Multi-Armed Bandit

DEPARTMENT OF COMPUTER SCIENCE
ST. FRANCIS XAVIER UNIVERSITY

CSCI-531 - Reinforcement Learning

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From Full RL to Bandit Problems

Why Start with Bandits?

- Multi-armed bandits are a **simplified version** of full RL

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- **Bandit**: Single state, multiple actions, immediate rewards

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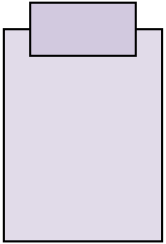
Understanding bandits $\rightarrow$ *Foundation for all RL*

What is the k-Armed Bandit Problem?

The Setup

- Agent has k different actions



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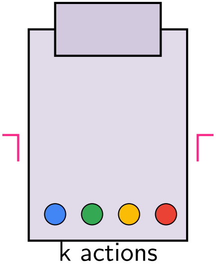

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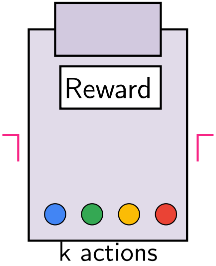

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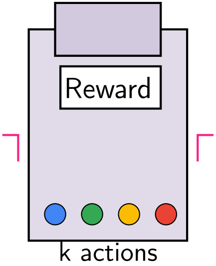
- **Objective**: Maximize expected total reward



Mathematical Formulation

Key Variables

- **Action at time** t: $A_t \in \{1, 2, \ldots, k\}$

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True vs Estimated Values

True value of action a**:**

$$q_*(a) = \mathbb{E}[R_t|A_t = a]$$

Estimated value at time t**:**

$$Q_t(a) \approx q_*(a)$$

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Challenge: We don't know $q_*(a)$ - must estimate from experience!

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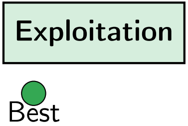

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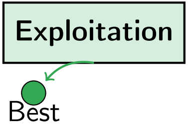

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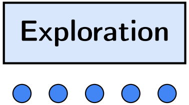

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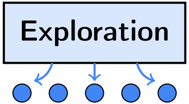

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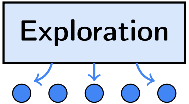

Cost: Lower immediate reward

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The Challenge: Balance exploration and exploitation

Action-Value Methods

The Approach

1. **Estimate** action values from experience

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How do we estimate action values?

Sample Average Method

Natural Estimation Approach

One natural way to estimate expected reward:

$$Q_t(a) = \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t}$$

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Mathematical Formulation

More formally:

$$Q_t(a) = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

where $\mathbb{1}_{A_i=a}$ is 1 if action a was taken at time i, 0 otherwise.

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Law of Large Numbers: As $n \to \infty$, $Q_t(a) \to q_*(a)$

Example: Estimating Action Values

Time:	1	2	3	4	5	6
Action:	A_2	A_1	A_3	A_4	A_3	A_1
Reward:	5	-1	2	10	1.5	1

Estimates:

$$Q_7(A_1) = \frac{-1+1}{2} = 0 \qquad Q_7(A_2) = \frac{5}{1} = 5$$

$$Q_7(A_3) = \frac{2+1.5}{2} = 1.75 \qquad Q_7(A_4) = \frac{10}{1} = 10$$

Best action so far: A_4 with $Q_7(A_4) = 10$

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The Strategy

Balance exploration and exploitation with a simple rule:

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- With probability $(1-\epsilon)$: Choose **greedy action**

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Simple but effective approach to exploration-exploitation

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Trade-off: Higher ϵ = more exploration, lower immediate reward

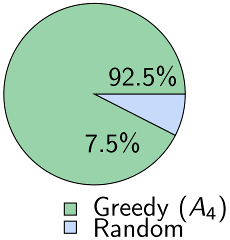
ϵ-Greedy Example

Current estimates:

$$Q_t(A_1) = 0$$
$$Q_t(A_2) = 5$$
$$Q_t(A_3) = 1.75$$
$$Q_t(A_4) = 10 \quad \leftarrow \text{Best}$$

With $\epsilon = 0.1$:

- Probability of choosing A_4: $0.9 + 0.1 \times 0.25 = 0.925$
- Probability of choosing any other action: $0.1 \times 0.25 = 0.025$



The Computational Challenge

Naive Implementation

Store all rewards and recompute average each time:

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Better approach needed.

