

Expansion and Implementation of a 3x3 Sobel and Prewitt Edge Detection Filter to a 5x5 Dimension Filter

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Abstract

Sobel and Prewitt edge detection is considered in this work. Technically Sobel operator is a discrete differentiation operator computing an approximation of the gradient of the image intensity function. The result of the Sobel operator is either the corresponding gradient vector or the norm of this vector. The Prewitt operator gives values that are symmetric around the center (x,y) , while Sobel operator gives weight to point lying closer to (x,y) .

The horizontal and vertical gradient matrices whose dimensions are 3x3 for Sobel and Prewitt method has been generally used in edge detection operation. In this paper, the standard 3x3 dimension Sobel and prewitt masks, which are based on the relationship between point in the Cartesian grid and its 8 neighbors where the gradient value represent the sum of two orthogonal values, is expanded to find edges using the matrices whose dimension are 5x5 and these matrices are applied on images using Matlab.

الملخص

في هذا البحث تم اعتماد طريقتي Sobel و Prewitt لكشف الحافة. تقنيا ان مشغل Sobel يعتبر مجزء تفاضلي يقوم بحساب تقريبي لميل دالة شدة الصورة وان نتائج هذا المشغل اما متجه الميل او العمود على ذلك المتجه. ان مشغل Prewitt يعطي قيم متماثلة حول المركز (x,y) بينما مشغل Sobel يعطي الوزن لنقطة تقع بالقرب من المركز (x,y) . ان مصفوفتي الانحدار الافقي والعمودي في طريقتي Sobel و Prewitt والتي ابعادها 3x3 تستخدم بصورة واسعة في عمليات كشف الحافة.

في هذا البحث تم استخدام المصفوفة القياسية 3x3 لكل من Sobel و Prewitt ، والمبنية على اساس العلاقة بين النقاط في شبكة الاحداثيات الكارتيزية ومجاوراتها الثمانية حيث ان قيمة الميل تمثل مجموع قيمتين متعامدتين، هذه المصفوفة تم توسيعها لايجاد الحافة باستخدام المصفوفة التي ابعادها 5x5. ان المصفوفات 3x3 و 5x5 الناتجة تم تطبيقها على الصور باستخدام لغة ال Matlab.

Introduction

Edge detection is by far the most common approach for detecting meaningful discontinuities in gray level [Gonzalez, 2000]. In many ways edge detection can be considered as the dual of image segmentation and also have been used by object recognition, and target tracking. The goal of edge detection is to find the boundary of objects of interest [Sangwine, 1998].

There are several edge detection methods such as (Sobel, Frei-chen, Prewitt, Canny, Robert). These methods determined the best gradient operator to detect sharp intensity variations [Ziou, 1997].

In computer vision, edge detection is traditionally implemented by convolving the signal with some form of linear filter usually a filter that approximate a first or second derivative operator [Umbaugh, 1998].

Sobel and Prewitt which is an edge detection method is considered because of its simplicity, and they implement their algorithm on the idea that edge can be detected as local maximum of the image convolved with a first derivative operator [Sobel,1990]. Sobel and prewitt edge detector uses two masks, one vertical and and one horizontal. These masks use generally a 3x3 matrices, which are extended in this work to 5x5 dimensions matrices. A Matlab is a high performance language for technical computing and is a product of a Mathworks company, matlab dealing with a lot of toolbox which has many functions and algorithms [Image Toolbox, 2002]. A set of 6 images is used to test a 3x3 and 5x5 Sobel and Prewitt edge detector in Matlab.

Generating and Expanding of Sobel edge detector

In standard Sobel operator for a 3x3 neighborhood, each simple central gradient estimate is a vector sum of a pair of orthogonal vectors [Sobel, 1990]. Each orthogonal vector is a directional derivative estimate multiplied by a unit vector specifying the derivative's direction. The vector sum of these simple gradient estimates amounts to a vector sum of the 8-directional derivative vectors.

The point on Cartesian grid and its eight neighbors having density values are shown below:

a	b	c
d	e	f
g	h	i

The directional derivative estimate vector G was defined such as:

$G = (\text{density difference}) / (\text{distance to neighbor})$.

This vector is determined such that the direction of G will be given by the unit vector to approximate neighbor, where the neighbors group into antipodal pairs: (a,i),(b,h),(c,g), (f,d) [sobel,1990].

