LEO K. C. CHEUNG

SHOWING, ANALYSIS AND THE TRUTH-FUNCTIONALITY OF LOGICAL NECESSITY IN WITTGENSTEIN'S *TRACTATUS*

ABSTRACT. This paper aims to explain how the *Tractatus* attempts to unify logic by deriving the truth-functionality of logical necessity from the thesis that a proposition shows its sense. I first interpret the Tractarian notion of showing as the displaying of what is intrinsic to an expression (or a symbol). Then I argue that, according to the *Tractatus*, the thesis that a proposition shows its sense implies the determinacy of sense, the possibility of the complete elimination of non-primitive symbols, the analyticity thesis and the strong analyticity thesis. The picture theory emerges as what provides the only acceptable account of an elementary proposition, subject to the constraint that a proposition must show its sense. The picture theory and the analyticity thesis then entail the contingency thesis (that an elementary proposition is contingent) and the independence thesis (that elementary propositions are mutually logically independent) which, together with the strong analyticity thesis, imply that all logical propositions are tautologies.

1. INTRODUCTION

After reading the manuscript of *Tractatus Logico-Philosophicus* 'twice carefully' in the summer of 1919,[1] in a letter to Wittgenstein, Russell refers to the thesis that 'all logical prop[osition]s are tautologies' as Wittgenstein's 'main contention' in the book.[2] The thesis, or TLP 6.1,[3] is very important in the *Tractatus* because it is the kernel of the attempt to unify logic via the truth-functionality of logical necessity. Wittgenstein, however, does not think it is his main contention, as one can see from his reply to Russell.

> ... Now I'm afraid you haven't really got hold of my main contention, to which the whole business of logical prop[osition]s is only a corollary. The main point is the theory of what can be expressed (gesagt; said[4]) by prop[osition]s – i.e., by language – (and, which comes to be the same, what can be *thought*) and what can not be expressed by prop[osition]s, but only shown (gezeigt); which, I believe, is the cardinal problem of philosophy.[5]

The main point of his main contention is the theory of what can be said and what cannot be said but only shown. What he calls 'the whole business' of logical propositions, which includes, of course, the thesis that Russell has mistakenly taken to be the main contention, is only a corollary

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to the theory. The above passage is widely known by commentators of the *Tractatus*, but none of them seems to have tried to explain in detail how Wittgenstein attempts to justify, correctly or incorrectly, the claim that the whole business of logical propositions is a corollary to his main contention.[6] Perhaps, they think there is simply no such attempt in the *Tractatus*. However, I think the *Tractatus* at least tries to explain how the thesis that all (true) logical propositions are (truth-functional) tautologies[7] follows from a major thesis belonging to the theory of what can be said and what cannot be said but only shown – the thesis that '[a] proposition *shows* its sense' (TLP 4.022) – as a corollary. In this paper, I shall explain how the *Tractatus* attempts to unify logic by deriving the truth-functionality of logical necessity from the thesis that a proposition shows its sense *alone*. This will be a partial vindication of what Wittgenstein says in the letter to Russell.

The truth-functionality of logical necessity can be deduced from the following three theses in a very straightforward manner.

(1) All (genuine) propositions and all logical propositions are truth-functions of elementary propositions.

(2) An elementary proposition is contingent.

(3) Elementary Propositions are mutually logically independent.

One may call (1), (2) and (3) 'the strong analyticity thesis', 'the contingency thesis' and 'the independence thesis', respectively. (The strong analyticity thesis is stronger than the thesis in TLP 5, which may be called 'the analyticity thesis', that '[a] proposition is a truth-function of elementary propositions'.) To see the deduction, first of all, note that if elementary propositions are contingent and mutually logically independent, then, for any truth-function of elementary propositions, each of the truth-possibilities constituted by the truth-possibilities of the component elementary propositions is genuine. Then the truth-function must be contingent, so far as it is neither a tautology nor a contradiction. A tautology is of course necessarily true, while a contradiction is not. Thus, if elementary propositions are contingent and mutually logically independent, then a truth-function of elementary propositions is necessarily true if and only if it is a tautology. If, in addition, all logical propositions are truth-functions of elementary propositions, then all (true) logical propositions are tautologies. This demonstrates how the truth-functionality of logical necessity can be deduced from the strong analyticity thesis, the contingency thesis and the independence thesis in a very straightforward manner. The focus of this paper will then be on how the *Tractatus* derives the three important theses from the thesis that a proposition shows its sense.

2. EXPRESSION AND SYMBOL

I shall begin with an explanation of the Tractarian notion of an expression, or a symbol, because, as I shall argue in the next section, showing, according to the *Tractatus*, is the displaying of what is intrinsic to an expression. In TLP 3.3s, any part of a proposition that characterizes its sense, or any sense-characterizing propositional part, is called 'an expression' or 'a symbol' (TLP 3.31, 3.331 and 3.34–3.3411). A proposition is an example of an expression that characterizes a sense. It is in a trivial sense a propositional part, and characterizes a sense by its having the sense. The term 'proposition' in the *Tractatus* is, however, ambiguous. It may refer to a thought with a sense (TLP 4) or a perceptible expression of a thought (TLP 3.1). When a proposition is said to be an expression, what this means is that a proposition is an expression of a thought. When a proposition is taken to be a thought, it may have different expressions but is not an expression itself. (In most of the cases, the relevant context will tell which meaning is intended.) A name is another example of a symbol that characterizes a sense. It is a proper essential constituent of a proposition (TLP 4.0312 and 4.22), which contributes to, and thus characterizes, the sense of the proposition by being a representative of an object (TLP 3.203 and 3.221).

The introduction of the notion of an expression in TLP 3.3s, however, should not be taken as providing a general definition because the relevant context is just about the nature of propositions and the place of names in propositional nexus. In fact, things other than sense-characterizing propositional parts like tautologies and contradictions are also expressions. The fact that tautologies and contradictions are taken to be expressions can be seen from TLP 5.525, where the term 'symbol' refers not only to a proposition but also to a tautology and a contradiction. Also, because '[t]he logical product of a tautology and a proposition say the same thing as the proposition' (TLP 4.465), tautologies, and, for a similar reason, contradictions as well, are not sense-characterizing. In what way are tautologies and contradictions expressions? In a suitable notation, tautologies and contradictions can be expressed as truth-functions, or truth-functional combinations, of elementary propositions (TLP 4.45–4.461). An elementary proposition, as a proposition, is, of course, a symbol that is a sense-characterizing propositional part. So tautologies and contradictions are 'sense-lacking' truth-functional combinations of sense-characterizing propositional parts. In fact, they are limiting cases of such combinations

