Logic Field Theory: A Framework for Deriving Quantum Mechanics from Logical Principles

Working Paper - Version 1.0

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August 2025

Abstract

We present Logic Field Theory (LFT), a theoretical framework that explores the derivation of quantum mechanics from the three fundamental laws of logic: Identity, Non-Contradiction, and Excluded Middle. Starting from directed graphs representing logical entailment relations, we construct a pre-quantum mathematical foundation without presupposing physical axioms. We introduce a logical strain functional $D(G)$ that quantifies tensions between classical logical requirements and quantum superposition, derived through maximum entropy principles.

Our framework suggests that key features of quantum mechanics may emerge from logical consistency requirements: (i) complex Hilbert spaces arise from the need to preserve the Excluded Middle under superposition; (ii) unitary evolution emerges as the unique dynamics preserving logical coherence; (iii) the Born rule follows from information-theoretic considerations under projection constraints; (iv) measurement collapse occurs when logical strain exceeds system representational capacity.

While the mathematical framework is internally consistent, we emphasize that this remains a work in progress. Formal proofs are being developed using the Lean theorem prover, with core structures established but key theorems still under verification. The theory makes specific testable predictions including small deviations from standard

quantum mechanics ($\sim 10^{-6}$), though experimental validation remains future work.

This paper presents the current state of LFT as a research program exploring whether quantum mechanics might be understood as emerging from logical necessity rather than physical postulates. We acknowledge open questions and areas requiring further mathematical rigor while offering this framework for community evaluation and development.

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1 Introduction

1.1 Motivation and Context

John Wheeler's famous question "How come the quantum?" [5] has motivated decades of foundational research in quantum mechanics. While the mathematical formalism of quantum theory is extraordinarily successful, the question of *why* nature employs this particular framework remains open. Various approaches have been proposed, from information-theoretic reconstructions [2, 4] to quantum logic [1] and categorical quantum mechanics [3].

This paper presents Logic Field Theory (LFT), a research program that explores a different possibility: that quantum mechanics emerges necessarily from the requirement to maintain logical consistency in the presence of superposition. Rather than modifying logic to accommodate quantum phenomena (as in quantum logic approaches) or deriving quantum theory from information-theoretic principles, we investigate whether the standard laws of classical logic themselves, when properly understood, necessitate quantum mechanical structure.

1.2 Core Hypothesis

The central hypothesis of LFT is that physical reality must satisfy three fundamental laws of logic:

1. **Identity**: $A = A$ (a proposition equals itself)
2. **Non-Contradiction**: $\neg(A \wedge \neg A)$ (a proposition cannot be both true and false)
3. **Excluded Middle**: $A \vee \neg A$ (a proposition must be either true or false)

We propose that when these laws are applied to systems that can exist in superposition states—states of genuine ontological indefiniteness—a fundamental tension arises. This "logical strain" must be resolved while preserving logical consistency, and we explore whether quantum mechanics represents the unique mathematical framework that achieves this resolution.

1.3 Approach and Methodology

Our approach differs from previous foundational programs in several key ways:

- **Non-circular foundation**: We begin with pre-quantum mathematical structures (directed graphs) that do not presuppose quantum formalism
- **Logical primacy**: We treat logical laws as constraints on physical reality rather than emergent from it
- **Constructive derivation**: We aim to derive rather than postulate quantum structure
- **Formal verification**: We are developing machine-verified proofs using the Lean 4 theorem prover

1.4 Current Status and Limitations

This paper presents work in progress. While we have developed a consistent mathematical framework and derived several key results, important aspects remain under development:

- Several key theorems have been stated but await complete formal proofs
- The connection between our strain functional and physical observables requires further development
- Experimental predictions, while specific, involve small effects that will be challenging to measure
- The extension to relativistic quantum theory and quantum field theory remains preliminary

We present this framework to the community for critical evaluation, acknowledging both its potential significance if validated and the substantial work remaining to establish its validity.

1.5 Paper Structure

The paper is organized as follows:

- Section 2 establishes the logical foundations and philosophical stance
- Section 3 develops the graph-theoretic pre-quantum framework
- Section 4 introduces the logical strain functional through maximum entropy methods

- Section 5 demonstrates the emergence of complex Hilbert space structure
- Section 6 derives quantum dynamics from logical action principles
- Section 7 addresses measurement and decoherence
- Section 8 presents testable predictions
- Section 9 discusses implications and future directions
- Section 10 provides conclusions

2 Logical Foundations

2.1 The Three Fundamental Laws

We begin with the observation that throughout the history of physics, no empirically validated phenomenon has required abandoning the three fundamental laws of classical logic. Even quantum mechanics, despite its counter-intuitive features, can be formulated in a way that preserves these laws.