The vector sum of this gradient is:

$$G = \frac{(c - g)}{R} \cdot \frac{[1,1]}{R} + \frac{(a - i)}{R} \cdot \frac{[-1,1]}{R} + (b - h) \cdot [0,1] + (f - d) \cdot [1,0] \quad (1)$$

Where $R = \sqrt{2}$ which represent the distance to neighbor.

$$G = \frac{(c - g)}{\sqrt{2}} \cdot \frac{[1,1]}{\sqrt{2}} + \frac{(a - i)}{\sqrt{2}} \cdot \frac{[-1,1]}{\sqrt{2}} + (b - h) \cdot [0,1] + (f - d) \cdot [1,0] \quad (2)$$

$$G = [(c - g - a + i)/2 + f - d, (c - g + a - i)/2 + b - h] \quad (3)$$

This vector is multiplied by 2 because of replacing the divide by 2. the resultant formula is given as follows: [Sobel,1990]

$$G' = 2 \cdot G = [(c - g - a + i) + 2 \cdot (f - d), (c - g + a - i) + 2 \cdot (b - h)] \quad (4)$$

The resultant weighting function for x and y components were obtained by using the above vector is shown as follows:

-1	0	1
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1	2	1
0	0	0
-1	-2	-1

G_x

-2	0	2
-1	0	1

G_y

It represents the two convolutions horizontal and vertical kernels G_x and G_y, respectively. We can expanded the dimension of the matrices above by using the definition of the gradient for 5x5 neighborhoods, were 11 directional gradients must be determined instead of 4 gradients as in the following figure:

a	b	c	d	e
f	g	h	i	j
k	l	m	n	o
p	r	s	t	u
v	w	x	y	z

In this case, the vector sum of this gradient is:

$$\begin{aligned}
 G = & \frac{(i-r)}{R_1} \cdot \frac{[1,1]}{R_1} + \frac{(g-t)}{R_1} \cdot \frac{[-1,1]}{R_1} + (h-s) \cdot [0,1] + (n-l) \cdot (1,0) + \\
 & \frac{(d-w)}{R_2} \cdot \frac{[1,2]}{R_2} + \frac{(b-y)}{R_2} \cdot \frac{[-1,2]}{R_2} + \frac{(e-v)}{R_3} \cdot \frac{[2,2]}{R_3} + \frac{(a-z)}{R_3} \cdot \frac{[-2,2]}{R_3} + \\
 & \frac{(j-p)}{R_2} \cdot \frac{[2,1]}{R_2} + \frac{(f-u)}{R_2} \cdot \frac{[-2,1]}{R_2} + \frac{(c-x)}{R_4} \cdot \frac{[0,2]}{R_4} + \frac{(o-k)}{R_4} \cdot \frac{[2,0]}{R_4}
 \end{aligned} \tag{5}$$

Where $R_1 = \sqrt{2}$, $R_2 = \sqrt{5}$, $R_3 = \sqrt{8}$, $R_4 = 2$

$$\begin{aligned}
G = & \frac{(i-r)}{\sqrt{2}} \cdot \frac{[1,1]}{\sqrt{2}} + \frac{(g-t)}{\sqrt{2}} \cdot \frac{[-1,1]}{\sqrt{2}} + (h-s) \cdot [0,1] + (n-l) \cdot (1,0) + \\
& \frac{(d-w)}{\sqrt{5}} \cdot \frac{[1,2]}{\sqrt{5}} + \frac{(b-y)}{\sqrt{5}} \cdot \frac{[-1,2]}{\sqrt{5}} + \frac{(e-v)}{\sqrt{8}} \cdot \frac{[2,2]}{\sqrt{8}} + \frac{(a-z)}{\sqrt{8}} \cdot \frac{[-2,2]}{\sqrt{8}} + \\
& \frac{(j-p)}{\sqrt{5}} \cdot \frac{[2,1]}{\sqrt{5}} + \frac{(f-u)}{\sqrt{5}} \cdot \frac{[-2,1]}{\sqrt{5}} + \frac{(c-x)}{2} \cdot \frac{[0,2]}{2} + \frac{(o-k)}{2} \cdot \frac{[2,0]}{2}
\end{aligned} \tag{6}$$

$$\begin{aligned}
G = & [(i-r-g+t)/2 + (n-l) + (d-w)/5 + (-b+y)/5 + 2(e-v)/8 + \\
& 2(-a+z)/8 + 2(j-p)/5 + 2(-f+u)/5 + 2(o-k)/4, \\
& (i-r+g-t)/2 + (h-s) + 2(d-w)/5 + 2(b-y)/5 + 2(e-v)/8 + \\
& 2(a-z)/8 + (j-p)/5 + (f-u)/5 + 2(c-x)/4]
\end{aligned} \tag{7}$$

$$\begin{aligned}
G = & [(i-r-g+t)/2 + (n-l) + (d-w-b+y)/5 + 2(e-v-a+z)/8 + \\
& 2(j-p-f+u)/5 + 2(o-k)/4, \\
& (i-r+g-t)/2 + (h-s) + 2(d-w+b-y)/5 + 2(e-v+a-z)/8 + \\
& (j-p+f-u)/5 + 2(c-x)/4]
\end{aligned} \tag{8}$$

