Questions:

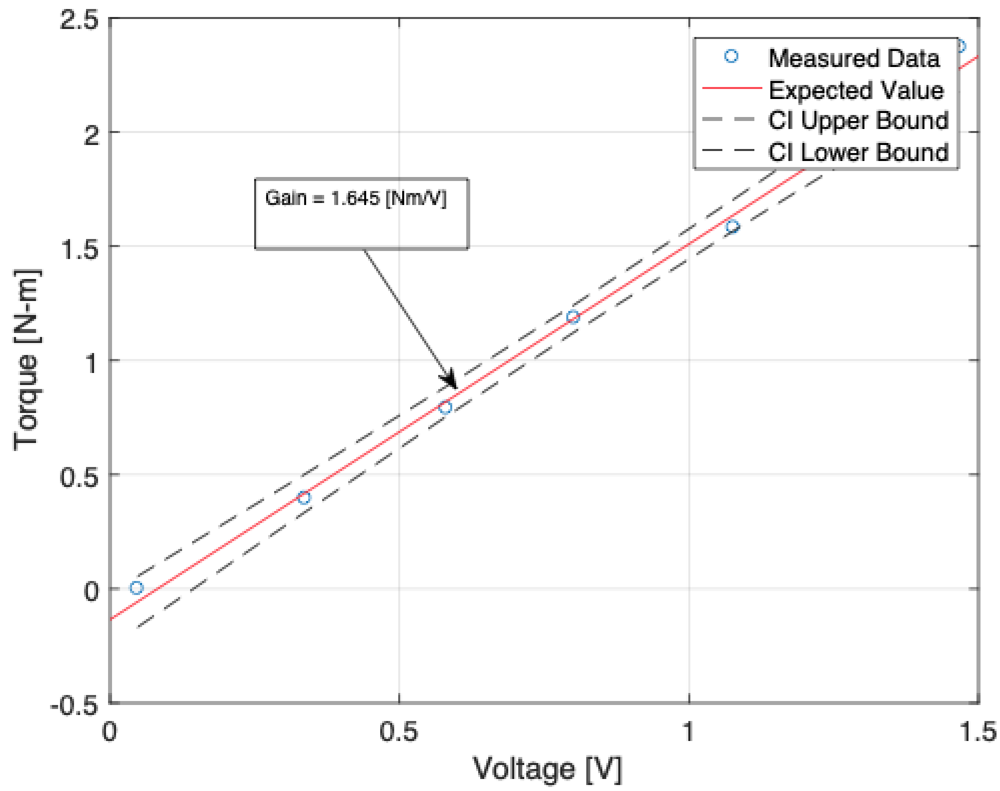
Part 1: Calibration

- **What is the calibration function for the Torque Sensor?** ○ **Show your data in an annotated figure**

○ **Make sure to account for all offsets and the one point you know to be true...**

○ **Characterize the error in the calibration output using a confidence interval with K = 2**

■ **Hint:** ***Force*** **=** ***F*****(*****V*****)±(*****K******σ*****F*****)**



The calibration function for the torque sensor is:

$$T_{motor} = G(V - V_{offset}) = (1.645)(V - 1.02)\ [Nm]$$

The vast majority of the values lie within the confidence interval, and as such the model is deemed to be sufficiently accurate. However, because some of the points lie outside of the confidence interval, the output values must be considered with caution.

Applying to engine torque:

$$T_{enginge} = -3G(V - V_{offset})$$

Plugging in numbers,

$$T_{enginge} = 4.935V - 5.036\ [Nm]$$

Part 2: Determine Motor Work, Friction Torque, and Flywheel Inertia

Question: Assuming constant tooth spacing (except for the missing tooth), what is the angle of each tooth as a function of tooth number?

- **Form of function should be Crankshaft angle [degrees] = function(Tooth Number)**
- **Assume the first tooth after the missing tooth (first to pass the sensor) is tooth 1**

35 teeth, 1 missing = 36 teeth total.
Crankshaft Angle [°] = $\Theta = 10$ * ((Tooth Number) + (Time since last tooth)/(Time between teeth))

Question: What is the position of the piston as a function of the crankshaft angle (in degrees)?

- **Form of function should be Piston displacement [mm] = function(Crankshaft angle [degrees])**
- **Use missing tooth as 0 degrees**

With TDC position = 0,

Stroke = S = |h(BDC)| = 37.2 mm

Piston Position = h = 0 mm at TDC and -37.2 mm at BDC

$$h = -\frac{S}{2} * cos(\Theta * \frac{\Pi}{180}) + \frac{S}{2}$$

With Θ = Crankshaft angle [°]

Plugging in #'s:

$$h\,[mm] = -18.6 * cos(\Theta * \frac{\Pi}{180}) + 18.6$$

With Θ = Crankshaft angle [°]

Low RPM and High RPM

acquire the following plots: Crankshaft Angle and Tooth#, Piston Displacement, Torque Waveform

Note: High is 500 RPM, Med is 300 RPM, Low is 200 RPM

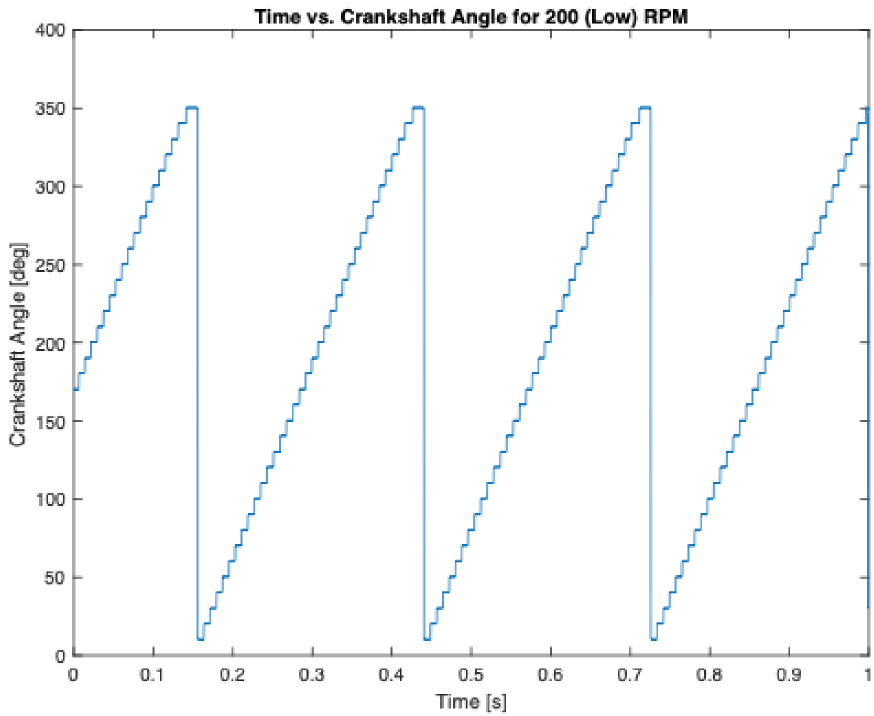
Carefully remove all the engine flywheels but one so there is only 1 plate attached to the

