AEROSP-530 Final Project

Winter 2023

Jacob Dewey

1. Introduction:

A government Request for Proposal (RFP) has been released for contractor study and response. The project is associated with a unique weapons delivery system and its focus is on engine performance at the cruise conditions indicated below as well as other requirements.

- Net Thrust at Cruise: 4800 lbf
- Cruise Conditions:
  - Altitude: 30 kft
  - Mach Number: 0.8
  - Ambient Pressure: 4.36 psia
  - Ambient Temperature: -44.4 ºC = 411.75 ºR
- Compressor Pressure Ratio: 8.0
- Maximum Cycle Temperature: 2559.7 ºR
- Cruise Mission Length: 110 minutes
- Cruise Mission Fuel Burn: ≤ 10,451.2 lbm (1600 gal assuming 1 gal = 6.532 lbm)

2. Cycle Analysis Model and Results:

To analyze the configuration and determine the feasibility of the engine, a cycle model made in MATLAB (attached in appendix A) was employed. To create the model, a number of assumptions and simplifications were made. It is assumed that the cooling flow of the turbine is introduced downstream of it after the turbine work has been extracted from it, it is also assumed that it enters the flow with the same temperature that it exited the compressor with. It is also assumed no horsepower is extracted from the turbine and that there are no windage losses in the compressor or turbine. It is also assumed that there is no pressure drag on the engine.

2.1 Cycle Analysis Model:

In order to determine the size of the engine given the above constraints, a method of bisection is employed to determine the appropriate mass flow (and thus engine diameter). If the value of the thrust is too large, the current mass flow rate is set as the new maximum and if the thrust is too low, the specified mass flow rate is set as the new minimum. This new mass flow rate is then simulated through the engine cycle analysis function that finds the values at each of the stations as well as the net thrust at cruise conditions. The cycle analysis follows the following form (outlined in basic terms):

- Station 0: Inlet Plane
  - $P_{t,0} = P_a\left(1 + \left(\frac{\gamma-1}{2}\right)M^2\right)^{\frac{\gamma}{\gamma-1}}$
  - $T_{t,0} = T_a\left(1 + \left(\frac{\gamma-1}{2}\right)M^2\right)$

    - $\dot{m}_0 = \dot{m}_a$
        - where $\dot{m}_a$ is the mass flow rate from the bisection loop
- Station 1: Diffuser Exit
    - $P_{t,1} = P_{t,0} * \left(\frac{P_{t,1}}{P_{t,0}}\right)$
    - $T_{t,1} = T_{t,0}$
    - $\dot{m}_1 = \dot{m}_0$
    - $T_1 = T_{t,1}\left(1 + \left(\frac{\gamma-1}{2}\right)M_1^2\right)^{-1}$
        - M_1 is the Mach number in the diffuser predetermined with prior data
    - $P_1 = P_{t,1}\left(1 + \left(\frac{\gamma-1}{2}\right)M_1^2\right)^{-\gamma/(\gamma-1)}$
    - $u_1 = M_1\sqrt{\gamma_c * g_c * R * T_1}$
    - $\rho_1 = P_1 * 144\left(\frac{in^2}{ft^2}\right)/(R * T_1)$
    - $A_1 = \frac{\dot{m}_1}{\rho_1 * u_1}$
- Station 2: Fan Exit (configuration is a turbojet so properties are the same as station 1)
    - $P_{t,2} = P_{t,1}$
    - $T_{t,2} = T_{t,1}$
    - $\dot{m}_2 = \dot{m}_1$
- Station 3: Compressor Exit (Before Bleeds)
    - $P_{t,3} = P_{t,2} * CPR$
    - $T_{t,3} = T_{t,2} * \left(\left(\frac{1}{\eta_c}\right)CPR^{\frac{\gamma-1}{\gamma}-1} + 1\right)$
        - η_c is the efficiency factor for the compressor.
    - $\dot{m}_3 = \dot{m}_2$
- Station 3.1: Compressor Exit (After Bleeds)
    - $P_{t,3.1} = P_{t,3}$
    - $T_{t,3.1} = T_{t,3}$
    - $\dot{m}_{cool} = \dot{m}_3 * \left(\frac{\dot{m}_{cool}}{\dot{m}_3}\right)$
    - $\dot{m}_{3.1} = \dot{m}_3 * \left(1 - \frac{\dot{m}_{leak}}{\dot{m}_3} - \frac{\dot{m}_{customer}}{\dot{m}_3}\right) - \dot{m}_{cool}$
- Station 4: Combustor Exit
    - $P_{t,4} = P_{t,3.1} * \left(\frac{P_{t,4}}{P_{t,3.1}}\right)$
    - $T_{t,4} = T_{t,4}$
        - $T_{t,4}$ is determined beforehand by the combustor limits.
    - $\dot{m}_f = \left(\dot{m}_{3.1}C_{p,h}\left(T_{t,4} - T_{t,3.1}\right)\right)/(LHV * \eta_b)$
        - η_b is the burner efficiency.
    - $\dot{m}_4 = \dot{m}_{3.1} + \dot{m}_f$
- Station 4.9: High Pressure Turbine Exit
    - $\dot{m}_{4.9} = \dot{m}_4$
    - $T_{t,4.9} = T_{t,4} - \left(\dot{m}_2 C_{p,c}\right)\frac{T_{t,3} - T_{t,2}}{\dot{m}_4 C_{p,h}}$