To simplify, vector is multiplied by 20. The resultant formula is given as follows:

$$\begin{aligned}
G' = 20 \cdot G = & [10(i-r-g+t+o-k) + 20(n-l) + 4(d-w-b+y) + 5(e-v-a+z) + \\
& 8(j-p-f+u), 10(i-r+g-t+c-x) + 20(h-s) + 8(d-w+b-y) + \\
& 5(e-v+a-z) + 4(j-p+f-u)]
\end{aligned} \tag{9}$$

The horizontal and vertical masks are obtained using the coefficients in the above equation are as follows:

5	8	10	8	5
4	10	20	10	4
0	0	0	0	0
-4	-10	-20	-10	-4
-5	-8	-10	-8	-5

G_x

-5	-4	0	4	5
-8	-10	0	10	8
-10	-20	0	20	10
-8	-10	0	10	8
-5	-4	0	4	5

G_y

Generating and Expansion of Prewitt edge detector:

Since the prewitt kernel are based on the idea of the contrast difference, so the directional derivates that estimate vector G was defined as the density differences as follows: [Phillips, 1994]

$$G = (c - g).[1,1] + (a - i).[-1,1] + (f - d).[1,0] + (b - h).[0,1] \quad (10)$$

$$G = (c - g - a + i + f - d), (c - g + a - i + b - h) \quad (11)$$

The horizontal and vertical masks for G_x and G_y which is obtained using the above vector and depend on the character in the 3x3 Cartesian grid as in Sobel are as follows:

1	1	1
0	0	0
-1	-1	-1

G_x

-1	0	1
-1	0	1
-1	0	1

G_y

Depending on the above definition of the gradient we can also expand 3x3 neighborhood to a 5x5 neighborhood as follows:

$$\begin{aligned}
G = & (i - r) \cdot [1,1] + (g - t) \cdot [-1,1] + (h - s) \cdot [0,1] + (n - l) \cdot [1,0] + (d - w) \cdot [1,2] + \\
& (b - y) \cdot [-1,2] + (e - v) \cdot [2,2] + (a - z) \cdot [-2,2] + (j - p) \cdot [2,1] + (f - u) \cdot [-2,1] + \\
& (c - x) \cdot [0,2] + (o - k) \cdot [2,0]
\end{aligned} \tag{12}$$

$$\begin{aligned}
G = & [(i - r - g + t + n - l + d - w - b + y) + 2(e - v - a + z + j - p - f + u + o - k), \\
& (i - r + g - t + h - s + 2(d - w) + 2(b - y) + 2(e - v) + 2(a - z) + (j - p + f - u) + \\
& 2(c - x)]
\end{aligned} \tag{13}$$

$$\begin{aligned}
G = & [(i - r - g + t + n - l + d - w - b + y) + 2(e - v - a + z + j - p - f + u + o - k), \\
& (i - r + g - t + h - s + j - p + f - u) + 2(d - w + b - y + e - v + a - z + c - x)]
\end{aligned} \tag{14}$$

The horizontal and vertical masks that obtained by using the coefficient in the above equation are:

2	2	2	2	2
1	1	1	1	1
0	0	0	0	0
-1	-1	-1	-1	-1
-2	-2	-2	-2	-2

G_x

-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2
-2	-1	0	1	2

G_y

Results and Conclusion

In this work, a set of 6 gray scale images has been tested with a 3x3 horizontal and vertical masks of Sobel and Prewitt. A 3x3 masks for these two operators are expanded to 5x5 dimensions and applied on the same original images as shown in figure (1) and figure (2), the results shows that the edges in the image detected using 5x5 masks appear more thick than that with a 3x3 also it is found that when applying a 3x3 mask for both operators the edge in some images is discontinued while in a 5x5 mask the edge is more continuous.

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(e)



(f)



(g)



Figure (1-a). (e) Original images. (f) The Implementation of 3x3 Sobel edge detector. (g) The Implementation of 5x5 sobel edge detector.

