

# Logic Field Theory: Deriving Quantum Field Theory from the Three Fundamental Laws of Logic

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## Abstract

We present Logic Field Theory (LFT), a framework deriving quantum field theory from the Three Fundamental Laws of Logic without assuming physical postulates. Starting from Identity, Non-Contradiction, and Excluded Middle, we show that quantum mechanics emerges as the unique mathematical structure consistent with logical coherence. Key results include: (1) necessity of complex amplitudes over real or quaternionic alternatives, (2) unique emergence of the  $U(1) \times SU(2) \times SU(3)$  gauge structure, (3) derivation of exactly three fermion generations, (4) the Born rule as a theorem rather than postulate, and (5) resolution of the measurement problem through logical strain dynamics. Most significantly, LFT predicts decoherence scaling as  $\tau_D \propto \xi^2$  (positive exponent) versus standard quantum mechanics'  $\tau_D \propto \xi^{-1}$  (negative exponent), enabling decisive experimental validation. All core derivations are formally verified in Lean 4.

**Keywords:** quantum foundations, measurement problem, decoherence, gauge theory, Lean verification

## 1. Introduction

The foundations of quantum mechanics remain contentious nearly a century after its formulation [1][2]. While the mathematical formalism yields precise predictions confirmed to extraordinary accuracy [3], fundamental questions persist: Why complex numbers? Why the Born rule? What constitutes measurement? Various interpretations—Copenhagen [4], Many-Worlds [5], de Broglie-Bohm [6], QBism [7]—offer different ontologies but maintain the same mathematical structure.

Recent work in quantum foundations has increasingly focused on deriving quantum theory from information-theoretic [8][9] or operational principles [10][11]. Hardy's axiomatization [12] and the reconstruction program of Chiribella, D'Ariano, and Perinotti [13] demonstrate that quantum theory can emerge from seemingly non-quantum assumptions. However, these approaches typically assume probabilistic structure or operational primitives that themselves require justification.

Logic Field Theory takes a more fundamental approach: deriving quantum mechanics from the Three Fundamental Laws of Logic (3FLL):

$$\begin{aligned} \text{Identity: } & A = A & (1) \\ \text{Non-Contradiction: } & \neg(A \wedge \neg A) & (2) \\ \text{Excluded Middle: } & A \vee \neg A & (3) \end{aligned}$$

We show these laws, when applied as constraints on admissible physical states, uniquely determine quantum mechanical structure. This paper presents the complete derivation, experimental predictions, and formal verification.

## 2. Theoretical Framework

### 2.1 The Admissibility Principle

**Definition 1** (Syntactic Space). Let  $\mathcal{S}$  denote the space of all formally describable configurations, represented as finite directed graphs with negation involution:

$$\mathcal{S} = \{G = (V, E, \tau) \mid V \subset \mathcal{L}, |V| < \infty, E \subseteq V \times V, \tau \circ \tau = \text{id}_V\}$$

where  $\mathcal{L}$  is the set of literals and  $\tau$  maps each vertex to its negation.

**Definition 2** (Logic Operator). The logic operator  $\mathbb{L} : \mathcal{S} \rightarrow \mathcal{S}$  enforces the 3FLL:

$$\mathbb{L}(G) = \begin{cases} G & \text{if } G \text{ satisfies 3FLL} \\ \emptyset & \text{otherwise} \end{cases}$$

**Theorem 1** (Admissible Set). The admissible configurations form the fixed-point set:

$$\mathcal{A} = \mathbb{L}(\mathcal{S}) = \{G \in \mathcal{S} \mid \mathbb{L}(G) = G\}$$

This establishes our fundamental principle: **physical reality consists only of logically admissible configurations.**

## 2.2 The Logical Strain Functional

Among admissible configurations, selection occurs via the logical strain functional:

**Definition 3** (Strain Functional). For configuration  $\psi \in \mathcal{A}$ :

$$D(\psi) = w_I v_I(\psi) + w_N v_N(\psi) + w_E v_E(\psi)$$

where:

- $v_I$ : internal contradiction strain (proximity to logical inconsistency)
- $v_N$ : non-classicality strain (logical indeterminacy)
- $v_E$ : external misfit strain (environmental coupling)

**Theorem 2** (Scale Invariance). Requiring scale invariance under  $\xi \rightarrow \lambda\xi$  uniquely determines:

$$w_I : w_N : w_E = \left( \frac{\xi}{\ell_0} \right)^2 : 1 : \left( \frac{\ell_0}{\xi} \right)^2$$

where  $\xi$  is the system's coherence length and  $\ell_0 \approx 1.6 \times 10^{-35}$  m is the fundamental length.

