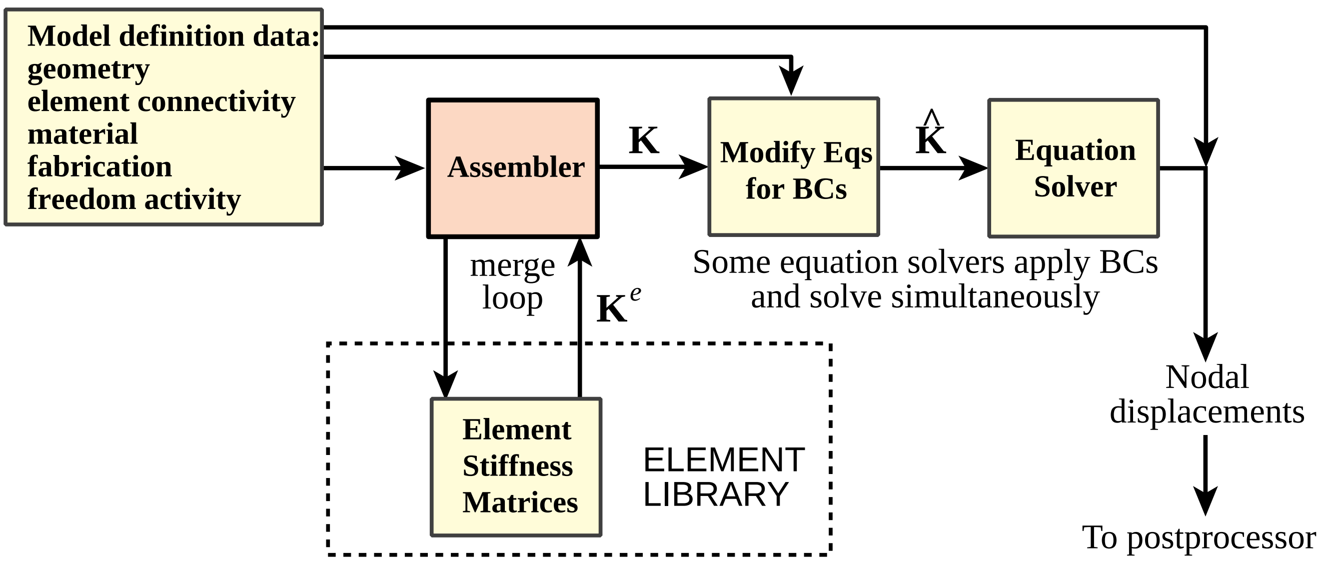
25
The Assembly Process

Role of the Assembler in a FEM Code



Simplified Assembly Process is Possible If

All elements are of the same type ; e.g. 2-node bars

The number and configuration of DOFs at each node is the same

There are no gaps in the node numbers

Restrictions removed in Chapter

There are no multifreedom constraints (MFCs)