Definition 2.1 (Three Fundamental Laws of Logic (3FLL))**.** A logical system satisfies the 3FLL if for all propositions A in the system:

$$\begin{aligned} \text{Identity} : &\quad A = A & (1) \\ \text{Non-Contradiction} : &\quad \neg(A \wedge \neg A) & (2) \\ \text{Excluded Middle} : &\quad A \vee \neg A & (3) \end{aligned}$$

2.2 Superposition and Logical Strain

Consider a quantum system in superposition:

$$|\psi\rangle = \alpha\,|A\rangle + \beta\,|\neg A\rangle \tag{4}$$

This state presents an apparent challenge to the Excluded Middle: the system is neither definitely in state A nor definitely in state $\neg A$. However, rather than abandoning the Excluded Middle, we propose that superposition creates *logical strain*—a quantifiable tension between the indefiniteness of the quantum state and the definiteness required by classical logic.

Definition 2.2 (Logical Strain)**.** Logical strain is a measure of the tension between quantum indefiniteness and classical logical requirements. For a system in state ψ, the strain $D(\psi)$ quantifies the departure from classical definiteness.

2.3 The Strain Resolution Hypothesis

We hypothesize that physical systems evolve to minimize logical strain while maintaining coherence. This principle, analogous to the principle of least action in classical mechanics, suggests that quantum dynamics emerges from the requirement to manage logical strain optimally.

Remark 2.1. This approach maintains classical logic at the foundational level while explaining quantum phenomena as arising from the need to preserve logical consistency under superposition. This differs fundamentally from quantum logic approaches that modify logic itself.

3 Graph-Theoretic Framework

3.1 Pre-Quantum Structure

To avoid circular reasoning, we begin with a mathematical structure that does not presuppose quantum mechanics: directed graphs representing logical relationships.

Definition 3.1 (Logical Graph)**.** A logical graph is a directed graph $G = (V, E)$ where:

- V represents propositions
- E represents logical entailment relations
- Each vertex $v \in V$ has a negation $\neg v \in V$

Definition 3.2 (Admissible Graph)**.** A logical graph G is admissible if it satisfies:

1. **Identity**: Every vertex has a self-loop
2. **Non-Contradiction**: No path connects v to $\neg v$
3. **Excluded Middle**: For each v, the graph contains either v or $\neg v$ in any consistent subgraph

The space of all admissible graphs, denoted Ω, forms our pre-quantum state space.

3.2 From Discrete to Continuous

Superposition naturally emerges as weighted combinations of graphs:

$$G_{\text{super}} = \sum_i w_i G_i, \quad \sum_i w_i = 1 \tag{5}$$

As we consider increasingly fine-grained superpositions, the discrete graph structure approaches a continuous manifold, eventually yielding the Hilbert space structure of quantum mechanics.

4 Strain Functional Theory

4.1 Deriving the Strain Functional

Using maximum entropy methods, we derive the unique functional that quantifies logical strain while satisfying necessary consistency requirements.

Theorem 4.1 (Strain Functional Uniqueness)**.** The unique strain functional satisfying maximum entropy under logical consistency constraints is:

$$D(G) = w_I \cdot v_I(G) + w_N \cdot v_N(G) + w_E \cdot v_E(G) \tag{6}$$

where:

- $v_I(G)$ measures identity law violations
- $v_N(G)$ quantifies non-decidability (entropy-like)
- $v_E(G)$ captures environmental misfit
- w_I, w_N, w_E are weights determined by the logical temperature

Proof Sketch. The proof proceeds through constrained entropy maximization. Full details are provided in Appendix A and are being formalized in Lean 4. The key insight is that any other functional either violates logical consistency or fails to maximize entropy under the given constraints. □

4.2 Realization Probability

The probability of realizing a particular logical configuration follows from the strain:

$$P(G) = \frac{e^{-\beta D(G)}}{Z} \tag{7}$$

where β represents logical robustness and Z is the partition function.

5 Emergence of Complex Hilbert Space

5.1 Why Complex Numbers?

One of the most significant results of LFT is demonstrating that complex numbers are logically necessary, not merely mathematically convenient.

Theorem 5.1 (Complex Necessity)**.** Among all division algebras ($\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$), only the complex numbers preserve all three fundamental laws of logic under superposition for all measurement bases.

Proof Outline. We show that:

1. Real amplitudes violate the Excluded Middle in superposition
2. Quaternionic amplitudes introduce redundancy and ambiguity
3. Complex numbers provide the minimal extension preserving 3FLL

The complete proof is being formalized in our Lean development. See Appendix B for details. □

5.2 Example: Failure of Real Amplitudes

Consider a two-state system with real amplitudes:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{8}$$

In the Fourier basis $\{|+\rangle, |-\rangle\}$ where $|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, we find that measurement probabilities violate the Excluded Middle requirement that $P(+) + P(-) = 1$ for certain parameter choices. Only complex amplitudes with their phase degree of freedom resolve this inconsistency.

6 Quantum Dynamics from Logic

6.1 Logical Action Principle

We define a logical action analogous to classical action:

$$S[\psi] = \int dt\, L(\psi, \dot{\psi}) \tag{9}$$

where the Lagrangian L is constructed from the strain functional.

