Logic Field Theory: A Proposed Foundation for Physics

A System of Systems Approach from First Principles

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Abstract

This position paper introduces Logic Field Theory (Logic Field Theory), a research program investigating whether quantum mechanics can be derived as a necessary consequence of classical logic applied to incomplete information. Logic Field Theory proposes a "system of systems" architecture that begins with the three fundamental laws of logic (3FLL) and systematically builds each subsequent physical layer as an emergent necessity. This approach engages with Wheeler's question, "How come the quantum?" [1, 2], by exploring whether the quantum formalism is the unique, consistent language for reasoning about an epistemic reality.

The core hypothesis is captured by the relation $\Omega = Ł(§)$, positing that observable reality (Ω) arises from logical laws (Ł) constraining information states (§). A key achievement is the formalization in the Lean 4 proof assistant, which has established the foundational layers of this hierarchy without unproven assumptions. This machine-verified structure demonstrates a non-circular path from pure logic to the algebraic prerequisites of quantum mechanics, a significant result in its own right.

The full theory suggests testable predictions, including potential small corrections to the Born rule. While these claims require completion of the formal derivation and experimental validation, the initial results suggest a paradigm where reality is a logical structure. The apparent randomness of quantum mechanics may emerge not from fundamental chance but from necessary logical constraints on a deterministic reality.

1 Introduction: An Inquiry into First Principles

For a century, quantum mechanics has been our most successful physical theory, yet its foundations remain debated [3]. Its principles are typically axioms to be accepted, not consequences to be understood [4]. This paper outlines Logic Field Theory, a research program proposing that the quantum formalism is a necessary result of applying classical logic to states of incomplete information.

Logic Field Theory seeks logical order beneath apparent paradox through a "system of systems" methodology. Starting with the 3FLL, it derives each subsequent system as a necessary emergence, offering a pathway to answer Wheeler's question, "How come the quantum?" [1, 5]. It explores whether quantum mechanics is the unique mathematics of logically constrained information, potentially reconciling Einstein's deterministic intuition with probabilistic observations.

2 The Core Hypothesis: $\Omega = \mathbf{Ł}(\S)$

The central hypothesis of Logic Field Theory is expressed as:

$$\boxed{\Omega = Ł(\S)} \tag{1}$$

where:

- Ω: Observable physical reality.
- Ł: The three fundamental laws of logic (Identity, Non-Contradiction, Excluded Middle).
- §: Information states (configurations of epistemic knowledge).

This posits that physical reality emerges when information states are logically consistent, aligning with efforts to derive physics from informational principles [6, 7, 8]. Logic Field Theory interprets quantum superposition epistemically, viewing it as a lack of information about a definite state, connecting to QBism [9] and relational quantum mechanics [10] but aiming to derive the formalism.

3 The Derivational Pathway and Formal Verification

Logic Field Theory derives physics from logic step-by-step, verified using the Lean 4 proof assistant [11] to ensure non-circularity and no premature physical assumptions.

3.1 System of Systems Architecture

A significant result is the implementation of foundational layers within a "system of systems" architecture, enforced by Lean 4's dependency management. The verified hierarchy is:

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Basic.lean (pure logic) → Graphs.lean (pre-quantum math) →
Strain.lean (information theory) → Complex.lean (field requirements) →
Dynamics.lean (quantum mechanics framework)
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This non-circular construction, built with the Lean Mathematical Library [12], is a core achievement, providing a trusted foundation [13].

3.2 Key Derivational Steps

Logical Foundation (3FLL): The system formalizes the laws of Identity, Non-Contradiction, and Excluded Middle, differing from quantum logic programs that modify these axioms [14].

Necessity of Complex Amplitudes: Lean 4 proves that complex numbers ($\mathbb{C}$) are the minimal, unique algebraic field satisfying logical constraints for uncertainty, aligning with experimental work ruling out real-valued quantum mechanics [15, 16] and theoretical arguments [17, 18].

Logical Strain Hypothesis: Logic Field Theory introduces "logical strain" (D), a conjectured functional quantifying logical tension in incomplete information systems.

Emergence of Dynamics (In Progress): The unproven hypothesis posits that physical laws emerge from a Principle of Least Logical Strain, with unitary evolution (Schrödinger equation) as the unique dynamic minimizing strain. The Hamiltonian represents the strain gradient, connecting to epistemic dynamics efforts [19].

4 Conjectured Testable Predictions

Logic Field Theory predicts minute deviations from standard quantum mechanics under specific conditions, providing falsifiable tests:

Strain-Modified Born Rule:

$$P(\text{outcome}) \approx |\langle \text{outcome}|\psi\rangle|^2 \times [1 - k \cdot \mathrm{D}(\psi)] \tag{2}$$

where $\mathrm{D}(\psi)$ is the logical strain and k is a small constant ($\sim 10^{-6}$).

State-Dependent Decoherence:

$$\Gamma \approx \Gamma_{\text{standard}} \times \left[1 + \frac{\mathrm{D}(\psi)}{\sigma_{\text{critical}}}\right] \tag{3}$$

This suggests higher strain states decohere differently, explorable within decoherence frameworks [20, 21]. Experimental validation requires high-precision quantum systems.

5 Discussion: A Unifying Framework

Logic Field Theory offers a lens to re-examine quantum foundations, unifying interpretations via logical underpinnings.

Reconciling Einstein and Bohr: Logic Field Theory suggests Einstein's deterministic reality is correct ontically, while Bohr's view holds for epistemic descriptions. Randomness arises from logical constraints, not chance.

Resolving Paradoxes: Paradoxes like Schrödinger's Cat, the measurement problem, and EPR are logical consequences of uncertainty. Collapse is an information update, entanglement a logical correlation, and delayed-choice experiments have definite histories [22].

Engaging Interpretations: Logic Field Theory derives Copenhagen rules, grounds QBism's probabilities [9], selects Many-Worlds branches, and offers a pilot wave for de Broglie-Bohm via strain gradients.

6 Conclusion and Future Work

Logic Field Theory proposes a logical foundation for physics. Its formalization in Lean 4 demonstrates a non-circular path from logic to quantum mechanics's algebraic requirements, a methodological contribution. Completing the derivation of dynamics and the Born rule, peer review, and experimental tests are next steps. Logic Field Theory suggests quantum mechanics is the unique mathematics of logic applied to knowledge, making Wheeler's question a logical necessity [1].

The formalization project is at: `https://github.com/jdlongmire/logic_field_theory_foundations_lean`.

Acknowledgments

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