

Brief Article

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1 First section

In one-dimension:

$$C_{i-\frac{1}{2}}(h_{i-1}^t + h_i^t) + C_{i+\frac{1}{2}}(h_{i+1}^t + h_i^t) + H_i(h_b^t - h_i^t) + \frac{S_{ijk}\Delta x\Delta y\Delta z}{\Delta t}(h_i^{t+1} - h_i^t) + w = 0 \quad (1)$$

simplify equation 1 to:

$$h_i^t - h_i^{t+1} = \frac{\Delta t}{S_{ijk}\Delta x\Delta y\Delta z} \left[C_{i-\frac{1}{2}}(h_{i-1}^t + h_i^t) + C_{i+\frac{1}{2}}(h_{i+1}^t + h_i^t) + H_i(h_b^t - h_i^t) + w \right] \quad (2)$$

rearrange equation 2 to:

$$h_i^{t+1} = h_i^t - \frac{\Delta t}{S_{ijk}\Delta x\Delta y\Delta z} \left[C_{i-\frac{1}{2}}(h_{i-1}^t + h_i^t) + C_{i+\frac{1}{2}}(h_{i+1}^t + h_i^t) - H_i h_i^t + H_i h_b^t + w \right] \quad (3)$$

Adding known boundary terms:

$$r_i^t = H_i h_b^t + w \quad (4)$$

Equation 3 in matrix-vector form:

$$\mathbf{h}^{t+1} = \mathbf{h}^t - \frac{\Delta t}{\Delta x\Delta y\Delta z} \mathbf{S}_{ijk} \left[(\mathbf{C} - \mathbf{H}) \mathbf{h}^t + \mathbf{r} \right] \quad (5)$$

Subject to the stability constraint:

$$\max \left(\frac{C_{ijk}\Delta t}{S_{ijk}\Delta x\Delta y\Delta z} \right) \leq 1 \quad (6)$$

rearrange equation 6 to determine the maximum stable time step length:

$$\Delta t_{max} = \frac{S_{ijk}\Delta x\Delta y\Delta z}{C_{ijk}} \quad (7)$$

1.1 A subsection

More text.