Answer the following questions, making sure to *give details and justify your answers.*
Answers without details or justification will not receive full credit, even if correct! All numerical answers must be stated to the proper number of significant figures and units.

The detailed analysis formulas for the uncertainty δf on a calculated quantity $f(x,y,z)$ that depends on measured quantities $x \pm \delta x$, $y \pm \delta y$, and $z \pm \delta z$ are:

$$\delta f = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\delta y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 (\delta z)^2}$$

and

$$\frac{\delta f}{f} = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 \left(\frac{\delta x}{f}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 \left(\frac{\delta y}{f}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 \left(\frac{\delta z}{f}\right)^2}$$

The detailed formulas for standard deviation and uncertainty of a measured independent quantity are

$$\sigma_x = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}} \qquad \delta x = \frac{\sigma}{\sqrt{N}}$$

Question 1: 18 **/ 22 pts**

You want to measure the centripetal acceleration, a_c, of an object on the rim of a merry-go-round rotating at a constant rate. To do that, you measure the angular positions θ_o at time t = 0 s and θ_1 at a later time t, and the radial position r of the object.

You can use these values and kinematic theory to calculate the angular displacement $\chi = \theta_1 - \theta_o$ and the centripetal acceleration

$a_c = \omega^2 r =$

$$a_c = \frac{\chi r}{t^2}$$

You measure the following values:

$\theta_0 = (0.24 \pm 0.02) rad$ $\qquad$ $\theta_1 = (4.47 \pm 0.07) rad$

$r = (8.5 \pm 0.1) m$ $\qquad$ $t = (14.7 \pm 0.2) s$

(a) Derive an analytic formula for the uncertainty $\delta\chi$ (2pts).

$\frac{a_c t^2}{r} = \chi$

$$\delta x = \sqrt{\left(\frac{\partial x}{\partial a_c}\right)^2 (\delta a_c)^2 + \left(\frac{\partial x}{\partial t}\right)^2 (\delta t)^2 + \left(\frac{\partial x}{\partial r}\right)^2 (\delta r)^2}$$

$$= \sqrt{\left(\frac{t^2}{r}\right)^2 (\delta a_c)^2 + \left(\frac{2a_c t}{r}\right)^2 (\delta t)^2 + \left(\frac{-a_c t^2}{r^2}\right)(\delta r)^2}$$

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$\sqrt{(\delta\theta_1)^2 + (\delta\theta_2)^2}$

$$a_c = \frac{(\theta_1 - \theta_0)r}{t^2} = \frac{\theta_1 r - \theta_0 r}{t^2}$$

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(b) Using the values given above calculate the value of $\delta\chi$ (2pts).

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(c) Derive an analytic formula for the uncertainty δa_c (6pts).

$$\delta a_c = \sqrt{\left(\frac{\partial a_c}{\partial \theta_1}\right)^2 (\delta\theta_1)^2 + \left(\frac{\partial a_c}{\partial \theta_0}\right)^2 (\delta\theta_0)^2 + \left(\frac{\partial a_c}{\partial r}\right)^2 (\delta r)^2 + \left(\frac{\partial a_c}{\partial t}\right)^2 (\delta t)^2}$$

$$= \sqrt{\left(\frac{r}{t^2}\right)^2 (\delta\theta_1)^2 + \left(\frac{-r}{t^2}\right)^2 (\delta\theta_0)^2 + \left(\frac{\theta_1 - \theta_0}{t^2}\right)^2 (\delta r)^2 + \left(\frac{-2(\theta_1 r - \theta_0 r)}{t^3}\right)^2 (\delta t)^2}$$

(d) Derive an analytic formula for the $\delta a_c / a_c$. Simplify it as much as you can (2pts).

$$\frac{\delta a_c}{a_c} = \sqrt{\frac{\left(\frac{r}{t^2}\right)^2 (\delta\theta_1)^2 + \left(\frac{-r}{t^2}\right)^2 (\delta\theta_0)^2 + \left(\frac{\theta_1 - \theta_0}{t^2}\right)^2 (\delta r)^2 + \left(\frac{-2(\theta_1 r - \theta_0 r)}{t^3}\right)^2 (\delta t)^2}{\left(\frac{(\theta_1 - \theta_0)r}{t^2}\right)^2}}$$

$$= \sqrt{\frac{1}{(\theta_1 - \theta_0)^2}(\delta\theta_1)^2 + \frac{1}{(\theta_1 - \theta_0)^2}(\delta\theta_0)^2 + \frac{1}{(r)^2}(\delta r)^2 + 4\frac{4}{(t)^2}(\delta t)^2}$$

χ^2 χ^2

(e) Using your formula above, calculate the magnitude $a_c \pm \delta a_c$ of the acceleration and $\delta a_c / a_c$ (6pts).