        - Extracts the necessary work for the compressor.
    - $P_{t,4.9} = P_{t,4} * \left(1 - \frac{T_{t,4.9}/T_{t,4}}{\eta_t}\right)^{\frac{\gamma_h}{\gamma_h - 1}}$
        - η_t is the turbine efficiency.
- Station 4.95: Recombination of cooling mass flow and core flow
    - $\dot{m}_{4.95} = \dot{m}_{4.9} + \dot{m}_{cool}$
    - $P_{t,4.95} = P_{t,4.9}$
    - $T_{t,4.95} = \left(\dot{m}_{4.9} T_{t,4.9} C_{p,h} - \dot{m}_{cool} C_{p,c} T_{t,3}\right) / \left(\dot{m}_{4.95} C_{p,h}\right)$
- Station 9: Nozzle Exit
    - $\dot{m}_9 = \dot{m}_{4.95}$
    - $T_{t,9} = T_{t,4.95}$
    - $P_9 = P_{amb}$
    - $T_{t,9i} = T_{t,9} \left(\frac{P_9}{P_{t,9}}\right)^{\frac{\gamma - 1}{\gamma}}$
    - $T_9 = T_{t,5} - C_v^2 \left(T_{t,5} - T_{t,9i}\right)$
    - $u_9 = \sqrt{2 * C_{p,h} * g_c * J * \left(T_{t,5} - T_9\right)}$
    - $M_9 = u_9 / \sqrt{\gamma_h * g_c * R * T_9}$
    - $P_{t,9} = P_9 \left(1 + \left(\frac{\gamma - 1}{2}\right) M_9^2\right)^{\frac{\gamma}{\gamma - 1}}$
- Results:
    - $D_1 = 24 \sqrt{\frac{A_1}{\pi}}$
    - $u_0 = M_\infty \sqrt{\gamma * g_c * R * T_0}$
    - $Thrust = \frac{1}{g_c} \left(\dot{m}_9 * u_9 - \dot{m}_0 u_0\right)$

2.2 Model Results:

Table 1: Total Temperature, Total Pressure, and Mass Flow Rate at Stations 0-9

Station	Total Temperature (R)	Total Pressure (psia)	Mass Flow Rate (lbm/s)
0	464.45	6.65	69.88
1	464.45	6.58	69.88
2	464.45	6.58	69.88
3	897.65	52.64	69.88
3.1	897.65	52.64	62.89
4	2559.67	50.01	64.44
4.9	2148.69	23.22	64.44
4.95	2065.41	23.22	68.64
9	2065.41	21.66	68.64

Cycle Analysis Results

- Thrust = 4800 lbf
- Thrust/$\dot{m}_2$: 68.69 lbf-s/lbm
- Total Fuel Burn at Cruise: 10,252.83 lbm
- Max diameter: 29.52 inches
- Nozzle Exit Velocity: 3060.23 ft/s

These results show the engine meets the desired performance metrics of thrust, size, fuel burn during cruise, and exhaust gas temperature. The next several sections will cover the detailed flow path design given the information of the cycle analysis. Some of the values may be slightly different than the full analysis results shown above but the results should be within a percentage point in accuracy.

3. Inlet Flowpath Design:

Given the static cruise conditions of the vehicle (shown in Sec. 1), we can size the inlet in order to "scoop up" the correct amount of air. This is done by finding the area, A_0 required for 70.17 lbm/s to enter the inlet. Using the area and a given Mass Flow Ratio (MFR), we can find the diameter of the leading edge of the inlet (otherwise known as the highlight). Seen below:

$$A_0 = \frac{\dot{m}_0 * R * T_{t,0}}{144 * P_{t,0} * V_0} = 3.084\ ft^2$$

$$D_0 = \sqrt{\frac{4A_0}{\pi}} = 23.78\ in$$

$$\frac{A_0}{A_{HL}} = 0.7 => A_{HL} = \frac{A_0}{0.7} = 4.406\ ft^2$$

$$D_{HL} = \sqrt{\frac{4A_{HL}}{\pi}} = 28.42\ in$$

Given a throat Mach number of 0.7 and the diameter of the stream tube necessary for the required amount of air, we can find the diameter of the throat required using the following:

$$\frac{A_0}{A_t} = \frac{A_0}{A^*}\frac{A^*}{A_t} = \sqrt{\frac{\frac{1}{M_0^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_0^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}}{\frac{1}{M_t^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_t^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}}} = 0.9487$$

$$\frac{A_0}{A_t} = \frac{D_0^2}{D_t^2} => D_t = \frac{D_0}{\sqrt{0.9487}} = 24.41\ in$$

The design point of the engine calls for a Mach number of 0.4 exiting the inlet. In order to achieve this, we need to know what the required area is to slow the flow down from the throat value of 0.7 to 0.4. Reformatting the equation above, we can write the ratio of the throat to the area exiting the inlet as follows:

$$\frac{A_2}{A_t} = \frac{A_2}{A^*}\frac{A^*}{A_t} = \sqrt{\frac{\frac{1}{M_2^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_2^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}}{\frac{1}{M_t^2}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_t^2\right)\right)^{\frac{\gamma+1}{\gamma-1}}}} = 1.453$$

$$\frac{A_2}{A_t} = \frac{D_2^2}{D_t^2} => D_2 = 1.453 * D_t = 29.43\ in$$

We can then find the length of the inlet necessary for this area change by using the given angle of 5 degrees and the following equation:

$$L = \frac{D_2 - D_t}{2\tan 5°} = 28.69\ in$$

Using the above information, diagrams of the stream tube in front of the inlet and the inlet itself can be drawn. These are seen in Figures 1 and 2 respectively.

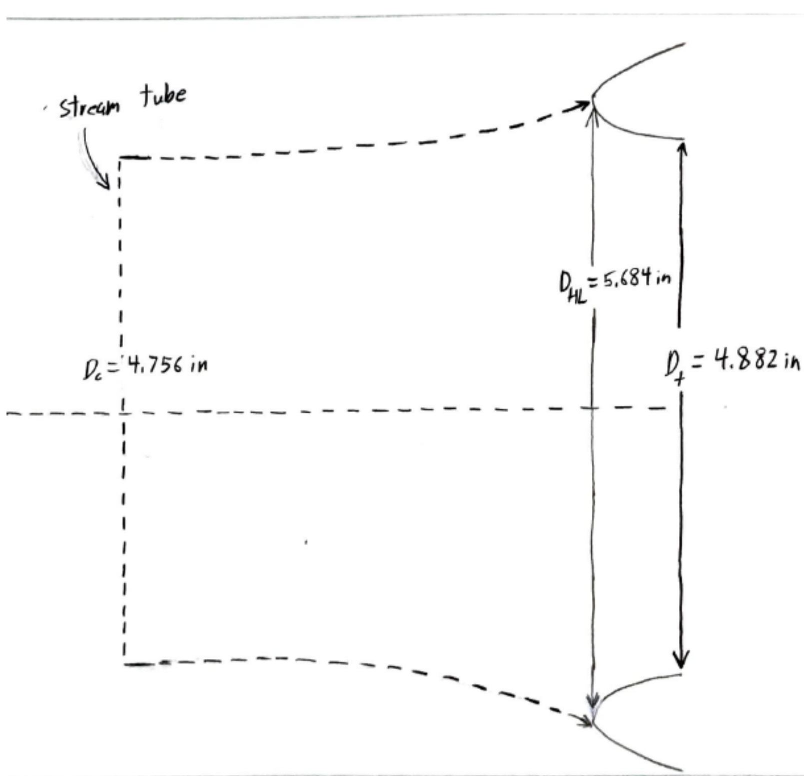


Figure 1: Stream Tube Diagram (1/5 Scale)

