Interaction between Model and Its Evolution Control in Surrogate-Assisted CMA Evolution Strategy (Supplementary Material)

Zbyněk Pitra
Czech Technical University, Faculty of Nuclear Sciences and Physical Engin.
Břehová 7, Prague, Czech Republic
z.pitra@gmail.com

Marek Hanuš
Czech Technical University, Faculty of Information Technology
Thákurova 9, Prague, Czech Republic
hanusm12@fit.cvut.cz

Jan Koza
Czech Technical University, Faculty of Information Technology
Thákurova 9, Prague, Czech Republic
kozajan@fit.cvut.cz

Jiří Tumpach
The Czech Academy of Sciences, Institute of Computer Science
Charles University
Malostran. n. 25, Prague, Czech Rep.
tumpji@gmail.com

Martin Holeňa
The Czech Academy of Sciences, Institute of Computer Science
Pod Vodárenskou věží 2, Prague, Czech Republic
martin@cs.cas.cz

ABSTRACT

Surrogate regression models have been shown as a valuable technique in evolutionary optimization to save evaluations of expensive black-box objective functions. Each surrogate modelling method has two complementary components: the employed model and the control of when to evaluate the model and when the true objective function, aka evolution control. They are often tightly interconnected, which causes difficulties in understanding the impact of each component on the algorithm performance. In this supplementary material for the original article *Interaction between Model and Its Evolution Control in Surrogate-Assisted CMA Evolution Strategy* [17], we report detailed results of experiments on the noiseless and noisy benchmarks of the Comparing-Continuous-Optimisers platform and a real-world simulation benchmark, all in the expensive scenario, where only a small budget of evaluations is available.

CCS CONCEPTS

• **Theory of computation** → **Continuous optimization**; *Nonconvex optimization*; *Bio-inspired optimization*;

KEYWORDS

black-box optimization, evolutionary optimization, surrogate modelling, evolution control, CMA-ES

1 INTRODUCTION

In [17], we have selected three most successful surrogate-assisted variants of the Covariance Matrix Adaptation Evolution Strategy (*CMA-ES*) [5, 11], which we consider to be the state-of-the-art evolutionary black-box optimizer: lmm-CMA [1, 14], DTS-CMA-ES [2], and lq-CMA-ES [6]. We have analysed what exactly constitutes the *evolution control* (EC) [2–4, 12, 13, 15, 16, 18] of each of those methods, relying apart from the published decriptions also on their publicly available implementations. That allowed us to subsequently implement all possible combinations of the regression models employed in those methods with the three specific evolution control strategies, and to compare the models and their controls separately.

We have experimentally compared all combinations of the models employed in those methods

- model using *Gaussian process* means ($\mathrm{GP}_{\mathrm{M}}^{\mathrm{fval}}$) [19],
- model using *Gaussian process Probability of Improvement* ($\mathrm{GP}_{\mathrm{M}}^{\mathrm{PoI}}$),
- locally-weighted full-quadratic model ($\mathrm{lmm}_{\mathrm{M}}$),
- linear-quadratic polynomial model (lq_{M}),

and their evolution controls

- lmm-CMA evolution control ($\mathrm{lmm}_{\mathrm{EC}}$),
- DTS-CMA-ES evolution control ($\mathrm{DT}_{\mathrm{EC}}$),
- lq-CMA-ES evolution control ($\mathrm{lq}_{\mathrm{EC}}$).

In this material, we report detailed results of the experimental comparison of all mentioned combinations on all the noiseless and noisy single-objective COCO benchmarks [7, 9, 10]. In addition, we also show the results on the real-world benchmark simulating energy gained from energy converter buoys [20].

2 EXPERIMENTAL INVESTIGATION OF INTERACTIONS BETWEEN MODEL AND ITS MANAGEMENT

2.1 Experimental Set-up

We have performed experimental evaluation on the 24 noiseless f_{1-24} and 30 noisy $f_{101-130}$ single-objective COCO benchmarks [7, 9, 10] in dimensions 2, 3, 5, 10, and 20, considering always 15 instances of each function. Moreover, we have also evaluated the benchmark f_{sim} simulating energy landscape based on a layout of wave energy converters [20] for 1, 2, 5, 6, 8, and 10 buoys, i. e., in dimensions 2, 4, 10, 12, 16, and 20, on all 24 combinations of the following

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simulation settings: simulation frequencies 1 and 2, buoy radiuses 2.0, 2.5, 3.2, and search space sizes $[10, 10]$, $[25, 25]$, $[50, 50]$, and $[100, 100]$. Each algorithm had a budget of 250 function evaluations per dimension (FE$/D$) to reach the target distance $\Delta f_T = 10^{-8}$ from the function optimum.

We use a budget-dependent quality measure of the compared algoritms, strongly influenced by the concept of target precision values used in the COCO framework [8]. In particular, we measure the subset of target precision values achievable by the assessed algorithm at the considered budget within an apriori selected set of target precision values. To incorporate all possible target values of tested benchmarks, we have symmetrically doubled on the logarithmic scale the size of the default set $[10^{-8}, 10^{2}]$ utilized in [8], which yields the interval $[10^{-13}, 10^{7}]$. The subset of achievable target precision values is measured with a measure that we denote Δ_f^{μ}: the Lebesgue measure of the logarithmic transfomation of this set, normalized to yield the maximal value 1 if all target precision values from $[10^{-13}, 10^{7}]$ are achievable for the algorithm at the considered budget. We consider two evaluation budgets (⅕ of the full budget and the full budget, i. e., 50FE$/D$ and 250FE$/D$) and 12 groups of functions (f_{1-5}, f_{6-9}, f_{10-14}, f_{15-19}, f_{20-24}, f_{1-24}, $f_{101-106}$, $f_{107-121}$, $f_{122-130}$, $f_{101-130}$, f_{sim}, and $f_{1-24,101-130,\text{sim}}$).

Results from experiments are presented in Figures 1–70 and also in Tables 1–71. Figures 1–70 show the dependence of the scaled best-achieved logarithms $\Delta_f^{\log}$ of median distances Δ_f^{med} to the optimal fitness value on the number of fitness evaluations divided by the dimension. Medians Δ_f^{med}, 1st, and 3rd quartiles are calculated from 15 (24 for f_{sim}) independent instances for each respective combination of algorithm, function, and dimension. The scaled logarithms of Δ_f^{med} are calculated as

$$\Delta_f^{\log} = \frac{\log \Delta_f^{\text{med}} - \Delta_f^{\text{MIN}}}{\Delta_f^{\text{MAX}} - \Delta_f^{\text{MIN}}} \log_{10}\left(1/10^{-8}\right) + \log_{10} 10^{-8},$$

where Δ_f^{MIN} (Δ_f^{MAX}) is the minimal (maximal) distance $\log \Delta_f^{\text{med}}$ found among all the compared algorithms for the particular function f and dimension D up to 250 FE$/D$. The resulting values are scaled to interval $[-8, 0]$, where -8 corresponds to Δ_f^{MIN} and 0 to Δ_f^{MAX}.

We test pairwise differences in performance measured with Δ_f^{μ} for all evolution control methods, surrogate models, and their combinations on the noisy and noiseless part of COCO framework and the simulation benchmark using the non-parametric two-sided Wilcoxon signed rank test with the Holm correction controlling the family-wise error. To better illustrate the differences of individual settings, we also count the percentage of instances at which one combination/evolution control/model had Δ_f^{μ} higher than the other. The pairwise score and the statistical significance of the pairwise differences are reported in Tables 1–71.

Results are structured as follows

(1) noiseless functions f_{1-24} in 2, 3, 5, 10, and 20D,
  (a) separable functions f_{1-5} in 2, 3, 5, 10, and 20D,
  (b) functions with low and moderate conditioning f_{6-9} in 2, 3, 5, 10, and 20D,
  (c) functions with high conditioning and unimodal f_{10-14} in 2, 3, 5, 10, and 20D,
  (d) multi-modal functions with adequate global structure f_{15-19} in 2, 3, 5, 10, and 20D,
  (e) multi-modal functions with weak global structure f_{20-24} in 2, 3, 5, 10, and 20D,
(2) noisy functions $f_{101-130}$ in 2, 3, 5, 10, and 20D,
  (a) functions with moderate noise $f_{101-106}$ in 2, 3, 5, 10, and 20D,
  (b) functions with severe noise $f_{107-121}$ in 2, 3, 5, 10, and 20D,
  (c) highly multi-modal functions with severe noise $f_{122-130}$ in 2, 3, 5, 10, and 20D,
(3) buoy placement simulation benchmark f_{sim} in 2, 4, 10, 12, 16, and 20D,
(4) all tested functions f_{1-24}, $f_{101-130}$, and f_{sim} in 2, 10, and 20D.

Table 1: A pairwise comparison of the evolution controls, models, and their combinations in 2*D* over the noiseless COCO benchmarks for different evaluation budgets. The percentage of wins of *i*-th algorithm against *j*-th algorithm over all benchmark instances is given in the *i*-th row and *j*-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}+GP_M^{fval}}$	—	—	35	44	**64**	52	33	43	43	52	38	42	47	48	38	44	**57**	48	51	44	**63**	**66**	49.9	43
$\mathrm{DT_{EC}+GP_M^{PoI}}$	**65**	56	—	—	**75**	**66**	55	55	61	**72**	47	51	64	58	59	55	**69**	**61**	**62**	49	**75**	**75**	**62**	56
$\mathrm{DT_{EC}+lq_M}$	36	48	25	34	—	—	25	41	38	51	26	44	31	44	24	40	43	52	39	43	49	**64**	31	42
$\mathrm{DT_{EC}+lmm_M}$	**67**	**58**	45	45	**75**	**59**	—	—	**63**	**61**	54	51	**61**	56	52	53	**69**	**59**	**68**	50.4	**78**	**71**	**65**	53
$\mathrm{lq_{EC}+GP_M^{fval}}$	57	48	39	28	**63**	49	37	39	—	—	42	42	48	40	42	39	**60**	44	**56**	38	**64**	**59**	55	39
$\mathrm{lq_{EC}+GP_M^{PoI}}$	**62**	**58**	53	49	**74**	**56**	46	49	**58**	**58**	—	—	57	49	49	48	**69**	54	**66**	49	**74**	**66**	**62**	46
$\mathrm{lq_{EC}+lq_M}$	53	53	36	42	**69**	**56**	39	44	52	**60**	43	51	—	—	42	48	**60**	54	**56**	49	**71**	**69**	**55**	49.6
$\mathrm{lq_{EC}+lmm_M}$	**62**	**56**	41	45	**76**	**60**	48	47	**58**	**61**	51	52	**58**	52	—	—	**70**	**56**	**66**	50.1	**75**	**69**	**65**	48
$\mathrm{lmm_{EC}+GP_M^{fval}}$	43	52	31	39	58	48	31	41	40	56	31	46	40	46	30	44	—	—	47	45	55	**63**	43	41
$\mathrm{lmm_{EC}+GP_M^{PoI}}$	49	**56**	38	51	**61**	**57**	32	49.6	44	**62**	34	51	44	51	34	49.9	53	55	—	—	**61**	**71**	45	48
$\mathrm{lmm_{EC}+lq_M}$	37	34	25	25	51	36	22	29	36	41	26	34	29	31	25	31	45	37	39	29	—	—	35	30
$\mathrm{lmm_{EC}+lmm_M}$	50.1	**58**	38	44	**69**	**58**	35	47	45	**61**	38	54	45	50.4	35	52	**57**	**59**	55	52	**65**	**70**	—	—

2*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	43	49.6	**58**	**54**
$\mathrm{lq_{EC}}$	**58**	50.4	—	—	**66**	**53**
$\mathrm{lmm_{EC}}$	42	46	34	47	—	—

2*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	43	43	**55**	52	39	41
$\mathrm{GP_M^{PoI}}$	**57**	**57**	—	—	**61**	**61**	48	49
$\mathrm{lq_M}$	45	48	39	39	—	—	34	40
$\mathrm{lmm_M}$	**61**	**59**	**52**	51	**66**	**60**	—	—

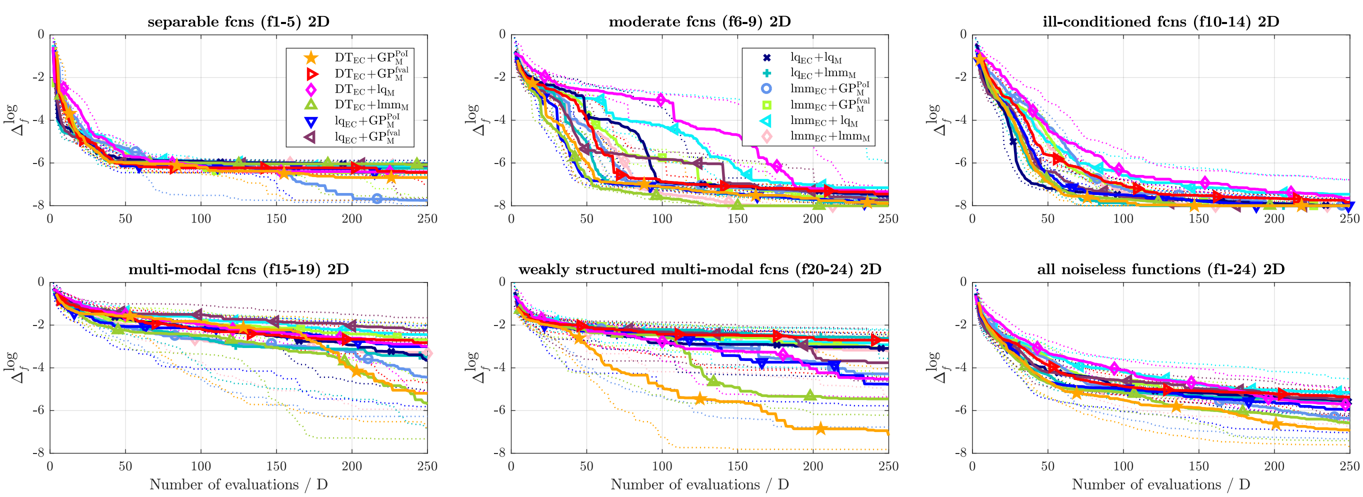

Figure 1: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless COCO benchmarks in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 2: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the noiseless COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3D	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	30	44	**70**	**59**	37	49	51	**60**	40	56	49	49.6	37	47	56	53	51	51	**66**	**69**	**58**	48
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	**70**	56	—	—	**84**	**65**	49	47	**64**	60	52	**61**	**62**	49	61	53	**70**	**63**	**63**	58	**72**	**64**	**71**	59
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	30	41	16	35	—	—	20	41	35	54	23	49	26	38	21	38	33	48	32	44	43	**60**	32	40
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**63**	51	51	53	**80**	**59**	—	—	**62**	**63**	54	**58**	**65**	53	**56**	49	**70**	**56**	**64**	49.6	**79**	**70**	**71**	51
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	49	40	36	40	**65**	46	38	37	—	—	42	42	49	37	43	37	**57**	44	49	39	**63**	58	**54**	39
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**60**	44	48	39	**77**	51	46	42	**58**	58	—	—	58	45	54	43	**66**	53	**61**	47	**73**	**62**	**65**	47
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	51	50.4	38	51	**74**	**62**	35	47	51	**63**	43	**55**	—	—	44	52	**58**	**57**	**55**	53	**69**	**71**	**55**	49
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	**63**	53	39	47	**79**	**62**	44	51	**57**	**63**	46	**57**	56	48	—	—	**65**	**57**	**62**	53	**77**	**69**	**68**	52
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	44	47	30	38	**67**	52	30	44	43	56	34	47	42	43	35	43	—	—	49.6	45	**62**	**60**	57	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	49	49	37	43	**68**	56	36	50.4	51	**61**	39	53	45	48	38	47	50.4	55	—	—	**61**	**68**	55	50
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	34	31	28	36	**57**	40	21	30	37	43	27	38	31	29	23	31	38	40	39	32	—	—	39	32
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	42	52	29	41	**68**	**60**	29	49	46	**61**	35	53	45	51	32	48	43	54	45	50	**61**	**68**	—	—

3D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	45	50.3	**57**	**55**
$\mathrm{lq_{EC}}$	**55**	49.8	—	—	**64**	**53**
$\mathrm{lmm_{EC}}$	43	45	36	47	—	—

3D	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	44	44	**60**	52	46	44
$\mathrm{GP^{PoI}_M}$	**56**	**56**	—	—	**63**	**58**	54	47
$\mathrm{lq_M}$	40	48	37	42	—	—	34	42
$\mathrm{lmm_M}$	**54**	**56**	46	**53**	**66**	**58**	—	—

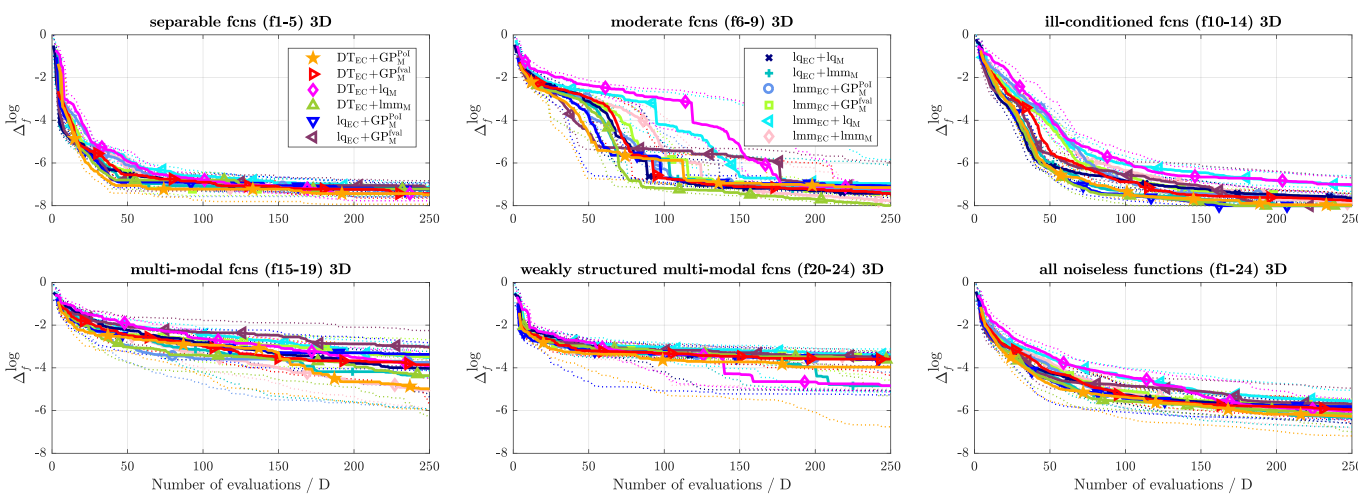

Figure 2: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless COCO benchmarks in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 3: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the noiseless COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$5D$	$\mathrm{DT_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	35	44	**67**	57	37	52	43	48	37	47	42	42	52	46	53	52	47	41	**62**	**64**	**59**	46
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	**65**	56	—	—	**77**	53	55	59	**62**	56	56	55	61	47	**68**	57	**62**	56	59	43	**73**	**65**	**68**	51
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	33	43	23	47	—	—	22	49.6	37	44	26	40	24	38	33	46	38	46	31	39	43	**58**	36	41
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**63**	48	45	41	**78**	50.4	—	—	54	49	43	48	51	40	**65**	45	**61**	49	**57**	40	**74**	**58**	**67**	41
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	57	52	38	44	**63**	56	46	51	—	—	43	48	49	43	58	49	**56**	53	52	45	**62**	**62**	**59**	47
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**63**	53	44	45	**74**	60	57	53	57	52	—	—	60	49	**65**	52	**64**	54	**56**	43	**70**	**62**	**65**	50.3
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**58**	**58**	39	53	**76**	**62**	49	**60**	51	**57**	40	51	—	—	**60**	57	**57**	**58**	**53**	51	**72**	**72**	**61**	53
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	48	54	32	43	**67**	54	35	55	42	51	35	48	40	43	—	—	49	54	46	44	**62**	**64**	**57**	50.1
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	47	48	38	44	**62**	54	39	51	44	47	36	46	43	42	51	46	—	—	48	37	**60**	**63**	59	44
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	53	**59**	41	57	**69**	**61**	43	**60**	48	**55**	44	57	47	49	54	56	52	**63**	—	—	**66**	**69**	**60**	54
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	38	36	27	35	57	42	26	42	38	38	30	38	28	28	38	36	40	37	34	31	—	—	42	34
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	41	54	32	49	**64**	**59**	33	**59**	41	53	35	49.7	39	47	43	49.9	41	56	40	46	58	**66**	—	—

$5D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	45	45	**55**	49.7
$\mathrm{lq_{EC}}$	**55**	**55**	—	—	**60**	**54**
$\mathrm{lmm_{EC}}$	45	50.3	40	46	—	—

$5D$	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	44	43	**59**	55	51	48
$\mathrm{GP^{PoI}_M}$	**56**	**57**	—	—	**65**	**58**	**61**	54
$\mathrm{lq_M}$	41	45	35	42	—	—	42	47
$\mathrm{lmm_M}$	49	52	39	46	**58**	**53**	—	—

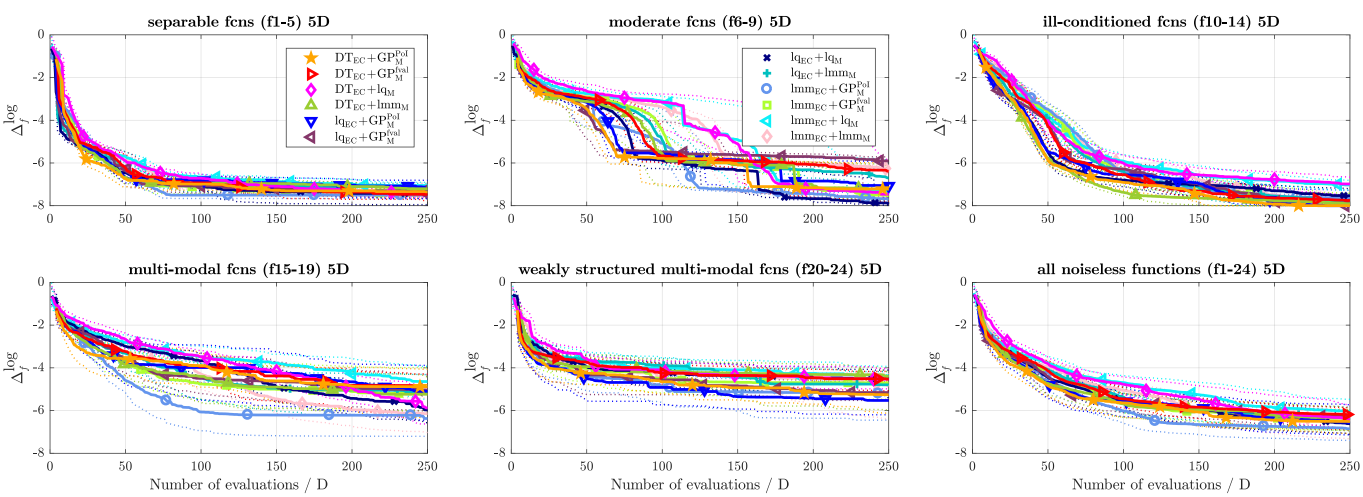

Figure 3: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless COCO benchmarks in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 4: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noiseless COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	30	42	**67**	**61**	51	**63**	42	55	45	56	48	49	**63**	**60**	**66**	58	**63**	48	**75**	**67**	**77**	58
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	**70**	58	—	—	**78**	**65**	**68**	**65**	**60**	**62**	**63**	**65**	63	54	**80**	**66**	**81**	63	**80**	59	**88**	**72**	**88**	58
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	33	39	22	35	—	—	30	55	31	48	33	49	33	37	45	58	48	48	46	39	**58**	61	**60**	50
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	49	37	33	35	**70**	45	—	—	42	46	43	45	47	37	**64**	52	**64**	44	**66**	37	**76**	48	**76**	41
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	58	45	40	38	**69**	52	58	54	—	—	48	49	58	42	**67**	55	**68**	51	**69**	47	**73**	54	**77**	47
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**55**	44	38	35	**68**	51	58	55	52	51	—	—	53	43	**63**	55	**67**	49	**69**	47	**75**	55	**74**	45
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	53	51	38	46	**67**	**63**	53	**63**	43	**58**	47	57	—	—	**64**	**69**	**64**	**59**	**64**	51	**77**	**68**	**74**	**61**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	38	40	20	34	55	42	36	48	33	45	38	45	36	31	—	—	**58**	44	**58**	40	**68**	49	**68**	40
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	34	42	19	37	52	52	36	56	32	49	33	51	36	41	42	56	—	—	52	36	**66**	**61**	**64**	53
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	37	52	20	41	54	**61**	34	**63**	31	53	31	53	36	49	42	**60**	48	**64**	—	—	61	**70**	**63**	**64**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	25	33	12	28	42	39	24	52	27	46	25	45	23	32	32	51	34	39	39	30	—	—	52	42
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	23	42	12	42	40	50	24	59	23	53	26	55	26	39	32	60	36	47	37	36	48	58	—	—

10*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	48	49.8	**68**	**54**
$\mathrm{lq_{EC}}$	52	50.2	—	—	**71**	**52**
$\mathrm{lmm_{EC}}$	32	46	29	48	—	—

10*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	47	42	**64**	**55**	**61**	**57**
$\mathrm{GP^{PoI}_M}$	53	**58**	—	—	**60**	**58**	**63**	**60**
$\mathrm{lq_M}$	36	45	40	42	—	—	49	**55**
$\mathrm{lmm_M}$	39	43	37	40	51	45	—	—

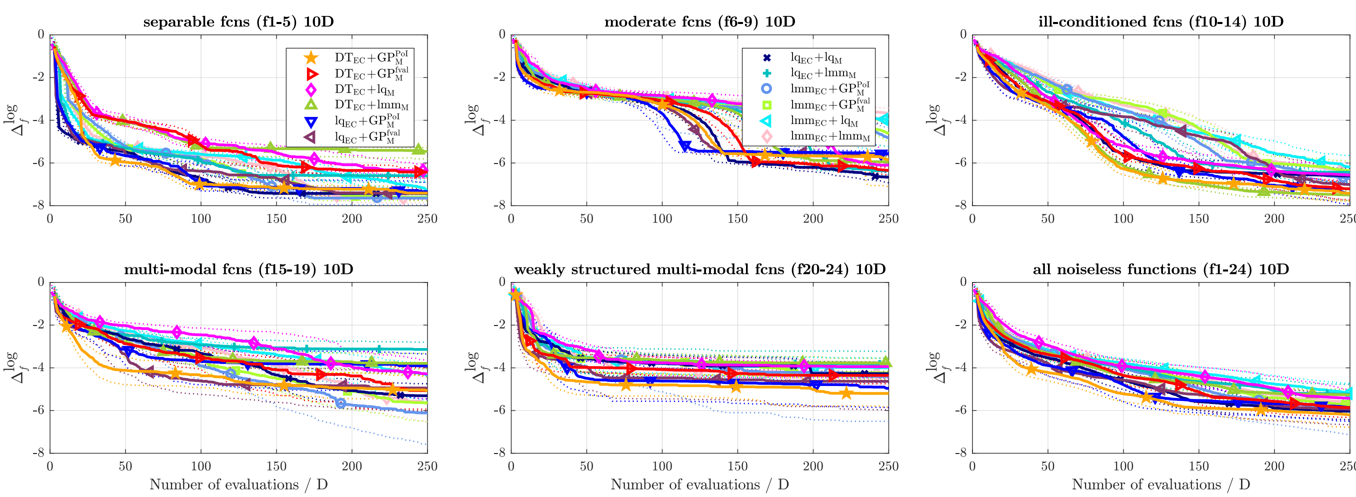

Figure 4: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless COCO benchmarks in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 5: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the noiseless COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	35	45	**60**	**62**	**62**	**64**	37	**59**	36	58	56	48	**75**	**76**	**64**	**57**	**63**	53	**69**	**62**	**76**	53
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	**65**	55	—	—	**71**	58	**72**	63	55	**65**	53	**65**	**68**	47	**79**	**69**	**80**	58	**84**	56	**75**	**61**	**82**	53
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	40	38	29	42	—	—	52	53	31	51	29	49	45	38	**64**	**64**	57	53	54	54	61	**65**	**66**	54
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	38	36	28	38	48	47	—	—	29	47	33	46	38	35	**65**	**63**	45	45	48	45	56	47	**67**	41
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	**63**	41	45	35	**69**	49	**71**	53	—	—	48	43	**62**	36	**76**	56	**70**	42	**72**	37	**73**	48	**76**	38
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	**64**	42	48	35	**71**	51	**68**	54	52	**57**	—	—	**61**	35	**75**	55	**73**	49	**74**	39	**68**	50.4	**76**	40
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	44	52	33	53	**55**	**62**	**62**	**65**	38	**64**	39	**65**	—	—	**71**	**78**	**59**	**63**	**60**	**58**	**67**	**70**	**73**	**58**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	25	24	21	31	36	36	35	37	24	44	25	45	29	22	—	—	37	45	38	40	42	43	55	38
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	36	43	20	42	43	47	55	55	30	**58**	27	51	41	37	**63**	55	—	—	51	39	57	51	**67**	45
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	37	47	16	44	46	46	52	55	28	**63**	26	**61**	40	42	**62**	60	49	**61**	—	—	51	**59**	**63**	51
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	31	38	25	39	39	35	44	53	28	52	32	49.6	33	30	**58**	57	43	49	49	41	—	—	**66**	39
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	24	48	18	47	34	46	33	59	24	**62**	24	**60**	28	42	45	**62**	33	**55**	37	49	34	**61**	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	**55**	**66**	**55**
$\mathrm{lq_{EC}}$	51	45	—	—	**66**	47
$\mathrm{lmm_{EC}}$	34	45	34	53	—	—

20*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	48	42	**59**	49.7	**68**	55
$\mathrm{GP^{PoI}_{M}}$	52	**58**	—	—	**58**	49	**70**	55
$\mathrm{lq_{M}}$	41	**50.3**	42	51	—	—	**63**	**57**
$\mathrm{lmm_{M}}$	32	45	30	45	37	43	—	—

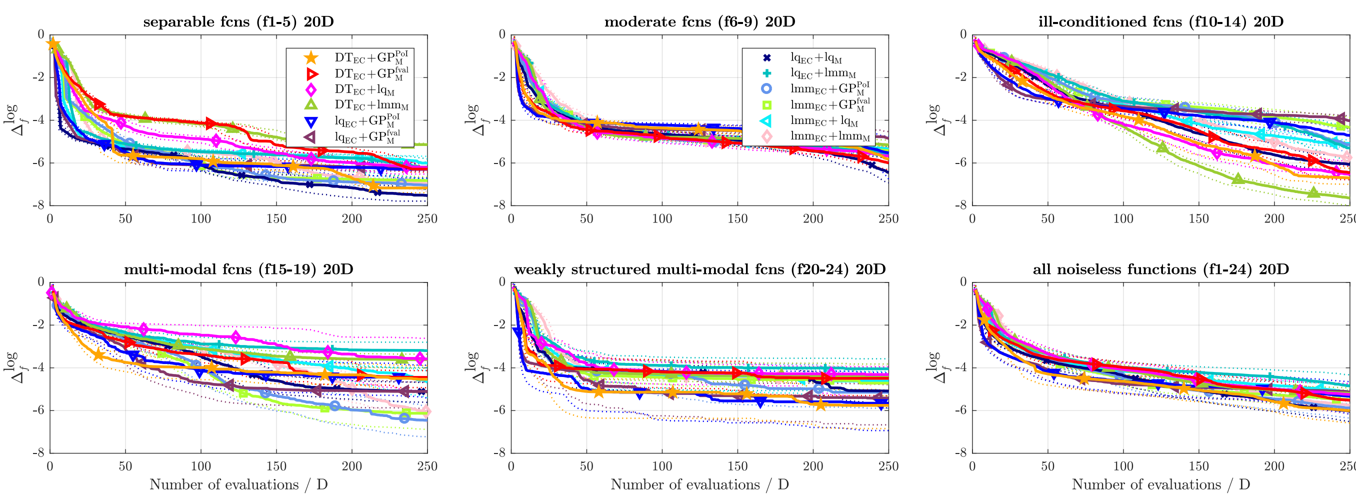

Figure 5: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless COCO benchmarks in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 6: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noiseless COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	33	44	**66**	**58**	44	54	43	**55**	39	52	48	47	53	**55**	**59**	**54**	**55**	47	**67**	**66**	**64**	49
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	**67**	**56**	—	—	**77**	**62**	**59**	**58**	**60**	**63**	**54**	**59**	**63**	51	**69**	**60**	**72**	**60**	**70**	53	**77**	**67**	**74**	**55**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	35	42	23	39	—	—	30	48	34	49.7	27	46	32	39	37	49	44	49.6	40	43	51	**62**	45	45
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**56**	46	41	42	**70**	**52**	—	—	50	**53**	45	49	**52**	44	**61**	**53**	**62**	**51**	**61**	44	**73**	**59**	**69**	45
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	**57**	45	40	37	**66**	50.3	50	47	—	—	45	45	53	40	**57**	47	**62**	47	**59**	41	**67**	**56**	**64**	42
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	**61**	48	46	41	**73**	**54**	**55**	51	**55**	**55**	—	—	**58**	44	**61**	51	**68**	52	**65**	45	**72**	**59**	**68**	46
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**52**	**53**	37	49	**68**	**61**	48	**56**	47	**60**	42	**56**	—	—	**56**	**61**	**60**	**58**	**58**	**52**	**71**	**70**	**64**	**54**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	47	45	31	40	**63**	51	39	47	43	**53**	39	50	44	39	—	—	**56**	**51**	**54**	46	**65**	**59**	**63**	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	41	46	28	40	**56**	50.4	38	49	38	**53**	32	48	40	42	44	49	—	—	49.7	41	**60**	**60**	**58**	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	45	53	30	47	**60**	**57**	39	56	41	**59**	35	**55**	42	48	46	54	50.3	**60**	—	—	**60**	**67**	**57**	53
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	33	34	23	33	49	39	27	41	33	44	28	41	29	30	35	41	40	40	40	33	—	—	47	36
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	36	51	26	45	**55**	**55**	31	55	36	**58**	32	**54**	36	46	37	**54**	42	**54**	43	47	**53**	**64**	—	—

2 − 20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	46	49.9	**61**	**53**
$\mathrm{lq_{EC}}$	**54**	50.1	—	—	**65**	**52**
$\mathrm{lmm_{EC}}$	39	47	35	48	—	—

2 − 20*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	45	43	**59**	53	53	49
$\mathrm{GP_M^{PoI}}$	**55**	**57**	—	—	**61**	**57**	**59**	**53**
$\mathrm{lq_M}$	41	47	39	43	—	—	44	48
$\mathrm{lmm_M}$	47	**51**	41	47	**56**	**52**	—	—

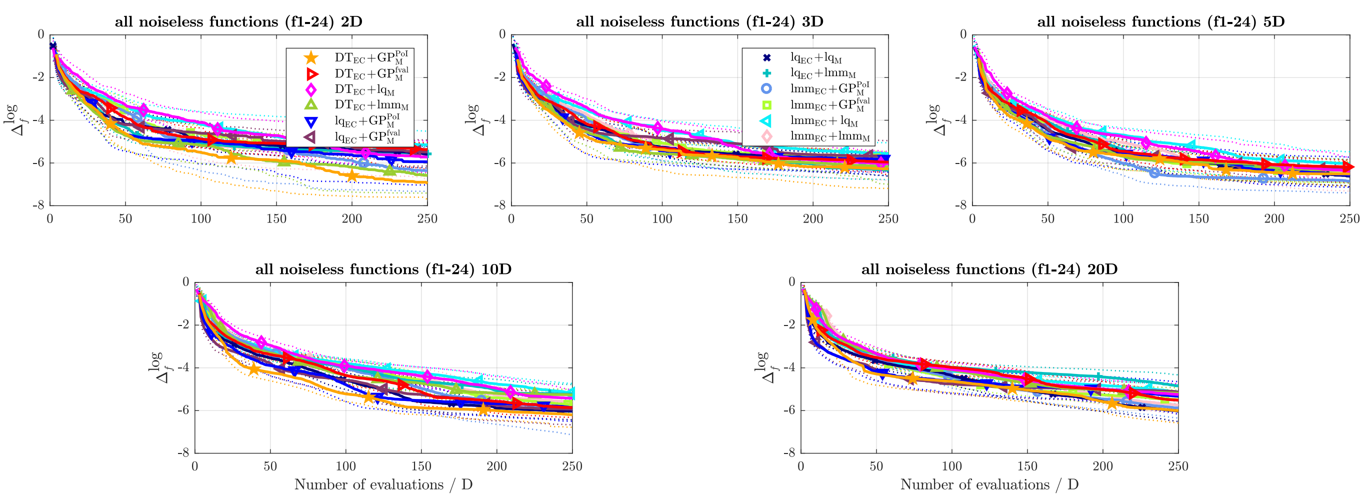

Figure 6: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless COCO benchmarks in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 7: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the noiseless separable COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	54	50	59	48	47	57	59	60	56	55	49	51	39	40	**69**	59	56	53	**66**	63	50	49
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	46	50	—	—	66	62	58	74	64	80	52	60	66	58	54	54	60	72	60	48	70	78	50	54
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	41	52	34	38	—	—	37	56	49	65	45	62	30	50	26	43	54	63	59	55	61	61	41	49
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	53	43	42	26	**63**	44	—	—	61	60	59	47	54	50	37	37	**67**	55	**64**	45	**75**	57	55	45
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	41	40	36	20	51	35	39	40	—	—	45	41	37	39	24	25	52	50	48	33	60	51	49	42
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	44	45	48	40	55	38	41	53	55	59	—	—	40	39	23	30	61	49	55	41	63	56	51	40
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	51	49	34	42	**70**	50	46	50	**63**	61	**60**	61	—	—	43	44	**70**	59	**67**	55	**71**	55	**61**	57
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	**61**	60	46	46	**74**	57	63	63	**76**	**75**	**77**	**70**	57	56	—	—	**78**	**69**	**73**	65	**78**	**67**	**69**	61
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	31	41	40	28	46	37	33	45	48	50	39	51	30	41	22	31	—	—	43	40	49	46	37	41
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	44	47	40	52	41	45	36	55	52	**67**	45	59	33	45	27	35	57	60	—	—	52	61	43	48
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	34	37	30	22	39	39	25	43	40	49	37	44	29	45	22	33	51	54	48	39	—	—	37	43
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	50	51	50	46	59	51	45	55	51	58	49	60	39	43	31	39	63	59	57	52	63	57	—	—

2*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	43	50.2	**61**	54
$\mathrm{lq_{EC}}$	**57**	49.8	—	—	**62**	52
$\mathrm{lmm_{EC}}$	39	46	38	48	—	—

2*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	45	42	48	44	36	41
$\mathrm{GP_M^{PoI}}$	55	**58**	—	—	49	52	36	44
$\mathrm{lq_M}$	52	**56**	**51**	48	—	—	39	48
$\mathrm{lmm_M}$	**64**	**59**	**64**	**56**	**61**	52	—	—

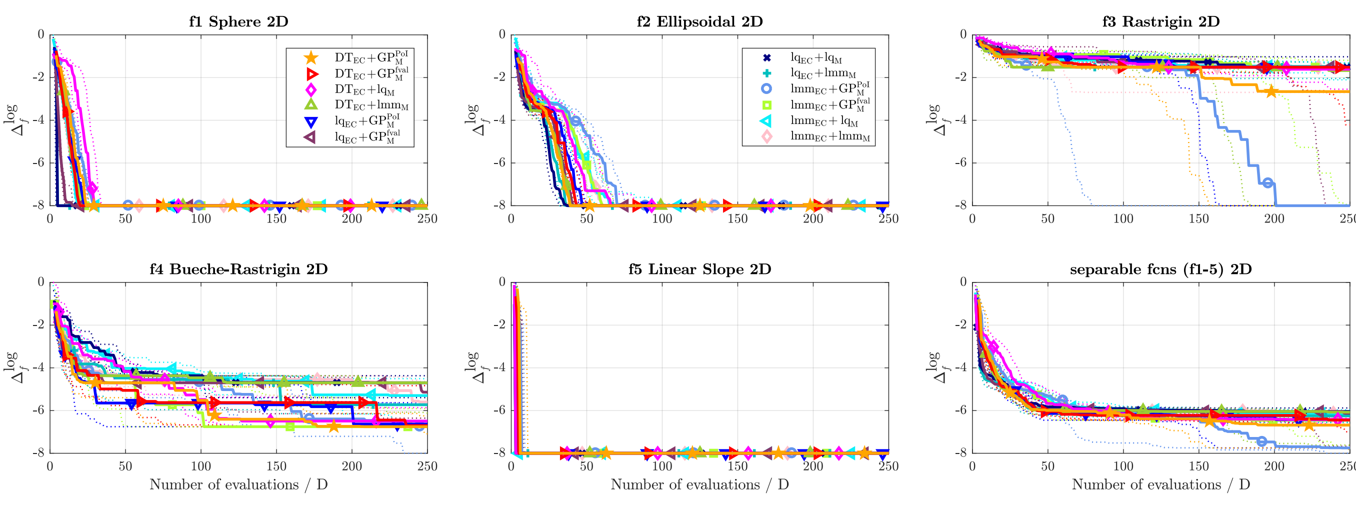

Figure 7: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 separable noiseless COCO benchmarks in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 8: A pairwise comparison of the evolution controls, models, and their combinations in 3*D* over the noiseless separable COCO benchmarks for different evaluation budgets. The percentage of wins of *i*-th algorithm against *j*-th algorithm over all benchmark instances is given in the *i*-th row and *j*-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	26	46	**70**	57	41	59	56	**70**	46	63	42	57	29	45	59	61	57	55	**63**	65	61	57
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	74	54	—	—	**78**	54	50	46	56	56	60	60	54	46	50	38	68	60	64	60	66	54	74	62
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	30	43	22	46	—	—	22	47	37	**63**	30	57	23	45	14	47	39	53	43	49	46	58	31	54
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	59	41	50	54	**78**	53	—	—	65	**69**	59	62	54	46	43	45	**69**	61	**72**	57	**75**	59	**69**	57
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	44	30	44	44	63	37	35	31	—	—	47	46	41	29	33	30	60	45	49	33	60	51	53	38
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	54	37	40	40	70	43	41	38	53	54	—	—	46	34	42	35	61	51	53	47	63	55	62	49
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	58	43	46	54	**77**	55	46	54	59	**71**	54	**66**	—	—	51	60	**66**	**65**	**71**	62	**78**	**69**	**61**	53
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	**71**	55	50	62	**86**	53	57	55	**67**	**70**	**58**	**65**	49	40	—	—	**71**	**65**	**77**	**63**	**79**	**69**	**70**	60
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	41	39	32	40	61	47	31	39	40	55	39	49	34	35	29	35	—	—	49	34	53	53	55	47
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	43	45	36	40	57	51	28	43	51	67	47	53	29	38	23	37	51	66	—	—	48	60	45	49
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	37	35	34	46	54	42	25	41	40	49	37	45	22	31	21	31	47	47	52	40	—	—	46	43
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	39	43	26	38	69	46	31	43	47	62	38	51	39	47	30	40	45	53	55	51	54	57	—	—

3*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	43	54	**59**	**59**
$\mathrm{lq_{EC}}$	**57**	46	—	—	**65**	55
$\mathrm{lmm_{EC}}$	41	41	35	45	—	—

3*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	45	41	55	46	43	46
$\mathrm{GP_M^{PoI}}$	55	59	—	—	51	48	45	43
$\mathrm{lq_M}$	45	**54**	49	52	—	—	40	50.2
$\mathrm{lmm_M}$	**57**	**54**	**55**	**57**	**60**	49.8	—	—