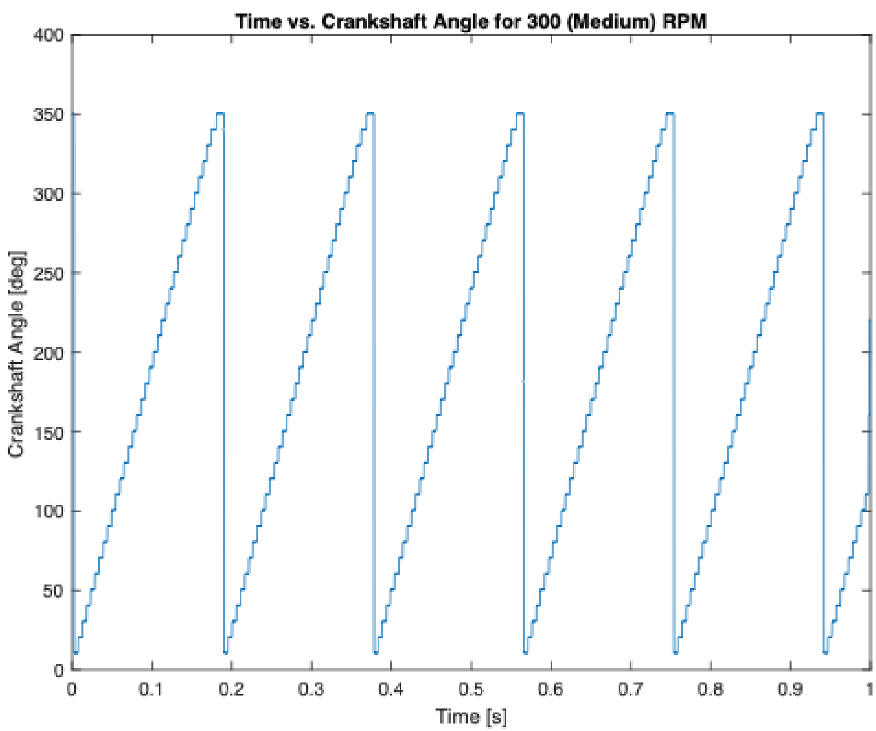
crankshaft at the right side

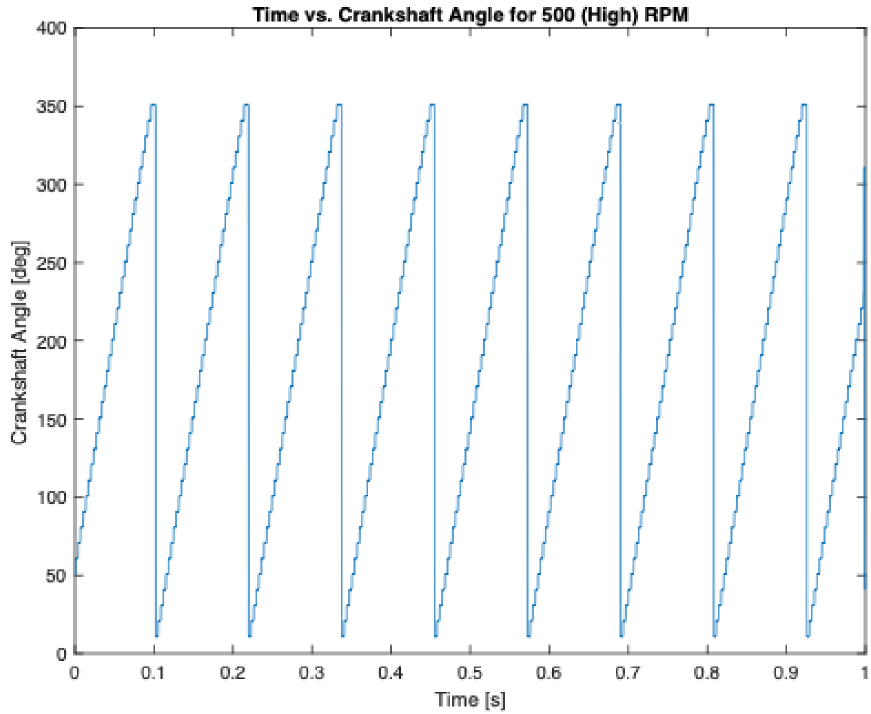
○ Repeat data acquisition only for the case of accelerating from rest to high RPM to obtain the following plots: Crankshaft Angle and Tooth #, Piston Displacement, Torque Waveform

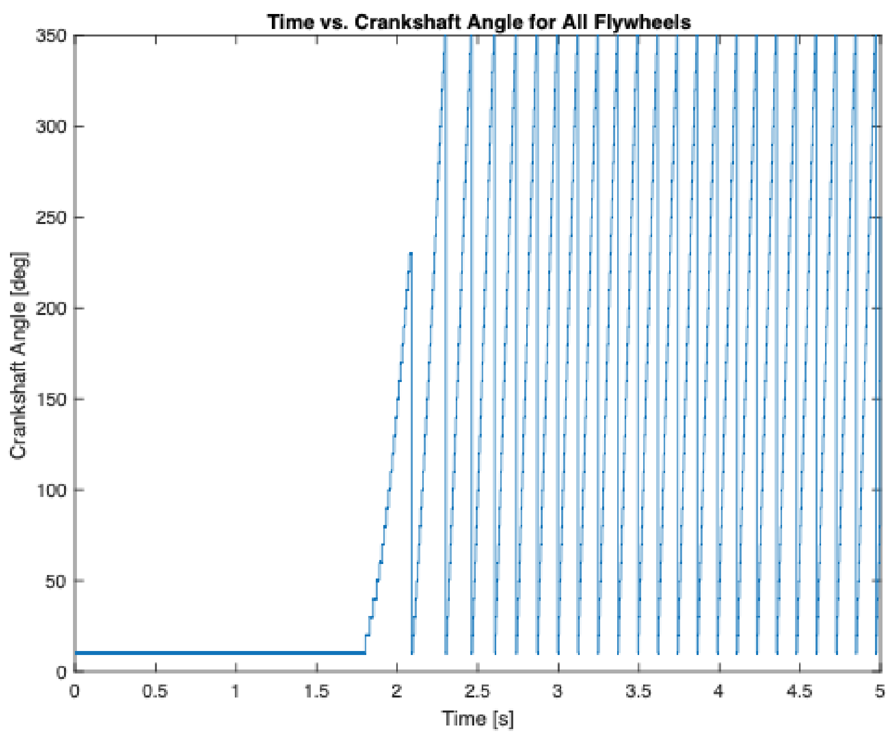
Required Plots:

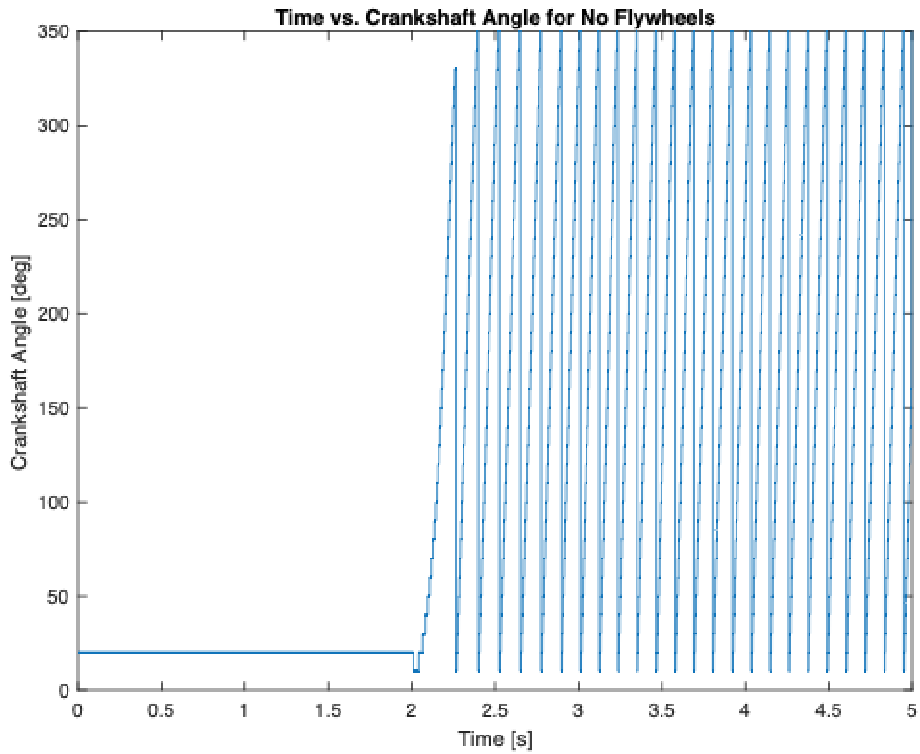
Time vs. Crankshaft Angle Plots


Time vs. Crankshaft Angle for 200 (Low) RPM
Crankshaft Angle [deg]
Time [s]
0
50
100
150
200
250
300
350
400
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