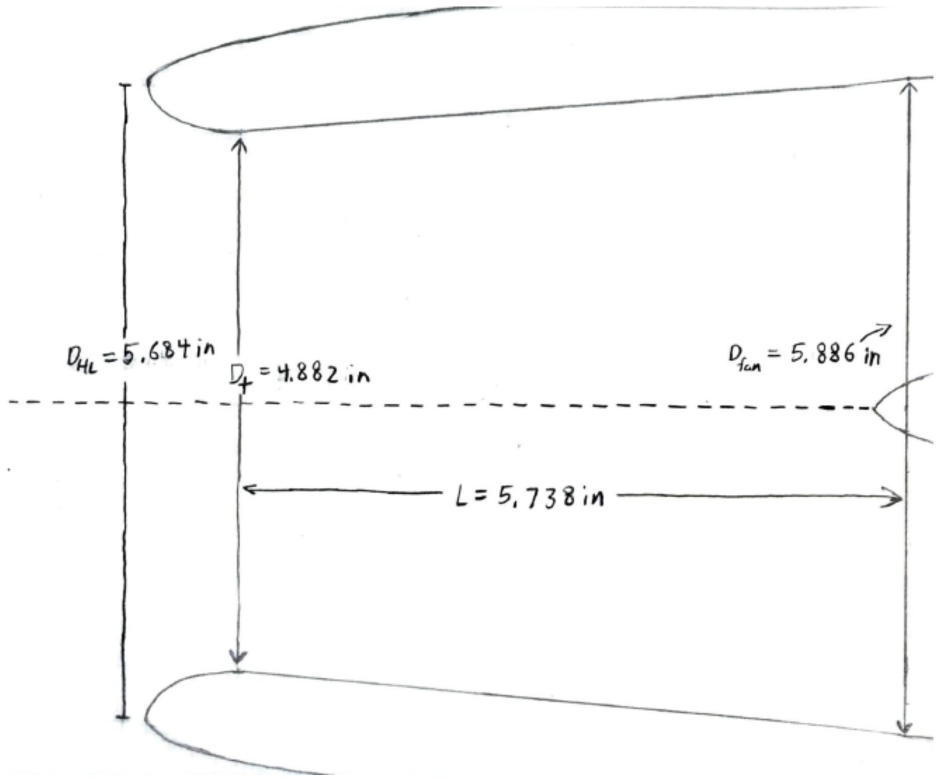


Figure 2: Inlet Diagram (1/5 Scale)

4. Compressor Flowpath Design:

Given the conditions of the air flowing out of the inlet and into the compressor, we can begin to calculate the initial velocity of the air flowing into the compressor. Since the3 initial M_2 value isn't known, an iterative approach must be taken to find a Mach number that is consistent with the mass flow rate and area of the compressor inlet. These values can be seen below:

$$M_2 = 0.5023;\ c_2 = 517.75\frac{ft}{s}$$

The same can be done with the compressor exit at station 3.1, this time to solve for the area as it is the unknown rather than the Mach number. This results in an $A_{3.1} = 0.9547\ ft^2$ and a hub diameter $D_h = 26.46$ in. Using the average of the tip (constant between stations) and hub diameters, we find the average pitchline diameter to be:

$$D_{p,avg} = \frac{1}{4}\left(2 * D_{tip} - D_{hub,3.1} - D_{hub,2}\right) = 26.46\ in$$

Assuming the compressor is operating at its maximum tip speed of 1500 ft/s, we find the resulting RPM of the compressor:

$$\omega = \frac{2 * V_{tip,max}}{D_{tip}} = 1217\frac{rad}{s} = 11{,}621\ RPM$$

Using the work equation (shown below) and taking U to be the tangential velocity of the pitchline, J and g_c to be conversion factors, and ϕ to be the work coefficient, we get the following:

$$\Delta h_{stg} = \frac{\phi * U^2}{2 * J * g_c} = 18.288\frac{BTU}{lbm}$$

With a total Δh value of 103.97 BTU/lbm needed, we find that a total of 6 stages are needed to achieve the desired compression. Given an aspect ratio of 2.5 for the rotor and stator blades, we find the average blade width to be equal to:

$$w_{blade,avg} = \frac{h_{avg}}{AR} = 2.0872\ in$$

Resulting in a compressor length of L = 30.786 in. The dimensions can be seen in Fig. 3 which is an axial diagram of the flowpath envelope of the compressor.

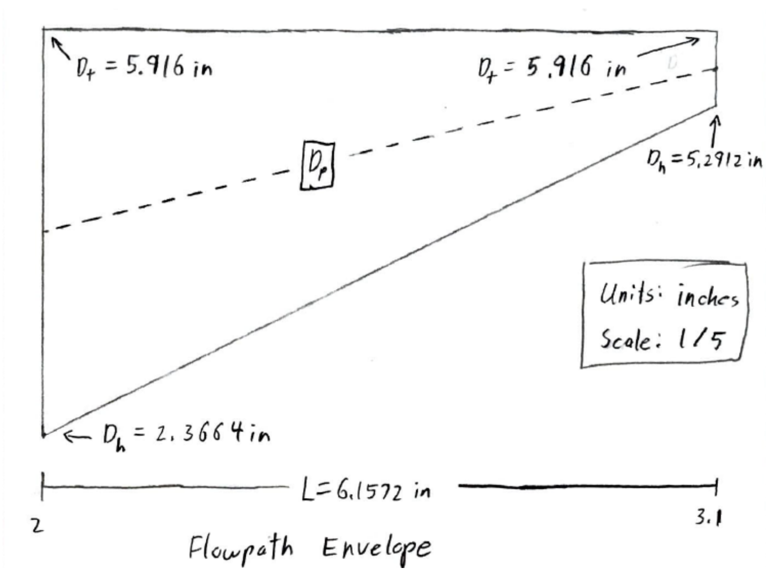


Figure 3: Compressor Flowpath Diagram (1/5 Scale)

