Chapter 13 Gravity

Thursday, April 27, 2023 12:06 PM

Gravity - Topics

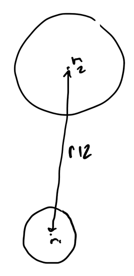
$\hat{r}$ = length of the magnitude

$|F| = -g \frac{m_1 m_2 \hat{r}}{|\vec{r}^2|}$

$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$

Gravitational force
Gravitational field
Energy
Keplers laws
Gravitational fields of Continuous Mass Distributions

$F = -G \frac{m_1 m_2}{|r|^2} \hat{r}$

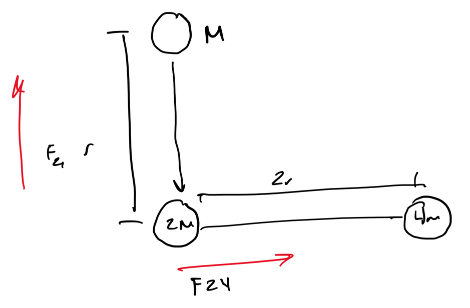


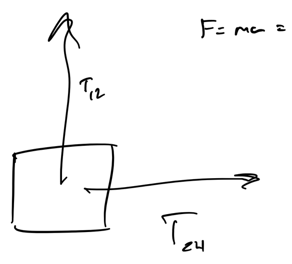
$\hat{r} = \frac{r_{12}}{|r_{12}|}$

$|\vec{F}_{12}| = \frac{G m_1 m_2}{|r_{12}|^2}$

they will have different acceleration

proportional to product of masses





$F = ma$ = resistance to a change in motion

$\Sigma F = m_1 a$

$F = G \frac{m_1 m_2}{\hat{r}^2} \hat{r}$

Principle of Equivalence

$F_t = \frac{G 2m \cancel{4}m}{(\cancel{2}r)^2} i + \frac{G 2m m}{r^2} j$

$2 \frac{G m^2}{r^2} i + \frac{G m^2 2}{r^2}$

$2 \frac{G m^2}{r^2} (i + j) \quad \theta = 45°$

Gravitational Field

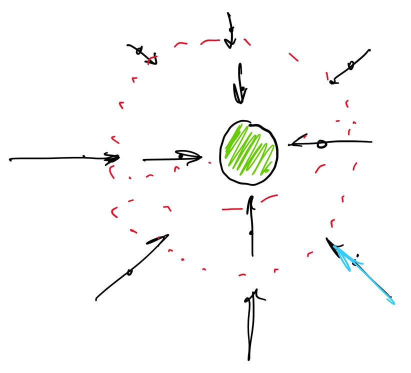
" Explain " action at a distance

- Ratio of gravitational force to a "test mass"

$\vec{g} = \frac{\vec{F}}{m} = \frac{\frac{g m_E m}{R_E^2}}{m} = \frac{G M_E}{R_E^2}$

$\vec{g} = \frac{\frac{-g m_1 m_2}{|r_{12}|^2} \hat{r}_{12}}{m_1}$

better term for gravity is "field strength"



Field lines

$\vec{g} = -G \frac{m}{r^2} \hat{r}$

$E = k + u$ - kinetic + potential = zero

$E = 0$ means we have enough energy to escape the gravitational field.

