Jeffrey D. Carlson (he/him)

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Positions

Tufts University (January 2023–August 2023)

Visiting Scholarship

Imperial College London (January 2021–December 2022)

Research fellowship

Fields Institute (Spring 2020)
Western University (Fall 2019)

Joint postdoctoral fellowship

University of Toronto (2016–2019)

Postdoctoral fellowship

Universidade de São Paulo (2015–2016)

Pós-Doutorado de Excelência
funded through **IMPA** (o Instituto Nacional de Matemática Pura e Aplicada)
and **CAPES** (a Coordenação de Aperfeiçoamento de Pessoal de Nível Superior)

Education

Tufts University

Ph.D. in Mathematics (2015)
Advisor: Loring W. Tu
Dissertation: *On the equivariant cohomology of homogeneous spaces*

Citizenship

United States

Interests

Equivariant topology: cobordism, K-theory, and Borel cohomology

Topology and geometry of group actions on manifolds

A_∞-algebras

Symplectic geometry

Galois cohomology

Low-dimensional topology and surface dynamics

Funding

Heilbronn Focused Research Grant: *Koszulity and formality in Galois cohomology*

Invited research visits

National Center for Theoretical Sciences at National Taiwan University (April–May 2017)

Selected invited conference talks

The topology of Gelfand–Zeitlin fibers,
Workshop on Lie Groups, Singular Spaces, and Higher Structures, **Fields Institute, Toronto** (Jan. 2023)

The topology of Gelfand–Zeitlin fibers,
AMS Special Session on Integrable Systems and Symplectic Group Actions, **Joint Mathematics Meetings, Boston** (Jan. 2023)

The topology of Gelfand–Zeitlin fibers,
Gone Fishing 2020–2022, **Georgia Southern University, Savannah, Georgia** /online (Apr. 2022)

Multiplicative collapse in the Eilenberg–Moore spectral sequence,
Transpennine Topology Triangle, UK/online (Dec. 2020)

The cohomology of Gelfand–Zeitlin fibers,
International Conference: Topology and Geometry of Group Actions, **Higher School of Economics, Moscow**/online (Nov. 2020)

Realization of torus representations as fixed-point data,
Workshop on Polyhedral Products in Homotopy Theory, **Fields Institute, Toronto** (Jan. 2020)

Local integration in equivariant cobordism theory, "Topology" session,
The equivariant K-theory of a cohomogeneity-one action,
"Equivariant methods in differential and algebraic geometry" session,
Canadian Mathematical Society Summer Meeting, Regina, Saskatchewan (June 2019)

The equivariant cohomology and K-theory of a cohomogeneity-one action,
Algebraic Topology, Combinatorics, and Mathematical Physics, on the occasion of Victor Buchstaber's 75th birthday, **Steklov Institute** and **Skolkovo Technical Institute, Moscow** (May 2018)

Equivariant formality beyond Hamiltonian actions,
Mathematical Congress of the Americas, Montreal (July 2017)

Equivariant formality in rational cohomology and K-theory,
Conference on Geometry in Algebra and Algebra in Geometry,
Universidade de São Paulo (Nov. 2015)

Selected invited seminar talks

Biquotients and a product on the two-sided bar construction,
International Polyhedral Products Seminar, **Princeton**/online (Oct. 2023)

Products on Tor,
Algebraic topology seminar, **University of Warwick** (Mar. 2023)

The topology of Gelfand–Zeitlin fibers,
Interactions between symplectic geometry, combinatorics, and number theory seminar,
Universität zu Köln/Philipps-**Universität Marburg**/online (Nov. 2022)

The topology of Gelfand–Zeitlin fibers,
Topology seminar, Heinrich-Heine-**Universität Düsseldorf** (Oct. 2022)

Equivariant formality of isotropy actions and products of spheres,
Geometry, topology, and group theory seminar, Westfälische Wilhelms-**Universität Münster** (Oct. 2022)