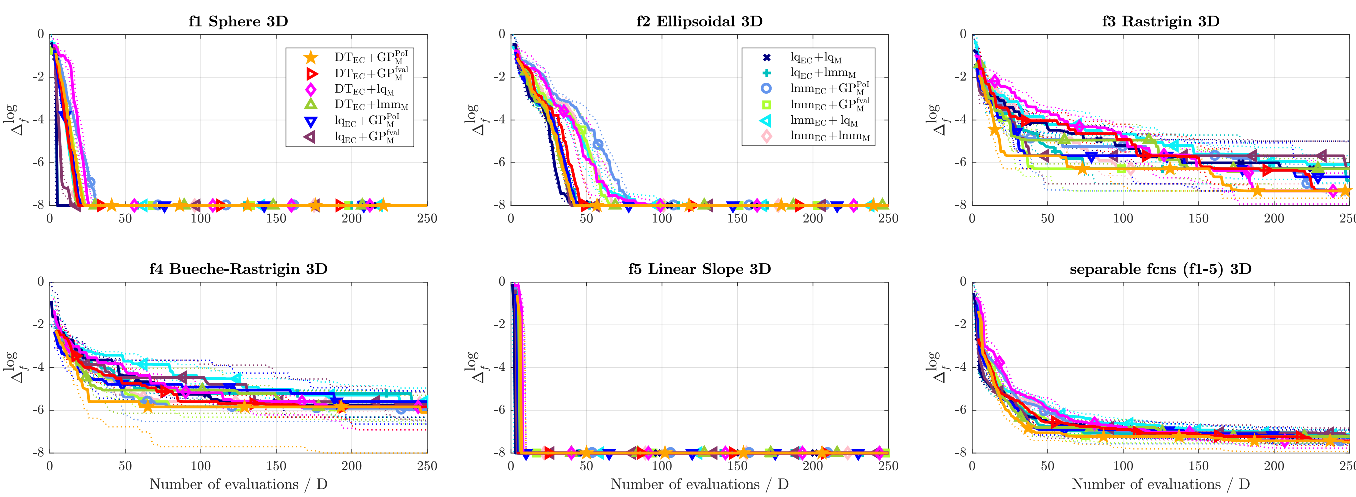

Figure 8: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 separable noiseless COCO benchmarks in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 9: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the noiseless separable COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

5D	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	30	62	69	59	42	57	55	**67**	55	**67**	33	36	51	54	57	**63**	52	57	**62**	**69**	58	55
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	70	38	—	—	74	42	54	54	**76**	52	74	70	56	36	74	54	72	64	64	48	66	58	**78**	54
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	31	41	26	58	—	—	26	59	47	62	37	**59**	16	39	33	53	49	59	47	56	47	62	42	55
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	58	43	46	46	**74**	41	—	—	**65**	55	62	58	45	29	**67**	49	**64**	53	**60**	45	**74**	54	**65**	45
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	45	33	24	48	53	38	35	45	—	—	47	52	27	23	49	50	51	54	45	45	53	51	53	46
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	45	33	26	30	63	41	38	42	53	48	—	—	39	22	51	39	55	49	44	35	59	43	50	37
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**67**	**64**	44	64	**84**	**61**	55	**71**	**73**	**77**	61	**78**	—	—	**65**	**67**	**69**	**72**	**64**	**73**	**75**	**77**	**64**	**65**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	49	46	26	46	67	47	33	51	51	50	49	61	35	33	—	—	59	55	52	53	**63**	58	62	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	43	37	28	36	51	41	36	47	49	46	45	51	31	28	41	45	—	—	45	40	56	52	59	47
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	48	43	36	52	53	44	40	55	55	55	56	65	36	27	48	47	55	60	—	—	61	55	52	49
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	38	31	34	42	53	38	26	46	47	49	41	57	25	23	37	42	44	48	39	45	—	—	38	47
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	42	45	22	46	58	45	35	55	47	54	50	63	36	35	38	49	41	53	48	51	62	53	—	—

5D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	53	**57**	**56**
$\mathrm{lq_{EC}}$	51	47	—	—	**58**	**54**
$\mathrm{lmm_{EC}}$	43	44	42	46	—	—

5D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	44	48	51	45	49.8	51
$\mathrm{GP_M^{PoI}}$	**56**	52	—	—	54	39	52	45
$\mathrm{lq_M}$	49	**55**	46	**61**	—	—	43	**58**
$\mathrm{lmm_M}$	50.2	49	48	55	57	42	—	—

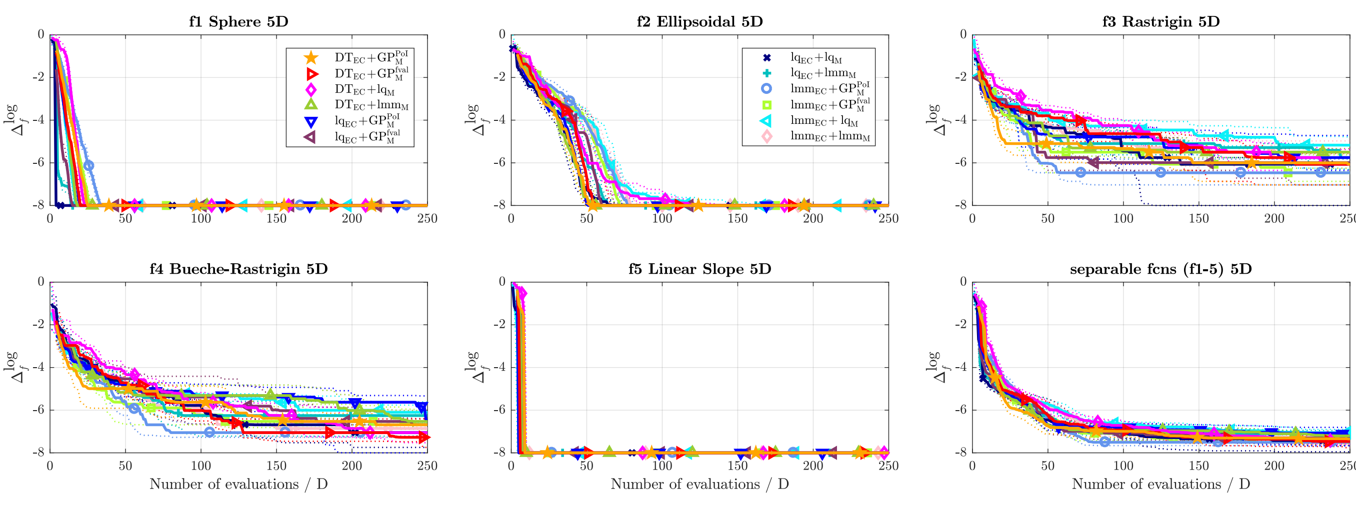

Figure 9: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 separable noiseless COCO benchmarks in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 10: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noiseless separable COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$10D$	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	16	48	55	53	51	**71**	44	55	33	61	23	44	47	57	47	49	57	43	63	57	60	44
DT_{EC}+GP_M^{PoI}	**84**	52	—	—	72	54	**88**	**80**	**88**	64	80	72	52	36	74	58	**88**	60	**88**	56	**86**	66	**74**	46
DT_{EC}+lq_M	45	47	28	46	—	—	47	**67**	43	55	36	59	21	35	47	61	43	43	54	42	54	57	54	42
DT_{EC}+lmm_M	49	29	12	20	53	33	—	—	43	35	32	42	23	19	47	46	43	27	55	25	63	38	60	21
lq_{EC}+GP_M^{fval}	56	45	12	36	57	45	57	**65**	—	—	34	53	29	29	48	53	60	42	**68**	38	56	41	57	32
lq_{EC}+GP_M^{PoI}	**67**	39	20	28	64	41	**68**	58	66	47	—	—	39	23	55	49	**67**	33	**77**	36	69	43	63	24
lq_{EC}+lq_M	**77**	**56**	48	64	**79**	**65**	**77**	**81**	**71**	**71**	**61**	**77**	—	—	**67**	**75**	**71**	**63**	**74**	58	**77**	**68**	**75**	55
lq_{EC}+lmm_M	53	43	26	42	53	39	53	54	52	47	45	51	33	25	—	—	**61**	44	**75**	42	**62**	41	54	29
lmm_{EC}+GP_M^{fval}	53	51	12	40	57	57	57	**73**	40	58	33	67	29	37	39	56	—	—	60	43	56	60	52	39
lmm_{EC}+GP_M^{PoI}	43	57	12	44	46	58	45	**75**	32	62	23	64	26	42	25	58	40	57	—	—	42	57	43	45
lmm_{EC}+lq_M	37	43	14	34	46	43	37	62	44	59	31	57	23	32	38	59	44	40	**58**	43	—	—	54	33
lmm_{EC}+lmm_M	40	56	26	54	46	58	40	**79**	43	**68**	37	**76**	25	45	46	**71**	48	**61**	57	55	46	**67**	—	—

$10D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	41	48	57	44
lq_{EC}	**59**	**52**	—	—	**67**	44
lmm_{EC}	43	**56**	33	**56**	—	—

$10D$	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	43	48	47	47	50.2	54
GP_M^{PoI}	**57**	52	—	—	45	42	55	52
lq_M	**53**	**53**	**55**	**58**	—	—	**56**	**58**
lmm_M	49.8	46	45	48	44	42	—	—

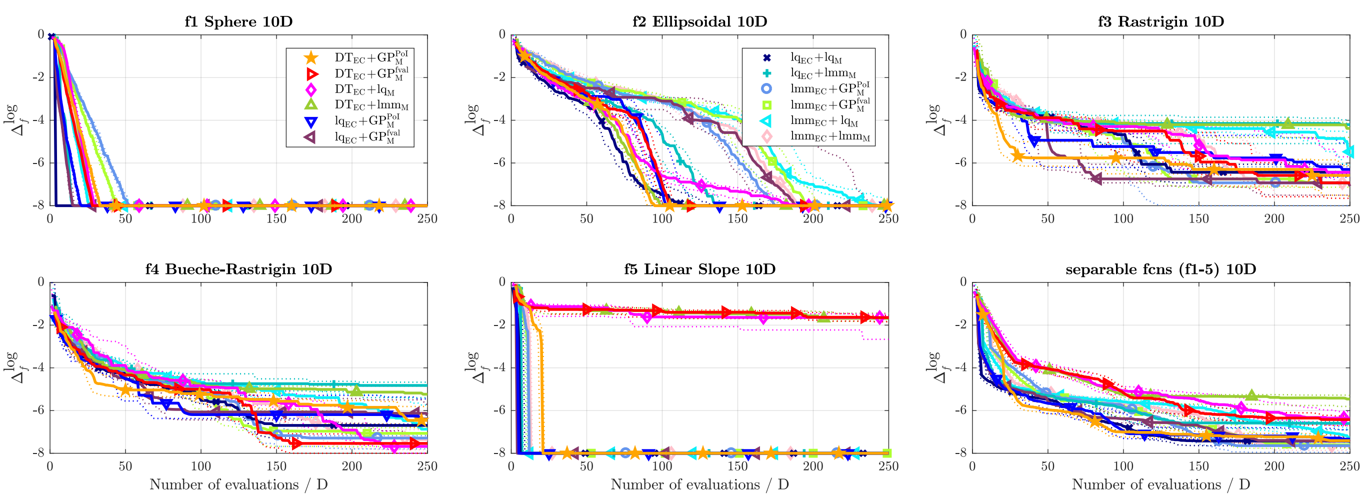

Figure 10: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 separable noiseless COCO benchmarks in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 11: A pairwise comparison of the evolution controls, models, and their combinations in 20D over the noiseless separable COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20D	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	24	52	43	60	55	**77**	35	67	28	63	20	41	59	68	49	52	51	48	45	65	57	49
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	**76**	48	—	—	52	48	72	**84**	72	68	64	64	44	36	**74**	66	**92**	56	**92**	52	70	62	**78**	42
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	57	40	48	52	—	—	**67**	67	47	67	47	60	36	32	67	67	63	43	68	45	57	63	60	48
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	45	23	28	16	33	33	—	—	35	32	35	31	19	13	55	44	41	24	47	32	41	29	49	28
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	**65**	33	28	32	53	33	65	68	—	—	40	37	32	11	49	37	65	15	65	13	43	33	56	13
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**72**	37	36	36	53	40	65	69	**60**	**63**	—	—	31	16	63	43	**77**	39	**79**	29	53	39	63	22
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**80**	59	56	64	**64**	**68**	**81**	**87**	**68**	**89**	**69**	**84**	—	—	**81**	**81**	74	**69**	**79**	**69**	**79**	**81**	**85**	**61**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	41	32	26	34	33	33	45	56	51	**63**	37	57	19	19	—	—	**61**	54	**57**	50	38	50	50	34
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	51	48	8	44	37	57	59	76	35	**85**	23	61	26	31	39	46	—	—	57	40	37	47	46	35
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	49	52	8	48	32	55	53	68	35	**87**	21	**71**	21	31	43	50	43	60	—	—	31	58	43	38
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	55	35	30	38	43	37	59	71	57	**67**	47	61	21	19	62	50	**63**	53	**69**	42	—	—	**69**	27
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	43	51	22	58	40	52	51	72	44	**87**	37	**78**	15	39	50	66	54	**65**	57	62	31	**73**	—	—

20D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	44	49	56	48
$\mathrm{lq_{EC}}$	**56**	51	—	—	**68**	40
$\mathrm{lmm_{EC}}$	44	52	32	60	—	—

20D	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	45	41	37	39	50	50
$\mathrm{GP^{PoI}_M}$	**55**	**59**	—	—	34	39	56	47
$\mathrm{lq_M}$	**63**	**61**	**66**	**61**	—	—	**72**	**58**
$\mathrm{lmm_M}$	50	50	44	53	28	42	—	—

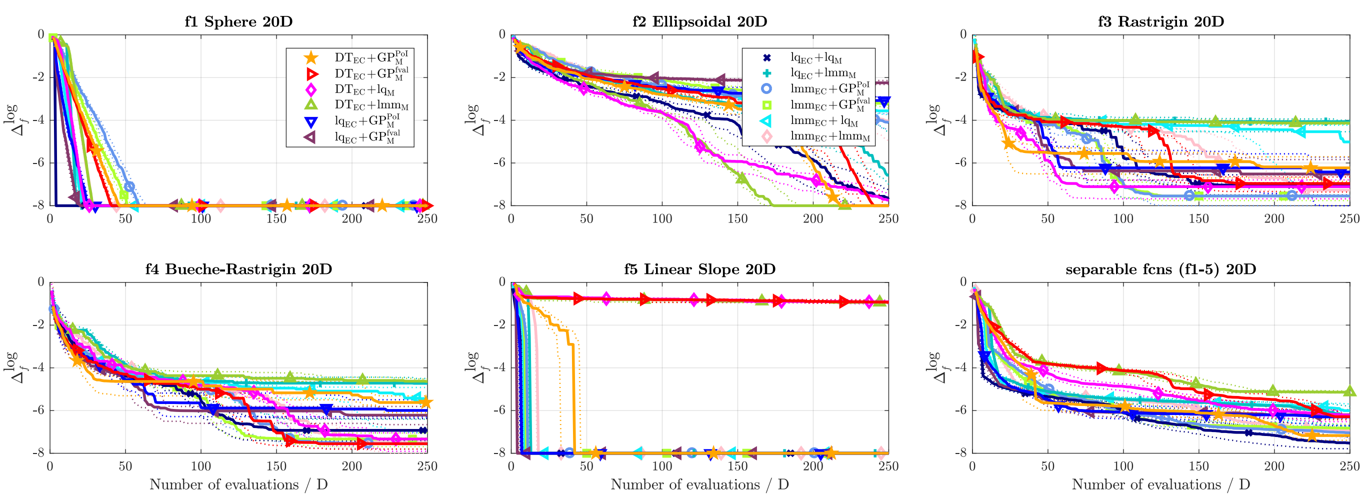

Figure 11: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 separable noiseless COCO benchmarks in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 12: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noiseless separable COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	DT_{EC}+GP^{fval}_{M}		DT_{EC}+GP^{PoI}_{M}		DT_{EC}+lq_{M}		DT_{EC}+lmm_{M}		lq_{EC}+GP^{fval}_{M}		lq_{EC}+GP^{PoI}_{M}		lq_{EC}+lq_{M}		lq_{EC}+lmm_{M}		lmm_{EC}+GP^{fval}_{M}		lmm_{EC}+GP^{PoI}_{M}		lmm_{EC}+lq_{M}		lmm_{EC}+lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP^{fval}_{M}	—	—	30	52	59	55	47	**64**	49.7	**64**	44	**62**	33	46	45	53	**56**	57	55	51	**60**	**64**	57	51
DT_{EC}+GP^{PoI}_{M}	**70**	48	—	—	**68**	52	64	**68**	**71**	**64**	**66**	**65**	54	42	65	54	**76**	**62**	**74**	53	**72**	**64**	**71**	52
DT_{EC}+lq_{M}	41	45	32	48	—	—	40	**59**	45	**62**	39	**59**	25	40	37	54	49.6	52	54	49.6	53	**60**	46	49.7
DT_{EC}+lmm_{M}	**53**	36	36	32	**60**	41	—	—	54	50.1	49	48	39	32	49.7	44	**57**	44	**59**	41	**66**	47	**60**	39
lq_{EC}+GP^{fval}_{M}	50.3	36	29	36	55	38	46	49.9	—	—	43	46	33	26	41	39	**58**	41	55	32	54	45	54	34
lq_{EC}+GP^{PoI}_{M}	56	38	34	35	**61**	41	51	52	**57**	54	—	—	39	27	47	39	**64**	44	**62**	38	**61**	47	**58**	35
lq_{EC}+lq_{M}	**67**	**54**	46	58	**75**	**60**	**61**	**68**	**67**	**74**	**61**	**73**	—	—	**61**	**65**	**70**	**66**	**71**	**63**	**76**	**70**	**69**	**58**
lq_{EC}+lmm_{M}	**55**	47	35	46	**63**	46	50.3	**56**	**59**	**61**	**53**	**61**	39	35	—	—	**66**	**57**	**67**	**54**	**64**	**57**	**61**	47
lmm_{EC}+GP^{fval}_{M}	44	43	24	38	50.4	48	43	56	42	**59**	36	56	30	34	34	43	—	—	51	39	50	52	49.7	42
lmm_{EC}+GP^{PoI}_{M}	45	49	26	47	46	50.4	41	**59**	45	**68**	38	**62**	29	37	33	46	49	**61**	—	—	47	58	45	46
lmm_{EC}+lq_{M}	40	36	28	36	47	40	34	53	46	**55**	39	53	24	30	36	43	50	48	**53**	42	—	—	49	39
lmm_{EC}+lmm_{M}	43	49	29	48	54	50.3	40	**61**	46	**66**	42	**65**	31	42	39	53	50.3	**58**	**55**	54	51	**61**	—	—

2 − 20*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	44	51	**58**	52
lq_{EC}	**56**	49	—	—	**64**	49
lmm_{EC}	42	48	36	51	—	—

2 − 20*D*	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	44	44	48	44	46	48
GP^{PoI}_{M}	**56**	**56**	—	—	47	44	49	46
lq_{M}	**52**	**56**	**53**	**56**	—	—	49.9	**55**
lmm_{M}	**54**	**52**	**51**	**54**	50.1	45	—	—

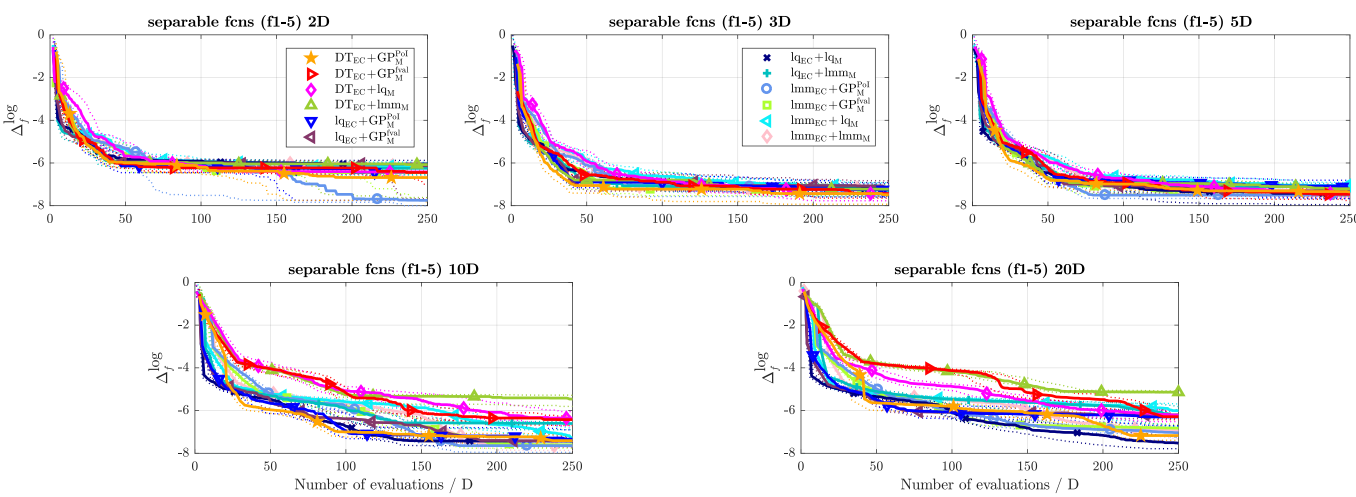

Figure 12: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless separable COCO benchmarks in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 13: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the noiseless COCO benchmarks with low or moderate conditioning for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	35	45	**76**	62	32	43	43	52	32	40	66	42	45	37	62	45	55	52	**65**	**70**	55	33
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	65	55	—	—	**85**	**80**	30	45	50	75	25	50	70	65	40	50	68	65	65	65	80	70	70	50
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	24	38	15	20	—	—	5	35	22	40	3	35	35	35	8	27	26	40	17	32	37	63	13	28
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**68**	57	70	55	**95**	**65**	—	—	68	63	59	58	**83**	53	70	55	**75**	60	**82**	62	**90**	**70**	**75**	45
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	57	48	50	25	**78**	60	32	37	—	—	33	40	68	37	54	33	65	47	66	48	**73**	**65**	60	27
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	**68**	60	75	50	**97**	**65**	41	42	67	60	—	—	**80**	53	62	47	**80**	58	**78**	60	**83**	**67**	**73**	35
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	34	58	30	35	**65**	**65**	17	47	33	63	20	47	—	—	27	45	33	53	33	62	55	**80**	33	45
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	55	63	60	50	**92**	**73**	30	45	46	67	38	53	**73**	55	—	—	70	53	67	55	**83**	**75**	70	32
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	38	55	33	35	**74**	60	25	40	35	53	20	42	67	47	30	47	—	—	50	55	63	**70**	43	33
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	45	48	35	35	**83**	68	18	38	34	52	22	40	67	38	33	45	50	45	—	—	**68**	**77**	42	28
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	35	30	20	30	63	37	10	30	27	35	17	33	45	20	17	25	37	30	32	23	—	—	27	18
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	45	**67**	30	50	**87**	**72**	25	55	40	**73**	27	65	67	55	30	68	57	67	58	**72**	**73**	**82**	—	—

$2D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	47	48	**59**	53
$\mathrm{lq_{EC}}$	53	53	—	—	**67**	55
$\mathrm{lmm_{EC}}$	42	48	33	45	—	—

$2D$	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	41	47	**69**	**56**	43	37
$\mathrm{GP_M^{PoI}}$	**59**	53	—	—	**76**	**67**	49	39
$\mathrm{lq_M}$	31	44	24	33	—	—	19	33
$\mathrm{lmm_M}$	57	**63**	51	**61**	**81**	**67**	—	—

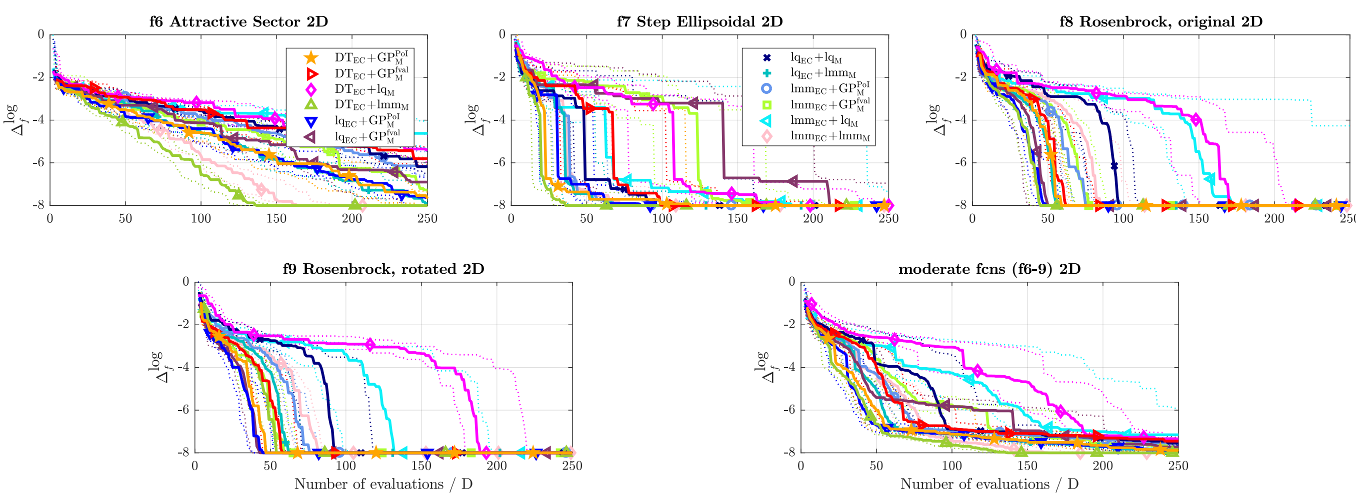

Figure 13: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 4 noiseless COCO benchmarks with low or moderate conditioning in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 14: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the noiseless COCO benchmarks with low or moderate conditioning for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	20	50	**78**	55	39	45	43	62	36	55	51	40	38	60	46	53	49	55	62	**75**	53	40
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	80	50	—	—	**100**	65	55	40	70	55	65	65	65	25	80	60	75	45	70	50	**80**	55	**75**	40
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	22	45	0	35	—	—	13	40	23	55	13	48	17	28	15	35	17	40	17	35	31	58	17	37
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	61	55	45	60	**88**	60	—	—	55	**63**	42	57	73	50	58	58	67	57	67	47	**83**	**73**	**73**	52
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	57	38	30	45	**77**	45	45	37	—	—	49	39	55	28	53	32	60	37	53	38	**67**	62	58	30
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	**64**	45	35	35	**87**	52	58	43	51	61	—	—	59	33	65	42	67	42	**73**	42	**80**	**67**	**73**	42
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	49	60	35	75	**83**	**72**	27	50	45	**72**	41	**67**	—	—	51	67	53	62	55	65	**72**	**87**	54	52
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	62	40	20	40	**85**	65	43	42	47	68	35	58	49	33	—	—	58	47	63	48	**73**	**68**	60	38
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	54	47	25	55	**83**	60	33	43	40	63	33	58	47	38	42	53	—	—	58	47	**75**	**68**	60	43
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	51	45	30	50	**83**	65	33	53	47	62	27	58	45	35	38	52	42	53	—	—	**67**	**78**	45	43
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	38	25	20	45	**69**	42	17	27	33	38	20	33	28	13	27	32	25	32	33	22	—	—	25	25
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	47	60	25	60	**83**	63	28	48	42	**70**	27	58	46	48	40	62	40	58	55	57	**75**	**75**	—	—

3*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	42	51	52	54
$\mathrm{lq_{EC}}$	**58**	49	—	—	**66**	51
$\mathrm{lmm_{EC}}$	48	46	34	49	—	—

3*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	49	44	**69**	51	51	40
$\mathrm{GP_M^{PoI}}$	51	56	—	—	**68**	57	**55**	42
$\mathrm{lq_M}$	31	49	32	43	—	—	29	44
$\mathrm{lmm_M}$	49	**60**	45	58	**71**	**56**	—	—

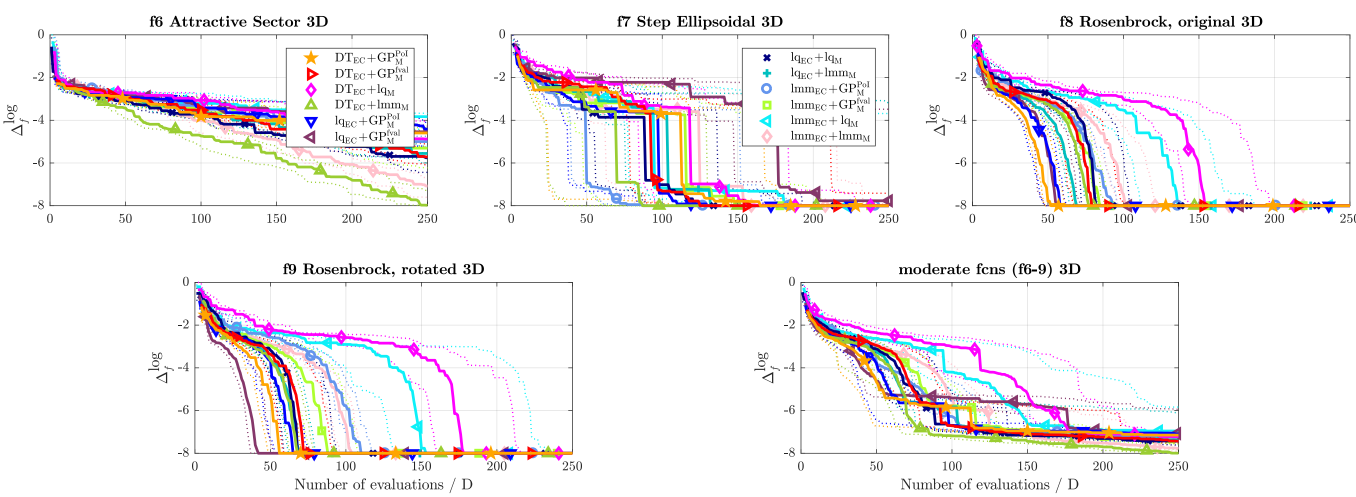

Figure 14: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 4 noiseless COCO benchmarks with low or moderate conditioning in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 15: A pairwise comparison of the evolution controls, models, and their combinations in 5D over the noiseless COCO benchmarks with low or moderate conditioning for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

5*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	55	35	**80**	60	53	53	38	54	41	52	39	23	58	38	68	54	62	38	**75**	62	68	47
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	45	65	—	—	**85**	50	60	60	70	60	65	60	60	40	65	55	65	45	55	30	75	70	70	55
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	20	40	15	50	—	—	17	52	32	48	22	43	13	22	22	33	25	38	15	30	38	53	17	40
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	47	47	40	40	**83**	48	—	—	42	55	33	47	35	28	43	32	57	45	57	33	**73**	47	70	34
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	62	46	30	40	**68**	52	58	45	—	—	52	47	60	27	53	36	57	38	57	33	**63**	50	55	33
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	59	48	35	40	**78**	57	67	53	48	53	—	—	56	33	63	45	**65**	45	58	30	**68**	52	**63**	42
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	61	**77**	40	60	**87**	**78**	65	**72**	40	**73**	44	67	—	—	63	67	67	67	67	65	**82**	**70**	**68**	**73**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	42	63	35	45	**78**	68	57	68	47	64	37	55	38	33	—	—	53	59	53	52	**77**	67	62	53
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	32	46	35	55	**75**	62	43	55	43	63	35	55	33	33	47	41	—	—	55	40	72	67	62	38
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	38	62	45	70	**85**	**70**	43	**67**	43	**67**	42	70	33	35	47	48	45	60	—	—	**70**	**75**	58	62
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	25	38	25	30	62	47	27	53	37	50	32	48	18	30	23	33	28	33	30	25	—	—	40	30
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	32	53	30	45	**83**	60	30	66	45	67	37	58	32	27	38	47	38	62	42	38	60	70	—	—

5*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	35	38	**59**	46
$\mathrm{lq_{EC}}$	**65**	**62**	—	—	**65**	48
$\mathrm{lmm_{EC}}$	42	55	35	52	—	—

5*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	54	42	**71**	51	56	43
$\mathrm{GP_M^{PoI}}$	46	**58**	—	—	**66**	54	**61**	54
$\mathrm{lq_M}$	29	49	34	46	—	—	40	49
$\mathrm{lmm_M}$	44	58	39	46	**60**	51	—	—

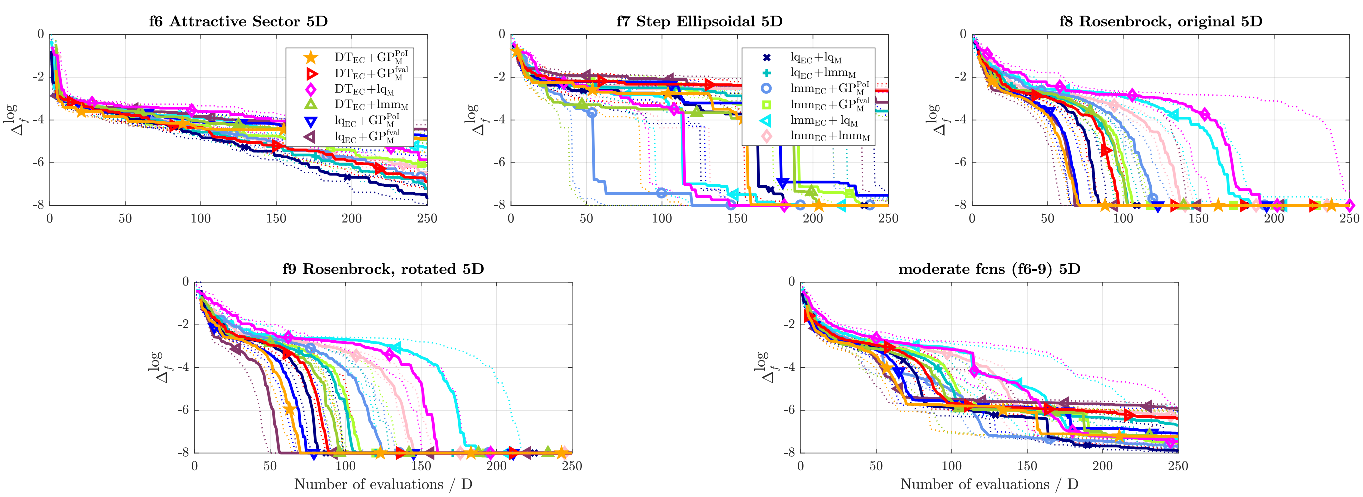

Figure 15: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 4 noiseless COCO benchmarks with low or moderate conditioning in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 16: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noiseless COCO benchmarks with low or moderate conditioning for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10D	$\mathrm{DT_{EC}+GP_M^{fval}}$		$\mathrm{DT_{EC}+GP_M^{PoI}}$		$\mathrm{DT_{EC}+lq_M}$		$\mathrm{DT_{EC}+lmm_M}$		$\mathrm{lq_{EC}+GP_M^{fval}}$		$\mathrm{lq_{EC}+GP_M^{PoI}}$		$\mathrm{lq_{EC}+lq_M}$		$\mathrm{lq_{EC}+lmm_M}$		$\mathrm{lmm_{EC}+GP_M^{fval}}$		$\mathrm{lmm_{EC}+GP_M^{PoI}}$		$\mathrm{lmm_{EC}+lq_M}$		$\mathrm{lmm_{EC}+lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}+GP_M^{fval}}$	—	—	40	40	**80**	57	54	52	43	**77**	52	**67**	52	28	70	45	73	**62**	62	63	**80**	**78**	**80**	**82**
$\mathrm{DT_{EC}+GP_M^{PoI}}$	60	60	—	—	70	70	50	45	55	85	35	75	40	45	55	40	65	60	60	55	80	65	**80**	55
$\mathrm{DT_{EC}+lq_M}$	20	43	30	30	—	—	17	47	30	**77**	33	63	38	35	35	52	43	62	38	53	60	**65**	67	**68**
$\mathrm{DT_{EC}+lmm_M}$	46	48	50	55	**83**	53	—	—	47	**82**	45	**73**	50	45	65	48	65	65	**68**	55	**80**	**60**	**87**	**65**
$\mathrm{lq_{EC}+GP_M^{fval}}$	57	23	45	15	70	23	53	18	—	—	50	46	57	17	65	22	60	47	57	40	**63**	43	**72**	45
$\mathrm{lq_{EC}+GP_M^{PoI}}$	48	33	65	25	67	37	55	27	50	54	—	—	50	25	53	37	55	48	57	48	**70**	47	**65**	48
$\mathrm{lq_{EC}+lq_M}$	48	73	60	55	62	65	50	55	43	**83**	50	**75**	—	—	53	68	58	**70**	57	70	**70**	**78**	73	**83**
$\mathrm{lq_{EC}+lmm_M}$	30	55	45	60	65	48	35	52	35	**78**	47	63	47	33	—	—	58	58	68	68	**85**	**78**	**85**	**77**
$\mathrm{lmm_{EC}+GP_M^{fval}}$	27	38	35	40	57	38	35	35	40	53	45	52	42	30	42	42	—	—	53	40	**77**	70	**75**	**78**
$\mathrm{lmm_{EC}+GP_M^{PoI}}$	38	37	40	45	62	47	32	45	43	60	43	52	43	30	32	32	47	60	—	—	63	**80**	**77**	**87**
$\mathrm{lmm_{EC}+lq_M}$	20	22	20	35	40	35	20	40	37	57	30	53	30	22	15	22	23	30	37	20	—	—	52	**70**
$\mathrm{lmm_{EC}+lmm_M}$	20	18	20	45	33	32	13	35	28	55	35	52	27	17	15	23	25	22	23	13	48	30	—	—

10D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	48	56	**72**	**63**
$\mathrm{lq_{EC}}$	53	45	—	—	**68**	**63**
$\mathrm{lmm_{EC}}$	28	37	32	38	—	—

10D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	50	43	**71**	48	**65**	51
$\mathrm{GP_M^{PoI}}$	50	58	—	—	**59**	55	**63**	59
$\mathrm{lq_M}$	29	52	41	45	—	—	41	**61**
$\mathrm{lmm_M}$	35	49	37	41	**59**	39	—	—

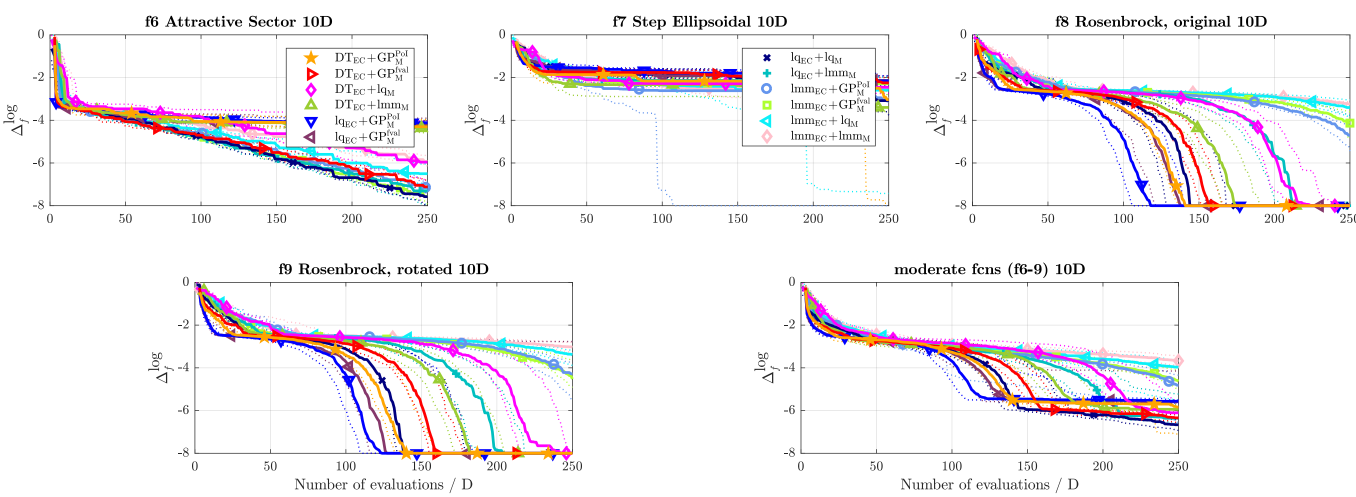

Figure 16: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 4 noiseless COCO benchmarks with low or moderate conditioning in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 17: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the noiseless COCO benchmarks with low or moderate conditioning for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	DT_{EC}+ GP^{fval}_{M}		DT_{EC}+ GP^{PoI}_{M}		DT_{EC}+ lq_{M}		DT_{EC}+ lmm_{M}		lq_{EC}+ GP^{fval}_{M}		lq_{EC}+ GP^{PoI}_{M}		lq_{EC}+lq_{M}		lq_{EC}+ lmm_{M}		lmm_{EC}+ GP^{fval}_{M}		lmm_{EC}+ GP^{PoI}_{M}		lmm_{EC}+ lq_{M}		lmm_{EC}+ lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP^{fval}_{M}	—	—	65	60	73	**72**	73	65	55	**70**	48	57	**72**	23	**82**	**80**	**78**	**77**	**78**	**77**	**83**	**82**	**87**	**78**
DT_{EC}+GP^{PoI}_{M}	35	40	—	—	60	60	60	65	30	**80**	30	60	45	35	55	65	50	60	65	60	55	55	70	55
DT_{EC}+lq_{M}	27	28	40	40	—	—	65	47	40	58	37	53	53	25	**77**	55	72	**77**	**72**	73	**75**	**82**	**92**	**82**
DT_{EC}+lmm_{M}	27	35	40	35	35	53	—	—	32	57	35	35	38	23	57	60	40	70	45	65	50	68	**78**	67
lq_{EC}+GP^{fval}_{M}	45	30	70	20	60	42	68	43	—	—	52	40	62	15	**68**	43	62	53	63	50	**70**	48	**78**	50
lq_{EC}+GP^{PoI}_{M}	52	43	70	40	63	47	65	65	48	60	—	—	55	27	58	52	58	55	62	52	65	50	**72**	50
lq_{EC}+lq_{M}	28	**77**	55	65	47	**75**	62	**77**	38	**85**	45	73	—	—	**67**	**85**	57	**85**	65	**83**	63	**85**	**80**	**87**
lq_{EC}+lmm_{M}	18	20	45	35	23	45	43	40	32	57	42	48	33	15	—	—	35	63	42	67	48	68	**68**	**68**
lmm_{EC}+GP^{fval}_{M}	22	23	50	40	28	23	60	30	38	47	42	45	43	15	65	37	—	—	58	35	60	55	**77**	67
lmm_{EC}+GP^{PoI}_{M}	22	23	35	40	28	27	55	35	37	50	38	48	35	17	58	33	42	65	—	—	45	**77**	**80**	**83**
lmm_{EC}+lq_{M}	17	18	45	45	25	18	50	32	30	52	35	50	37	15	52	32	40	45	55	23	—	—	**70**	63
lmm_{EC}+lmm_{M}	13	22	30	45	8	18	22	33	22	50	28	50	20	13	32	32	23	33	20	17	30	37	—	—

20*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	**53**	53	**76**	**74**
lq_{EC}	48	48	—	—	**64**	**65**
lmm_{EC}	24	27	36	35	—	—

20*D*	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	56	41	**65**	47	**73**	58
GP^{PoI}_{M}	44	59	—	—	51	53	**68**	**67**
lq_{M}	35	53	49	47	—	—	**67**	**65**
lmm_{M}	27	42	32	33	33	35	—	—

