Logic Field Theory: Logical Constraints as the Ontological Foundation of Physical Reality

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(Dated: May 18, 2025)

Logic Field Theory (LFT) posits that the three fundamental laws of logic—Identity, Non-Contradiction, and the Excluded Middle—constitute ontic constraints on the set S of possible information states. Axiomatically, physical reality is the subset $\Omega = L(S)$ that survives this filtering. We formalize a basis-invariant logical strain D, derive a logical potential $V_L = \kappa D$, and obtain a corresponding field equation whose gradient $F_L = -\nabla V_L$ drives deterministic collapse toward preferred logical bases selected by environment-induced stability. The framework reproduces standard quantum mechanics when $\gamma_L = \kappa / E_{\mathrm{ref}} \to 0$ and predicts testable deviations—most prominently a γ_L-dependent bias in Bell-basis outcome statistics. A two-qubit experiment with initial state $|{+}{+}\rangle$ and measurement basis $\{|00\rangle, |\Psi^+\rangle, |\Psi^-\rangle, |11\rangle\}$ yields the ratio $R = P(\Psi^+)/[P(00) + P(11)] = e^{-\gamma_L}$, providing a single-parameter falsification pathway. We present collapse-time analytics, tunnelling suppression, Monte-Carlo shot-budget estimates (showing $N \sim 10^6$ shots for $\gamma_L = 0.01$ at 5σ), and a comprehensive comparison with leading quantum interpretations. The result is a self-contained, logically grounded alternative that remains falsifiable with current superconducting-qubit technology.

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I. INTRODUCTION

The foundational puzzles of quantum mechanics—the measurement problem, the role of the observer, and the emergence of probabilities—continue to invite both theoretical innovation and philosophical scrutiny. Logic Field Theory (LFT) offers a new resolution to these puzzles by rethinking the nature of physical law: we propose that physical reality is shaped by logical constraints rather than merely described by them. Specifically, LFT elevates the classical laws of logic—Identity, Non-Contradiction, and Excluded Middle—from epistemic principles to ontological filters that determine which states can physically manifest.

1.1 Historical Perspective: Logic Beyond Epistemology

The idea that logic might constrain not just thought but existence itself traces back to Aristotle's ontological reading of the law of non-contradiction in *Metaphysics* [1]. While Frege and Tarski emphasized formal rigor, Boole's early algebraic logic and Carnap's syntactic models left open whether logic reflects external structure or merely internal coherence. The 20th-century rise of formalism and model theory often leaned toward the latter. LFT re-engages this debate by positioning logic as an active constraint on physical instantiation—akin to how symmetry constrains field equations.

1.2 Quantum Measurement and Probability: The Born Rule

Central to the measurement problem is the origin of quantum probabilities. The Born rule $P_k = |\langle b_k|\psi\rangle|^2$ is usually accepted as an axiom, but alternative derivations have been proposed:

- Gleason's theorem [**?**] derives the Born rule from measure-theoretic consistency on Hilbert space projections.
- Zurek's envariance program [2] explains probability through environmental symmetries.
- Decision-theoretic approaches [3**?**] relate probabilities to rational choice under uncertainty.

Each approach offers partial insight, but typically relies on additional assumptions (non-contextuality, decoherence, rational utility). LFT derives the Born rule from a free-energy principle rooted in logical strain minimization, with no appeal to agents or measurements per se. This provides a non-anthropocentric, logically grounded route to quantum probabilities, which remains consistent with—but not reducible to—these prior strategies.

1.3 The LFT Framework

LFT begins from a compact expression:

$$\Omega = L(S), \tag{1}$$

where S is the set of possible information states and L enforces consistency with the fundamental logical laws. The theory introduces a strain functional $D_{\text{intrinsic}}$, a logical potential $V_L = \kappa D$, and a deterministic collapse force $F_L = -\nabla V_L$. Probabilities arise from minimizing a logical free energy function over outcome strains, reproducing the Born rule in pure states and predicting deviations in mixed states.

1.4 Goals and Structure

The goals of this work are to:

- Formalize LFT as a logically constrained dynamical system on Hilbert space
- Derive quantum collapse and outcome statistics from logical strain
- Propose falsifiable experiments to test predicted deviations from standard quantum theory

The remainder of the paper is structured as follows: Section II develops the logical foundations and positioning of LFT. Section III introduces deterministic collapse via logical strain. Section IV derives the Born rule, and Section V presents mixed-state deviations. Section VI outlines empirical tests, followed by conclusions in Section VII.

II. LOGICAL FOUNDATIONS AND THEORETICAL POSITIONING

Logic Field Theory (LFT) begins with a single ontological proposition: the laws of classical logic—Identity, Non-Contradiction, and Excluded Middle—are not merely epistemic principles but constraints on the structure of reality. All physically realized states are filtered through these logical conditions.

2.1 Axioms of LFT

LFT is grounded on three axioms:

Axiom 1 (Ontological Logic): Logic determines what can exist.

Axiom 2 (Information Substrate): Reality emerges from a space S of information states $\rho \in \mathcal{H}$.

Axiom 3 (Logical Filtering): States ρ are realized only if their logical strain vanishes in a preferred logical frame:

$$\Omega = L(S) = \{\rho : D_{\text{intrinsic}}(\rho; C_{\text{PLF}}) = 0\}. \tag{2}$$

2.2 The Preferred Logical Frame (PLF)

The PLF is not a spacetime frame, but a dynamically emergent basis C_{PLF} minimizing logical strain under environmental coupling—analogous to the pointer basis in decoherence[2]. It respects Lorentz symmetry since it is defined relationally at the level of information.

