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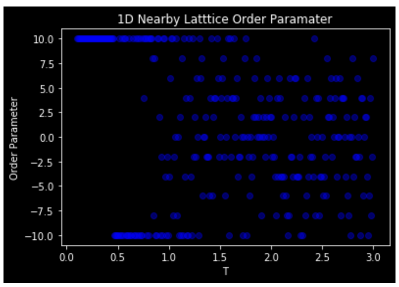
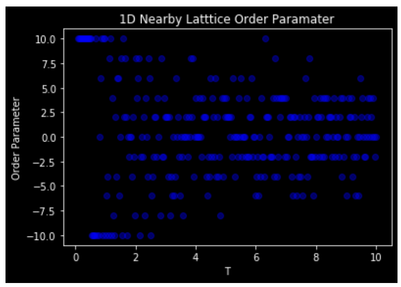
Complex Physics

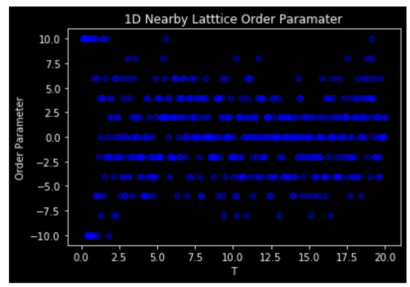
Midterm 1

1. *Order Parameter verus Temperature*

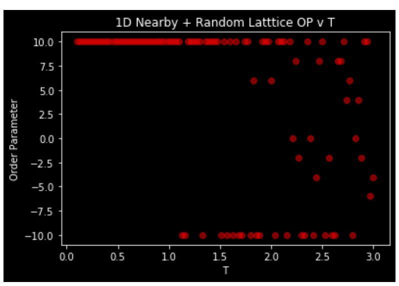
The order parameter versus T for a neighbor-based 1D Ising Lattice is given here (starting +1 J>0). As one can see and as Ising hypothesized, there is no first-order phase transition (spontaneous magnetism) as presented by higher-ordered models (besides T<<1). I used a mid-ranged lattice (2 digits) with 150-450 temperature steps below (with ranging upper temp bounds to ensure distribution was stable) and 10^3-10^5 sweeps per MC.

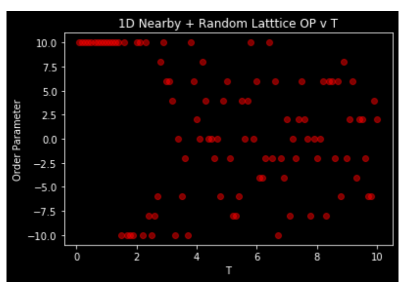


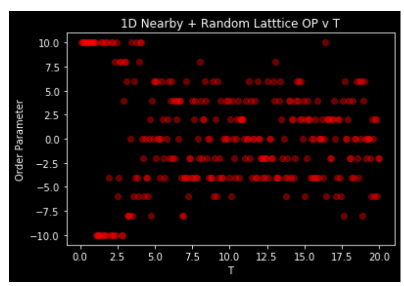




When we introduce two additional points i1 and i2 to be randomly affected in addition to the neighbors, we alter the typical 1D model and see different results, where this drop from uniformity (call it Tc for simplicity) occurs a a different time yet at more discrete temperatures- with different distributions around 0. Resistance to collapse makes sense here, since the random swapping resists the model's natural progress of unpolarization..