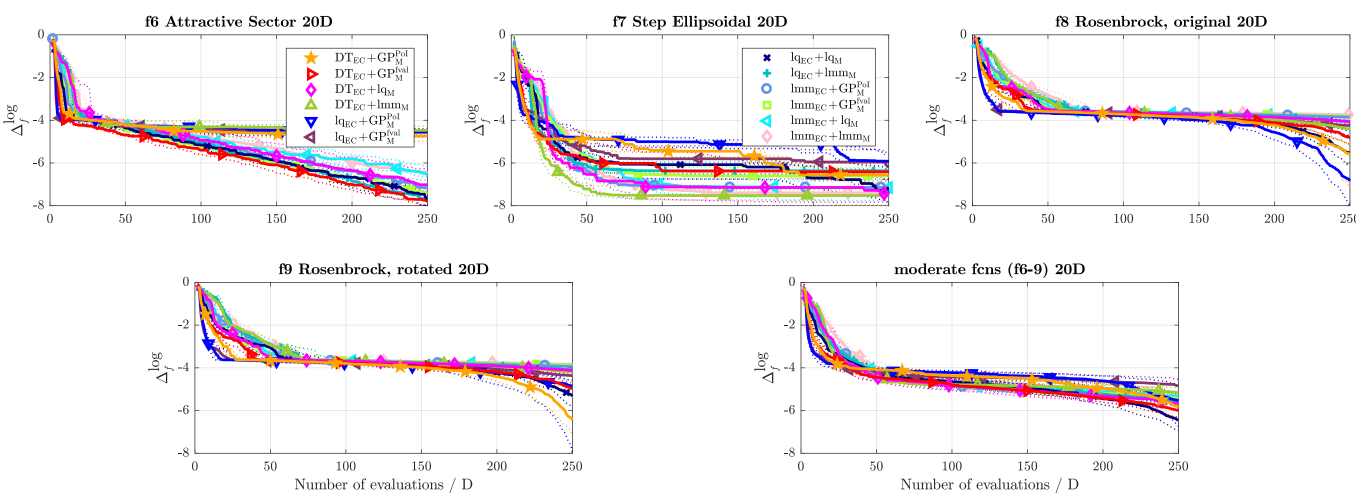

Figure 17: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 4 noiseless COCO benchmarks with low or moderate conditioning in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 18: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noiseless COCO benchmarks with low or moderate conditioning for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	43	46	**78**	**61**	50.3	52	45	**63**	42	54	56	31	59	52	**66**	58	61	57	**73**	**73**	**69**	56
DT_{EC}+GP_M^{PoI}	57	54	—	—	**80**	65	51	51	55	**71**	44	62	56	42	59	54	**65**	55	**63**	52	**74**	**63**	**73**	51
DT_{EC}+lq_M	23	39	20	35	—	—	23	44	29	56	22	49	31	29	31	40	37	51	32	45	48	**64**	41	51
DT_{EC}+lmm_M	49.7	48	49	49	**77**	56	—	—	49	**64**	43	54	**56**	40	**59**	51	**61**	59	**64**	52	**75**	**64**	**77**	53
lq_{EC}+GP_M^{fval}	55	37	45	29	**71**	44	51	36	—	—	47	42	**60**	25	**59**	33	**61**	44	**59**	42	**67**	54	**65**	37
lq_{EC}+GP_M^{PoI}	**58**	46	56	38	**78**	51	57	46	53	**58**	—	—	**60**	34	**60**	44	**65**	49.7	**66**	46	**73**	**56**	**69**	43
lq_{EC}+lq_M	44	**69**	44	58	**69**	**71**	44	**60**	40	**75**	40	**66**	—	—	52	**66**	54	**67**	55	**69**	**68**	**80**	**62**	**68**
lq_{EC}+lmm_M	41	48	41	46	**69**	**60**	42	49	41	**67**	40	56	48	34	—	—	55	56	59	58	**73**	**71**	**69**	54
lmm_{EC}+GP_M^{fval}	35	42	36	45	**64**	49	39	41	39	56	35	50.3	46	33	45	44	—	—	55	43	**69**	**66**	**63**	52
lmm_{EC}+GP_M^{PoI}	39	43	37	48	**68**	55	36	48	41	**58**	34	54	45	31	42	42	45	57	—	—	**63**	**77**	**60**	**61**
lmm_{EC}+lq_M	27	27	26	37	52	36	25	36	33	46	27	44	32	20	27	29	31	34	37	23	—	—	43	41
lmm_{EC}+lmm_M	31	44	27	49	**59**	49	24	48	35	**63**	31	57	38	32	31	46	37	48	40	39	**57**	**59**	—	—

2 − 20*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	45	49	**63**	**58**
lq_{EC}	**55**	51	—	—	**66**	**56**
lmm_{EC}	37	42	34	44	—	—

2 − 20*D*	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	49.9	43	**69**	51	**58**	46
GP_M^{PoI}	50.1	**57**	—	—	**64**	**57**	**59**	52
lq_M	31	49	36	43	—	—	39	51
lmm_M	43	**54**	41	48	**61**	50	—	—

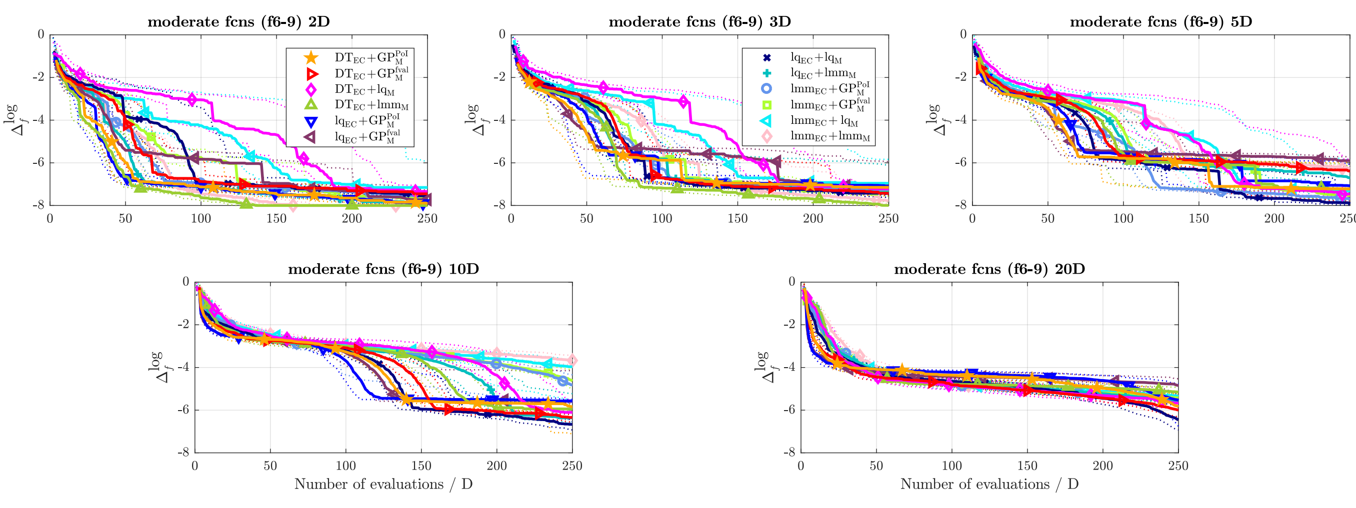

Figure 18: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 4 noiseless COCO benchmarks with low and moderate conditioning in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 19: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the noiseless COCO benchmarks with high conditioning and unimodal benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	24	48	**80**	65	16	48	24	36	21	36	32	45	17	49	64	41	**67**	41	68	**75**	67	51
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	**76**	52	—	—	**92**	56	64	56	68	48	44	28	64	52	68	56	**88**	48	**88**	40	**92**	**72**	**88**	48
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	20	35	8	44	—	—	5	40	13	28	12	28	7	33	3	35	39	37	40	36	39	63	32	36
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**84**	52	36	44	**95**	**60**	—	—	56	44	52	43	55	48	47	52	**88**	48	**95**	44	**89**	**80**	**88**	51
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	**76**	64	32	52	**87**	**72**	44	56	—	—	51	51	52	47	47	55	**91**	51	**88**	49	**85**	**79**	**84**	56
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**79**	64	56	72	**88**	**72**	48	57	49	49	—	—	53	52	44	57	**85**	56	**95**	55	**89**	**80**	**80**	52
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**68**	55	36	48	**93**	**67**	45	52	48	53	47	48	—	—	36	59	**91**	53	**95**	52	**95**	**81**	**91**	57
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	**83**	51	32	44	**97**	**65**	53	48	53	45	56	43	64	41	—	—	**92**	53	**96**	43	**93**	**77**	**93**	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	36	59	12	52	61	**63**	12	52	9	49	15	44	9	47	8	47	—	—	**65**	52	51	**81**	48	49
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	33	59	12	60	60	**64**	5	56	12	51	5	45	5	48	4	57	35	48	—	—	44	**75**	37	51
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	32	25	8	28	61	37	11	20	15	21	11	20	5	19	7	23	49	19	**56**	25	—	—	45	20
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	33	49	12	52	**68**	**64**	12	49	16	44	20	48	9	43	7	49	52	51	**63**	49	55	**80**	—	—

$2D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	28	39	**66**	50.4
$\mathrm{lq_{EC}}$	**72**	**61**	—	—	**93**	**59**
$\mathrm{lmm_{EC}}$	34	49.6	7	41	—	—

$2D$	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	53	51	**61**	**64**	37	51
$\mathrm{GP^{PoI}_M}$	47	49	—	—	55	**62**	44	54
$\mathrm{lq_M}$	39	36	45	38	—	—	29	40
$\mathrm{lmm_M}$	**63**	49	**56**	46	**71**	**60**	—	—

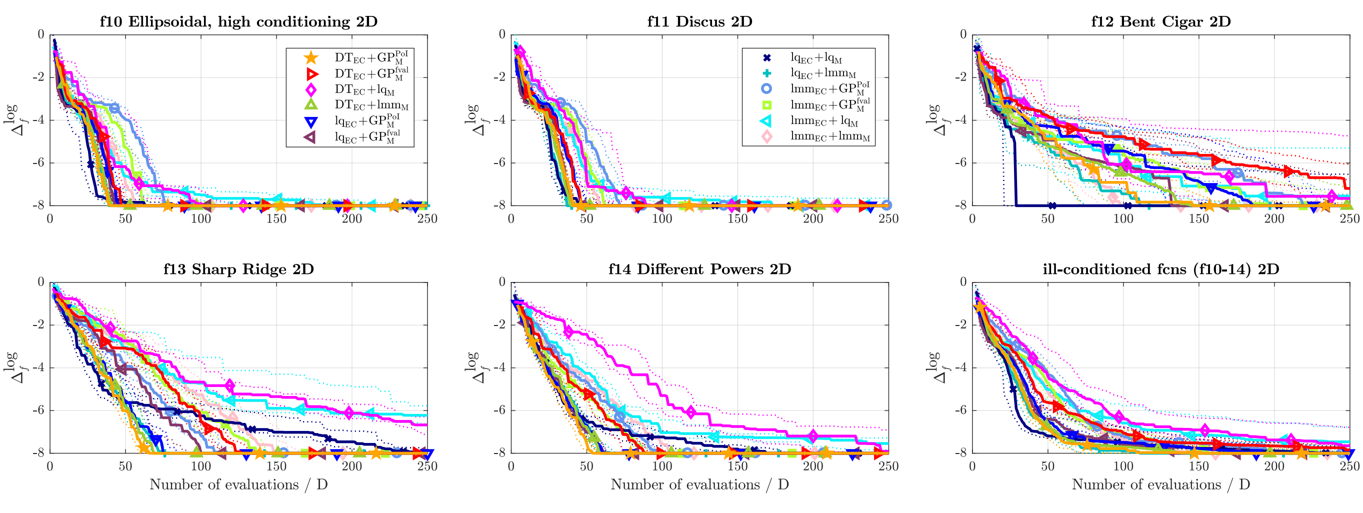

Figure 19: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless COCO benchmarks with high conditioning and unimodal in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 20: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the noiseless COCO benchmarks with high conditioning and unimodal benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3D	$\mathrm{DT_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}+\mathrm{lq_M}$		$\mathrm{DT_{EC}}+\mathrm{lmm_M}$		$\mathrm{lq_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}+\mathrm{lq_M}$		$\mathrm{lq_{EC}}+\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}+\mathrm{lq_M}$		$\mathrm{lmm_{EC}}+\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}+\mathrm{GP^{fval}_M}$	—	—	8	40	**71**	**72**	13	47	35	45	20	48	29	45	21	31	64	40	**68**	49	**64**	**84**	**68**	41
$\mathrm{DT_{EC}}+\mathrm{GP^{PoI}_M}$	**92**	60	—	—	**92**	72	56	52	64	60	56	60	72	60	72	60	**96**	72	**84**	64	**76**	**88**	**92**	72
$\mathrm{DT_{EC}}+\mathrm{lq_M}$	29	28	8	28	—	—	4	29	25	33	9	31	11	27	13	20	33	28	41	32	40	56	49	27
$\mathrm{DT_{EC}}+\mathrm{lmm_M}$	**87**	53	44	48	**96**	**71**	—	—	**67**	48	59	52	68	55	**64**	43	**95**	39	**96**	45	**95**	**80**	**89**	47
$\mathrm{lq_{EC}}+\mathrm{GP^{fval}_M}$	**65**	55	36	40	**75**	**67**	33	52	—	—	33	44	52	52	47	41	**75**	47	**73**	59	**73**	**80**	**72**	45
$\mathrm{lq_{EC}}+\mathrm{GP^{PoI}_M}$	**80**	52	44	40	**91**	**69**	41	48	**67**	56	—	—	59	59	60	41	**89**	57	**91**	60	**91**	**80**	**83**	51
$\mathrm{lq_{EC}}+\mathrm{lq_M}$	**71**	55	28	40	**89**	**73**	32	45	48	48	41	41	—	—	52	36	**92**	40	**93**	47	**91**	**79**	**88**	47
$\mathrm{lq_{EC}}+\mathrm{lmm_M}$	**79**	**69**	28	40	**87**	**80**	36	57	53	59	40	59	48	64	—	—	**89**	61	**89**	63	**87**	**87**	**88**	57
$\mathrm{lmm_{EC}}+\mathrm{GP^{fval}_M}$	36	60	4	28	67	**72**	5	61	25	53	11	43	8	60	11	39	—	—	64	55	55	**80**	65	48
$\mathrm{lmm_{EC}}+\mathrm{GP^{PoI}_M}$	32	51	16	36	59	**68**	4	55	27	41	9	40	7	53	11	37	36	45	—	—	48	**79**	49	47
$\mathrm{lmm_{EC}}+\mathrm{lq_M}$	36	16	24	12	60	44	5	20	27	20	9	20	9	21	13	13	45	20	52	21	—	—	53	16
$\mathrm{lmm_{EC}}+\mathrm{lmm_M}$	32	59	8	28	51	**73**	11	53	28	55	17	49	12	53	12	43	35	52	51	53	47	**84**	—	—

3D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	38	40	**66**	49
$\mathrm{lq_{EC}}$	**62**	**60**	—	—	**86**	**61**
$\mathrm{lmm_{EC}}$	34	51	14	39	—	—

3D	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	43	48	59	**68**	42	45
$\mathrm{GP^{PoI}_M}$	**57**	52	—	—	59	**69**	55	45
$\mathrm{lq_M}$	41	32	41	31	—	—	36	27
$\mathrm{lmm_M}$	**58**	55	45	55	**64**	**73**	—	—

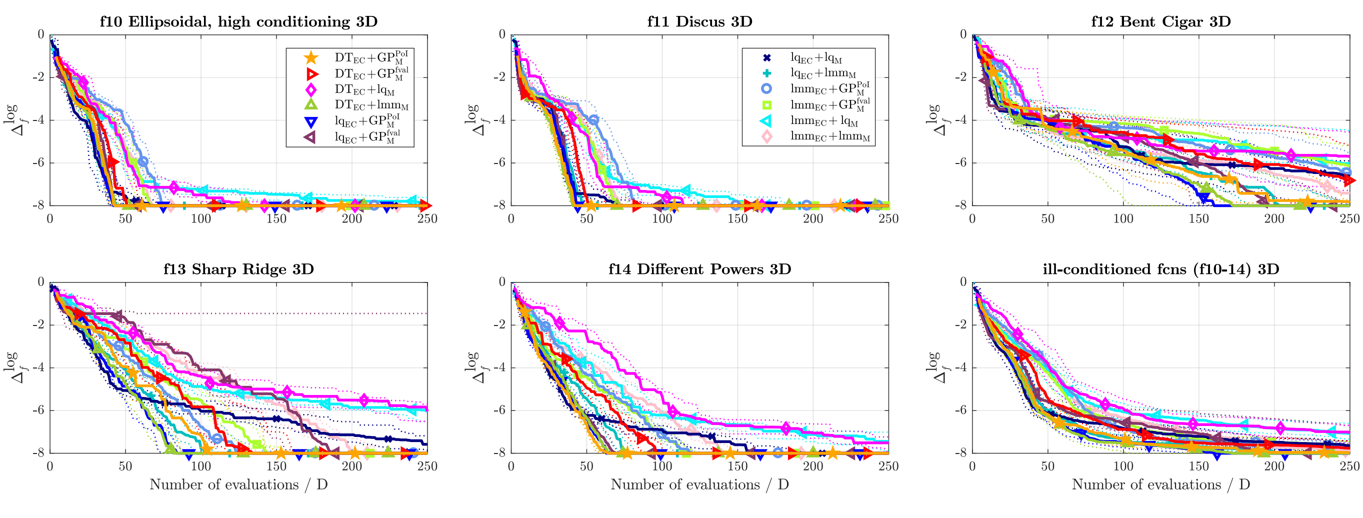

Figure 20: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless COCO benchmarks with high conditioning and unimodal in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 21: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the noiseless COCO benchmarks with high conditioning and unimodal benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

5D	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	20	36	55	**67**	9	41	47	45	23	49	17	59	57	40	**65**	47	**71**	47	55	**73**	**68**	44
DT_{EC}+GP_M^{PoI}	**80**	64	—	—	**84**	72	56	76	**76**	52	68	60	56	68	**80**	60	**80**	72	**92**	64	**76**	76	**80**	56
DT_{EC}+lq_M	45	33	16	28	—	—	11	23	51	27	29	28	21	33	41	29	55	32	56	35	51	41	**61**	24
DT_{EC}+lmm_M	**91**	59	44	24	**89**	**77**	—	—	**73**	57	45	61	49	**64**	**88**	51	**92**	59	**97**	57	**88**	**76**	**89**	57
lq_{EC}+GP_M^{fval}	53	55	24	48	49	**73**	27	43	—	—	37	55	21	57	63	48	**65**	55	**72**	53	57	**77**	**69**	57
lq_{EC}+GP_M^{PoI}	**77**	51	32	40	**71**	**72**	55	39	**63**	45	—	—	49	63	**71**	48	**80**	53	**80**	51	**71**	**75**	**80**	57
lq_{EC}+lq_M	**83**	41	44	32	**79**	**67**	51	36	**79**	43	51	37	—	—	**88**	41	**92**	44	**96**	41	**89**	**72**	**92**	37
lq_{EC}+lmm_M	43	60	20	40	59	**71**	12	49	37	52	29	52	12	59	—	—	59	53	**69**	51	52	**72**	67	59
lmm_{EC}+GP_M^{fval}	35	53	20	28	45	**68**	8	41	35	45	20	47	8	56	41	47	—	—	64	44	39	**72**	57	52
lmm_{EC}+GP_M^{PoI}	29	53	8	36	44	**65**	3	43	28	47	20	49	4	59	31	49	36	56	—	—	36	**68**	45	55
lmm_{EC}+lq_M	45	27	24	24	49	59	12	24	43	23	29	25	11	28	48	28	61	28	**64**	32	—	—	65	25
lmm_{EC}+lmm_M	32	56	20	44	39	**76**	11	43	31	43	20	43	8	63	33	41	43	48	55	45	35	**75**	—	—

5D	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	54	45	**71**	50
lq_{EC}	46	**55**	—	—	**75**	**59**
lmm_{EC}	29	50	25	41	—	—

5D	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	46	47	38	**65**	43	47
GP_M^{PoI}	**54**	53	—	—	49	**66**	**58**	55
lq_M	**62**	35	51	34	—	—	55	30
lmm_M	**57**	53	42	45	45	**70**	—	—

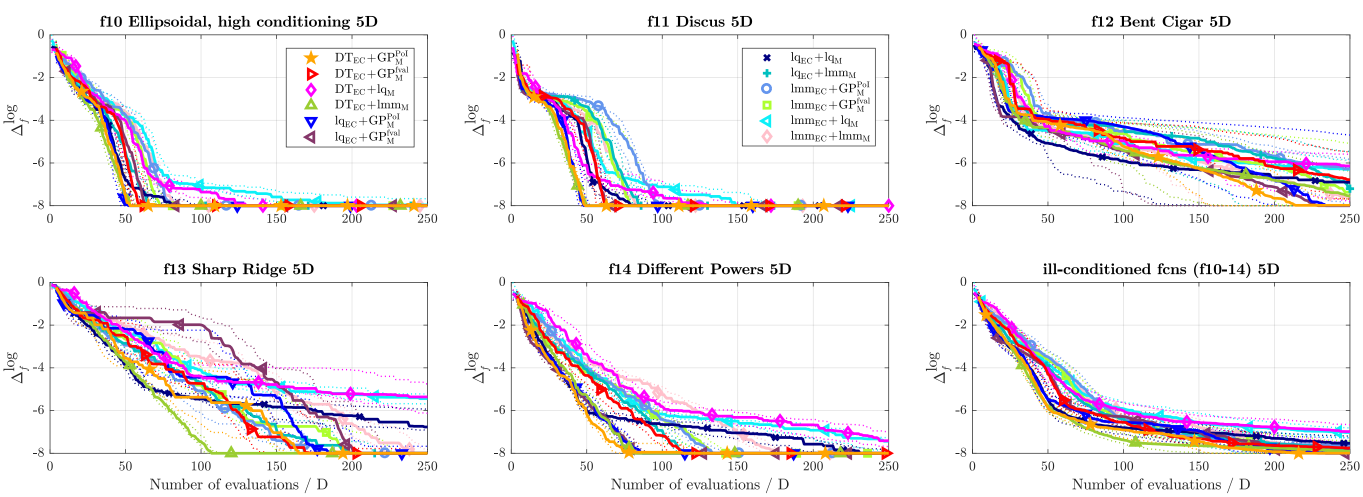

Figure 21: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless COCO benchmarks with high conditioning and unimodal in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 22: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noiseless COCO benchmarks with high conditioning and unimodal benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	36	40	59	**84**	43	52	52	**69**	40	56	49	**72**	**79**	**61**	**89**	**83**	**85**	**68**	**88**	**89**	**95**	**80**
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	64	60	—	—	**80**	**84**	60	52	64	64	60	64	64	**72**	**96**	64	**100**	**88**	**92**	**80**	**100**	**96**	**100**	**80**
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	41	16	20	16	—	—	21	25	44	45	41	31	39	35	**72**	39	**87**	53	**83**	32	**95**	**69**	**93**	59
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	57	48	40	48	**79**	**75**	—	—	56	**71**	51	57	52	**64**	**91**	59	**97**	**76**	**100**	**67**	**96**	**81**	**100**	**68**
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	48	31	36	36	56	55	44	29	—	—	40	32	55	47	**73**	39	**81**	55	**81**	47	**88**	**60**	**95**	60
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	60	44	40	36	**59**	**69**	49	43	60	**68**	—	—	57	**63**	**79**	51	**89**	**67**	**88**	**69**	**96**	**80**	**99**	**68**
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	51	28	36	28	61	65	48	36	45	53	43	37	—	—	**84**	48	**95**	**65**	**93**	53	**97**	**76**	**99**	**72**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	21	39	4	36	28	61	9	41	27	61	21	49	16	52	—	—	**76**	61	**72**	51	**71**	**68**	**84**	57
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	11	17	0	12	13	47	3	24	19	45	11	33	5	35	24	39	—	—	52	31	53	60	**65**	57
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	15	32	8	20	17	68	0	33	19	53	12	31	7	47	28	49	48	**69**	—	—	55	**76**	65	**69**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	12	11	0	4	5	31	4	19	12	40	4	20	3	24	29	32	47	40	45	24	—	—	**65**	39
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	5	20	0	20	7	41	0	32	5	40	1	32	1	28	16	43	35	43	35	31	35	61	—	—

10*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**60**	**55**	**94**	**74**
$\mathrm{lq_{EC}}$	40	45	—	—	**88**	**64**
$\mathrm{lmm_{EC}}$	6	26	12	36	—	—

10*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	45	33	56	**64**	**60**	49
$\mathrm{GP^{PoI}_{M}}$	55	**67**	—	—	**59**	**71**	**70**	**59**
$\mathrm{lq_{M}}$	44	36	41	29	—	—	**57**	37
$\mathrm{lmm_{M}}$	40	51	30	41	43	**63**	—	—

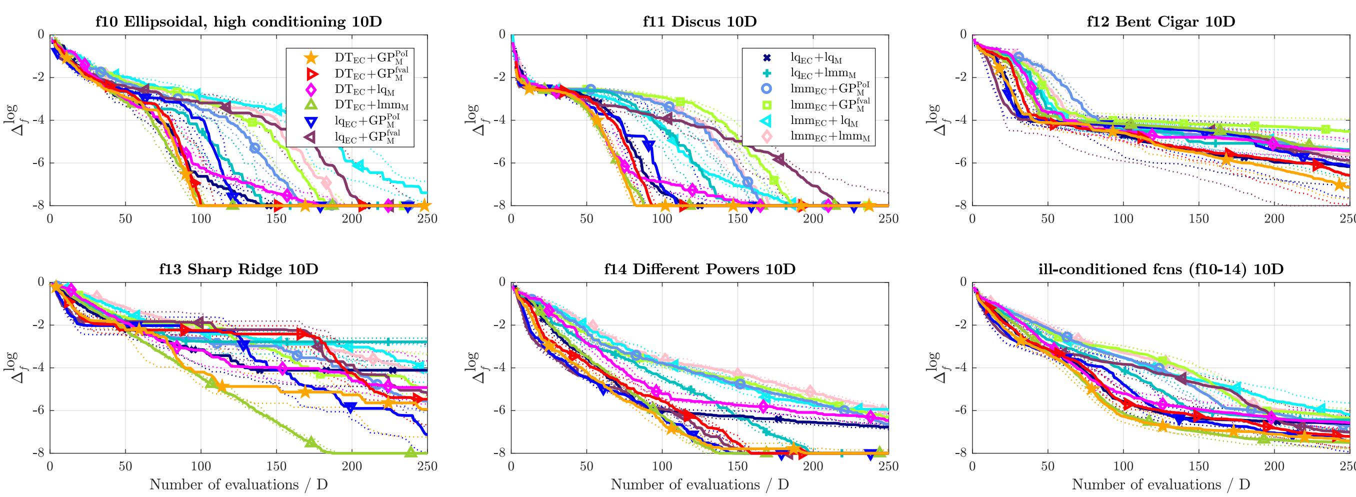

Figure 22: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless COCO benchmarks with high conditioning and unimodal in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 23: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the noiseless COCO benchmarks with high conditioning and unimodal benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	32	36	52	64	**59**	33	31	**88**	27	**76**	**76**	**71**	**89**	**80**	**91**	76	**85**	**79**	**83**	**77**	**87**	**65**
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	68	64	—	—	60	68	60	32	44	**88**	40	**80**	**84**	72	**88**	**84**	**96**	**84**	**96**	**84**	**92**	**84**	**92**	68
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	48	36	40	32	—	—	45	19	28	**75**	31	**57**	44	51	**81**	**67**	**71**	**83**	**67**	**81**	**80**	**80**	**91**	**65**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	41	**67**	40	68	55	**81**	—	—	27	**95**	35	**93**	51	**81**	**87**	**95**	**69**	**92**	**71**	**93**	**73**	**88**	**92**	**80**
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	**69**	12	56	12	**72**	25	**73**	5	—	—	49	21	**79**	39	**93**	35	**92**	47	**88**	32	**96**	39	**91**	29
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**73**	24	60	20	**69**	43	**65**	7	51	**79**	—	—	**83**	45	**92**	45	**95**	59	**91**	43	**91**	52	**97**	40
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	24	29	16	28	56	49	49	19	21	**61**	17	**55**	—	—	**79**	57	**81**	**76**	**65**	**73**	**77**	**75**	**81**	57
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	11	20	12	16	19	33	13	5	7	**65**	8	55	21	43	—	—	35	**73**	36	60	48	59	67	49
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	9	24	4	16	29	17	31	8	8	53	5	41	19	24	65	27	—	—	52	31	55	31	**68**	21
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	15	21	4	16	33	19	29	7	12	**68**	9	57	35	27	64	40	48	**69**	—	—	52	45	**69**	29
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	17	23	8	16	20	20	27	12	4	**61**	9	48	23	25	52	41	45	69	48	55	—	—	**71**	35
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	13	35	8	32	9	35	8	20	9	**71**	3	**60**	19	43	33	51	32	**79**	31	**71**	29	**65**	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	52	**78**	**88**	**79**
$\mathrm{lq_{EC}}$	48	22	—	—	**82**	53
$\mathrm{lmm_{EC}}$	12	21	18	47	—	—

20*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	48	27	**62**	44	**73**	30
$\mathrm{GP^{PoI}_M}$	52	**73**	—	—	**66**	49	**78**	37
$\mathrm{lq_M}$	38	**56**	34	51	—	—	**65**	37
$\mathrm{lmm_M}$	27	**70**	22	**63**	35	**63**	—	—

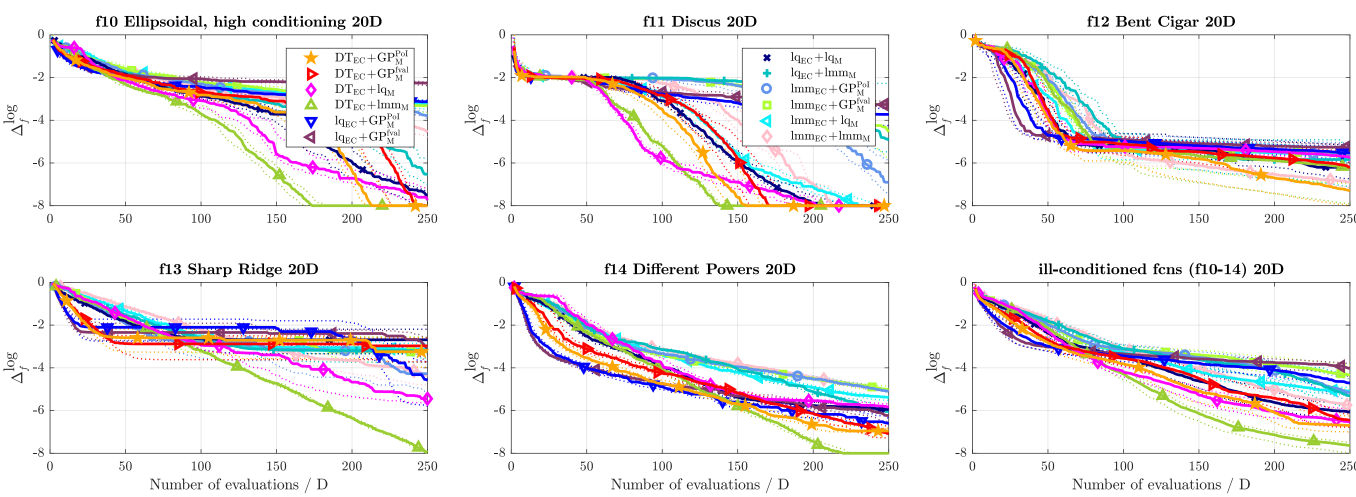

Figure 23: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless COCO benchmarks with high conditioning and unimodal in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 24: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noiseless COCO benchmarks with high conditioning and unimodal benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	24	40	**63**	**70**	28	44	38	**57**	26	53	41	**58**	53	52	**75**	**57**	**75**	**57**	**71**	**80**	**77**	**56**
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	**76**	60	—	—	**82**	**70**	59	54	**63**	**62**	54	58	68	**65**	**81**	**65**	**92**	**73**	**90**	**66**	**87**	**83**	**90**	**65**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	37	30	18	30	—	—	17	27	32	42	25	35	24	36	42	38	57	47	**57**	43	61	**62**	**65**	42
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**72**	**56**	41	46	**83**	**73**	—	—	**56**	**63**	48	**61**	55	**62**	**75**	**60**	**88**	**63**	**92**	**61**	**88**	**81**	**92**	**61**
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	**62**	43	37	38	**68**	**58**	44	37	—	—	42	41	52	48	**65**	43	**81**	51	**81**	48	**80**	**67**	**82**	49.6
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	**74**	47	46	42	**75**	**65**	52	39	**58**	**59**	—	—	**60**	56	**69**	49	**88**	**58**	**89**	55	**87**	**73**	**88**	54
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**59**	42	32	35	**76**	**64**	45	38	48	52	40	44	—	—	**68**	48	**90**	**56**	**89**	53	**90**	**77**	**90**	54
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	47	48	19	35	**58**	**62**	25	40	35	57	31	51	32	52	—	—	**70**	**61**	**73**	53	**70**	**73**	**80**	55
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	25	43	8	27	43	53	12	37	19	49	12	42	10	44	30	39	—	—	**59**	42	50.4	**65**	**61**	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	25	43	10	34	43	**57**	8	39	19	52	11	45	11	47	27	47	41	**58**	—	—	47	**69**	53	50.1
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	29	20	13	17	39	38	12	19	20	33	13	27	10	23	30	27	49.6	35	**53**	31	—	—	**60**	27
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	23	44	10	35	35	**58**	8	39	18	50.4	12	46	10	46	20	45	39	**54**	47	49.9	40	**73**	—	—

2 − 20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	46	**52**	**77**	**61**
$\mathrm{lq_{EC}}$	**54**	48	—	—	**85**	**59**
$\mathrm{lmm_{EC}}$	23	39	15	41	—	—

2 − 20*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	47	41	**55**	**61**	51	44
$\mathrm{GP^{PoI}_M}$	**53**	**59**	—	—	**58**	**64**	**61**	49.9
$\mathrm{lq_M}$	45	39	42	36	—	—	48	34
$\mathrm{lmm_M}$	49	**56**	39	50.1	**52**	**66**	—	—

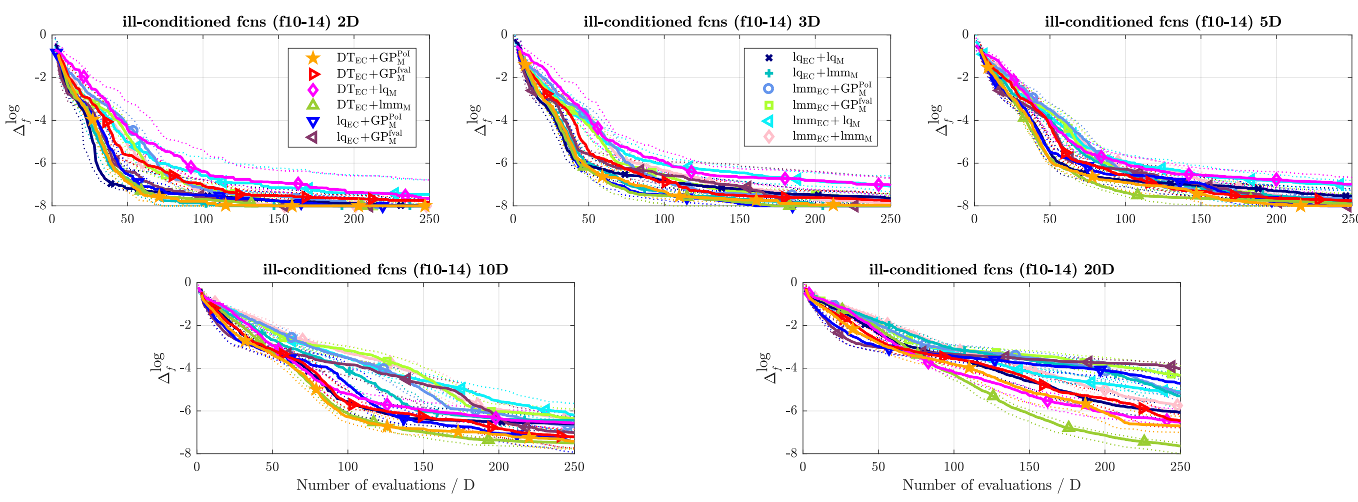

Figure 24: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless COCO benchmarks with high conditioning and unimodal in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 25: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the noiseless multi-modal COCO benchmarks with adequate global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$DT_{EC}+GP^{fval}_{M}$		$DT_{EC}+GP^{PoI}_{M}$		$DT_{EC}+lq_{M}$		$DT_{EC}+lmm_{M}$		$lq_{EC}+GP^{fval}_{M}$		$lq_{EC}+GP^{PoI}_{M}$		$lq_{EC}+lq_{M}$		$lq_{EC}+lmm_{M}$		$lmm_{EC}+GP^{fval}_{M}$		$lmm_{EC}+GP^{PoI}_{M}$		$lmm_{EC}+lq_{M}$		$lmm_{EC}+lmm_{M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$DT_{EC}+GP^{fval}_{M}$	—	—	32	36	57	45	37	27	51	61	44	44	48	47	40	36	47	51	36	35	**71**	71	36	33
$DT_{EC}+GP^{PoI}_{M}$	68	64	—	—	56	68	60	52	60	72	48	60	56	60	60	44	52	62	44	44	68	76	40	56
$DT_{EC}+lq_{M}$	43	55	44	32	—	—	31	31	53	**63**	31	49	36	43	29	35	37	57	29	44	56	**72**	25	41
$DT_{EC}+lmm_{M}$	63	**73**	40	48	**69**	**69**	—	—	**68**	**81**	49	63	63	61	52	55	57	**69**	51	53	**81**	**83**	57	63
$lq_{EC}+GP^{fval}_{M}$	49	39	40	28	47	37	32	19	—	—	32	40	37	34	35	29	41	33	31	23	49	43	35	29
$lq_{EC}+GP^{PoI}_{M}$	56	56	52	40	**69**	51	51	37	**68**	60	—	—	57	43	56	40	59	51	43	34	**75**	60	56	46
$lq_{EC}+lq_{M}$	52	53	44	40	64	57	37	39	63	**66**	43	57	—	—	44	41	49	61	36	44	**76**	**75**	40	49
$lq_{EC}+lmm_{M}$	60	64	40	56	**71**	65	48	45	65	**71**	44	60	56	59	—	—	64	62	49	48	**77**	**76**	49	56
$lmm_{EC}+GP^{fval}_{M}$	53	49	48	38	63	43	43	31	59	67	41	49	51	39	36	38	—	—	33	35	59	61	47	41
$lmm_{EC}+GP^{PoI}_{M}$	64	65	56	56	**71**	56	49	47	**69**	**77**	57	66	64	56	51	52	67	65	—	—	**87**	**79**	55	56
$lmm_{EC}+lq_{M}$	29	29	32	24	44	28	19	17	51	57	25	40	24	25	23	24	41	39	13	21	—	—	20	27
$lmm_{EC}+lmm_{M}$	64	67	60	44	**75**	59	43	37	**65**	**71**	44	54	60	51	51	44	53	59	45	44	**80**	**73**	—	—

$2D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	46	53	52	**60**
lq_{EC}	54	47	—	—	52	49
lmm_{EC}	48	40	48	51	—	—

$2D$	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	33	37	51	47	40	32
GP^{PoI}_{M}	**67**	**63**	—	—	**70**	**62**	56	49
lq_{M}	49	53	30	38	—	—	32	33
lmm_{M}	**60**	**68**	44	51	**68**	**67**	—	—

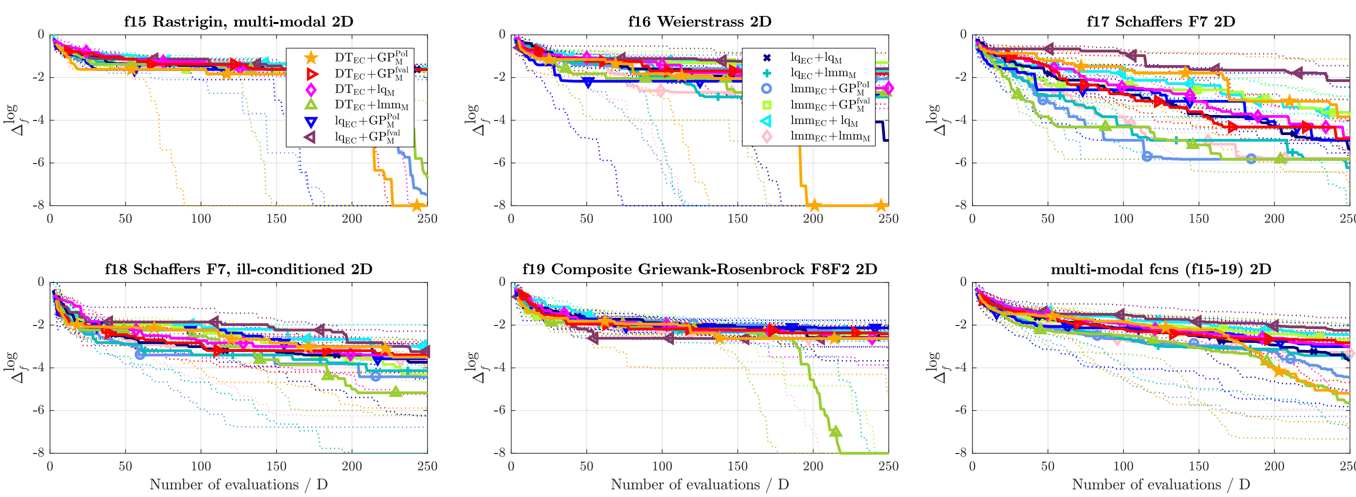

Figure 25: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with adequate global structure in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 26: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the noiseless multi-modal COCO benchmarks with adequate global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3*D*	DT_{EC}+GP^{fval}_{M}		DT_{EC}+GP^{PoI}_{M}		DT_{EC}+lq_{M}		DT_{EC}+lmm_{M}		lq_{EC}+GP^{fval}_{M}		lq_{EC}+GP^{PoI}_{M}		lq_{EC}+lq_{M}		lq_{EC}+lmm_{M}		lmm_{EC}+GP^{fval}_{M}		lmm_{EC}+GP^{PoI}_{M}		lmm_{EC}+lq_{M}		lmm_{EC}+lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP^{fval}_{M}	—	—	56	56	**77**	50	33	43	60	**66**	49	65	61	49	47	44	49	57	29	51	**76**	61	49	39
DT_{EC}+GP^{PoI}_{M}	44	44	—	—	68	56	24	32	64	60	40	68	64	52	60	40	48	60	36	44	72	48	44	36
DT_{EC}+lq_{M}	23	50	32	44	—	—	9	38	37	**67**	27	63	33	43	21	43	24	67	12	53	48	**71**	28	33
DT_{EC}+lmm_{M}	**67**	57	76	68	**91**	62	—	—	**69**	**75**	**65**	**68**	**77**	56	**71**	51	**68**	65	45	55	**91**	**77**	**75**	41
lq_{EC}+GP^{fval}_{M}	40	34	36	40	63	33	31	25	—	—	41	44	45	31	41	31	39	37	25	28	60	43	39	25
lq_{EC}+GP^{PoI}_{M}	51	35	60	32	**73**	37	35	32	59	56	—	—	60	37	49	36	45	52	41	39	**72**	45	51	34
lq_{EC}+lq_{M}	39	51	36	48	**67**	57	23	44	55	**69**	40	63	—	—	29	49	35	62	17	46	60	**65**	33	41
lq_{EC}+lmm_{M}	53	56	40	60	**79**	57	29	49	59	**69**	51	**64**	**71**	51	—	—	47	59	31	49	**76**	**64**	61	47
lmm_{EC}+GP^{fval}_{M}	51	43	52	40	**76**	33	32	35	61	63	55	48	**65**	38	53	41	—	—	33	41	**76**	53	59	33
lmm_{EC}+GP^{PoI}_{M}	**71**	49	64	56	**88**	47	55	45	**75**	**72**	59	61	**83**	54	**69**	51	67	59	—	—	**84**	65	**79**	51
lmm_{EC}+lq_{M}	24	39	28	52	52	29	9	23	40	57	28	55	40	35	24	36	24	47	16	35	—	—	28	28
lmm_{EC}+lmm_{M}	51	61	56	64	**72**	67	25	59	61	**75**	49	**66**	67	59	39	53	41	67	21	49	**72**	**72**	—	—

3*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	53	55	55	55
lq_{EC}	47	45	—	—	50	47
lmm_{EC}	45	45	50	53	—	—

3*D*	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	40	45	**66**	45	44	36
GP^{PoI}_{M}	**60**	55	—	—	**71**	52	**58**	42
lq_{M}	34	55	29	48	—	—	22	38
lmm_{M}	**56**	**64**	42	**58**	**78**	**62**	—	—