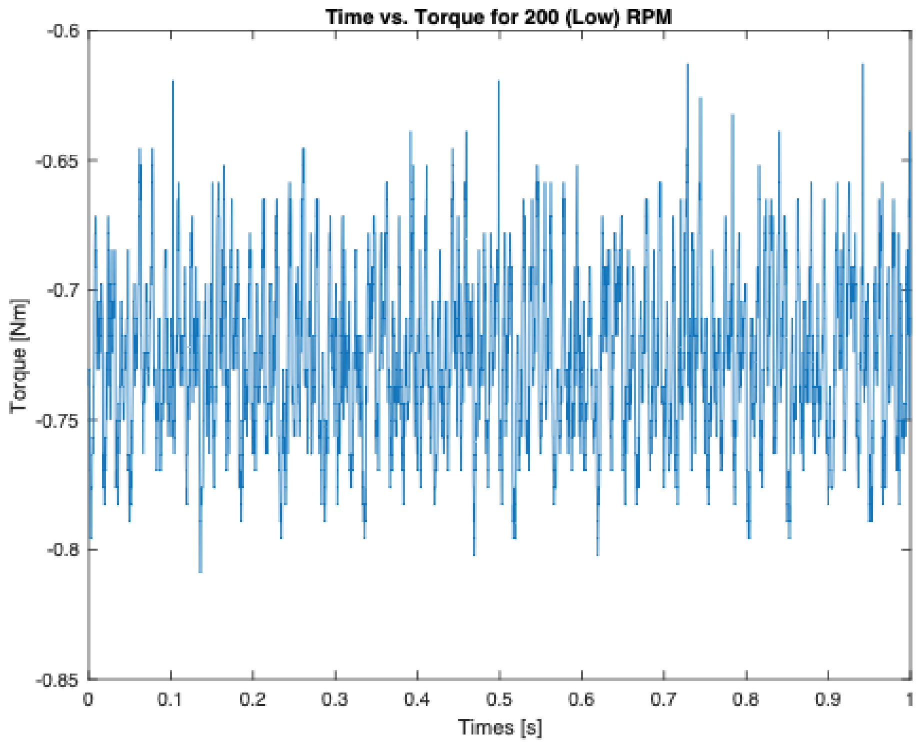

Time vs. Crankshaft Angle for 300 (Medium) RPM
Crankshaft Angle [deg]
Time [s]
0
50
100
150
200
250
300
350
400
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

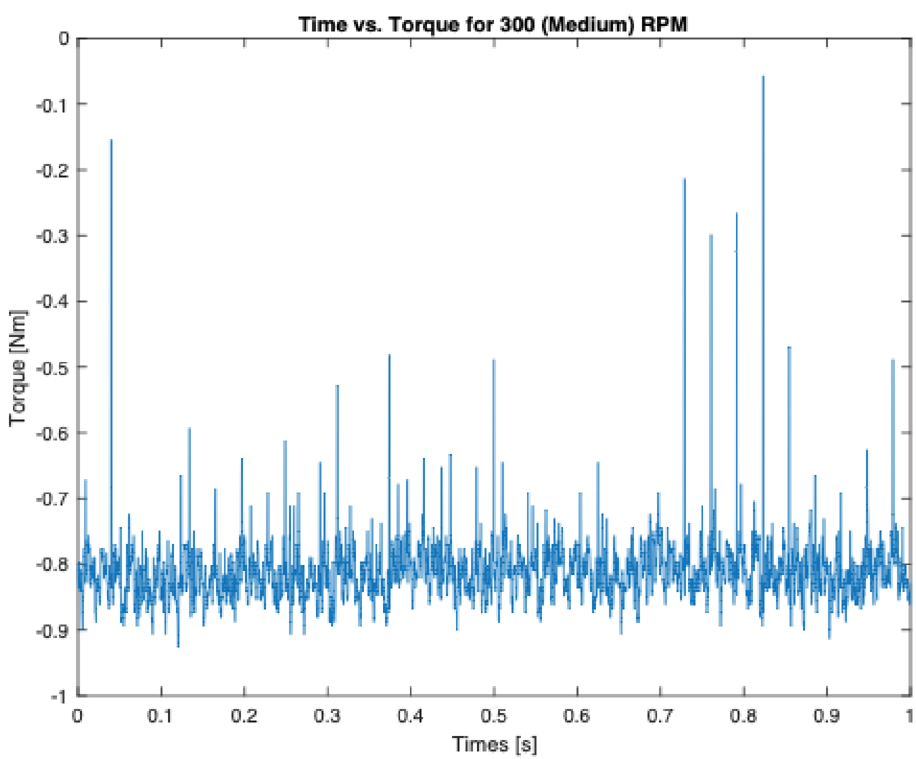

Time vs. Crankshaft Angle for 500 (High) RPM
Crankshaft Angle [deg]
Time [s]
0
50
100
150
200
250
300
350
400
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

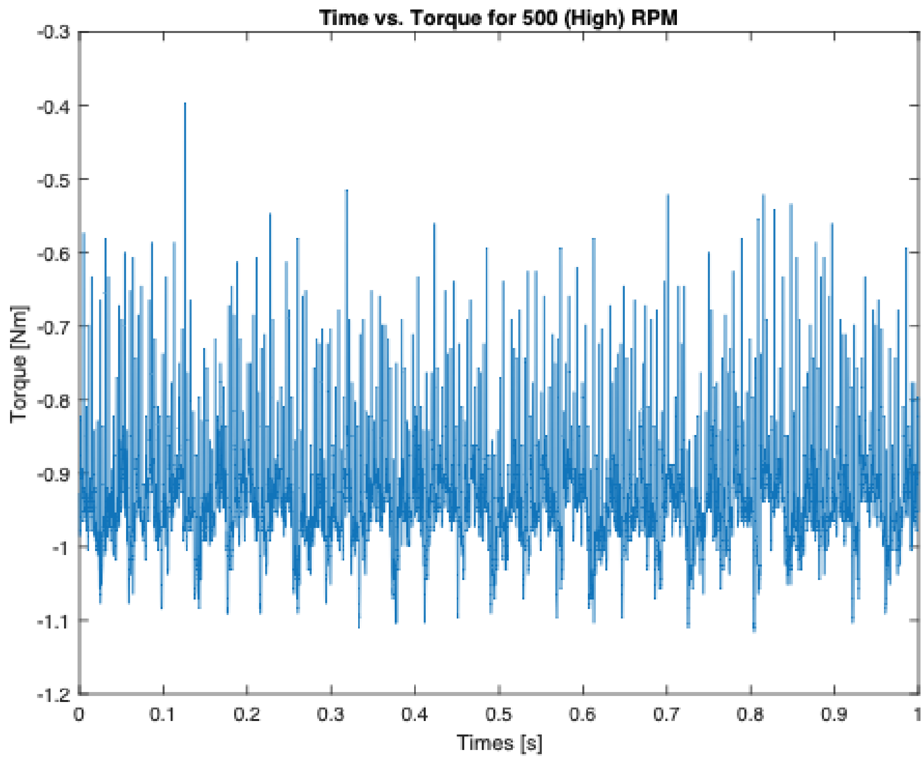

Time vs. Crankshaft Angle for All Flywheels
Crankshaft Angle [deg]
Time [s]
0
50
100
150
200
250
300
350
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