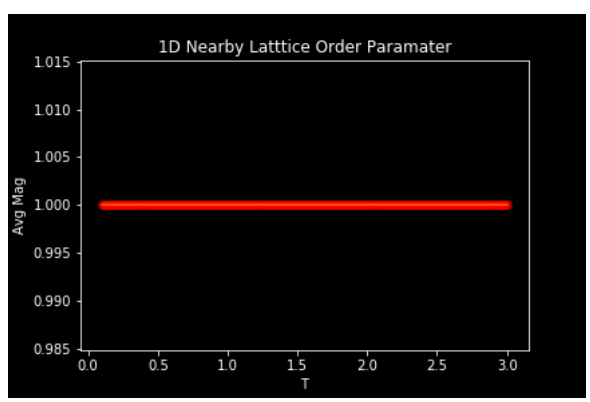


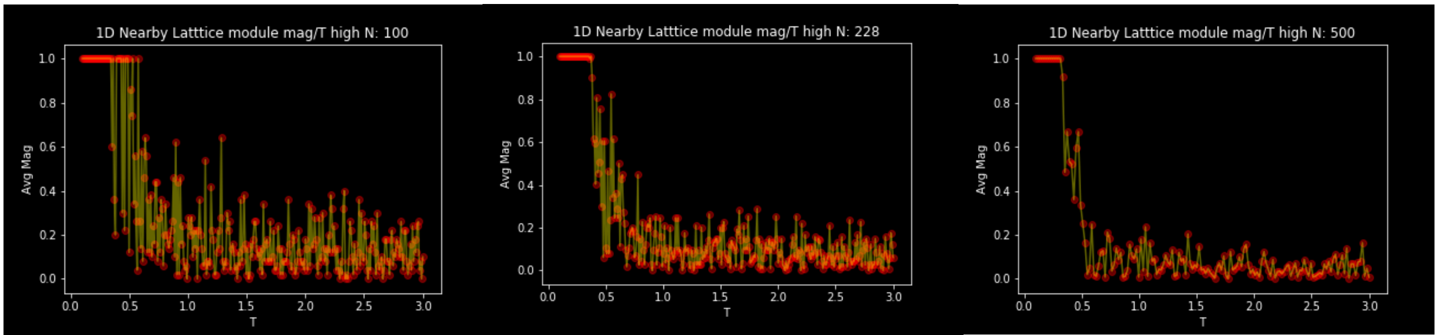


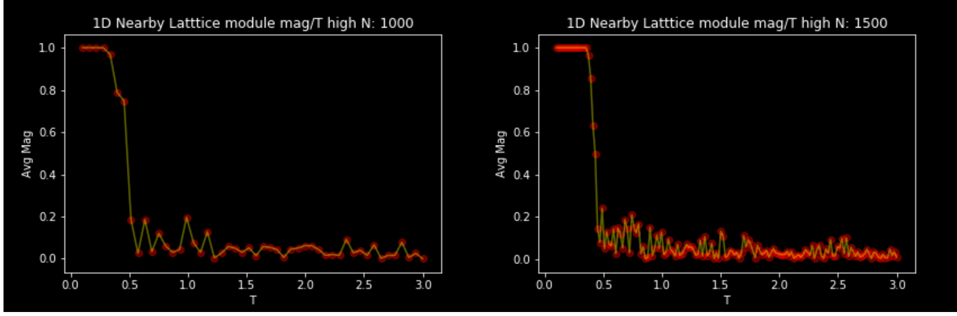


2. *Average mag versus T at varying N*

As seen on the right, when we start to take this problem to thermal equilibrium and model against average magnetization, we receive odd results. It turns out that as N -> infinity, the system begins to favour disorder and the critical temperature to this second-order order/disorder transition continues to fall. I continued to analyze this limit, which one can see below.