(TLP 4.466–4.4661). 'They are part of the symbolism, much as '0' is part of the symbolism of arithmetic' (TLP 4.4611). If one considers, say, the fact that $0 + 2 = 2$, then one can easily see that although '0' does not characterize any 'integral quantity', it is still a symbol (an integer) in the symbolism of arithmetic. Similarly, because the logical product of a tautology and a proposition is identical with the proposition (TLP 4.465), the tautology is still a symbol in the symbolism, although it does not characterize a sense. Hence, since tautologies and contradictions are limiting cases of the truth-functional combination of sense-characterizing propositional parts, just like '0' which is the limiting case of the addition of integral quantities, they are also expressions in the relevant symbolism. In conclusion, *an expression (or a symbol) is either a sense-characterizing propositional part, or a limiting case of the truth-functional combination of sense-characterizing propositional parts*.[8]

3. SHOWING

As an introduction to the notion of showing in the *Tractatus*, the following passage from 'Notes Dictated to G. E. Moore in Norway' is very helpful:

> ...In any ordinary proposition, e.g., "Moore good", this *shews* but does not say that "Moore" is to the left of "good"; and *here what* is shewn can be *said* by another proposition. But this only applies to that *part* of what is shewn which is arbitrary. The *logical* properties which it shews are not arbitrary, and that it has these cannot be said in any proposition.[9]

The point here is that 'Moore good' shows that 'Moore' is to the left of 'good'. But what is shown here can be said, and a different sign without such a feature may equally serve the same purpose. Nevertheless, the essential logical properties belonging to the proposition cannot be said but only shown. The example 'Moore good' here is obviously an expression that is a sense-characterizing propositional part. The main point can then be put in this way: what is shown by a proposition as a sign may be said but what is shown by a proposition *as an expression* cannot be said. This suggests that when Wittgenstein writes in TLP 2.1414 that '[w]hat *can* be shown, *cannot* be said', he means what can be shown by *an expression* (or *a symbol*) cannot be said. An expression, or a symbol, is what shows.

What concerns this paper is not why what can be shown cannot be said, but the notion of showing. What is the nature of showing (via an expression)? To answer this, let me consider two cases of showing. The first case is that '[a] proposition *shows* its sense', or that '[a] proposition *shows* how things stand *if* it is true' (TLP 4.022). Owing to factors like 'the outward form of clothing' and 'the tacit conventions' of language (TLP

4.002), there may be expressions of the proposition from which one cannot see how things stand if it is true. So, not all expressions of a thought show its sense. But an expression, or a symbol, is what shows. Hence, the thesis that a proposition shows its sense can be formulated as follows:

(4) A proposition (a thought) must have an expression that shows its sense.

In other words, a proposition can always be expressed in such a way that it displays[10] or shows its sense such that one can see how things stand if it is true by inspecting the expression alone (TLP 4.023), even though not all expressions of the proposition show its sense. The second case is that, in a suitable notation, a logical proposition shows that it is a tautology (TLP 6.127[11]). It also shows the logical properties of the world via its structural properties (TLP 6.113–6.12 and 6.122). In a suitable notation, the essential features of the logical proposition like being a tautology, the structural properties of the tautology, etc., can be seen from the relevant propositional sign or, in some cases, from the relevant truth-table (TLP 4.442–4.46). A logical proposition displays or shows its essential features via a suitable expression. The discussion of the two cases also offers an illustration of TLP 5.525:

> . . . The certainty, possibility, or impossibility of a situation is not expressed by a proposition, but by the expression's being a tautology, a proposition with sense or a contradiction. The precedent to which we are constantly inclined to appeal must reside in the symbol itself.

The point here is that in order to grasp what is essential to a tautology, a proposition or a contradiction, one must appeal to what resides in the relevant symbol. I suggest calling what both resides in and is essential to a symbol 'what is intrinsic to' the symbol. It is then reasonable to hold that, for the *Tractatus, showing is the displaying of what is intrinsic to an expression or a symbol*.

It is important to note that showing, or the displaying of what is intrinsic to a symbol, is not restricted to things like visual displaying. For example, whatever enables the language user to grasp the logical form of a symbol is whatever enables showing. In the *Tractatus*, the label for this is 'use'.

> In order to recognize a symbol by its sign we must observe how it is used with a sense.
>
> A sign does not determine a logical form unless it is taken together with its logico-syntactical employment. (TLP 3.326–3.327)

Use in the *Tractatus* is merely showing or displaying the logical form of a symbol. In the case of a proposition's showing its sense, that is, (4), the sense is shown via the actual use of the proposition. The analysis of symbols is one way to display the logical forms of symbols. In fact, as it is clear from TLP 3.32s, especially TLP 3.325, that the notion of use in the *Tractatus* is taken to be broad enough to include the analysis of symbols. This intimate relation between use and analysis, or, rather, between showing and analysis, as one will see, is crucial in the derivation of the truth-functionality of logical necessity from the thesis that a proposition shows its sense.

4. NON-PRIMITIVE SYMBOL AND LOGICAL ANALYSIS

Not all expressions of a proposition show its sense. The *Tractatus* indeed holds that an expression of a proposition containing what may be called ‘a non-primitive symbol’ does not show its sense. To see this, note that the Tractarian notion of a primitive symbol can be defined as a symbol which cannot be dissected via a definition in a nontrivial manner (TLP 3.26).[12,13] Similarly, a symbol capable of being dissected via a definition may be called ‘a non-primitive symbol’. In this section, I shall offer an investigation of the nature of a non-primitive symbol. There are three major conclusions:

(5) An expression of a proposition containing a non-primitive symbol does not show its sense.

(6) Logical analysis is the process of the elimination of non-primitive symbols, which is also the syntactical dissection process of the semantic content of non-primitive symbols.

(7) Every non-primitive symbol can be replaced by other symbols via logical analysis.

The conclusions (5)–(7) are deduced from the thesis that a proposition shows its sense, or (4), and a basic idea that the *Tractatus* has accepted uncritically. The basic idea can be found in TLP 5.14–5.141, which says to the effect that two logically equivalent propositions are one and the same proposition. I suggest formulating it in terms of the notion of an expression as follows.

[α] Two logically equivalent (propositional) expressions are expressions of the same proposition.

Owing to the fact that this paper is exegetical in nature, I would not comment on [α], but would focus on the role that [α] plays in the *Tractatus*’ proof of the truth-functionality of logical necessity.