Taking the values from above for RPM, c_2, and the flow properties, a set of velocity vector diagrams for the first stage of the compressor can be calculated. Since the pitchline diameter changes as you move further through the compressor, multiple pitchline diameters are used. These pitchline diameters result in multiple tangential velocities at the pitchline seen below:

$$U_1 = \frac{\omega * D_{p,1}}{2} = 1049.94\frac{ft}{s}$$

$$U_2 = \frac{\omega * D_{p,2}}{2} = 1083.40\frac{ft}{s}$$

Assuming that the flow entering the compressor has no swirl component to it ($\alpha_1 = 0°$), we the relative velocity of station 1 of the first stage of the compressor is as follows:

$$w_{z,1} = c_{z,1};\ w_{\theta,1} = -U_1$$

With the values for the first station found, we can move to station 2. Since station 1 has no tangential component in its velocity, the Δh for the stage can be rearranged as follows:

$$c_{\theta,2} = \frac{\Delta h}{U_2}$$

Using a bisection solver using the mass flow rate and area, we can then find $c_{z,2}$ before finding the relative velocity vector at station 2 where:

$$w_{z,2} = c_{z,2};\ w_{\theta,1} = c_{\theta,2} - U_1$$

Then, using the change in pressure (combined with the losses we should expect) that we should measure, we can determine the accuracy of the method. If the value is off (say that it's short of the expected value), simply increase the pressure commensurate with the difference between the target pressure and the measured pressure and follow back through the method. This results in the following values:

$$c_{z,1} = 517.93\frac{ft}{s};\ c_{\theta,1} = 0\frac{ft}{s}$$

$$w_{z,1} = 517.93\frac{ft}{s};\ w_{\theta,1} = -1049.94\frac{ft}{s}$$

$$c_{z,2} = 427.87\frac{ft}{s};\ c_{\theta,2} = 318.66\frac{ft}{s}$$

$$w_{z,2} = 427.87\frac{ft}{s};\ w_{\theta,2} = -764.74\ \frac{ft}{s}$$

$$\alpha_2 = 36.68°;\ \beta_2 = -60.77°$$

$$|c_2| = 533.50\frac{ft}{s};\ |w_2| = 767.74\frac{ft}{s}$$

$$P_{t,3} = 9.3053\ psia$$

Our predicted $P_{t,3}$ value is now within 0.003% of the value that it should be, therefore, our solution is close enough. The final compressor stage efficiency can be found as follows:

$$\eta_c = \frac{\left(\frac{P_{t,3}}{P_{t,2}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{T_{t,3}}{T_{t,1}}\right) - 1} = 0.84124$$

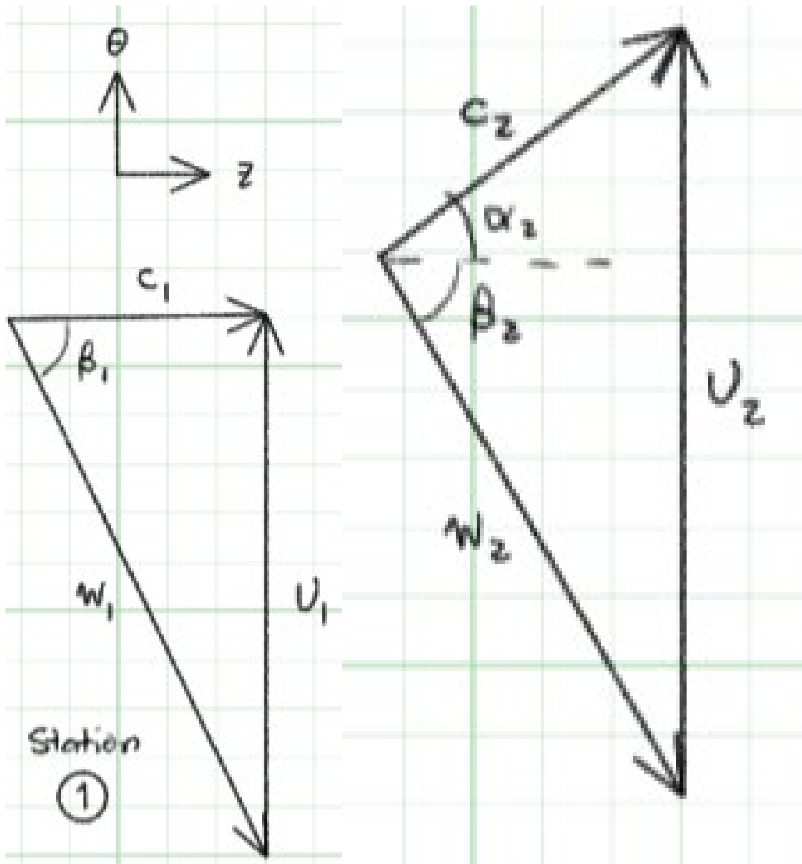


Figure 4: First Stage Compressor Vector Diagrams

5. Combustor Flowpath Design:

For the combustor, an annular design is used due to its advantages in providing more uniform combustion, being shorter, and having less surface area. It also has low pressure losses. To determine the flowpath dimensions seen in Fig. 5, the following steps are taken.