The master stiffness matrix is stored as a full symmetric matrix

Not addressed in Chapter

Assemblers Presented in Chapter

Simplified Assembler

Meets all restrictions of previous slide

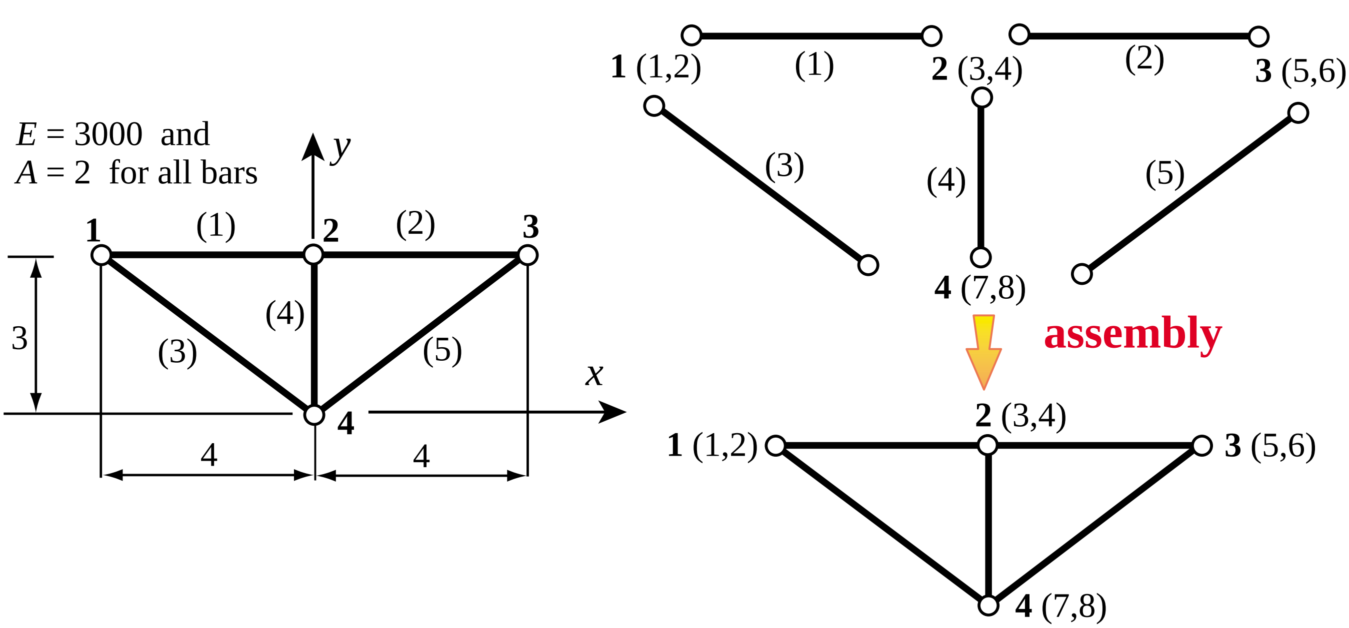
MET Assembler

Allows multiple element types

MET-VFC Assembler

**Allows multiple element types &
variable freedom configurations at nodes
(in particular, gaps in node numbers)**

Simplified Assembler Example: Plane Truss Structure



Global DOF numbers written in parenthesis after node number

Plane Truss Assembly Process

Start by clearing the master stiffness array K

$$\mathbf{K} = \begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix} \begin{array}{c} \mathit{1}\\ \mathit{2}\\ \mathit{3}\\ \mathit{4}\\ \mathit{5}\\ \mathit{6}\\ \mathit{7}\\ \mathit{8} \end{array}$$

Global DOF numbers (aka global equation numbers)

Form stiffness of bar (1)

$$\begin{bmatrix} 1500 & 0 & -1500 & 0\\ 0 & 0 & 0 & 0\\ -1500 & 0 & 1500 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix} \boxed{\begin{array}{c} \mathit{1}\\ \mathit{2}\\ \mathit{3}\\ \mathit{4} \end{array}}$$

Element Freedom Table (EFT)

and merge

$$\begin{bmatrix} 1500 & 0 & -1500 & 0 & 0&0&0&0\\ 0 & 0 & 0 & 0 & 0&0&0&0\\ -1500 & 0 & 1500 & 0 & 0&0&0&0\\ 0 & 0 & 0 & 0 & 0&0&0&0\\ 0 & 0 & 0 & 0 & 0&0&0&0\\ 0 & 0 & 0 & 0 & 0&0&0&0\\ 0 & 0 & 0 & 0 & 0&0&0&0\\ 0 & 0 & 0 & 0 & 0&0&0&0 \end{bmatrix} \begin{array}{c} \mathit{1}\\ \mathit{2}\\ \mathit{3}\\ \mathit{4}\\ \\ \\ \\ \\ \end{array}$$

Plane Truss Assembly Process (cont'd)