Let me begin with the discussion in TLP 3.24, which is about a special type of non-primitive symbol, namely, propositional elements signifying complexes. Consider the proposition 'The broom is in the corner' – an example taken from *Philosophical Investigations*.[14] It can be expressed as 'fa', where 'a' is a symbol for the broom and 'f –' short for '– is in the corner'. The broom is an example of what is called 'a complex' in the *Tractatus*, and thus 'a' is a propositional element signifying a complex. 'A complex can be given only by its description, which will be right or wrong' (TLP 3.24). For instance, the broom is given by the description 'The broomstick is fixed in the brush'. Let 'b' and 'c' be propositional elements signifying the broomstick and the brush, respectively, and '–R—' be short for '– is fixed in —'. The description can then be expressed as 'bRc'. The broom exists if 'bRc' is true; and does not if it is false. The broomstick's being fixed in the brush, which is described by 'bRc', belongs to the internal complexity of the broom. Now suppose that the broomstick and the brush are both in the corner. If the broomstick is fixed in the brush, then the proposition 'fa' is true. If not, according to the *Tractatus*, it is false, as '[a] proposition that mentions a complex will not be nonsensical, if the complex does not exist, but simply false' (TLP 3.24).[15] Thus, in order to understand the sense of 'fa' completely, one must know the relevant internal complexity of the broom. Hence, the internal complexity is sense-contributing, that is, counts towards the sense of 'fa'. Moreover, the *Tractatus* holds that '[e]very statement about complexes can be resolved into a statement about their constituents and into the propositions that describe the complexes completely' (TLP 2.0201). For instance, 'The broom is in the corner' can be resolved into 'The broomstick is in the corner, the brush is in the corner and the broomstick is fixed in the brush', that is, 'fa' into 'fb.fc.bRc'. Because of [α], the resolution can be expressed as: fa $\equiv$ (fb.fc.bRc).[16] The expression can be seen as a contextual definition of 'a' by means of the other symbols.[17] Since 'bRc' is a description of the complex a, the resolution of 'fa' into 'fb.fc.bRc' can be seen as the syntactical dissection, at least to a certain extent, of the internal complexity of the complex a. However, the sense-contributing internal complexity of a is not syntactically dissected in 'fa'. Thus, the expression 'fa' does not show the sense-contributing internal complexity of a. It can at most show an incomplete sense, or part of the sense, of the proposition. What is missing here gives rise to an intrinsic indeterminateness from which a propositional element signifying a complex, that is, 'a', can be detected. The point here is stated in TLP 3.24:

When a propositional element signifies a complex, this can be seen from an indeterminateness in the propositions in which it occurs. In such cases we *know* that the proposition leaves something undetermined ...

It follows that, first, an expression of a proposition with an element signifying a complex has an intrinsic indeterminateness, and thus does not show its sense. Second, with respect to the case of propositional elements signifying complexes, logical analysis is the syntactical dissection process of the sense-contributing internal complexity of complexes.

Propositional elements signifying complexes are referential. Indeed, one may identify the referent of a referential non-primitive symbol with a complex.[18] (This amounts to saying that the internal complexity of a complex must be sense-contributing.) However, not all non-primitive symbols are referential, as one can see from TLP 5.42 ('The interdefinability of Frege's and Russell's 'primitive signs' of logic is enough to show that they are not primitive signs ... ') and the Grundgedanke,[19] or the thesis that logical constants are not referential, in TLP 4.0312. For example, logical constants like disjunction and the quantifiers are non-primitive symbols because, as it is well known, they can be defined contextually in terms of other logical constants. This, together with the Grundgedanke, means that they are non-referential non-primitive symbols (see, also, TLP 4.441 and 5.4). The generalization of the major conclusions of the above discussion on propositional elements signifying complexes to the case of non-primitive symbols in general is indeed straightforward. Suppose that 'a' in 'fa' is a non-primitive symbol, which may or may not be referential. The contextual definition 'fa ≡ (fb.fc.bRc)' indicates the dissection of the non-primitive symbol 'a' by means of the other symbols. This, one may say, is the syntactical dissection of the semantic content of the non-primitive symbol 'a'. 'a' need not be referential; but if it does signify a complex, the contextual definition also indicates the syntactical dissection of the internal complexity of the complex. In general, a non-primitive symbol, by definition, has syntactically dissectible semantic content, no matter whether it is referential or not. Because a non-primitive symbol can be dissected in terms of other symbols via a contextual definition, its semantic content, though syntactically dissectible, is not syntactically dissected in any expression of a proposition containing the non-primitive symbol. Hence, an expression of a proposition containing a non-primitive symbol has an intrinsic indeterminateness, from which the presence of the non-primitive symbol can be detected, and thus fails to show its complete sense. In conclusion, first, an expression of a proposition containing a non-primitive symbol does not show its sense, that is, (5). Second, the dissection of a non-primitive symbol, say, 'a', via a contextual definition, say, 'fa $\equiv$ (fb.fc.bRc)', is a stage of analysis. Hence, logical analysis is the process of the elimination, or replacement, of non-primitive symbols via contextual definitions, and is also the syntactical dissection process of the semantic content of non-primitive symbols, that is, (6). (In the case of propositional elements signifying complexes, logical analysis is also the syntactical dissection process of the internal complexities of complexes.) Third, since one can always dissect a non-primitive symbol in terms of other symbols via a contextual definition, every non-primitive symbol can be replaced by other symbols via analysis, that is, (7).

5. ANALYSIS AND THE ELIMINATION OF NON-PRIMITIVE SYMBOLS

For the *Tractatus*, the employment of non-primitive symbols in language is inessential and probably just one of those tactic conventions mentioned in TLP 4.002. In fact, the *Tractatus* argues for the thesis that all non-primitive symbols can be eliminated from language or, rather, from the relevant symbolism. Note that (7) alone does not imply the possibility of the complete elimination of non-primitive symbols because what replace a non-primitive symbol at a stage of logical analysis may also be non-primitive symbols. Actually, the *Tractatus* derives the possibility of the complete elimination of non-primitive symbols from the thesis that a proposition shows its sense. This consists in two steps:

(8) The thesis that a proposition shows its sense guarantees the determinacy of sense.

(9) The determinacy of sense guarantees the possibility of the complete elimination of all non-primitive symbols or, equivalently, the possibility of a complete analysis for every proposition.

Let me consider (8) first. I shall not define the notion of determinacy in entries like TLP 3.23–3.261. What is needed here is the sufficient condition suggested by TLP 3.251: If the sense of a proposition can be set out clearly, then the sense of the proposition is determinate. Note that if a proposition shows its sense, then, in a suitable symbolism, one understands its complete sense by inspecting the expression of the proposition alone. In this case, its sense is set out clearly. The sufficient condition is fulfilled. Therefore, the thesis that a proposition shows its sense entails the thesis that sense is determinate, that is, (8).

I shall now turn to (9). It is said in TLP 3.2–3.203 and 3.23 that the determinacy of sense guarantees the possibility of names, and vice versa.