First, the total density of the flow leaving the compressor is found using the ideal gas law with the total temperature and pressure exiting the compressor. Using $\rho_{t,3.1}$, a reference velocity (initially 90 ft/s), and the mass flow rate entering the combustor, the reference area can be found. This value is then divided by pi and the pitchline diameter to find the reference height of the combustor which is then added to the pitchline diameter to find an estimate of the inner and outer casing diameters ensuring they fall within the established limits of a minimum of 13 inches and a maximum of 42 inches respectively. We then assume that the mass flow is divided evenly between the inner casing, the outer casing, and the dome resulting in a value of 20.953 lbm/s for each of the 3 paths. Setting an initial passage velocity of 150 ft/s, the passage area can be found using the following equation:

$$A_{pass} = \frac{\dot{m}_{pass}}{\rho_{t,3.1} V_{pass}}$$

This area is equal for both the inner and outer casings. Using the following equations, we can estimate the diameter of the inner and outer linings using the following equations respectively:

$$D_{il} = \sqrt{\frac{4A_{pass}}{\pi} + D_{ic}^2}$$

$$D_{ol} = \sqrt{D_{oc}^2 - \frac{4A_{pass}}{\pi}}$$

The dome height can then be found using the following:

$$H_D = \frac{1}{2}(D_{ol} - D_{il})$$

Combined with the pitchline diameter, we can find the dome area before using the continuity equation to calculate the dome velocity. Checking that it is below the limit of 80 ft/s, we can now find the length of the combustor using our desired L/D of 2.25. Assuming that the combustor is split into 2 halves, one that is perfectly cylindrical and the other that has a diameter that tapers until the turbine inlet tip and hub diameters are reached, the volume can be calculated using the following equation:

$$Vol = 0.5 * \pi * D_p * L_c \left(\frac{3H_D + H_T}{2}\right)$$

Combining the calculated volume with the pressure exiting the compressor (in units of atm), the lower heating value, and the mass flow rate of the fuel, we can find the space rate using the following equation:

$$SR = \frac{3600 * \dot{m}_{fuel} * LHV}{Vol * P_{t,3.1}}$$

On our initial pass, we find that the space rate (12,960,000 BTU/(hr-atm-ft^3)) is outside of the desired range of (8-10) * 10^6. Given that extending the length to the limit of 2.5 times the dome height isn't sufficient to bring the space rate to within the desired range, we lower the dome velocity to 40 ft/s (original value was 51.074 ft/s) and recalculate the previous values (documented below).

$$A_D = \frac{V_{D,old}}{V_{D,new}} A_{D,old} = 476.27\ in^2$$

$$H_D = \frac{A_D}{\pi D_P} = 5.4105\ in$$

$$L_c = 2.25 H_D = 12.174\ in$$

$$V = 2.9179\ ft^3$$

$$SR_{new} = \frac{V_{old}}{V_{new}} SR_{old} = 9{,}899{,}500\ \frac{BTU}{hr - atm - ft^3}$$

With the new space rate within bounds, the values for the passages can be determined (the areas are unchanged, but the diameters and heights change). The calculations for these can be found below:

$$D_{ol} = D_p + H_D = 33.431\ in$$

$$D_{il} = D_P - H_D = 22.610\ in$$

$$D_{il} = \sqrt{\frac{4A_{pass}}{\pi} + D_{ic}^2} = 18.894\ in$$

$$D_{ol} = \sqrt{D_{oc}^2 - \frac{4A_{pass}}{\pi}} = 35.767\ in$$

We can then find the number of fuel injectors by dividing the pitchline circumference by the dome height and rounding up as needed.

$$Num.\,Injectors = \frac{\pi D_p}{H_D} = 16.27$$

Thus, the combustor requires 17 injectors space around the pitchline of the combustor. Fig. 5 below shows values above displayed in diagram form.

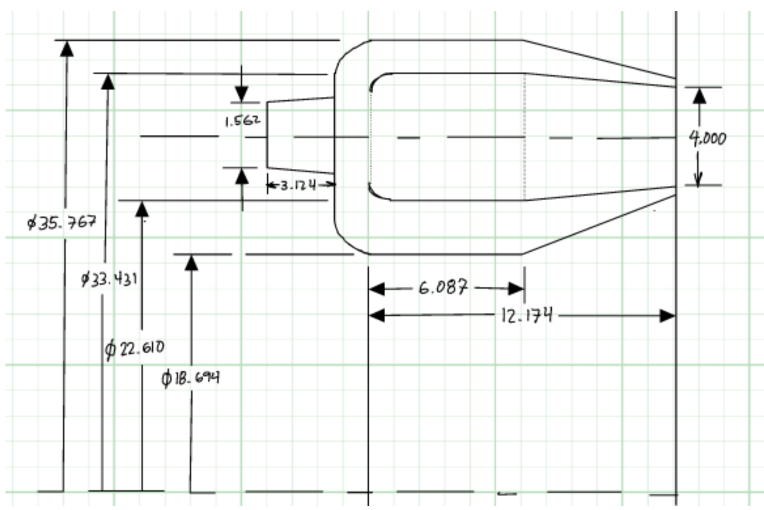


Figure 5: Combustor Flowpath Diagram

6. Turbine Flowpath Design:

Using the wheel speed limit of 1650 ft/s at the tip and an angular velocity of 1216.74 rad/s from the 11,619 rpm design point, the diameter for the tip of the rotor can be found using the equation:

$$D_t = \frac{2v_t}{\omega}$$

Plugging in our values for tip speed and angular velocity, we find the tip diameter at the turbine inlet (and thus the entirety of the single stage turbine) to be 32.546 in which is approximately the design point of the turbine at 32.55 in. With the design hub diameter of 25.76 in, we can find the pitchline diameter by averaging the two resulting in a value of:

$$D_p = 29.153\ in$$

Combining this value with the angular velocity gives a rotor speed U_p = 1477.98 ft/s at the pitchline. Using the following equation, we can find the pitchline loading of the turbine:

$$\psi_p = \frac{g_c J \Delta h}{2U_p^2}$$

Where Δh is the required work from the turbine, and g_c and J are conversion factors to ensure the unit's match. After plugging in the relevant values, we find the pitchline loading $\psi_p = 0.6459$, which is below the maximum value of 0.65 that prior data indicates cannot be exceeded to meet the desired turbine efficiency. Fig. 6a shows a diagram of the flowpath with the indicated hub and tip diameters above as well as the axial widths of the rotor and stator blades.

