Dynamic Programming

DEPARTMENT OF COMPUTER SCIENCE
ST. FRANCIS XAVIER UNIVERSITY

CSCI-531 - Reinforcement Learning

Fall 2025

From Theory to Practice: Computing Optimal Policies

What We Know So Far

- How to model problems as **MDPs**

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How do we actually compute these optimal policies?

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The Approach Menu

- **Dynamic Programming**: Complete knowledge of MDP

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- **Dynamic Programming**: Complete knowledge of MDP
- **Monte Carlo Methods**: Learn from experience

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The Approach Menu

- **Dynamic Programming**: Complete knowledge of MDP
- **Monte Carlo Methods**: Learn from experience
- **Temporal Difference Learning**: Combines both approaches

Why Start with Dynamic Programming?

Three Key Reasons

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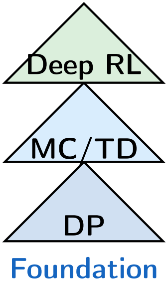

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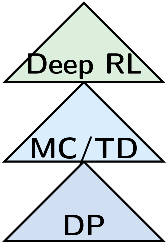

Foundation

What do we need to calculate to obtain optimal policies?

Policy Iteration: The Intuitive Approach

Core Idea

Start with any policy, then repeatedly improve it until optimal

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Two Alternating Steps

1. **Policy Evaluation**: How good is current policy?

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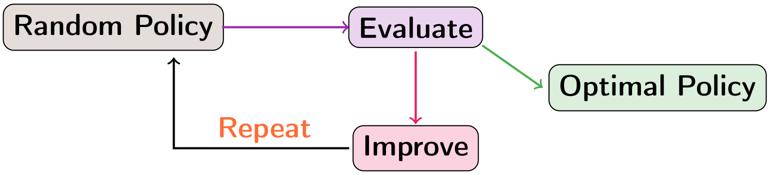

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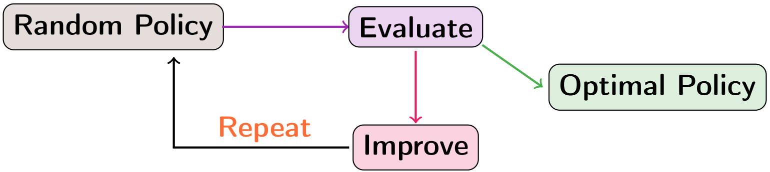

Each improvement step makes the policy better (or keeps it optimal)

Policy Evaluation: Computing Value Functions

The Challenge

Given policy π, compute $v_\pi(s)$ for all states using:

$$v_\pi(s) = \sum_a \pi(a|s) \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma v_\pi(s') \right]$$

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- This equation is **circular**!

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The Problem

- This equation is **circular**!
- Value of state s depends on values of future states s'
- We have a system of $|S|$ equations with $|S|$ unknowns

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1. Start with initial guesses $v_0(s)$ (often zeros)

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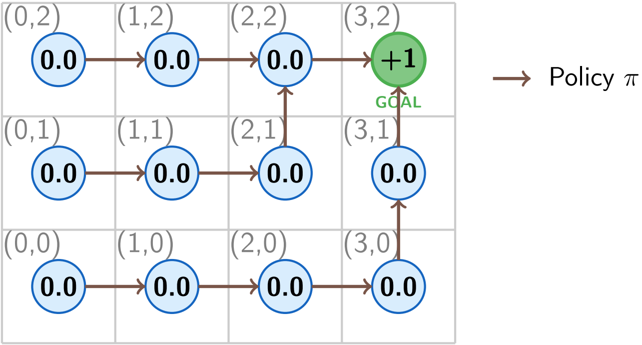
1. Start with initial guesses $v_0(s)$ (often zeros)
2. Iteratively improve estimates using Bellman equation
3. Continue until values stabilize

