Brief Article

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December 16, 2012

1 First section

For a one-dimension model where NROW = NLAY = 1:

$$\begin{aligned} C_{i-\frac{1}{2},j,k}\left(h^t_{i-1,j,k}-h^t_{i,j,k}\right)+C_{i+\frac{1}{2}}\left(h^t_{i+1,j,k}-h^t_{i,j,k}\right)+H_i\left(h^t_b-h^t_{i,j,k}\right)\\ +\frac{S^t_{i,j,k}\Delta x_{i,j,k}\Delta y_{i,j,k}\Delta z_{i,j,k}}{\Delta t}\left(h^{t+1}_{i,j,k}-h^t_{i,j,k}\right)+w=0 \end{aligned} \tag{1}$$

simplify equation 1 to:

$$\begin{aligned} h^t_{i,j,k}-h^{t+1}_{i,j,k}=\frac{\Delta t}{S^t_{i,j,k}\Delta x_{i,j,k}\Delta y_{i,j,k}\Delta z_{i,j,k}}\Big[C_{i-\frac{1}{2},j,k}\left(h^t_{i-1,j,k}-h^t_i\right)+C_{i+\frac{1}{2},j,k}\left(h^t_{i+1,j,k}-h^t_{i,j,k}\right)\\ +H_i\left(h^t_b-h^t_{i,j,k}\right)+w_{i,j,k}\Big] \end{aligned} \tag{2}$$

Set:

$$\hat{s}^t_{i,j,k}=\frac{\Delta t}{S_{i,j,k}\Delta x_{i,j,k}\Delta y_{i,j,k}\Delta z_{i,j,k}} \tag{3}$$

rearrange equation 2 to:

$$\begin{aligned} h^{t+1}_{i,j,k}=h^t_{i,j,k}-\hat{s}^t_{i,j,k}\Big[C_{i-\frac{1}{2},j,k}\left(h^t_{i-1,j,k}-h^t_{i,j,k}\right)\\ +C_{i+\frac{1}{2},j,k}\left(h^t_{i+1,j,k}-h^t_{i,j,k}\right)-H_ih^t_{i,j,k}+H_{i,j,k}h^t_b+w_{i,j,k}\Big] \end{aligned} \tag{4}$$

Adding known boundary terms:

$$r^t_i=H_{i,j,k}h^t_b+w_{i,j,k} \tag{5}$$

Equation 4 in matrix-vector form:

$$\mathbf{h}^{t+1}=\mathbf{h}^t-\hat{\mathbf{s}}\left[\left(\mathbf{C}-\mathbf{H}\right)\mathbf{h}^t+\mathbf{r}\right] \tag{6}$$

Subject to the stability constraint:

$$max\left(\frac{C_{ijk}\Delta t}{S_{ijk}\Delta x\Delta y\Delta z}\right)\leq 1 \tag{7}$$

rearrange equation 7 to determine the maximum stable time step length:

$$\Delta t_{max}=\frac{S_{ijk}\Delta x\Delta y\Delta z}{C_{ijk}} \tag{8}$$

1.1 A subsection

More text.