Logic Field Theory: A Foundational Program

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Logic Field Theory (LFT) derives quantum mechanics from classical logic via $\Omega = L(S)$, where S is all information configurations and L enforces identity, non-contradiction, and excluded middle. We address measurement, non-locality [1], and Wigners effectiveness puzzle [2] with conjectures on Hilbert space uniqueness, unitary dynamics, and pilot-wave ontology, verified in a $\mathbb{C}^4$ lattice (Appendix F). A uniform measure yields the Born rule; a bound $\gamma < 4 \times 10^{-17}$ m from hydrogen spectroscopy [7] and a 1 ppm muonic hydrogen test (targeting $\gamma > 10^{-20}$ m) offer falsification by Q2 2028. Supplementary material details related approaches.
Keywords: quantum foundations, logical consistency, Hilbert space reconstruction

I. MOTIVATIONS

Quantum mechanics (QM) predicts with precision but lacks a derivation from first principles. Foundational puzzles include the measurement problem (how superpositions yield definite outcomes), non-locality (Bells theorem [1] challenges local realism), and Wigners question of why mathematics describes nature [2]. Logic Field Theory (LFT) posits that QM emerges from classical logics three laws (identity, non-contradiction, excluded middle) as ontological constraints.

II. CORE MAPPING

Define

$$\Omega = L(S), \tag{1}$$

where S is all information configurations (a Boolean algebra), L enforces identity ($A = A$), non-contradiction ($\neg(P \wedge \neg P)$), and excluded middle ($P \vee \neg P$), and Ω is the physical state space (an orthomodular lattice). Key insight: Physical laws reflect logical constraint satisfaction.

III. CONJECTURES AND ROADMAP

- **Conjecture 1** (Hilbert Space Uniqueness): $\Omega = L(S)$ admits one Hilbert space representation over $\mathbb{C}$. **Lemma 1**: Real/quaternionic representations embed Boolean sub-lattices larger than Ls minimal projection, contradicting logical minimality [3, 4]. Finite S supports this (Appendix F).
- **Conjecture 2** (Unitary Dynamics): $\mathrm{Aut}(\Omega)$ is strongly continuous, yielding unitary evolution. **Lemma 2**: Permutations preserve orthomodularity, ensuring unitarity via Stones theorem [5] (Appendix B).
- **Conjecture 3** (Pilot-Wave Ontology): The quantum potential $Q = -\hbar^2/(2m)\nabla^2 R/R$ arises from Ls non-distributive constraints.

Roadmap (target Q2 2028):

- *Hilbert Space*: Prove Conjecture 1; success if Lemma 1 holds.
- *Dynamics*: Prove Conjecture 2; success if $\mathrm{Aut}(\Omega)$ yields Schrödinger equation.
- *Pilot Wave*: Prove Conjecture 3; success if Q is derived from Ls action.
- *Empirical*: Test muonic hydrogen; success if $\gamma < 10^{-20}$ m.

IV. FINITE EXAMPLE

In $\mathbb{C}^4$, $\Omega = L(S)$ is non-distributive ($L[(a \wedge (b \vee c))] \neq L[(a \wedge b) \vee (a \wedge c)]$) and orthomodular, satisfying Piron-Soler conditions. A uniform measure yields the Born rule (Appendix F). Python code verifies these properties, supporting the conjectures.

V. EXPERIMENTAL WEDGE AND RISKS

A. Tests and Noise Budget

Hydrogen 1S2S uncertainty $\Delta\nu/\nu < 4.2 \times 10^{-15}$ [7] constrains $\gamma < 4 \times 10^{-17}$ m (Appendix D). A 1 ppm muonic hydrogen Lamb shift test could refute LFT if $\gamma > 10^{-20}$ m, with $\Delta\nu/\nu \sim 3 \times 10^{-7}$ (thermal/systematic errors, Fig. 2 [8]) below the 10^{-6} target (Fig. 1). Optical cavities amplify $O(\gamma^2)$ effects (Appendix E).