Policy Evaluation Algorithm

Algorithm Policy Evaluation

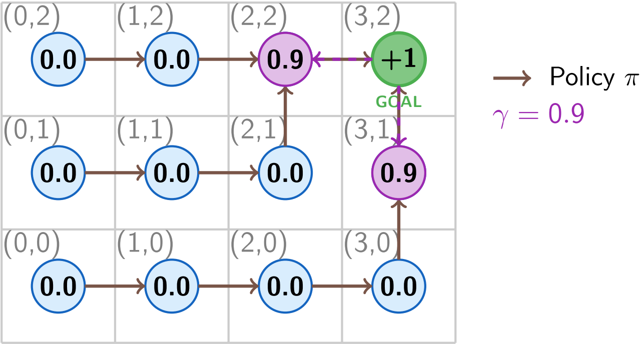
Initialize: $V(s) \in \mathbb{R}$ arbitrarily, $V(\text{terminal}) = 0$
repeat
    $\Delta \leftarrow 0$
    for each state s **do**
        $v \leftarrow V(s)$
        $V(s) \leftarrow \sum_a \pi(a|s) \sum_{s'} p(s'|s,a)[r + \gamma V(s')]$
        $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
    end for
until $\Delta < \theta$ (small threshold)

Policy Evaluation: Intuition



Iteration 0: Initialize all values to 0

Policy Evaluation: Intuition



Iteration 0: Initialize all values to 0

Iteration 1: Values propagate to immediate neighbors

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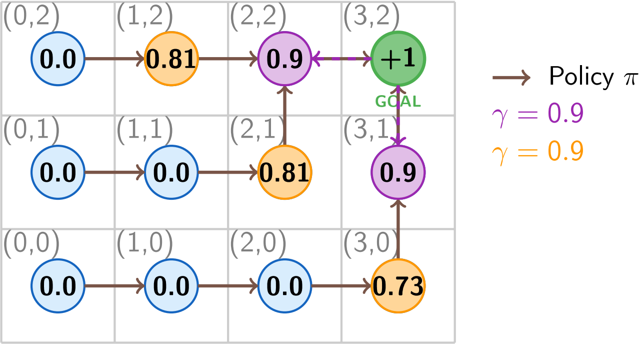

Iteration 0: Initialize all values to 0
Iteration 1: Values propagate to immediate neighbors
Iteration 2: Values continue propagating (discounted)

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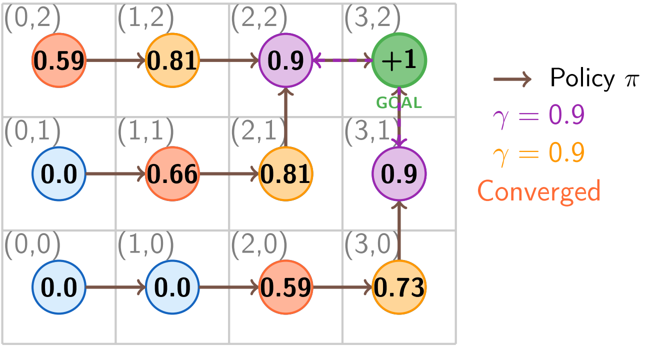

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Convergence: Optimal values for given policy

Policy Improvement: Making Better Decisions

The Question

Given policy π and its value function v_π, can we find a better policy?

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The Approach: Be Greedy!

For each state s, examine all possible actions:

$$q_\pi(s,a) = \sum_{s'} p(s'|s,a)\left[r(s,a) + \gamma v_\pi(s')\right]$$

"What happens if I take action a in state s, then follow policy π?"

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Improved Policy

Create new policy π' by being greedy:

$$\pi'(s) = \arg\max_a q_\pi(s, a) = \arg\max_a \sum_{s'} p(s'|s, a) \left[r(s, a) + \gamma v_\pi(s') \right]$$

Policy Improvement Theorem

Theorem

If π and π' are policies such that for all $s \in S$:

$$q_\pi(s, \pi'(s)) \geq v_\pi(s)$$

Then policy π' must be as good as or better than π:

$$v_{\pi'}(s) \geq v_\pi(s) \text{ for all } s \in S$$

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Guaranteed improvement or stability!

