Two-Phase Scalable Mixed-Integer Path Planning for UAVs

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1 Introduction

As a consequence of ever-increasing automation in our daily lives, more and more machines have to interact with and unpredictable environment and other actors within that environment. One of the sectors that seems like it will change dramatically in the near future is the transportation industry. Autonomous cars are actually starting to appear on public roads, autonomous truck convoys are being tested and large retail distributors like Amazon are investing heavily into delivering order by drones instead of courier. While these developments look promising, there are still many challenges that prevent these systems from being widely deployed.

One such challenge, which is especially important for areal vehicles, is path planning. Even though most modern quadrocopters are capable of flying by themselves, they are unable to generate a flight path that will get them to their destination reliably. Classic graph-based shortest path algorithms like Dijkstra's algorithm its many variants fail to take momentum and other factors into account. Mixed Integer Linear Programming (MILP) is one approach that shows promising results, however it is currently severely limited by computational complexity.

1.1 Motivation and Goal for the Thesis

One of the main advantages of using a constraint optimization approach like MILP is that they are extremely extendable by design. A system based on this can be deployed in many different scenarios with different goals and constraints without the need for significant changes to the algorithms that drive it. The solvers that construct the final path are general solvers which take constraints and a target function as input. This input can be generated by end users in the field to match their specific requirements, making the software controlling the drones as flexible as the hardware.

That flexibility is also the main limitation of using constraint optimization. The solvers are general purpose, which make them very slow compared to more direct approaches. They need to be carefully guided solve all but the most basic scenarios in a reasonable amount of time. While there have been some good results on small scales, I could not find any attempts at planning paths on the order of kilometers or more. Practical use cases involving drones often involve several minutes of flight and can cover several kilometers, so a path planner must be able to work at such a scale. This is the main goal of this thesis: To demonstrate how a MILP approach can be scaled to scenarios with a much larger scope, while preserving the advantages that make it interesting.

1.2 Structure of the Thesis

Section 1.3 summarizes the previous work that has been done in the field. The previous work in the field shows a common design to modeling the path planning problem as a MILP problem. This design forms the core of the approach in this thesis as well. Section 2 shows the implementation of this common design and explores the critical limitations to this approach.

Section 3 proposes a solution to these limitation. By finding a rough initial path, the planning problem can be split into smaller segments. Solving these segments on their own is significantly easier and can still enforce all the constraints. This approach is much faster than previous techniques, but at the cost of no longer finding the global optimum.

During the development of this algorithm, finding and solving bugs and other unwanted behavior proved to be a significant challenge. A visualization tool was developed to make it easier to see how the algorithm operates. Section 4 goes into detail of how nearly every variable in the MILP problem

EXAMPLE LINEAR PROBLEM HERE

was visualized and how this information can be interpreted.

To demonstrate the flexibility of the approach, section 5 showcases some possible extensions that can be added with relative ease. Some of these have been fully implemented to look at the impact on the solution. This section should demonstrate that the path planner discussed in this thesis is a modular strategy built out of several different algorithms. The specific algorithms discussed are just one way of doing things, and can be easily swapped out for other, more advanced, algorithms.

Section 6 analyzes the performance of the path planner in several different scenarios. It also looks at how the extensions which have been implemented affect the both the performance and quality of the planner. Finally, section 7 summarizes the main observations in this thesis and concludes whether or not the goals have been realized.

1.3 Context

Mixed integer linear programming is and extension of linear programming. In a linear programming problem, there is a single (linear) target function which needs to be minimized or maximized by the solver. A problem typically also contains a number linear inequalities which constrain the values of the variables in the target function. When all the variables are real, problems like this can be solved in polynomial time. However, only convex search spaces can be modeled this way. By adding obstacles to the world the drone has to navigate, the search space becomes non-convex. To express non-convex search spaces, integer variables are required[8]. A linear problem with integer variables is called a mixed integer linear problem. These integers make it possible to model logical expressions, which in turn enable approximations of non-linear function to be used.

fixed horizon vs receding horizon [1]

differential flatness [4] [6] [2] [7]

curve approaches [3] [5]

2 Modeling Path Planning as a MILP problem

3 Segmentation of the MILP problem

4 Visualization of the solution

5 Extensions

6 Analysis and Results

7 Summary and Conclusions

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