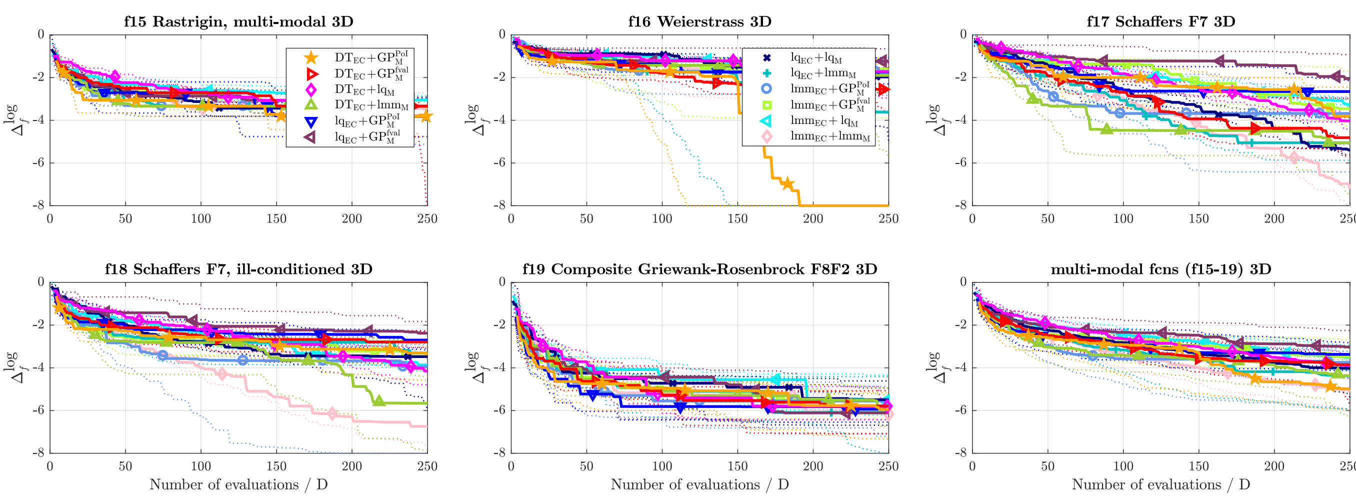

Figure 26: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with adequate global structure in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 27: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the noiseless multi-modal COCO benchmarks with adequate global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

5D	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	48	52	**69**	43	33	56	43	41	40	47	65	40	43	54	37	53	19	33	**68**	59	55	32
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	52	48	—	—	64	32	48	36	36	52	36	48	68	36	44	40	32	40	20	24	76	56	40	24
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	31	57	36	68	—	—	17	64	25	53	23	53	31	52	27	64	17	59	8	37	36	**76**	20	40
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	67	44	52	64	**83**	36	—	—	56	45	49	47	**72**	36	65	54	47	55	32	33	**84**	64	**67**	28
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	57	59	64	48	**75**	47	44	55	—	—	43	52	**69**	48	56	50	45	56	32	40	**69**	60	59	34
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	60	53	64	52	**77**	47	51	53	57	48	—	—	**72**	51	59	52	49	55	36	41	**69**	55	60	43
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	35	60	32	64	69	48	28	64	31	52	28	49	—	—	32	59	23	61	11	42	64	**72**	35	43
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	57	46	56	60	**73**	36	35	46	44	50	41	48	**68**	41	—	—	36	53	20	29	**72**	59	56	37
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	63	47	68	60	**83**	41	53	45	55	44	51	45	**77**	39	64	47	—	—	33	25	**79**	68	**68**	30
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	**81**	**67**	**80**	76	**92**	63	**68**	**67**	**68**	60	**64**	59	**89**	58	**80**	71	**67**	**75**	—	—	**96**	**75**	**84**	49
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	32	41	24	44	64	24	16	36	31	40	31	45	36	28	28	41	21	32	4	25	—	—	27	19
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	45	**68**	60	76	**80**	60	33	**72**	41	**66**	40	57	**65**	57	44	63	32	**70**	16	51	**73**	**81**	—	—

5D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	45	49	44	49
$\mathrm{lq_{EC}}$	55	51	—	—	50	51
$\mathrm{lmm_{EC}}$	56	51	50	49	—	—

5D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	39	41	**72**	53	52	45
$\mathrm{GP_M^{PoI}}$	**61**	59	—	—	**81**	58	**68**	48
$\mathrm{lq_M}$	28	47	19	42	—	—	25	47
$\mathrm{lmm_M}$	48	55	32	52	**75**	53	—	—

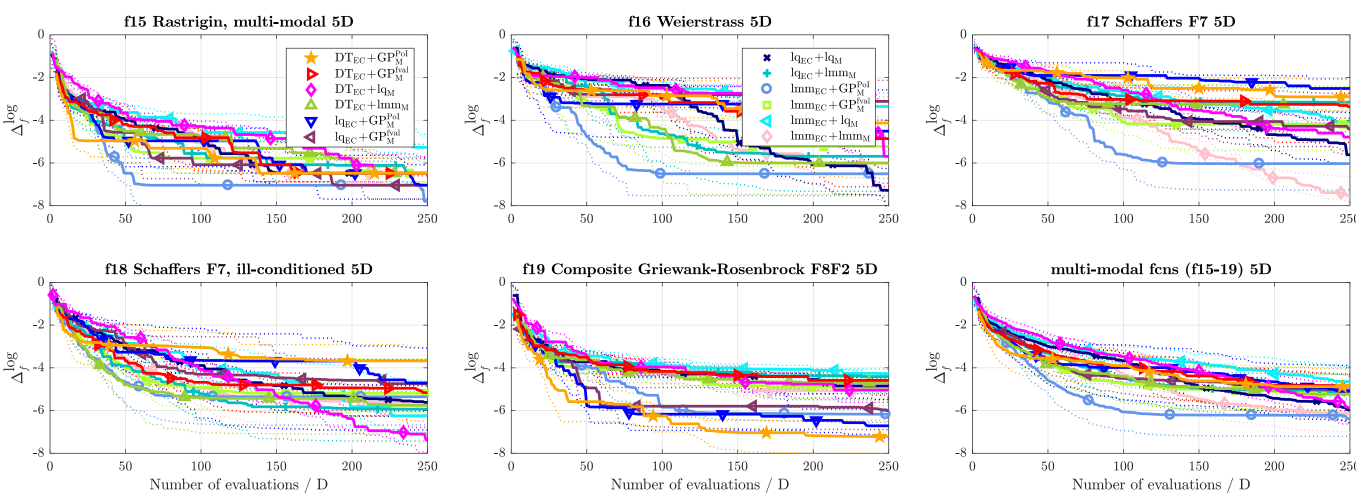

Figure 27: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with adequate global structure in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 28: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noiseless multi-modal COCO benchmarks with adequate global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$10D$	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	20	52	**77**	57	49	**68**	40	44	53	**64**	56	48	**65**	**77**	61	39	61	25	**76**	53	**75**	32
DT_{EC}+GP_M^{PoI}	80	48	—	—	**100**	48	64	64	56	52	72	56	**84**	36	**92**	**88**	**80**	40	**84**	40	**96**	56	**92**	36
DT_{EC}+lq_M	23	43	0	52	—	—	23	64	12	36	17	59	24	36	28	**75**	24	36	19	21	33	53	39	25
DT_{EC}+lmm_M	51	32	36	36	**77**	36	—	—	36	24	47	37	57	25	**71**	61	**61**	23	**64**	12	**80**	29	**76**	16
lq_{EC}+GP_M^{fval}	60	56	44	48	**88**	64	64	76	—	—	59	67	**76**	45	**79**	**83**	**71**	45	**72**	47	**84**	55	**80**	35
lq_{EC}+GP_M^{PoI}	47	36	28	44	**83**	41	53	63	41	33	—	—	61	31	**71**	68	59	29	57	22	**75**	39	**73**	21
lq_{EC}+lq_M	44	52	16	64	**76**	64	43	**75**	24	55	39	**69**	—	—	64	**88**	47	41	44	32	**76**	63	**64**	40
lq_{EC}+lmm_M	35	23	8	12	**72**	25	29	39	21	17	29	32	36	12	—	—	40	11	33	5	63	21	60	8
lmm_{EC}+GP_M^{fval}	39	61	20	60	**76**	**64**	39	**77**	29	55	41	**71**	53	59	60	**89**	—	—	49	35	**80**	**71**	**71**	45
lmm_{EC}+GP_M^{PoI}	39	**75**	16	60	**81**	**79**	36	**88**	28	53	43	**78**	56	68	**67**	**95**	51	65	—	—	**76**	**76**	72	59
lmm_{EC}+lq_M	24	47	4	44	**67**	47	20	**71**	16	45	25	61	24	37	37	**79**	20	29	24	24	—	—	39	25
lmm_{EC}+lmm_M	25	**68**	8	64	**61**	**75**	24	**84**	20	**65**	27	**79**	36	60	40	**92**	29	55	28	41	61	**75**	—	—

$10D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	48	48	**60**	36
lq_{EC}	52	52	—	—	**66**	34
lmm_{EC}	40	**64**	34	**66**	—	—

$10D$	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	49	51	**78**	58	**66**	**65**
GP_M^{PoI}	51	49	—	—	**73**	53	**70**	**63**
lq_M	22	42	27	47	—	—	42	**59**
lmm_M	34	35	30	37	**58**	41	—	—

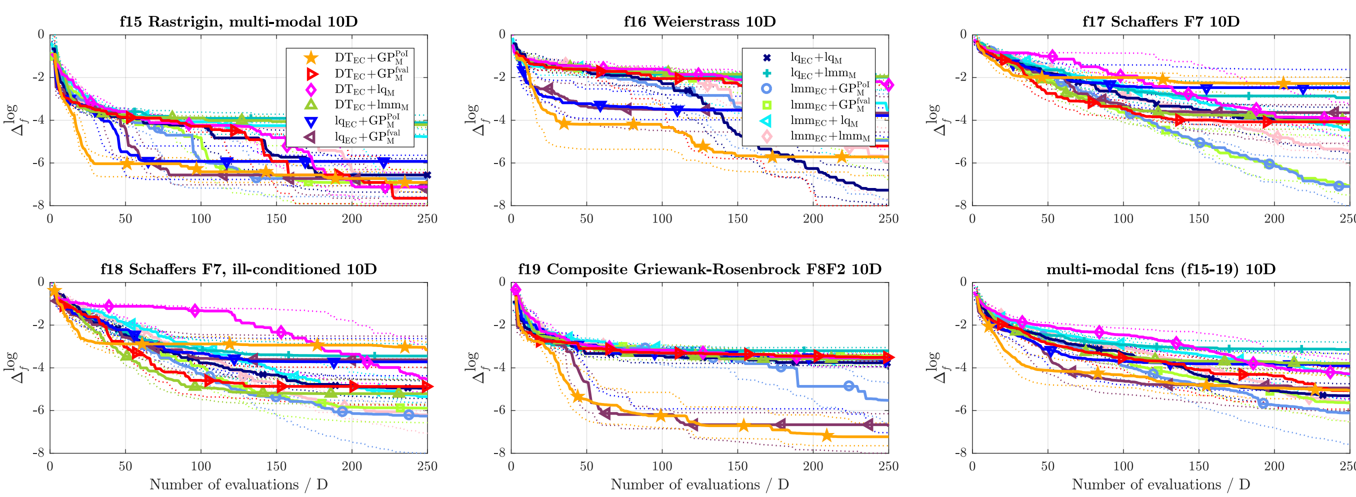

Figure 28: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with adequate global structure in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 29: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the noiseless multi-modal COCO benchmarks with adequate global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	36	48	**72**	61	**71**	**75**	40	43	41	56	**61**	48	**76**	**77**	51	36	52	20	**79**	40	**80**	25
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	64	52	—	—	**96**	56	**84**	64	68	36	56	48	**84**	28	**88**	56	76	28	**80**	20	**84**	44	**88**	28
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	28	39	4	44	—	—	37	65	15	24	7	39	36	36	37	64	27	27	25	27	40	52	39	28
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	29	25	16	36	**63**	35	—	—	25	27	28	36	41	23	**63**	65	29	13	28	8	60	16	60	8
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	60	57	32	64	**85**	**76**	**75**	73	—	—	52	61	**73**	48	**88**	**83**	**67**	32	**65**	28	**84**	52	**84**	32
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	59	44	44	52	**93**	61	**72**	64	48	39	—	—	**64**	31	**77**	67	64	27	64	20	**72**	49	**76**	28
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	39	52	16	72	**64**	64	59	**77**	27	52	36	69	—	—	**64**	**91**	37	36	41	21	60	51	**61**	35
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	24	23	12	44	**63**	36	37	35	12	17	23	33	36	9	—	—	20	4	23	4	37	16	41	9
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	49	**64**	24	72	**73**	**73**	**71**	**87**	33	**68**	36	**73**	63	**64**	**80**	**96**	—	—	44	44	**75**	**67**	**83**	49
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	48	**80**	20	**80**	**75**	**73**	**72**	**92**	35	**72**	36	**80**	59	**79**	**77**	**96**	56	56	—	—	**71**	**65**	**72**	55
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	21	60	16	56	**60**	48	40	**84**	16	48	28	51	40	49	63	**84**	25	33	29	35	—	—	60	27
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	20	**75**	12	72	**61**	**72**	40	**92**	16	**68**	24	**72**	39	**65**	59	**91**	17	51	28	45	40	**73**	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	47	48	53	31
$\mathrm{lq_{EC}}$	53	52	—	—	**58**	28
$\mathrm{lmm_{EC}}$	47	**69**	42	**72**	—	—

20*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	46	52	**73**	**59**	**80**	**69**
$\mathrm{GP^{PoI}_M}$	54	48	—	—	**71**	49	**76**	**61**
$\mathrm{lq_M}$	27	41	29	51	—	—	54	61
$\mathrm{lmm_M}$	20	31	24	39	46	39	—	—

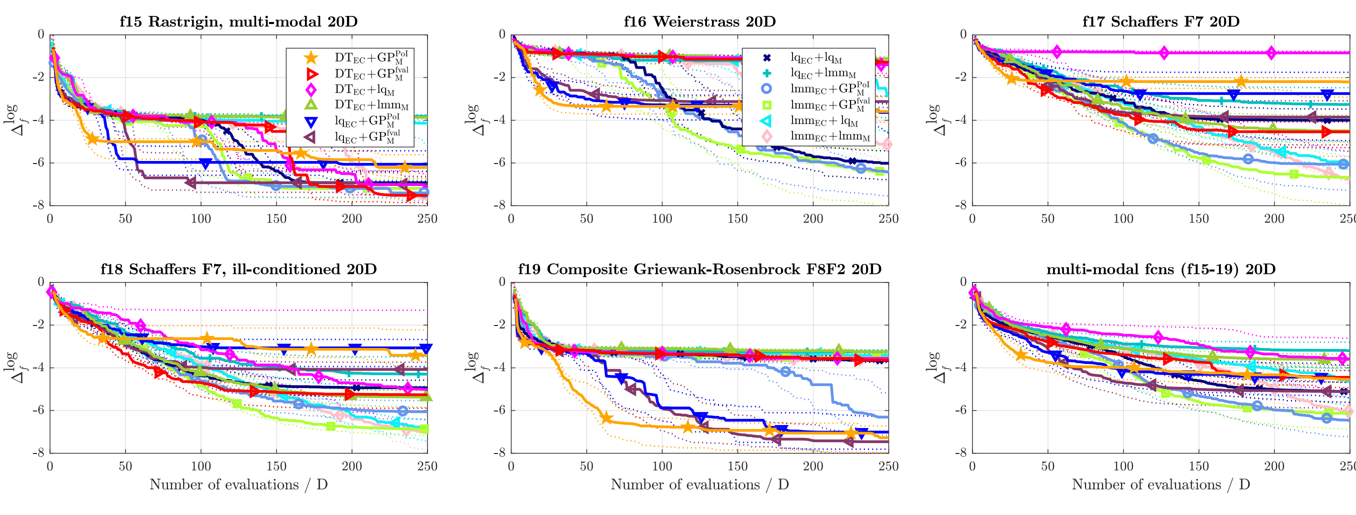

Figure 29: Medians (solid) and 1^st/3^rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with adequate global structure in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 30: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noiseless multi-modal COCO benchmarks with adequate global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20D	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	38	49	**71**	51	45	54	47	51	46	55	**58**	46	54	58	49	47	39	33	**74**	57	**59**	32
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	62	51	—	—	**77**	52	56	50	57	54	50	56	**71**	42	**69**	54	58	46	53	34	**79**	56	61	36
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	29	49	23	48	—	—	23	52	29	49	21	53	32	42	29	56	26	49	19	37	43	**65**	30	34
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**55**	46	44	50	**77**	48	—	—	51	50	48	50	**62**	40	**64**	57	53	45	44	32	**79**	**54**	**67**	31
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	53	49	43	46	**71**	51	49	50	—	—	45	53	**60**	41	60	55	53	40	45	33	**69**	50	59	31
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	54	45	50	44	**79**	47	52	50	55	47	—	—	**63**	38	**62**	53	55	43	48	31	**73**	50	**63**	34
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	42	54	29	58	**68**	**58**	38	60	40	**59**	37	**62**	—	—	47	**65**	38	52	30	37	**67**	**65**	47	42
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	46	42	31	46	**71**	44	36	43	40	45	38	47	53	35	—	—	41	38	31	27	**65**	47	54	31
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	51	53	42	54	**74**	51	47	55	47	**60**	45	**57**	**62**	48	**59**	62	—	—	39	36	**74**	**64**	**65**	40
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	**61**	**67**	47	**66**	**81**	**63**	56	**68**	**55**	**67**	52	**69**	**70**	**63**	**69**	**73**	**61**	**64**	—	—	**83**	**72**	**72**	54
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	26	43	21	44	**57**	35	21	46	31	50	27	50	33	35	35	53	26	36	17	28	—	—	35	25
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	41	**68**	39	64	**70**	**66**	33	**69**	41	**69**	37	**66**	53	**58**	46	**69**	35	**60**	28	46	**65**	**75**	—	—

2 − 20D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	48	51	**53**	46
$\mathrm{lq_{EC}}$	52	49	—	—	**55**	42
$\mathrm{lmm_{EC}}$	47	**54**	45	**58**	—	—

2 − 20D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	41	45	**68**	52	**57**	50
$\mathrm{GP_M^{PoI}}$	**59**	**55**	—	—	**73**	**55**	**66**	53
$\mathrm{lq_M}$	32	48	27	45	—	—	35	48
$\mathrm{lmm_M}$	43	50	34	47	**65**	**52**	—	—

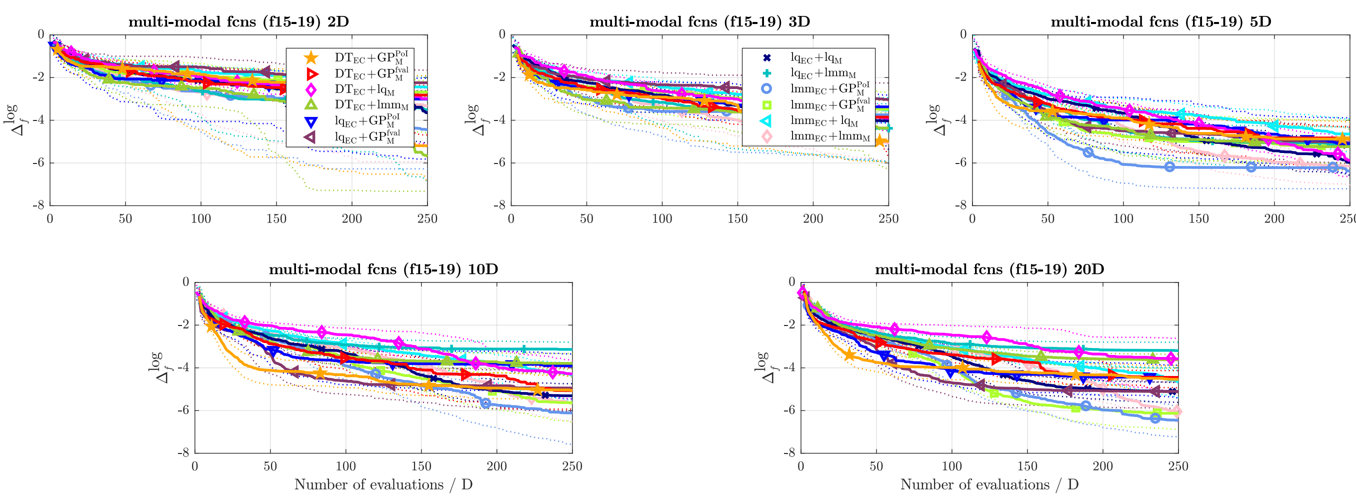

Figure 30: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 multi-modal noiseless COCO benchmarks with adequate global structure in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 31: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the noiseless multi-modal COCO benchmarks with weak global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$DT_{EC}+GP^{fval}_{M}$		$DT_{EC}+GP^{PoI}_{M}$		$DT_{EC}+lq_{M}$		$DT_{EC}+lmm_{M}$		$lq_{EC}+GP^{fval}_{M}$		$lq_{EC}+GP^{PoI}_{M}$		$lq_{EC}+lq_{M}$		$lq_{EC}+lmm_{M}$		$lmm_{EC}+GP^{fval}_{M}$		$lmm_{EC}+GP^{PoI}_{M}$		$lmm_{EC}+lq_{M}$		$lmm_{EC}+lmm_{M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$DT_{EC}+GP^{fval}_{M}$	—	—	28	40	48	43	32	37	40	50	35	36	44	52	49	56	47	44	41	39	47	52	43	45
$DT_{EC}+GP^{PoI}_{M}$	72	60	—	—	76	68	56	44	60	**84**	60	56	64	56	68	72	76	60	52	52	68	76	64	72
$DT_{EC}+lq_{M}$	52	57	24	32	—	—	41	41	47	59	36	42	47	56	51	60	53	60	44	44	49	60	40	51
$DT_{EC}+lmm_{M}$	68	63	44	56	59	59	—	—	64	57	51	47	56	**67**	59	**66**	60	63	51	50	59	64	52	61
$lq_{EC}+GP^{fval}_{M}$	60	50	40	16	53	41	36	43	—	—	45	35	48	44	51	50	53	43	49	37	53	57	47	41
$lq_{EC}+GP^{PoI}_{M}$	65	64	40	44	64	58	49	53	55	65	—	—	60	59	63	**64**	60	54	63	58	64	65	51	53
$lq_{EC}+lq_{M}$	56	48	36	44	53	44	44	33	52	56	40	41	—	—	59	49	49	41	47	35	56	55	48	39
$lq_{EC}+lmm_{M}$	51	44	32	28	49	40	41	34	49	50	37	36	41	51	—	—	44	42	47	41	45	52	43	39
$lmm_{EC}+GP^{fval}_{M}$	53	56	24	40	47	40	40	37	47	57	40	46	51	59	56	58	—	—	46	45	55	59	40	41
$lmm_{EC}+GP^{PoI}_{M}$	59	61	48	48	56	56	49	50	51	63	37	42	53	65	53	59	54	55	—	—	57	64	48	53
$lmm_{EC}+lq_{M}$	53	48	32	24	51	40	41	36	47	43	36	35	44	45	55	48	45	41	43	36	—	—	45	41
$lmm_{EC}+lmm_{M}$	57	55	36	28	60	49	48	39	53	59	49	47	52	61	57	61	60	59	52	47	55	59	—	—

$2D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	49.6	**57**	49.6	55
lq_{EC}	50.4	43	—	—	54	49
lmm_{EC}	50.4	45	46	51	—	—

$2D$	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	43	40	50.4	48	41	42
GP^{PoI}_{M}	**57**	**60**	—	—	**61**	**62**	55	**56**
lq_{M}	49.6	52	39	38	—	—	48	44
lmm_{M}	**59**	58	45	44	52	56	—	—

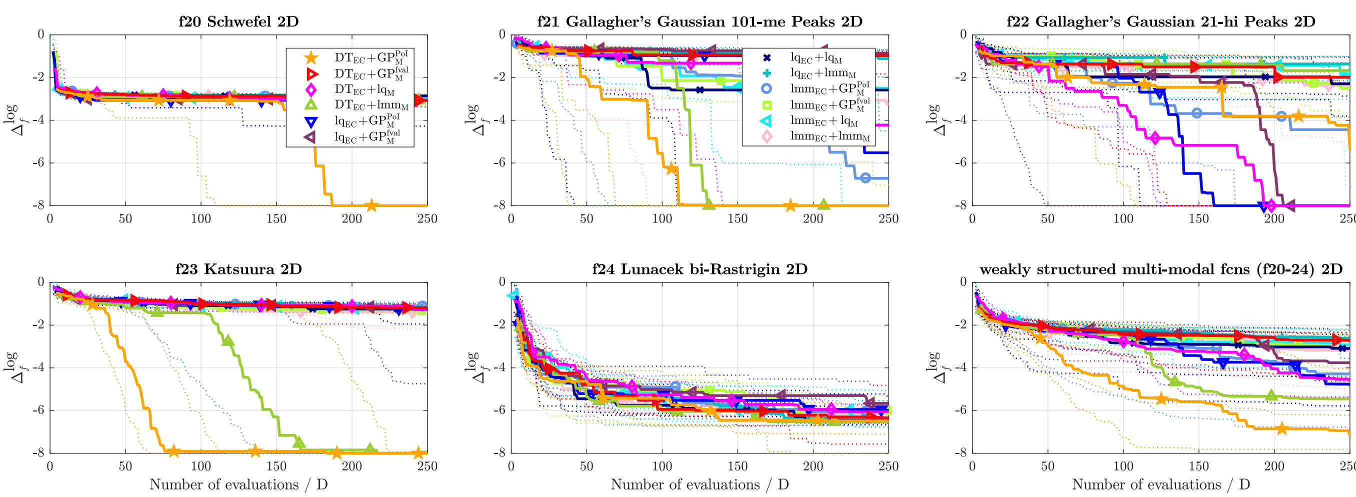

Figure 31: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with weak global structure in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 32: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the noiseless multi-modal COCO benchmarks with weak global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3*D*	$\mathrm{DT_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}+\mathrm{lq_M}$		$\mathrm{DT_{EC}}+\mathrm{lmm_M}$		$\mathrm{lq_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}+\mathrm{lq_M}$		$\mathrm{lq_{EC}}+\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}+\mathrm{lq_M}$		$\mathrm{lmm_{EC}}+\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}+\mathrm{GP^{fval}_M}$	—	—	36	28	56	62	57	49	61	57	49	49	61	55	49	57	59	54	49	47	64	63	57	61
$\mathrm{DT_{EC}}+\mathrm{GP^{PoI}_M}$	64	72	—	—	84	80	60	64	68	68	40	52	56	56	48	68	64	72	64	68	68	72	72	**80**
$\mathrm{DT_{EC}}+\mathrm{lq_M}$	44	38	16	20	—	—	51	51	49	51	36	48	45	45	41	47	48	53	43	47	49	57	32	49
$\mathrm{DT_{EC}}+\mathrm{lmm_M}$	43	51	40	36	49	49	—	—	51	59	44	49	52	56	45	51	52	57	43	43	51	60	49	57
$\mathrm{lq_{EC}}+\mathrm{GP^{fval}_M}$	39	43	32	32	51	49	49	41	—	—	41	37	51	42	43	47	51	53	43	39	57	53	47	55
$\mathrm{lq_{EC}}+\mathrm{GP^{PoI}_M}$	51	51	60	48	64	52	56	51	59	63	—	—	64	59	56	60	67	60	48	47	60	61	59	59
$\mathrm{lq_{EC}}+\mathrm{lq_M}$	39	45	44	44	55	55	48	44	49	58	36	41	—	—	40	50	45	58	40	45	44	59	40	54
$\mathrm{lq_{EC}}+\mathrm{lmm_M}$	51	43	52	32	59	53	55	49	57	53	44	40	60	50	—	—	56	49	48	42	68	57	60	53
$\mathrm{lmm_{EC}}+\mathrm{GP^{fval}_M}$	41	46	36	28	52	47	48	43	49	47	33	40	55	42	44	51	—	—	45	51	52	48	45	58
$\mathrm{lmm_{EC}}+\mathrm{GP^{PoI}_M}$	51	53	36	32	57	53	57	57	57	61	52	53	60	55	52	58	55	49	—	—	59	60	53	58
$\mathrm{lmm_{EC}}+\mathrm{lq_M}$	36	37	32	28	51	43	49	40	43	47	40	39	56	41	32	43	48	52	41	40	—	—	40	45
$\mathrm{lmm_{EC}}+\mathrm{lmm_M}$	43	39	28	20	68	51	51	43	53	45	41	41	60	46	40	47	55	42	47	42	60	55	—	—

3*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	51	54	57
$\mathrm{lq_{EC}}$	51	49	—	—	51	53
$\mathrm{lmm_{EC}}$	46	43	49	47	—	—

3*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	42	41	53	51	48	52
$\mathrm{GP^{PoI}_M}$	**58**	**59**	—	—	**65**	**62**	55	**60**
$\mathrm{lq_M}$	47	49	35	38	—	—	44	49
$\mathrm{lmm_M}$	52	48	45	40	56	51	—	—

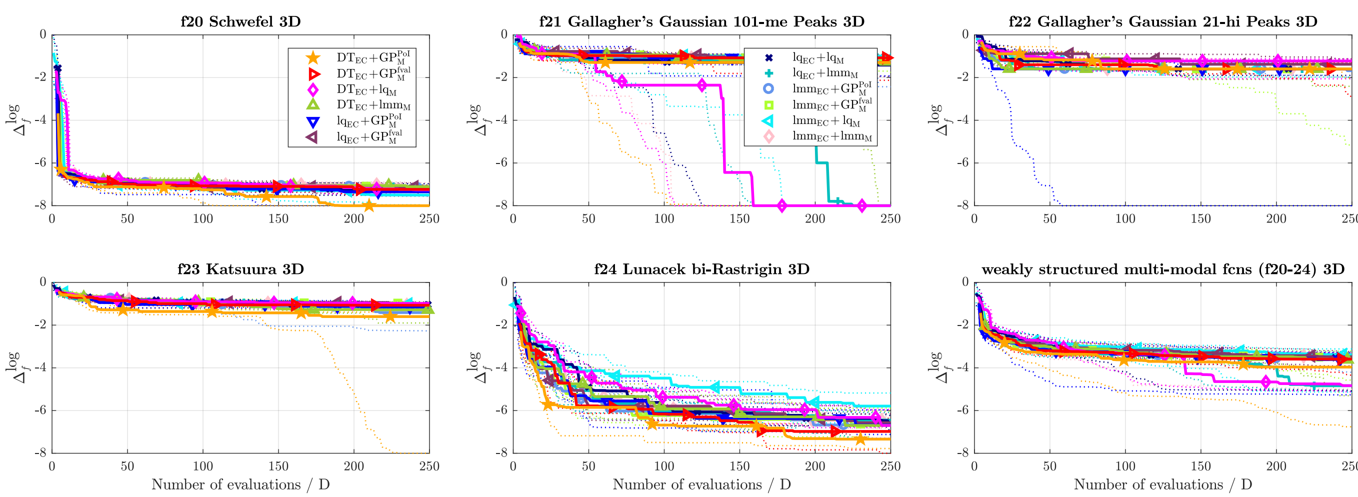

Figure 32: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with weak global structure in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 33: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the noiseless multi-modal COCO benchmarks with weak global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$5D$	$\mathrm{DT_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}+\mathrm{lq_M}$		$\mathrm{DT_{EC}}+\mathrm{lmm_M}$		$\mathrm{lq_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}+\mathrm{lq_M}$		$\mathrm{lq_{EC}}+\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}+\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}+\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}+\mathrm{lq_M}$		$\mathrm{lmm_{EC}}+\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}+\mathrm{GP^{fval}_M}$	—	—	28	32	64	57	51	53	32	35	25	21	52	49	53	43	41	44	35	31	52	57	48	51
$\mathrm{DT_{EC}}+\mathrm{GP^{PoI}_M}$	72	68	—	—	80	68	56	68	52	64	40	36	64	52	76	76	60	56	64	48	72	68	72	68
$\mathrm{DT_{EC}}+\mathrm{lq_M}$	36	43	20	32	—	—	39	51	27	31	17	16	36	43	39	47	40	42	27	33	41	57	35	46
$\mathrm{DT_{EC}}+\mathrm{lmm_M}$	49	47	44	32	61	49	—	—	32	35	25	25	52	42	59	39	44	34	37	30	52	47	45	39
$\mathrm{lq_{EC}}+\mathrm{GP^{fval}_M}$	**68**	65	48	36	**73**	69	68	65	—	—	39	35	71	59	67	57	60	59	55	51	**68**	68	60	61
$\mathrm{lq_{EC}}+\mathrm{GP^{PoI}_M}$	**75**	**79**	60	64	**83**	**84**	**75**	**75**	61	65	—	—	**81**	72	**79**	**75**	**71**	65	64	56	**81**	**83**	72	70
$\mathrm{lq_{EC}}+\mathrm{lq_M}$	48	51	36	48	64	57	48	58	29	41	19	28	—	—	53	52	36	48	31	37	52	67	48	51
$\mathrm{lq_{EC}}+\mathrm{lmm_M}$	47	57	24	24	61	53	41	61	33	43	21	25	47	48	—	—	40	51	35	37	49	65	41	52
$\mathrm{lmm_{EC}}+\mathrm{GP^{fval}_M}$	59	56	40	44	60	58	56	66	40	41	29	35	64	52	60	49	—	—	44	38	59	59	51	54
$\mathrm{lmm_{EC}}+\mathrm{GP^{PoI}_M}$	**65**	**69**	36	52	**73**	**67**	63	**70**	45	49	36	44	69	63	65	63	56	62	—	—	**65**	**72**	61	59
$\mathrm{lmm_{EC}}+\mathrm{lq_M}$	48	43	28	32	59	43	48	53	32	32	19	17	48	33	51	35	41	41	35	28	—	—	41	49
$\mathrm{lmm_{EC}}+\mathrm{lmm_M}$	52	49	28	32	65	54	55	61	40	39	28	30	52	49	59	48	49	46	39	41	59	51	—	—

$5D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	42	39	45	47
$\mathrm{lq_{EC}}$	**58**	**61**	—	—	54	58
$\mathrm{lmm_{EC}}$	55	53	46	42	—	—

$5D$	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	39	36	**64**	**58**	56	55
$\mathrm{GP^{PoI}_M}$	**61**	**64**	—	—	**74**	**71**	**68**	**67**
$\mathrm{lq_M}$	36	42	26	29	—	—	44	51
$\mathrm{lmm_M}$	44	45	32	33	56	49	—	—

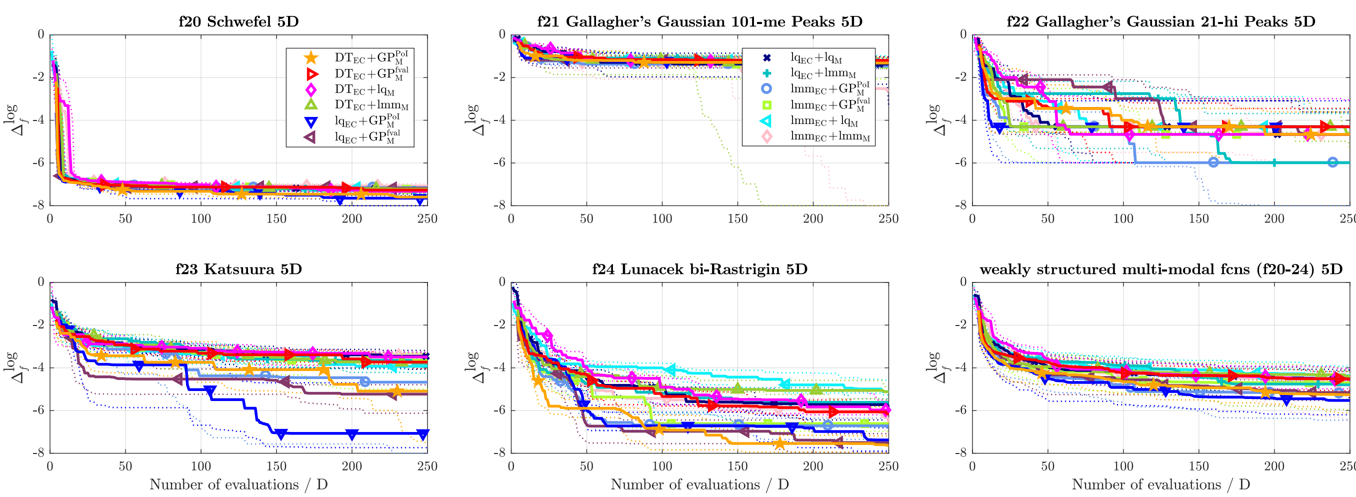

Figure 33: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with weak global structure in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 34: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noiseless multi-modal COCO benchmarks with weak global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10D	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	40	28	68	52	59	**69**	32	35	49	36	59	51	53	57	61	57	51	45	**71**	59	75	56
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	60	72	—	—	68	68	72	**80**	36	48	60	60	68	80	76	76	68	68	72	64	76	76	**92**	72
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	32	48	32	32	—	—	41	71	24	34	35	38	47	45	40	61	44	49	35	47	49	59	47	59
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	41	31	28	20	59	29	—	—	31	25	39	21	52	31	49	45	53	35	45	30	63	33	59	38
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	68	65	64	52	**76**	**66**	69	**75**	—	—	57	47	71	69	68	71	67	67	64	63	73	68	**80**	63
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	51	64	40	40	**65**	62	61	**79**	43	53	—	—	59	69	53	65	64	66	63	60	**64**	66	**69**	63
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	41	49	32	20	53	55	48	**69**	29	31	41	31	—	—	48	66	49	56	49	43	63	59	59	58
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	47	43	24	24	60	39	51	55	32	29	47	35	52	34	—	—	53	47	43	41	65	45	61	38
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	39	43	32	32	56	51	47	65	33	33	36	34	51	44	47	53	—	—	45	31	65	48	59	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	49	55	28	36	65	53	55	**70**	36	37	37	40	51	57	57	59	55	69	—	—	71	63	60	63
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	29	41	24	24	51	41	37	67	27	32	36	34	37	41	35	55	35	52	29	37	—	—	51	50
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	25	44	8	28	53	41	41	62	20	37	31	37	41	42	39	62	41	49	40	37	49	50	—	—

10D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	44	43	**58**	53
$\mathrm{lq_{EC}}$	**56**	57	—	—	**63**	**56**
$\mathrm{lmm_{EC}}$	42	47	37	44	—	—

10D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	49.7	37	**68**	**56**	**62**	**64**
$\mathrm{GP_M^{PoI}}$	50.3	**63**	—	—	**65**	**67**	**59**	**67**
$\mathrm{lq_M}$	32	44	35	33	—	—	47	**62**
$\mathrm{lmm_M}$	38	36	41	33	53	38	—	—

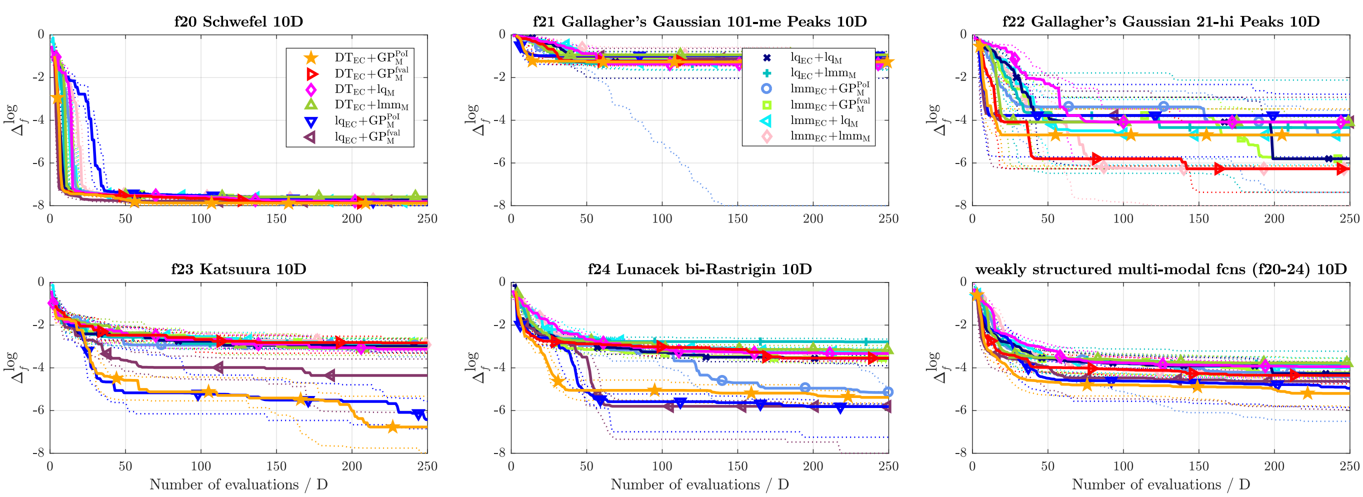

Figure 34: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with weak global structure in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 35: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the noiseless multi-modal COCO benchmarks with weak global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	24	32	61	55	53	71	29	32	39	39	53	51	69	**76**	53	48	53	46	57	51	69	49
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	76	68	—	—	84	60	80	68	56	56	68	72	76	60	**84**	72	80	64	**84**	64	72	60	80	72
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	39	45	16	40	—	—	48	66	27	33	27	36	56	43	61	64	55	43	44	45	57	52	56	54
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	47	29	20	32	52	34	—	—	27	27	31	31	43	30	64	53	45	30	48	31	52	36	56	29
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	**71**	**68**	44	44	**73**	**67**	**73**	**73**	—	—	49	54	65	63	**79**	**79**	63	**66**	**75**	65	**69**	69	**69**	**70**
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	61	61	32	28	**73**	**64**	**69**	**69**	51	46	—	—	69	54	**81**	**68**	**69**	67	**71**	55	61	61	**71**	**63**
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	47	49	24	40	44	57	57	**70**	35	37	31	46	—	—	64	**77**	45	55	52	49	53	60	56	55
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	31	24	16	28	39	36	36	47	21	21	19	32	36	23	—	—	33	32	36	26	40	28	51	35
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	47	52	20	36	45	57	55	**70**	37	34	31	33	55	45	67	68	—	—	47	45	57	56	64	56
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	47	54	16	36	56	55	52	**69**	25	35	29	45	48	51	64	**74**	53	55	—	—	53	55	56	57
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	43	49	28	40	43	48	48	64	31	31	39	39	47	40	60	**72**	43	44	47	45	—	—	61	49
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	31	51	20	28	44	46	44	71	31	30	29	37	44	45	49	65	36	44	44	43	39	51	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	52	45	58	45
$\mathrm{lq_{EC}}$	48	55	—	—	**59**	54
$\mathrm{lmm_{EC}}$	42	55	41	46	—	—

20*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	45	47	**61**	58	**65**	**69**
$\mathrm{GP^{PoI}_{M}}$	55	53	—	—	**65**	55	**70**	**63**
$\mathrm{lq_{M}}$	39	42	35	45	—	—	58	**64**
$\mathrm{lmm_{M}}$	35	31	30	37	42	36	—	—