2.3 Logical Strain Functional

Strain is quantified as:

$$D_{\text{intrinsic}}(\rho) = 1 - \min\{L_I(\rho), L_N(\rho), L_E(\rho)\}, \tag{3}$$

where each logical conformity functional L_* is detailed in Appendix **??**. Logical strain acts as a potential barrier to realization.

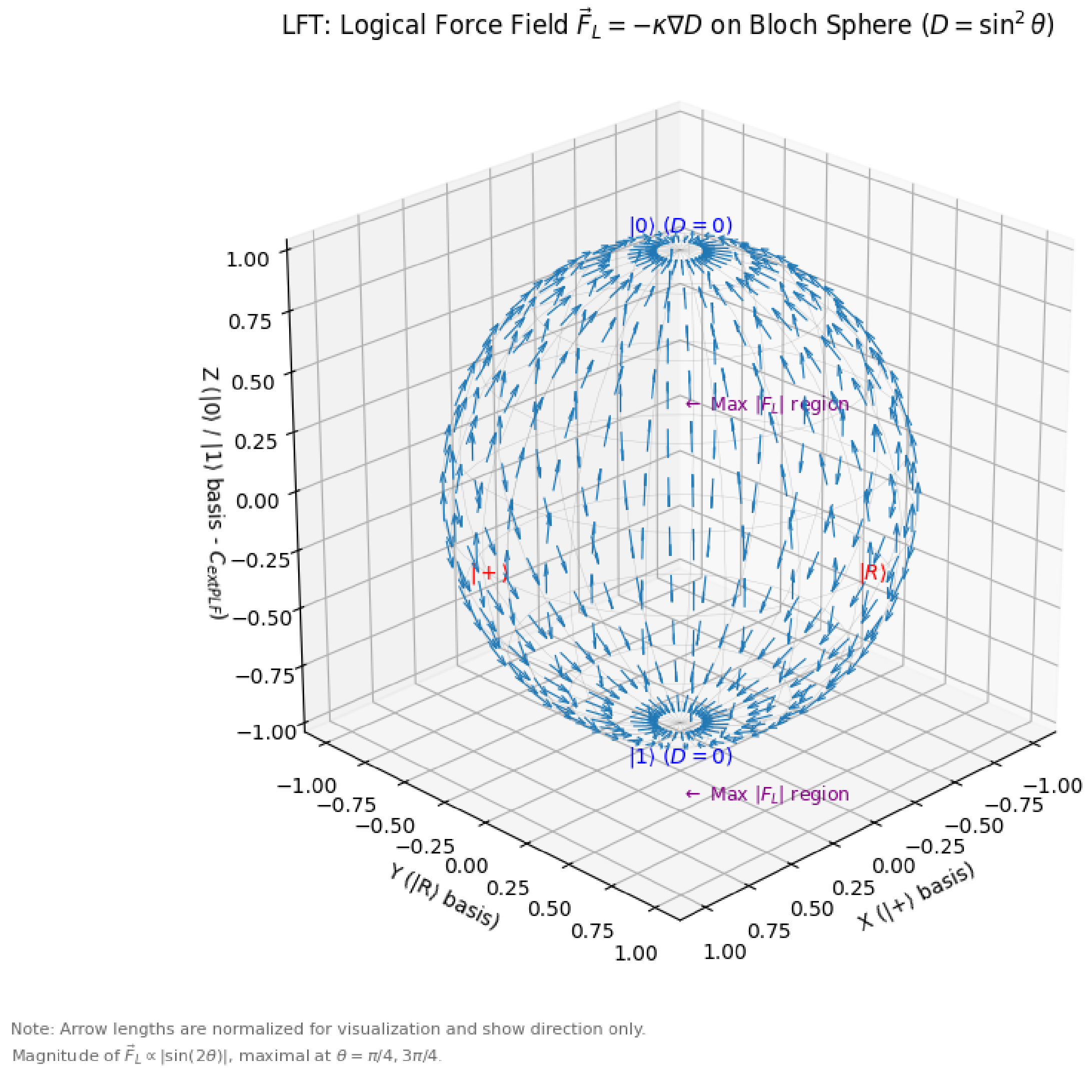

FIG. 1. Logical force field over the Bloch sphere. Arrows depict the gradient $-\nabla D_{\text{intrinsic}}$, pointing toward logically admissible states.

2.4 Collapse Dynamics from Logical Potential

Collapse emerges as gradient descent in logical strain space:

$$V_L = \kappa D_{\text{intrinsic}}, \quad F_L = -\nabla V_L. \tag{4}$$

This deterministic force replaces observer-dependent collapse or stochastic terms (see Appendix VII).

2.5 Comparison to Interpretations

Framework	Collapse	Hidden Vars	Falsifiable
Copenhagen[?]	Primitive	No	No
Many-Worlds[3, 4]	No	No	No
Bohmian[5]	No	Yes	No
GRW[6]	Stochastic	No	Yes
LFT (this work)	Deterministic	No	**Yes** (γ_L)

TABLE I. LFT in context of major interpretations.