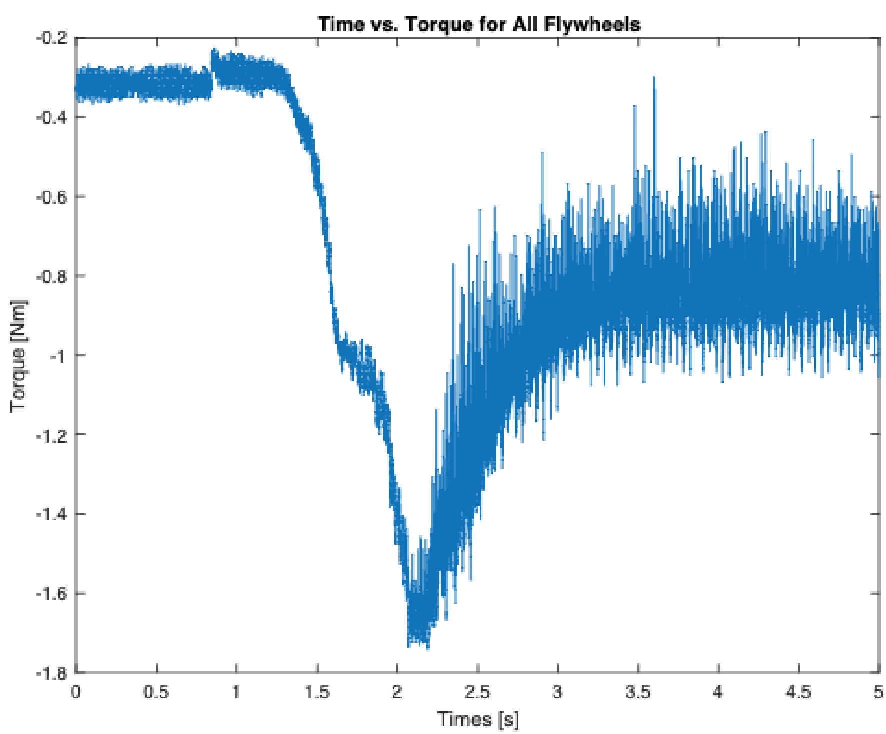


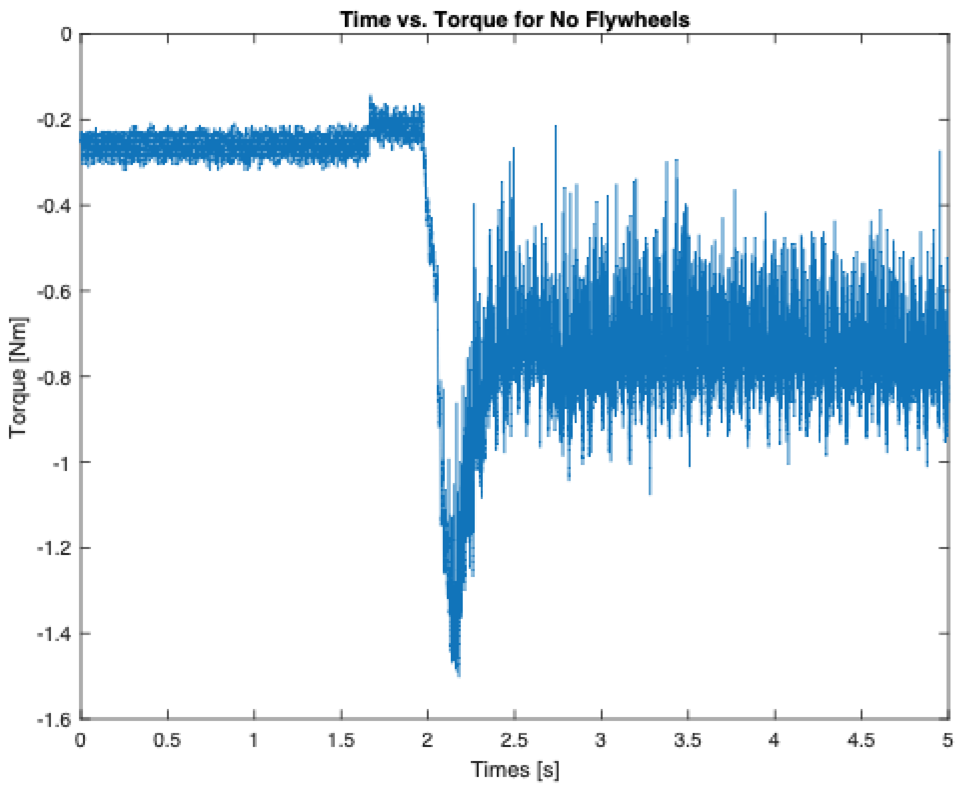
Time vs. Torque Plots


Time vs. Torque for 200 (Low) RPM
Torque [Nm]
Times [s]
-0.6
-0.65
-0.7
-0.75
-0.8
-0.85
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

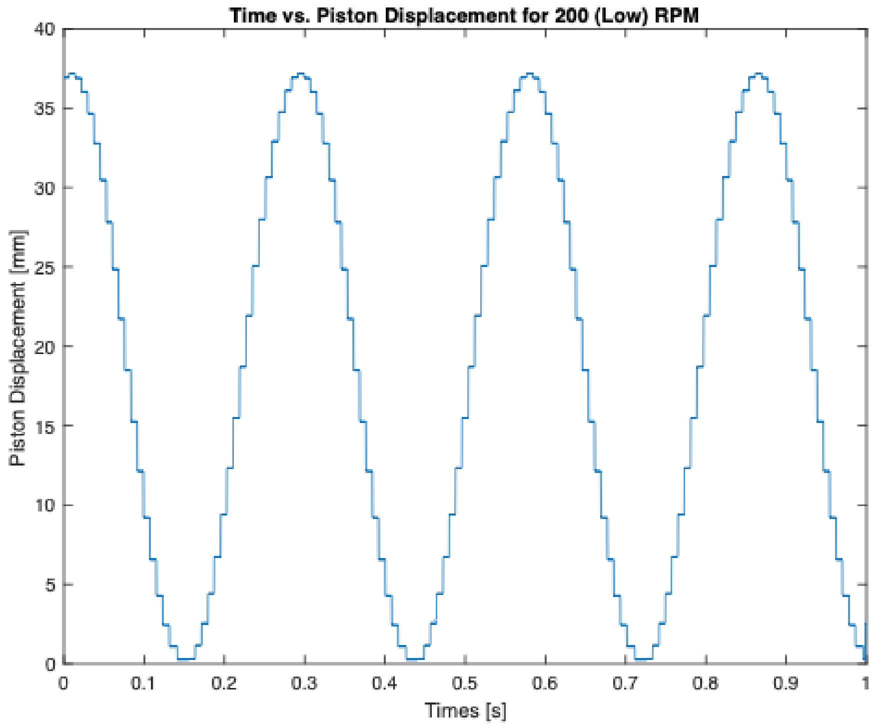

Time vs. Torque for 300 (Medium) RPM
Torque [Nm]
Times [s]
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

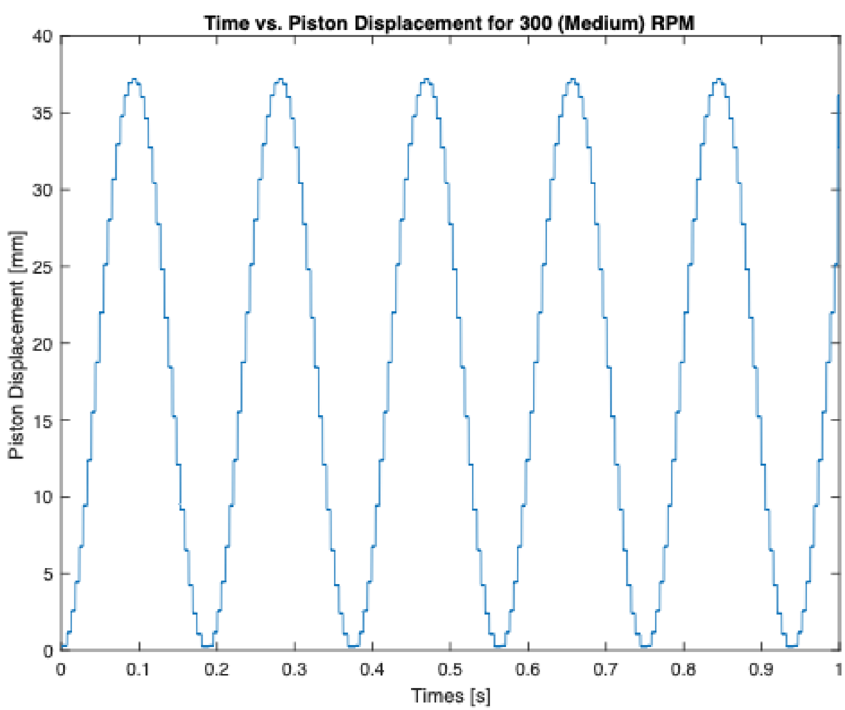

Time vs. Torque for 500 (High) RPM
Torque [Nm]
Times [s]
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
-1.1
-1.2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