## 3. Emergence of Quantum Structure

### 3.1 Complex Hilbert Space Necessity

**Theorem 3** (Complex Necessity). The admissible set  $\mathcal{A}$  admits a faithful representation only in complex Hilbert space  $\mathcal{H}_{\mathbb{C}}$ .

*Proof sketch:* Real representations cannot capture orientation in logical cycles (violating Excluded Middle), while quaternionic representations violate commutativity required for independent subsystems. See [\[14\]](#) for Gleason-type arguments and [\[15\]](#) for the role of complex structure in quantum mechanics.  $\square$

### 3.2 Unitary Evolution

**Theorem 4** (Uniqueness of Unitary Dynamics). Logical coherence preservation requires evolution via one-parameter unitary groups:

$$U(t) = \exp(-i\hat{H}t/\hbar)$$

This follows from Wigner's theorem [\[16\]](#) on symmetry representations and Stone's theorem [\[17\]](#) on one-parameter groups. The Hamiltonian  $\hat{H}$  emerges as the generator of logical-coherence-preserving

transformations.

### 3.3 Born Rule Derivation

**Theorem 5** (Born Rule). The unique probability measure consistent with logical constraints is:

$$P(k|\psi) = |\langle k|\psi\rangle|^2$$

*Proof:* Apply Gleason's theorem [\[18\]](#) restricted to the admissible set  $\mathcal{A}$ , extended by Busch [\[19\]](#) to include 2-dimensional subspaces. The Born measure is the unique normalized, non-contextual probability assignment.  $\square$

## 4. Gauge Structure and the Standard Model

### 4.1 Emergence of Gauge Groups

**Theorem 6** (Gauge Structure). Local logical coherence requires exactly three gauge symmetries:

$$\text{Cyclic consistency} \Rightarrow \text{U}(1) \quad (4)$$

$$\text{Binary distinction} \Rightarrow \text{SU}(2) \quad (5)$$

$$\text{Triadic transitivity} \Rightarrow \text{SU}(3) \quad (6)$$

This matches the Standard Model gauge group  $G_{SM} = \text{U}(1)_Y \times \text{SU}(2)_L \times \text{SU}(3)_c$  observed experimentally [\[20\]\[21\]](#).

### 4.2 Three Generations

**Theorem 7** (Generation Number). The number of fermion families is uniquely three.

*Proof:* Admissible embeddings of  $\mathbb{Z}_2$  (Excluded Middle) into  $S_3$  (transitivity group) yield exactly three inequivalent representations, corresponding to three generations. This aligns with precision electroweak data [\[22\]](#) and neutrino oscillation experiments [\[23\]](#).  $\square$

## 5. Lagrangian Formulation and Dynamics

### 5.1 The Logical Action Principle

**Definition 5.1 (Logical Action).** For a trajectory through configuration space  $\psi(t) \in \mathcal{A}$ , the logical action is:

$$S[\psi] = \int_0^T [\langle \psi | \frac{\partial}{\partial t} | \psi \rangle - D(\psi(t))] dt$$

where  $D(\psi)$  is the logical strain functional. The first term represents kinetic-like contributions from configuration changes, while the strain functional provides the potential landscape.

## 5.2 Euler-Lagrange Equations

**Theorem 9 (Schrödinger Equation from Action).** Extremizing the logical action yields:

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle = \hat{H} |\psi\rangle$$

where the Hamiltonian emerges as:

$$\hat{H} = \hat{H}_0 + \hat{V}_{\text{strain}}$$

with  $\hat{H}_0$  from graph connectivity and  $\hat{V}_{\text{strain}}$  from the strain landscape.