With the geometry verified, a set of vector diagrams that satisfy the requirements and the degree of reaction for the stage will be determined. Starting with the pitchline velocity and the pitchline loading determined above, the value for $c_{\theta,2}$ can be found using the following:

$$c_{\theta,2} = 2\psi_p U_p$$

This results in $c_{\theta,2} = 1909.25\ ft/s$. Subtracting U_p from this value give us the relative speed of $w_{\theta,2} = 431.40\ ft/s$. Given the design point of $\alpha_3 = 0, i.e.\ c_{\theta,3} = 0\ ft/s$ (so that there is no swirl in the flow downstream of the turbine), we find that $w_{\theta,3} = U_p = -1477.98\ ft/s$. By using our knowledge that the area of the turbine is a constant and the fact that the mass flow rate through the turbine is also a constant, we can solve for the axial velocities and Mach numbers by using the known mass flow rate combined with the tangential velocities. By iterating through Mach numbers using a method of bisection, we find that the velocities for Fig. 6b are as follows:

$$c_{\theta,2} = 1909.38\frac{ft}{s}, c_{z,2} = 842.53\frac{ft}{s}$$

$$w_{\theta,2} = 431.40\frac{ft}{s}, w_{z,2} = 842.53\frac{ft}{s}$$

$$w_{\theta,3} = 1136.30\frac{ft}{s}, w_{z,3} = 1477.98\frac{ft}{s}$$

We can then find the relevant angles using the following:

$$\alpha_2 = \tan^{-1}\frac{c_{\theta,2}}{c_{z,2}} = +66.19°$$

$$\beta_2 = \tan^{-1}\frac{w_{\theta,2}}{w_{z,2}} = +27.14°$$

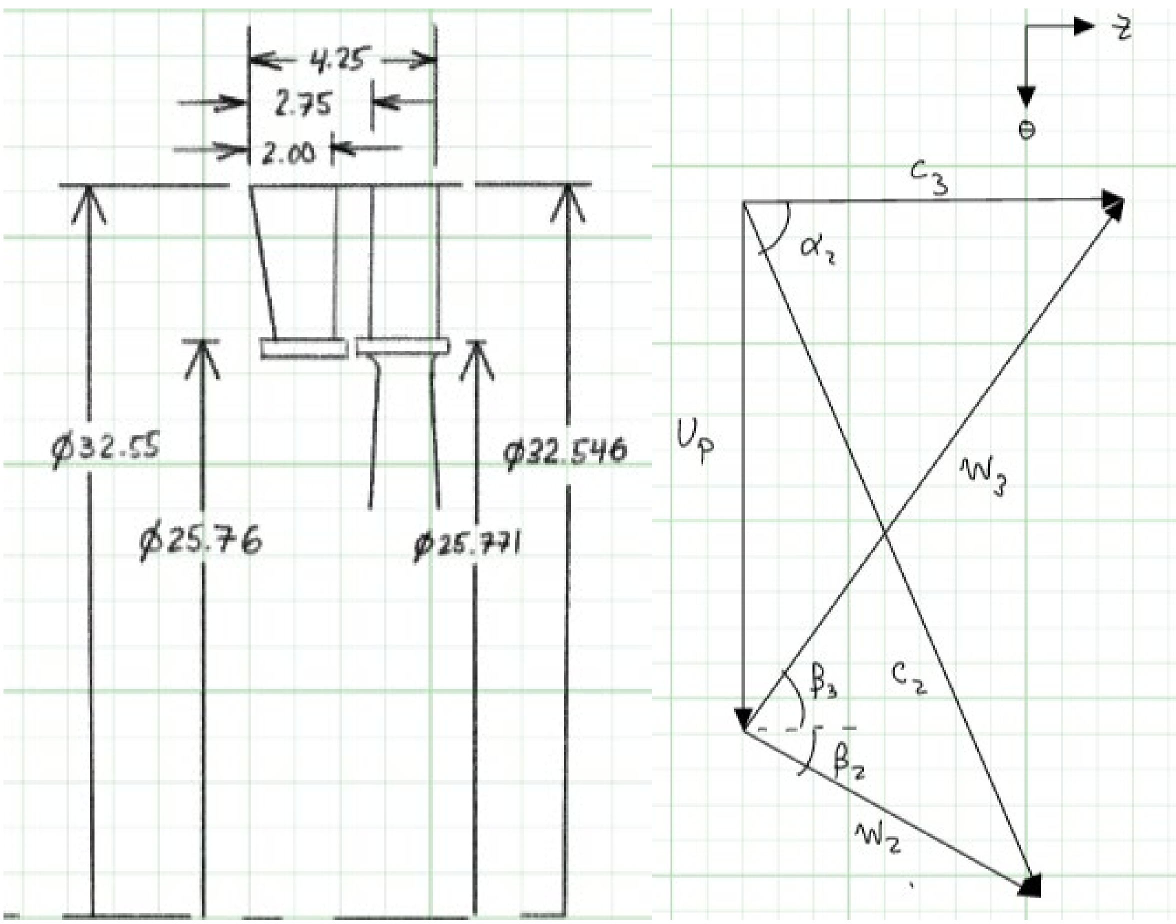
$$\beta_3 = \tan^{-1}\frac{w_{\theta,3}}{w_{z,3}} = -57.26°$$

The pressure and temperature values for stations 1-3 can be seen in Table 2. Using these values, we can find the degree of reaction using the following:

$$°R = \frac{T_{s,2} - T_{s,3}}{T_s - T_{s,3}} = 0.35235$$

Table 2: Pressure and Temperature values across Turbine Stage

Station	T_s (Rankine)	T_t (Rankine)	P_s (psia)	P_t (psia)
1	2535.21	2560	48.130	50.04
2	2242.85	2560	29.482	50.04
3	2046.52	2149	19.106	23.23



a) Turbine Flowpath Diagram **b)** Turbine Vector Diagram

Figure 6: Turbine Engineering Diagrams

7. Nozzle Flowpath Design Parameters:

Given that $M_{4.95} = 0.548145$ and that $A_{4.95} = 2.1531$ ft^2, we can find the required throat area to reach sonic conditions using the following equation for the Mach-Area relation:

$$\frac{A}{A^*} = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M}$$

By rearranging the equation and plugging in the aforementioned values for $M_{4.95}$ and $A_{4.95}$, we find the throat area, $A^* = 1.7062$ ft^2. Using a nozzle efficiency value of 0.96629, we can use the pressure ratio between P_9 (which is matched to the ambient pressure outside of the nozzle) and $P_{t,4.95}$ to calculate the static temperature at the exit, T_9. This temperature is then used to find the flow velocity and Mach number at station 9. With the Mach number and the throat area known, the above equation can then be used to find the exit area. This results in a value of 2.3513 ft^2.