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Complete Process

1. **Initialize**: Start with arbitrary policy π_0

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Convergence Guarantee

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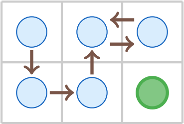
- Each improvement makes policy strictly better (unless optimal)
- Must converge to optimal policy in finite steps

Policy Iteration Example

Step 1: Random Policy



Random actions

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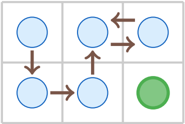
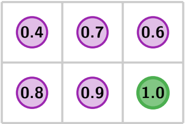


Random actions

Step 2: Evaluate



Computed values

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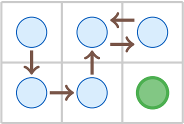

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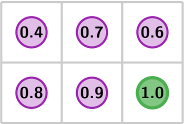

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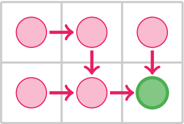
Step 3: Improve



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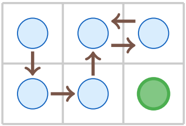

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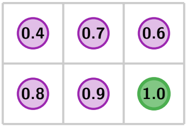

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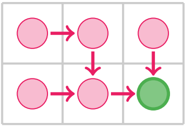

Greedy actions

Key Insight

Each iteration makes the policy strictly better until optimal

Value Iteration: Cutting the Bottleneck

Policy Iteration's Bottleneck

- Full policy evaluation requires multiple sweeps through all states

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Skip the separate policy - work directly with values!

Value Iteration Formula

Key Difference

Instead of following a fixed policy, always choose the best action:

$$v_{k+1}(s) = \max_a \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma v_k(s') \right]$$

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Policy Iteration says:

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Connection to Bellman Optimality

This directly solves the Bellman optimality equation:

$$v^*(s) = \max_a \sum_{s'} p(s'|s,a)\left[r(s,a) + \gamma v^*(s')\right]$$

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Key Advantages

- **Simpler**: No separate policy evaluation phase

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     - $V(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)[r + \gamma V(s')]$
     - $\Delta \leftarrow \max(\Delta, |v - V(s)|)$
3. **Until** $\Delta < \theta$ (convergence threshold)
4. **Extract policy**: $\pi(s) = \arg\max_a \sum_{s'} p(s'|s,a)[r + \gamma V(s')]$

Key Advantages

- Simpler: No separate policy evaluation phase
- More efficient: Often faster per iteration

Value Iteration vs Policy Iteration

Aspect	Policy Iteration	Value Iteration
Approach	Evaluate then improve	Combined eval+improve
Per iteration	Slower (full eval)	Faster (one step)
Total iterations	Fewer	More
Memory	Store policy + values	Store values only
Implementation	More complex	Simpler
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When to Use Which?

- **Policy Iteration**: When actions are expensive to compute
- **Value Iteration**: When state space is large
- **In practice**: Value iteration often preferred for simplicity

Computational Complexity Analysis

Time Complexity per Iteration

For both algorithms, each iteration requires:

- Loop over all states: $O(|S|)$

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Space Complexity

- **Value Iteration**: $O(|S|)$ for value function
- **Policy Iteration**: $O(|S|)$ for values + $O(|S|)$ for policy
- Both need to store MDP: $O(|S|^2|A|)$ for transition probabilities

Computational Complexity Analysis

Scalability Issues

- **Curse of dimensionality**: State space grows exponentially
- **Memory requirements**: Storing full transition model
- **Exact methods**: Limited to small-medium problems

Handling Large State Spaces

The Problem

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- **Linear**: $V(s) \approx \theta^T \phi(s)$
  - Feature vector $\phi(s)$, learned weights θ

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- **Decision Trees**: Partition state space

From Dynamic Programming to Model-Free RL

DP Limitations in Practice

- **Model requirement**: Often don't know T and R

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Key Insight

These methods use the same core principles as DP, but learn from experience instead of using a model

Summary

What We've Learned

- **Policy Iteration**: Alternate evaluation and improvement
- **Value Iteration**: Combined evaluation and improvement
- **Theoretical guarantees**: Convergence to optimal policies
- **Practical considerations**: Complexity and scalability

Next Topics

- Monte Carlo methods for model-free learning
- Temporal difference learning (TD, Q-Learning, SARSA)
- Function approximation and deep reinforcement learning
- Policy gradient methods