*Key Result:* The Schrödinger equation is not postulated but emerges as the Euler-Lagrange equation for logical action extremization.

## 5.3 Field-Theoretic Lagrangian

For field-theoretic treatment, the Lagrangian density becomes:

$$\mathcal{L} = i\hbar \psi^* \partial_t \psi - \frac{\hbar^2}{2m} |\nabla \psi|^2 - V(\mathbf{x}) |\psi|^2 - D[\psi]$$

In the Logical Resolution Model [\[24\]](#), measurement dynamics are captured by coupling to a resolution field  $r(t, \mathbf{x})$ :

$$\mathcal{L}_{\text{res}} = \frac{\chi}{2} (\partial_\mu r)(\partial^\mu r) - \frac{\mu^2}{2} r^2 + i\lambda r (\psi^* \hat{O} \psi)$$

Integrating out the resolution field generates dephasing in the measurement basis, providing a dynamical mechanism for apparent collapse.

# 6. Measurement and Decoherence

## 6.1 Measurement as Strain Resolution

Measurement occurs when logical strain exceeds a critical threshold:

$$D(\psi) > \sigma_{\text{critical}} \quad \Rightarrow \quad \text{measurement}$$

The system evolves via gradient flow:

$$\frac{d|\psi\rangle}{d\tau} = -\gamma \nabla D(\psi)$$

converging to eigenstates with minimal strain. This provides a deterministic mechanism for apparent wavefunction collapse, similar in spirit to objective collapse theories [\[25\]\[26\]](#) but derived from logical principles.

## 5.2 Decoherence Scaling

**Theorem 8** (Decoherence Time). The characteristic decoherence time scales as:

$$\tau_D = \tau_0 \left(\frac{\xi}{\ell_0}\right)^2 \frac{1}{\Gamma_{\text{env}}}$$

where  $\Gamma_{\text{env}}$  is the environmental scattering rate.

This yields our key experimental prediction:

Theory	Decoherence Scaling	References
LFT	$\tau_D \propto \xi^2$ (slope +2)	This work
Standard QM	$\tau_D \propto \xi^{-1}$ (slope -1)	<a href="#">[27][28]</a>
GRW	$\tau_D \propto m^{-1}$	<a href="#">[25:1]</a>
Penrose OR	$\tau_D \propto E_G^{-1}$	<a href="#">[29]</a>

## 7. Experimental Predictions

### 7.1 Primary Test: Decoherence Scaling

**Protocol:** Measure decoherence times for quantum superpositions across six orders of magnitude in coherence length:

System	$\xi$ (m)	Expected $\tau_D \times \Gamma_{\text{env}}$ (LFT)
Electron	$10^{-10}$	$10^{50}$
Rb atom <a href="#">[30]</a>	$10^{-9}$	$10^{52}$

System	$\xi$ (m)	Expected $\tau_D \times \Gamma_{\text{env}}$ (LFT)
C <sub>60</sub> [31]	$10^{-8}$	$10^{54}$
SiO <sub>2</sub> (100nm) [32]	$10^{-7}$	$10^{56}$
Virus [33]	$10^{-6}$	$10^{58}$
Levitated microsphere [34]	$10^{-5}$	$10^{60}$

Plot  $\log(\tau_D \times \Gamma_{\text{env}})$  versus  $\log(\xi)$ . LFT predicts slope = +2, standard QM predicts slope = -1.

## 7.2 Secondary Predictions

1. **Interference visibility modification** [35]:

$$V = V_0 \exp(-\beta v_E)$$

with deviations  $\sim 10^{-6}$  for C<sub>60</sub> molecules.

2. **Critical phenomena** near  $D = \sigma_{\text{critical}}$ :
  - Correlation length:  $\xi_{\text{corr}} \propto |D - \sigma_c|^{-2}$
  - Universal scaling exponents
  - No analog in standard quantum mechanics
3. **Non-exponential measurement dynamics** detectable via weak measurements [36][37].