The *Tractatus* would also hold that the possibility of names guarantees the possibility of a complete analysis for every proposition, and vice versa (TLP 3.2–3.201, 3.24 and 3.25). Since TLP 3.24 is about the elimination of propositional elements signifying complexes, the possibility of names here is equivalent to the possibility of the complete elimination of propositional elements signifying complexes. Clearly, the main points here can be generalized to the case of non-primitive symbols. Hence, the *Tractatus* would hold that the determinacy of sense guarantees the possibility of the complete elimination of non-primitive symbols or, equivalently, the possibility of a complete analysis for every proposition, that is, (9). One can indeed construct a proof of (9) from the text of the *Tractatus*. To prove the first part of (9), suppose, on the contrary, that not all non-primitive symbols can be eliminated. It suffices to prove that there is a proposition whose sense is indeterminate. Since a non-primitive symbol is always capable of being eliminated, what does it mean by saying that not all non-primitive symbols can be eliminated? The only possibility is that there exists a proposition such that its process of analysis never comes to an end. Thus, every step of the endless process of analysis results in an expression containing at least one non-primitive symbol. Such an expression of the proposition, as mentioned before, has an intrinsic indeterminateness and thus does not show its sense. One cannot see from the expression, or from the analysis process, a complete sense. So, each step of the analysis does not set out the complete sense (TLP 3.251). There is simply no expression, which can set out a complete sense. For, otherwise, the process of analysis would stop. The sense of the proposition is then indeterminate. This proves that the determinacy of sense guarantees the possibility of the complete elimination of non-primitive symbols. It remains to prove the second part of (9). The thesis that every proposition has a unique complete analysis (TLP 3.25) entails that it is possible to establish an adequate symbolism, that is, one capable of expressing sense, without using any non-primitive symbols. Thus, the possibility of a complete analysis for every proposition guarantees the possibility of the complete elimination of non-primitive symbols. To prove the converse, note that if all non-primitive symbols can be eliminated, then it is possible to establish an adequate symbolism without any non-primitive symbols. Once the symbolism is adopted, every proposition can be expressed as a combination of primitive symbols, and thus is completely analyzed. This completes the proof.

6. PRIMITIVE SYMBOLS AND NAMES

When logical analysis eliminates all non-primitive symbols, what are left are primitive symbols, that is, those without any syntactically dissectible semantic content. Recall that propositional elements signifying complexes are referential non-primitive symbols, and non-primitive logical constants are non-referential non-primitive symbols. The complete elimination of all referential non-primitive symbols gives rise to the referential primitive symbols. Names are defined to be the referential primitive symbols in completely analyzed expressions of propositions (TLP 3.201–3.202). The meaning of a name is then that which contributes to (the characterization of) the senses of the propositions in which it occurs. The *Tractatus* identifies an object with the meaning of a name (TLP 3.203), and a name with the representative of an object (TLP 3.22). I take this move as defining a name to be referential. Moreover, for the *Tractatus*, the complete elimination of non-referential non-primitive symbols, or non-primitive logical constants, gives rise to a single primitive logical constant, which is symbolized in the Tractarian system by N introduced in TLP 5.5. I have already explained the nature of N and various issues concerning its expressive capability elsewhere,[20] and I shall not repeat the details here. It should be noticed that, in the Tractarian system, N is the primitive logical constant, or the sole fundamental logical constant by means of which other logical constants can be defined. (Note that, however, N is *not* the sole logical constant, or the one and only general primitive logical sign in logic, in TLP 5.47–5.472. According to the Tractatus, 'the real general primitive signs are not '$p \vee q$', '$(\exists x).fx$', etc. but the most general form of their combinations' (TLP 5.46). A general primitive sign is then the most general form of the combinations of a logical sign. Since the Tractarian system has N as its sole primitive logical constant, all logical constants are unified via the general form of the combinations of N. Thus, there is the one and only one general primitive logical sign, or the sole logical constant, which is not N but the general form of the combinations of N. Since what concerns this paper is the primitive logical constant and not the sole logical constant, I shall leave the latter here.) By the Grundgedanke, the primitive logical constant N is also non-referential. Consequently, first, the *Tractatus* holds this:

(10) The end products of the complete elimination of non-primitive symbols are the referential primitive symbols – names, and the non-referential primitive symbol – the primitive logical constant.

Second, symbols can be classified into referential primitive symbols (names), the non-referential primitive symbol (the primitive logical constant or, in the Tractarian system, N), referential non-primitive symbols (propositional elements signifying complexes), and non-referential non-primitive symbols (non-primitive logical constants).

Before proceeding further, I would like to point out that, for the *Tractatus*, a name, as a primitive symbol, by definition, cannot have any syntactically dissectible semantic content. It is also clear from TLP 2.02–2.021 that an object, or the meaning of name, is simple[21] or devoid of any internal complexity. Thus, the *Tractatus* holds this:

(11) A name has no syntactically dissectible semantic content, and its meaning, that is, an object, is devoid of internal complexity.

In fact, (11) follows from the thesis that a proposition shows its sense directly. To see this, note that, since an object is the meaning of a primitive symbol, it cannot have syntactically dissectible internal complexity. So, either an object is simple or it has internal complexity which is not syntactically dissectible. It remains to reject the second disjunct. First, if an object had sense-contributing internal complexity which is not syntactically dissectible, then what contributes to the relevant sense could not be syntactically dissected and there would be no way to display the complete sense via an expression in an intrinsic manner. The thesis that a proposition shows its sense would then be violated. Thus, an object cannot have sense-contributing internal complexity which is not syntactically dissectible. Second, recall that an object, or the meaning of a name, is defined to be linguistically dependent, that is, as that which contributes to the senses of the propositions in which the relevant name occurs. Then, by definition, whatever belongs to the nature of an object must be sense-contributing. Thus, an object cannot have any, so to speak, non-sense-contributing internal complexity which is not syntactically dissectible. Hence, an object cannot have any internal complexity which is not syntactically dissectible, and the second disjunct is rejected. Therefore, an object is devoid of internal complexity – it is simple.

7. ELEMENTARY PROPOSITIONS AND THE STRONG ANALYTICITY THESIS

The complete removal of non-primitive symbols gives rise to names and the primitive logical constant. The result of the complete elimination of non-primitive symbols from an expression of a proposition is then either a 'minimal' sense-expressing unit, or a truth-function of 'minimal' sense-expressing units (TLP 5.2341). A 'minimal' sense-expressing unit, of course, cannot be a proper truth-function. Otherwise, that would be question-begging. It must be either merely a set of names or an immediate combination of names. (The first disjunct, as one will see, is to be rejected.) Either way, it contains names only. Names, as primitive symbols, have no syntactically dissectible semantic content, and thus can only refer to things devoid of any internal complexities. They are like labels for indistinguishable balls inside a bag. Just as indistinguishable balls inside a bag can convey no sense, a set of names also cannot express a sense, unless the names are connected to one another in a determinate way. Thus, a set of names cannot express a sense (TLP 3.142). Hence, a 'minimal' sense-expressing unit must be an immediate combination of names. It is an expression of what the *Tractatus* calls 'an elementary proposition'. This explains why an elementary proposition is or, rather, can be expressed as an immediate combination (nexus or concatenation) of names (TLP 4.211–4.221).