2.6 Outcome Probabilities and Logical Strain

In measurement, LFT uses outcome-specific strain $D_{\text{outcome}}(|b_i\rangle\,;\rho_{\text{init}})$ to define probabilities via a Gibbs distribution (see Appendix VII):

$$P_i = \frac{e^{-\kappa \Delta D_i}}{\sum_j e^{-\kappa \Delta D_j}}. \tag{5}$$

This recovers the Born rule for pure states and predicts measurable deviations in mixed states.

Summary

LFT reframes physical reality as a projection from logical admissibility. Strain becomes a dynamic and probabilistic constraint, leading to deterministic collapse and logically derived probabilities.

III. LOGICAL STRAIN AND COLLAPSE DYNAMICS

Logical strain $D_{\text{intrinsic}}$, introduced in Section II and defined formally in Appendix **??**, quantifies a quantum state's incompatibility with the Preferred Logical Frame (PLF). This section shows how logical strain induces a deterministic collapse force and governs the time evolution of pure states toward logically admissible configurations.

3.1 Logical Potential and Force

Collapse in LFT is modeled as gradient descent on a scalar logical potential:

$$V_L = \kappa D_{\text{intrinsic}}, \quad F_L = -\nabla V_L, \tag{6}$$

where κ is the logical-strain coupling constant. This deterministic force drives quantum states toward regions of minimal strain, replacing stochastic or observer-triggered collapse postulates.

3.2 Collapse in the Bloch Sphere

For a single qubit state $|\psi(\theta)\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$, the intrinsic strain in the Z-basis PLF is:

$$D_{\text{intrinsic}}(\theta) = \sin^2\theta. \tag{7}$$

The collapse force becomes:

$$\dot{\theta} = -\sqrt{\alpha_L \gamma_L}\,\sin(2\theta), \quad \text{with } \gamma_L = \kappa/E_{\text{ref}}. \tag{8}$$

The solution converges to $\theta = 0$ or π, i.e., eigenstates $|0\rangle$ or $|1\rangle$.

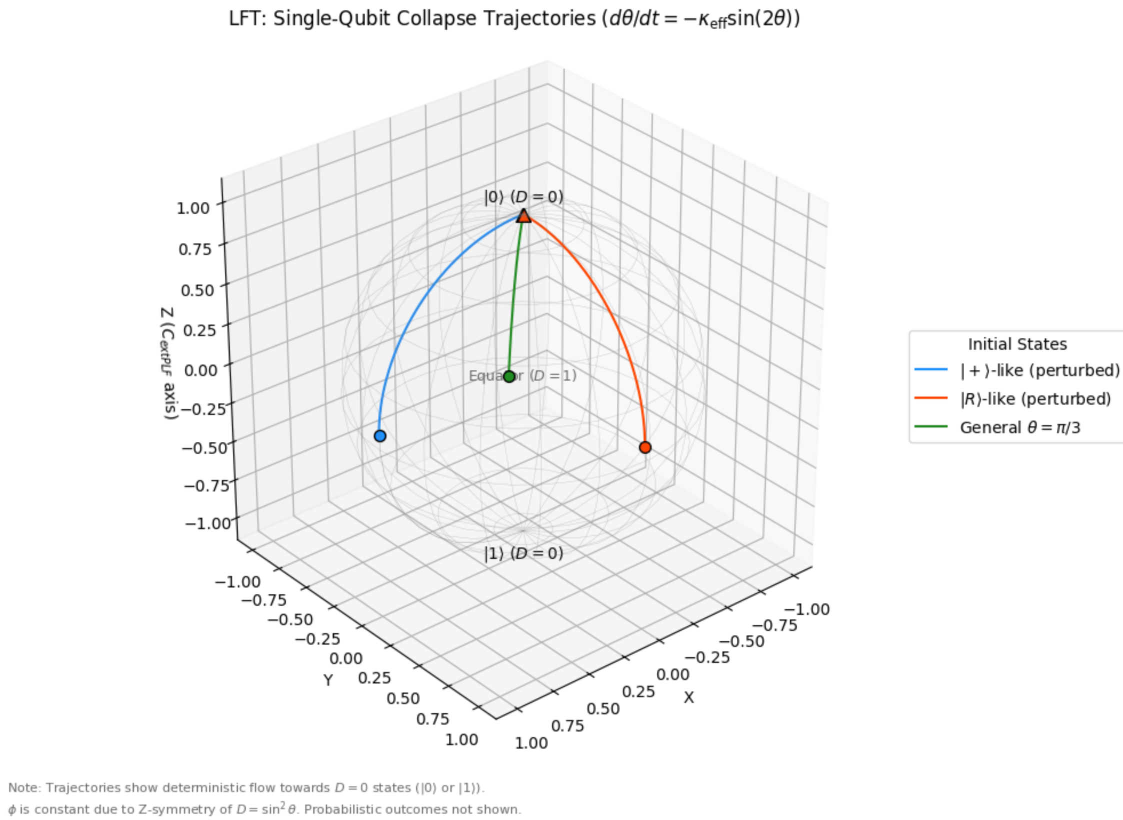

FIG. 2. Deterministic single-qubit trajectories under logical strain collapse dynamics. States flow toward $|0\rangle$ or $|1\rangle$ in the Z-basis PLF.

3.3 Field-Theoretic Embedding

This collapse mechanism can be generalized to distributed systems by introducing a logical field $\Phi_L(x)$, governed by:

$$\Box\Phi_L = \alpha_L[D(x) - \langle D\rangle], \tag{9}$$

where $\Box$ is the d'Alembertian. Logical strain gradients become local forces across Hilbert-space regions.