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Mathematical Derivation

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Key RL Update Pattern

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This pattern appears throughout all of reinforcement learning!

Complete k-Armed Bandit Algorithm

Algorithm ϵ-Greedy k-Armed Bandit

Initialize:
for $a = 1$ to k **do**
    $Q(a) \leftarrow 0$
    $N(a) \leftarrow 0$
end for
Loop forever:
$$A \leftarrow \begin{cases} \arg\max_a Q(a) & \text{with probability } 1 - \epsilon \\ \text{random action} & \end{cases}$$
$R \leftarrow \text{bandit}(A)$
$N(A) \leftarrow N(A) + 1$
$Q(A) \leftarrow Q(A) + \frac{1}{N(A)}[R - Q(A)]$

Beyond Sample Averages: Constant Step-Size

Limitation of Sample Average

Sample average method: $Q_{n+1} = Q_n + \frac{1}{n}[R_n - Q_n]$

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Replace $\frac{1}{n}$ with constant $\alpha \in (0, 1]$:

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Benefits:

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- Called **weighted average**

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&= (1-\alpha)Q_n + \alpha R_n \\
&= (1-\alpha)[(1-\alpha)Q_{n-1} + \alpha R_{n-1}] + \alpha R_n \\
&= (1-\alpha)^2 Q_{n-1} + \alpha(1-\alpha)R_{n-1} + \alpha R_n \\
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\end{aligned}
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Weighted Average Derivation

Expanding the Constant Step-Size Update

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Weight of reward R_i: $\alpha(1-\alpha)^{n-i}$

- Recent rewards have weight close to α
- Weights decay exponentially into the past
- Sum of all weights $= 1$ (true weighted average)

Upper Confidence Bound (UCB) Action Selection

Limitation of ϵ-Greedy

- Random exploration among **all** non-greedy actions

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- Select action with highest **upper confidence bound**

Understanding the UCB Formula

$$A_t = \arg\max_a \left[\underbrace{Q_t(a)}_{\text{Exploitation}} + \underbrace{c\sqrt{\frac{\ln t}{N_t(a)}}}_{\text{Exploration Bonus}} \right]$$

where $c > 0$ controls the degree of exploration

Exploration Bonus Analysis

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Exploration Bonus Analysis

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- t **increases**: More total trials $\rightarrow$ should explore more
- c **parameter**: Controls exploration vs exploitation trade-off

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Intuition

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- Actions with many trials get low bonus (low uncertainty)
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No parameters to tune over time - UCB adapts automatically!

UCB Example

Current state after 8 time steps:

$$Q_8(A_1) = 0, \quad N_8(A_1) = 2 \quad Q_8(A_2) = 5, \quad N_8(A_2) = 1$$
$$Q_8(A_3) = 1.75, \quad N_8(A_3) = 3 \quad Q_8(A_4) = 10, \quad N_8(A_4) = 2$$

UCB values with $c = 2$:

$$UCB_8(A_1) = 0 + 2\sqrt{\frac{\ln 8}{2}} = 0 + 2\sqrt{1.04} = 2.04$$

$$UCB_8(A_2) = 5 + 2\sqrt{\frac{\ln 8}{1}} = 5 + 2\sqrt{2.08} = 7.88$$

$$UCB_8(A_3) = 1.75 + 2\sqrt{\frac{\ln 8}{3}} = 1.75 + 2\sqrt{0.69} = 3.42$$

$$UCB_8(A_4) = 10 + 2\sqrt{\frac{\ln 8}{2}} = 10 + 2\sqrt{1.04} = 12.04$$

Gradient Bandit Algorithms

Different Approach

Instead of estimating action values, learn **action preferences**:

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Softmax Action Selection

$$\pi_t(a) = P(A_t = a) = \frac{e^{H_t(a)}}{\sum_{b=1}^{k} e^{H_t(b)}}$$

where $\pi_t(a)$ is the probability of selecting action a at time t.