Form stiffness of bar (2)

$$\begin{bmatrix} 1500 & 0 & -1500 & 0 \\ 0 & 0 & 0 & 0 \\ -1500 & 0 & 1500 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \mathit{3} \\ \mathit{4} \\ \mathit{5} \\ \mathit{6} \end{matrix}$$

and merge

$$\begin{bmatrix} 1500 & 0 & -1500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1500 & 0 & 3000 & 0 & -1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1500 & 0 & 1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ \mathit{3} \\ \mathit{4} \\ \mathit{5} \\ \mathit{6} \\ \\ \\ \end{matrix}$$

Form stiffness of bar (3)

$$\begin{bmatrix} 768 & -576 & -768 & 576 \\ -576 & 432 & 576 & -432 \\ -768 & 576 & 768 & -576 \\ 576 & -432 & -576 & 432 \end{bmatrix} \begin{matrix} \mathit{1} \\ \mathit{2} \\ \mathit{7} \\ \mathit{8} \end{matrix}$$

and merge

$$\begin{bmatrix} 2268 & -576 & -1500 & 0 & 0 & 0 & -768 & 576 \\ -576 & 432 & 0 & 0 & 0 & 0 & 576 & -432 \\ -1500 & 0 & 3000 & 0 & -1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1500 & 0 & 1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -768 & 576 & 0 & 0 & 0 & 0 & 768 & -576 \\ 576 & -432 & 0 & 0 & 0 & 0 & -576 & 432 \end{bmatrix} \begin{matrix} \mathit{1} \\ \mathit{2} \\ \\ \\ \\ \\ \mathit{7} \\ \mathit{8} \end{matrix}$$

Plane Truss Assembly Process (cont'd)

Form stiffness of bar (4) **and merge**

$$\begin{array}{c c} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2000 & 0 & -2000 \\ 0 & 0 & 0 & 0 \\ 0 & -2000 & 0 & 2000 \end{bmatrix} & \begin{matrix} \boldsymbol{3} \\ \boldsymbol{4} \\ \boldsymbol{7} \\ \boldsymbol{8} \end{matrix} \end{array}$$

$$\begin{array}{c c} \begin{bmatrix} 2268 & -576 & -1500 & 0 & 0 & 0 & -768 & 576 \\ -576 & 432 & 0 & 0 & 0 & 0 & 576 & -432 \\ -1500 & 0 & 3000 & 0 & -1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2000 & 0 & 0 & 0 & -2000 \\ 0 & 0 & -1500 & 0 & 1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -768 & 576 & 0 & 0 & 0 & 0 & 768 & -576 \\ 576 & -432 & 0 & -2000 & 0 & 0 & -576 & 2432 \end{bmatrix} & \begin{matrix} \\ \\ \boldsymbol{3} \\ \boldsymbol{4} \\ \\ \\ \boldsymbol{7} \\ \boldsymbol{8} \end{matrix} \end{array}$$

Form stiffness of bar (5) **and merge**

$$\begin{array}{c c} \begin{bmatrix} 768 & 576 & -768 & -576 \\ 576 & 432 & -576 & -432 \\ -768 & -576 & 768 & 576 \\ -576 & -432 & 576 & 432 \end{bmatrix} & \begin{matrix} \boldsymbol{5} \\ \boldsymbol{6} \\ \boldsymbol{7} \\ \boldsymbol{8} \end{matrix} \end{array}$$

$$\begin{array}{c c} \begin{bmatrix} 2268 & -576 & -1500 & 0 & 0 & 0 & -768 & 576 \\ -576 & 432 & 0 & 0 & 0 & 0 & 576 & -432 \\ -1500 & 0 & 3000 & 0 & -1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2000 & 0 & 0 & 0 & -2000 \\ 0 & 0 & -1500 & 0 & 2268 & 576 & -768 & -576 \\ 0 & 0 & 0 & 0 & 576 & 432 & -576 & -432 \\ -768 & 576 & 0 & 0 & -768 & -576 & 1536 & 0 \\ 576 & -432 & 0 & -2000 & -576 & -432 & 0 & 2864 \end{bmatrix} & \begin{matrix} \\ \\ \\ \\ \boldsymbol{5} \\ \boldsymbol{6} \\ \boldsymbol{7} \\ \boldsymbol{8} \end{matrix} \end{array}$$

