Q-Learning** for optimal performance
- ▶ Both are **foundations** for advanced methods

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

## Algorithm Choice

- ▶ Use **SARSA** when safety matters
- ▶ Use **Q-Learning** for optimal performance
- ▶ Both are **foundations** for advanced methods

## Next Steps

- ▶ Function approximation (Deep Q-Networks)

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

## Algorithm Choice

- ▶ Use **SARSA** when safety matters
- ▶ Use **Q-Learning** for optimal performance
- ▶ Both are **foundations** for advanced methods

## Next Steps

- ▶ Function approximation (Deep Q-Networks)
- ▶ Policy gradient methods

# Key Takeaways

## Fundamental Concepts

- ▶ **TD Error:** Measures prediction accuracy
- ▶ **Bootstrapping:** Learn from estimates
- ▶ **Online Learning:** Update at every step

## Algorithm Choice

- ▶ Use **SARSA** when safety matters
- ▶ Use **Q-Learning** for optimal performance
- ▶ Both are **foundations** for advanced methods

## Next Steps

- ▶ Function approximation (Deep Q-Networks)
- ▶ Policy gradient methods

Questions?

# Questions?

Thank you for your attention!

Next: Function Approximation Methods