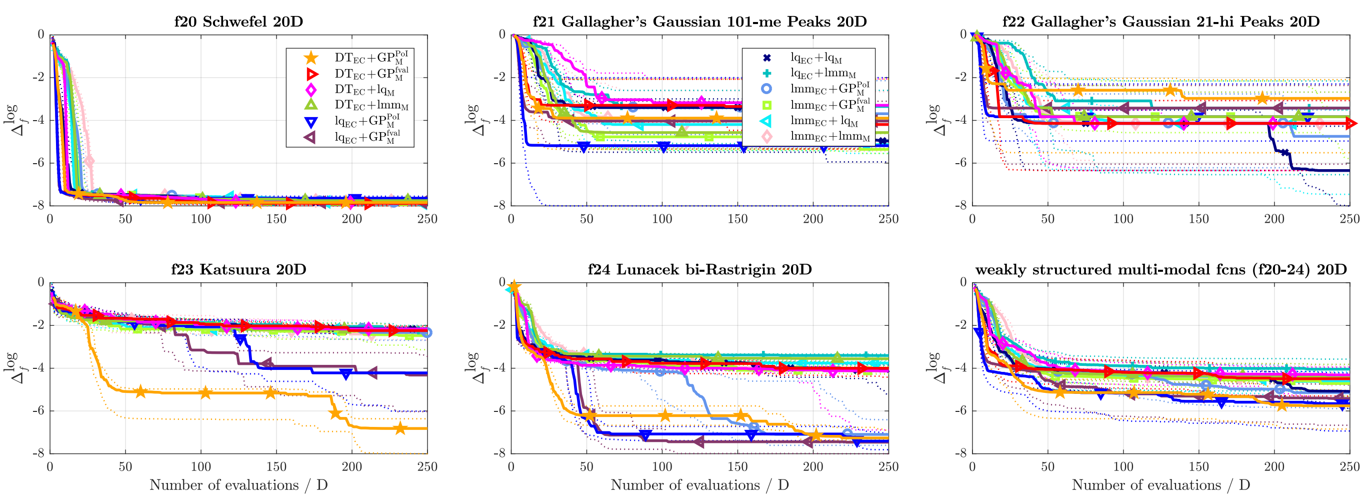

Figure 35: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 5 noiseless multi-modal COCO benchmarks with weak global structure in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 36: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noiseless multi-modal COCO benchmarks with weak global structure for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20D	DT_{EC}+GP^{fval}_{M}		DT_{EC}+GP^{PoI}_{M}		DT_{EC}+lq_{M}		DT_{EC}+lmm_{M}		lq_{EC}+GP^{fval}_{M}		lq_{EC}+GP^{PoI}_{M}		lq_{EC}+lq_{M}		lq_{EC}+lmm_{M}		lmm_{EC}+GP^{fval}_{M}		lmm_{EC}+GP^{PoI}_{M}		lmm_{EC}+lq_{M}		lmm_{EC}+lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP^{fval}_{M}	—	—	31	32	**59**	54	50.4	56	39	42	39	36	54	51	55	58	52	49	46	41	58	56	58	52
DT_{EC}+GP^{PoI}_{M}	**69**	**68**	—	—	**78**	**69**	**65**	**65**	**54**	**64**	54	55	**66**	**61**	**70**	**73**	**70**	**64**	**67**	59	**71**	**70**	**76**	**73**
DT_{EC}+lq_{M}	41	46	22	31	—	—	44	56	35	42	30	36	46	46	46	56	48	49	38	43	49	57	42	52
DT_{EC}+lmm_{M}	49.6	44	35	35	**56**	44	—	—	41	41	38	35	51	45	55	51	51	44	45	37	55	48	52	45
lq_{EC}+GP^{fval}_{M}	**61**	**58**	46	36	**65**	58	59	59	—	—	46	42	61	55	**61**	**61**	59	57	57	51	**64**	**63**	61	58
lq_{EC}+GP^{PoI}_{M}	**61**	**64**	46	45	**70**	**64**	**62**	**65**	54	58	—	—	**67**	**63**	**66**	**66**	**66**	**62**	**62**	55	**66**	**67**	**64**	**61**
lq_{EC}+lq_{M}	46	49	34	39	54	54	49	55	39	45	33	37	—	—	53	59	45	52	44	42	54	60	50.1	51
lq_{EC}+lmm_{M}	45	42	30	27	54	44	45	49	39	39	34	34	47	41	—	—	45	44	42	37	54	49	51	43
lmm_{EC}+GP^{fval}_{M}	48	51	30	36	52	51	49	56	41	43	34	38	55	48	55	56	—	—	45	42	58	54	52	52
lmm_{EC}+GP^{PoI}_{M}	54	**59**	33	41	**62**	**57**	55	**63**	43	49	38	45	56	**58**	**58**	**63**	55	**58**	—	—	**61**	**63**	56	**58**
lmm_{EC}+lq_{M}	42	44	29	30	51	43	45	52	36	37	34	33	46	40	46	51	42	46	39	37	—	—	48	47
lmm_{EC}+lmm_{M}	42	48	24	27	**58**	48	48	55	39	42	36	39	49.9	49	49	**57**	48	48	44	42	52	53	—	—

2 − 20D	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	47	47	53	51
lq_{EC}	53	53	—	—	**56**	54
lmm_{EC}	47	49	44	46	—	—

2 − 20D	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	44	40	**59**	**54**	54	**56**
GP^{PoI}_{M}	**56**	**60**	—	—	**66**	**64**	**62**	**63**
lq_{M}	41	46	34	36	—	—	48	54
lmm_{M}	46	44	38	37	**52**	46	—	—

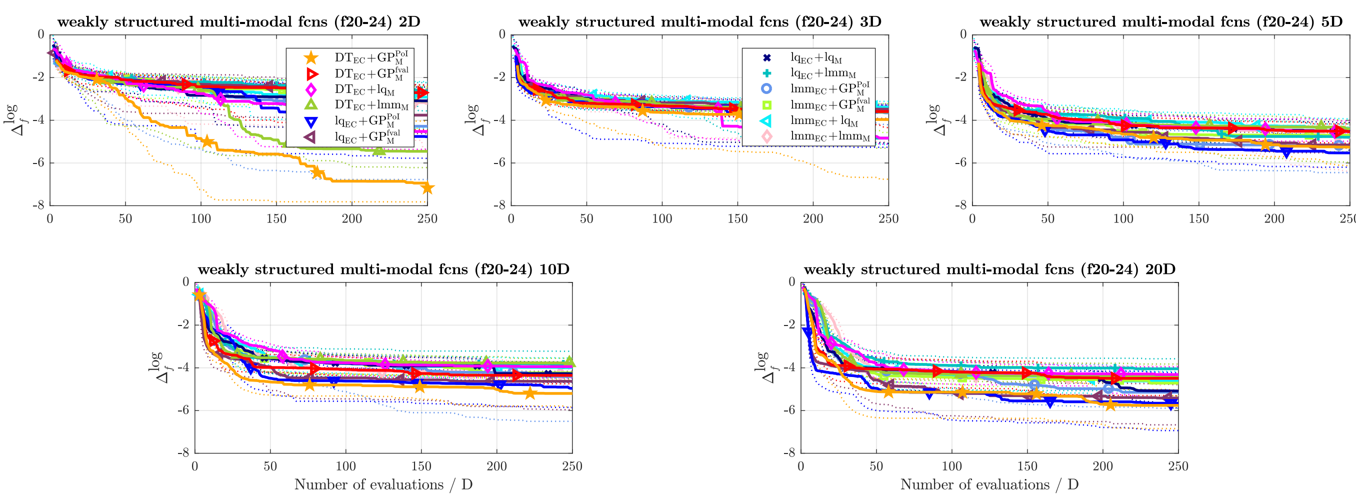

Figure 36: Medians (solid) and 1[st]/3[rd] quartiles (dotted) of the distances to the optima of 5 multi-modal noiseless COCO benchmarks with weak global structure in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 37: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the noisy COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2D	$\mathrm{DT_{EC}+GP^{fval}_{M}}$		$\mathrm{DT_{EC}+GP^{PoI}_{M}}$		$\mathrm{DT_{EC}+lq_{M}}$		$\mathrm{DT_{EC}+lmm_{M}}$		$\mathrm{lq_{EC}+GP^{fval}_{M}}$		$\mathrm{lq_{EC}+GP^{PoI}_{M}}$		$\mathrm{lq_{EC}+lq_{M}}$		$\mathrm{lq_{EC}+lmm_{M}}$		$\mathrm{lmm_{EC}+GP^{fval}_{M}}$		$\mathrm{lmm_{EC}+GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}+lq_{M}}$		$\mathrm{lmm_{EC}+lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}+GP^{fval}_{M}}$	—	—	51	49	59	49	42	51	49.6	**59**	49.6	54	49	39	44	44	52	**59**	48	52	**60**	52	49.6	**62**
$\mathrm{DT_{EC}+GP^{PoI}_{M}}$	49	51	—	—	59	54	40	54	50	**59**	49.8	**59**	50.2	39	44	46	51	**63**	49	**56**	**58**	56	47	**59**
$\mathrm{DT_{EC}+lq_{M}}$	41	51	41	46	—	—	34	49.8	44	**58**	44	**56**	41	41	39	43	45	**62**	38	54	49	56	41	**59**
$\mathrm{DT_{EC}+lmm_{M}}$	**58**	49	**60**	46	**66**	50.2	—	—	**58**	**55**	**60**	55	58	39	56	43	**60**	**60**	**57**	54	**65**	49.6	**61**	**59**
$\mathrm{lq_{EC}+GP^{fval}_{M}}$	50.4	41	50	41	56	42	42	45	—	—	49	50	46	33	46	41	51	52	50	48	55	43	49	50
$\mathrm{lq_{EC}+GP^{PoI}_{M}}$	50.4	46	50.2	41	56	44	40	45	51	50	—	—	49	33	47	37	52	52	50.2	47	59	44	51	49.6
$\mathrm{lq_{EC}+lq_{M}}$	51	**61**	49.8	**61**	**59**	**59**	42	**61**	54	**67**	51	**67**	—	—	48	**54**	52	**69**	47	**64**	**58**	**63**	49	**69**
$\mathrm{lq_{EC}+lmm_{M}}$	56	**56**	56	54	**61**	57	44	**57**	54	**59**	53	**63**	52	46	—	—	57	**65**	54	**58**	**60**	**57**	55	**64**
$\mathrm{lmm_{EC}+GP^{fval}_{M}}$	48	41	49	37	55	38	40	40	49	48	48	48	48	31	43	35	—	—	46	42	56	42	49.6	49
$\mathrm{lmm_{EC}+GP^{PoI}_{M}}$	52	48	51	44	**62**	46	43	46	50	52	49.8	53	53	36	46	42	54	**58**	—	—	**60**	45	52	56
$\mathrm{lmm_{EC}+lq_{M}}$	40	48	42	44	51	44	35	50.4	45	**57**	41	**56**	42	37	40	43	44	**58**	40	55	—	—	43	54
$\mathrm{lmm_{EC}+lmm_{M}}$	50.4	38	53	41	59	41	39	41	51	50	49	50.4	51	31	45	36	50.4	51	48	44	57	46	—	—

2D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	50.4	**53**	**58**
$\mathrm{lq_{EC}}$	51	49.6	—	—	**54**	**57**
$\mathrm{lmm_{EC}}$	47	43	46	43	—	—

2D	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	49	47	53	41	46	47
$\mathrm{GP^{PoI}_{M}}$	51	**53**	—	—	**56**	44	46	49
$\mathrm{lq_{M}}$	47	**59**	44	**56**	—	—	41	**52**
$\mathrm{lmm_{M}}$	**54**	**53**	**54**	51	**59**	48	—	—

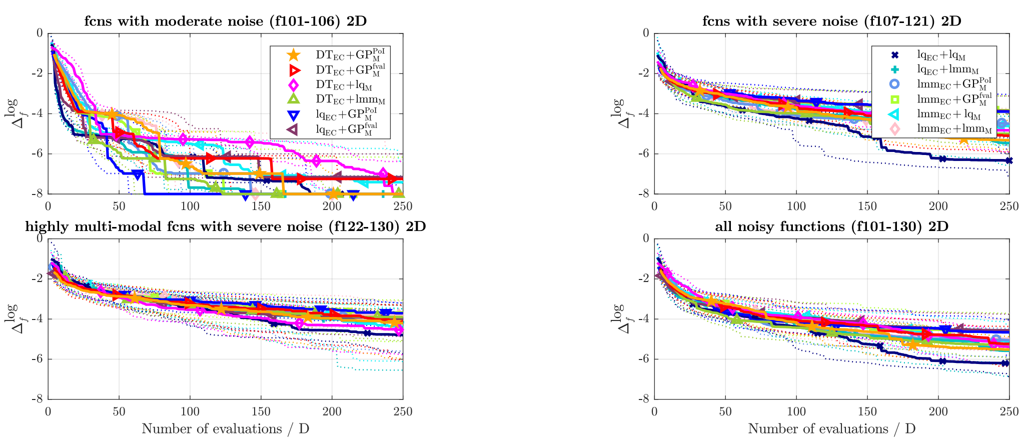

Figure 37: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 30 noisy COCO benchmarks in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 38: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the noisy COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3D	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	48	45	52	45	41	51	50	**57**	49	**55**	45	38	48	43	50	**60**	43	49.8	55	50.4	47	54
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	52	55	—	—	59	47	42	**57**	48	**60**	48	**60**	47	44	49	48	53	**64**	48	55	55	55	48	**58**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	48	**55**	41	53	—	—	38	**60**	47	**65**	47	**61**	42	47	46	51	45	**65**	41	**57**	49	**61**	41	**60**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**59**	49	**58**	43	**62**	40	—	—	**56**	**57**	**60**	55	55	38	**59**	38	**57**	**57**	57	49.6	**63**	48	54	55
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	50	43	52	40	53	35	44	43	—	—	48	49	46	33	48	35	52	52	46	44	56	43	46	46
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	51	45	52	40	53	39	40	45	52	51	—	—	47	35	50.4	37	53	**58**	46	45	53	43	47	48
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	55	**62**	53	**56**	**58**	53	45	**62**	54	**67**	53	**65**	—	—	50.4	52	56	**67**	47	**58**	58	**59**	49.8	**60**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	52	**57**	51	52	54	49	41	**62**	52	**65**	49.6	**63**	49.6	48	—	—	57	**64**	49	**57**	57	**59**	47	**60**
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	50	40	47	36	55	35	43	43	48	48	47	42	44	33	43	36	—	—	43	41	55	39	48	44
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	57	50.2	52	45	59	43	43	50.4	54	**56**	54	55	53	42	51	43	57	**59**	—	—	57	48	51	53
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	45	49.6	45	45	51	39	37	52	44	**57**	47	**57**	42	41	43	41	45	**61**	43	52	—	—	43	55
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	53	46	52	42	59	40	46	45	54	54	53	52	50.2	40	53	40	52	**56**	49	47	57	45	—	—

3D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49.6	50.4	50.3	**57**
$\mathrm{lq_{EC}}$	50.4	49.6	—	—	51	**54**
$\mathrm{lmm_{EC}}$	49.7	43	49	46	—	—

3D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	46	45	51	39	45	43
$\mathrm{GP_M^{PoI}}$	54	**55**	—	—	54	43	48	49
$\mathrm{lq_M}$	49	**61**	46	**57**	—	—	44	**55**
$\mathrm{lmm_M}$	**55**	**57**	**52**	51	**56**	45	—	—

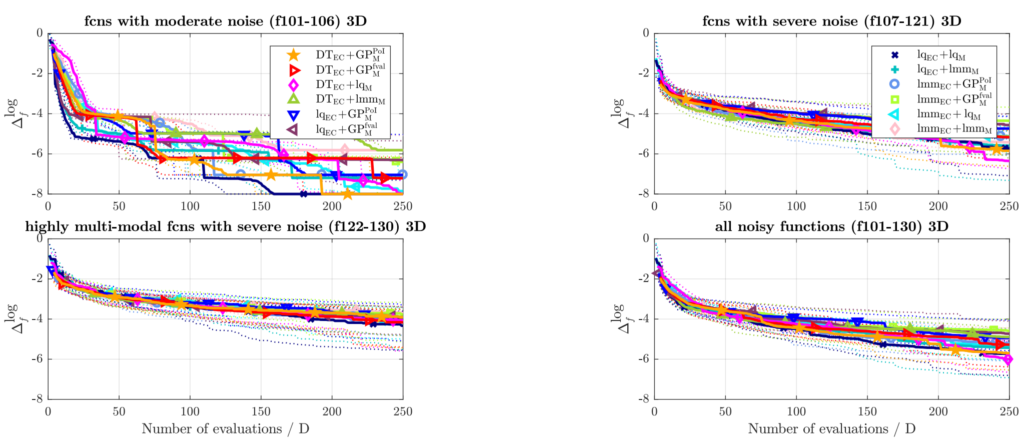

Figure 38: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 30 noisy COCO benchmarks in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 39: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the noisy COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

5*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	51	47	**58**	41	43	**60**	**56**	**61**	52	**60**	44	38	47	48	46	**58**	46	52	55	50	51	49.8
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	49	53	—	—	58	47	45	**64**	52	**63**	53	**65**	45	45	47	53	46	**59**	48	**55**	57	**57**	48	56
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	42	**59**	42	53	—	—	38	**66**	49	**72**	46	**71**	35	49	39	**57**	38	**62**	36	**60**	47	**60**	38	**58**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	57	40	55	36	**62**	34	—	—	**60**	53	**57**	52	48	33	53	42	52	50.2	50.4	40	**62**	44	52	41
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	44	39	48	37	51	28	40	47	—	—	50.2	50	42	30	40	37	44	46	43	38	52	41	45	39
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	48	40	47	35	54	29	43	48	49.8	50	—	—	41	31	44	41	45	46	44	43	51	40	43	43
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	56	**62**	**55**	**55**	**65**	51	52	**67**	**58**	**70**	**59**	**69**	—	—	52	**54**	52	**64**	54	**60**	**61**	**62**	57	**59**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	53	52	53	47	**61**	43	47	**58**	**60**	**63**	**56**	**59**	48	46	—	—	45	**56**	45	54	**61**	52	50	54
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	54	42	54	41	**62**	38	48	49.8	**56**	54	55	54	48	36	55	44	—	—	50.4	47	58	43	55	48
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	54	48	52	45	**64**	40	49.6	60	**57**	**62**	**56**	**57**	46	40	55	46	49.6	53	—	—	**60**	49.6	53	48
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	45	50	43	43	53	40	38	**56**	48	**59**	49	**60**	39	38	39	48	42	**57**	40	50.4	—	—	41	49
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	49	50.2	52	44	**62**	42	48	**59**	55	**61**	**57**	**57**	43	41	50	46	45	52	47	52	**59**	51	—	—

5*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	**54**	48	**53**
$\mathrm{lq_{EC}}$	51	46	—	—	49.8	51
$\mathrm{lmm_{EC}}$	52	47	50.2	49	—	—

5*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	51	48	53	38	46	48
$\mathrm{GP^{PoI}_{M}}$	49	**52**	—	—	53	43	47	51
$\mathrm{lq_{M}}$	47	**62**	47	**57**	—	—	43	**57**
$\mathrm{lmm_{M}}$	**54**	**52**	**53**	49	**57**	43	—	—

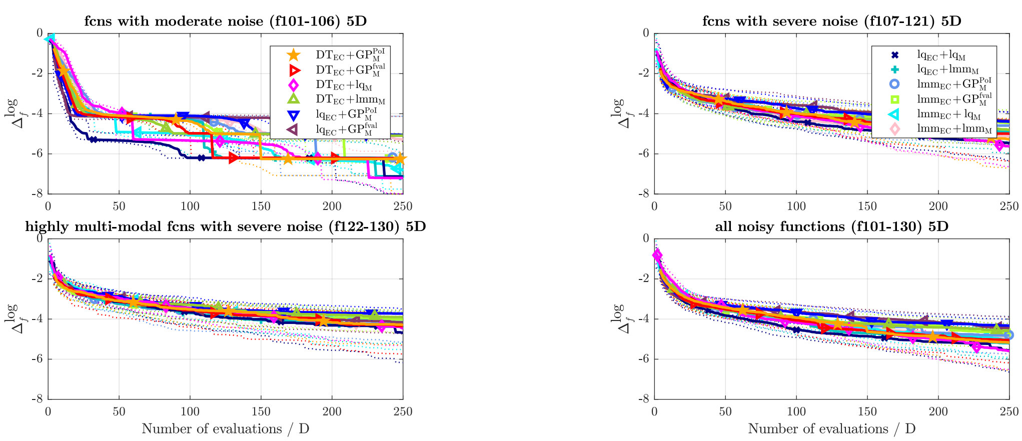

Figure 39: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 30 noisy COCO benchmarks in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 40: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the noisy COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10D	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	51	48	**60**	46	52	**62**	**56**	**59**	54	**61**	47	41	**58**	**62**	56	49	55	44	**63**	53	**60**	48
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	49	52	—	—	**59**	49.8	53	**66**	**58**	**63**	53	**63**	46	46	**59**	**63**	58	52	58	47	**62**	**56**	**60**	52
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	40	54	41	50.2	—	—	41	**62**	49.8	**65**	47	**64**	36	45	52	**64**	46	53	49.8	51	53	**56**	52	**56**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	48	38	47	34	**59**	38	—	—	**59**	50.2	54	46	43	29	**59**	52	54	42	55	35	**60**	42	**60**	39
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	44	41	42	37	50.2	35	41	49.8	—	—	46	50.2	37	32	53	54	48	42	49	35	53	41	51	38
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	46	39	47	37	53	36	46	54	54	49.8	—	—	41	33	56	53	52	41	52	35	56	42	51	39
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	53	**59**	54	54	**64**	55	**57**	**71**	**63**	**68**	**59**	**67**	—	—	**63**	**70**	**61**	**57**	**60**	54	**65**	**61**	**66**	**58**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	42	38	41	37	48	36	41	48	47	46	44	47	37	30	—	—	52	42	52	37	56	38	52	41
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	44	51	42	48	54	47	46	**58**	52	**58**	48	**59**	39	43	48	**58**	—	—	53	45	56	53	**57**	52
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	45	**56**	42	53	50.2	49	45	**65**	51	**65**	48	**65**	40	46	48	**63**	47	55	—	—	55	**58**	56	56
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	37	47	38	44	47	44	40	**58**	47	**59**	44	**58**	35	39	44	**62**	44	47	45	42	—	—	49	47
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	40	52	40	48	48	44	40	**61**	49	**62**	49	**61**	34	42	48	**59**	43	48	44	44	51	53	—	—

10D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	**55**	**57**	48
$\mathrm{lq_{EC}}$	49	45	—	—	**54**	45
$\mathrm{lmm_{EC}}$	43	**52**	46	**55**	—	—

10D	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	50	48	51	44	54	**56**
$\mathrm{GP_M^{PoI}}$	50	**52**	—	—	52	47	**55**	**58**
$\mathrm{lq_M}$	49	**56**	48	**53**	—	—	51	**60**
$\mathrm{lmm_M}$	46	44	45	42	49	40	—	—

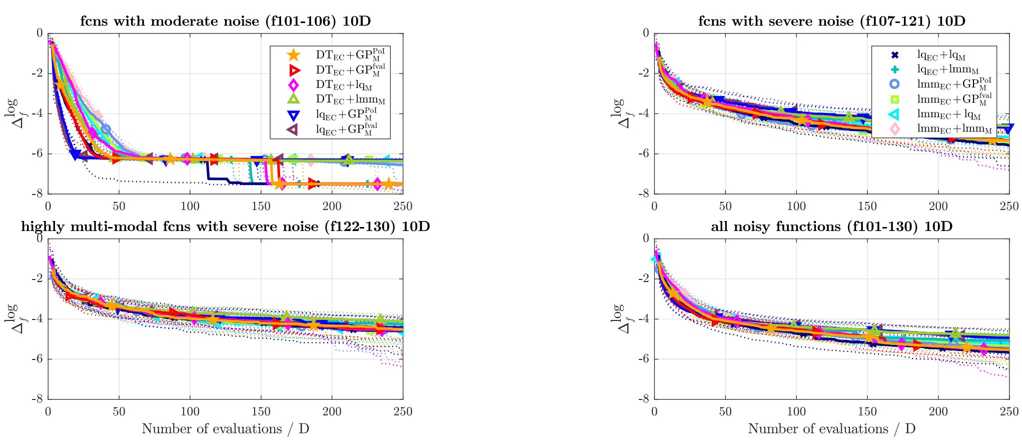

Figure 40: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 30 noisy COCO benchmarks in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 41: A pairwise comparison of the evolution controls, models, and their combinations in 20D over the noisy COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	49	50.4	**59**	50	**67**	**69**	**59**	**62**	55	**59**	57	49.6	**66**	**66**	**57**	54	**56**	55	**64**	**59**	**63**	49
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	51	49.6	—	—	**59**	46	**66**	**68**	**60**	**58**	55	**60**	**60**	50	**70**	**66**	**58**	51	**58**	52	**67**	**58**	**65**	49
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	41	50	41	54	—	—	**60**	**67**	51	**62**	44	**60**	46	51	**66**	**65**	51	**58**	49.6	**58**	57	**59**	**59**	54
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	33	31	34	32	40	33	—	—	42	38	39	38	37	33	52	46	40	36	40	35	45	34	51	30
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	41	38	40	42	49	38	**58**	**62**	—	—	42	49.8	47	39	**57**	59	47	42	51	40	55	46	54	40
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	45	41	45	40	56	40	**61**	**62**	**58**	50.2	—	—	51	39	**63**	**60**	55	44	53	45	**59**	47	**61**	42
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	43	50.4	40	50	54	49	**63**	**67**	53	**61**	49	**61**	—	—	**65**	**65**	55	53	56	55	**63**	**59**	**61**	50.4
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	34	34	30	34	34	35	48	54	43	41	37	40	35	35	—	—	36	39	38	37	43	37	49.8	31
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	43	46	42	49	49	42	**60**	**64**	53	58	45	56	45	47	**64**	**61**	—	—	54	49.6	57	54	**63**	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	44	45	42	48	50.4	42	**60**	**65**	49	**60**	47	55	44	45	**62**	**63**	46	50.4	—	—	59	56	**64**	48
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	36	41	33	42	43	41	55	**66**	45	54	41	53	37	41	**57**	**63**	43	46	41	44	—	—	56	44
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	37	51	35	51	41	46	49	**70**	46	**60**	39	**58**	39	49.6	50.2	**69**	37	54	36	52	44	**56**	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**53**	**55**	**56**	49
$\mathrm{lq_{EC}}$	47	45	—	—	**53**	44
$\mathrm{lmm_{EC}}$	44	51	47	**56**	—	—

20*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	49	49.9	**55**	48	**62**	**58**
$\mathrm{GP^{PoI}_{M}}$	51	50.1	—	—	**56**	47	**64**	**58**
$\mathrm{lq_{M}}$	45	**52**	44	**53**	—	—	**60**	**59**
$\mathrm{lmm_{M}}$	38	42	36	42	40	41	—	—

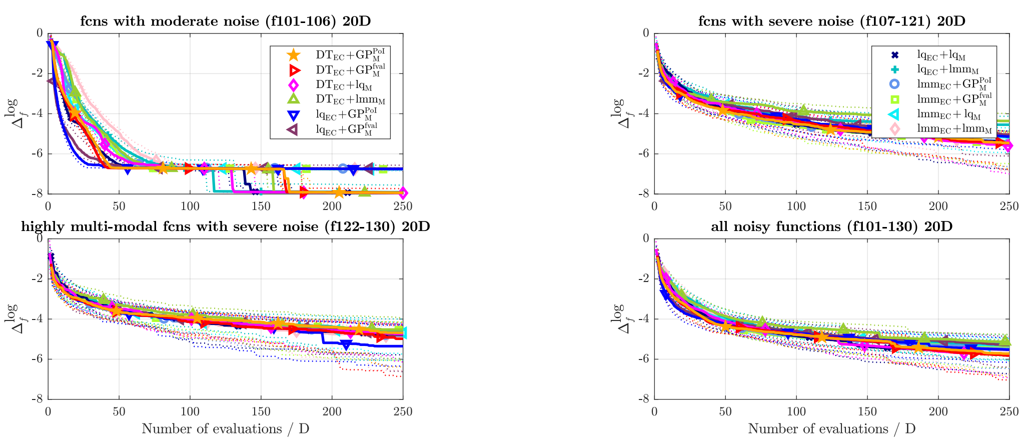

Figure 41: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 30 noisy COCO benchmarks in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 42: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the noisy COCO benchmarks for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	50.1	48	**58**	46	49	**59**	**54**	**60**	52	**58**	48	41	53	52	52	**56**	49.7	51	**60**	**53**	**54**	**52**
DT_{EC}+GP_M^{PoI}	49.9	**52**	—	—	**59**	49	49	**62**	**54**	**61**	**52**	**61**	49.7	45	**54**	**55**	53	**58**	52	**53**	**60**	**56**	**54**	**55**
DT_{EC}+lq_M	42	**54**	41	51	—	—	42	**61**	48	**65**	46	**62**	40	47	48	**56**	45	**60**	43	**56**	51	**58**	46	**58**
DT_{EC}+lmm_M	51	41	51	38	**58**	39	—	—	**55**	51	**54**	49	48	34	**56**	44	**53**	49	52	43	**59**	43	**56**	45
lq_{EC}+GP_M^{fval}	46	40	46	39	52	35	45	49	—	—	47	49.8	44	34	49	45	48	47	48	41	54	43	49	42
lq_{EC}+GP_M^{PoI}	48	42	48	39	54	38	46	51	53	50.2	—	—	46	34	52	46	51	48	49	43	56	43	51	44
lq_{EC}+lq_M	52	**59**	50.3	**55**	**60**	**53**	52	**66**	**56**	**66**	**54**	**66**	—	—	**56**	**59**	**55**	**62**	**53**	**58**	**61**	**61**	**57**	**59**
lq_{EC}+lmm_M	47	48	46	45	52	44	44	**56**	51	**55**	48	**54**	44	41	—	—	49	**53**	48	49	**55**	49	51	50.1
lmm_{EC}+GP_M^{fval}	48	44	47	42	**55**	40	47	51	52	**53**	49	52	45	38	51	47	—	—	49.5	45	**56**	46	**54**	48
lmm_{EC}+GP_M^{PoI}	50.3	49	48	47	**57**	44	48	**57**	**52**	**59**	51	**57**	47	42	52	51	50.5	**55**	—	—	**58**	51	**55**	52
lmm_{EC}+lq_M	40	47	40	44	49	42	41	**57**	46	**57**	44	**57**	39	39	45	51	44	**54**	42	49	—	—	46	49.7
lmm_{EC}+lmm_M	46	48	46	45	54	42	44	**55**	51	**58**	49	**56**	43	41	49	49.9	46	**52**	45	48	54	50.3	—	—

2 − 20*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	50.4	**53**	**53**	**53**
lq_{EC}	49.6	47	—	—	**52**	**50.2**
lmm_{EC}	47	47	48	49.8	—	—

2 − 20*D*	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	49	47	53	42	51	50.4
GP_M^{PoI}	**51**	**53**	—	—	**54**	45	**52**	**53**
lq_M	47	**58**	46	**55**	—	—	48	**57**
lmm_M	49	49.6	48	47	52	43	—	—

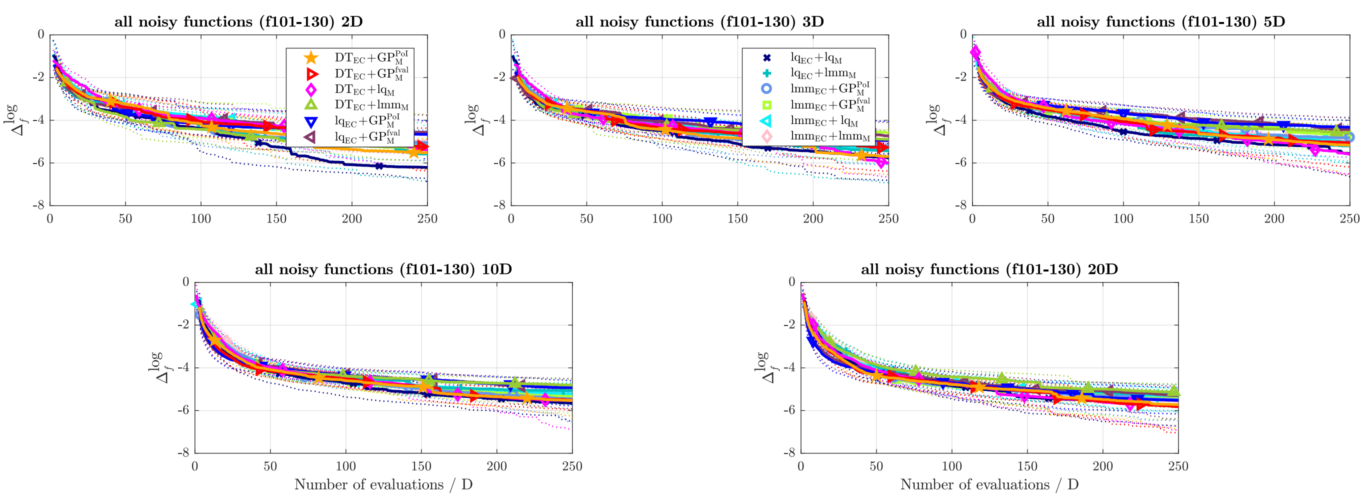

Figure 42: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 30 noisy COCO benchmarks in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 43: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the COCO benchmarks with moderate noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$DT_{EC}+GP_M^{fval}$		$DT_{EC}+GP_M^{PoI}$		$DT_{EC}+lq_M$		$DT_{EC}+lmm_M$		$lq_{EC}+GP_M^{fval}$		$lq_{EC}+GP_M^{PoI}$		$lq_{EC}+lq_M$		$lq_{EC}+lmm_M$		$lmm_{EC}+GP_M^{fval}$		$lmm_{EC}+GP_M^{PoI}$		$lmm_{EC}+lq_M$		$lmm_{EC}+lmm_M$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$DT_{EC}+GP_M^{fval}$	—	—	47	51	**74**	59	43	42	44	53	37	41	52	31	36	44	46	54	48	41	64	49	46	47
$DT_{EC}+GP_M^{PoI}$	53	49	—	—	69	59	38	43	49	56	41	47	47	28	40	44	48	51	51	39	61	47	51	44
$DT_{EC}+lq_M$	26	41	31	41	—	—	27	39	31	48	32	40	28	27	24	32	29	44	30	41	41	48	36	46
$DT_{EC}+lmm_M$	57	58	62	57	**73**	61	—	—	50	57	50	47	50	31	52	53	56	54	58	49	67	52	59	50
$lq_{EC}+GP_M^{fval}$	56	47	51	44	69	52	50	43	—	—	43	42	46	26	41	42	46	43	54	39	53	38	47	39
$lq_{EC}+GP_M^{PoI}$	63	59	59	53	**68**	60	50	53	57	58	—	—	54	34	48	47	59	58	60	50	70	57	63	50
$lq_{EC}+lq_M$	48	**69**	53	**72**	**72**	**73**	50	**69**	54	**74**	46	66	—	—	36	60	47	**67**	47	**66**	63	**70**	50	**72**
$lq_{EC}+lmm_M$	64	56	60	56	**76**	68	48	47	59	58	52	53	64	40	—	—	59	56	62	48	**72**	52	68	56
$lmm_{EC}+GP_M^{fval}$	54	46	52	49	71	56	44	46	54	57	41	42	53	33	41	44	—	—	58	46	62	47	60	53
$lmm_{EC}+GP_M^{PoI}$	52	59	49	61	70	59	42	51	46	61	40	50	53	34	38	52	42	54	—	—	68	58	49	54
$lmm_{EC}+lq_M$	36	51	39	53	59	52	33	48	47	62	30	43	37	30	28	48	38	53	32	42	—	—	41	49
$lmm_{EC}+lmm_M$	54	53	49	56	64	54	41	50	53	61	37	50	50	28	32	44	40	47	51	46	59	51	—	—

$2D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	41	45	49	48
lq_{EC}	**59**	**55**	—	—	**59**	55
lmm_{EC}	51	52	41	45	—	—

$2D$	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	49	46	61	44	48	46
GP_M^{PoI}	51	**54**	—	—	**64**	50.4	45	48
lq_M	39	**56**	36	49.6	—	—	34	49
lmm_M	52	**54**	55	52	**66**	51	—	—

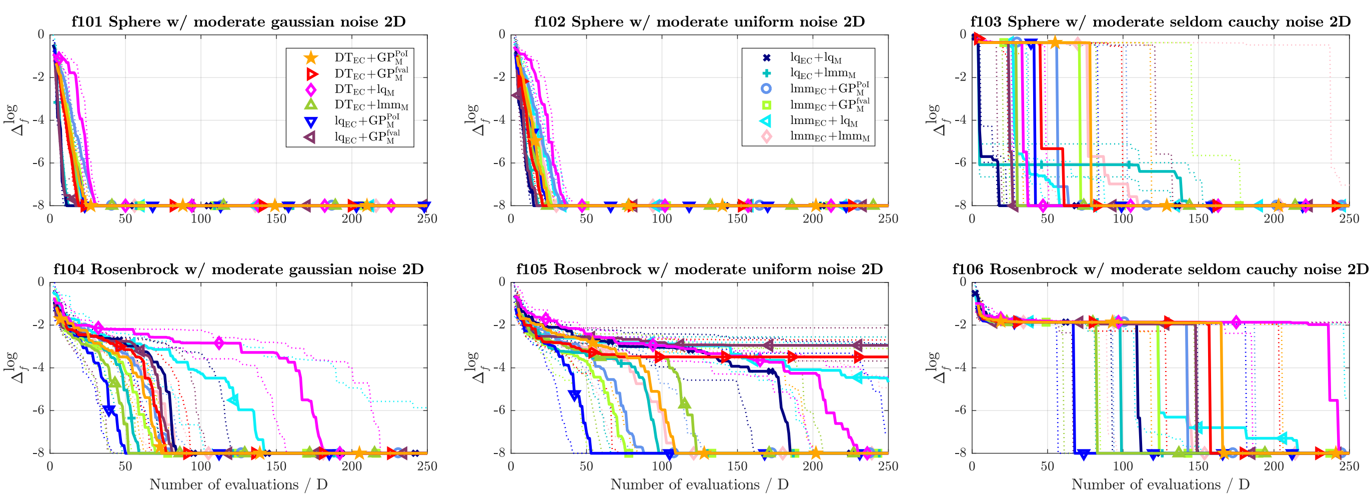

Figure 43: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 6 noisy COCO benchmarks with moderate noise in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 44: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the COCO benchmarks with moderate noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$3D$	$\text{DT}_\text{EC}+\text{GP}^\text{fval}_\text{M}$		$\text{DT}_\text{EC}+\text{GP}^\text{PoI}_\text{M}$		$\text{DT}_\text{EC}+\text{lq}_\text{M}$		$\text{DT}_\text{EC}+\text{lmm}_\text{M}$		$\text{lq}_\text{EC}+\text{GP}^\text{fval}_\text{M}$		$\text{lq}_\text{EC}+\text{GP}^\text{PoI}_\text{M}$		$\text{lq}_\text{EC}+\text{lq}_\text{M}$		$\text{lq}_\text{EC}+\text{lmm}_\text{M}$		$\text{lmm}_\text{EC}+\text{GP}^\text{fval}_\text{M}$		$\text{lmm}_\text{EC}+\text{GP}^\text{PoI}_\text{M}$		$\text{lmm}_\text{EC}+\text{lq}_\text{M}$		$\text{lmm}_\text{EC}+\text{lmm}_\text{M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\text{DT}_\text{EC}+\text{GP}^\text{fval}_\text{M}$	—	—	47	41	63	49	47	52	50	53	40	46	39	38	46	43	50	53	54	48	58	41	53	51
$\text{DT}_\text{EC}+\text{GP}^\text{PoI}_\text{M}$	53	59	—	—	**70**	56	47	59	50	61	38	51	40	43	37	59	58	**68**	61	52	57	57	59	60
$\text{DT}_\text{EC}+\text{lq}_\text{M}$	37	51	30	44	—	—	36	54	31	56	31	46	20	41	33	50	39	56	39	46	43	52	36	56
$\text{DT}_\text{EC}+\text{lmm}_\text{M}$	53	48	53	41	64	46	—	—	46	52	44	46	47	34	48	37	51	58	58	39	59	48	53	54
$\text{lq}_\text{EC}+\text{GP}^\text{fval}_\text{M}$	50	47	50	39	69	44	54	48	—	—	40	44	50	40	49	44	54	51	56	38	58	39	59	46
$\text{lq}_\text{EC}+\text{GP}^\text{PoI}_\text{M}$	60	54	62	49	**69**	54	56	54	60	56	—	—	52	42	56	50	64	62	62	48	63	49	61	60
$\text{lq}_\text{EC}+\text{lq}_\text{M}$	61	62	60	57	**80**	59	53	**66**	50	**60**	48	58	—	—	48	59	60	**69**	58	58	66	60	64	58
$\text{lq}_\text{EC}+\text{lmm}_\text{M}$	54	57	63	41	67	50	52	63	51	56	44	50	52	41	—	—	66	60	64	52	60	51	60	53
$\text{lmm}_\text{EC}+\text{GP}^\text{fval}_\text{M}$	50	47	42	32	61	44	49	42	46	49	36	38	40	31	34	40	—	—	53	36	58	44	52	42
$\text{lmm}_\text{EC}+\text{GP}^\text{PoI}_\text{M}$	46	52	39	48	61	54	42	61	44	62	38	52	42	42	36	48	47	64	—	—	48	51	43	54
$\text{lmm}_\text{EC}+\text{lq}_\text{M}$	42	59	43	43	57	48	41	52	42	61	37	51	34	40	40	49	42	56	52	49	—	—	51	57
$\text{lmm}_\text{EC}+\text{lmm}_\text{M}$	47	49	41	40	64	44	47	46	41	54	39	40	36	42	40	47	48	58	57	46	49	43	—	—

$3D$	DT_EC		lq_EC		lmm_EC	
#FEs/D	50	250	50	250	50	250
DT_EC	—	—	39	46	52	53
lq_EC	**61**	54	—	—	**61**	53
lmm_EC	48	47	39	47	—	—

$3D$	$\text{GP}^\text{fval}_\text{M}$		$\text{GP}^\text{PoI}_\text{M}$		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
$\text{GP}^\text{fval}_\text{M}$	—	—	47	40	57	44	49	46
$\text{GP}^\text{PoI}_\text{M}$	53	**60**	—	—	57	49.6	49	54
lq_M	43	**56**	43	50.4	—	—	45	**57**
lmm_M	51	54	51	46	55	43	—	—

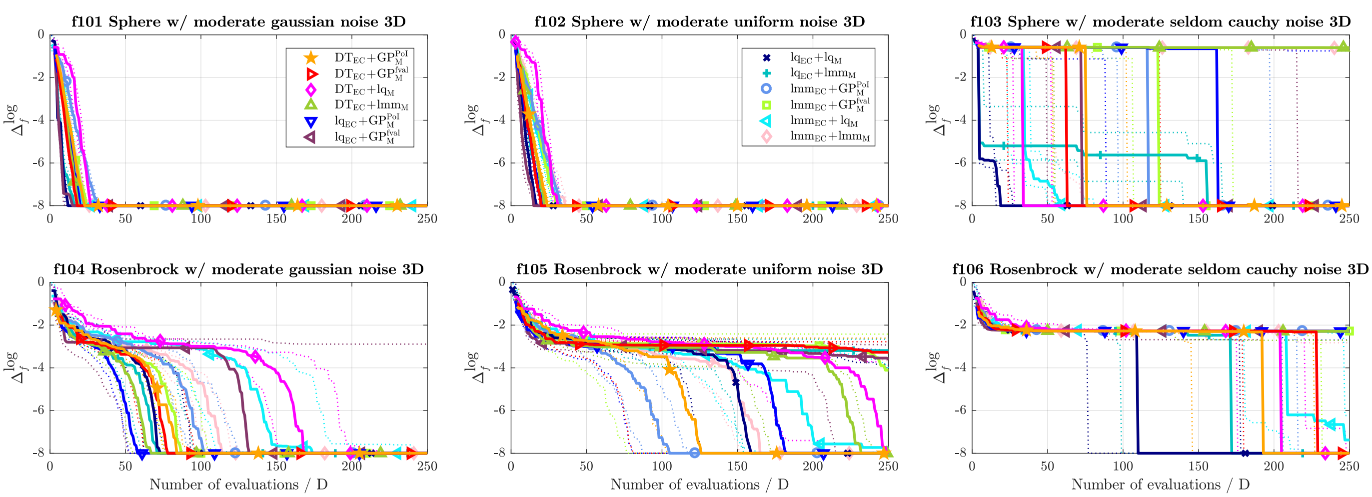

Figure 44: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 6 noisy COCO benchmarks with moderate noise in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 45: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the COCO benchmarks with moderate noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$5D$	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	50	48	**64**	40	38	59	49	62	41	51	39	34	56	51	44	52	48	44	53	42	63	54
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	50	52	—	—	**70**	49	41	**67**	50	**71**	42	56	38	47	52	57	46	57	53	56	59	54	58	**67**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	36	60	30	51	—	—	28	**63**	39	**68**	29	59	18	44	31	56	29	**64**	27	58	33	47	33	59
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	62	41	59	33	**72**	37	—	—	56	57	51	47	42	38	61	47	51	54	56	36	62	42	59	50
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	51	38	50	29	**61**	32	44	43	—	—	51	37	39	33	49	30	46	39	50	27	60	31	58	39
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	59	49	58	44	**71**	41	49	53	49	**63**	—	—	41	41	62	58	56	51	53	41	53	49	58	54
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	61	**66**	**62**	53	**82**	56	58	**62**	61	**67**	59	59	—	—	**62**	58	58	**59**	**68**	59	**68**	57	**73**	**63**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	44	49	48	43	**69**	44	39	53	51	**70**	38	42	38	42	—	—	36	52	43	44	59	41	56	57
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	56	48	54	43	**71**	36	49	46	54	61	44	49	42	41	64	48	—	—	60	48	59	40	58	54
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	52	56	47	44	**73**	42	44	64	50	**73**	47	59	32	41	57	56	40	52	—	—	57	50	56	61
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	47	58	41	46	**67**	53	38	58	40	**69**	47	51	32	43	41	59	41	60	43	50	—	—	50	60
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	37	46	42	33	**67**	41	41	50	42	61	42	46	27	37	44	43	42	46	44	39	50	40	—	—