Plane Truss Assembly Process (cont'd)

Because all elements have been processed

$$\mathbf{K} = \begin{bmatrix} 2268 & -576 & -1500 & 0 & 0 & 0 & -768 & 576 \\ -576 & 432 & 0 & 0 & 0 & 0 & 576 & -432 \\ -1500 & 0 & 3000 & 0 & -1500 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2000 & 0 & 0 & 0 & -2000 \\ 0 & 0 & -1500 & 0 & 2268 & 576 & -768 & -576 \\ 0 & 0 & 0 & 0 & 576 & 432 & -576 & -432 \\ -768 & 576 & 0 & 0 & -768 & -576 & 1536 & 0 \\ 576 & -432 & 0 & -2000 & -576 & -432 & 0 & 2864 \end{bmatrix}$$

is the *master stiffness matrix*

Eigenvalue check shows 3 zeros.

Plane Truss Assembler Module

```
PlaneTrussMasterStiffness[nodxyz_,elenod_,elemat_,elefab_,
  eleopt_]:=Module[{numele=Length[elenod],numnod=Length[nodxyz],
  e,ni,nj,eft,i,j,ii,jj,ncoor,Em,A,options,Ke,K},
  K=Table[0,{2*numnod},{2*numnod}];
  For [e=1, e<=numele, e++, {ni,nj}=elenod[[e]];
       eft={2*ni-1,2*ni,2*nj-1,2*nj};
       ncoor={nodxyz[[ni]],nodxyz[[nj]]};
       Em=elemat[[e]]; A=elefab[[e]]; options=eleopt;
       Ke=PlaneBar2Stiffness[ncoor,Em,A,options];
       For [i=1, i<=4, i++, ii=eft[[i]];
           For [j=i, j<=4, j++, jj=eft[[j]];
               K[[jj,ii]]=K[[ii,jj]]+=Ke[[i,j]] ];
           ];
      ];  Return[K]
  ];
```

Plane Truss Assembler Script & Results

```
nodxyz={{-4,3},{0,3},{4,3},{0,0}};
elenod= {{1,2},{2,3},{1,4},{2,4},{3,4}};
elemat= Table[3000,{5}]; elefab= Table[2,{5}]; eleopt= {True};
K=PlaneTrussMasterStiffness[nodxyz,elenod,elemat,elefab,eleopt];
Print["Master Stiffness of Plane Truss of Fig 25.2:"];
K=Chop[K]; Print[K//MatrixForm];
Print["Eigs of K=",Chop[Eigenvalues[N[K]]]];
```

Master Stiffness of Plane Truss of Fig 25.2:

$$\begin{pmatrix} 2268. & -576. & -1500. & 0 & 0 & 0 & -768. & 576. \\ -576. & 432. & 0 & 0 & 0 & 0 & 576. & -432. \\ -1500. & 0 & 3000. & 0 & -1500. & 0 & 0 & 0 \\ 0 & 0 & 0 & 2000. & 0 & 0 & 0 & -2000. \\ 0 & 0 & -1500. & 0 & 2268. & 576. & -768. & -576. \\ 0 & 0 & 0 & 0 & 576. & 432. & -576. & -432. \\ -768. & 576. & 0 & 0 & -768. & -576. & 1536. & 0 \\ 576. & -432. & 0 & -2000. & -576. & -432. & 0 & 2864. \end{pmatrix}$$