(e)



(f)



(g)



Figure (1-b). (e) Original images. (f) The Implementation of 3x3 Sobel edge detector.
(g) The Implementation of 5x5 sobel edge detector.

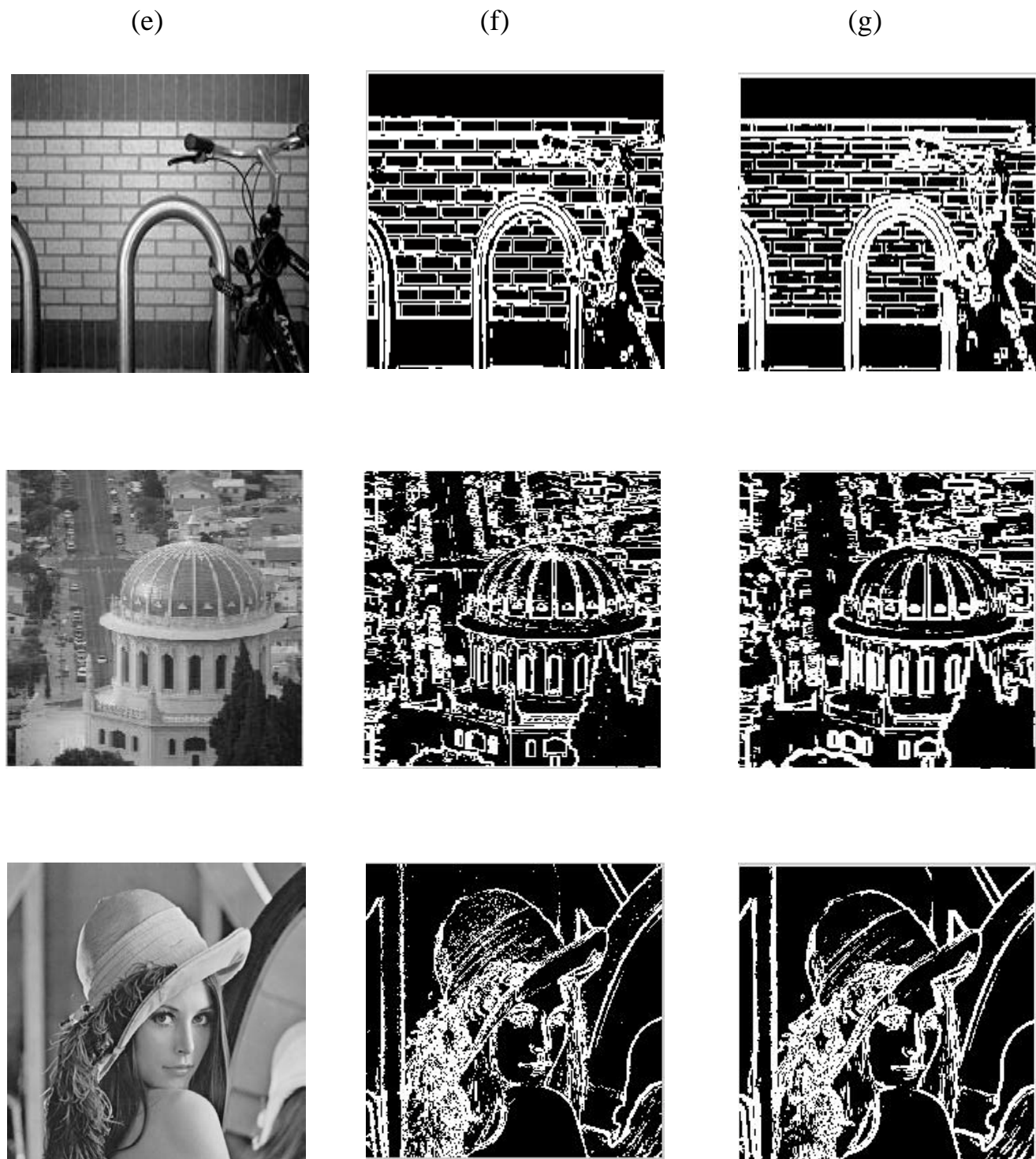


Figure (2-a). (e) Original images. (f) The Implementation of 3x3 Prewitt edge detector.
 (g) The Implementation of 5x5 Prewitt edge detector.



Figure (2-b). (e) Original images. (f) The Implementation of 3x3 Prewitt edge detector. (g) The Implementation of 5x5 Prewitt edge detector.