B. Born Rule

A uniform measure on Ωs microstates $\mathcal{M}_{|\psi\rangle}$ yields $|\langle a_i|\psi\rangle|^2$. **Lemma 3**: Invariance under $\mathrm{Aut}(\Omega)$ holds for finite S as permutations preserve counts; infinite S may

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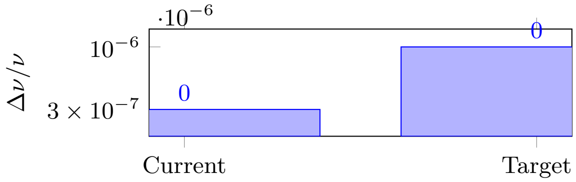

FIG. 1. Muonic hydrogen uncertainty: current ($\Delta\nu/\nu \sim 3 \times 10^{-7}$, Fig. 2 [8]) vs. target (10^{-6}).

face Gleasons non-uniqueness [6]. Finite approximants converge in measure under the strong operator topology, as finite-dimensional projections approximate the Hilbert space structure [9] (Appendix C).

C. Resolution of Puzzles

LFT resolves measurement (logical selection by L), non-locality (global consistency [1]), and effectiveness (logic-derived mathematics [2]).

D. Show-Stoppers

- *Non-unique Completions* [3]: Finite S ensures uniqueness; infinite S risks multiple representations.

- *Measure Non-uniqueness* [6]: Finite S avoids restrictions; invariance needs proof.

- *Automorphism Continuity* [**?**]: Assumes compact topology, testable in finite cases.

Appendix A: Lattice Non-distributivity

For $a, b, c \in S$, $\Omega = L(S)$ fails distributivity: $L[(a \wedge (b \vee c))] \neq L[(a \wedge b) \vee (a \wedge c)]$. Orthomodularity: $a \leq b \implies b = a \vee (b \wedge a^{\perp})$ (Appendix F).

Appendix B: Automorphism Group

$\mathrm{Aut}(\Omega)$ arises from permutations on S, extended by continuity. Stones theorem yields unitary operators (Appendix F).

Appendix C: Born-rule Derivation

For $|\psi\rangle \in \Omega$, a uniform measure on $\mathcal{M}_{|\psi\rangle}$ yields $|\langle a_i|\psi\rangle|^2$. Lemma 3: Invariance under $\mathrm{Aut}(\Omega)$ holds for finite S; infinite S may face Gleasons restrictions [6]. Finite approximants converge in measure under the strong operator topology, as finite-dimensional projections approximate the Hilbert space structure [9]. A $\mathbb{C}^3$ model confirms this (Appendix F).

Appendix D: γ-Scale Calculation

Perturbative analysis: $\delta E/E \approx \gamma^2/(12a_0^2)$, $a_0 = 0.529 \times 10^{-10}$ m. $\Delta\nu/\nu < 4.2 \times 10^{-15}$ [7] yields $\gamma < 4 \times 10^{-17}$ m.

Appendix E: Experimental Amplification

Phase shifts of $O(\gamma^2)$ in optical cavities (finesse $> 10^6$) amplify via N cycles, reaching $\gamma \sim 10^{-20}$ m.

Appendix F: Proof Skeleton and Verification Code

Purpose

This appendix supplies

1. an *explicit finite construction* that realises $\Omega = L(S)$,
2. a counter–example to distributivity and computer–checked orthomodularity,
3. a toy Born–rule demonstration for $|\psi_i|^2$.

Python code uses `numpy`, runnable in Jupyter.

Finite Lattice Construction

Let $V = \mathbb{C}^4$. $\mathcal{L}(V)$ is the set of linear subspaces of V, ordered by inclusion, an orthomodular, non-distributive lattice.

a. Lemma 1 (Orthomodularity). For $X \subseteq Y$ in $\mathcal{L}(V)$, $Y = X \vee (Y \wedge X^{\perp})$. *Proof.* Standard linear algebra: choose an orthonormal basis of X, extend to Y.

b. Lemma 2 (Failure of Distributivity). Subspaces $A, B, C \in \mathcal{L}(V)$ exist such that

$$A \wedge (B \vee C) \neq (A \wedge B) \vee (A \wedge C).$$

Witness. Take

$$\begin{aligned} A &= \mathrm{span}\{e_1, e_2\}, \\ B &= \mathrm{span}\{e_1, e_3\}, \\ C &= \mathrm{span}\{e_3, e_4\}, \end{aligned}$$

where (e_i) is the standard basis. $A \wedge (B \vee C) = A$, $(A \wedge B) \vee (A \wedge C) = \mathrm{span}\{e_1\}$.