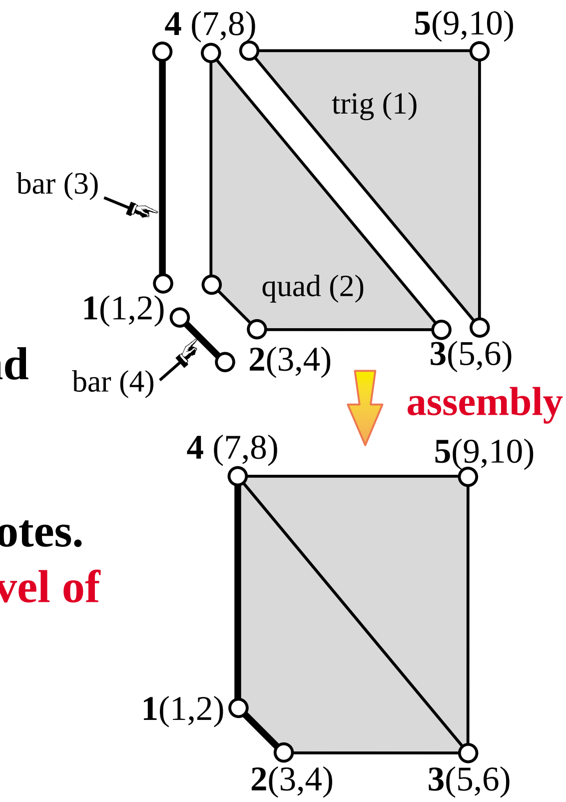
Eigs of K={5007.22, 4743.46, 2356.84, 2228.78, 463.703, 0, 0, 0}

Multiple Element Type (MET) Assembler

Useful for problems such as this plane stress example.

Three element types: bar, triangle & quadrilateral, but all nodes have 2 DOFs (u_x, u_y) and no numbering gaps are allowed.

For implementation details see Notes. Here we go directly to the next level of assembler (most complicated type considered in Chapter)



MET-VFC Assembler (allows Multiple Element Types & Variable Freedom Configuration)

Allows element type mixing in one FEM model
Nodes may have different freedom configurations identified by a signature

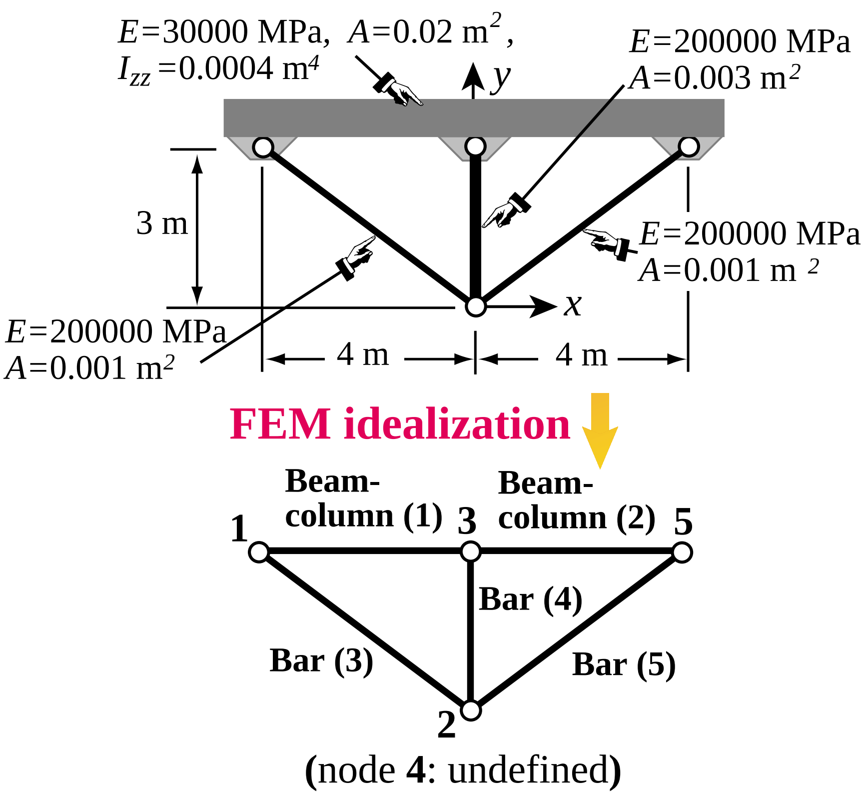
Additional data structures needed

For the MET part:
- **Element Type List**

For the VFC part:
- **Node Freedom Arrangement**
- **Node Freedom Signature**
- **Node Freedom Allocation table**
- **Node Freedom Map table**
- **Element Freedom Signature**