If an expression of an elementary proposition is not suitably analyzed, it might not be an immediate combination of names. To facilitate future discussions, the notion of a completely analyzed expression is needed.

(12) An expression of a proposition is said to be 'completely analyzed' if it is either an immediate combination of names, or a truth-functional combination of immediate combinations of names which does not contain any logically equivalent components.[22]

The constraint of not containing equivalent components here is to ensure that cases like '$\sim\ \sim$P' and 'P $\vee$ P $\vee$ Q' are not completely analyzed expressions. Armed with the notion of a completely analyzed expression, it is clear that the *Tractatus* would hold this:

(13) Every completely analyzed expression of an elementary proposition is an immediate combination of names.

This entails that a completely analyzed expression of an elementary proposition does not contain any sign of any logical constant.

The elimination of all non-primitive symbols in an expression of a proposition gives rise to a completely analyzed expression of the proposition, which is either an immediate combination of names, or the result of applying the primitive logical operation, symbolized by the primitive logical constant, on immediate combinations of names. The completely analyzed expression can always be seen as the result of applying the primitive logical operation on immediate combinations of names, provided that the convention that an elementary proposition be taken as the result of applying, for zero time, the primitive logical operation on the elementary proposition itself be adopted. The possibility of the complete elimination of non-primitive symbols via analysis then entails what may be called ‘the analyticity thesis’ in TLP 5:

(14) Every proposition can be analyzed into (or expressed as) a truth-functional combination, or a truth-function, of elementary propositions.

Since the focus of this paper is not on the primitive logical constant, nor on the primitive logical operation that it symbolizes, for the sake of convenience, (14) is formulated in the weaker form of not involving the notion of the primitive logical constant. By (14), it is always possible to establish an adequate symbolism containing merely names and logical constants. The employment of the names and the logical constants of the adequate symbolism produce truth-functional combinations of elementary propositions, which need not be propositions. For they may be, say, tautologies. If a logical proposition is expressed by means of an adequate symbolism containing names and logical constants only, it must be framed in terms of those names and logical constants. This also entails that all non-primitive symbols in an expression of a logical proposition can be eliminated via the same definitions that are used to eliminate the same non-primitive symbols from expressions of propositions in which they occur. The following then holds for logical propositions:

(15) Every logical proposition can be analyzed into a truth-functional combination of elementary propositions.

The conjunction of (14) and (15) is exactly the strong analyticity thesis, that is, (1). It is then explained how, according to the *Tractatus*, the strong analyticity thesis follows from the possibility of the complete elimination of non-primitive symbols, which, in turn, is derived from the thesis that a proposition shows its sense (and [α]).

8. THE PICTURE THEORY AND THE CONTINGENCY OF ELEMENTARY PROPOSITIONS

The picture theory states that ‘[a] proposition is a picture of reality’ (TLP 4.01). The *Tractatus* seems to take the picture theory as what provides the only mechanism capable of accounting for an elementary proposition’s expressing its sense, subject to the constraint that a proposition must show its sense. (This is in accordance with the insight in TLP 4.02–4.024 and 4.21 that a proposition is a picture of reality because we can understand its sense without its having been explained to us, and we can do that because a proposition shows its sense.) Because of the more general setting of the picture theory, the constraint should be formulated in a more general manner, as follows.

(16) Whatever expresses a sense must show the sense.

In this way, the picture theory can be seen as being implied by (16), or, in the special case of an elementary proposition, by the thesis that a proposition shows its sense.

A completely analyzed expression of an elementary proposition contains names connected to one another. It has a structure. In the Tractarian terminology, it is a fact. What is a fact? The *Tractatus* calls an immediate combination of the meanings of names, or objects, 'a state of affairs' (TLP 2.01 and 2.03). The determinate way that objects are connected to one another in a state of affairs is called 'the structure', whose possibility is called 'the form', of the state of affairs (TLP 2.032–2.033). In general, the *Tractatus* seems to call a determinate way of combination 'a structure', and a possibility of structure, or a combinatorial possibility, 'a form'. An object has its combinatorial possibility, and thus has a form (TLP 2.0141). A possible state of affairs has a form, and, if it exists, has a structure. A fact is defined as the existence of states of affairs (TLP 2), and thus has a form and a structure as well. In general, an actual determinate combination of things is a fact. An elementary proposition, or, rather, each of its completely analyzed expressions, is an immediate combination of names. Thus, it is a fact, and has a structure and a form as well (TLP 3.14). How can a fact express a sense, subject to the constraint that whatever expresses a sense must show its sense, that is, (16)? The answer is to be found in the picture theory. According to the *Tractatus*, a fact becomes a picture when objects are correlated with its constituent elements, or when a pictorial relationship is established, in a certain manner:

> That is how a picture is attached to reality; it reaches right out to it.
>
> It is laid against reality like a measure.
>
> Only the end-points of the graduating lines actually touch the object that is to be measured.
>
> So a picture, conceived in this way, also includes the pictorial relationship, which makes it into a picture.
>
> The pictorial relationship consists of the correlations of the picture's elements with things.
>
> These correlations are, as it were, the feelers of the picture's elements, with which the picture touches reality. (TLP 2.1511–2.1515)

The talk of ‘measure’ and ‘the graduating lines’ here is to emphasize the following constraint presented by the form of the fact, which the element-object correlation must satisfy:

[β] Only objects having the same form as that of a constituent element of a picture can be correlated to the element.

The constraint [β] ensures that the element-object correlation determines a state of affairs of the same form as that of the picture. It is in this way that a picture represents the possibility of a state of affairs, or a possible state of affairs (TLP 2.2–2.203). In a picture, the fact that its elements are related to one another represents that objects[23] are related to one another *in the same determinate way*:

> The fact that the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way.
>
> Let us call this connexion of its elements the structure of the picture, and let us call the possibility of this structure the pictorial form of the picture. (TLP 2.15)

A fact is made into a picture, and its form, which is constituted by the form of its constituent elements, becomes a pictorial form, when its constituent elements denote objects such that it represents a state of affairs of the same form. The form of a picture acts as a constraint, that is, [β], such that only state of affairs of the same form can be represented. In particular, an elementary proposition, as a picture, asserts that the meanings of its constituent names are connected to one another in the same determinate way as its constituent names. That is, it asserts the existence of the possible state of affairs represented by means of its constituent names and its form (TLP 2.201–2.202 and 4.21).