$5D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	43	52	48	51
$\mathrm{lq_{EC}}$	**58**	48	—	—	**56**	48
$\mathrm{lmm_{EC}}$	53	49	44	52	—	—

$5D$	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	54	44	54	38	48	48
$\mathrm{GP^{PoI}_M}$	46	**56**	—	—	56	47	53	**62**
$\mathrm{lq_M}$	46	**62**	44	**53**	—	—	47	**60**
$\mathrm{lmm_M}$	52	52	47	38	53	40	—	—

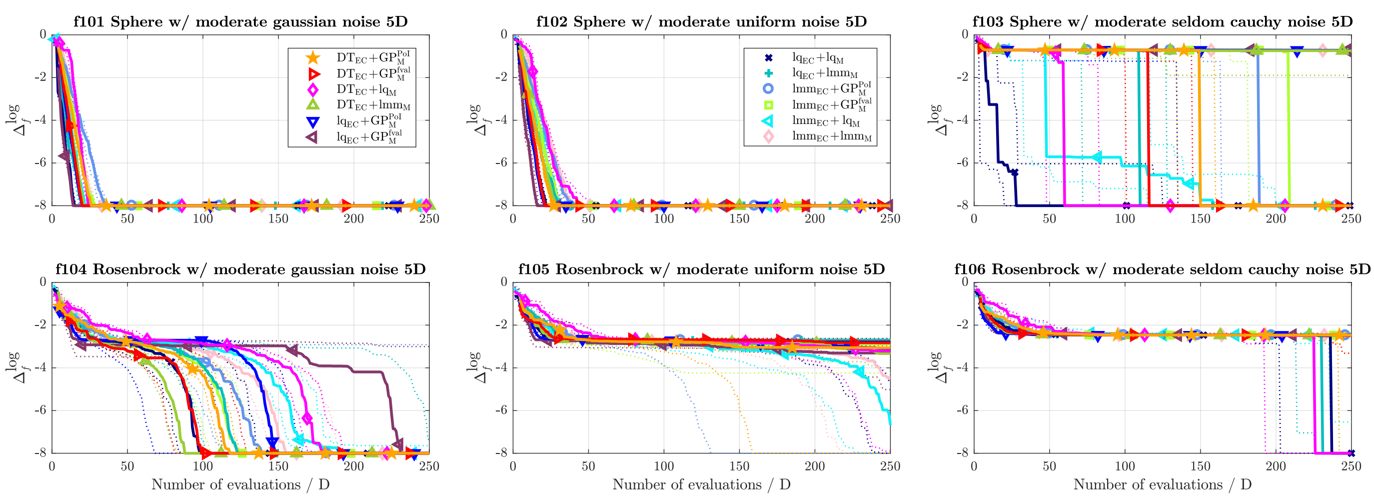

Figure 45: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 6 noisy COCO benchmarks with moderate noise in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 46: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the COCO benchmarks with moderate noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	54	44	**73**	43	48	46	50	**62**	49	**67**	38	37	**78**	57	**80**	62	**83**	47	**81**	58	**76**	51
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	46	56	—	—	**71**	49	46	49	53	**69**	48	**71**	38	44	**76**	54	**84**	**67**	**87**	50	**81**	64	**80**	60
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	27	57	29	51	—	—	23	44	32	**74**	28	**69**	17	42	57	57	64	**71**	**76**	63	**67**	**66**	**70**	68
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	52	54	54	51	**77**	56	—	—	60	**69**	50	**59**	39	43	**81**	58	**79**	71	**83**	57	**83**	64	**83**	57
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	50	38	47	31	**68**	26	40	31	—	—	43	42	39	27	**74**	33	**79**	49	**87**	31	**74**	43	**77**	37
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	51	33	52	29	**72**	31	50	41	57	58	—	—	46	26	**80**	41	**83**	52	**90**	32	**83**	47	**77**	31
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**62**	63	62	56	**83**	58	61	57	61	**73**	54	**74**	—	—	**87**	**63**	**89**	**66**	**94**	60	**92**	**63**	**90**	63
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	22	43	24	46	43	43	19	42	26	**67**	20	59	13	37	—	—	**64**	61	**77**	47	**72**	57	**69**	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	20	38	16	33	36	29	21	29	21	51	17	48	11	34	36	39	—	—	**72**	33	54	43	**68**	43
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	17	53	13	50	24	37	17	43	13	**69**	10	**68**	6	40	23	53	28	67	—	—	39	62	52	62
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	19	42	19	36	33	34	17	36	26	57	17	53	8	37	28	43	46	57	**61**	38	—	—	**64**	43
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	24	49	20	40	30	32	17	43	23	63	23	**69**	10	37	31	49	32	57	48	38	36	57	—	—

10*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	**58**	**79**	**59**
$\mathrm{lq_{EC}}$	51	42	—	—	**83**	49
$\mathrm{lmm_{EC}}$	21	41	18	51	—	—

10*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	**57**	40	**56**	38	**63**	41
$\mathrm{GP^{PoI}_M}$	43	**60**	—	—	52	46	**59**	51
$\mathrm{lq_M}$	44	**62**	48	54	—	—	**58**	50.4
$\mathrm{lmm_M}$	37	**59**	41	49	42	49.6	—	—

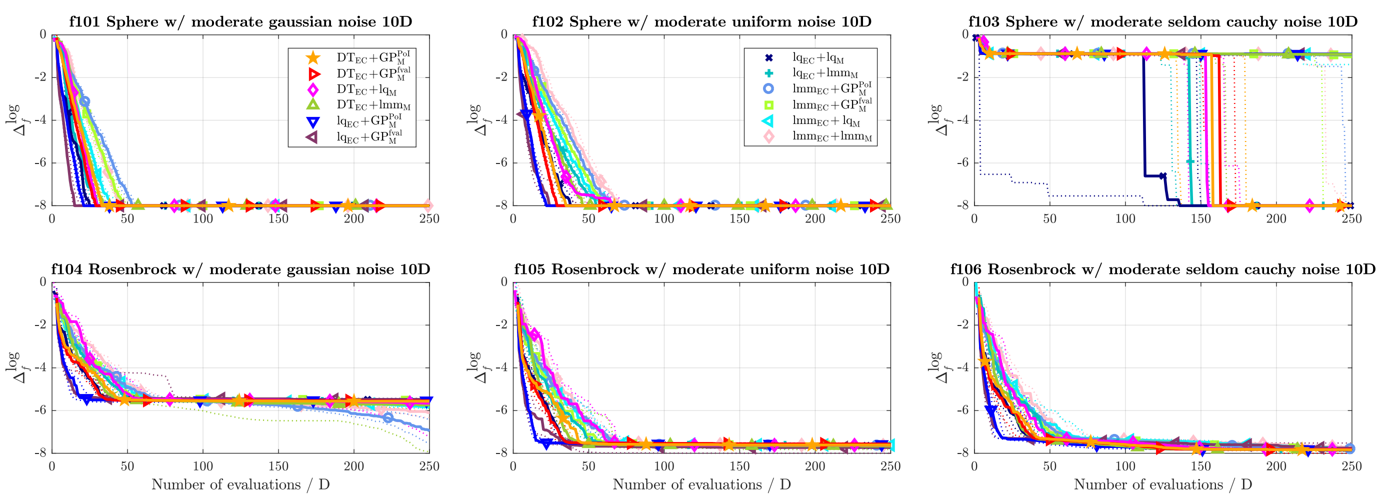

Figure 46: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 6 noisy COCO benchmarks with moderate noise in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 47: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the COCO benchmarks with moderate noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	51	44	**73**	49	**82**	53	57	**68**	54	**64**	**76**	52	**84**	59	**83**	63	**83**	63	**79**	**64**	**93**	**63**
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	49	56	—	—	**78**	49	**82**	54	**60**	**70**	54	**66**	**77**	49	**94**	66	**86**	**58**	**89**	58	**82**	**62**	**93**	**61**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	27	51	22	51	—	—	**69**	54	34	**70**	24	**66**	38	50	**83**	63	**70**	59	**74**	62	**63**	**63**	**96**	**64**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	18	47	18	46	31	46	—	—	22	**63**	24	**59**	26	51	**66**	52	42	57	51	56	39	**59**	**86**	52
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	43	32	40	30	**66**	30	**78**	37	—	—	42	51	69	37	**82**	40	**78**	43	**83**	34	**73**	41	**92**	41
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	46	36	46	34	**76**	34	**76**	41	58	49	—	—	**72**	34	**87**	48	**78**	46	**81**	46	**74**	43	**94**	49
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	24	48	23	51	**62**	50	**74**	49	31	**63**	28	**66**	—	—	**83**	57	**79**	60	**86**	56	**76**	60	**92**	63
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	16	41	6	34	17	37	34	48	18	**60**	13	52	17	43	—	—	24	53	33	52	30	54	**78**	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	17	37	14	42	30	41	58	43	22	57	22	54	21	40	**76**	47	—	—	**70**	51	49	47	**91**	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	17	37	11	42	26	38	49	44	17	66	19	54	14	44	**67**	48	30	49	—	—	38	50	**91**	56
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	21	36	18	38	37	37	61	41	27	59	26	57	24	40	**70**	46	51	53	62	50	—	—	**92**	51
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	7	37	7	39	4	36	14	48	8	59	6	51	8	37	22	49	9	49	9	44	8	49	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**54**	**59**	**80**	**59**
$\mathrm{lq_{EC}}$	46	41	—	—	**78**	50
$\mathrm{lmm_{EC}}$	20	41	22	50	—	—

20*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	54	49	**64**	44	**85**	48
$\mathrm{GP^{PoI}_M}$	46	51	—	—	**63**	44	**87**	53
$\mathrm{lq_M}$	36	**56**	37	56	—	—	**81**	54
$\mathrm{lmm_M}$	15	52	13	47	19	46	—	—

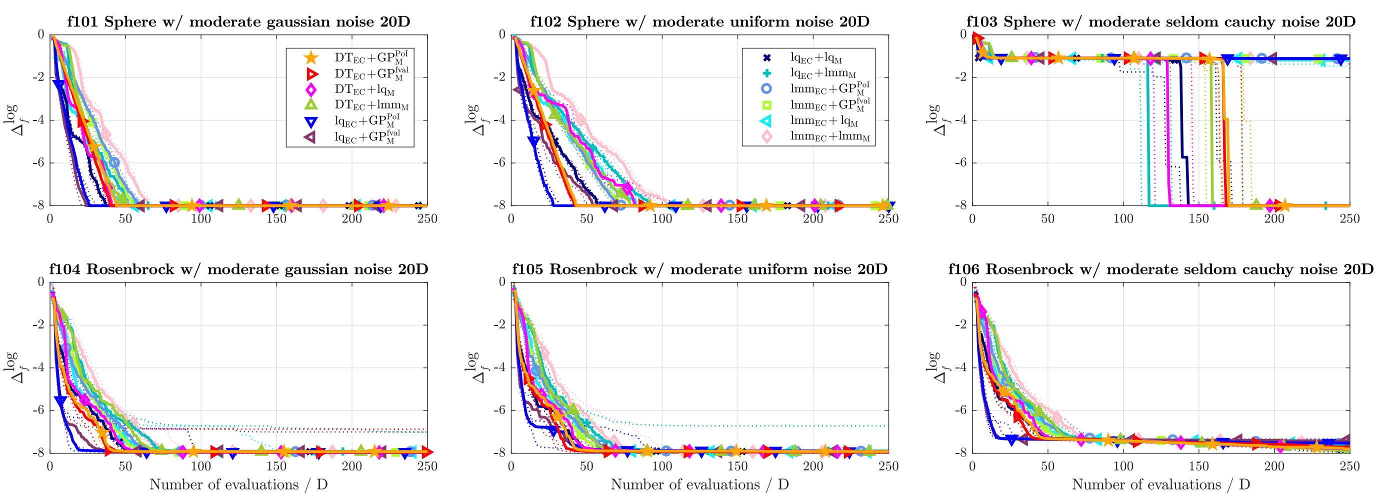

Figure 47: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 6 noisy COCO benchmarks with moderate noise in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 48: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the COCO benchmarks with moderate noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20D	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	49.8	46	**70**	48	52	50.4	50	**60**	44	54	49	38	**60**	51	**61**	**57**	**63**	49	**67**	51	**66**	53
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	50.2	54	—	—	**72**	52	51	54	52	**65**	45	**58**	48	42	**60**	56	**64**	**60**	**68**	51	**68**	**57**	**68**	**58**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	30	52	28	48	—	—	36	51	34	**63**	29	**56**	24	41	46	52	46	**59**	49	54	49.6	55	54	**58**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	48	49.6	49	46	**64**	49	—	—	47	**60**	44	51	41	40	**62**	49	**56**	**59**	**61**	47	**62**	53	**68**	53
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	50	40	48	35	**66**	37	53	40	—	—	44	43	48	32	**59**	38	**60**	45	**66**	34	**64**	38	**66**	40
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	56	46	55	42	**71**	44	**56**	49	**56**	**57**	—	—	53	36	**66**	49	**68**	54	**69**	43	**69**	49	**71**	49
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	51	**62**	52	**58**	**76**	**59**	**59**	**60**	52	**68**	47	**64**	—	—	**63**	**59**	**66**	**64**	**70**	**60**	**73**	**62**	**74**	**64**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	40	49	40	44	54	48	38	51	41	**62**	34	51	37	41	—	—	49.8	**56**	**56**	49	59	51	**66**	54
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	39	43	36	40	54	41	44	41	40	55	32	46	34	36	50.2	44	—	—	**63**	43	56	44	**66**	49
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	37	51	32	49	51	46	39	53	34	**66**	31	57	30	40	44	51	37	**57**	—	—	49.8	54	**58**	**58**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	33	49	32	43	50.4	45	38	47	36	**62**	31	51	27	38	41	49	44	56	50.2	46	—	—	**60**	52
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	34	47	32	42	46	42	32	47	34	**60**	29	51	26	36	34	46	34	51	42	42	40	48	—	—

2 − 20D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	45	**52**	**62**	**54**
$\mathrm{lq_{EC}}$	**55**	48	—	—	**67**	**51**
$\mathrm{lmm_{EC}}$	38	46	33	49	—	—

2 − 20D	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	52	44	**58**	42	**59**	46
$\mathrm{GP^{PoI}_M}$	48	**56**	—	—	**58**	47	**58**	54
$\mathrm{lq_M}$	42	**58**	42	**53**	—	—	**53**	**54**
$\mathrm{lmm_M}$	41	**54**	42	46	47	46	—	—

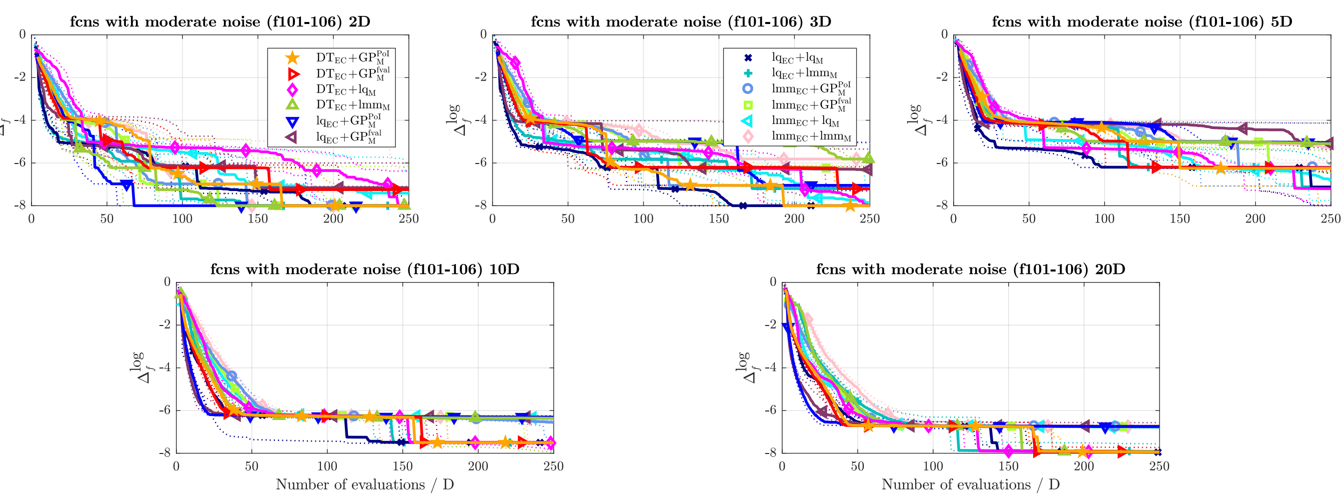

Figure 48: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 6 noisy COCO benchmarks with moderate noise in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 49: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	53	49.8	58	50.2	40	54	50.2	**62**	52	**58**	47	40	43	44	54	**64**	48	57	62	56	49	**63**
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	47	50.2	—	—	60	53	36	56	52	**64**	52	**62**	51	41	42	45	52	**68**	48	**60**	59	**60**	43	**60**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	42	49.8	40	47	—	—	28	49	43	**63**	42	**60**	38	44	36	44	45	**66**	36	**59**	47	**59**	32	**60**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**60**	46	**64**	44	**72**	51	—	—	**63**	**58**	**67**	56	**60**	40	55	40	**64**	**64**	59	**58**	**69**	52	**60**	**61**
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	49.8	38	48	36	57	37	37	42	—	—	48	51	44	32	44	37	49	52	48	50.2	56	42	44	49
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	48	42	48	38	58	40	33	44	52	49	—	—	45	32	46	32	48	51	48	46	60	41	44	45
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	53	**60**	49	**59**	62	**56**	40	**60**	56	**68**	55	**68**	—	—	53	53	55	**73**	47	**68**	60	**64**	46	**68**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	57	56	58	55	**64**	56	45	**60**	56	**63**	54	**68**	47	47	—	—	57	**70**	53	**62**	**60**	**61**	52	**64**
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	46	36	48	32	55	34	36	36	51	48	52	49	45	27	43	30	—	—	44	40	56	41	44	44
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	52	43	52	40	**64**	41	41	42	52	49.8	52	54	53	32	47	38	56	**60**	—	—	**60**	43	48	52
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	38	44	41	40	53	41	31	48	44	**58**	40	59	40	36	40	39	44	**59**	40	57	—	—	40	50.2
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	51	37	57	40	**68**	40	40	39	56	51	56	55	54	32	48	36	56	56	52	48	60	49.8	—	—

$2D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	52	52	**61**
$\mathrm{lq_{EC}}$	51	48	—	—	52	**57**
$\mathrm{lmm_{EC}}$	48	39	48	43	—	—

$2D$	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	47	53	41	43	45
$\mathrm{GP_M^{PoI}}$	52	**53**	—	—	55	43	43	47
$\mathrm{lq_M}$	47	**59**	45	**57**	—	—	40	**51**
$\mathrm{lmm_M}$	**57**	**55**	**57**	**53**	**60**	49	—	—

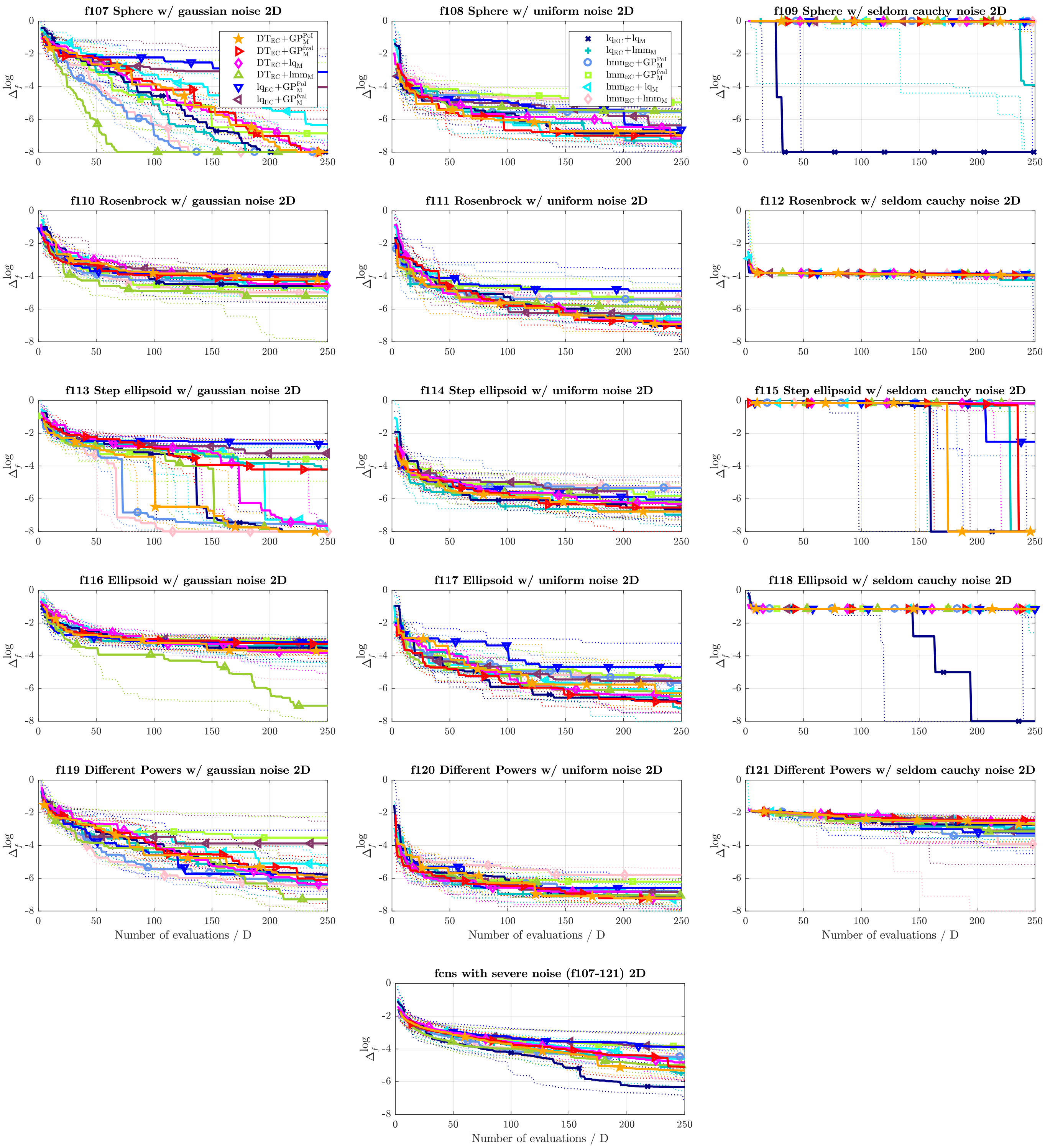

Figure 49: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 15 noisy COCO benchmarks with severe noise in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 50: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	49	43	48	38	36	49	56	**61**	51	**58**	44	36	49	38	52	**64**	38	49	53	51	38	52
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	51	57	—	—	55	44	36	57	51	**66**	49.8	**65**	49	44	52	44	52	**65**	44	56	52	55	43	54
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	52	**62**	45	56	—	—	34	**64**	56	**76**	51	**68**	44	52	47	51	47	**74**	40	**64**	52	**64**	39	**62**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**64**	51	**64**	43	**66**	36	—	—	**65**	**64**	**66**	**60**	59	41	**64**	36	**62**	**60**	58	52	**65**	48	55	52
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	44	39	49	34	44	24	35	36	—	—	47	48	39	28	42	26	49	51	42	44	49	39	36	38
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	49	42	50.2	35	49	32	34	40	53	52	—	—	43	32	48	30	50.2	59	41	43	49.8	39	40	41
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	56	**64**	51	56	56	48	41	**59**	**61**	**72**	**57**	**68**	—	—	53	51	57	**69**	42	**59**	54	**60**	43	**59**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	51	**62**	48	56	53	49	36	**64**	58	**74**	52	**70**	47	49	—	—	57	**72**	44	**61**	57	**63**	39	60
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	48	36	48	35	53	26	38	40	51	49	49.8	41	43	31	43	28	—	—	40	41	53	33	42	36
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	**62**	51	56	44	60	36	42	48	**58**	56	**59**	57	58	41	56	39	60	**59**	—	—	57	47	49	48
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	47	49	48	45	48	36	35	52	51	**61**	50.2	**61**	46	40	43	37	47	**67**	43	53	—	—	36	52
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	**62**	48	57	46	61	38	45	48	**64**	**62**	**60**	**59**	57	41	61	40	58	**64**	51	52	**64**	48	—	—

3*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**53**	**53**	51	**59**
$\mathrm{lq_{EC}}$	47	47	—	—	46	**53**
$\mathrm{lmm_{EC}}$	49	41	**54**	47	—	—

3*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	45	44	47	33	40	37
$\mathrm{GP^{PoI}_M}$	**55**	**56**	—	—	52	41	44	45
$\mathrm{lq_M}$	**53**	**67**	48	**59**	—	—	41	**56**
$\mathrm{lmm_M}$	**60**	**63**	**56**	**55**	**59**	44	—	—

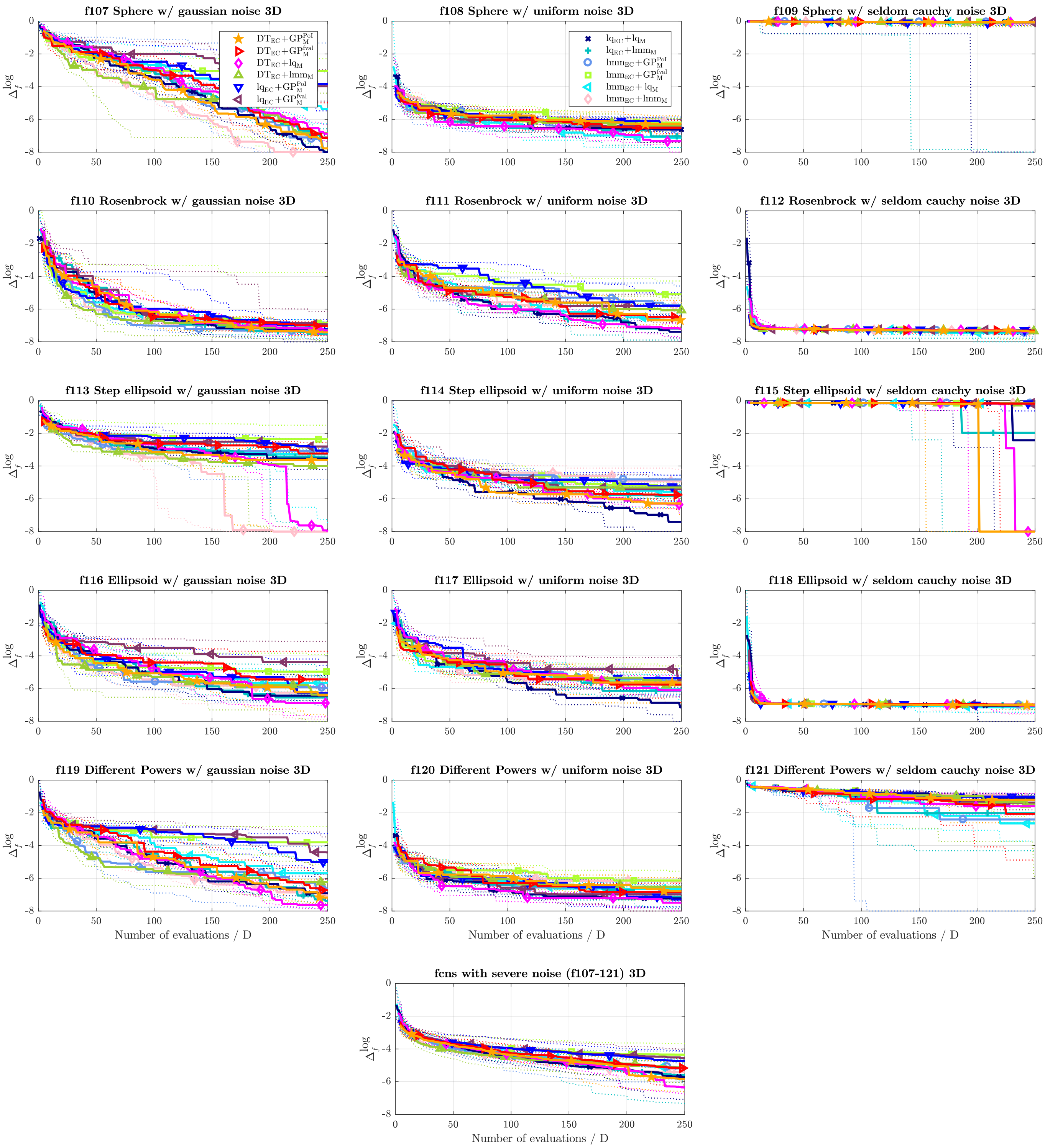

Figure 50: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 15 noisy COCO benchmarks with severe noise in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 51: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$5D$	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	53	43	54	38	40	57	55	**65**	55	**63**	43	38	45	47	43	**60**	44	56	54	53	45	47
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	47	57	—	—	53	48	40	**63**	52	**66**	55	**70**	44	46	47	51	45	**60**	46	**59**	53	59	46	52
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	46	**62**	47	52	—	—	39	**68**	52	**80**	52	**77**	36	51	41	**60**	41	**67**	39	**64**	50.2	**65**	38	55
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	60	43	60	37	61	32	—	—	**63**	**60**	**62**	57	52	34	57	43	55	55	54	46	**64**	46	53	40
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	45	35	48	34	48	20	37	40	—	—	52	48	42	24	39	33	42	46	41	40	49	39	41	32
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	45	37	45	30	48	23	38	43	48	52	—	—	38	29	40	36	41	48	44	45	50.2	38	41	40
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	57	**62**	56	54	**64**	49	48	**66**	**58**	**76**	**62**	**71**	—	—	53	53	54	**66**	54	**64**	**61**	**63**	56	57
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	55	53	53	49	59	40	43	57	**61**	**67**	**60**	**64**	47	47	—	—	44	**58**	48	58	60	55	45	49.8
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	57	40	55	40	59	33	45	45	58	54	59	52	46	34	56	42	—	—	48	46	58	44	55	44
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	56	44	54	41	61	36	46	54	**59**	60	56	55	46	36	52	42	52	54	—	—	62	47	51	42
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	46	47	47	41	49.8	35	36	54	51	**61**	49.8	**62**	39	37	40	45	42	56	38	53	—	—	39	43
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	55	53	54	48	62	45	47	**60**	**59**	**68**	**59**	**60**	44	43	55	50.2	45	**56**	49	**58**	**61**	57	—	—

$5D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	**57**	48	**56**
$\mathrm{lq_{EC}}$	49	43	—	—	48	51
$\mathrm{lmm_{EC}}$	52	44	52	49	—	—

$5D$	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	51	46	51	36	45	45
$\mathrm{GP_M^{PoI}}$	49	**54**	—	—	51	41	44	47
$\mathrm{lq_M}$	49	**64**	49	**59**	—	—	44	**55**
$\mathrm{lmm_M}$	**55**	**55**	**56**	**53**	**56**	45	—	—

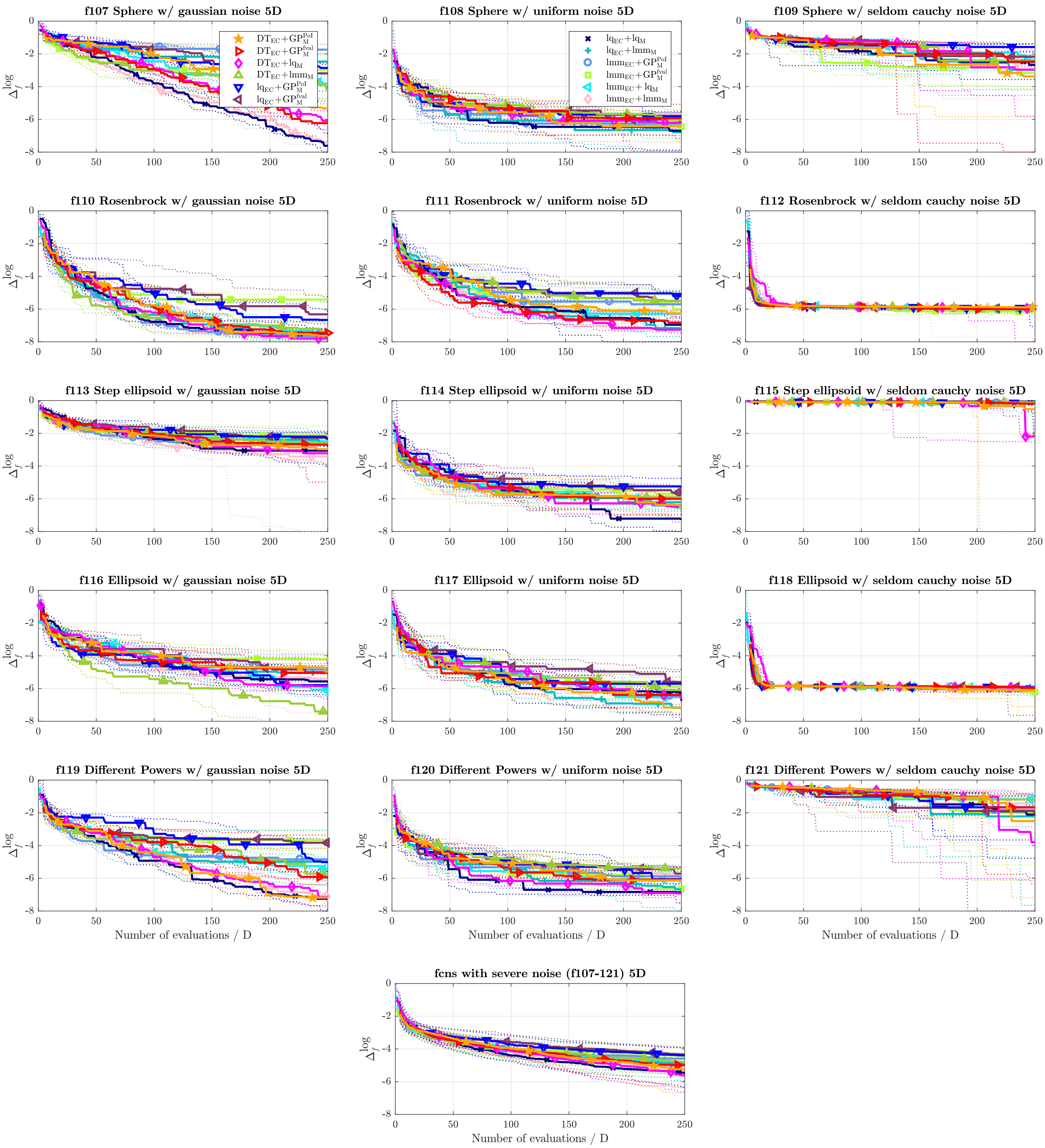

Figure 51: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 15 noisy COCO benchmarks with severe noise in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 52: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	52	48	58	48	51	**68**	**64**	**63**	57	**61**	48	40	54	**66**	49.8	46	49.8	43	61	55	56	46
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	48	52	—	—	56	49.8	52	**70**	**66**	**67**	56	**68**	44	46	52	**68**	49	48	52	46	58	58	57	48
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	42	52	44	50.2	—	—	42	**70**	**59**	**67**	52	**65**	40	47	48	**68**	40	48	43	47	48	57	47	55
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	49	32	48	30	58	30	—	—	**64**	45	59	43	47	25	54	53	48	30	52	28	57	36	57	32
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	36	37	34	33	41	33	36	55	—	—	43	55	30	28	43	58	33	36	35	31	43	37	40	32
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	43	39	44	32	48	35	41	57	57	45	—	—	37	29	47	55	41	32	41	31	48	40	44	35
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	52	60	56	54	**60**	53	53	**75**	**70**	**72**	**63**	**71**	—	—	57	**75**	53	53	52	52	60	**63**	**64**	56
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	46	34	48	32	52	32	46	47	**57**	42	53	45	43	25	—	—	48	32	48	31	55	33	51	34
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	50.2	54	51	52	**60**	52	52	**70**	**67**	**64**	**59**	**68**	47	47	52	**68**	—	—	51	47	59	**60**	57	56
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	50.2	**57**	48	54	**57**	53	48	**72**	**65**	**69**	**59**	**69**	48	48	52	**69**	49	53	—	—	60	**62**	**60**	54
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	39	45	42	42	52	43	43	**64**	**57**	**63**	52	**60**	40	37	45	**67**	41	40	40	38	—	—	45	44
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	44	54	43	52	53	45	43	**68**	**60**	**68**	56	**65**	36	44	49	**66**	43	44	40	46	55	56	—	—

10*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	53	**58**	52	45
$\mathrm{lq_{EC}}$	47	42	—	—	46	41
$\mathrm{lmm_{EC}}$	48	**55**	**54**	**59**	—	—

10*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	49	49.9	49	45	50.4	**61**
$\mathrm{GP^{PoI}_{M}}$	51	50.1	—	—	51	47	53	**60**
$\mathrm{lq_{M}}$	51	**55**	49	53	—	—	48	**63**
$\mathrm{lmm_{M}}$	49.6	39	47	40	52	37	—	—

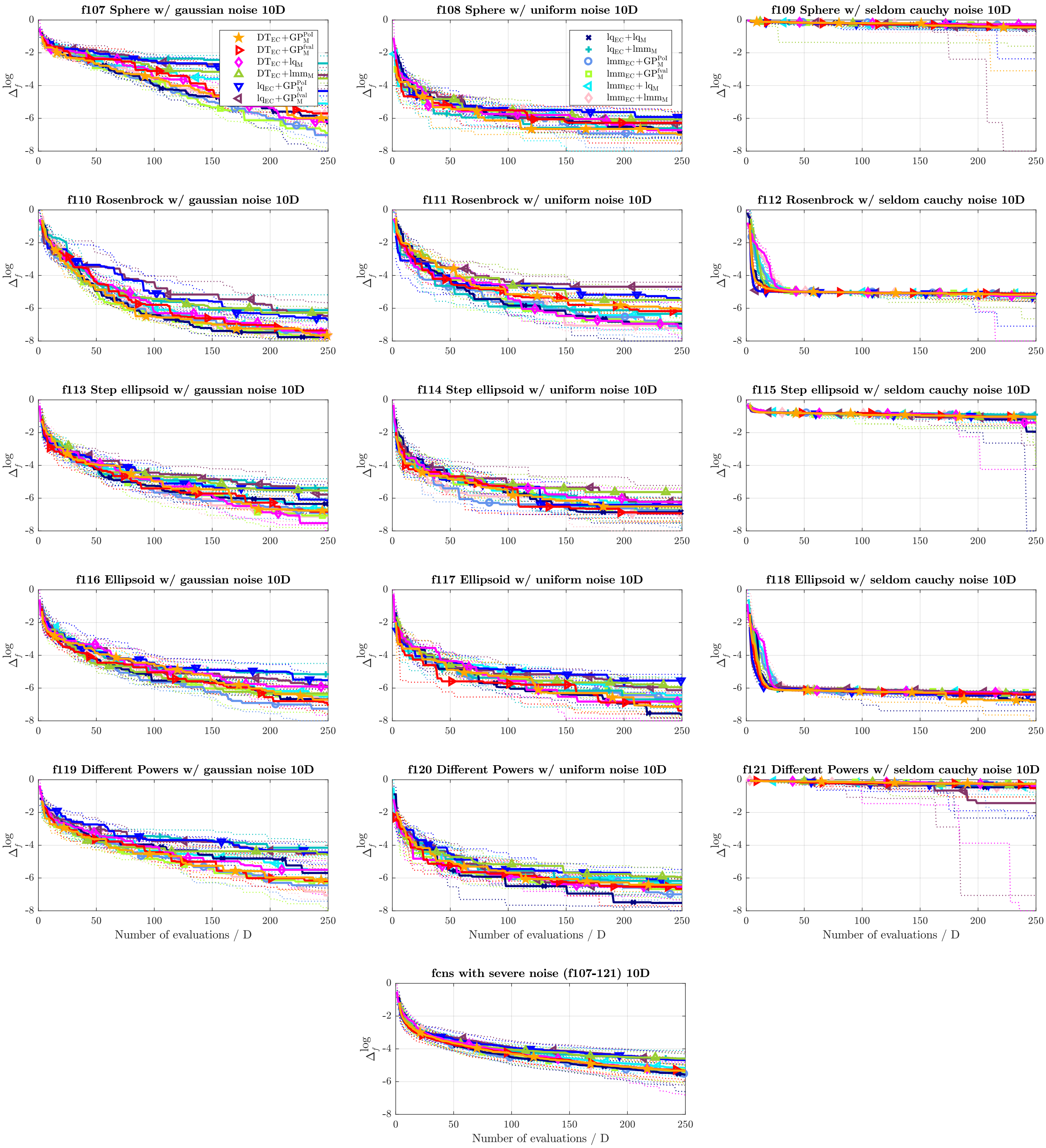

Figure 52: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 15 noisy COCO benchmarks with severe noise in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 53: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_{M}}$	—	—	47	51	55	44	**62**	**78**	**61**	**62**	58	**60**	53	47	**64**	**72**	50.2	46	50.2	52	**64**	58	57	43
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	53	49	—	—	53	43	**63**	**76**	**64**	**58**	57	**65**	58	51	**68**	**74**	51	48	51	50.2	**66**	60	62	45
$\mathrm{DT_{EC}}$+$\mathrm{lq_{M}}$	45	56	47	57	—	—	**59**	**76**	58	**66**	50.2	**66**	48	54	**65**	**71**	47	58	45	58	58	**61**	55	55
$\mathrm{DT_{EC}}$+$\mathrm{lmm_{M}}$	38	22	37	24	41	24	—	—	48	29	40	30	40	22	51	43	40	24	37	25	48	23	44	20
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_{M}}$	39	38	36	42	42	34	52	**71**	—	—	42	50.2	39	35	49	**67**	38	36	39	36	50.2	45	42	33
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	42	40	43	35	49.8	34	**60**	**70**	58	49.8	—	—	46	39	58	**65**	51	39	44	40	54	47	53	36
$\mathrm{lq_{EC}}$+$\mathrm{lq_{M}}$	47	53	42	49	52	46	**60**	**78**	61	**65**	54	**61**	—	—	**64**	**72**	49	49	49	55	**63**	**60**	59	49
$\mathrm{lq_{EC}}$+$\mathrm{lmm_{M}}$	36	28	32	26	35	29	49	57	51	33	42	35	36	28	—	—	37	28	36	27	44	28	44	22
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_{M}}$	49.8	54	49	52	53	42	**60**	**76**	**62**	**64**	49	**61**	51	51	**63**	**72**	—	—	49.8	51	**61**	59	58	47
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_{M}}$	49.8	48	49	49.8	55	42	**63**	**75**	**61**	**64**	56	60	51	45	**64**	**73**	50.2	49	—	—	**65**	60	61	47
$\mathrm{lmm_{EC}}$+$\mathrm{lq_{M}}$	36	42	34	40	42	39	52	**77**	49.8	55	46	53	37	40	56	**72**	39	41	35	40	—	—	45	41
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_{M}}$	43	57	38	55	45	45	**56**	**80**	58	**67**	47	**64**	41	51	**56**	**78**	42	53	39	53	55	**59**	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**54**	**56**	51	44
$\mathrm{lq_{EC}}$	46	44	—	—	47	39
$\mathrm{lmm_{EC}}$	49	**56**	**53**	**61**	—	—

20*D*	$\mathrm{GP^{fval}_{M}}$		$\mathrm{GP^{PoI}_{M}}$		$\mathrm{lq_{M}}$		$\mathrm{lmm_{M}}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_{M}}$	—	—	46	51	51	46	**56**	**64**
$\mathrm{GP^{PoI}_{M}}$	54	49	—	—	**55**	47	**61**	**63**
$\mathrm{lq_{M}}$	49	54	45	53	—	—	**56**	**63**
$\mathrm{lmm_{M}}$	44	36	39	37	44	37	—	—