Detailed definitions in Notes. Here most are introduced through an application example

Trussed Frame Structure to Illustrate MET-VFC Assembly



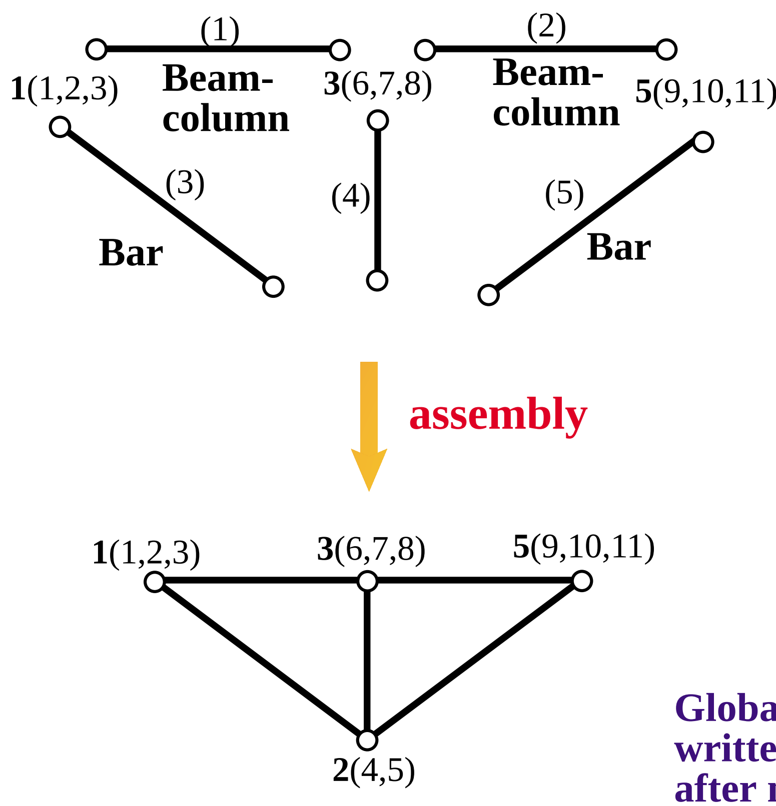
Two element types: Beam-column & bar

Nodes 1, 3 and 5 have 3 DOFs each

Node 2 has 2 DOF

Node 4 is not defined (numbering gap)

Trussed Frame Structure (cont'd)



Global DOF numbers written in parenthesis after node number

Some Definitions

Node Freedom Arrangement (NFA): $u_x, u_y, u_z, \theta_x, \theta_y, \theta_z$
(standard in general-purpose 3D FEM codes)
***position never changes*: u_x always at #1, u_y always at #2, etc**

Node Freedom Signature (NFS): a sequence of six zeros and ones packed into an integer:

1 freedom at that NFA position is allocated,
0 freedom at that NFA position is not used

110001: means u_x, u_y, θ_z allocated but u_z, θ_x, θ_y not used

A zero NFS means node is undefined or an orientation node.

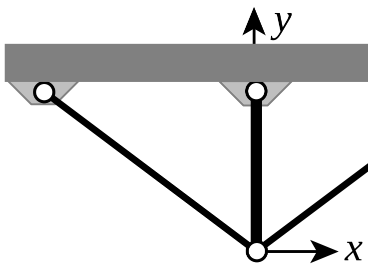
More Definitions

The lists of the NFS for all nodes is the Node Freedom Allocation Table or NFAT (program name: nodfat)