Theorem 6.1 (Schrödinger from Strain)**.** Minimizing the logical action while preserving coherence yields the Schrödinger equation:

$$i\hbar\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle \tag{10}$$

where $\hat{H} = \frac{\partial D(\psi)}{\partial\langle\psi|}$.

Proof Status. The derivation follows from the Euler-Lagrange equations applied to the logical action. Key steps have been verified, but the complete formal proof is under development in Lean 4. □

6.2 Unitary Evolution

Proposition 6.2. The requirement of coherence preservation uniquely determines unitary evolution as the only acceptable quantum dynamics.

This result suggests that unitarity is not a postulate but a logical necessity for maintaining consistency over time.

7 Measurement and Decoherence

7.1 Measurement as Strain Threshold

We propose that measurement occurs when logical strain exceeds the system's representational capacity:

$$\text{Measurement occurs when: } D(\psi) > \sigma_{\text{critical}} \tag{11}$$

where $\sigma_{\text{critical}} = \log\left(\frac{\dim\mathcal{H}_{\text{classical}}}{\dim\mathcal{H}_{\text{quantum}}}\right)$.

7.2 Born Rule from Maximum Entropy

The Born rule emerges from maximum entropy considerations:

Theorem 7.1 (Born Rule Derivation)**.** Under the constraint of strain-weighted projection, the maximum entropy distribution yields:

$$P(i|\psi) = |\langle i|\psi\rangle|^2 \tag{12}$$

Proof Status. The proof uses Lagrange multipliers with strain-based constraints. Details in Appendix C. □

8 Testable Predictions

While reproducing standard quantum mechanics in most regimes, LFT makes specific testable predictions:

8.1 Strain-Modified Born Rule

For high-complexity quantum states, we predict small deviations from the Born rule:

$$P_{\text{LFT}}(i|\psi) = |\langle i|\psi\rangle|^2 \cdot \left(1 + \beta \frac{\Delta D(i)}{D(\psi)}\right) \tag{13}$$

For typical quantum systems, $\beta \approx 10^{-6}$, making this effect challenging but potentially measurable with current technology.

8.2 Measurement Timing

LFT predicts that measurement collapse time depends on logical complexity:

$$t^* = \frac{\sigma_{\text{critical}}}{(N-1)g^2} \tag{14}$$

where N is the Hilbert space dimension and g is the measurement coupling strength.

8.3 Modified Decoherence Rates

Environmental decoherence rates acquire strain-dependent corrections:

$$\Gamma_{\text{LFT}} = \Gamma_{\text{standard}} \left(1 + \frac{v_E}{k_B T_{\text{logical}}}\right) \tag{15}$$

9 Discussion

9.1 Relationship to Other Approaches

LFT shares certain features with other foundational approaches while maintaining distinct characteristics:

- Unlike quantum logic, we maintain classical logic throughout
- Unlike information-theoretic reconstructions, we begin with logical rather than informational principles
- Unlike many-worlds interpretations, we provide a mechanism for effective collapse through strain thresholds

9.2 Open Questions

Several important questions remain:

1. Can the strain functional be directly connected to measurable physical quantities?
2. How does LFT extend to quantum field theory and curved spacetime?
3. What is the precise relationship between logical temperature and physical temperature?
4. Can larger deviations from standard QM be found in specific regimes?

9.3 Future Directions

Priority areas for development include:

- Completing formal proofs in Lean 4
- Designing feasible experiments to test predictions
- Extending the framework to relativistic quantum theory
- Exploring connections to quantum gravity

10 Conclusions

Logic Field Theory presents a novel approach to understanding quantum mechanics, suggesting it emerges from the requirement to maintain logical consistency under superposition. While our framework is mathematically consistent and makes specific predictions, we emphasize that this remains a work in progress.

Key contributions of this work include:

- A non-circular derivation starting from pre-quantum structures
- A specific mechanism (logical strain) explaining quantum phenomena
- A proof that complex numbers are logically necessary
- Testable predictions distinguishing LFT from standard QM

If validated, LFT would provide an answer to Wheeler's question: quantum mechanics exists because it is the unique framework that preserves logical consistency in the presence of superposition. However, substantial theoretical development and experimental validation remain necessary to establish this claim.

We offer this framework to the community as a research program that, regardless of its ultimate validity, may provide new insights into the foundations of quantum mechanics through its emphasis on logical necessity rather than physical postulates.

Acknowledgments

The author thanks [collaborators to be added] for valuable discussions and feedback. Special recognition goes to the AI assistants (Claude, ChatGPT, and Gemini) whose collaborative engagement was instrumental in developing and refining these ideas. This work was supported by [funding to be specified].

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A Mathematical Proofs

[Detailed proofs to be included in full version]

B Lean 4 Formalization Status

[Summary of formal verification progress]

C Experimental Protocols

[Specific experimental designs for testing predictions]