# Surfing: Iterative Optimization Over Incrementally Trained Deep Networks

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## Main Idea

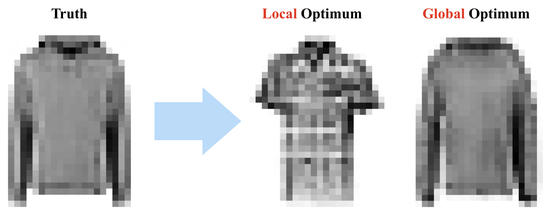
We minimize

$$f_\theta(x) = \frac{1}{2}\|AG_\theta(x) - Ay\|^2 \tag{1}$$

$A \in \mathbb{R}^{m\times n}$ matrix, $y \in \mathbb{R}^n$ the true signal ($m \ll n$), $G_\theta$ generative network.

**Goal**: Find the minimizer $\hat{x}$ and recover y by $G_\theta(\hat{x})$.

**PROBLEM**: $f_\theta(x)$ is usually non-convex; gradient descent cannot find global minimum.

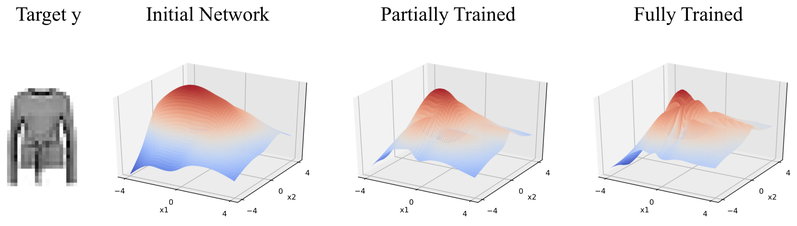



**OUR RESULTS**:

- Propose a novel algorithm called surfing outperforms GD
- Theoretical analysis is provided to ensure its convergence

## Surfing Algorithm

The landscape of $f_\theta(x)$ is nice at the beginning and getting wavy and bumpy as $G_\theta$ is trained.




**Motivations**: Use intermediate networks from training process and track the global optimum

**Algorithm**:

A sequence of network $G_0, G_1, \ldots, G_T$. Write

$$f_t(x) = \frac{1}{2}\|AG_t(x) - Ay\|^2, \quad \forall\, t \in [T]. \tag{2}$$

Minimize $f_0$ to obtain the minimizer $x_0$, then apply gradient descent on $f_t$ for $t = 1, 2, ..., T$ iteratively, initializing at the minimizer $x_{t-1}$.

## Theoretical Results

Consider $G$ with ReLU activation,

$$G(x,\theta) = V\sigma(W_d \ldots \sigma(W_2\sigma(W_1 x + b_1) + b_2) \ldots + b_d).$$

Our results:

1. If $\theta_0$ is Gaussian and $G$ is sufficiently expansive, then w.h.p., all critical points of $f_0(x)$ belong to a small neighborhood around 0.

   Proof technique:

   - $G$ is piecewise linear, gradient of $f_0$ has explicit expression.
   - Concentration bounds give $\nabla f_0(x) = 2^{-d}x + O(\varepsilon(1+\|x\|))$.
   - Find descent direction $v$ s.t. $D_v f_0(x) < 0$ for all $\|x\| > O(\varepsilon)$.

2. Consider a network flow $G^s(x) = G(x, \theta(s))$ for $s \in [0, S]$ and corresponding objective $f^s(x) = f(x, \theta(s))$. If the weights of $G^s$ is bounded and the global minimizer of $f^s$ is unique and Lipschitz-continuous, then the projected-gradient surfing can keep track of the minimizer with a small time discretization step $\delta$ of $G^s$.

   Projected-gradient surfing:

   - Identify all the linear pieces $\{P_1, \ldots, P_l\}$ for current $G_t$ that could contain global minimizer of $f_t$.
   - Apply projected gradient descent $x \leftarrow \mathrm{Proj}_P(x - \eta\nabla f_t(x))$ for each $P_i$.