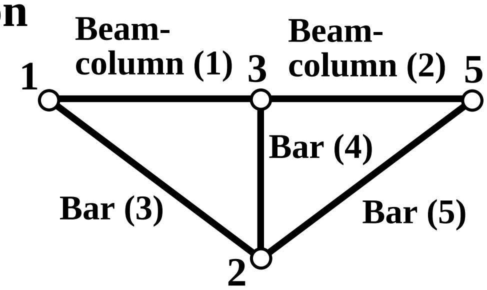
Adding node freedom counts taken from the NFAT one builds the Node Freedom Map Table or NFMT (program name: nodfmt)**. The** *n*-th **entry of NFMT points to the global DOF number *before* the first global DOF for node** *n* (*0* **if** *n*=1)

The Element Freedom Signature or EFS is a list of freedoms contributed to by the element, in node-by-node packed integer form

NFAT and NFMT for Trussed Frame Structure



FEM idealization

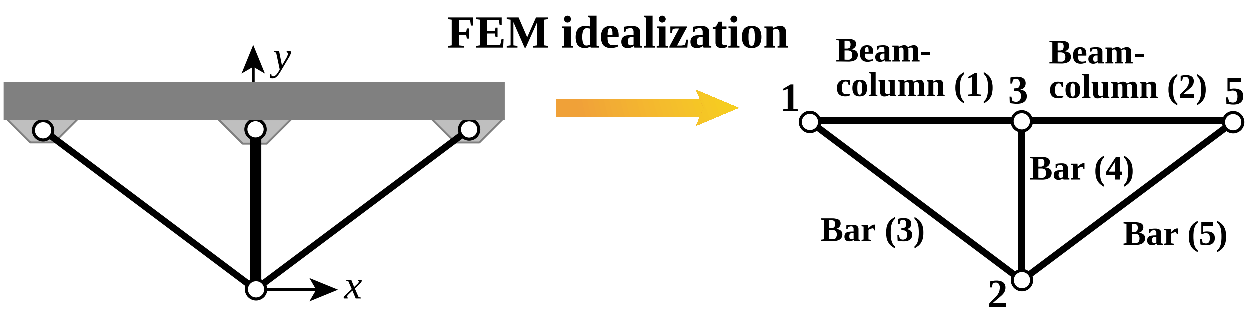


NFAT	{ 110001,	110000,	110001,	000000,	110001}
DOF count	{ 3,	2,	3,	0,	3}
NFMT	{ 0,	3,	5,	8,	8}

EFS for beam-columns: {110001,110001}
EFS for bars: {110000,110000}

From this info the Element Freedom Table (EFT) of each element may be constructed on the fly by the assembler (next slides)

Element Freedom Tables of Trussed Frame



Elem	Type	Nodes	EFS	EFT
(1)	Beam-column	{1,3}	{110001,110001}	{1,2,3,6,7,8}
(2)	Beam-column	{3,5}	{110001,110001}	{6,7,8,9,10,11}
(3)	Bar	{1,2}	{110000,110000}	{1,2,4,5}
(4)	Bar	{2,3}	{110000,110000}	{4,5,6,7}
(5)	Bar	{2,5}	{110000,110000}	{4,5,9,10}

Trussed Frame Assembly Process

Beam-column (1)

$$\mathbf{K}^{(1)} = \begin{bmatrix} 150. & 0. & 0. & -150. & 0. & 0. \\ 0. & 22.5 & 45. & 0. & -22.5 & 45. \\ 0. & 45. & 120. & 0. & -45. & 60. \\ -150. & 0. & 0. & 150. & 0. & 0. \\ 0. & -22.5 & -45. & 0. & 22.5 & -45. \\ 0. & 45. & 60. & 0. & -45. & 120. \end{bmatrix} \begin{array}{c} \mathit{1} \\ \mathit{2} \\ \mathit{3} \\ \mathit{6} \\ \mathit{7} \\ \mathit{8} \end{array}$$