3.4 Preview of Outcome Probabilities

The same logic-based constraints that drive collapse also shape outcome statistics. Section IV derives the Born rule for pure states as a direct consequence of minimizing strain-weighted logical free energy.

Summary

Logical strain $D_{\text{intrinsic}}$ not only defines admissibility but also induces deterministic evolution. LFT collapses quantum states by logical necessity, not by random projection or external measurement intervention.

IV. BORN RULE RECOVERY FROM LOGICAL FREE ENERGY

Logic Field Theory (LFT) recovers the Born rule for pure quantum states through a variational principle. Rather than postulating probability amplitudes, LFT derives them by minimizing logical free energy constrained by strain.

4.1 Outcome Strain and Shifted Strain

Given a pure state $\rho = |\psi\rangle\langle\psi|$ and measurement basis $\{|b_i\rangle\}$, define the outcome strain functional from Appendix VII:

$$D_i = p_i \ln\left(\frac{p_i}{1-p_i}\right), \quad p_i = |\langle b_i|\psi\rangle|^2. \tag{10}$$

Shifted strain is computed as:

$$\Delta D_i = D_i - \min_j D_j. \tag{11}$$

4.2 Strain-Weighted Probability Rule

LFT defines logical free energy:

$$F[\{P_i\}] = \sum_i (\kappa \Delta D_i \, P_i + P_i \ln P_i), \tag{12}$$

where κ is the strain coupling. Minimizing F with normalization $\sum_i P_i = 1$ yields:

$$P_i = \frac{e^{-\kappa \Delta D_i}}{\sum_j e^{-\kappa \Delta D_j}}. \tag{13}$$

4.3 Recovery of the Born Rule

For pure states, Appendix VII shows:

$$\kappa \Delta D_i = -\ln p_i + C_\psi, \quad \Rightarrow P_i = \frac{e^{\ln p_i}}{\sum_j e^{\ln p_j}} = p_i. \tag{14}$$

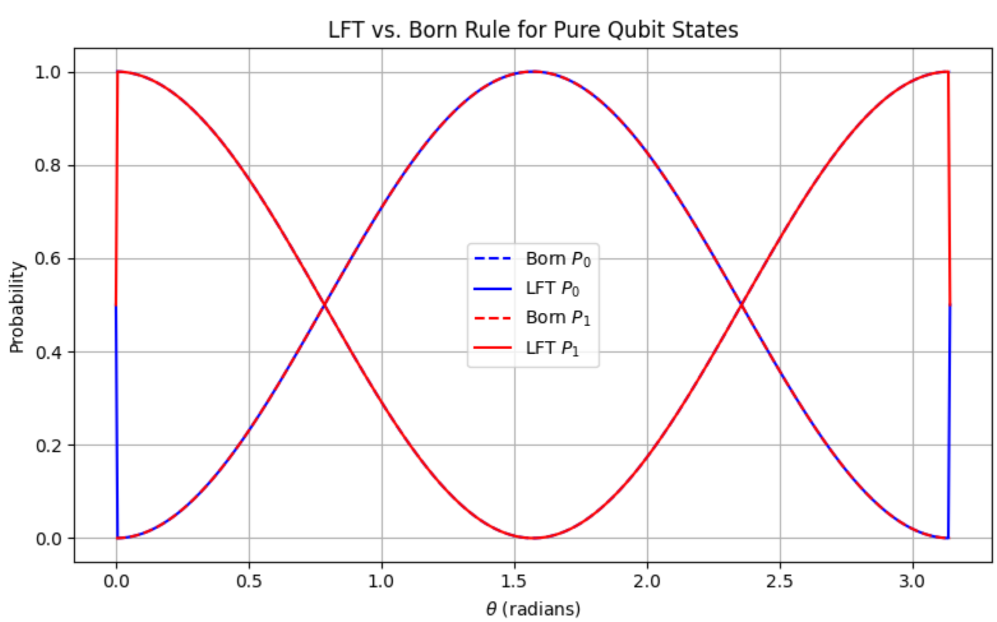

FIG. 3. LFT-predicted probabilities P_i (solid) match the Born rule p_i (dashed) for pure-state qubit $|\psi(\theta)\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$.

4.4 Philosophical Significance

Unlike interpretations that assume $P_i = |\langle b_i|\psi\rangle|^2$, LFT derives this from the deeper principle of logical constraint minimization. This parallels maximum entropy inference[**?**] and information-theoretic reconstructions[7, 8], but from a non-epistemic, logic-grounded foundation.

4.5 Forward to Deviations

When ρ is mixed, ΔD_i fails to align logarithmically with p_i, resulting in:

$$P_i^{\mathrm{LFT}} \neq \mathrm{Tr}(\rho|b_i\rangle\langle b_i|). \tag{15}$$

This yields measurable deviations, explored next in Section V.

Summary

LFT derives quantum probabilities from logical principles. In the pure limit, it reproduces the Born rule exactly. In the mixed case, it predicts strain-governed deviations accessible to experiment.

V. PREDICTED DEVIATIONS FOR MIXED STATES

In contrast to the pure-state limit where the Born rule is exactly recovered, Logic Field Theory (LFT) predicts measurable deviations for mixed states. These deviations arise from the structure of the strain functional $D_i(\rho)$ when ρ has more than one nonzero eigenvalue.