what does it mean if $e < 0$,

$\frac{1}{2} m v_i^2 + \frac{-G m_E m}{R_E} = 0 + 0$

$\frac{1}{2} m v^2 = k_E$

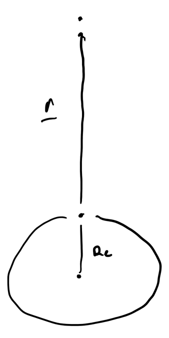
$k_E = \frac{G m_E m}{R_E}$

$V_{eff} = \frac{-G m_1 m_2}{r_{12}}$

$\frac{1}{2} \cancel{m} v_i^2 = \frac{G m_E \cancel{m}}{r_E}$

$\sqrt{v_i^2} = \sqrt{\frac{2 G M_E}{r_E}}$

$\boxed{V_i = \sqrt{\frac{2 G M_E}{r_E}}}$



$V_{esc} = \sqrt{\frac{2GM}{r_E}}$

Dom's Universe

$G = 10 nm^2$
$R = 1.0$
$M = 10 kg$

$\frac{G M_1 m}{r^2} = \frac{m v^2}{r}$

$\boxed{v = \sqrt{\frac{Gm}{r}}}$

$\sqrt{\frac{(10nm)(10kg)}{1.0}}$

Energy of an object orbiting in a circle

$\frac{1}{2} m v^2 - \frac{G M m}{r}$

$\frac{1}{2} m \frac{Gm}{r} - \frac{G m n}{r} =$

$\boxed{\frac{1}{2} m \frac{Gm}{r}}$

$E = -Gm$

$\Delta E = \Delta u + \Delta k$

ΔE

Force $= \frac{G M m}{r^2}$

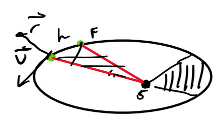
Field $= g = \frac{GM}{r^2}$

$1\ a.u. = 1.5 \times 10^{11}$

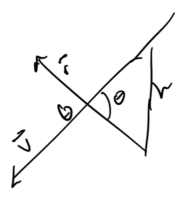
Keplers Laws

- Orbits of planets are ellipses
- Equal areas in equal times
- $\frac{T^2}{R^3}$ constant for orbits around common center.

$\tau = \bar{r} \times \bar{F}$



$\Delta a = \frac{1}{2} r h$



angular momentum =
$r \times p$
$r v m \sin\theta$

$\frac{Gm}{r^2} = \left(\frac{2\pi r}{T}\right)^2 \frac{1}{r}$

$\frac{Gm}{r^2} = \frac{4\pi^2 r}{T^2}$

$\boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{Gm}}$

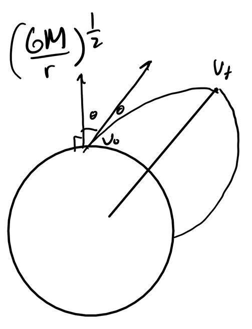
Orbit and trajectory problems

L is constant
$L_a = L_p$
Energy is Constant
$U_a + K_a = U_p + K_p$

$-\frac{Gm}{r_a} + \frac{1}{2}$

A rocket is fired at 60° to the local vertical

$V_0 = \left(\frac{GM}{r}\right)^{\frac{1}{2}}$



$\frac{1}{2} m v_0^2 + \frac{-G M_E m}{r_E} = \frac{1}{2} m V_f^2 + \frac{-G M_E m}{r_{max}}$

$\frac{\cancel{m} v_0 r_E \sin\theta = m v_f r_{max}}{\cancel{m} r_{max}}$

$\boxed{\frac{V_0 r_E \sin\theta}{r_{max}} = V_f}$

$\frac{1}{2} m V_0^2 + \frac{-G M_E m}{r_E} = \frac{1}{2} m \left(\frac{V_0 R_E \sin\theta}{r_{max}}\right)^2 + \frac{-G M_E m}{r_{max}}$

- Sub in V_0

$\frac{1}{2} \cancel{m} \frac{\cancel{G}m}{R_E} + \frac{-G M_E m}{R_E} = \frac{1}{2} m \left(\frac{G M_E}{R_E}\right) \left[\frac{R_E^2 \sin^2\theta}{r^2_{max}}\right]$

$-\frac{G M_E m}{r_{max}}$

$-\frac{1}{2R_E} = \frac{R_E \sin\theta^2}{2 r^2_{max}} - \frac{1}{r_{max}}$

$\frac{-r^2_{max}}{r_E} = R_E \sin^2\theta - 2 r_{max}$

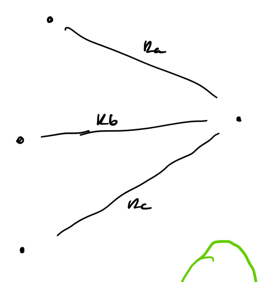
$0 = R_E^2 - 2 R_E r_{max} + r_{max}^2$

$r_{max} = 2R_E \pm \sqrt{4R_E^2 - 4R_E^2 \sin\theta}$

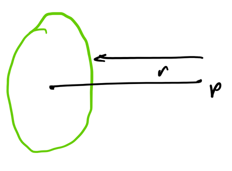
$\boxed{= R_E(1 \pm \cos\theta)}$

Gravitational Field

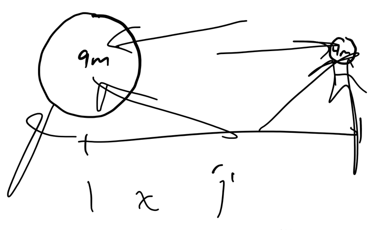
(Energy)
(Force)
Due to Distributed Force