EFT

Beam-column (2)

$$\mathbf{K}^{(2)} = \begin{bmatrix} 150. & 0. & 0. & -150. & 0. & 0. \\ 0. & 22.5 & 45. & 0. & -22.5 & 45. \\ 0. & 45. & 120. & 0. & -45. & 60. \\ -150. & 0. & 0. & 150. & 0. & 0. \\ 0. & -22.5 & -45. & 0. & 22.5 & -45. \\ 0. & 45. & 60. & 0. & -45. & 120. \end{bmatrix} \begin{array}{c} \mathit{6} \\ \mathit{7} \\ \mathit{8} \\ \mathit{9} \\ \mathit{10} \\ \mathit{11} \end{array}$$

Bar (3)

$$\mathbf{K}^{(3)} = \begin{bmatrix} 25.6 & -19.2 & -25.6 & 19.2 \\ -19.2 & 14.4 & 19.2 & -14.4 \\ -25.6 & 19.2 & 25.6 & -19.2 \\ 19.2 & -14.4 & -19.2 & 14.4 \end{bmatrix} \begin{array}{c} \mathit{1} \\ \mathit{2} \\ \mathit{4} \\ \mathit{5} \end{array}$$

Trussed Frame Assembly Process (cont'd)

Bar (4)

$$\mathbf{K}^{(4)} = \begin{array}{c} \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 200. & 0 & -200. \\ 0 & 0 & 0 & 0 \\ 0 & -200. & 0 & 200. \end{array}\right] \end{array} \begin{array}{c} \boxed{\begin{array}{c} \mathit{4} \\ \mathit{5} \\ \mathit{6} \\ \mathit{7} \end{array}} \end{array}$$

EFT

Bar (5)

$$\mathbf{K}^{(5)} = \left[\begin{array}{cccc} 25.6 & 19.2 & -25.6 & -19.2 \\ 19.2 & 14.4 & -19.2 & -14.4 \\ -25.6 & -19.2 & 25.6 & 19.2 \\ -19.2 & -14.4 & 19.2 & 14.4 \end{array}\right] \boxed{\begin{array}{c} \mathit{4} \\ \mathit{5} \\ \mathit{9} \\ \mathit{10} \end{array}}$$

Master Stiffness Matrix

$$\mathbf{K} = \left[\begin{array}{rrrrrrrrrrr} 175.6 & -19.2 & 0 & -25.6 & 19.2 & -150. & 0 & 0 & 0 & 0 & 0 \\ -19.2 & 36.9 & 45. & 19.2 & -14.4 & 0 & -22.5 & 45. & 0 & 0 & 0 \\ 0 & 45. & 120. & 0 & 0 & 0 & -45. & 60. & 0 & 0 & 0 \\ -25.6 & 19.2 & 0 & 51.2 & 0 & 0 & 0 & 0 & -25.6 & -19.2 & 0 \\ 19.2 & -14.4 & 0 & 0 & 228.8 & 0 & -200. & 0 & -19.2 & -14.4 & 0 \\ -150. & 0 & 0 & 0 & 0 & 300. & 0 & 0 & -150. & 0 & 0 \\ 0 & -22.5 & -45. & 0 & -200. & 0 & 245. & 0 & 0 & -22.5 & 45. \\ 0 & 45. & 60. & 0 & 0 & 0 & 0 & 240. & 0 & -45. & 60. \\ 0 & 0 & 0 & -25.6 & -19.2 & -150. & 0 & 0 & 175.6 & 19.2 & 0 \\ 0 & 0 & 0 & -19.2 & -14.4 & 0 & -22.5 & -45. & 19.2 & 36.9 & -45. \\ 0 & 0 & 0 & 0 & 0 & 0 & 45. & 60. & 0 & -45. & 120. \end{array}\right] \begin{array}{c} \mathit{1} \\ \mathit{2} \\ \mathit{3} \\ \mathit{4} \\ \mathit{5} \\ \mathit{6} \\ \mathit{7} \\ \mathit{8} \\ \mathit{9} \\ \mathit{10} \\ \mathit{11} \end{array}$$

HW #10 Problem: Write Assembler for Plate Reinforced Trussed Frame