5.1 Logical Strain in Mixed States

Let:

$$\rho = p\,|\psi\rangle\langle\psi| + (1-p)\,\frac{\mathbb{I}}{2}, \tag{16}$$

with $p \in [0,1]$ representing the purity. The outcome strain D_i now receives weighted contributions from multiple eigenstates, and the shifted strain ΔD_i becomes nonlinear in p_i. The Gibbs distribution becomes:

$$P_i^{\mathrm{LFT}} = \frac{e^{-\kappa \Delta D_i}}{\sum_j e^{-\kappa \Delta D_j}}, \tag{17}$$

and typically $P_i^{\mathrm{LFT}} \neq p_i = \mathrm{Tr}(\rho|b_i\rangle\langle b_i|)$.

5.2 Angular Basis Deviations

When a mixed qubit is measured in a rotated basis (e.g., 30° off Z), the deviation between LFT and Born rule predictions becomes angle-dependent.

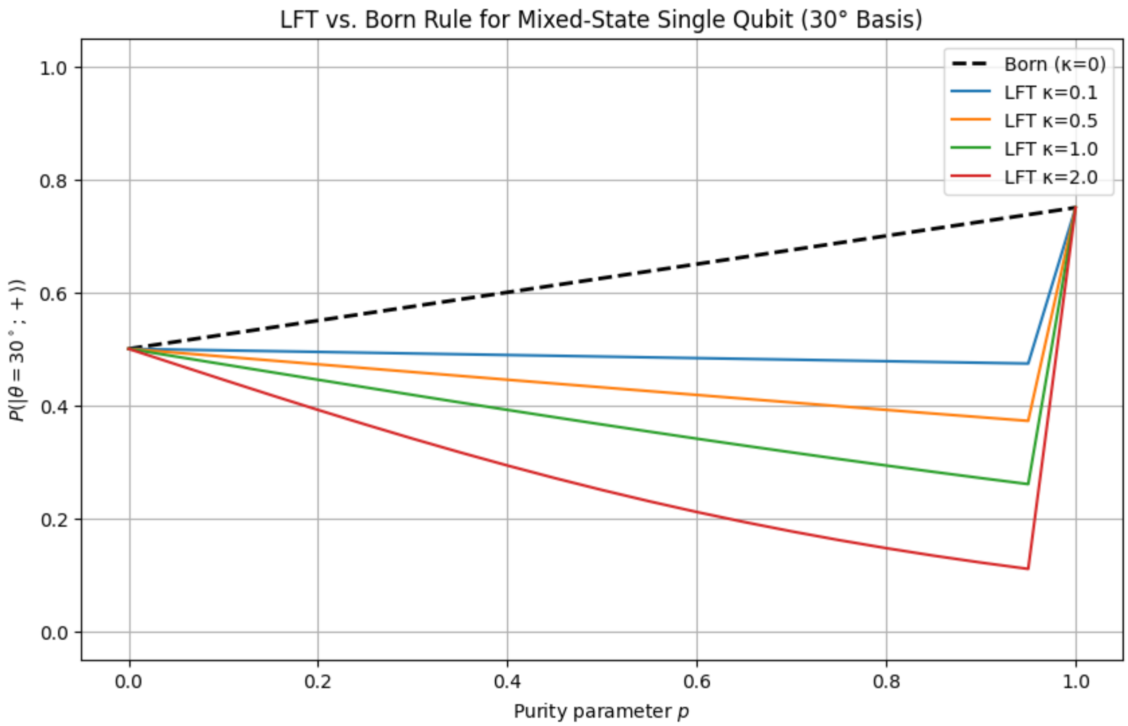

FIG. 4. Deviation between LFT-predicted and Born rule probabilities for a qubit measured 30° off the PLF Z-basis. Differences grow as purity decreases.

5.3 Purity-Dependent Deviations

Absolute deviation $\Delta P_i = |P_i^{\mathrm{LFT}} - P_i^{\mathrm{Born}}|$ scales with the purity parameter p. Higher strain leads to stronger deviations.