The sense of a picture, or of an elementary proposition, in particular, is introduced naturally in TLP 2.221 as the possible state of affairs that it represents. For the *Tractatus*, an elementary proposition, as required by the constraint [β] in the construction of a picture, *shows* a sense, or the state of affairs that it represents, by making use of what is intrinsic to it alone. To see this, note that the constituent names of an elementary proposition are representatives of simple objects. Since those simple objects have no internal complexity, no other proposition can syntactically dissect the semantic content of their representatives. They are indistinguishable among objects of the same form. To distinguish them from other objects of different forms (denoted by the constituent names of other elementary propositions, of course), what is needed is the pictorial form, which is intrinsic to the elementary proposition, alone. Therefore, the form and the constituent names, which are intrinsic to an elementary proposition, are sufficient to not only represent, but also show the possible state of affairs, that is, its sense.

Because a picture represents its sense, the notion of truth and falsehood can be introduced, naturally, as follows (and what is about an elementary proposition in TLP 4.21 and 4.25 is just a special case).

(17) An elementary proposition (or picture, in general) is true if its sense, or the state of affairs that it represents, exists, and is false otherwise (TLP 4.022–4.024).

Although what an elementary proposition represents is completely determined by its form and its constituent elements in an intrinsic manner, the existence of what it represents is, however, extrinsic. Nothing intrinsic to an elementary proposition can tell whether what it represents exists or not. As a result, the definition of the truth-relation in (17) and the way that an elementary proposition represents its sense, as described by the picture theory, guarantee the contingency thesis, that is, (2), or that:

(18) Elementary propositions (or pictures, in general) are contingent (TLP 2.21–2.225 and 4.464).

This explains how the contingency thesis follows from the picture theory, which, in turn, is deduced from the thesis that a proposition shows its sense (and [α]).

9. THE MUTUAL LOGICAL INDEPENDENCE OF ELEMENTARY PROPOSITIONS

'When propositions have no truth-arguments in common with one another, we call them independent of one another. Two elementary propositions give one another the probability $\frac{1}{2}$' (TLP 5.152). Given the definition of probability in TLP 5.15–5.151, this proves that the *Tractatus* holds that two elementary propositions have no truth-arguments in common with one another, and thus are logically independent.[24] I shall argue that, for the *Tractatus*, the mutual logical independence of elementary propositions, that is, the independence thesis or (3), is derived from the thesis that a proposition shows its sense (and [α]). To begin, note that the independence thesis is equivalent to the conjunction of these three theses:

(19) An elementary proposition cannot imply another elementary proposition (or, equivalently, TLP 5.134).

(20) An elementary proposition cannot imply the denial of another elementary proposition (or, equivalently, TLP 4.211[25]).

(21) The denial of an elementary proposition cannot imply another elementary proposition.

The *Tractatus* seems to take (19) mainly as a logical consequence of what is developed before TLP 5.134, especially TLP 5.12–5.133. To see this, consider the theses that '[a] proposition affirms every proposition that follows from it' (TLP 5.124), and that '[w]hen the truth of one proposition follows from the truth of others, we can see this from the structure of the propositions' (TLP 5.13). That is, a proposition occurs in another proposition that implies it, so to speak, 'structurally'. It is clear from TLP 5.54 that by 'structurally' here, the *Tractatus* would mean 'truth-functionally'. Thus, a proposition occurs in another proposition that implies it truth-functionally. Now suppose, for reductio, that an elementary proposition, say, 'P', implies another elementary proposition, say, 'Q'. Then 'Q' occurs in 'P' truth-functionally. This would contradict the supposition that 'P' be an elementary proposition. Therefore, an elementary proposition cannot imply another elementary proposition, that is, (19). The theses (20) and (21), perhaps with a little complication, are supposed to be deduced from similar premisses in a similar manner. This describes the basic structures of the proofs only. The *Tractatus*, following the style of its author, who is very reluctant to write down the details of any argument, contains neither the details of the proofs, nor that of the justifications of the premisses. I shall fill in the details now.

The first premiss is from TLP 5.54 which, in terms of the notion of a completely analyzed expression, can be formulated as follows:

(22) A completely analyzed expression of a proposition can only occur in a completely analyzed expression of another proposition truth-functionally (that is, as a base of a truth-functional operation).

This is indeed a corollary to the analyticity thesis. To see this, consider a completely analyzed expression of a proposition. By the analyticity thesis, it is a truth-functional combination of immediate combinations of names. However, a proper part of an immediate combination of names cannot be an expression of a proposition, otherwise an immediate combination of names would no longer be a 'minimal' sense-expressing propositional part. The only completely analyzed expression that it can contain is then either one of its component immediate combinations of names or a truth-functional combination of some, or all, of them. Hence, a completely analyzed expression of a proposition can only occur in a completely analyzed expression of another proposition truth-functionally. This completes the proof. (After stating the thesis in TLP 5.54, the *Tractatus* deals with what it considers as apparent counterexamples in TLP 5.541–5.5423 immediately. I shall not discuss the *Tractatus*' solution here.[26])

The second premiss has not been spelt out in the *Tractatus* explicitly. It can be formulated as follows:

(23) No completely analyzed expression of an elementary proposition can contain a completely analyzed expression of another proposition.

Remember that a completely analyzed expression of an elementary proposition is an immediate combination of names, that is, (13). By (22), it can only contain an expression of another proposition truth-functionally. So, as an immediate combination of names, it cannot contain an expression of any proposition, except itself. Therefore, no completely analyzed expression of an elementary proposition can contain a completely analyzed expression of another proposition, that is, (23). (23), just like (22), is also a corollary to the analyticity thesis.

The third premiss is a special case of the statement that a proposition occurs in another proposition that implies it truth-functionally, which is derived from TLP 5.12, 5.124 and 5.54. It can be formulated as follows:

(24) If a proposition implies an elementary proposition (or the denial of an elementary proposition), then every completely analyzed expression of the former contains a completely analyzed expression of the latter truth-functionally.

This is a corollary to the picture theory and (22). To see this, suppose that a proposition, say, 'P', implies another proposition, say, 'Q', and that 'Q' is an elementary proposition (or the denial of an elementary proposition, respectively). Without loss of generality, let 'fa' (or '∼fa', respectively) be a completely analyzed expression of 'Q', where 'f' and 'a' are names. Then 'P' must affirm what 'fa' (or '∼fa', respectively) affirms. That is, the truth-conditions, or the sense, of 'P' must, so to speak, contain that of 'fa' (or '∼fa', respectively). By the picture theory, the truth-conditions of 'fa' (or '∼fa', respectively) are completely determined by this:

> 'fa' is true if it is the case that fa; and false if otherwise (or '∼fa' is true if it is not the case that fa; and false if otherwise, respectively).