$\boxed{g = \vec{g_a} + \vec{g_b} + \vec{g_c}}$



$d\vec{g} = -G\left(\frac{dm}{r^2}\right)\hat{r}$



fields carry energy

gravitons photons

$-g \cdot \frac{9m}{x^2}$

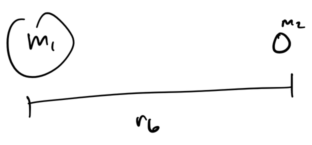
$\frac{(x-h)^2}{a} + \frac{(y-k)^2}{b}$

$\boxed{x = \frac{3}{5}}$

$-\frac{\cancel{G} 9 \cancel{m}}{x^2} + \frac{4 \cancel{g} \cancel{m}}{(x-1)^2} = 0$

Gravitational u

$\Delta u = -w_c$



$W_g = W_c = \int F \cdot d\vec{s} = \int \frac{G m_1 m_2}{r^2} \hat{r} \cdot d\vec{s}$

$= \int G \frac{m_1 m_2}{r^2} \hat{r} \cdot \vec{dr}\, d\theta$

$(n+1) \cdot a \cdot b^{n+1}$

$\frac{G m_1 m_2}{r^2} = G m_1 m_2 \cdot r^{-2}$

$-U_a = \frac{G m_1 m_2}{r_a} \quad U_a = -\frac{G m_1 m_2}{r_a}$

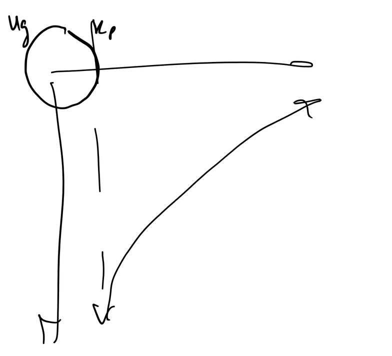
$\Delta u = u_f - u_i$

$= -\frac{G m_E m}{R_E} - \frac{G m_E M}{R_E + h}$

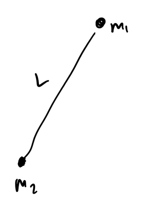
$(1 \pm x)^n$

$\left(1 + \frac{h}{R_E}\right)^{-1} = \left[\left(1 - \frac{h}{R_E}\right)\right]$

$-\frac{G M_E}{R_E} m h = mgh$



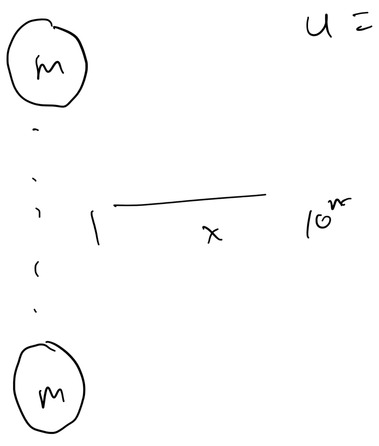
$W_1 = U_f - U_i = \frac{-G m_1 m}{L}$



$W_1 = -\frac{G m_1 m_2}{L}$

$W_2 = -\frac{G m_1 m_3}{L} - \frac{G m_1 m_2}{L}$

$W_t = -\frac{G m_1 m_2}{L} - \frac{G m_1 m_3}{L} - \frac{G m_2 m_3}{L}$



$u = \frac{-2GMm}{\sqrt{x^2 + a^2}}$

$F = \frac{-du}{dx}$

$\frac{V'w - Vw'}{w^2}$

$\frac{-2GMm}{\sqrt{x^2 + a^2}}$