Gradient Bandit Algorithms

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Initial Conditions

- Start with $H_1(a) = 0$ for all actions
- Initially: $\pi_1(a) = \frac{1}{k}$ (uniform distribution)

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Key Insight

- No need to estimate action values $q_*(a)$

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- Only **relative** differences matter

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Preference Properties

- Only **relative** differences matter
- Adding constant to all preferences: no effect on probabilities
- If $H_t(a) > H_t(b)$, then $\pi_t(a) > \pi_t(b)$

Gradient Bandit Update Rules

Stochastic Gradient Ascent Updates

For the selected action:

$$H_{t+1}(A_t) = H_t(A_t) + \alpha(R_t - \bar{R}_t)(1 - \pi_t(A_t))$$

For all other actions $a \neq A_t$**:**

$$H_{t+1}(a) = H_t(a) - \alpha(R_t - \bar{R}_t)\pi_t(a)$$

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- Baseline $\bar{R}_t$ provides context - what's "good" vs "bad"

Thompson Sampling: Bayesian Approach

The Bayesian Philosophy

- Maintain **belief distributions** over true action values

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- **Natural exploration**: High uncertainty $\rightarrow$ more sampling variation
- **Theoretically optimal** for many reward distributions

Bandit Algorithm Comparison

Algorithm	**Exploration**	**Parameters**	**Cost**	**Best Use**
ϵ-greedy	Random	ϵ	Very Low	Simple baseline
UCB	Confidence	c	Low	Principled exploration
Gradient	Preference	α, baseline	Medium	Varying scales
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- ϵ**-greedy**: Need interpretable, simple baseline
- **UCB**: Want theoretical guarantees without parameter tuning
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- **Thompson**: Want near-optimal exploration

Theoretical Performance Guarantees

Regret Analysis

Regret = Total reward loss compared to always choosing optimal action

$$\text{Regret}_T = T \cdot r^* - \sum_{t=1}^{T} R_t$$

where $r^* = \max_a q_*(a)$ is the optimal expected reward.

Theoretical Performance Guarantees

Algorithm Regret Bounds

Algorithm	**Regret Bound**
Random	$O(T)$
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Key insight: Sublinear regret bounds ($O(T)$) mean the algorithm eventually finds the optimal action!

From Bandits to Full Reinforcement Learning

What Bandits Taught Us

- **Exploration vs Exploitation** trade-off

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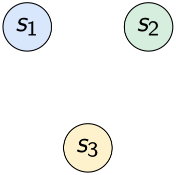
- **Stationary environment**: World doesn't change

The Jump to Full Reinforcement Learning

Full RL Removes Limitations

- **Multiple States**: Different situations



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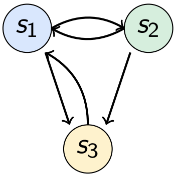

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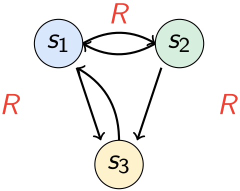

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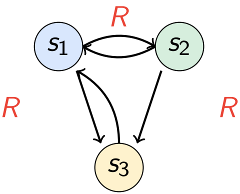

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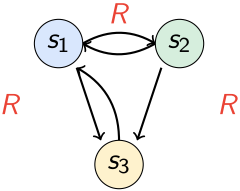

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- **Bandit**: $Q(a)$ - value of action a

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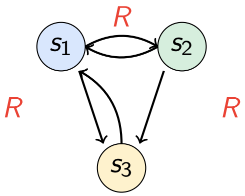

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- **Bandit**: $Q(a)$ - value of action a
- **Full RL**: $Q(s, a)$ - value of action a in state s

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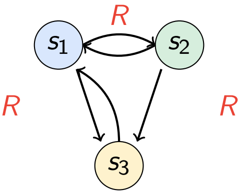

Mathematical Connection

- **Bandit**: $Q(a)$ - value of action a
- **Full RL**: $Q(s, a)$ - value of action a in state s

Every state in RL = separate bandit problem!

Summary

Next: Markov Decision Processes

- States, actions, transitions, and rewards
- Policies and value functions
- Bellman equations
- Dynamic programming solutions

The exploration-exploitation foundation is set!