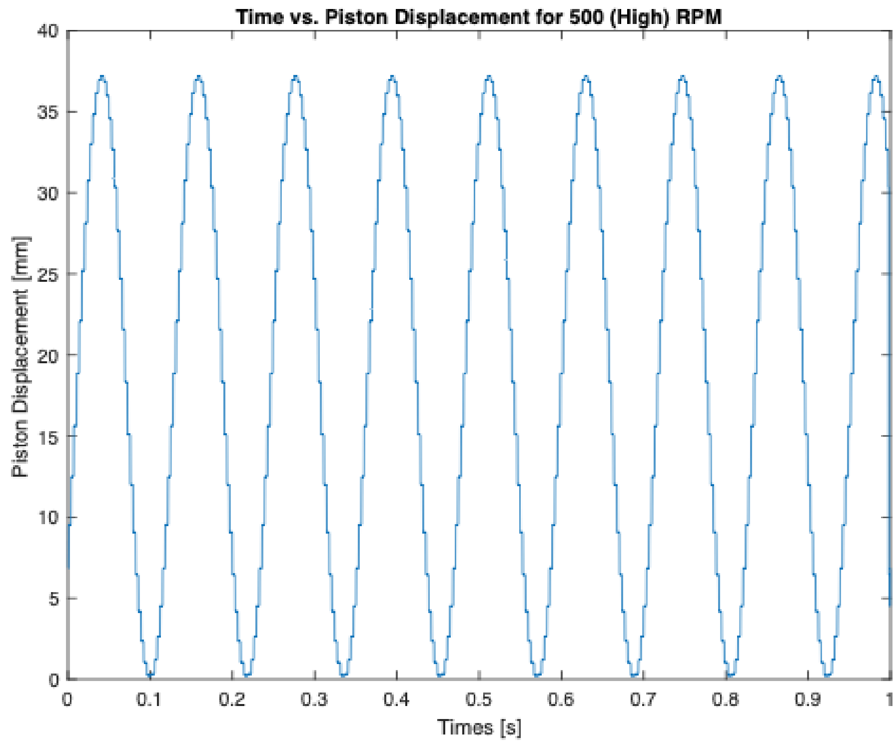

Time vs. Torque for All Flywheels
Torque [Nm]
Times [s]
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5



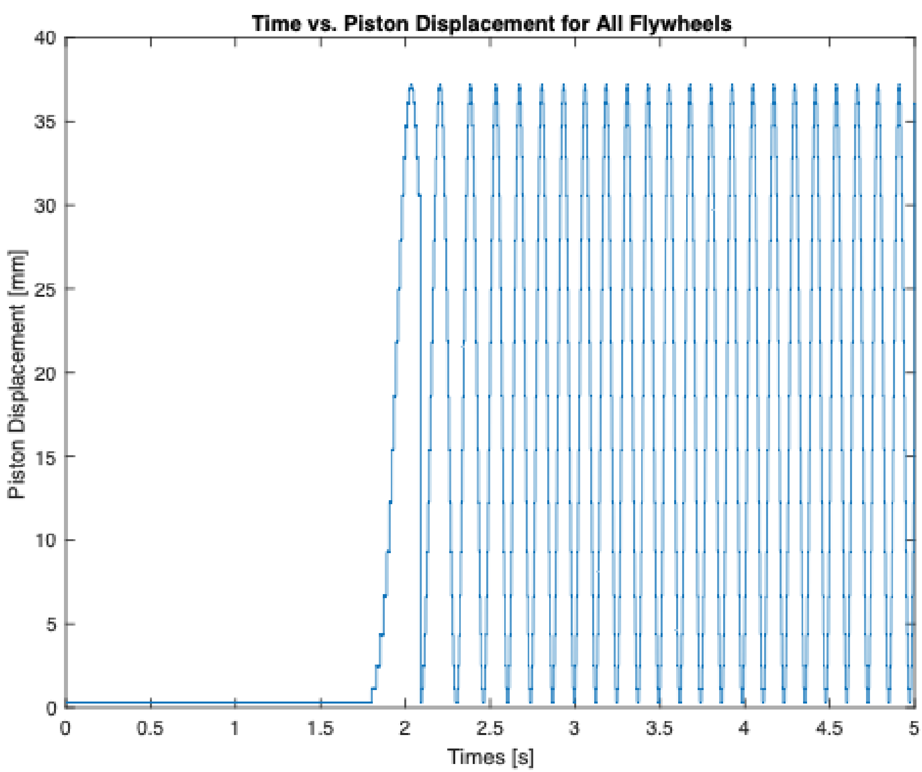
Time vs. Piston Displacement Plots


Time vs. Piston Displacement for 200 (Low) RPM
Piston Displacement [mm]
Times [s]
0
5
10
15
20
25
30
35
40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

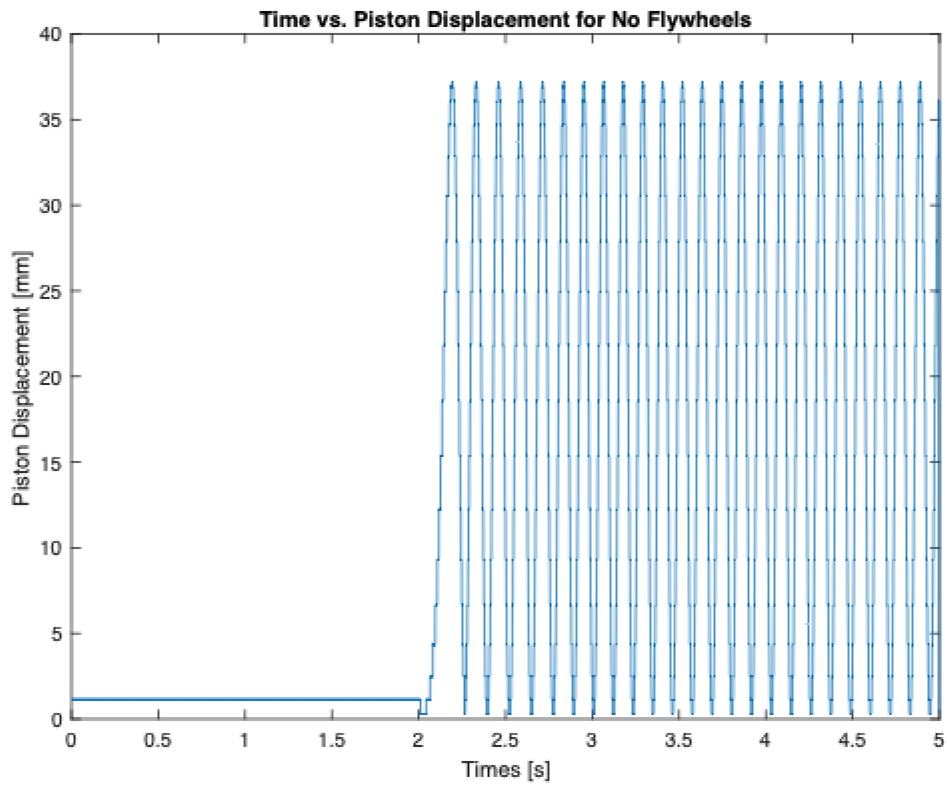

Time vs. Piston Displacement for 300 (Medium) RPM
Piston Displacement [mm]
Times [s]
0
5
10
15
20
25
30
35
40
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



Time vs. Piston Displacement for 500 (High) RPM
Piston Displacement [mm]
Times [s]



Time vs. Piston Displacement for All Flywheels
Piston Displacement [mm]
Times [s]



Follow-up Questions:

- **How accurate was your torque calibration? Does the torque vs. time data support your calibration? If yes, explain why. If not, explain how you would adjust your calibration for your remaining analysis.**

Our torque calibration was accurate, with an R^2 value of 0.9959 and an RMSE of 0.0598. The torque vs. time data does indeed support our calibration values. This is because the values are in the trend of the expected values - as RPM increases, torque decreases. This trend is observed in the above figure. Thus the calibration is deemed to be sufficiently accurate for the remaining analysis.