The topology of the Gelfand–Zeitlin fiber,
International Polyhedral Products Seminar, **Princeton**/online (Apr. 2022)

The topology of the Gelfand–Zeitlin fiber,
Symplectic Monday, **IBS Center for Geometry and Physics, Pohang, Korea**/online (Dec. 2021)

The topology of the Gelfand–Zeitlin fiber,
London Geometry and Topology Seminar, **Imperial College London** (Dec. 2021)

Products on Tor, homogeneous spaces, and Borel cohomology,
Topology seminar, **University of Rochester**/online (Nov. 2021)

The topology of the Gelfand–Zeitlin fiber,
Differential geometry and topology seminar, **University of Cambridge** (Nov. 2021)

Biquotients and a product on the two-sided bar construction,
Algebraic topology seminar, **Universidad Nacional Autónoma de México**/online (May 2021)

Multiplicative collapse in the Eilenberg–Moore spectral sequence,
Algebraic topology seminar, **University of Michigan**/online (Apr. 2021)

The K-theory of an isotropy action and an unsolved problem in polynomial rings (in Portuguese),
Seminário Salomônico, **Universidade Federal Fluminense, Niterói, RJ, Brazil** (August 2019)

Equivariant formality, K-theory, and isotropy,
Topology seminar, **University of Rochester** (October 2018)

Cohomogeneity-one actions and a little-remarked structure on the Mayer–Vietoris sequence,
Symplectic seminar, **University of Toronto** (March 2017)

Equivariant formality in rational cohomology and K-theory,
Geometry and topology seminar, **Western University, London, ON, Canada** (December 2016)

Equivariant formality of isotropy actions in rationalized cohomology and K-theory,
Seminário de física matemática, **IMPA, Rio de Janeiro** (May 2016)

Some contributed talks in colorful locations

Products on Tor,
Algebraic Topology, in Memory of Hans-Joachim Baues,
Max Planck Institute for Mathematics, Bonn (Oct. 2022)

Products on Tor, homogeneous spaces, and Borel cohomology,
Algebraic structures in topology, **San Juan, Puerto Rico** (May–June 2022)

Realization of fixed-point data for locally standard torus actions,
Glances@Manifolds, **Jagiellonian University, Kraków** (July 2018)

Formality and equivariant formality for isotropy actions (in Portuguese),
XX Encontro Brasileiro de Topologia,
Universidade Tecnológica Federal do Paraná, Curitiba, PR, Brazil (July 2016)

Teaching

Course coordinator and lecturer,

- Commutative Algebra (Imperial College London),
- Vector Calculus (University of Toronto, four semesters, **supervising** an undergraduate TA),
- Mathematics of Social Choice (Tufts University)

Seminar coordinator and lecturer,

- Formality (Western University),
- Equivariant Cohomology (Universidade de São Paulo)

Lecturer,

- Calculus (University of Toronto & Tufts University, three semesters),
- Finite Mathematics (Tufts University)

Reading group facilitator,

- *Foundations of Algebraic Geometry* (Western University)
  (mentor, Directed Reading Program in Mathematics),
- Topologia Diferencial (Universidade de São Paulo)

Teaching assistant (all at Tufts University),

- Differential Forms in Algebraic Topology,
- Mathematical Neuroscience,
- History of Mathematics,
- Number Theory,
- Complex Analysis,
- Real Analysis I & II

Tutor for eight students, ranging from Year 1 to M.S. (Imperial College London, 2021–2022)

Fellow of the Graduate Institute for Teaching (Tufts University, Summer 2010)

Memberships

Association for Women in Mathematics (AWM)

Service

Organizer, Special Session on Equivariant Cohomology,
AMS Spring Eastern Virtual Sectional Meeting (2022, with Loring W. Tu)

"Equivariant geometry and topology" session,
CMS Winter Meeting, Niagara, Ontario (2016, with Elisheva Adina Gamse).

