Jonathan Hurwitz
804258351

① a) $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

P	Q	$\neg P$	$\neg Q$	$P \Rightarrow \neg Q$	$Q \Rightarrow \neg P$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

$\therefore$ equivalent

α	β	$\alpha \Rightarrow \beta$
0	0	1
0	1	1
1	0	0
1	1	1

b) $P \Leftrightarrow \neg Q$, $\underbrace{((P \wedge \neg Q) \vee (\neg P \wedge Q))}_{\beta}$

$\downarrow$

$\underbrace{(P \Rightarrow \neg Q) \wedge (\neg Q \Rightarrow P)}_{\alpha}$

P	Q	$\neg P$	$\neg Q$	$P \Rightarrow \neg Q$	$\neg Q \Rightarrow P$	$P \wedge \neg Q$	$\neg P \wedge Q$	$(P\Rightarrow\neg Q) \wedge (\neg Q \Rightarrow P) = \alpha$	$(P \wedge \neg Q) \vee (\neg P \wedge Q) = \beta$
0	0	1	1	1	0	0	0	0	0
0	1	1	0	1	1	0	1	1	1
1	0	0	1	1	1	1	0	1	1
1	1	0	0	0	1	0	0	0	0

$\therefore$ equivalent

② a) $(S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$

S	F	$\neg S$	$\neg F$	$S \Rightarrow F$	$\neg S \Rightarrow \neg F$	$\alpha = (S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)$
0	0	1	1	1	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	1
1	1	0	0	1	1	1

$\therefore$ $((S \Rightarrow F) \Rightarrow (\neg S \Rightarrow \neg F)) = \alpha$ is not valid since one world is false. α has at least one world that is T, so it is not unsatisfiable but it is satisfiable (neither).

b) $(S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$

S	F	H	$(S \Rightarrow F)$	$(S \vee H)$	$(S \vee H) \Rightarrow F$	$\alpha = (S \Rightarrow F) \Rightarrow ((S \vee H) \Rightarrow F)$
0	0	0	1	0	1	1
0	0	1	1	1	0	0
0	1	0	1	0	1	1
0	1	1	1	1	1	1
1	0	0	0	1	0	1
1	0	1	0	1	0	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

$\Rightarrow$ sentence α is not valid but it is not unsat because there is at least one true world.

$\therefore$ α is satisfiable (neither)

c) $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$

$\Downarrow$

$$\Big(((S \wedge H) \Rightarrow F) \Rightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))\Big) \wedge \Big(((S \Rightarrow F) \vee (H \Rightarrow F)) \Rightarrow ((S \wedge H) \Rightarrow F)\Big)$$

S	H	F	$(S \wedge H)$	$(S \wedge H) \Rightarrow F$	$(S \Rightarrow F)$	$(H \Rightarrow F)$	$(S \Rightarrow F) \vee (H \Rightarrow F)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	1	0	1
0	1	1	0	1	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

α $(S \wedge H) \Rightarrow F$	β $(S \Rightarrow F) \vee (H \Rightarrow F)$	$\alpha \Rightarrow \beta$	$\beta \Rightarrow \alpha$	$(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)$
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
1	1	1	1	1
0	0	1	1	1
1	1	1	1	1

$\therefore$ $((S \wedge H) \Rightarrow F) \Leftrightarrow ((S \Rightarrow F) \vee (H \Rightarrow F))$ is valid because every world is true.

3) Mythical $\Rightarrow$ Immortal

$\neg$ Mythical $\Rightarrow$ $\neg$ ~~Immortal~~ $\wedge$ Mammal (must be both, so AND)

Immortal $\vee$ Mammal $\Rightarrow$ Horned

Horned $\Rightarrow$ magical

a)

M = mythical

I = immortal

H = horned

A = mammal

G = magical

KB:

$M \Rightarrow I$ ①

$\neg M \Rightarrow \neg I \wedge A$ ②

$I \vee A \Rightarrow H$ ③

$H \Rightarrow G$ ④

b) ① $M \Rightarrow I \longrightarrow \neg M \vee I$

② $\neg M \Rightarrow \neg I \wedge A \longrightarrow M \vee (\neg I \wedge A) = (M \vee \neg I) \wedge (M \vee A)$

③ $I \vee A \Rightarrow H \longrightarrow (\neg I \wedge \neg A) \vee H = (\neg I \vee H) \wedge (\neg A \vee H)$

④ $H \Rightarrow G \longrightarrow \neg H \vee G$

$$\boxed{\begin{array}{ll} ⑤ & \neg M \vee I \\ ⑥ & (M \vee \neg I) \wedge (M \vee A) \\ ⑦ & (\neg I \vee H) \wedge (\neg A \vee H) \\ ⑧ & \neg H \vee G \end{array}}$$

want to find M, G, H

c) ⑨ $I \vee (\neg I \wedge A)$ ① ② $= (I \vee A)$

⑩ $(\neg I \wedge \neg A) \vee G$ ③ ④

⑪ $H \wedge \neg G$ ④ negate

⑫ $(\neg I \wedge \neg A) \vee H$ ⑩ ⑪

⑬ H ⑨ ⑫

⑭ G ⑧ ⑬

↓

We can prove unicorn is horned + magical but not that it is mythical.