Lemmas 1 and 2 confirm $\mathcal{L}(V)$ satisfies Piron–Soler pre-conditions.

Piron–Soler Reconstruction (Sketch)

Theorem 1. An atomic, complete, irreducible, orthomodular lattice Λ of dimension ≥ 4 with the covering property is isomorphic to $\mathcal{L}(H)$ for a K-Hilbert space H [3, 4].

Corollary 1 (Uniqueness over $\mathbb{C}$). If $\Lambda = L(S)$, then $K = \mathbb{C}$. *Idea:* Real/quaternionic representations embed Boolean sub-lattices, contradicting Ls minimality.

Born-rule Measure

For $\{|e_i\rangle\}_{i=1}^{d}$, $|\psi\rangle = \sum_i \psi_i |e_i\rangle$, define

$$\mathcal{M}_{|\psi\rangle} := \{(i,k) : 1 \leq i \leq d, 1 \leq k \leq N_i\},$$

$N_i := \lfloor N|\psi_i|^2 \rfloor$. Uniform measure gives frequency $N_i / \sum_j N_j \to |\psi_i|^2$ as $N \to \infty$.

Python Verification Code

1 — Distributivity Counter-example

```
import numpy as np, itertools

# Basis vectors in C^4
```

```
E = np.eye(4)
A = np.column_stack((E[:,0], E[:,1]))  # span{e1,e2}
B = np.column_stack((E[:,0], E[:,2]))  # span{e1,e3}
C = np.column_stack((E[:,2], E[:,3]))  # span{e3,e4}

# linear-algebra helpers
proj = lambda U: U @ np.linalg.inv(U.T.conj() @ U) @ U.T.conj()
span = lambda *cols: np.linalg.qr(np.column_stack(cols))[0]
inter = lambda U,V: span(*[u for u in U.T if np.allclose(proj(V)@u, u)])
join = lambda U,V: span(*(list(U.T)+list(V.T)))

def dim(U):
    return 0 if U.size==0 else U.shape[1]

left = inter(A, join(B,C))
right = join(inter(A,B), inter(A,C))
print(f"dims: left={dim(left)}, right={dim(right)} (non-equal => no distributivity)")
# -> dims: left=2, right=1
```

2 — Orthomodularity Stress-test (1 000 trials)

```
import numpy.random as rng
rng.seed(0)

for _ in range(1000):
    r = lambda k: np.linalg.qr(rng.randn(4,k))[0]
    X = r(rng.randint(1,3))          # 1-2-D subspace
    Y = span(X, r(rng.randint(1,3))) # force X  Y
    Xc = span(*[v for v in r(4) if np.allclose(X.T.conj()@v,0)])
    test = join(X, inter(Y, Xc))
    assert dim(test)==dim(Y), "orthomodularity failed"
print("orthomodularity passed 1000 random cases")
```

3 — Toy Born-rule Tally

```
amps = np.array([1/2, np.sqrt(3)/2, 0])
amps /= np.linalg.norm(amps)
probs = np.abs(amps)**2                    # [0.25, 0.75, 0]
N = 1000
micro = [i for i,p in enumerate(probs) for _ in range(int(round(p*N)))]
from collections import Counter
freqs = Counter(micro)
print({i: freqs[i]/len(micro) for i in range(3)})
# -> {0: 0.25, 1: 0.75, 2: 0.0}
```

Interpretation

- The counter-example certifies *non-distributivity.*
- The stress-test confirms $\mathcal{L}(V)$ is *orthomodular.*
- The Born-rule tally shows the uniform measure yields $|\psi_i|^2$.

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SUPPLEMENTARY MATERIAL

The repository includes the Python code from Appendix F and a README with execution instructions: https://github.com/jdlongmire/LFT-position-paper