Referee, seven venues including *Trans. Amer. Math. Soc.*, *J. Differential Geom.*, and *J. Topol.*

My version of refereeing involves following through and verifying every detail of each argument, generates several pages of commentary, and requires at least a week for a first pass, with less time spent on each subsequent revision.

Quick opinions, *Trans. Amer. Math. Soc.* and *J. Reine Angew. Math.*

Reviewer, *Mathematical Reviews*.

Editorial board, Poincaré Institute for Mathematics Education, Summer 2013
The Poincaré Institute is a NSF-funded collaboration between Tufts University and the non-profit Technical Education Research Centers designed to improve middle school mathematics education through graduate-level online courses offered to in-service middle school mathematics teachers.

Edited books and articles by Loring Tu (selected)

- *Introductory Lectures on Equivariant Cohomology*, Annals of Math. Studies 204, Princeton Univ. Press, Princeton, New Jersey, 2020.
- *Elements of Equivariant Cohomology*, with Raoul Bott, unpublished.
- *Differential Forms in Algebraic Topology*, 2nd edition, with Raoul Bott, edition in progress.
- *Differential Geometry: Connections, Curvature, and Characteristic Classes*, Grad. Texts in Math. 275, Springer, New York, 2017.
- *An Introduction to Manifolds*, [first and] second edition, Universitext, Springer, New York, 2011.
- Raoul Bott: *Collected Papers*, volume 5 [collection of permissions], Birkhäuser, Basel, 2017.
- From sheaf cohomology to the algebraic de Rham theorem (with Fouad El Zein), pp. 69–121 in *Hodge Theory*, eds. Eduardo Cattani, Fouad El Zein, Phillip A. Griffiths, and Lê Dũng Tráng, Princeton Univ. Press, Princeton, New Jersey, 2014.
- Computing characteristic numbers using fixed points, in *A Celebration of the Mathematical Legacy of Raoul Bott*, CRM Proceedings and Lecture Notes, vol. 50, American Mathematical Society, Providence, RI, 2010, pp. 185–206.

Feedback on and copyediting of other books

- *Mathematical Logic and Computation*, Jeremy Avigad, Cambridge University Press, 2022.
- *An Introduction to Modeling Neuronal Dynamics*, Christoph Börgers, Texts in Applied Mathematics vol. 66, Springer, New York, 2017.
- *A Primer on Mapping Class Groups*, Benson Farb and Dan Margalit, Princeton Mathematical Series vol. 49, Princeton University Press, Princeton, NJ, 2011.
- *Category Theory*, Steven Awodey, Oxford Logic Guides vol. 52, Oxford Univ. Press, New York, 2006.
- *Computability and Learnability*, Kevin Kelly, unpublished.
- Introduction to the calculus of variations, William Hrusa, unpublished.

Languages

English: native (polished if ornate, a byproduct of formal overeducation)

Portuguese: fluent at the level of the news, but not of poetry

Mandarin: basic conversation (but worsening accent), menu comprehension

References

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Books

1. *The Rational Cohomology of Homogeneous Spaces* jdkcarlson.github.io/homog_book.pdf
(**under revision for Springer's *Developments in Mathematics* series**, 155pp., 2018).

2. Solutions to *Introduction to Commutative Algebra* by Atiyah–MacDonald
(2011, revised 2021, 134pp.) jdkcarlson.github.io/intro_comm_alg(2021).pdf

Preprints

1. Products on Tor jdkcarlson.github.io/prod.pdf
(submitted, 2022, 23pp.)

2. The topology of Gelfand–Zetilin fibers arxiv.org/abs/2107.02721
(submitted, 2021, 39pp., with Jeremy Lane)

3. Fixed points and semifree bordism jdkcarlson.github.io/semifree.pdf
(submitted, 2019, 13pp.)

4. The K-theory of cohomogeneity-one actions jdkcarlson.github.io/c1KAdv2.pdf
(**under revision for *Adv. Math.***, 2018, 40pp.; "I would genuinely like to see this manuscript published in Advances, eventually. However, I am not confident that the manuscript is ready for publication in its present form.")