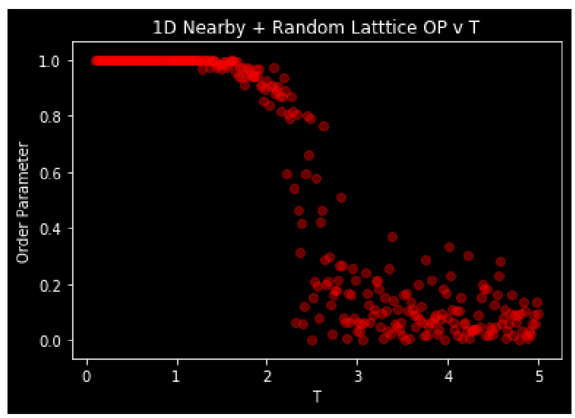


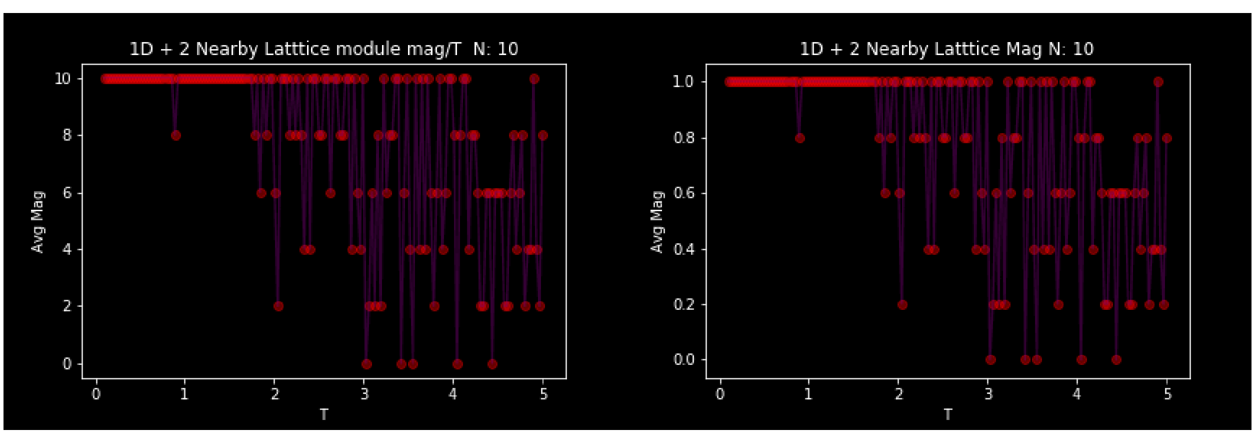




As these N values continued to increase, we begin to both a) get a clearer picture/less variation of the critical curve between phases and b) move back the point at which the system hits Tc. These changes are incredibly slight at smaller ns, but partiularily noticeable 10^3 onward. Looking here, this system seems to have a decent Tc range between .35 and .75 for the vast majority of n values, settling before 10^5 factors somewhere at the beginning of that range.

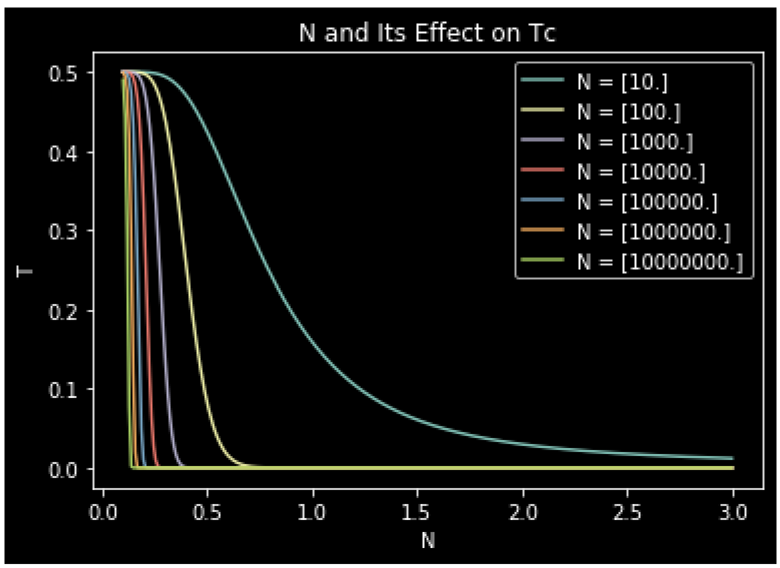
The same trend is true for the neighbor+random situation, wherein taking increasing Ns and reviewing their avg mag vs temperature gives us an idea of their transitions. However, as previously mentioned, this case had a much clearer transition than the solo neighbor one (slower change), so besides some scaling it's fairly easy to see the same critical temperature just looking at the unnormalized distribution. It's Tc was also higher, and it is likewise resistant to tthe lowering of Tc by N. The plot on the right represents N=200, with clear curve indication.





Optional Part (ties into 3)

What do we make of this? Well, as we look at the partition function it's easy to see that as we take N to infinity, the whole average magnetization becomes a step function, from 1 at T=0 to 0 above. That is, ferromagnetism does not handle well in 1D models at out thermo extreme. We might analytically see that by generalizing our entire lattice' partition into Z = Sum[Exp[Beta*J*(sigma(N-1)sigma(N),[sigmaN=+-1]]]. Through some manipulation using cosh we end up disregarding our sigmas (free-end BC and evenness) and can generalize this into 2(2coshBetaJ)**(N-1). We want to know the probability of finding all spins up, which we know yields an Energy of -JN+J. Since we have our partition we can just use Boltzmann to say that the probability all our spins are up is P(all_up)= ½ Exp[BetaJ(N-1)] / Z. Naturally these results will change depending on the configuration and interaction (such as figure 1b), but these adjustments are fairly trivial (at least for simple changes, not for additional rows, additional spins etc. then it is not trivial :/)



3. *Extra Credit*

A 1D nearest-neighbor lattice in the Ising model will not yield a critical temperature for the traditional phase transition system- however, as explained, it does suggest a second-order order/disorder phase transition (seen in anomalous specific heat). Why doesn't 1D have a typical phase transition? For a system to be ordered and stable, the change in free energy must be greater than zero (for non-0 Temps), but as N -> inf (thermal equilibrium) in our 1D configuration, the change in energy - change in entropy will always be negative, and the system prefers disorder; energy spent in a case of introducing boundaries is minimized by placing additional borders at each of the N lattice points, but N -> inf, so there is no change in symmetry or phase transition possible. When we increase the sphere of influence to the two surrounding the neighbors, or next-neighbors, we double the possible change in energy (-4J, 0 or 4J to -8J, 0 or 8J), meaning that the Free energy minimization will have to "work harder" than before, in that the cost of additional boundaries relative to the balancing entropy is no longer guaranteed.