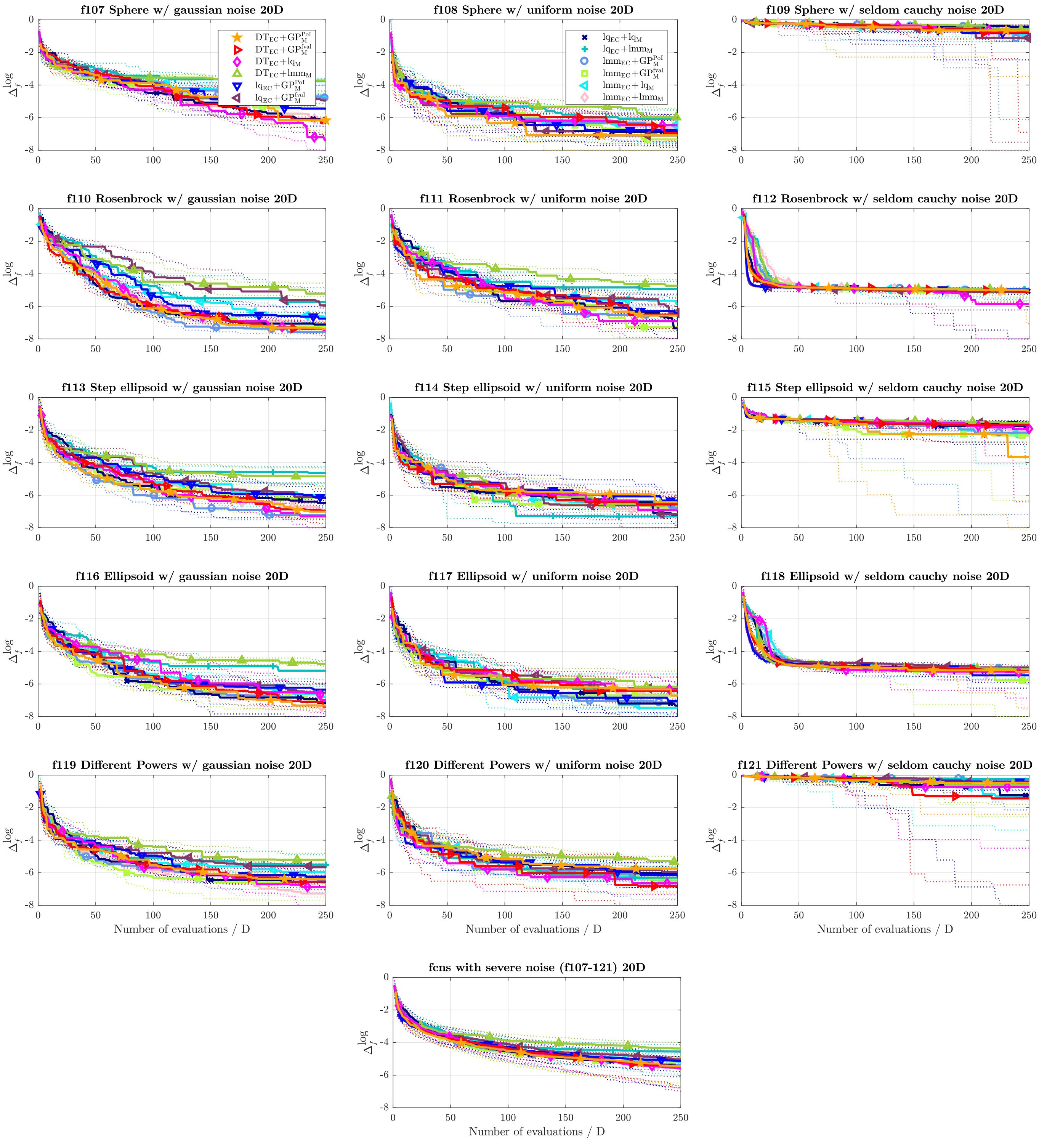

Figure 53: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 15 noisy COCO benchmarks with severe noise in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 54: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2-20D$	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	51	47	55	44	46	**61**	**57**	**63**	**54**	**60**	47	40	51	53	49.9	**56**	46	52	**59**	**54**	49	50.3
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	49	53	—	—	55	48	45	**64**	**57**	**64**	**54**	**66**	49	46	52	**56**	49.9	**58**	48	**54**	**57**	**58**	50	52
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	45	**56**	45	**52**	—	—	40	**66**	**54**	**70**	49	**67**	41	49	47	**59**	44	**63**	41	**58**	51	**61**	42	**57**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**54**	39	**55**	36	**60**	34	—	—	**61**	51	**59**	49	52	32	**56**	43	54	47	52	42	**61**	41	54	41
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	43	37	43	36	46	30	39	49	—	—	46	51	39	29	43	44	42	44	41	40	49.5	40	41	37
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	46	40	46	34	51	33	41	51	54	49	—	—	42	32	48	44	46	46	44	41	52	41	44	39
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	53	**60**	51	**54**	**59**	51	48	**68**	**61**	**71**	**58**	**68**	—	—	**56**	**61**	54	**62**	49	**59**	**60**	**62**	54	**58**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	49	47	48	44	53	41	44	**57**	**57**	**56**	52	**56**	44	39	—	—	49	**52**	46	48	55	48	46	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	50.1	44	50.1	42	**56**	37	46	53	**58**	**56**	**54**	54	46	38	51	48	—	—	46	45	**57**	47	51	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	**54**	48	**52**	46	**59**	42	48	**58**	**59**	**60**	**56**	**59**	51	41	**54**	52	**54**	**55**	—	—	**61**	52	54	49
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	41	46	43	42	49	39	39	**59**	50.5	**60**	48	**59**	40	38	45	52	43	**53**	39	48	—	—	41	46
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	51	49.7	50	48	**58**	43	46	**59**	**59**	**63**	**56**	**61**	46	42	**54**	54	49	**54**	46	51	**59**	54	—	—

$2-20D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**52**	**55**	51	**53**
$\mathrm{lq_{EC}}$	48	45	—	—	48	48
$\mathrm{lmm_{EC}}$	49	47	**52**	52	—	—

$2-20D$	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	47	50.2	40	47	50.3
$\mathrm{GP_M^{PoI}}$	**52**	**53**	—	—	53	44	49	52
$\mathrm{lq_M}$	49.8	**60**	47	**56**	—	—	46	**57**
$\mathrm{lmm_M}$	**53**	49.7	51	48	**54**	43	—	—

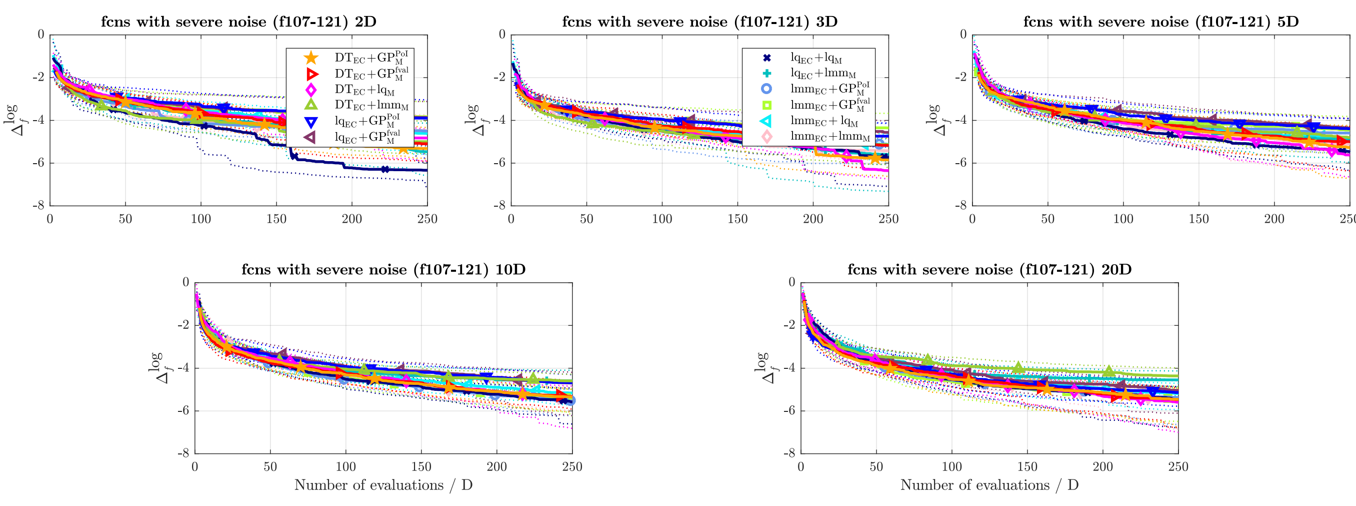

Figure 54: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 15 noisy COCO benchmarks with severe noise in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 55: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the highly multi-modal COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	50.4	47	49	41	44	51	52	56	53	56	50.4	44	53	42	55	56	48	52	56	48	53	**69**
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	49.6	53	—	—	52	51	48	58	48	53	53	**61**	51	43	50.4	47	51	**64**	47	59	56	53	53	**67**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	51	59	48	49	—	—	47	59	53	58	53	**59**	56	47	53	49.6	55	**65**	48	55	57	56	60	**66**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	56	49	52	42	53	41	—	—	53	49.6	56	59	61	41	59	41	58	55	52	53	58	44	64	**63**
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	48	44	52	47	47	42	47	50.4	—	—	53	53	49	41	52	45	59	56	51	51	56	47	57	59
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	47	44	47	39	47	41	44	41	47	47	—	—	53	36	47	39	53	49.6	48	47	51	41	54	56
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	49.6	56	49	57	44	53	39	59	51	59	47	**64**	—	—	48	51	52	**65**	46	58	51	57	55	**69**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	47	58	49.6	53	47	50.4	41	59	48	55	53	**61**	52	49	—	—	55	**62**	49.6	59	49.6	53	53	**71**
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	45	44	49	36	45	35	42	45	41	44	47	50.4	48	35	45	38	—	—	43	44	51	39	51	55
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	52	48	53	41	52	45	48	47	49	49	52	53	54	42	50.4	41	57	56	—	—	54	41	61	**62**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	44	52	44	47	43	44	42	56	44	53	49	**59**	49	43	50.4	47	49	**61**	46	59	—	—	49	**62**
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	47	31	47	33	40	34	36	37	43	41	46	44	45	31	47	29	49	45	39	38	51	38	—	—

$2D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	55	51	**56**	**58**
$\mathrm{lq_{EC}}$	45	49	—	—	53	**58**
$\mathrm{lmm_{EC}}$	44	42	47	42	—	—

$2D$	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	49	48	49.6	40	49	50.4
$\mathrm{GP_M^{PoI}}$	51	52	—	—	53	43	52	53
$\mathrm{lq_M}$	50.4	**60**	47	**57**	—	—	48	**57**
$\mathrm{lmm_M}$	51	49.6	48	47	52	43	—	—

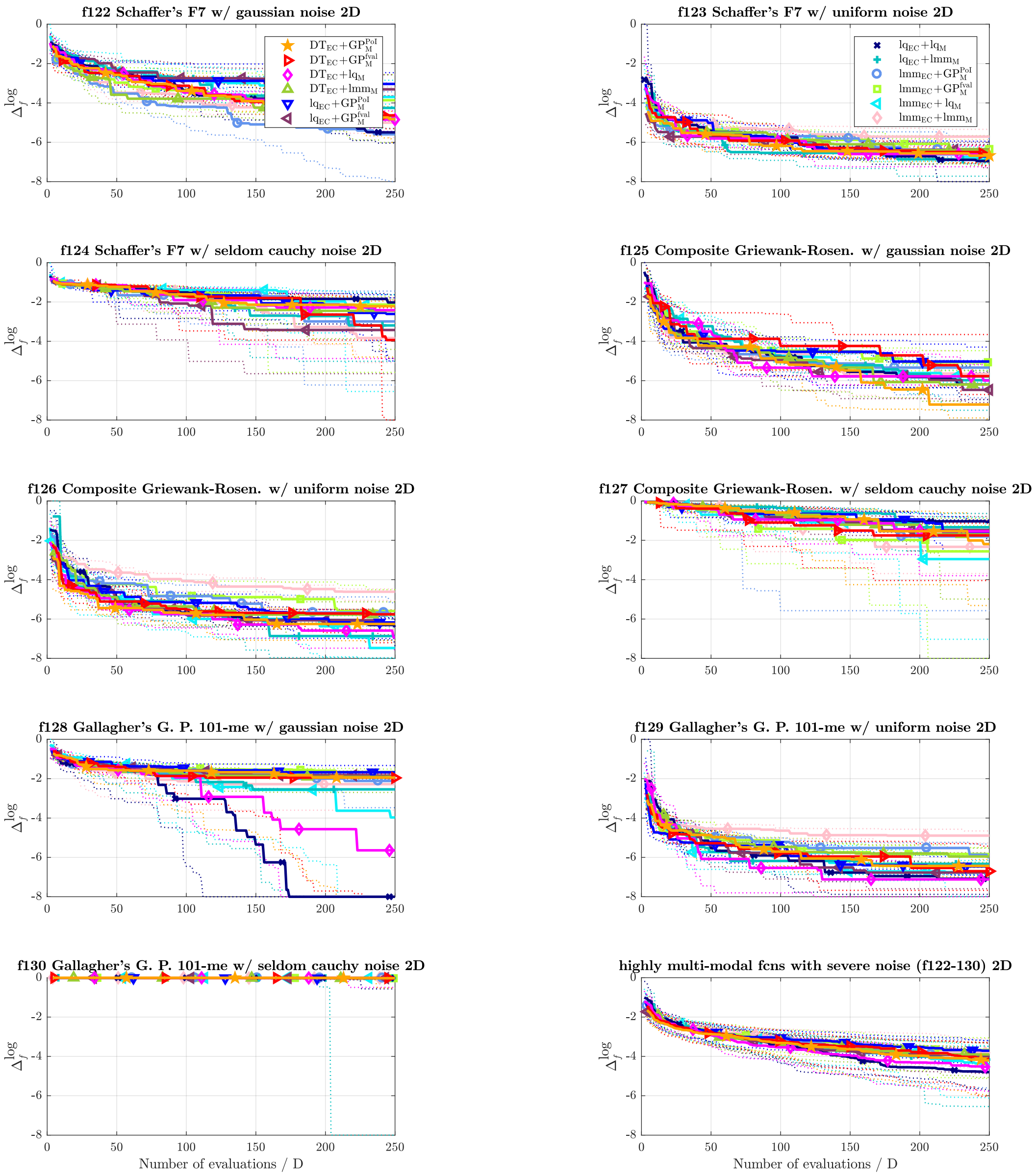

Figure 55: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 9 highly multi-modal noisy COCO benchmarks with severe noise in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 56: A pairwise comparison of the evolution controls, models, and their combinations in $3D$ over the highly multi-modal COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

3*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	46	51	53	53	45	54	41	53	53	56	51	42	47	51	46	56	44	53	57	56	58	59
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	54	49	—	—	57	47	48	55	42	49.6	51	56	48	46	53	48	50.4	59	47	53	59	55	49.6	**63**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	47	47	43	53	—	—	46	57	43	53	51	59	53	43	52	53	47	56	44	52	49	61	47	60
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	55	46	52	45	54	43	—	—	47	47	59	53	53	37	58	42	53	52	56	52	63	49	53	59
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	59	47	58	50.4	57	47	53	53	—	—	55	54	56	36	56	44	55	54	48	47	**67**	53	55	59
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	47	44	49	44	49	41	41	47	45	46	—	—	49.6	36	52	40	49	53	44	47	53	47	49.6	53
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	49	58	52	54	47	57	47	**63**	44	**64**	50.4	**64**	—	—	48	48	51	**63**	47	58	59	58	52	**64**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	53	49	47	52	48	47	42	58	44	56	48	60	52	52	—	—	50.4	54	46	53	56	59	52	**66**
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	54	44	49.6	41	53	44	47	48	45	46	51	47	49	37	49.6	46	—	—	43	44	56	45	53	57
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	56	47	53	47	56	48	44	48	52	53	56	53	53	42	54	47	57	56	—	—	61	47	59	62
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	43	44	41	45	51	39	37	51	33	47	47	53	41	42	44	41	44	55	39	53	—	—	49	57
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	42	41	50.4	37	53	40	47	41	45	41	50.4	47	48	36	48	34	47	43	41	38	51	43	—	—

3*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	49	49	**57**
$\mathrm{lq_{EC}}$	49	51	—	—	52	**56**
$\mathrm{lmm_{EC}}$	51	43	48	44	—	—

3*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	49.9	55	45	52	52
$\mathrm{GP_M^{PoI}}$	52	50.1	—	—	56	43	53	52
$\mathrm{lq_M}$	45	**55**	44	**57**	—	—	48	**54**
$\mathrm{lmm_M}$	48	48	47	48	52	46	—	—

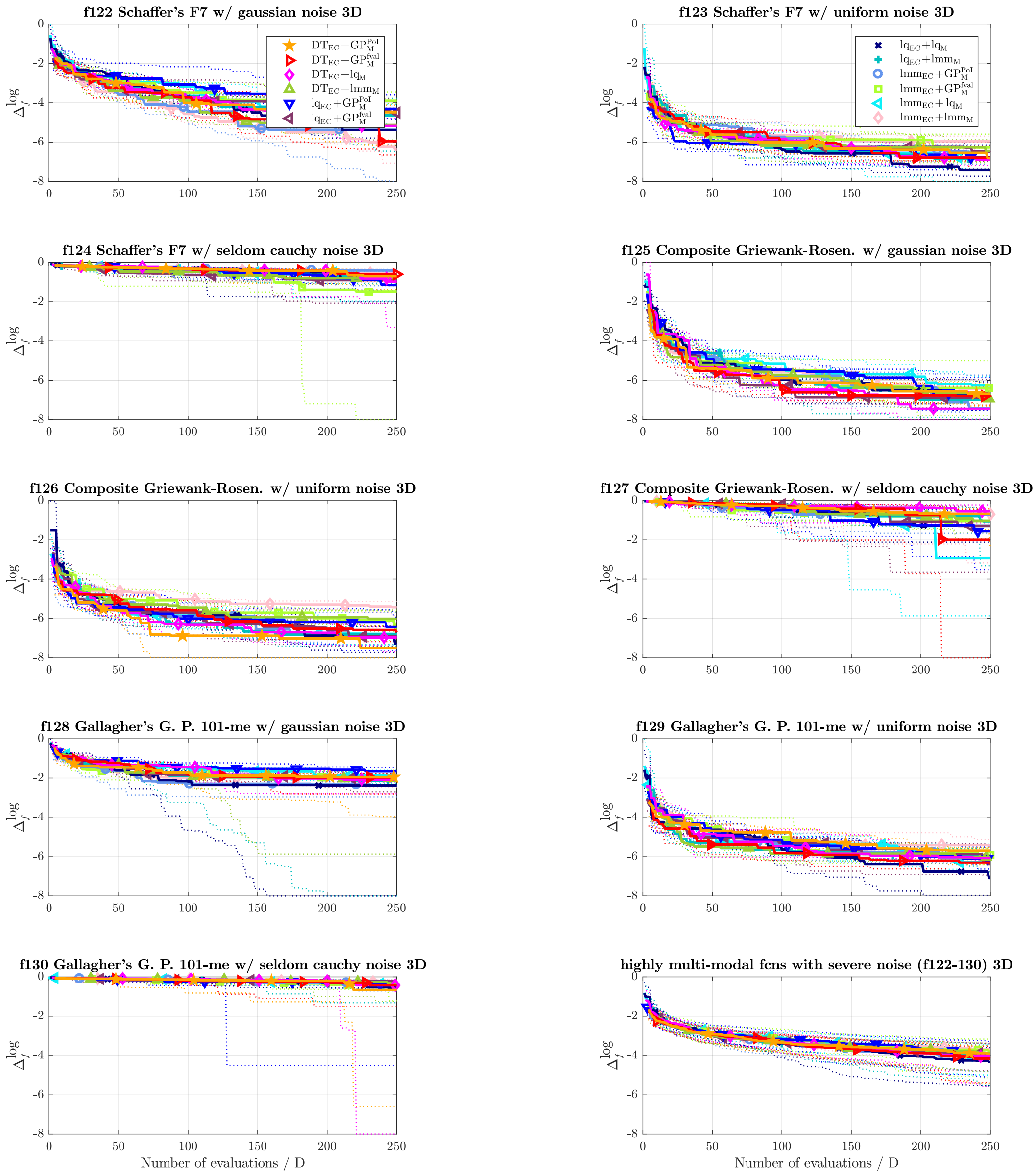

Figure 56: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 9 highly multi-modal noisy COCO benchmarks with severe noise in 3D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 57: A pairwise comparison of the evolution controls, models, and their combinations in $5D$ over the highly multi-modal COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

5*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	49	53	61	47	53	**65**	62	55	56	**61**	50.4	39	46	48	50.4	56	48	49.6	59	50.4	53	51
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	51	47	—	—	59	46	56	**64**	54	52	56	**64**	53	41	44	53	48	59	49	48	62	55	47	56
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	39	53	41	54	—	—	44	**65**	50.4	61	49	**68**	45	49.6	39	55	38	53	37	53	52	60	40	63
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	47	35	44	36	56	35	—	—	58	40	52	49	45	28	41	36	49	40	41	34	57	41	46	35
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	38	45	46	48	49.6	39	42	60	—	—	47	**61**	46	37	36	47	47	52	41	44	53	50.4	43	49
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	44	39	44	36	51	32	48	51	53	39	—	—	45	28	39	39	43	40	38	40	51	39	38	40
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	49.6	61	47	59	55	50.4	55	**72**	54	**63**	55	**72**	—	—	41	55	45	**64**	43	53	58	**63**	47	59
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	54	52	56	47	61	45	59	**64**	**64**	53	**61**	**61**	59	45	—	—	53	55	42	53	63	53	54	58
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	49.6	44	52	41	62	47	51	60	53	48	57	60	55	36	47	45	—	—	49	49	56	44	53	49.6
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	52	50.4	51	52	63	47	59	**66**	59	56	**62**	**60**	57	47	58	47	51	51	—	—	58	54	54	50.4
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	41	49.6	38	45	48	40	43	59	47	49.6	49	**61**	42	37	37	47	44	56	42	46	—	—	38	53
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	47	49	53	44	60	37	54	**65**	57	51	**62**	60	53	41	46	42	47	50.4	46	49.6	62	47	—	—

5*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	51	49	49.8
$\mathrm{lq_{EC}}$	49	49	—	—	49	53
$\mathrm{lmm_{EC}}$	51	50.2	51	47	—	—

5*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	54	54	43	47	54
$\mathrm{GP_M^{PoI}}$	52	46	—	—	54	43	49.6	51
$\mathrm{lq_M}$	46	**57**	46	**57**	—	—	41	**58**
$\mathrm{lmm_M}$	53	46	50.4	49	**59**	42	—	—

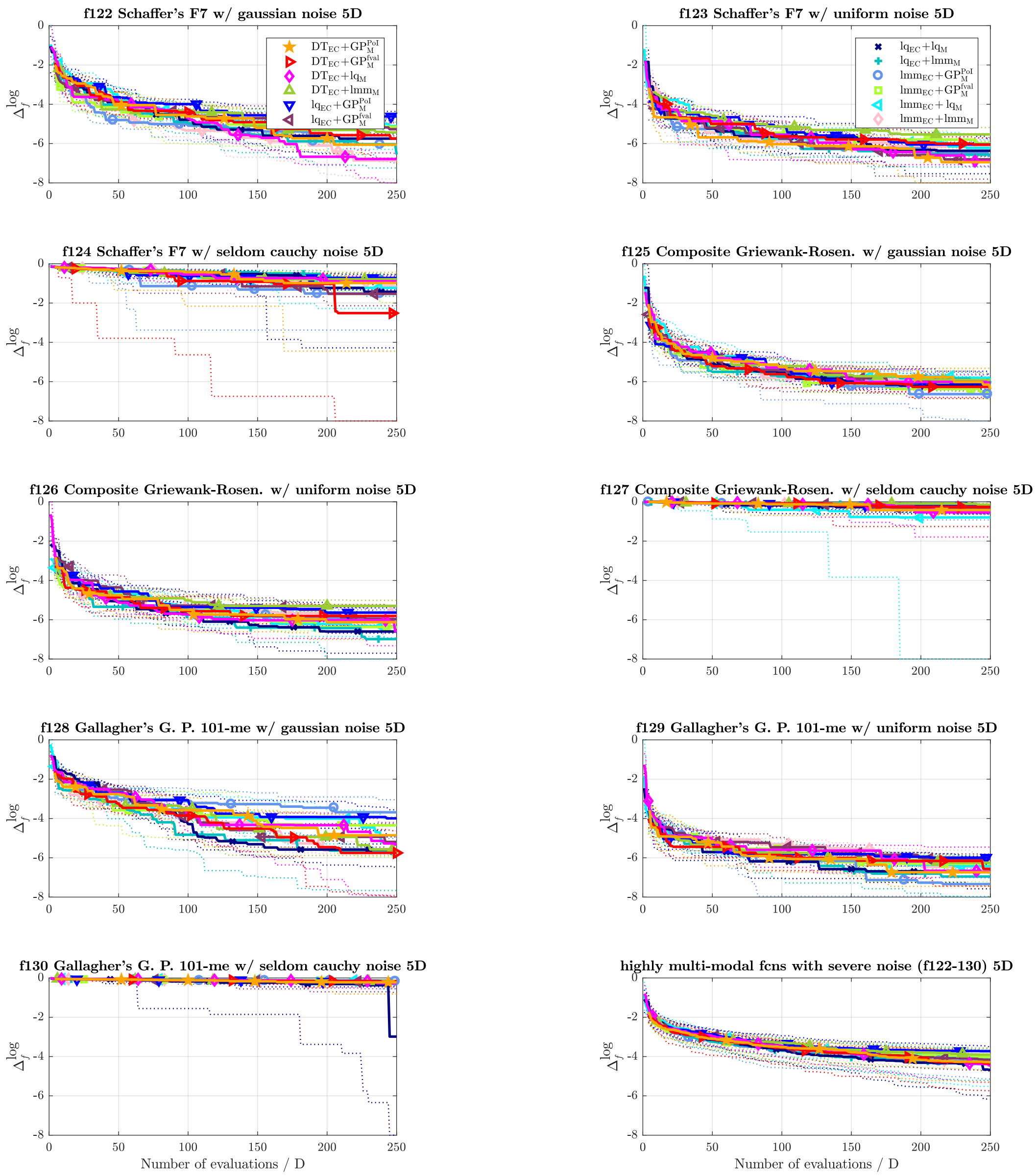

Figure 57: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 9 highly multi-modal noisy COCO benchmarks with severe noise in 5D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 58: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the highly multi-modal COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	47	49	54	44	57	63	48	50.4	53	56	51	45	53	58	51	46	45	43	54	47	54	48
DT_{EC}+GP_M^{PoI}	53	51	—	—	56	50.4	61	**69**	49	53	53	49.6	54	46	59	60	55	48	51	47	57	49	53	52
DT_{EC}+lq_M	46	56	44	49.6	—	—	50.4	**61**	46	56	51	59	43	43	56	63	44	49	44	49	53	48	47	51
DT_{EC}+lmm_M	43	37	39	31	49.6	39	—	—	49	46	48	43	39	27	53	47	49	42	42	33	48	36	49	39
lq_{EC}+GP_M^{fval}	52	49.6	51	47	54	44	51	54	—	—	53	47	48	44	56	60	52	49	48	43	56	47	52	47
lq_{EC}+GP_M^{PoI}	47	44	47	50.4	49	41	52	57	47	53	—	—	44	43	54	56	49	48	45	43	50.4	43	45	49.6
lq_{EC}+lq_M	49	55	46	54	57	57	61	**73**	52	56	56	57	—	—	59	**68**	55	59	51	53	56	55	54	60
lq_{EC}+lmm_M	47	42	41	40	44	37	47	53	44	40	46	44	41	32	—	—	49.6	46	43	41	46	34	43	46
lmm_{EC}+GP_M^{fval}	49	54	45	52	56	51	51	58	48	51	51	52	45	41	50.4	54	—	—	44	49.6	54	47	49	50.4
lmm_{EC}+GP_M^{PoI}	55	57	49	53	56	51	58	**67**	52	57	55	57	49	47	57	59	56	50.4	—	—	59	50.4	51	55
lmm_{EC}+lq_M	46	53	43	51	47	52	52	**64**	44	53	49.6	57	44	45	54	66	46	53	41	49.6	—	—	44	54
lmm_{EC}+lmm_M	46	52	47	48	53	49	51	61	48	53	55	50.4	46	40	57	54	51	49.6	49	45	56	46	—	—

10*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	49	47	51	45
lq_{EC}	51	53	—	—	49	48
lmm_{EC}	49	55	51	52	—	—

10*D*	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	48	49	52	45	54	**58**
GP_M^{PoI}	52	51	—	—	53	48	55	**60**
lq_M	48	55	47	52	—	—	51	**61**
lmm_M	46	42	45	40	49	39	—	—

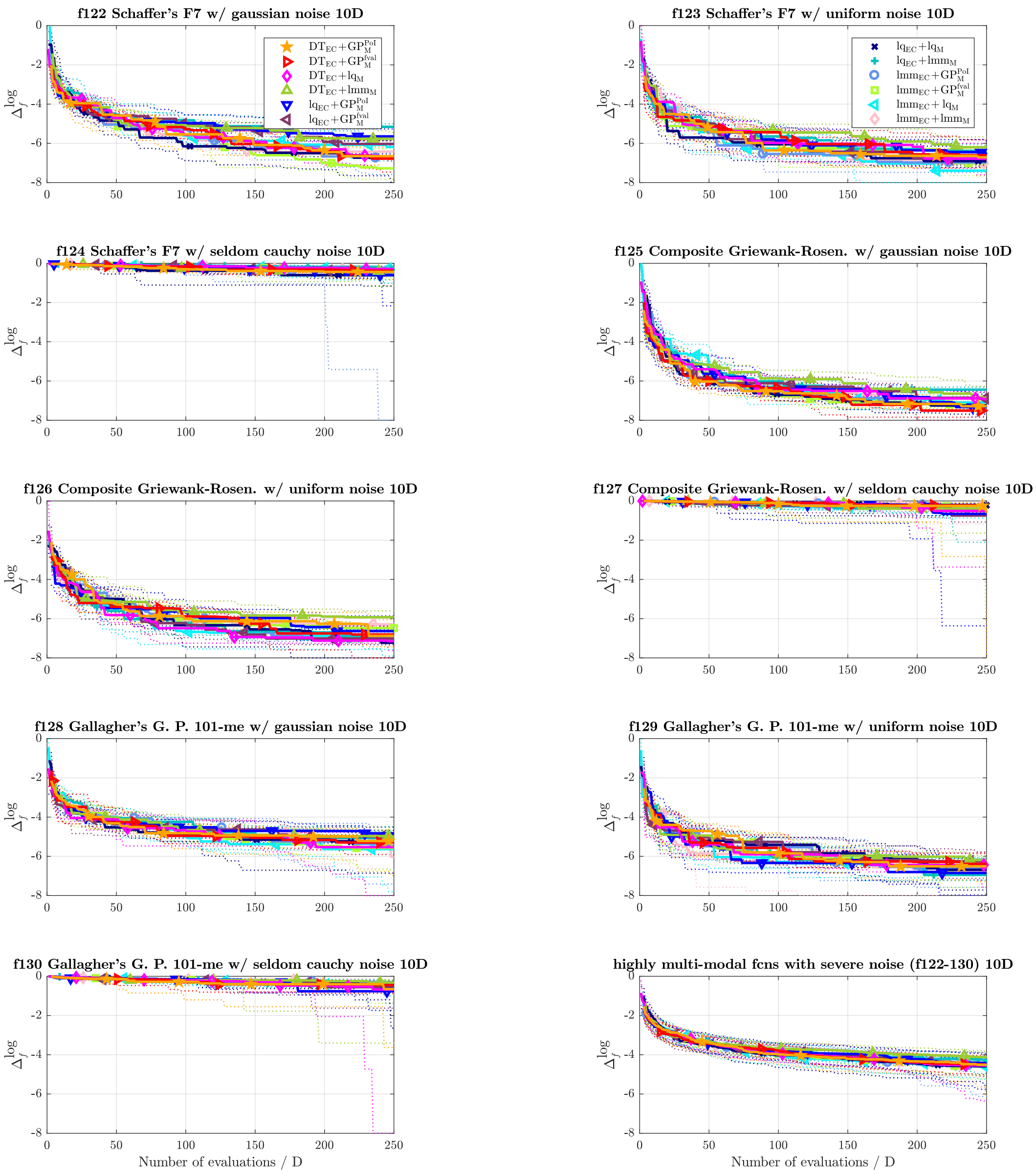

Figure 58: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 9 highly multi-modal noisy COCO benchmarks with severe noise in 10D for all compared EC − model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 59: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the highly multi-modal COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	DT_{EC}+GP_M^{fval}		DT_{EC}+GP_M^{PoI}		DT_{EC}+lq_M		DT_{EC}+lmm_M		lq_{EC}+GP_M^{fval}		lq_{EC}+GP_M^{PoI}		lq_{EC}+lq_M		lq_{EC}+lmm_M		lmm_{EC}+GP_M^{fval}		lmm_{EC}+GP_M^{PoI}		lmm_{EC}+lq_M		lmm_{EC}+lmm_M	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP_M^{fval}	—	—	51	53	58	60	64	**65**	58	59	50.4	56	53	52	56	61	50.4	61	47	55	54	58	54	50.4
DT_{EC}+GP_M^{PoI}	49	47	—	—	57	48	60	63	55	51	50.4	48	53	49.6	56	53	52	51	49	51	58	53	53	46
DT_{EC}+lq_M	42	40	43	52	—	—	56	61	49.6	51	47	47	47	47	55	57	45	56	41	56	53	51	41	47
DT_{EC}+lmm_M	36	35	40	37	44	39	—	—	46	36	45	36	40	40	46	48	38	41	36	36	45	36	41	33
lq_{EC}+GP_M^{fval}	42	41	45	49	50.4	49	54	**64**	—	—	42	48	47	47	54	58	43	53	49	48	50.4	49.6	48	49
lq_{EC}+GP_M^{PoI}	49.6	44	49.6	52	53	53	55	**64**	58	52	—	—	47	43	54	59	48	52	49	53	56	49.6	52	49.6
lq_{EC}+lq_M	47	48	47	50.4	53	53	60	60	53	53	53	57	—	—	53	58	48	56	47	55	56	56	44	44
lq_{EC}+lmm_M	44	39	44	47	45	43	54	52	46	42	46	41	47	42	—	—	42	48	43	44	49	39	41	34
lmm_{EC}+GP_M^{fval}	49.6	39	48	49	55	44	62	59	57	47	52	48	52	44	58	52	—	—	52	46	57	49.6	52	41
lmm_{EC}+GP_M^{PoI}	53	45	51	49	59	44	64	**64**	51	52	51	47	53	45	57	56	48	54	—	—	62	54	52	43
lmm_{EC}+lq_M	46	42	42	47	47	49	55	64	49.6	50.4	44	50.4	44	44	51	61	43	50.4	38	46	—	—	49	44
lmm_{EC}+lmm_M	46	49.6	47	54	59	53	59	**67**	52	51	48	50.4	56	56	59	66	48	59	48	57	51	56	—	—

20*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	50.2	50.4	48	49
lq_{EC}	49.8	49.6	—	—	47	49
lmm_{EC}	52	51	53	51	—	—

20*D*	GP_M^{fval}		GP_M^{PoI}		lq_M		lmm_M	
#FEs/D	50	250	50	250	50	250	50	250
GP_M^{fval}	—	—	48	49	54	52	57	55
GP_M^{PoI}	52	51	—	—	55	48	55	55
lq_M	46	48	45	52	—	—	53	54
lmm_M	43	45	45	45	47	46	—	—

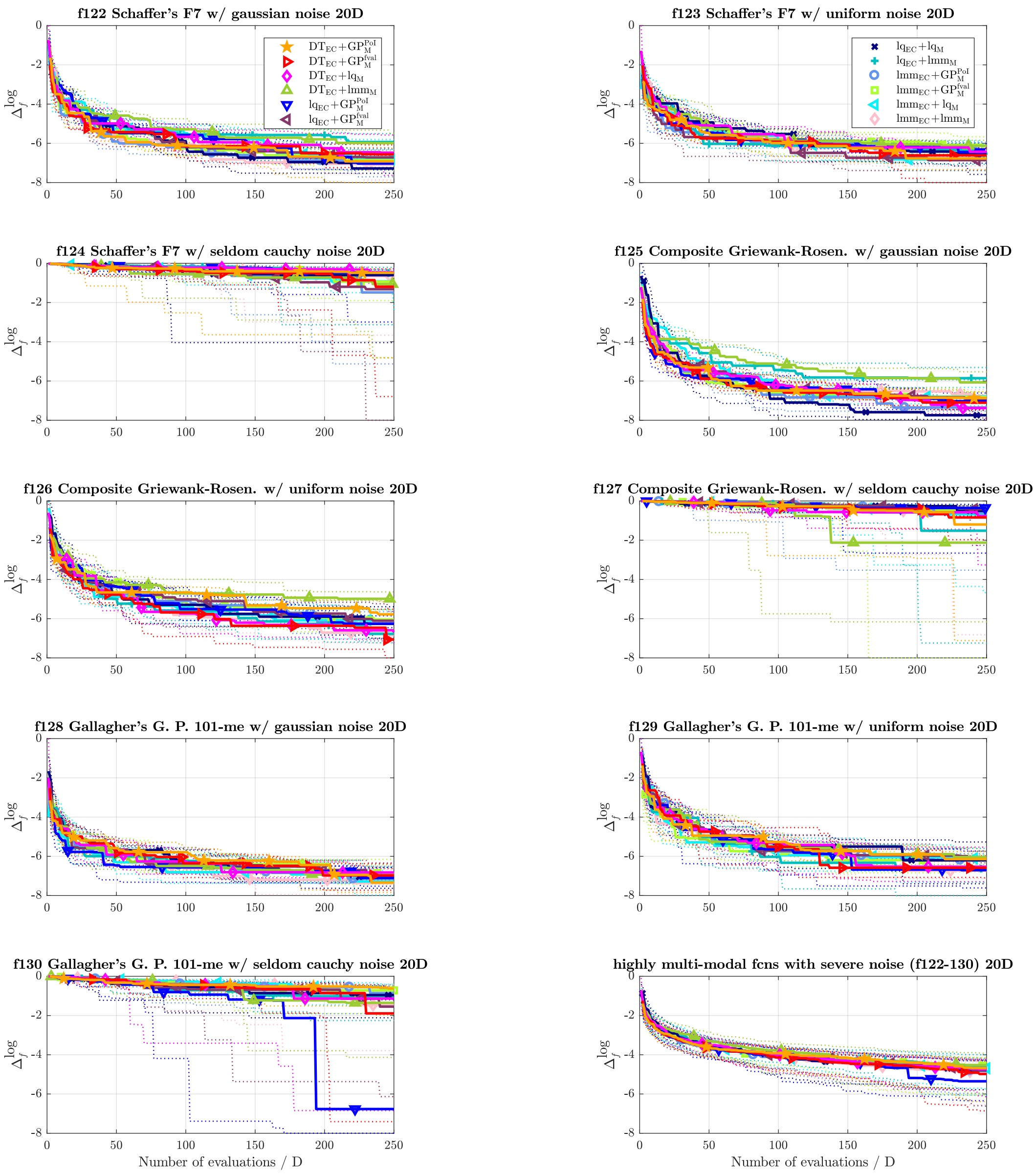

Figure 59: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 9 highly multi-modal noisy COCO benchmarks with severe noise in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 60: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $3D$, $5D$, $10D$ and $20D$ over the highly multi-modal COCO benchmarks with severe noise for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	49	51	55	49	53	**60**	52	55	53	**57**	51	44	51	52	51	**55**	47	50.4	56	52	54	**55**
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	51	49	—	—	56	48	55	**62**	49.6	52	53	**56**	52	45	52	52	51	**56**	49	52	**58**	53	51	**57**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	45	51	44	52	—	—	49	**60**	48	**56**	50.4	**59**	49	46	51	55	46	**56**	43	53	53	55	47	**57**
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	47	40	45	38	51	40	—	—	51	44	52	48	48	35	52	43	49	46	45	41	54	41	51	46
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	48	45	50.4	48	52	44	49	**56**	—	—	50.1	53	49	41	51	51	51	53	47	47	56	49	51	52
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	47	43	47	44	49.6	41	48	52	49.9	47	—	—	48	37	49	47	48	49	45	46	52	44	48	49.6
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	49	**56**	48	55	51	54	52	**65**	51	**59**	52	**63**	—	—	49.9	56	50.2	**61**	47	**55**	56	**58**	51	**59**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	49	48	48	48	49	45	48	**57**	49	49	51	**53**	50.1	44	—	—	49.9	53	45	49.9	53	48	49	**55**
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	49	45	49	44	54	44	51	54	49	47	52	51	49.8	39	50.1	47	—	—	46	47	55	45	52	51
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	53	49.6	51	48	57	47	55	**59**	53	53	55	**54**	53	45	55	50.1	54	53	—	—	**59**	49	55	**55**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	44	48	42	47	47	45	46	**59**	44	51	48	**56**	44	42	47	52	45	55	41	51	—	—	46	54
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	46	45	49	43	53	43	49	54	49	48	52	50.4	49	41	51	45	48	49	45	45	54	46	—	—

2 − 20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	49.8	51	**52**
$\mathrm{lq_{EC}}$	49	50.2	—	—	50.1	**53**
$\mathrm{lmm_{EC}}$	49	48	49.9	47	—	—

2 − 20*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	50	53	45	52	**54**
$\mathrm{GP_M^{PoI}}$	52	50	—	—	54	45	53	**54**
$\mathrm{lq_M}$	47	**55**	46	**55**	—	—	48	**57**
$\mathrm{lmm_M}$	48	46	47	46	52	43	—	—

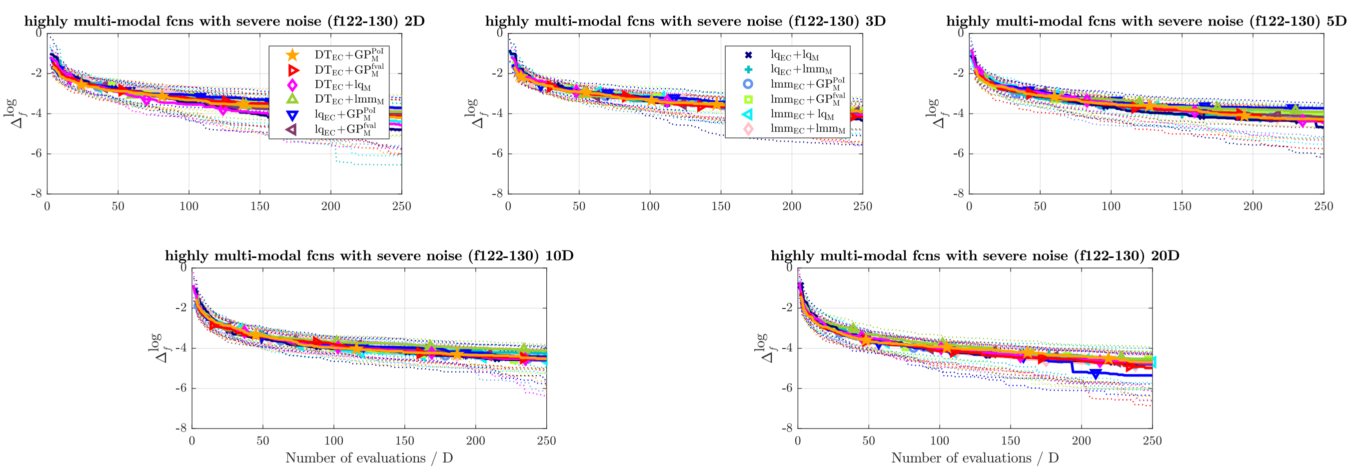

Figure 60: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 9 highly multi-modal noisy COCO benchmarks with severe noise in 2, 3, 5, 10, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 61: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	50	50	52	50	52	50	52	50	52	50	52	52	52	50	50	50	52	52	52	52	54	52
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	50	50	—	—	52	50	52	50	52	50	52	50	52	52	52	50	50	50	52	52	52	52	54	52
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	48	50	48	50	—	—	50	50	50	50	50	50	50	52	50	50	48	50	50	52	50	52	52	52
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	48	50	48	50	50	50	—	—	50	50	50	50	50	52	50	50	48	50	50	52	50	52	52	52
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	48	50	48	50	50	50	50	50	—	—	50	50	50	52	50	50	48	50	50	52	50	52	52	52
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	48	50	48	50	50	50	50	50	50	50	—	—	50	52	50	50	48	50	50	52	50	52	52	52
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	48	48	48	48	50	48	50	48	50	48	50	48	—	—	50	48	48	48	50	50	50	50	52	50
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	48	50	48	50	50	50	50	50	50	50	50	50	50	52	—	—	48	50	50	52	50	52	52	52
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	50	50	50	50	52	50	52	50	52	50	52	50	52	52	52	50	—	—	52	52	52	52	54	52
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	48	48	48	48	50	48	50	48	50	48	50	48	50	50	50	48	48	48	—	—	50	50	52	50
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	48	48	48	48	50	48	50	48	50	48	50	48	50	50	50	48	48	48	50	50	—	—	52	50
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	46	48	46	48	48	48	48	48	48	48	48	48	48	50	48	48	46	48	48	50	48	50	—	—