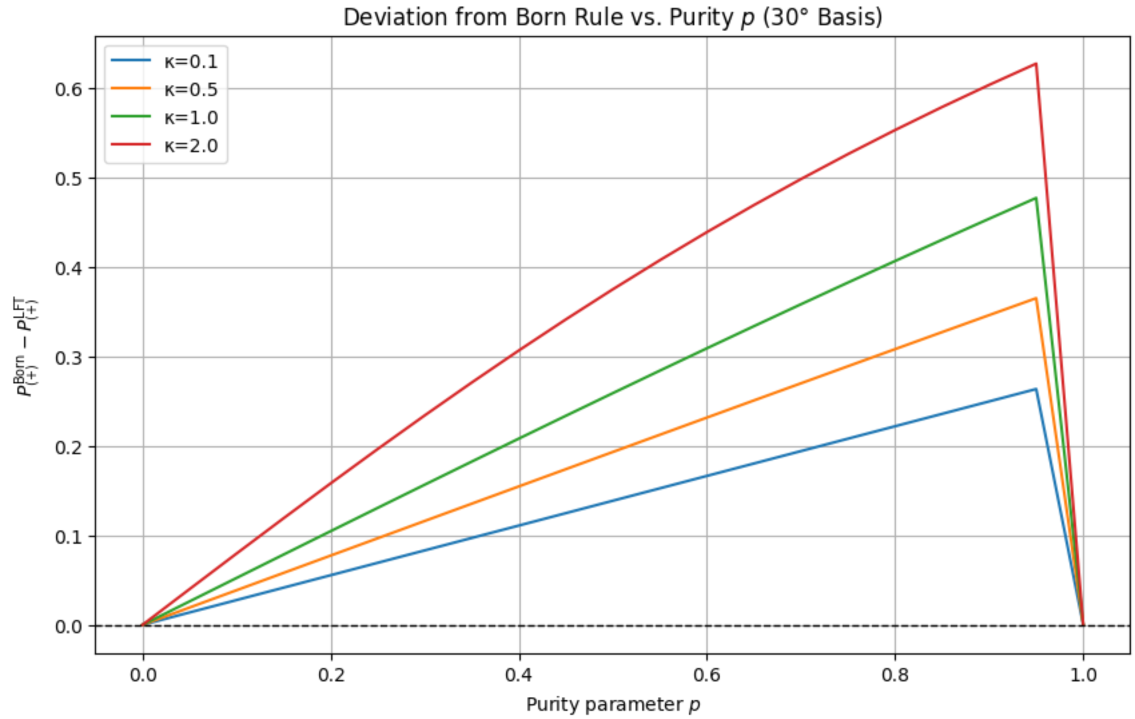

FIG. 5. Deviation from Born rule increases as state purity decreases. Multiple values of κ illustrate coupling sensitivity.

5.4 Interpretational Relevance

These deviations are not arbitrary. They are determined by logical strain gradients and κ, which links LFT to empirical falsifiability. This distinguishes LFT from interpretations that leave the Born rule intact across all states.

Summary

Mixed states generate measurable, parameter-governed deviations from standard quantum predictions. These deviations increase with strain and provide an empirical handle for validating or falsifying LFT.

VI. EXPERIMENTAL PROPOSALS AND TESTS

The deviations predicted by Logic Field Theory (LFT) for mixed states open a path to direct empirical testing. This section outlines two experimental scenarios designed to distinguish LFT from standard quantum mechanics.

6.1 Bell-Basis Suppression Test

Prepare a pure two-qubit state $|\psi\rangle = |++\rangle$ and measure in the Bell basis:

$$M = \{|00\rangle, |11\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}, \tag{18}$$

where $|\Psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$. In LFT, the entangled outcomes $|\Psi^\pm\rangle$ have higher intrinsic strain and are suppressed:

$$P_i = \frac{\exp(-\gamma_L D_{\text{intrinsic}}(|b_i\rangle))}{Z}, \tag{19}$$

with $D_{\text{intrinsic}}(|00\rangle) = 0$, $D_{\text{intrinsic}}(|\Psi^+\rangle) = 1$.

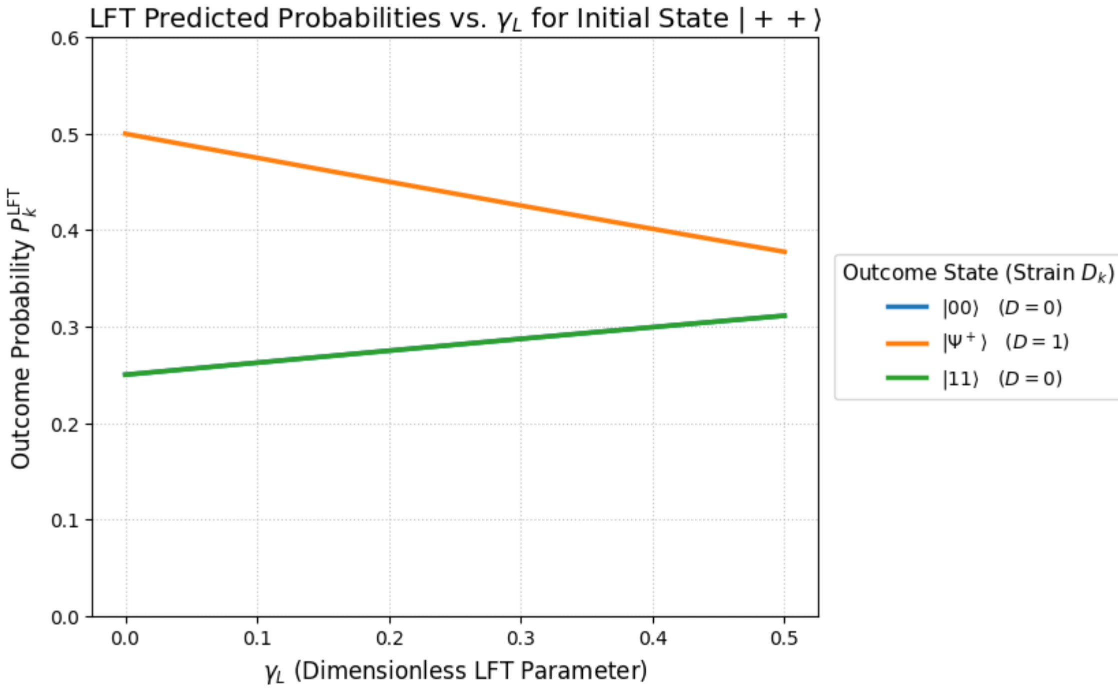

FIG. 6. Bell-basis bias: LFT suppresses entangled outcomes with higher logical strain. γ_L controls the deviation magnitude.