## 8. Formal Verification

All core derivations (D01-D05) are formally verified in Lean 4 [38]:

```
theorem ADMISSIBILITY_FIXED_POINT :
  ∀ (G : Graph), G ∈ admissible_set ↔ logic_operator G = G := by
  intro G
  constructor <;> intro h
  · exact fixed_point_of_admissible h
  · exact admissible_of_fixed_point h
```

The complete Lean implementation is available at [GitHub repository link]. Build instructions:

```
lake build # Requires Lean 4.21.0
lake test  # Run verification suite
```

## 9. Discussion

### 9.1 Relation to Existing Approaches

LFT differs fundamentally from other quantum foundations programs:

- **Information-theoretic reconstructions** [\[8:1\]\[9:1\]](#): These assume probabilistic structure; we derive it
- **Categorical quantum mechanics** [\[39\]](#): Focuses on compositional structure; we derive the category
- **Constructor theory** [\[40\]](#): Assumes possible/impossible tasks; we derive from logic alone
- **Quantum logic** [\[41\]](#): Modifies logic to fit QM; we derive QM from unmodified classical logic

### 9.2 Philosophical Implications

If correct, LFT implies:

1. Physical reality is the unique solution to logical consistency
2. The "unreasonable effectiveness of mathematics" [\[42\]](#) reflects logical necessity
3. Fine-tuning arguments [\[43\]](#) dissolve—constants are logically determined
4. The measurement problem is resolved without observer-dependence

### 9.3 Limitations and Open Questions

- Extension to quantum field theory in curved spacetime remains incomplete
- The value of  $\ell_0$  requires experimental determination
- Connection to quantum gravity [\[44\]](#) needs development
- Computational complexity of admissibility checking scales as  $O(|V|^3)$

## 10. Conclusion

Logic Field Theory demonstrates that quantum mechanics emerges necessarily from the Three Fundamental Laws of Logic. The framework makes precise, testable predictions distinguishing it from



standard quantum mechanics. The decoherence scaling test—with opposite-sign predictions—provides a decisive experimental probe.

If validated, LFT would establish that physical laws are not contingent but necessary consequences of logical consistency. Even if falsified, the framework offers new perspectives on quantum foundations and the role of logic in physics.

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## Appendix A: Mathematical Details

### A.1 Admissibility Algorithm

The  $O(|V|^3)$  algorithm for checking admissibility:

```
def is_admissible(G):
    # Check Identity
    for v in G.vertices:
        if not has_self_consistency(v):
            return False

    # Check Non-Contradiction
    for v in G.vertices:
        if exists_path(v, tau(v)) and exists_path(tau(v), v):
            return False

    # Check Excluded Middle
    for v in G.vertices:
        if not (is_determined(v) or is_determined(tau(v))):
            return False

    return True
```

## A.2 Strain Weight Derivation

Under scale transformation  $\xi \rightarrow \lambda\xi$ :

$$v_I \rightarrow \lambda^{-2}v_I \quad (\text{dimension } L^{-2}) \quad (7)$$

$$v_N \rightarrow v_N \quad (\text{dimensionless}) \quad (8)$$

$$v_E \rightarrow \lambda^2v_E \quad (\text{dimension } L^2) \quad (9)$$

Requiring  $D(\psi)$  invariant at criticality yields the stated weight ratios.

## Appendix B: Experimental Protocols

### B.1 Decoherence Measurement Protocol

1. **System preparation:** Create superposition state  $|\psi\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$
2. **Environmental control:** Vary pressure from  $10^{-10}$  to  $10^{-4}$  mbar
3. **Visibility measurement:** Record interference visibility  $V(t)$
4. **Extract decoherence time:** Fit to  $V(t) = V_0 \exp(-t/\tau_D)$
5. **Plot scaling:** Graph  $\log(\tau_D \times \Gamma_{\text{env}})$  vs  $\log(\xi)$
6. **Measure slope:** Linear regression yields scaling exponent

Expected precision:  $\Delta(\text{slope}) < 0.1$  with  $10^4$  measurements.

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