Appendix A: Cycle Analysis MATLAB Code

```
%% Problem Constants:

% Flight Conditions and Fluid Properties

amb.Alt = 30e3; % Altitude [ft]

amb.tBurn = 110*60; % Time at Cruise [s];

amb.fuelDens = 6.532; % Fuel Density [lbm/gal]

amb.Ma = 0.8; % Freestream Mach Number

amb.M1 = 0.4; % Mach Number at Inlet Exit

amb.Pa = 4.36; % Ambient Pressure [psia]

amb.Ta = convtemp(-44.4,'C','R'); % Ambient Temperature [Rankine]

amb.gamc = 1.4; % Specific Heat Ratio (cold)

amb.gamh = 1.3333; % Specific Heat Ratio (hot)

amb.R = 53.35; % Gas Constant [(ft-lbf)/(lbm-R)]

amb.J = 778; % Energy Constant [(ft-lbf)/Btu]

amb.gc = 32.174; % Gravitational Constant [(ft-lbm)/(lbf-s^2)]

amb.LHV = 18550; % Fuel Lower Heating Value [Btu/(lbm-fuel)]

amb.Fsg = 0.8165; % Fuel Specific Gravity

amb.Cpc = amb.R/(1-1/amb.gamc)/amb.J; % Specific Heat (cold) [(ft-lbf)/(lbm-R)]
```

```
amb.Cph = amb.R/(1-1/amb.gamh)/amb.J; % Specific Heat (hot) [(ft-lbf)/(lbm-R)]

% Component Information:
comp.nr = 0.99; % Inlet Recovery (Pt1/Pt0)
comp.ncf = 0.88; % Fan Efficiency
comp.nc = 0.87; % Compressor Efficiency
comp.mlm2 = 0.01; % Compressor Leakage Flow (m_leak/m_2)
comp.mcm2 = 0.06; % Turbine Cooling Flow (m_cool/m_2)
comp.mbm2 = 0.03; % Customer Bleed Flow (m_bleed/m_2)
comp.Pt4Pt3 = 0.95; % Combustor Pressure Drop (Pt4/Pt3.1)
comp.nb = 0.995; % Combustor Efficiency
comp.nt = 0.92; % Core Turbine Efficiency
comp.ntf = 0.925; % Fan Turbine Efficiency
comp.Cv = 0.983; % Core Exhaust Velocity Coefficient
comp.Cvf = 0.985; % Fan Exhaust Velocity Coefficient

% Limitations and Other:
lim.maxTt3 = convtemp(450,'F','R'); % Max Compressor Discharge Temperature [R]
lim.maxTt495 = convtemp(890,'C','R'); % Max Exhaust Gas Temperature (EGT) [R]
lim.maxTt9 = convtemp(880,'C','R'); % Max Core Exhaust Nozzle Gas Temperature [R]
lim.maxD1 = 30; % Maximum Internal Flowpath Diameter [in]
lim.maxFuelVol = 1600; % Maximum Fuel [gal]
lim.maxFuelm = amb.fuelDens*lim.maxFuelVol;

%% Cycle Analysis:
var.BPR = 0; % Bypass Ratio (m3/m2)
var.FPR = 1; % Fan Pressure Ratio (Pt13/Pt1 and Pt2/Pt1)
var.CPR = 8; % Compressor Pressure Ratio (Pt3/Pt2)
var.Tt4 = convtemp(2100,'F','R'); % Combustor Exit Temperature [Rankine]
var.target = 4800;
```

```
var.text = "Design: Configuration 1";
[Config1,var] = TurbojetConfig(amb,comp,var);
disp(' ')
dispSolution(Config1,var.BPR,var.text);


%% Functions:
% Turbofan/Turbojet Cycle Analysis
function [out] = TurbofanSim(amb,comp,var)
% Station 0 (Inlet Plane):
out.Pt0 = amb.Pa*(1 + (amb.gamc-1)*amb.Ma^2/2)^(amb.gamc/(amb.gamc-1)); % Total
Pressure at Station 0
out.Tt0 = amb.Ta*(1 + (amb.gamc-1)*amb.Ma^2/2); % Total Temperature at Station 0
out.m0 = var.mdota; % Mass Flow Rate at Station 0

% Station 1 (Diffuser Exit):
out.Pt1 = out.Pt0*comp.nr;
out.Tt1 = out.Tt0;
out.m1 = out.m0;
out.Ts1 = out.Tt1/(1 + (amb.gamc - 1)*amb.M1^2/2);
out.Ps1 = out.Pt1/(1 + (amb.gamc - 1)*amb.M1^2/2)^(amb.gamc/(amb.gamc-1));
out.a1 = sqrt(amb.gamc*amb.gc*amb.R*out.Ts1);
out.u1 = amb.M1*out.a1;
out.rhos1 = out.Ps1*144/(amb.R*out.Ts1);
out.A1 = out.m1/(out.rhos1*out.u1);

% Station 2 (Fan Exit):
out.Pt2 = out.Pt1*var.FPR;
out.Tt2 = out.Tt1*((1/comp.ncf)*(var.FPR^((amb.gamc-1)/amb.gamc) - 1) + 1);
out.m2 = out.m1/(var.BPR + 1);
```

```
% Station 3 (Compressor Exit Before Bleeds):
out.Pt3 = out.Pt2*var.CPR;
out.Tt3 = out.Tt2*((1/comp.nc)*(var.CPR^((amb.gamc-1)/amb.gamc) - 1) + 1);
out.m3 = out.m2;

% Station 3.1 (Compressor Exit After Bleeds):
out.Pt31 = out.Pt3;
out.Tt31 = out.Tt3;
out.mcool = out.m2*comp.mcm2;
out.m31 = out.m3*(1 - comp.mbm2 - comp.mlm2) - out.mcool;

% Station 4 (Combustor Exit)
out.Pt4 = out.Pt3*comp.Pt4Pt3;
out.Tt4 = var.Tt4;
out.mf = (out.m31*amb.Cph*(out.Tt4-out.Tt31))/(amb.LHV*comp.nb); %-amb.Cph*out.Tt4);
out.m4 = out.m31 + out.mf;

% Station 4.9 (High-Pressure Turbine Exit):
out.m49 = out.m4;
out.Tt49 = out.Tt4 - out.m2*amb.Cpc*(out.Tt3 - out.Tt2)/(out.m4*amb.Cph);