## Experiments

1. $f(x) = \frac{1}{2}\|G(x) - G(x_*)\|^2$. (Invert a generator)

Compared with regular gradient descent (ADAM), surfing has

- Higher proportion of convergence
- Comparable computations

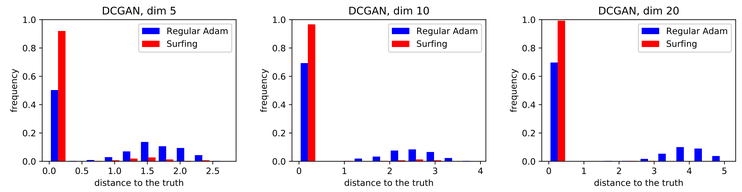



Figure 1: Distribution of distance between solution $\hat{x}$ and the truth $x_*$.

| Input dimension | | 5 | 10 | 20 | 5 | 10 | 20 | 5 | 10 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Model | DCGAN | | | WGAN | | | WGAN-GP | | |
| % successful | Regular Adam | 48.3 | 68.7 | 80.0 | 56.0 | 84.3 | 90.3 | 47.0 | 64.7 | 64.7 |
| | Surfing | 78.3 | 98.7 | 96.3 | 81.7 | 97.3 | 99.3 | 83.7 | 95.7 | 97.3 |
| # iterations | Regular Adam | 618 | 4560 | 18937 | 464 | 1227 | 3702 | 463 | 1915 | 15445 |
| | Surfing | 741 | 6514 | 33294 | 547 | 1450 | 4986 | 564 | 2394 | 25991 |

Table 1: Percentages of solutions $\hat{x}$ satisfying $\|\hat{x} - x_*\| < 0.01$.

2. $f(x) = \frac{1}{2}\|AG(x) - AG(x_*)\|^2$. (Compressed sensing, $y$ in the range of $G$)
   Surfing find global optimum more easily than regular GD.

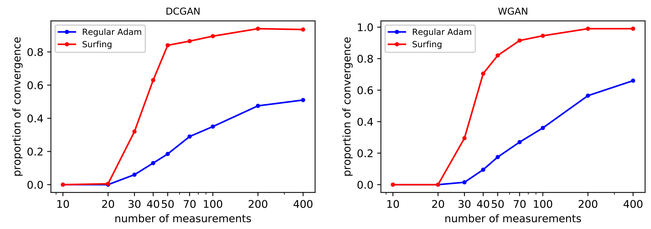



Figure 2: Compressed sensing setting for exact recovery.

3. $f(x) = \frac{1}{2}\|AG(x) - Ay\|^2$, $y$ from test data.
   Surfing improves the reconstruction of given signal $y$.

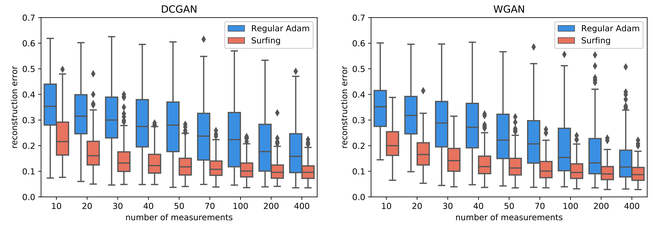



Figure 3: Compressed sensing setting for approximation, or rate-distortion.

The reconstruction error is given by $\sqrt{\frac{1}{n}\|G(\hat{x}) - y\|^2}$.

## Future Work

1. Can we bridge the gap between practice and theory. For now, we use projected-gradient surfing in our analysis.
2. Sometimes, the objective function (1) with trained network does have a nice landscape. How do we understand this phenomenon?
3. If we are able to regularize the training process of $G$, we can just minimize (1) by simple gradient descent.
4. The idea of surfing can probably apply to other optimization problems, where the objective evolves incrementally and has favorable properties at initial stage.