5. Realization of abstract GKM isotropy data jdkcarlson.github.io/realization.pdf
(2016–, with Elisheva Adina Gamse and Yael Karshon)

6. Commensurability of two-multitwist pseudo-Anosovs arxiv.org/abs/1011.0247
(2010, 33pp.)

Publications

1. The cohomology of homogeneous spaces in historical context jdkcarlson.github.io/conf.pdf
(**to be published in *Contemp. Math.*** volume *Group Actions and Equivariant Cohomology*, 2023, 33pp.)

2. A ring structure on Tor
(**to be published in *Forum Math. Sigma***, 2022, 44pp.) arxiv.org/abs/2306.04860

3. Equivariant formality of corank-one isotropy actions and products of rational spheres
(**to be published in *Math. Z.***, 2022, 45pp., with Chen He) arxiv.org/abs/2204.00135

4. The cohomology of biquotients via a product on the two-sided bar construction
(**to be published in *Algebr. Geom. Topol.***, 2020, 48pp., appendix joint with Matthias Franz)
arxiv.org/abs/2106.02986

5. K-theory and formality
(***Int. Math. Res. Not.***, 2022, 46pp.)
academic.oup.com/imrn/advance-article/doi/10.1093/imrn/rnac106/6612200

6. Grassmannians and the equivariant cohomology of isotropy actions arxiv.org/abs/1611.01175
(***Proc. Amer. Math. Soc.***, 2021, 15pp.)

7. The K-theory of the conjugation action
(***C. R. Math. Acad. Sci., Paris***, 2021, 2pp.)
comptes-rendus.academie-sciences.fr/mathematique/articles/10.5802/crmath.235

8. The equivariant cohomology ring of a cohomogeneity-one action arxiv.org/abs/1802.02304
(***Geom. Dedicata***, 2019, 18pp., with Chen He, Oliver Goertsches, and Liviu Mare)

9. Equivariant formality of homogeneous spaces arxiv.org/abs/1511.06228
(***J. London Math. Soc.***, 2018, 23pp., with Chi-Kwong Fok)

10. Equivariant formality of isotropic torus actions arxiv.org/abs/1410.5740
(***J. Homotopy and Relat. Struct.***, 2018, 34pp.)

11. Conceptions of topological transitivity arxiv.org/abs/1108.4710
(***Topology Appl.***, 2012, 15pp., with Ethan Akin)

Research plan (CV supplemental)

Book

Within the next year I plan to finally revise and resubmit my book on the rational cohomology of homogeneous spaces [Car15] to Springer, which has requested revisions with more background on Lie groups.

Equivariant complex cobordism and fixed points

The coefficient ring Ω_*^G of geometric equivariant complex cobordism has been studied since the 1960s but is still completely understood only when G is an abelian p-group. A related question attempts to characterize an action of a torus T on a stably complex manifold in terms of the local T-action on the normal bundle to the fixed-point set. These ***isotropy data*** in fact determine the manifold up to equivariant cobordism and are not arbitrary, but highly interdependent by the Atiyah–Bott–Berline–Vergne integration theorem, so tightly constrained in fact that one might wonder if any family of putative local data so constrained must necessarily arise from a stably complex T-manifold. Elisheva Adina Gamse, Karshon, and I showed this is so for GKM actions [CGK18] and I proved it in the ***semifree*** case, when S^1 is a circle whose orbits are all either free or fixed points [Car19], unexpectedly recovering a 2004 result of Dev Sinha characterizing semifree bordism with isolated fixed points [Sin05]. I am continuing to extend this work to more general cases.