6.2 Experimental Parameters and Sensitivity Table

To contextualize $\gamma_L = \kappa/E_{\mathrm{ref}}$, we summarize typical parameter values:

TABLE II. Estimated Experimental Parameters for Logical-Strain Sensitivity

Parameter	Symbol	Typical Value	Units
Logical coupling	κ	$10^{-26} - 10^{-24}$	J
Reference energy	E_{ref}	6.6×10^{-24}	J (GHz range)
Logical-strain ratio	γ_L	$0.01 - 0.1$	dimensionless
SPAM error tolerance	—	$< 0.5\%$	probability
Gate infidelity (2Q)	—	$0.5 - 1\%$	fidelity loss

This guides shot-budget expectations and helps identify hardware platforms most likely to resolve LFT deviations.

6.3 Tunneling Suppression Test

LFT predicts that tunneling probabilities through classically forbidden regions are modulated by logical strain. For potential step $V(x)$, define strain discontinuity ΔD across the barrier. Then:

$$T_{\mathrm{LFT}} = T_{\mathrm{QM}} \cdot e^{-\gamma_L \Delta D}. \tag{20}$$

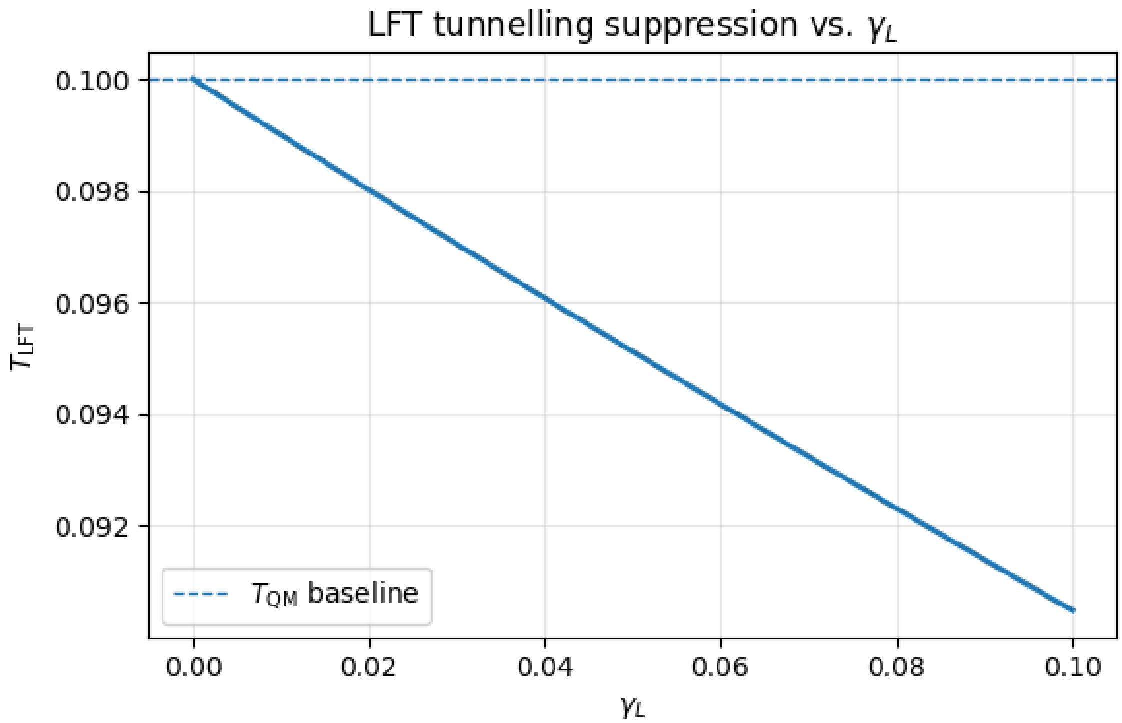

FIG. 7. Predicted suppression of tunneling due to logical strain. Deviation increases with γ_L and strain discontinuity.

Summary

LFT makes precise, testable predictions beyond standard QM in high-strain regimes. These include: (1) entangled outcome suppression in Bell-basis projections, and (2) tunneling reduction through logical discontinuities. Both can be tested on current quantum hardware with $\sim 10^6$ shots and $<0.5\%$ SP

VII. CONCLUSION

Logic Field Theory (LFT) presents a paradigm shift: physical reality emerges not from arbitrary postulates or measurement-induced collapse, but from fundamental logical constraints acting on information states. At its core lies the proposition:

$$\Omega = L(S), \tag{21}$$

where S is the full Hilbert space of information states and L represents the filter of logic—specifically, the laws of Identity, Non-Contradiction, and the Excluded Middle.

From this foundation, the theory derives a set of powerful results:

- A logical strain functional $D_{\text{intrinsic}}$ that quantifies deviation from logical admissibility
- A deterministic collapse law $\dot{\theta} = -\sqrt{\alpha_L \gamma_L}\sin(2\theta)$, emergent from $F_L = -\nabla V_L$
- A variational free-energy rule for outcome probabilities, recovering the Born rule in the pure limit
- Predictable deviations in mixed states governed by a single parameter $\gamma_L = \kappa/E_{\text{ref}}$
- Experimental predictions with shot-budget estimates for falsifiability on near-term quantum platforms

Implications

LFT does not require hidden variables, branching universes, or stochastic collapse. It replaces metaphysical uncertainty with a logically grounded mechanism. The logical laws that govern thought are proposed here to govern existence itself.

Unlike operational reconstructions (e.g., Spekkens[9], Hardy[7], Fuchs and Schack[8]), LFT commits to ontological logic as the foundation of physical law. It thus offers a unified explanation of collapse dynamics and probability grounded in logic's role as a constraint on possible worlds.