- **Create a complete mechanical model based on the Work-Kinetic Energy equation**

relating flywheel (and crankshaft) energy change to motor and friction work

- $E_{fly,\,b} - E_{fly,\,a} = W_{motor,\,a->b} + W_{friction,\,a->b}$
- $E_{fly,\,n} = \frac{1}{2} * I_{fly} * \omega^2_{fly,\,a}$
- Numerically integrate motor torque to get $W_{motor,\,a->b}$

$$\Delta E_{fly} = E_{fly,\,b} - E_{fly,\,a} = W_{motor,\,a->b} + W_{friction,\,a->b}$$

$$E_{fly,\,b} = \frac{1}{2} * I_{fly} * \omega^2_{fly,\,b}$$

$$E_{fly,\,a} = \frac{1}{2} * I_{fly} * \omega^2_{fly,\,a}$$

$$\frac{1}{2} * I_{fly} * \omega^2_{fly,\,b} - \frac{1}{2} * I_{fly} * \omega^2_{fly,\,a} = W_{motor,\,a->b} + W_{friction,\,a->b}$$

At steady state, $\omega_{fly,\,a} = \omega_{fly,\,b}$. So, $W_{motor,\,a->b} = -W_{friction,\,a->b}$ at steady state.

$$W_{friction,\,a->b} = -138.1980\ [J]$$

Thus,

$$W_{motor,\,a->b} = 138.1980\ [J]$$

● What is the torque due to friction acting on the system as it rotates?

○ Hint: How can you isolate the torque in the energy balance you created and what operational condition must be set to make this calculable?

$$W_{motor,\,a->b} = \int_a^b T(\Theta)\, d\Theta$$

As $W_{motor,\,a->b} = -W_{friction,\,a->b}$, for steady state,

Thus,

$$W_{friction,\,a->b} = -\int_a^b T(\Theta)\, d\Theta$$

Integrating, the torque can be isolated in this energy balance. The operational condition that must be set to make this calculable is that steady state must be achieved.

$T_{friction} = -0.8697\ [Nm]$

- **What is *Ifly*?**
  - **Calculate this quantity using the energy balance derived earlier.**
  - **What operational conditions are required in order to keep *Ifly* in the energy balance?**
  - **Use *T friction* from your previous results**

$T_{friction} = -0.8697\ [Nm]$

Operational conditions to keep Ifly in this energy balance: not steady state, thus,
$\omega_{fly,a} \neq \omega_{fly,b}$

Calculating,

$\omega_{fly,b} = \omega_{SS} = 500\ RPM = 52.359\ rad/s$

$E_{fly,b} = \frac{1}{2} * I_{fly} * \omega^2_{fly,b} = W_{motor,a->b} + W_{friction,a->b}$

Solving for I_{fly}, numerically integrating for $W_{motor,a->b}$:

$I_{fly} = (2 * (W_{motor,a->b} + T_{friction} * \Delta\Theta_{a->b})) / (\omega^2_{fly,b})$

$I_{fly,experimental} = (2 * (138.1980 - 122.6538))/(52.359^2) = 0.0113\ [kg * m^2]$

- **Compare your value of *Ifly* to a theoretical value consisting of 4 disks. Each flywheel plate has a mass $m = 1.0141 kg$ and radius $r = 0.07 m$. What do you think**

accounts for the difference between your calculated value and the theoretical value from the flywheel plates?

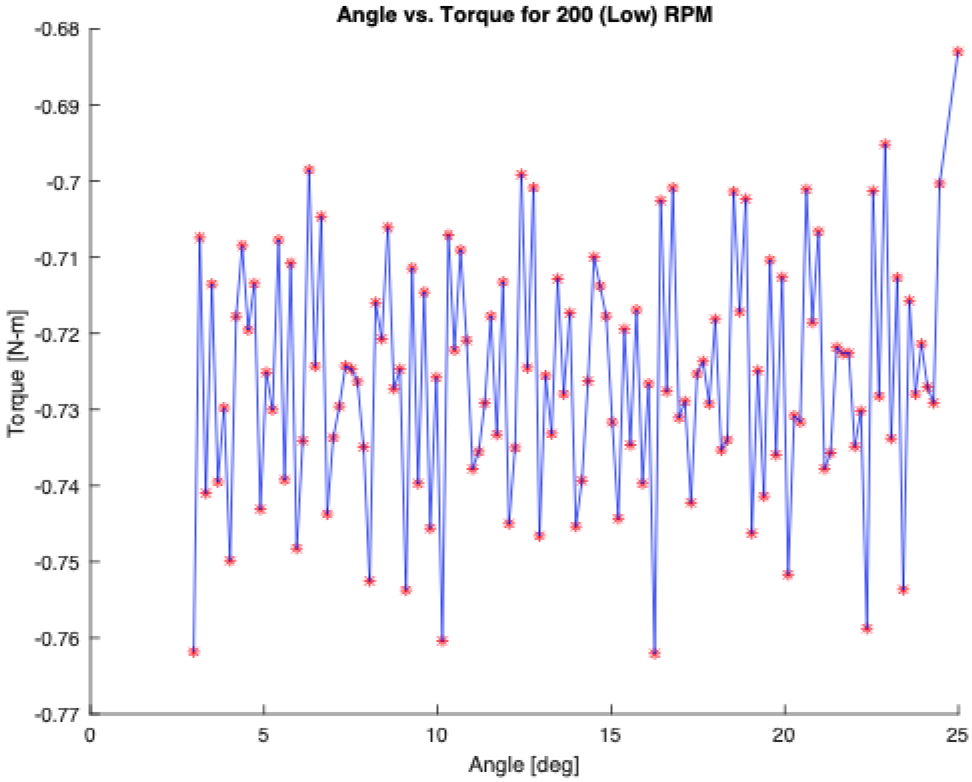
- **You may not throw human error under the bus. You are better than that.**

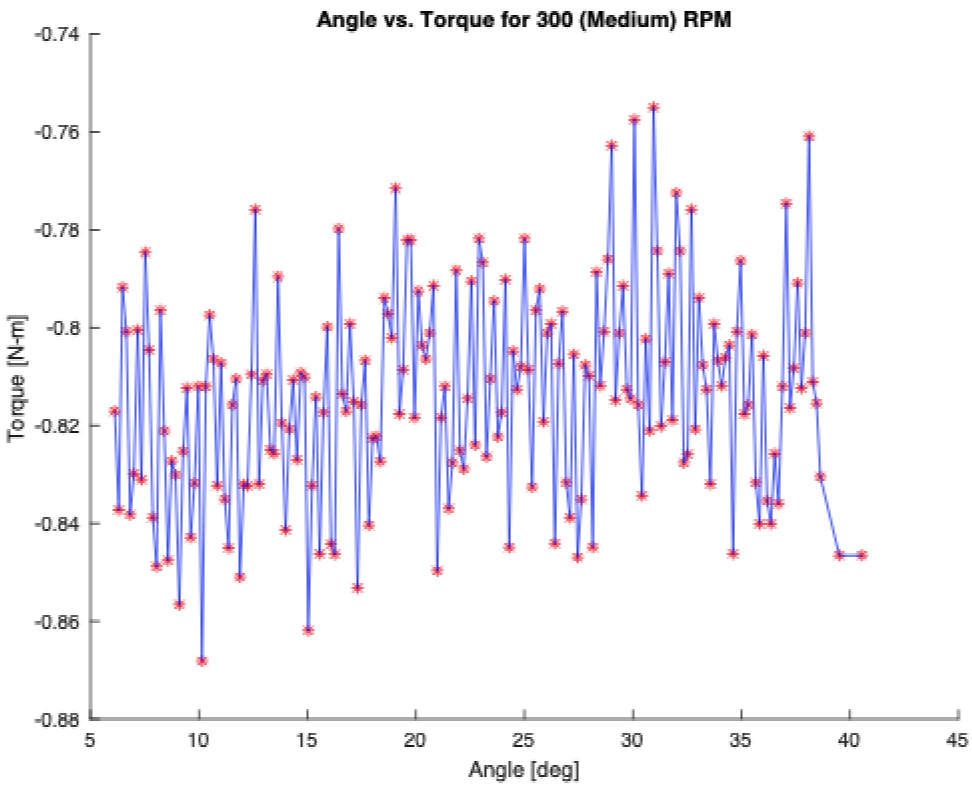
$$I_{fly,\ theoretical} = (\#\ plates) * (\frac{1}{2} * mass_{fly} * radius_{fly}^{2}) = 0.009938\ [kg * m^{2}]$$

