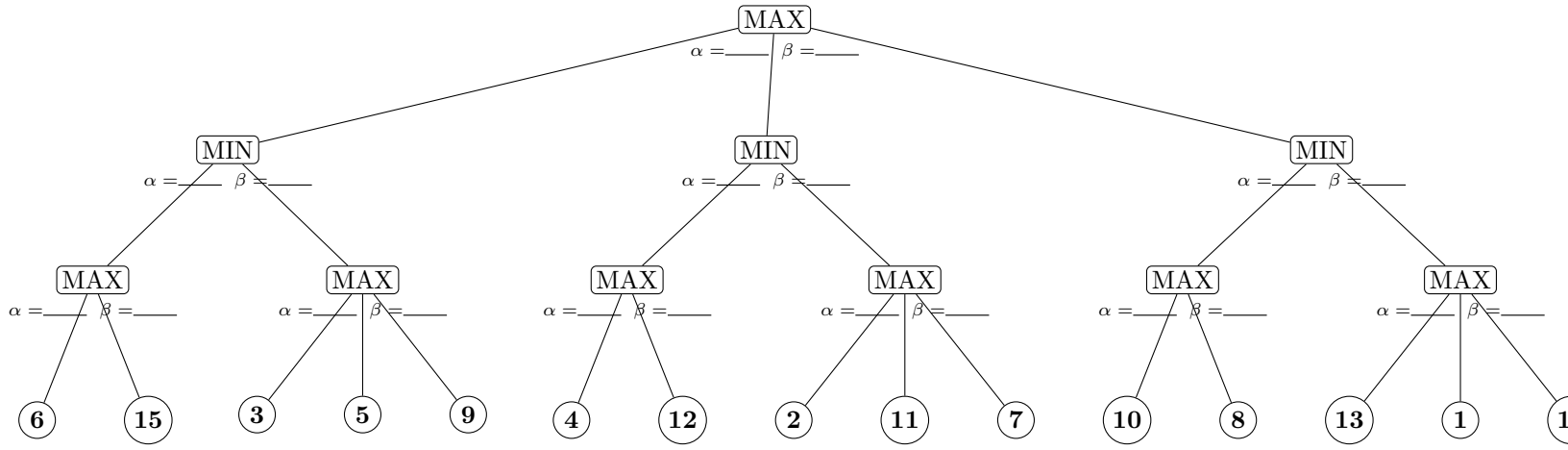
3-Ply Alpha–Beta Pruning Worksheet — Set 2

Root MAX → MIN → MAX. Write , at each internal node.

Alpha–beta recap

- Keep bounds: α (best for MAX so far), β (best for MIN so far).
- MAX: update $\alpha = \max(\alpha, v)$; MIN: update $\beta = \min(\beta, v)$.
- Prune when $\alpha \geq \beta$ at a node.
- Start at root with $(-\infty, +\infty)$; traverse left to right.

Exercise C — Left-to-right traversal



Root (MAX) initial $(\alpha, \beta) = (-\infty, +\infty)$

Child	Result	α	β
Left MIN			
Middle MIN			
Right MIN			

Visit order (leaves):

Pruned subtrees:

Root value: Chosen move:

Answer key on back

Answer Key — Set 2

Values of each internal node (left-to-right):

- Left MIN: children MAX values 15 and 9 $\Rightarrow \min(15, 9) = \mathbf{9}$.
- Middle MIN: children MAX values 12 and 11 $\Rightarrow \min(12, 11) = \mathbf{11}$.
- Right MIN: children MAX values 10 and 14 $\Rightarrow \min(10, 14) = \mathbf{10}$.
- Root MAX: $\max(9, 11, 10) = \mathbf{11}$, choose the middle branch.

Alpha–beta trace (concise):

1. Root: $(\alpha, \beta) = (-\infty, +\infty)$.
2. Enter Left MIN with $(-\infty, +\infty)$.
   - MAX A [6,15] with $(-\infty, +\infty) \to 15$; Left MIN $\beta = 15$.
   - MAX B [3,5,9] with $(-\infty, 15) \to 9$; Left MIN $\beta = \min(15, 9) = 9$.
   - Left MIN returns 9.

   Root $\alpha = \max(-\infty, 9) = 9$.
3. Enter Middle MIN with $(\alpha, \beta) = (9, +\infty)$.
   - MAX C [4,12] with $(9, +\infty) \to 12$; Middle MIN $\beta = 12$.
   - MAX D [2,11,7] with $(9, 12) \to 11$; Middle MIN $\beta = \min(12, 11) = 11$.
   - Middle MIN returns 11.

   Root $\alpha = \max(9, 11) = 11$.
4. Enter Right MIN with $(\alpha, \beta) = (11, +\infty)$.
   - MAX E [10,8] with $(11, +\infty) \to 10$; Right MIN $\beta = 10$.
   - Cutoff: inherited $\alpha = 11 \geq \beta = 10$ at Right MIN, so prune MAX F entirely.
   - Right MIN returns 9? No, it returns current $\beta = \mathbf{10}$.

   Root final $\alpha = \max(11, 10) = \mathbf{11}$.

Visit order of leaves:

6, 15, 3, 5, 9, 4, 12, 2, 11, 7, 10, 8.

Pruned subtrees:

Entire right-branch second child MAX F with leaves $[13, 1, 14]$.