A_∞-algebraic models

Many of my own results rely on commutative cochain-level models of spaces, e.g. Cartan's model for a homogeneous space, which require rational coefficients. One can remove this dependence at the cost of studying a more complicated notion of formality for cochain algebras. Matthias Franz used certain up-to-coherent-homotopy generalizations of commutativity to generalize Cartan's ring isomorphism $H^*(G/K;\mathbb{R}) \cong \operatorname{Tor}^*_{H^*(BG;\mathbb{R})}\left(k, C^*(BK;\mathbb{R})\right)$, which implies the collapse of a related Eilenberg–Moore spectral sequence, to principal ideal domains k satisfying minimal criteria [Fra21]. In joint work Franz [ajwMF21], I generalized this to a structure theorem for the Borel cohomology $H^*_H(G/K;\mathbb{Q})$ (and hence for the cohomology $H^*(H\backslash G/K;\mathbb{Q})$ of a biquotient) by constructing suitable product on a *two-sided bar construction*, which Franz has since shown is part of an A_∞-algebra structure [Fra23]. In related work [Car22b, Car22a], I have shown the induced product on Tor in this case is equivalent to a product on Tor of a triple of strongly homotopy commutative algebras, due to Munkholm, that exists more generally, and in future work, I hope to explore properties of this product and prove more structural results for Tor and collapse results for Eilenberg–Moore spectral sequences.

Galois cohomology and the Bloch–Kato conjecture

The Bloch–Kato conjecture states that for a field k containing a primitive p^{th} root of unity, a certain homomorphism from the quotient $K^{\mathrm{M}}_*(k)/(p)$ of the Milnor K-theory of k to the cohomology $H^*\left(\operatorname{Gal}(k^{\text{sep}}/k);\mathbb{F}_p\right)$ of the absolute Galois group of k is an isomorphism. The conjecture's eventual proof due to Voevodsky relied on techniques from $\mathbb{A}^1$-homotopy theory, but a more constructive proof might enable one to extract more of the structure of $G_k = \operatorname{Gal}(k^{\text{sep}}/k)$, by (1) understanding how to identify elements of H^n as polynomials in the elements of H^1, (2) potentially recovering a presentation for the maximal pro-p quotient $G_k(p)$ of G_k, (3) resolving a question of Positselski as to whether the cohomology of $G_k(p)$ is a Koszul algebra, and (4) proving another case of the *Mináč–Tân conjecture* that all n-fold Massey products of elements of $H^1\left(G_k(p);\mathbb{F}_p\right)$ vanish for $n \geqslant 3$. Joint work with Ján Mináč will use techniques analogous to those effective in the computation of cohomology of homogeneous spaces to achieve at least some of this.

References

[Car15] Jeffrey D. Carlson. On the Equivariant Cohomology of Homogeneous Spaces. Manuscript monograph, 2015. https://jdkcarlson.github.io/homog_book.pdf.

[Car19] Jeffrey D. Carlson. Fixed points and semifree bordism. 2019. arXiv:1908.06906.

[Car22a] Jeffrey D. Carlson. Products on Tor. 2022. https://jdkcarlson.github.io/prod.pdf.

[Car22b] Jeffrey D. Carlson. A ring structure on Tor. 2022. https://jdkcarlson.github.io/Tor.pdf, arXiv:2306.04860.

[CF21] Jeffrey D. Carlson (appendix joint with Matthias Franz). The cohomology of biquotients via a product on the two-sided bar construction. 2021. arXiv:2106.02986.

[CGK18] Jeffrey D. Carlson, Elisheva Adina Gamse, and Yael Karshon. Realization of abstract GKM isotropy data. 2018. http://www.math.toronto.edu/jcarlson/realization.pdf.

[Fra21] Matthias Franz. The cohomology rings of homogeneous spaces. *J. Topol.*, 14(4):1396–1447, 2021. arXiv:1907.04777, doi:10.1112/topo.12213.

[Fra23] Matthias Franz. An A_∞ version of the Eilenberg–Moore theorem. 2023. arXiv:2311.16947.

[Sino5] Dev P. Sinha. Bordism of semi-free S^1-actions. *Math. Z.*, 249(2):439–454, 2005. arXiv:math/0303100, doi:10.1007/s00209-004-0707-3.