$2D$	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	51	51	52
$\mathrm{lq_{EC}}$	49	49	—	—	50	51
$\mathrm{lmm_{EC}}$	49	48	50	49	—	—

$2D$	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	51	51	51	51	52	51
$\mathrm{GP_M^{PoI}}$	49	49	—	—	51	51	51	50
$\mathrm{lq_M}$	49	49	49	49	—	—	51	49
$\mathrm{lmm_M}$	48	49	49	50	49	51	—	—

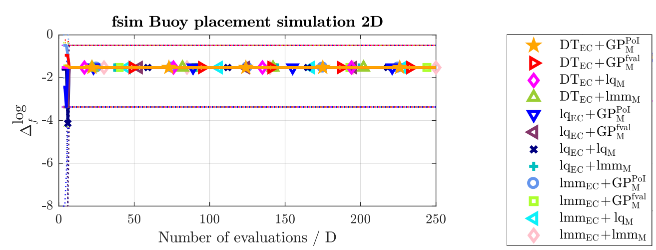

Figure 61: Medians (solid) and $1^{\mathrm{st}}/3^{\mathrm{rd}}$ quartiles (dotted) of the distances to the optima of simulation benchmark in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 62: A pairwise comparison of the evolution controls, models, and their combinations in $4D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

4D	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	48	42	42	44	46	46	60	52	56	42	54	46	42	44	56	54	56	48	65	42	65	44
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	52	58	—	—	50	52	48	52	58	56	65	48	60	58	42	54	56	56	65	56	65	50	63	50
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	58	56	50	48	—	—	52	54	63	56	65	50	67	54	48	48	56	60	69	54	65	46	58	52
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	54	54	52	48	48	46	—	—	58	50	60	44	60	56	52	50	58	50	67	50	67	44	63	52
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	40	48	42	44	38	44	42	50	—	—	50	42	40	50	35	46	46	52	54	48	44	42	54	50
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	44	58	35	52	35	50	40	56	50	58	—	—	48	56	40	54	48	56	63	58	52	50	54	56
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	46	54	40	42	33	46	40	44	60	50	52	44	—	—	38	48	46	52	58	48	56	44	54	50
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	58	56	58	46	52	52	48	50	65	54	60	46	63	52	—	—	58	58	67	54	69	42	71	54
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	44	46	44	44	44	40	42	50	54	48	52	44	54	48	42	42	—	—	56	48	60	40	46	48
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	44	52	35	44	31	46	33	50	46	52	38	42	42	52	33	46	44	52	—	—	54	42	40	50
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	35	58	35	50	35	54	33	56	56	58	48	50	44	56	31	58	40	60	46	58	—	—	44	56
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	35	56	38	50	42	48	38	48	46	50	46	44	46	50	29	46	54	52	60	50	56	44	—	—

4D	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**61**	51	62	52
$\mathrm{lq_{EC}}$	39	49	—	—	59	52
$\mathrm{lmm_{EC}}$	38	48	41	48	—	—

4D	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	51	44	47	44	42	47
$\mathrm{GP^{PoI}_M}$	49	56	—	—	51	50	42	52
$\mathrm{lq_M}$	53	56	49	50	—	—	44	53
$\mathrm{lmm_M}$	58	53	**58**	48	56	47	—	—

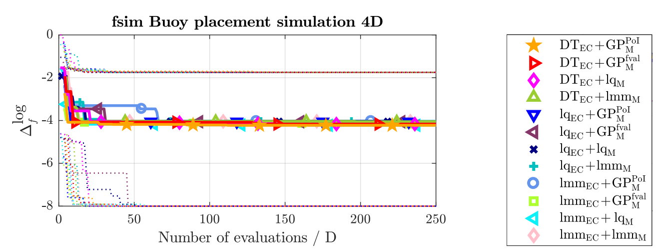

Figure 62: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of simulation benchmark in 4D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 63: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	42	60	75	60	33	52	83	69	67	69	71	58	71	75	46	75	42	63	58	65	58	56
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	58	40	—	—	71	58	44	48	79	73	71	60	75	63	71	75	50	58	54	54	63	65	63	56
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	25	40	29	42	—	—	17	48	79	67	54	60	58	65	50	71	25	63	33	50	29	42	33	48
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	67	48	56	52	83	52	—	—	83	**88**	79	75	79	58	83	75	67	52	67	58	71	35	79	44
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	17	31	21	27	21	33	17	13	—	—	38	42	25	33	21	46	17	35	25	27	21	25	17	29
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	33	31	29	40	46	40	21	25	63	58	—	—	42	44	46	63	23	38	33	38	29	33	46	31
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	29	42	25	38	42	35	21	42	75	67	58	56	—	—	33	63	13	40	38	33	29	40	25	40
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	29	25	29	25	50	29	17	25	79	54	54	38	67	38	—	—	25	33	38	33	33	17	33	25
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	54	25	50	42	75	38	33	48	**83**	65	77	63	**88**	60	75	67	—	—	54	50	63	40	58	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	58	38	46	46	67	50	33	42	75	73	67	63	63	67	63	67	46	50	—	—	54	42	50	46
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	42	35	38	35	71	58	29	65	79	**75**	71	67	71	60	67	83	38	60	46	58	—	—	42	50
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	42	44	38	44	67	52	21	56	83	**71**	54	69	75	60	67	**75**	42	54	50	54	58	50	—	—

10*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	**74**	**67**	52	54
$\mathrm{lq_{EC}}$	26	33	—	—	28	34
$\mathrm{lmm_{EC}}$	48	46	**72**	**66**	—	—

10*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	44	51	54	44	38	48
$\mathrm{GP_M^{PoI}}$	56	49	—	—	56	48	47	52
$\mathrm{lq_M}$	46	56	44	52	—	—	31	53
$\mathrm{lmm_M}$	63	52	53	48	69	47	—	—

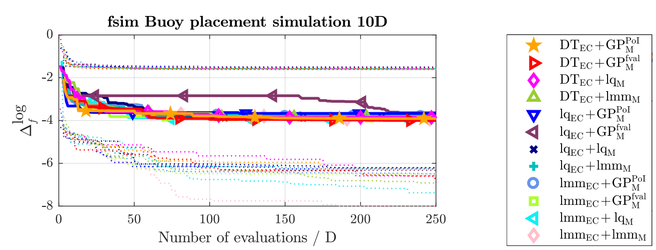

Figure 63: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of simulation benchmark in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 64: A pairwise comparison of the evolution controls, models, and their combinations in $12D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

12D	$DT_{EC}+GP^{fval}_{M}$		$DT_{EC}+GP^{PoI}_{M}$		$DT_{EC}+lq_{M}$		$DT_{EC}+lmm_{M}$		$lq_{EC}+GP^{fval}_{M}$		$lq_{EC}+GP^{PoI}_{M}$		$lq_{EC}+lq_{M}$		$lq_{EC}+lmm_{M}$		$lmm_{EC}+GP^{fval}_{M}$		$lmm_{EC}+GP^{PoI}_{M}$		$lmm_{EC}+lq_{M}$		$lmm_{EC}+lmm_{M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$DT_{EC}+GP^{fval}_{M}$	—	—	46	48	71	**83**	33	40	67	83	71	62	71	46	65	68	42	44	33	38	65	44	60	40
$DT_{EC}+GP^{PoI}_{M}$	54	52	—	—	**92**	83	56	31	79	75	86	74	79	42	82	71	63	40	46	42	71	35	67	40
$DT_{EC}+lq_{M}$	29	17	8	17	—	—	17	13	42	50	43	43	29	17	24	41	17	8	21	25	25	13	21	17
$DT_{EC}+lmm_{M}$	67	60	44	69	**83**	**88**	—	—	71	83	71	71	63	46	71	82	33	63	38	60	63	48	54	56
$lq_{EC}+GP^{fval}_{M}$	33	17	21	25	58	50	29	17	—	—	62	38	46	23	29	41	25	17	29	29	38	25	33	17
$lq_{EC}+GP^{PoI}_{M}$	29	38	14	26	57	57	29	29	38	62	—	—	38	31	24	50	10	38	10	36	29	33	24	38
$lq_{EC}+lq_{M}$	29	54	21	58	71	**83**	38	54	54	**77**	62	69	—	—	47	85	25	44	29	56	46	48	46	40
$lq_{EC}+lmm_{M}$	35	32	18	29	76	59	29	18	71	59	76	50	53	15	—	—	35	26	12	29	41	32	35	29
$lmm_{EC}+GP^{fval}_{M}$	58	56	38	60	83	**92**	67	38	**75**	**83**	**90**	62	**75**	56	65	74	—	—	42	48	67	46	67	48
$lmm_{EC}+GP^{PoI}_{M}$	67	63	54	58	79	75	63	40	71	71	90	64	71	44	88	71	58	52	—	—	75	52	63	40
$lmm_{EC}+lq_{M}$	35	56	29	65	75	**88**	38	52	63	75	71	67	54	52	59	68	33	54	25	48	—	—	35	52
$lmm_{EC}+lmm_{M}$	40	60	33	60	79	**83**	46	44	67	83	76	62	54	60	65	71	33	52	38	60	65	48	—	—

12D	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	**62**	62	42	39
lq_{EC}	38	38	—	—	29	33
lmm_{EC}	58	**61**	**71**	**67**	—	—

12D	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	49	45	61	51	45	43
GP^{PoI}_{M}	51	55	—	—	70	57	50	39
lq_{M}	39	49	30	43	—	—	32	46
lmm_{M}	55	57	50	61	**68**	54	—	—

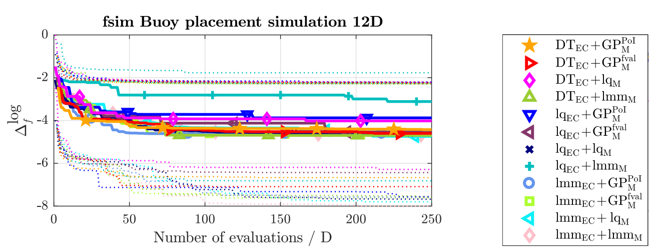

Figure 64: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of simulation benchmark in 12D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 65: A pairwise comparison of the evolution controls, models, and their combinations in $16D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

16*D*	DT_{EC}+$\text{GP}^{\text{fval}}_{\text{M}}$		DT_{EC}+$\text{GP}^{\text{PoI}}_{\text{M}}$		DT_{EC}+lq_{M}		DT_{EC}+lmm_{M}		lq_{EC}+$\text{GP}^{\text{fval}}_{\text{M}}$		lq_{EC}+$\text{GP}^{\text{PoI}}_{\text{M}}$		lq_{EC}+lq_{M}		lq_{EC}+lmm_{M}		lmm_{EC}+$\text{GP}^{\text{fval}}_{\text{M}}$		lmm_{EC}+$\text{GP}^{\text{PoI}}_{\text{M}}$		lmm_{EC}+lq_{M}		lmm_{EC}+lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+$\text{GP}^{\text{fval}}_{\text{M}}$	—	—	63	60	**83**	75	54	31	71	59	89	78	63	29	62	62	25	17	17	19	33	25	33	17
DT_{EC}+$\text{GP}^{\text{PoI}}_{\text{M}}$	38	40	—	—	**79**	67	63	15	53	53	89	67	63	17	69	54	29	8	17	15	33	13	29	15
DT_{EC}+lq_{M}	17	25	21	33	—	—	29	13	29	35	56	56	33	4	8	23	4	4	4	0	4	8	17	4
DT_{EC}+lmm_{M}	46	69	38	**85**	71	**88**	—	—	53	65	67	100	54	58	23	85	4	46	4	38	21	25	17	38
lq_{EC}+$\text{GP}^{\text{fval}}_{\text{M}}$	29	41	47	47	71	65	47	35	—	—	78	50	65	41	42	42	18	24	18	15	35	15	29	18
lq_{EC}+$\text{GP}^{\text{PoI}}_{\text{M}}$	11	22	11	33	44	44	33	0	22	50	—	—	22	11	33	22	0	0	0	0	22	11	11	0
lq_{EC}+lq_{M}	38	71	38	83	67	**96**	46	42	35	59	78	89	—	—	23	77	21	38	8	25	17	25	17	31
lq_{EC}+lmm_{M}	38	38	31	46	**92**	77	77	15	58	58	67	78	77	23	—	—	23	15	23	8	15	8	15	15
lmm_{EC}+$\text{GP}^{\text{fval}}_{\text{M}}$	**75**	**83**	71	**92**	**96**	**96**	**96**	54	82	76	100	100	**79**	63	77	85	—	—	42	50	67	42	58	42
lmm_{EC}+$\text{GP}^{\text{PoI}}_{\text{M}}$	**83**	**81**	**83**	**85**	**96**	**100**	**96**	63	**82**	85	100	100	**92**	75	77	92	58	50	—	—	75	35	58	54
lmm_{EC}+lq_{M}	67	75	67	**88**	**96**	92	79	75	65	85	78	89	**83**	75	85	92	33	58	25	65	—	—	42	63
lmm_{EC}+lmm_{M}	67	**83**	71	**85**	**83**	**96**	83	63	71	82	89	100	**83**	69	85	85	42	58	42	46	58	38	—	—

16*D*	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	49	44	16	19
lq_{EC}	51	56	—	—	14	19
lmm_{EC}	**84**	**81**	**86**	**81**	—	—

16*D*	$\text{GP}^{\text{fval}}_{\text{M}}$		$\text{GP}^{\text{PoI}}_{\text{M}}$		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
$\text{GP}^{\text{fval}}_{\text{M}}$	—	—	56	54	**72**	54	53	38
$\text{GP}^{\text{PoI}}_{\text{M}}$	44	46	—	—	**68**	45	56	32
lq_{M}	28	46	32	55	—	—	33	46
lmm_{M}	47	63	44	**68**	**67**	54	—	—

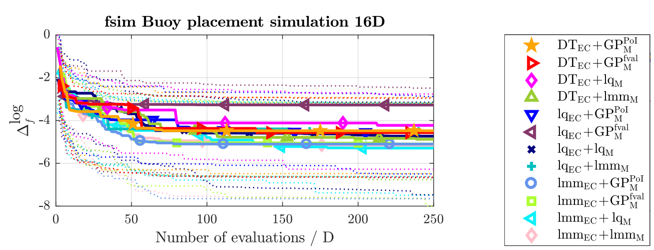

Figure 65: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of simulation benchmark in 16D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 66: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	54	54	67	67	75	67	58	33	71	57	67	13	50	8	4	4	0	7	29	8	21	8
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	46	46	—	—	54	63	50	50	58	38	71	14	58	4	50	0	8	0	7	13	17	0	25	8
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	33	33	46	38	—	—	33	33	38	25	57	43	33	0	17	17	8	0	0	0	4	0	21	4
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	25	33	50	50	67	67	—	—	67	33	71	43	50	8	20	10	0	0	0	0	17	0	8	0
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	42	67	42	63	63	75	33	67	—	—	57	57	58	21	50	33	13	13	13	7	33	8	25	17
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	29	43	29	86	43	57	29	57	43	43	—	—	29	43	43	43	0	29	0	0	29	29	14	29
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	33	**88**	42	**96**	67	**100**	50	**92**	42	79	71	57	—	—	50	67	13	25	7	33	29	29	25	42
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	50	92	50	**100**	83	83	80	90	50	67	57	57	50	33	—	—	8	17	0	27	33	33	17	50
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	**96**	**96**	**92**	**100**	**92**	**100**	**100**	**100**	**88**	**88**	100	71	**88**	75	92	83	—	—	40	53	79	58	83	63
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	**100**	**93**	**93**	87	**100**	**100**	**100**	**100**	**87**	**93**	100	100	**93**	67	**100**	73	60	47	—	—	87	27	80	80
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	**71**	**92**	**83**	**100**	**96**	**100**	83	**100**	67	**92**	71	71	71	71	67	67	21	42	13	73	—	—	50	54
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	79	**92**	**75**	**92**	**79**	**96**	92	**100**	75	**83**	86	71	75	58	83	50	17	38	20	20	50	46	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	45	15	5	4
$\mathrm{lq_{EC}}$	55	**85**	—	—	18	24
$\mathrm{lmm_{EC}}$	**95**	**96**	**82**	**76**	—	—

20*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	50	54	**68**	49	73	56
$\mathrm{GP_M^{PoI}}$	50	46	—	—	61	48	62	62
$\mathrm{lq_M}$	32	51	39	52	—	—	46	52
$\mathrm{lmm_M}$	27	44	38	38	54	48	—	—

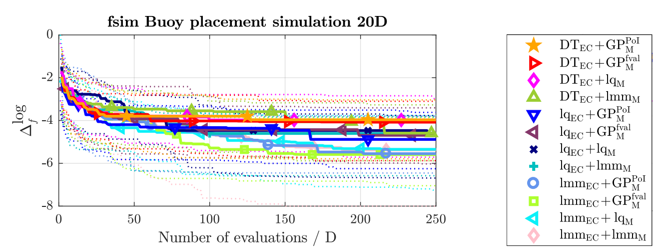

Figure 66: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of simulation benchmark in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 67: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $4D$, $10D$, $12D$, $16D$ and $20D$ over the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2-20D$	DT_{EC}+GP^{fval}_{M}		DT_{EC}+GP^{PoI}_{M}		DT_{EC}+lq_{M}		DT_{EC}+lmm_{M}		lq_{EC}+GP^{fval}_{M}		lq_{EC}+GP^{PoI}_{M}		lq_{EC}+lq_{M}		lq_{EC}+lmm_{M}		lmm_{EC}+GP^{fval}_{M}		lmm_{EC}+GP^{PoI}_{M}		lmm_{EC}+lq_{M}		lmm_{EC}+lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP^{fval}_{M}	—	—	50.3	52	**65**	**63**	47	46	**65**	58	64	57	63	41	57	54	37	41	36	40	50.3	39	49	36
DT_{EC}+GP^{PoI}_{M}	49.7	48	—	—	**66**	62	52	40	**64**	58	**70**	56	**65**	39	60	54	43	35	42	40	50	36	50	37
DT_{EC}+lq_{M}	35	37	34	38	—	—	33	35	51	48	54	51	45	32	37	46	26	31	31	32	30	27	34	30
DT_{EC}+lmm_{M}	53	54	48	**60**	**67**	**65**	—	—	**64**	**64**	**66**	**62**	**60**	50	55	61	38	47	41	47	51	37	49	44
lq_{EC}+GP^{fval}_{M}	35	42	36	42	49	52	36	36	—	—	52	44	46	36	37	44	28	32	34	32	37	28	35	31
lq_{EC}+GP^{PoI}_{M}	36	43	30	44	46	49	34	38	48	56	—	—	42	43	40	51	28	41	34	39	38	39	40	40
lq_{EC}+lq_{M}	37	**59**	35	**61**	55	**68**	40	50	54	**64**	58	**57**	—	—	40	62	27	41	33	41	38	39	36	42
lq_{EC}+lmm_{M}	43	46	40	46	**63**	54	45	39	**63**	56	60	49	60	38	—	—	36	37	37	38	43	32	42	39
lmm_{EC}+GP^{fval}_{M}	**63**	**59**	**57**	**65**	**74**	**69**	**62**	53	**72**	**68**	**72**	**59**	**73**	59	**64**	63	—	—	48	50	65	46	61	49.7
lmm_{EC}+GP^{PoI}_{M}	**64**	**60**	58	**60**	**69**	**68**	59	53	**66**	**68**	**66**	**61**	**67**	59	63	**62**	52	50	—	—	64	42	56	51
lmm_{EC}+lq_{M}	49.7	**61**	50	**64**	**70**	**73**	49	**63**	**63**	**72**	**62**	**61**	**62**	**61**	57	**68**	35	54	36	58	—	—	44	54
lmm_{EC}+lmm_{M}	51	**64**	50	**63**	**66**	**70**	51	**56**	**65**	**69**	**60**	**60**	**64**	58	**58**	**61**	39	50.3	44	49	56	46	—	—

$2-20D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	**58**	50.5	39	38
lq_{EC}	42	49.5	—	—	35	37
lmm_{EC}	**61**	**62**	**65**	**63**	—	—

$2-20D$	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	50.1	49	**59**	49	49	47
GP^{PoI}_{M}	49.9	51	—	—	**59**	49.9	50.1	47
lq_{M}	41	51	41	50.1	—	—	39	50
lmm_{M}	51	**53**	49.9	**53**	**61**	50	—	—

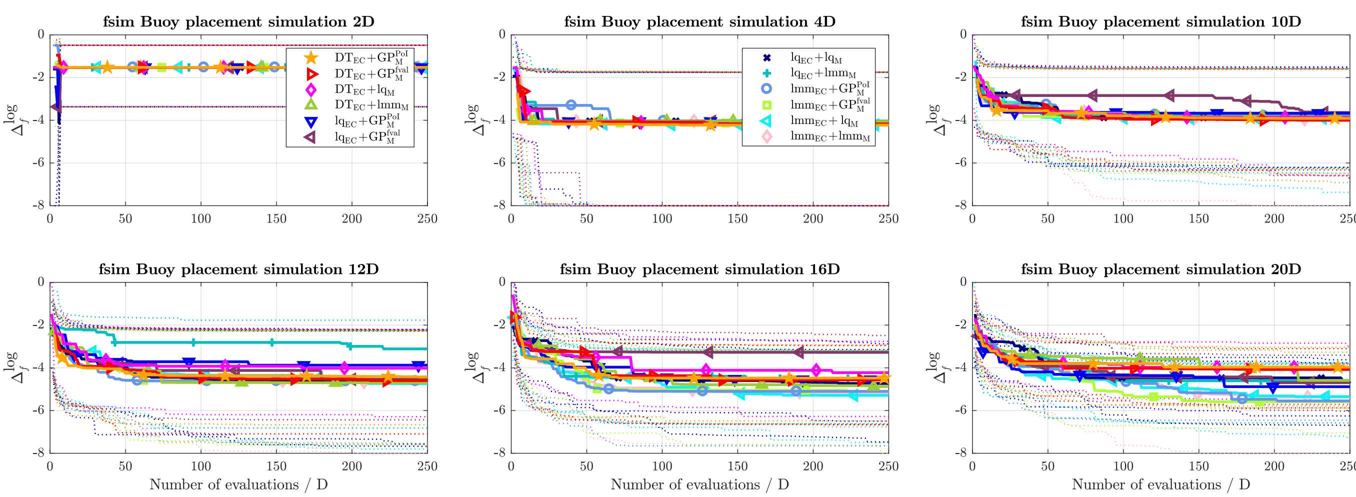

Figure 67: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of simulation benchmark in 2, 4, 10, 12, 16, and 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 24 independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 68: A pairwise comparison of the evolution controls, models, and their combinations in $2D$ over the all COCO benchmarks and the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

$2D$	DT_{EC}+GP^{fval}_{M}		DT_{EC}+GP^{PoI}_{M}		DT_{EC}+lq_{M}		DT_{EC}+lmm_{M}		lq_{EC}+GP^{fval}_{M}		lq_{EC}+GP^{PoI}_{M}		lq_{EC}+lq_{M}		lq_{EC}+lmm_{M}		lmm_{EC}+GP^{fval}_{M}		lmm_{EC}+GP^{PoI}_{M}		lmm_{EC}+lq_{M}		lmm_{EC}+lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
DT_{EC}+GP^{fval}_{M}	—	—	48	48	**61**	51	38	47	47	**55**	45	49	48	43	42	44	54	**54**	49	49	**61**	**58**	49.8	53
DT_{EC}+GP^{PoI}_{M}	52	52	—	—	**62**	**56**	43	54	52	**61**	49	**57**	53	43	47	48	54	**62**	51	**54**	**61**	**59**	51	**58**
DT_{EC}+lq_{M}	39	49	38	44	—	—	30	46	41	**55**	36	50.5	37	43	33	42	44	**57**	39	49	49	**59**	37	51
DT_{EC}+lmm_{M}	**62**	**53**	**57**	46	**70**	54	—	—	**60**	**58**	**57**	**53**	**59**	47	54	47	**64**	**59**	**61**	53	**71**	**59**	**62**	**56**
lq_{EC}+GP^{fval}_{M}	53	45	48	39	**59**	45	40	42	—	—	46	46	47	37	44	40	**55**	49	53	44	**59**	49.8	51	46
lq_{EC}+GP^{PoI}_{M}	**55**	51	51	43	**64**	49.5	43	47	**54**	**54**	—	—	53	41	48	42	**59**	53	**57**	48	**65**	54	**55**	48
lq_{EC}+lq_{M}	52	**57**	47	**57**	**63**	**57**	41	**53**	53	**63**	47	**59**	—	—	46	51	**55**	**62**	51	**57**	**64**	**65**	52	**60**
lq_{EC}+lmm_{M}	**58**	**56**	53	52	**67**	**58**	46	53	**56**	**60**	52	**58**	54	49	—	—	**62**	**61**	**59**	**55**	**66**	**62**	**59**	**57**
lmm_{EC}+GP^{fval}_{M}	46	46	46	38	**56**	43	36	41	45	51	41	47	45	38	38	39	—	—	47	44	55	51	47	46
lmm_{EC}+GP^{PoI}_{M}	51	51	49	46	**61**	51	39	47	47	**56**	43	52	49	43	41	45	53	**56**	—	—	**60**	**56**	49	52
lmm_{EC}+lq_{M}	39	42	39	41	51	41	29	41	41	50.2	35	46	36	35	34	38	45	49	40	44	—	—	40	43
lmm_{EC}+lmm_{M}	50.2	47	49	42	**63**	49	38	44	49	**54**	45	52	48	40	41	43	53	**54**	51	48	**60**	**57**	—	—

$2D$	DT_{EC}		lq_{EC}		lmm_{EC}	
#FEs/D	50	250	50	250	50	250
DT_{EC}	—	—	47	50.1	**54**	**56**
lq_{EC}	**53**	49.9	—	—	**59**	**55**
lmm_{EC}	46	44	41	45	—	—

$2D$	GP^{fval}_{M}		GP^{PoI}_{M}		lq_{M}		lmm_{M}	
#FEs/D	50	250	50	250	50	250	50	250
GP^{fval}_{M}	—	—	47	46	**54**	46	43	44
GP^{PoI}_{M}	**53**	**54**	—	—	**58**	51	47	49
lq_{M}	46	**54**	42	49	—	—	38	47
lmm_{M}	**57**	**56**	**53**	51	**62**	**53**	—	—

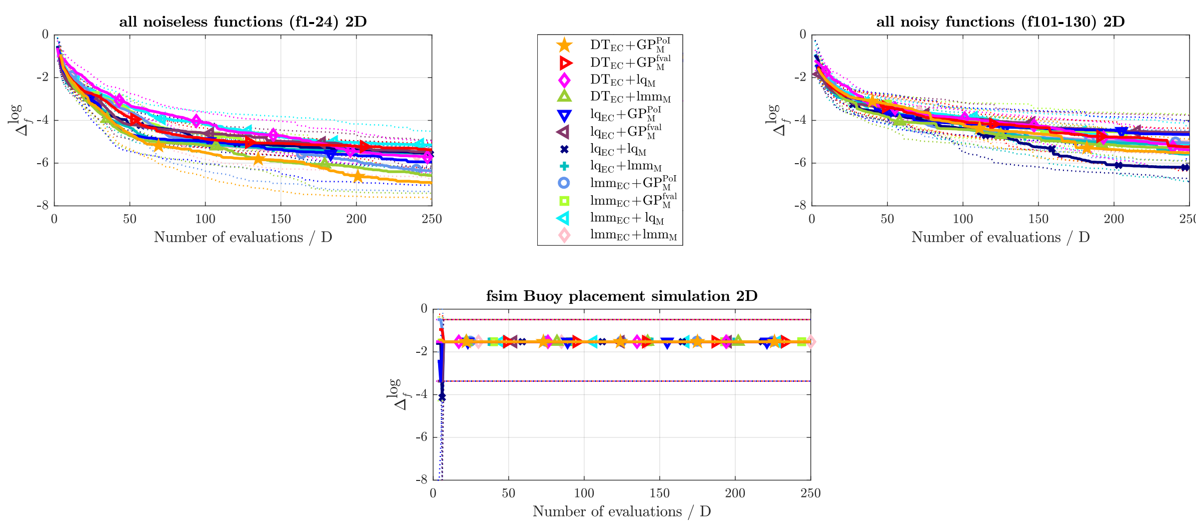

Figure 68: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless, 30 noisy and 1 simulation benchmarks in 2D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 (24 for f_{sim}) independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 69: A pairwise comparison of the evolution controls, models, and their combinations in $10D$ over the all COCO benchmarks and the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

10*D*	$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP^{fval}_M}$	—	—	46	47	**64**	53	51	**62**	51	**58**	51	**59**	48	45	**61**	**61**	**60**	54	**58**	46	**68**	**60**	**67**	52
$\mathrm{DT_{EC}}$+$\mathrm{GP^{PoI}_M}$	54	53	—	—	**63**	53	**56**	**65**	**59**	**63**	**56**	**63**	50.3	48	**64**	**64**	**62**	55	**63**	49.8	**67**	**60**	**66**	53
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	36	47	37	47	—	—	35	**59**	42	**58**	41	**57**	36	42	49	**61**	46	51	48	45	**55**	**58**	**54**	53
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	49	38	44	35	**65**	41	—	—	52	49	49.8	47	46	33	**62**	53	**59**	43	**60**	37	**67**	44	**67**	40
$\mathrm{lq_{EC}}$+$\mathrm{GP^{fval}_M}$	49	42	41	37	**58**	42	48	51	—	—	46	49	46	37	**58**	54	**56**	46	**57**	40	**61**	46	**61**	41
$\mathrm{lq_{EC}}$+$\mathrm{GP^{PoI}_M}$	49	41	44	37	**59**	43	50.2	53	54	51	—	—	46	37	**58**	54	**58**	44	**59**	40	**63**	48	**61**	41
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	52	**55**	49.7	52	**64**	**58**	**54**	**67**	**54**	**63**	54	**63**	—	—	**63**	**70**	**61**	**57**	**61**	52	**69**	**63**	**68**	**59**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	39	39	36	36	51	39	38	47	42	46	42	46	37	30	—	—	53	43	**54**	38	**61**	42	**59**	40
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{fval}_M}$	40	46	38	45	54	49	41	**57**	44	54	42	56	39	43	47	**57**	—	—	53	41	**61**	**56**	**60**	52
$\mathrm{lmm_{EC}}$+$\mathrm{GP^{PoI}_M}$	42	54	37	50.2	52	**55**	40	**63**	43	**60**	41	**60**	39	48	46	**62**	47	**59**	—	—	58	**63**	**59**	**59**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	32	40	33	40	45	42	33	56	39	54	37	52	31	37	39	58	39	44	42	37	—	—	50.1	45
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	33	48	34	47	46	47	33	**60**	39	**59**	39	**59**	32	41	41	**60**	40	48	41	41	49.9	**55**	—	—

10*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	**53**	**61**	50.2
$\mathrm{lq_{EC}}$	49	47	—	—	**61**	47
$\mathrm{lmm_{EC}}$	39	49.8	39	53	—	—

10*D*	$\mathrm{GP^{fval}_M}$		$\mathrm{GP^{PoI}_M}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP^{fval}_M}$	—	—	49	46	**57**	49	**56**	**56**
$\mathrm{GP^{PoI}_M}$	51	**54**	—	—	**55**	51	**58**	**59**
$\mathrm{lq_M}$	43	51	45	49	—	—	49	**58**
$\mathrm{lmm_M}$	44	44	42	41	51	42	—	—

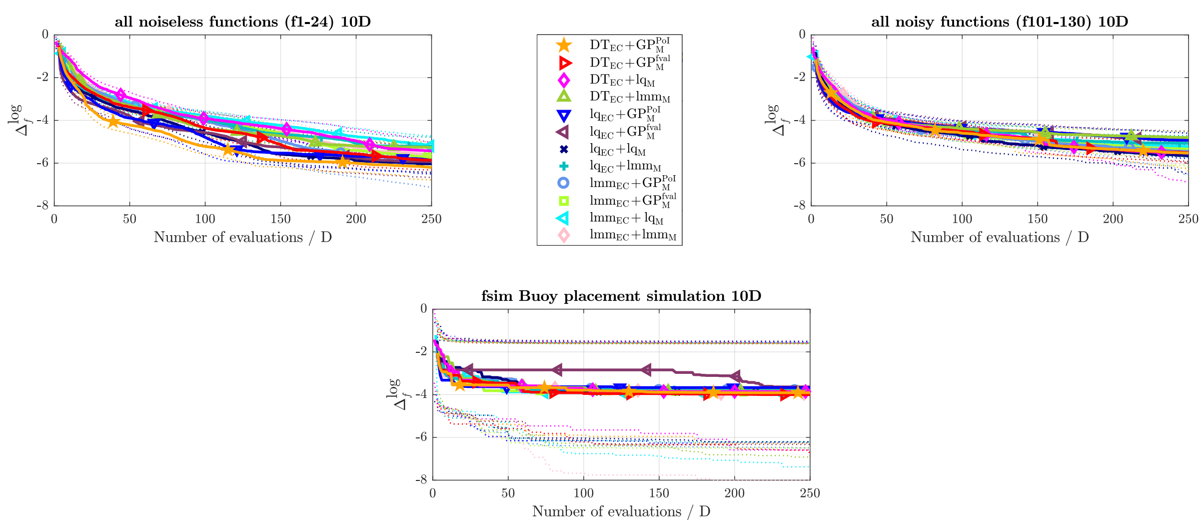

Figure 69: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless, 30 noisy and 1 simulation benchmarks in 10D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 (24 for f_{sim}) independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 70: A pairwise comparison of the evolution controls, models, and their combinations in $20D$ over the all COCO benchmarks and the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

20*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	46	49	**60**	56	**65**	**67**	49.8	**60**	47	**59**	**57**	48	**69**	**70**	**58**	**54**	**58**	**53**	**65**	**59**	**67**	49.5
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	54	51	—	—	**61**	49	**67**	**66**	**59**	**59**	**54**	**60**	**62**	47	**71**	**65**	**61**	50.3	**62**	52	**66**	**56**	**67**	48
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	40	44	39	51	—	—	**56**	**60**	42	**56**	38	**55**	45	44	**64**	**64**	52	**54**	51	55	**58**	**60**	**61**	53
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	35	33	33	34	44	40	—	—	37	42	36	41	38	33	**58**	**53**	42	39	43	39	49	39	**57**	35
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	50.2	40	41	41	**58**	44	**63**	58	—	—	45	47	54	37	**65**	57	**56**	41	**59**	38	**62**	46	**62**	38
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	53	41	46	40	**62**	45	**64**	**59**	**55**	**53**	—	—	**55**	37	**68**	**58**	**63**	46	**62**	42	**63**	48	**67**	41
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	43	52	38	53	**55**	**56**	**62**	**67**	46	**63**	45	**63**	—	—	**67**	**71**	**55**	**57**	**57**	**56**	**64**	**63**	**65**	**53**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	31	30	29	35	36	36	42	47	35	43	32	42	33	29	—	—	36	41	38	38	42	40	52	34
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	42	46	39	49.7	48	46	**58**	**61**	44	**59**	37	54	45	43	**64**	**59**	—	—	53	45	**58**	53	**65**	46
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	42	47	38	48	49	45	**57**	**61**	41	**62**	38	**58**	43	44	**62**	**62**	47	**55**	—	—	56	**57**	**64**	49.8
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	35	41	34	44	42	40	51	**61**	38	54	37	52	36	37	**58**	**60**	42	47	44	43	—	—	**60**	42
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	33	50.5	33	52	39	47	43	**65**	38	**62**	33	**59**	35	47	48	**66**	35	**54**	36	50.2	40	**58**	—	—

20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	51	**54**	**59**	50
$\mathrm{lq_{EC}}$	49	46	—	—	**58**	45
$\mathrm{lmm_{EC}}$	41	50	42	**55**	—	—

20*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	47	**57**	49	**65**	**57**
$\mathrm{GP_M^{PoI}}$	**52**	**53**	—	—	**57**	48	**66**	**57**
$\mathrm{lq_M}$	43	**51**	43	**52**	—	—	**61**	**58**
$\mathrm{lmm_M}$	35	43	34	43	39	42	—	—

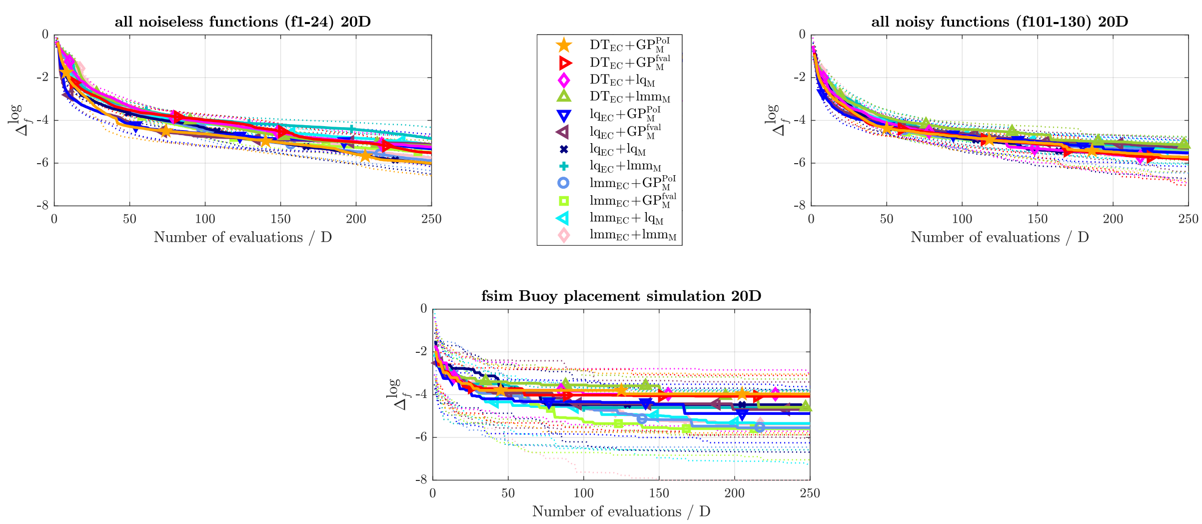

Figure 70: Medians (solid) and 1st/3rd quartiles (dotted) of the distances to the optima of 24 noiseless, 30 noisy and 1 simulation benchmarks in 20D for all compared EC — model combinations. The $\log_{10}$ of medians/quartiles were calculated across 15 (24 for f_{sim}) independent function instances for each combination and linearly scaled to $[-8, 0]$. Results summarized over all considered functions were obtained through averaging of these log-statistics.

Table 71: A pairwise comparison of the evolution controls, models, and their combinations in $2D$, $10D$ and $20D$ over the all COCO benchmarks and the simulation benchmark for different evaluation budgets. The percentage of wins of i-th algorithm against j-th algorithm over all benchmark instances is given in the i-th row and j-th column. The numbers in bold mark the row algorithm being significantly better than the column algorithm according to the two-sided Wilcoxon signed rank test with the Holm correction at family-wise significance level $\alpha = 0.05$.

2 − 20*D*	$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{DT_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{DT_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{DT_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lq_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$		$\mathrm{lq_{EC}}$+ $\mathrm{lmm_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{fval}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{GP_M^{PoI}}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lq_M}$		$\mathrm{lmm_{EC}}$+ $\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250	50	250
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{fval}}$	—	—	47	47	**61**	52	47	**56**	49.9	**58**	47	**55**	49	44	53	**53**	**55**	**54**	**52**	49	**62**	**58**	**58**	51
$\mathrm{DT_{EC}}$+$\mathrm{GP_M^{PoI}}$	**53**	**53**	—	—	**63**	52	51	**60**	**56**	**61**	**53**	**61**	53	46	**57**	**56**	**56**	**57**	**55**	**52**	**63**	**58**	**58**	**54**
$\mathrm{DT_{EC}}$+$\mathrm{lq_M}$	39	48	37	48	—	—	36	**55**	42	**58**	38	**55**	37	43	43	53	44	**54**	41	49.7	50.3	**59**	45	51
$\mathrm{DT_{EC}}$+$\mathrm{lmm_M}$	**53**	44	49	40	**64**	45	—	—	**53**	**52**	50.5	49.7	50.5	39	**58**	48	**56**	49.6	**55**	44	**65**	49.8	**61**	45
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{fval}}$	50.1	42	44	39	**58**	42	47	48	—	—	46	47	48	36	52	46	**54**	46	**52**	41	**59**	48	**55**	42
$\mathrm{lq_{EC}}$+$\mathrm{GP_M^{PoI}}$	**53**	45	47	39	**62**	45	49.5	50.3	**54**	**53**	—	—	51	39	**56**	48	**58**	49	**56**	44	**62**	50	**58**	45
$\mathrm{lq_{EC}}$+$\mathrm{lq_M}$	**51**	**56**	47	**54**	**63**	**57**	49.5	**61**	**52**	**64**	49	**61**	—	—	**55**	**60**	**56**	**60**	**54**	**55**	**65**	**64**	**59**	**57**
$\mathrm{lq_{EC}}$+$\mathrm{lmm_M}$	47	47	43	44	**57**	47	42	**52**	48	**54**	44	**52**	45	40	—	—	**52**	**52**	**50.1**	47	**59**	**53**	**56**	48
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{fval}}$	45	46	44	43	**56**	46	44	50.4	46	**54**	42	51	44	40	48	48	—	—	49.5	43	**58**	52	**56**	47
$\mathrm{lmm_{EC}}$+$\mathrm{GP_M^{PoI}}$	48	51	45	48	**59**	50.3	45	**56**	48	**59**	44	**56**	46	45	49.9	53	50.5	**57**	—	—	**59**	**58**	**56**	**53**
$\mathrm{lmm_{EC}}$+$\mathrm{lq_M}$	38	42	37	42	49.7	41	35	**50.2**	41	**52**	38	50	35	36	41	47	42	48	41	42	—	—	46	44
$\mathrm{lmm_{EC}}$+$\mathrm{lmm_M}$	42	49	42	46	**55**	49	39	**55**	45	**58**	42	**55**	41	43	44	52	44	**53**	44	47	**54**	**56**	—	—

2 − 20*D*	$\mathrm{DT_{EC}}$		$\mathrm{lq_{EC}}$		$\mathrm{lmm_{EC}}$	
#FEs/D	50	250	50	250	50	250
$\mathrm{DT_{EC}}$	—	—	49	**52**	**55**	**53**
$\mathrm{lq_{EC}}$	**51**	48	—	—	**57**	**51**
$\mathrm{lmm_{EC}}$	45	47	43	49	—	—

2 − 20*D*	$\mathrm{GP_M^{fval}}$		$\mathrm{GP_M^{PoI}}$		$\mathrm{lq_M}$		$\mathrm{lmm_M}$	
#FEs/D	50	250	50	250	50	250	50	250
$\mathrm{GP_M^{fval}}$	—	—	48	46	**56**	47	52	49.7
$\mathrm{GP_M^{PoI}}$	**52**	**54**	—	—	**57**	49	**55**	**53**
$\mathrm{lq_M}$	44	**53**	43	**51**	—	—	46	**53**
$\mathrm{lmm_M}$	48	**50.3**	45	47	**54**	47	—	—

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