Then ‘P’ must also mention f and a in the same combination as presented by their representatives, that is, ‘f’ and ‘a’ in ‘fa’ (or ‘∼fa’, respectively). Therefore, every completely analyzed expression of ‘P’ contains a propositional sign, which is a completely analyzed expression, of ‘fa’, that is, ‘Q’, (or ‘∼fa’, that is, ‘∼Q’, respectively), and, by (22), must contain it truth-functionally.

The proofs of (19), (20) and (21) are now straightforward. To prove (19), suppose, on the contrary, that there are two different elementary propositions, say, ‘P’ and ‘Q’, such that ‘P’ implies ‘Q’. Then, by (24), every completely analyzed expression of ‘P’ contains a completely analyzed expression of ‘Q’ truth-functionally. Because of (23), ‘P’ could not be an elementary proposition. This contradicts the supposition that ‘P’ be an elementary proposition. Therefore, an elementary proposition cannot imply another elementary proposition, that is, (19).

To prove (20), suppose, on the contrary, that there are two different elementary propositions, say, ‘P’ and ‘Q’, such that ‘P’ implies ‘∼Q’. Suppose further that ‘Q*’ is a completely analyzed expression of ‘Q’. Then ‘∼Q*’ is a completely analyzed expression of ‘∼Q’. Because of (24), as ‘P’ implies ‘∼Q’, every completely analyzed expression of ‘P’ contains a completely analyzed expression of ‘∼Q’ truth-functionally. Because of (23), the only possibility is that ‘P’ and ‘∼Q’ are one and the same proposition. Then ‘∼Q*’ is also a completely analyzed expression of ‘P’. But ‘∼Q*’ is not an immediate combination of names. Since a completely analyzed expression of an elementary proposition must be an immediate combination of names, that is, (13), ‘P’ could not be an elementary proposition. This contradicts the supposition that ‘P’ be an elementary proposition. Hence, an elementary proposition cannot imply the denial of another elementary proposition, that is, (20).

To prove (21), suppose, on the contrary, that there are two different elementary propositions, say, ‘P’ and ‘Q’, such that ‘∼P’ implies ‘Q’. Suppose further that ‘P*’ is a completely analyzed expression of ‘P’. Then ‘∼P*’ is also a completely analyzed expression of ‘∼P’ and, by (24), ‘∼P*’ contains a completely analyzed expression of ‘Q’, say, ‘Q*’. Since ‘∼P*’ contains ‘Q*’ and ‘P*’ and ‘Q*’ are both immediate combinations of names, ‘P*’ also contains ‘Q*’ truth-functionally. This contradicts (24). Consequently, the denial of an elementary proposition cannot imply another elementary proposition, that is, (21).

I have now explained how the independence thesis, or the conjunction of (19), (20) and (21), follows from the three premisses (22), (23) and (24), and how the three premisses follow from the analyticity thesis and the picture theory, which, in turn, are deduced from the thesis that a proposition shows its sense (and [α]).

10. THE TRUTH-FUNCTIONALITY OF LOGICAL NECESSITY AND THE UNIFICATION OF LOGIC

The thesis that a proposition shows its sense, as demonstrated in the above discussion, plays a crucial role in the *Tractatus*' proof of the truth-functionality of logical necessity. The determinacy of sense, the possibility of the complete elimination of non-primitive symbols, the analyticity thesis and the strong analyticity thesis are all corollaries to the thesis that a proposition shows its sense and the basic idea that logically equivalent expressions are expressions of the same proposition, that is, [α]. The picture theory emerges as what provides the only mechanism capable of accounting for an elementary proposition's expressing its sense, subject to the constraint that a proposition must show its sense, and thus can be seen as being implied by the thesis that a proposition shows its sense. The analyticity thesis and the picture theory then entail the contingency thesis and the independence thesis which, with the help of the strong analyticity thesis, as explained in Section 1, in turn, imply the truth-functionality of logical necessity, by means of which logic is unified. In this way, the truth-functionality of logical necessity is derived from the thesis that a proposition shows its sense and the basic idea [α]. In the *Tractatus*, [α] is accepted uncritically. Therefore, it appears to the author of the *Tractatus* that the truth-functionality of logical necessity follows from the thesis that a proposition shows its sense *alone*.

The *Tractatus* has dealt with a number of apparent counter-examples against the truth-functionality of logical necessity. Some are eventually excluded from the category of logical propositions, while the rest are seen as non-tautological expressions of logical propositions, which, upon analysis, would emerge as tautological. In TLP 6.31, 6.32–6.321, 6.34, 6.35–6.36, etc., the *Tractatus*, for various reasons, judges that the law of induction, the principle of sufficient reason, the law of causality, the law of continuity, the law of least action, etc., are not laws of logic. I shall not discuss the *Tractatus*' treatment of those cases here. For the case of non-tautological expressions of logical propositions, however, I shall discuss an example created from the case of the broom in the corner and another example treated in TLP 6.375–6.3751. Consider the non-tautological expression 'If the broom is in the corner, then the broomstick is in the corner' or, employing the terminology in Section 4, '$fa \supset fb$'. The *Tractatus*, while agreeing that it is an expression of a logical proposition, would insist that it contains non-primitive symbols. If some of those non-primitive symbols are replaced via analysis, a tautological expression of the logical proposition would emerge. In the present case, upon analysis, the non-primitive symbol ‘a’ can be replaced, and the tautological expression ‘(fb.fc.bRc) ⊃ fb’ would be obtained. The expression ‘(fb.fc.bRc) ⊃ fb’ need not be completely analyzed. It may still contain non-primitive symbols. What the *Tractatus* would say is just that when the expression ‘fa ⊃ fb’ is suitably analyzed, a tautological expression of the logical proposition would be found. I think this is also capable of explaining how the *Tractatus* deals with the apparent counter-example in TLP 6.375–6.3751. Although the example is the impossibility of the simultaneous presence of two colours at the same place in the visual field, without loss of generality, one can consider the necessity of its denial. Let ‘c’ and ‘d’ be symbols signifying the two colours, or, rather, colour patches, respectively, and ‘g–’ be short for ‘– is simultaneously present at the same place in the visual field’. The apparent counter-example is then the expression ‘∼(gc.gd)’. It is clearly a non-tautological expression of a logical proposition. The possibility of the simultaneous presence of two colours at the same place in the visual field ‘is ruled out by the logical structure of colour’ (TLP 6.3751). Presumably, the *Tractatus* would hold that the necessity truth of ‘∼(gc.gd)’ is ‘generated’ by the logical structure of colour. This amounts to saying that the symbols ‘c’ and ‘d’ are non-primitive symbols. The *Tractatus* would also hold that when the semantic contents of ‘c’ and ‘d’ are suitably syntactically dissected, or when ‘∼(gc.gd)’ is suitably analyzed, a tautological expression of the logical proposition would emerge. The *Tractatus* does not explain how the analysis should be carried out. This is perhaps understandable because the author of the *Tractatus* believes that the truth-functionality of logical necessity is a conclusion deduced from the solid doctrine that a proposition shows its sense.[27]