$$a_c = \frac{\chi r}{t^2} = \frac{(\theta_1 - \theta_0)r}{t^2} = \left(\frac{4.47\text{rad} - 0.24\text{rad}}{(14.7s)^2}\right)(8.5m) = 0.166389005 m/s^2 = 0.17 m/s^2$$

$$\delta a_c = \sqrt{\left(\frac{8.5m}{(14.7s)^2}\right)^2 (0.07rad)^2 + \left(\frac{8.5m}{(14.7s)^2}\right)^2 (0.02rad)^2 + \left(\frac{4.23rad}{(14.7s)^2}\right)^2 (0.1m)^2 + \left(\frac{-2(35.955\,\text{rad·m})}{(14.7s)^3}\right)^2 (0.2s)^2}$$

$$= \sqrt{(0.0000007582 + 0.000000619 + 0.000003832 + 0.000020499)\,m^2/s^4} = 0.00570369\,m/s^2$$

$$\frac{\delta a_c}{a_c} = \frac{0.00570369 m/s^2}{0.166389005 m/s^2} = 0.03427$$

$$\boxed{a_c = (0.166 \pm 0.006)\,m/s^2} \qquad \boxed{\frac{\delta a_c}{a_c} = 0.03}$$

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Note that questions (f) and (g) use values independent from those of question (a) and (b).

(f) Does an experimental value $a = (9.84 \pm 0.08)m/s^2$ agree with a theoretical value of $9.79 m/s^2$? Why (2pts)?

Yes, because the experimental value has a true value that ranges from $9.76 m/s^2$ to $9.92 m/s^2$, which overlaps with the theoretical value of $9.79 m/s^2$.

(g) If two separate experiments give the values $a_1 = (9.35 \pm 0.03)m/s^2$ and $a_2 = (9.27 \pm 0.04)m/s^2$, can you say that they agree with each other? Why (2pts)?

No, because the ~~value of~~ a_1 has a true value that ranges from $9.32 m/s^2$ to $9.38 m/s^2$ which does not overlap with the true value of a_2 which ranges from $9.24 m/s^2$ to $9.31 m/s^2$.

Question 2: 6 / 6 pts

Suppose you measure the time it takes an object on a merry-go-round to complete one cycle when it is r=6.30m from its center. The values you got are displayed in the table below. Your job is to calculate the average value of the measured period, its standard deviation, and its uncertainty. All answers must be stated to the proper number of significant figures and units.

$\frac{2\pi r}{T}$

Table 1

	1	2	3	4	5	6	7	8
t (s)	44.2	44.8	45.4	44.7	45.3	44.3	45.5	43.9

~~Circumference~~ $\Sigma t(s) = 358.1 s$ $\quad$ $Avg = \frac{\Sigma t(s)}{N} = \frac{358.1s}{8} = 44.7625 sec$ $\quad$ $\boxed{Avg = 44.76 sec}$

$$\sigma_T = \sqrt{\frac{(44.2s-44.76s)^2 + (44.8s-44.76sec)^2 + (45.4s-44.76sec)^2 - (44.7s-44.76s)^2 + (45.3s-44.76s)^2 + (44.3s-44.76s)^2 + (45.5s-44.76s)^2 + (43.9s-44.76s)^2}{7}}$$

$$= \sqrt{(0.3136s^2)+(0.0016s^2)+(0.4096s^2)+(0.0036s^2)+(0.2916s^2)+(0.2116s^2)+(0.5476s^2)+(0.7396s^2)/7}$$

$= 0.599857s$ $\quad$ $\boxed{\sigma_T = 0.6 sec}$ $\quad$ $\boxed{(44.8 \pm 0.2)s}$

$\delta T = \frac{\sigma}{\sqrt{N}} = \frac{0.599857s}{\sqrt{8}} = 0.21208152 1s$

$\boxed{\delta T = 0.2s}$

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Question 3: ___4___ **/ 8pts**