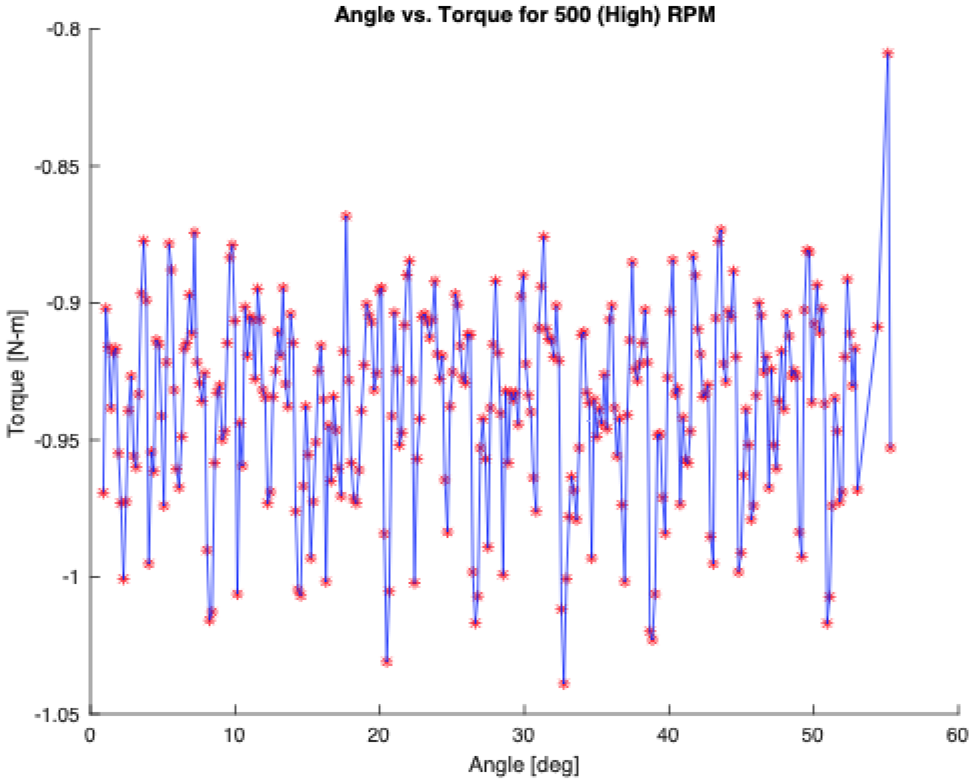
The theoretical and experimental flywheel moments of inertia are close enough, and match within reason for the purposes of this lab. Because the theoretical calculation makes the assumption of a solid, cylindrical flywheel (and this is not the case in actuality). There are bolt holes and other small abnormalities in material shape. Further, work due to friction of the piston was not accounted for in the theoretical model. These could account for the small difference between the calculated and theoretical value from the flywheel plates.

- **Is the assumption made in lecture about frictional torque being constant at all RPMs a valid assumption?**
  - **Prove or disprove this assumption with your data!**

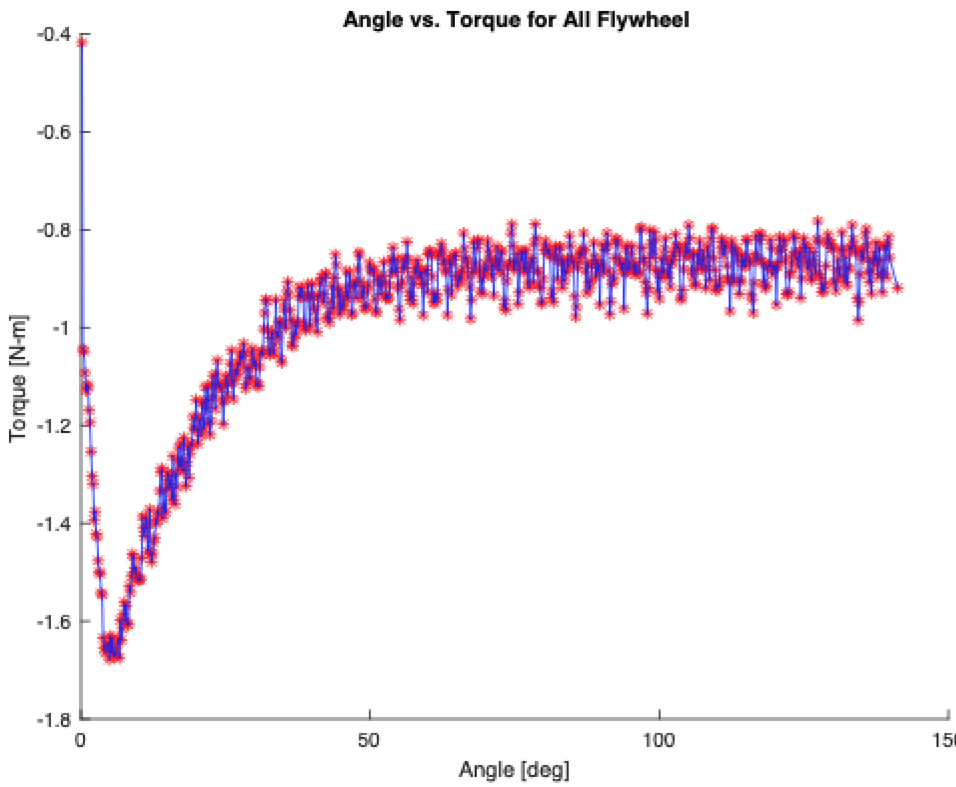
Torque vs. Angular Displacement Plots


Angle vs. Torque for 200 (Low) RPM
Torque [N-m]
Angle [deg]
-0.68
-0.69
-0.7
-0.71
-0.72
-0.73
-0.74
-0.75
-0.76
-0.77
0
5
10
15
20
25

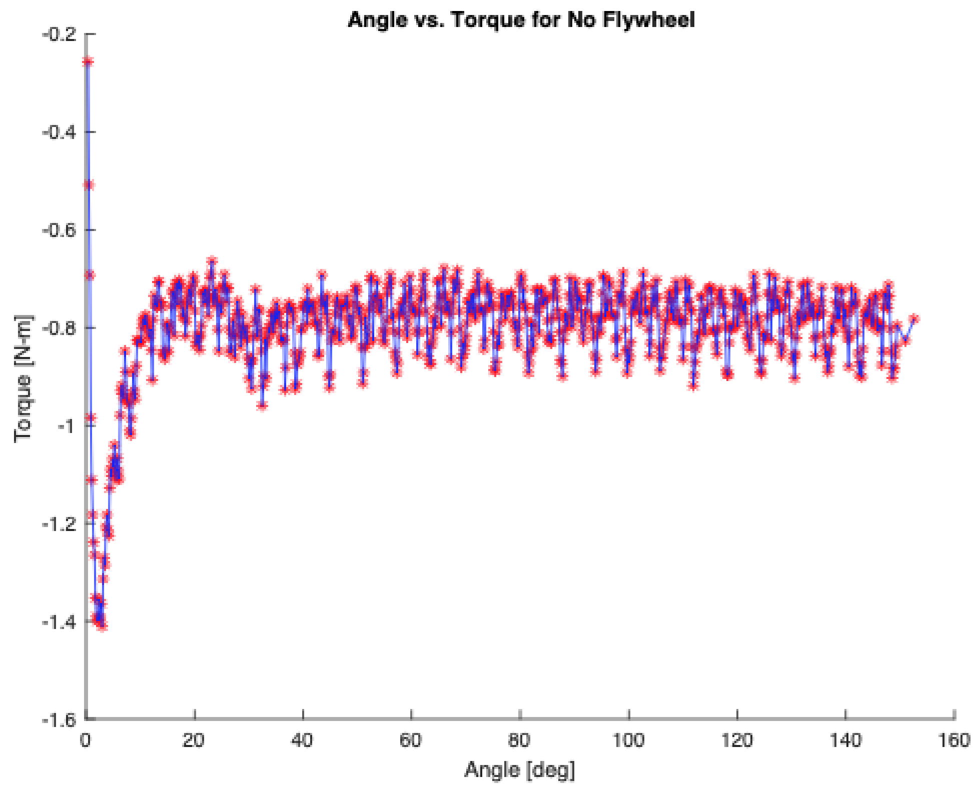

Angle vs. Torque for 300 (Medium) RPM
Torque [N-m]
Angle [deg]
-0.74
-0.76
-0.78
-0.8
-0.82
-0.84
-0.86
-0.88
5
10
15
20
25
30
35
40
45