One can see from the proof of the truth-functionality of logical necessity and the treatment of the non-tautological expressions of logical propositions that there is a basic insight in the *Tractatus*’ attempt to achieve the unity of logic. The basic insight is that the fact that a proposition shows its sense guarantees the possibility of, so to speak, ‘pulling down’ all that is essential to logic syntactically via analysis. The elimination of non-primitive symbols, the syntactical dissection of the semantic content of non-primitive symbols (including the internal complexities of complexes), etc., all amount to one thing – the ‘pulling down’ of whatever essential to logic syntactically. This explains why all non-tautological expressions of logical propositions are replaceable by tautological expressions. For what belongs to the semantic content of non-primitive symbols which contributes to the necessary truth of a non-tautological expression can be syntactically dissected via analysis, and a tautological expression would emerge. The basic insight, when applied to the present case, is then that the fact that a proposition shows its sense guarantees that whatever essential to logic and belonging to the semantic content of the non-primitive symbols in a non-tautological expression of a logical proposition can be syntactically dissected via analysis (or in use; cf., Section 3), and reside in a tautological expression of the same logical proposition.

The discussions in this paper demonstrate, at least to a certain extent, how serious Wittgenstein was when he told Russell that the whole business of logical proposition is only a corollary to the theory of what can be said and what cannot be said but only shown.

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NOTES

1 For details, see Wittgenstein (1995, 118–119, 121).

2 Russell wrote, '... I am convinced you are right in your main contention, that logical props are tautologies, which are not true in the sense that substantial props are true ...'. See Wittgenstein (1995, 121).

3 In this paper, I follow the usual practice of employing 'TLP' as the abbreviation for '*Tractatus Logico-Philosophicus*', and referring to the propositions of the *Tractatus* by the relevant decimal numbers. Pears and McGuinness' translation is used in this paper.

4 It is more preferable to translate 'gesagt' into 'said'.

5 Wittgenstein (1995, 124).

6 Many works on the *Tractatus* mention or refer to the passage in which Wittgenstein claims that the whole business of logical propositions is a corollary to his main contention, but do not try to explain how the *Tractatus* justifies the claim. For example, the passage is mentioned in Anscombe (1971, 161), in Black (1964, 188) but the sentence 'Now I'm afraid ... is only a corollary' is omitted, in Griffin (1964, 18) and in a footnote in Pears (1987, 124).

7 In this paper, 'tautology', 'contradiction' and 'logical proposition' mean truth-functional tautology, truth-functional contradiction and true logical proposition, respectively.

8 Strictly speaking, '... the truth-functional combination of propositions' should be used, instead. For the truth-functional combination of things other than propositions makes no sense.

9 Wittgenstein (1979, 111).

[10] The term 'display' occurs twice in the *Tractatus* – in TLP 2.172 and 4.121. It is used in more or less the same way as 'show'. For example, '... Propositions *show* the logical form of reality. They display it' (TLP 4.121).

[11] Although TLP 6.127 does not mention the case of a contradiction, it is clear that the *Tractatus* would also hold that a contradiction shows that it is a contradiction.

[12] In TLP 3.26, Wittgenstein uses the term 'primitive sign', instead of 'primitive symbol'. He should have used 'primitive symbol'. For, given the notion of a symbol introduced later in TLP 3.31, one can see that the discussion in TLP 3.26 clearly refers to a name as a symbol rather than to its sign. Indeed, he also holds that '[a] sign is what can be perceived of a symbol' (TLP 3.32). Probably, he would have employed the term 'primitive symbol' in TLP 3.26 if not because of the fact that the notion of a symbol is a later introduction in TLP 3.31.

[13] The *Tractatus* does not use the phrase 'in a nontrivial manner'. It is added here in order to exclude the case of the mere replacement of the sign instead of the symbol.

[14] Entry 60 in Wittgenstein (1997).

[15] I would not consider how the *Tractatus* justifies TLP 3.24 here, as it suffices to serve the purpose here by pointing out that if the broomstick is not fixed in the brush, then 'fa' is either nonsensical or, as the *Tractatus* takes it, false.

[16] In the entry dated 5.9.14 in *Notebooks 1914–16*, see Wittgenstein (1979, 4), there is the sentence, or formula '$\phi(a).\phi(b).aRb = Def\phi[aRb]$'. Cf., also, Kenny (1973, 79–80).

[17] This also explains how in general '[t]he contraction of a symbol for a complex into a simple symbol can be expressed in a definition' (TLP 3.24).

[18] It is clear that, throughout the *Tractatus*, object and complex are the only candidates of the referents of symbols. An object cannot be the referent of a non-primitive symbol because, as one will see, it cannot contribute to the syntactically dissectible semantic content of the non-primitive symbol.

[19] I have explained elsewhere the proofs of the Grundgedanke in the *Tractatus*. For details, see Cheung (1999, 395–410).

[20] See Cheung (2000, 247–261).

[21] It is clear from TLP 2.02 and 2.021 that something is simple if and only if it is not composite.

[22] This definition is clearly compatible with the one offered in TLP 3.2–3.21, and is indeed more general and specific.

[23] Judging from the content of TLP 2.151-2.1515, 'things' here can be taken as objects.

[24] Cf., Black (1964, 247–248).

[25] Clearly, TLP 4.211, that is, 'It is a sign of a proposition's being elementary that there can be no elementary proposition contradicting it', means that an elementary proposition cannot be 'excluded' from another elementary proposition, and this is equivalent to (20). Cf., Griffin (1964, 76–77).

[26] For a discussion of the issue, see Fogelin (1995, 74–77).

[27] Things become very different when the colour exclusion case is seen as a genuine counter-example against the view of the truth-functionality of logical necessity in the 1929 paper 'Some Remarks on Logic Form'. See Wittgenstein (1973, 31–37).

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Department of Religion and Philosophy
Hong Kong Baptist University
Kowloon Tong
Hong Kong
E-mail: kccheung@hkbu.edu.hk