% out.Tt49 = out.Tt4 + (out.Tt49i - out.Tt4)*comp.nt;
out.Pt49 = out.Pt4*(1 - (1 - out.Tt49/out.Tt4)/comp.nt)^(amb.gamh/(amb.gamh - 1));

% Station 4.95 (Low Pressure Turbine Entrance):
out.m495 = out.m49 + out.mcool;
out.Tt495 = (out.m49*amb.Cph*out.Tt49 + out.mcool*amb.Cpc*out.Tt3)/(out.m495*amb.Cph);
out.Pt495 = out.Pt49;

% Station 5 (Low Pressure Turbine Exit):
```

```
out.m5 = out.m495;
out.Tt5 = out.Tt495 - (out.m1*amb.Cpc*(out.Tt2-out.Tt1))/(out.m5*amb.Cph);
% out.Tt5 = out.Tt495 + (out.Tt5i - out.Tt495)*comp.ntf;
out.Pt5 = out.Pt495*(1 - (1 - out.Tt5/out.Tt495)/comp.ntf)^(amb.gamh/(amb.gamh - 1));

% Station 9; (Nozzle Exit):
out.m9 = out.m5;
out.Tt9 = out.Tt5;
out.P9 = amb.Pa;
out.T9i = out.Tt9*(out.P9/out.Pt5)^((amb.gamh-1)/amb.gamh);
out.T9 = out.Tt5 - comp.Cv^2*(out.Tt5-out.T9i);
out.u9 = sqrt(2*amb.Cph*amb.gc*amb.J*(out.Tt5 - out.T9));
out.M9 = out.u9/sqrt(amb.gamh*amb.gc*amb.R*out.T9);
out.Pt9 = out.P9*(1 + 0.5*(amb.gamh-1)*out.M9^2)^(amb.gamh/(amb.gamh-1));

% Station 13 (Bypass Fan Exit):
out.Pt13 = out.Pt1*var.FPR;
out.Tt13i = out.Tt1*(var.FPR^((amb.gamc-1)/amb.gamc));
out.Tt13 = out.Tt1 + (out.Tt13i - out.Tt1)/comp.ncf;
out.m13 = var.BPR*out.m1/(var.BPR + 1);

% Station 19 (Fan Nozzle Exit):
out.m19 = out.m13;
out.Tt19 = out.Tt13;
out.P19 = amb.Pa;
out.T19i = out.Tt19*(out.P19/out.Pt13)^((amb.gamc-1)/amb.gamc);
out.T19 = out.Tt13 - comp.Cvf^2*(out.Tt13-out.T19i);
out.u19 = sqrt(2*amb.Cpc*amb.gc*amb.J*(out.Tt13 - out.T19));
out.M19 = out.u19/sqrt(amb.gamc*amb.gc*amb.R*out.T19);
out.Pt19 = out.P19*(1 + 0.5*(amb.gamc-1)*out.M19^2)^(amb.gamc/(amb.gamc-1));
```

```
% Processing:
out.D1 = 2*12*sqrt(out.A1/pi);
out.u0 = amb.Ma*sqrt(amb.R*amb.gamc*amb.gc*amb.Ta);
out.thrust = (out.m9*out.u9 + out.m19*out.u19 - out.m0*out.u0)/amb.gc;
out.specThrust = out.thrust/out.m2;
out.fuelBurned = out.mf*amb.tBurn;
end

% Finds the Mass Flow That Satisfies the Target Thrust Value
function [out,var] = TurbojetConfig(amb,comp,var)
mdotvec = [0,200];
margin = 1e-2;
out.thrust = 0;
while abs(out.thrust-var.target) > margin
   var.mdot2 = mean(mdotvec);
   var.mdota = var.mdot2*(var.BPR+1);
   [out] = TurbofanSim(amb,comp,var);

   if out.thrust > var.target+margin
      mdotvec(2) = var.mdot2;
   elseif out.thrust < var.target - margin
      mdotvec(1) = var.mdot2;
   end
end
end

% Print Outputs:
function dispSolution(out,BPR,text)
disp('-----------------------------------')
```

```
disp(text)
disp('-----------------------------------')
disp('          Tt      pt       mdot')
disp('Station    (R)    (psia)   (lbm/s)')
disp('_____________________________________')
fprintf('0       %4.2f    %4.2f     %4.2f \n',out.Tt0,out.Pt0,out.m0)
fprintf('1       %4.2f    %4.2f     %4.2f \n',out.Tt1,out.Pt1,out.m1)
fprintf('2       %4.2f    %4.2f     %4.2f \n',out.Tt2,out.Pt2,out.m2)
fprintf('3       %4.2f    %4.2f     %4.2f \n',out.Tt3,out.Pt3,out.m3)
fprintf('3.1      %4.2f    %4.2f     %4.2f \n',out.Tt31,out.Pt31,out.m31)
fprintf('4       %4.2f    %4.2f     %4.2f \n',out.Tt4,out.Pt4,out.m4)
fprintf('4.9      %4.2f    %4.2f     %4.2f \n',out.Tt49,out.Pt49,out.m49)
fprintf('4.95     %4.2f    %4.2f     %4.2f \n',out.Tt495,out.Pt495,out.m495)
if BPR > 0
   fprintf('5       %4.2f    %4.2f     %4.2f \n',out.Tt5,out.Pt5,out.m5)
end
fprintf('9       %4.2f    %4.2f     %4.2f \n',out.Tt9,out.Pt9,out.m9)

if BPR > 0
   fprintf('13      %4.2f    %4.2f     %4.2f \n',out.Tt13,out.Pt13,out.m13)
   fprintf('19      %4.2f    %4.2f     %4.2f \n',out.Tt19,out.Pt19,out.m19)
end
disp(' ')
fprintf('u9 =  %1.2f ft/s\n',out.u9)
if BPR > 0
fprintf('u19 = %1.2f ft/s\n',out.u19)
end
fprintf('Thrust: %1.2f lbf \n',out.thrust)
fprintf('Specific Thrust: %1.2f lbf-s/lbm \n',out.specThrust)
fprintf('Fuel Burned: %1.2f lbm \n',out.fuelBurned)
```

```
fprintf('Max Diameter: %1.2f in \n',out.D1)
fprintf('Tt3:   %1.2f R \n',out.Tt3)
fprintf('Tt495: %1.2f R \n',out.Tt495)
fprintf('Tt9:   %1.2f R \n',out.Tt9)
end
```