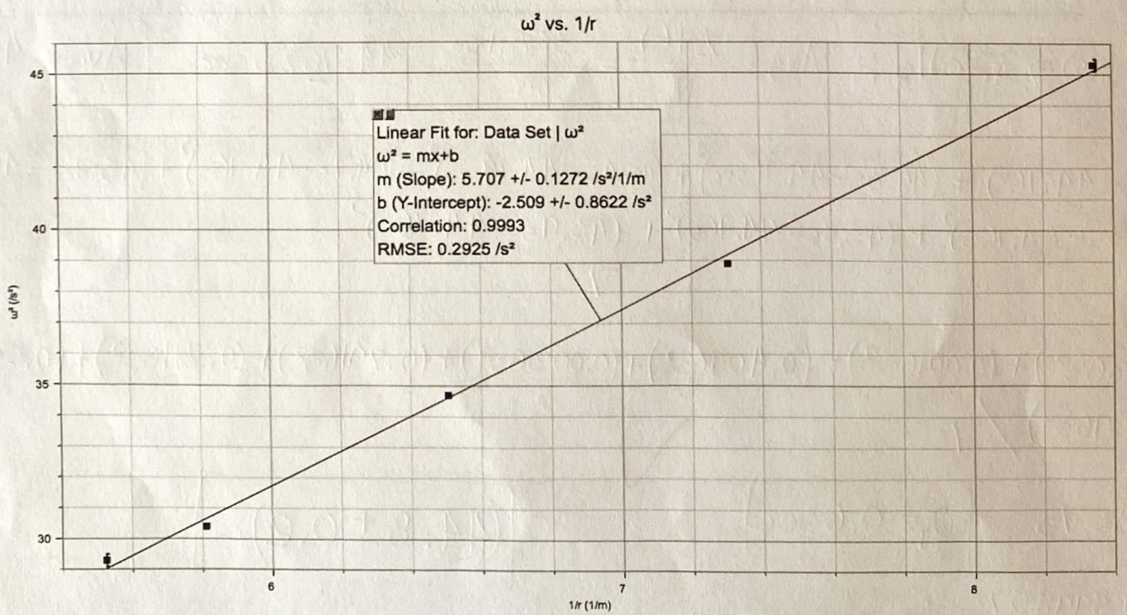
Suppose you want to measure the coefficient of static friction between an object and its surface. You determine the angular frequency in which maximum frictional force acting an object on a turntable occurs as a function of r. For a given the object location, r, from the center of a turntable, you measure the maximum angular velocity that the object can achieve before it slips relative to the turntable. A data table along with a graph of ω^2 vs. 1/r is shown below. The equation that relates the coefficient of static friction with the maximum angular speed can be derived from:

$$f_s = F_C$$
$$\mu_s mg = mr\omega^2$$
$$\mu_s = \frac{r\omega^2}{g}$$

Using the curve fit results determine the value of μ_s along with its uncertainty to the correct number of significant figures and units.

Table 2

Radius r (cm)	1/r (m^{-1})	Maximum Angular Speed ω (rad/s)	ω^2 (rad^2/s^2)
18.21	5.491	5.41	29.3
17.46	5.727	5.53	30.6
15.43	6.481	5.88	34.6
13.73	7.283	6.25	39.1
11.98	8.347	6.71	45.0



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$\mu_s = \frac{r\omega^2}{g}$ slope $= \frac{\omega^2}{\frac{1}{r}} = \omega^2 r = (5.707 \pm 0.1272)\ m/s^2$

$= (5.7 \pm 0.1)\ m/s^2$

$\mu_s = \frac{5.7 m/s^2}{9.8006 m/s^2} = 0.581597045$

Chose Data point that was on curve fit line

radius = 15.43 cm

= 0.1543m

$\mu_s = 0.58$

$\omega^2 = 34.6$

$\delta\mu_s = \sqrt{\left(\frac{\partial \mu_s}{\partial r}\right)^2 (\delta r)^2 + \left(\frac{\partial \mu_s}{\partial \omega}\right)^2 (\delta\omega)^2}$

$\delta\mu_s = \sqrt{\left(\frac{\omega^2}{g}\right)^2 (\delta r)^2 + \left(\frac{2r\omega}{g}\right)^2 (\delta\omega)^2}$

$= \sqrt{\left(\frac{(34.6\ rad^2/s^2)}{9.8006\ m/s^2}\right)^2 (0.0001 m)^2 + \left(\frac{2(0.1543 m)(5.88 rad/s)}{9.8006\ m/s^2}\right)^2 (0.01 rad/s)^2}$

$= \sqrt{0.0000001215 + 0.000003428}$

$= 0.001884941$

$\delta\mu_s = 0.002$

NO

(-4)

$\boxed{\mu_s = 0.58\cancel{2} \pm 0.002}$

$\mu = \frac{slope}{g}$

$\delta\mu = \frac{\delta slope}{g}$

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Question 4: ___3___ / 4pts

Miscellaneous Questions

(a) Calculate the following to the correct number of significant figures and units using the rules for significant figures:

$$\frac{1.840x10^{11}\frac{N}{m^2} - 1.90x10^{11}\frac{N}{m^2}}{1.840x10^{11}\frac{N}{m^2}}$$

$\frac{-6\times10^{9}\ N/m^2}{1.840\times10^{11}\ N/m^2}$

$\boxed{-0.03}$

(b) Give an example demonstrating how the data and results of an experiment be used to indicate whether the dominant source of error in an experiment is systematic?

The dominant source of error in a system is systematic in the case of the Accelerating Cart System Lab. As the mass of the hanger increased, the systematic error of friction decreased and this was depicted in the trend of the percent error decreasing. The percent error was calculated in the data and results, and by comparing the value of ...

Compare % uncertainty to % error

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