Angle vs. Torque for 500 (High) RPM
-0.8
-0.85
-0.9
-0.95
-1
-1.05
Torque [N-m]
0
10
20
30
40
50
60
Angle [deg]



Angle vs. Torque for All Flywheel
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
-1.8
Torque [N-m]
0
50
100
150
Angle [deg]



The highest torque values at each phase is about -0.73 at 200 RPM, -0.85 at 300 RPM, and -0.95 at 500 RPM. Since the work due to air is negligible, and the equation is $W_{motor, a->b} = -W_{friction, a->b}$, work done by the motor is equal to negative work done by friction. The torque of each RPM is frictional torque, which decreases as RPM increases. Therefore, the assumption of being constant frictional torque at all RPM is not a valid assumption.

- **What is the effect of removing the flywheels on the acceleration of the engine? How can you support your claim from the data and from theory?**

As shown by the above angle vs. torque plots, removing the flywheels results in less torque required to reach steady state. This is shown by the average steady state torque decreasing from -0.73 [Nm] to -0.95 [Nm]. The system also reaches steady state more quickly, but the system is ultimately underdamped. Attaching the flywheels shifts to an overdamped system.

- **What is the crankshaft angle at the beginning of each stroke of the engine in the 4 stroke cycle? What is the state of each valve (Intake and Exhaust) during each stroke?**

1. Intake
    a. Starts at 0°
    b. Intake valve open
    c. Exhaust valve closed
2. Compression
    a. Starts at 180°
    b. Intake valve closed
    c. Exhaust valve closed
3. Power
    a. Starts at 360°
    b. Intake valve closed
    c. Exhaust valve closed
4. Exhaust
    a. Starts at 540 °)
    b. Intake valve closed
    c. Exhaust valve open

Code

```
% Least Squares Regression Script
```

```
%% Initialize Script
clear all
close all
clc
%% Read in data from excel
xvals = [0.0473875; 0.3366975; 0.5808975; 0.8017175; 1.0771175; 1.3083675; 1.4680775];
yvals =  [0; 0.395335152; 0.790670304; 1.186005456; 1.581340608; 1.97667576; 2.372010912];


%% Reference Quantities
%Calculate some useful quantities for statistical analysis.

numMeas = length(xvals); % Number of points in regression
L = 2;  % for polynomial fits, L is the order of the polynomial minus 1; L=2 for linear, L=3 for quadratic, etc.
dof = numMeas-L; % Degrees of Freedom based on desired fit

alpha = .046; % Alpha value for 95.4% confidence level
% k = 2.859; % Critical coverage factor for dof=4 at 95.4% confidence
% k = 2.021; % Critical coverage factor for dof=98 at 95.4% confidence
% k = 1.998; % Critical coverage factor for dof=998 at 95.4% confidence
k = tinv(1-(alpha/2),dof); % Calculate the exact coverage factor for current situation

%% Prediction/Confidence Intervals from models

[xFit,FitStats] = fit(xvals,yvals,'poly1'); % Desired fit, last digit of third argument is the desired order of fit
xFitCI = predint(xFit,xvals,1-alpha,'functional','off'); % Calculation of the confidence intervals of single shot data
% xFitPI = predint(xFit,xvals,1-alpha,'observation','off'); % Calculation of the prediction intervals of single shot data

%% Plots
figure(1)
plot(xvals, yvals, 'o')
hold on
plot(xFit)
hold on
plot(xvals,xFitCI,'--k')
% hold on
% plot(xvals,xFitPI,'--g')
xlabel('Voltage [V]')
ylabel('Torque [N-m]')
legend('Measured Data','Expected Value','CI Upper Bound','CI Lower Bound')
grid on
set(gca,'FontSize',14)

% Vals

[coeff] = coeffvalues(xFit);
gain_Nm_V = coeff(1)
vertical_offset_Nm = coeff(2)
```

```
%% 1 - time vs angle plots
data = xlsread('No-Head Characterization - Remote.xlsx',1);

times = data(:,1);
```

```
low_rpm_angle = data(:,3);
med_rpm_angle = data(:,6);
high_rpm_angle = data(:,9);
allfly_angle = data(:,12);
nofly_angle = data(:,15);
times_fly = data(:,10);

figure(1)
plot(times,low_rpm_angle)
xlabel('Time [s]')
ylabel('Crankshaft Angle [deg]')
title('Time vs. Crankshaft Angle for 200 (Low) RPM')

figure(2)
plot(times,med_rpm_angle)
xlabel('Time [s]')
ylabel('Crankshaft Angle [deg]')
title('Time vs. Crankshaft Angle for 300 (Medium) RPM')

figure(3)
plot(times,high_rpm_angle)
xlabel('Time [s]')
ylabel('Crankshaft Angle [deg]')
title('Time vs. Crankshaft Angle for 500 (High) RPM')

figure(4)
plot(times_fly,allfly_angle)
xlabel('Time [s]')
ylabel('Crankshaft Angle [deg]')
title('Time vs. Crankshaft Angle for All Flywheels')

figure(5)
plot(times_fly,nofly_angle)
xlabel('Time [s]')
ylabel('Crankshaft Angle [deg]')
title('Time vs. Crankshaft Angle for No Flywheels')

%% 2 - torque

data2 = xlsread('No-Head Characterization - Remote.xlsx',2);

times2 = data2(:,1);
low_t = data2(:,3);
med_t = data2(:,6);
high_t = data2(:,9);
fly_times2 = data2(:,10);
allfly_t = data2(:,12);
nofly_t = data2(:,15);

figure(6)
plot(times2,low_t)
xlabel('Times [s]')
ylabel('Torque [Nm]')
title('Time vs. Torque for 200 (Low) RPM')

figure(7)
plot(times2,med_t)
xlabel('Times [s]')
ylabel('Torque [Nm]')
title('Time vs. Torque for 300 (Medium) RPM')
```

```
figure(8)
plot(times2,high_t)
xlabel('Times [s]')
ylabel('Torque [Nm]')
title('Time vs. Torque for 500 (High) RPM')

figure(9)
plot(fly_times2,allfly_t)
xlabel('Times [s]')
ylabel('Torque [Nm]')
title('Time vs. Torque for All Flywheels')

figure(10)
plot(fly_times2,nofly_t)
xlabel('Times [s]')
ylabel('Torque [Nm]')
title('Time vs. Torque for No Flywheels')

%% 3 - piston position

data3 = xlsread('No-Head Characterization - Remote.xlsx',3);

times3 = data3(:,1);
low_disp = data3(:,3);
med_disp = data3(:,6);
high_disp = data3(:,9);
fly_times3 = data3(:,10);
allfly_disp = data3(:,12);
nofly_disp = data3(:,15);

figure(11)
plot(times3,low_disp)
xlabel('Times [s]')
ylabel('Piston Displacement [mm]')
title('Time vs. Piston Displacement for 200 (Low) RPM')

figure(12)
plot(times3,med_disp)
xlabel('Times [s]')
ylabel('Piston Displacement [mm]')
title('Time vs. Piston Displacement for 300 (Medium) RPM')

figure(13)
plot(times3,high_disp)
xlabel('Times [s]')
ylabel('Piston Displacement [mm]')
title('Time vs. Piston Displacement for 500 (High) RPM')

figure(14)
plot(fly_times3,allfly_disp)
xlabel('Times [s]')
ylabel('Piston Displacement [mm]')
title('Time vs. Piston Displacement for All Flywheels')

figure(15)
plot(fly_times3,nofly_disp)
xlabel('Times [s]')
ylabel('Piston Displacement [mm]')
title('Time vs. Piston Displacement for No Flywheels')
```