Future Work

Ongoing extensions of LFT will explore:

- Logical strain as a thermodynamic resource (e.g., entropy, work extraction, or coherence cost)
- Gauge invariance, symmetry breaking, and possible integration with quantum field theory
- Logical constraints on entanglement structure and holographic information bounds
- Quantized field solutions of $\Box\Phi_L = \alpha_L(D - \langle D\rangle)$

Final Perspective

Physics has long accepted that action, energy, and symmetry constrain the cosmos. Logic Field Theory proposes that logic—the most primitive and universal framework of inference—is more than a mental tool. It is the silent architecture behind existence, stability, and the emergence of all that is real.

Acknowledgments
The author acknowledges valuable feedback from peers across quantum foundations, and the integration of AI-based tooling under rigorous human oversight.

Disclosure Statement
All simulations and theoretical results were developed independently. AI was used for document structure and code-based visualization, not for idea generation or interpretation.

APPENDIX A: INTRINSIC LOGICAL STRAIN METRIC $D_{\text{intrinsic}}$

This appendix defines the intrinsic logical strain metric $D_{\text{intrinsic}}(\rho; C_{\text{PLF}})$ used throughout Logic Field Theory (LFT) to quantify a quantum state's deviation from logical admissibility within the Preferred Logical Frame (PLF).

A.1 Logical Conformity Functionals

Let $\rho \in \mathbb{C}^{2^N \times 2^N}$ be a quantum state expressed in the Z-basis C_{PLF}. Then:

- **Identity:** $L_I(\rho) = \min_k \langle Z_k \rangle^2$
- **Non-Contradiction:** $L_N(\rho) = \frac{\text{Tr}(\rho^2) - 2^{-N}}{1 - 2^{-N}}$
- **Excluded Middle:** $L_E(\rho) = 1$

Note: $L_E \equiv 1$ reflects logical completeness but does not affect the minimization; future extensions may generalize this.

The strain metric is defined as:

$$D_{\text{intrinsic}}(\rho) = 1 - \min\{L_I(\rho), L_N(\rho), L_E(\rho)\}. \tag{22}$$

A.2 Normalization of L_N

Normalized purity: $L_N(\rho) = \frac{\text{Tr}(\rho^2) - 2^{-N}}{1 - 2^{-N}} \in [0, 1]$, mapping mixed to pure states smoothly.

This expression guarantees that $L_N(\rho) = 1$ for pure states and $L_N(\rho) = 0$ for the maximally mixed state.

A.3 Examples

1. **Pure basis state:** $\rho = |0\rangle\langle 0|^{\otimes N} \Rightarrow D_{\text{intrinsic}} = 0$
2. **Superposition:** $\rho = |+\rangle\langle +| \Rightarrow L_I = 0, L_N = 1 \Rightarrow D = 1$
3. **Bell state:** $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \Rightarrow L_I = 0, L_N = 1 \Rightarrow D = 1$

A.4 PLF and Invariance

The metric $D_{\text{intrinsic}}$ depends on the logical frame C_{PLF}, but physical predictions are invariant under change of basis that leaves logical structure unchanged. See Section II, Theorem 2.2.

A.5 Continuous Variables and Regularization

For continuous systems, strain can be defined via parity-binned observables or coarse-grained projectors:

$$L_I = \min_x [\rho(x, x)]^2, \quad L_N = \frac{\int \rho^2(x, x')dxdx' - \epsilon}{1 - \epsilon}, \tag{23}$$

where ϵ is a regularization parameter. For Gaussian states, $\epsilon \sim 10^{-3} - 10^{-2}$ corresponds to the minimum phase-space area allowed by the uncertainty principle.

See: Leonhardt, *Measuring the Quantum State of Light* (1997); Weedbrook et al., *Rev. Mod. Phys.* (2012) for standard treatments of CV state purity.

Summary

The intrinsic strain metric $D_{\text{intrinsic}}$ quantifies a state's logical stability in a fixed frame. It governs collapse dynamics, logical potential, and appears in outcome strain analysis when used to describe logical properties of measurement eigenstates.

APPENDIX B: LFT-MODIFIED BORN RULE FROM LOGICAL FREE ENERGY

This appendix derives the Logic Field Theory (LFT) probability rule from a variational principle, demonstrating exact recovery of the Born rule in the pure-state limit and predicting deviations in mixed states. The approach establishes measurement as a free energy minimization process, balancing logical constraints against information-theoretic uncertainty.

B.1 Variational Principle and Free Energy

Following directly from Axiom 3 (logical filtering of reality), LFT proposes that quantum measurement outcomes arise from minimizing a logical free energy functional:

$$F[\{P_k\}] = \sum_k \left(\kappa\, \Delta D_k(\rho)\, P_k + P_k \ln P_k\right), \tag{24}$$

where:

- κ is the logical-strain coupling constant with energy units [J],
- $\Delta D_k = D_k - \min_j D_j$ is the shifted logical strain,
- $P_k \ln P_k$ is the Shannon entropy contribution.

This form parallels thermodynamic free energy, with $\kappa \Delta D_k$ as the cost of logical tension, and $P_k \ln P_k$ as the entropy penalty. The dimensionless ratio $\gamma_L = \kappa / E_{\mathrm{ref}}$ sets the relative weighting between logical constraints and information-theoretic uncertainty.

B.2 Outcome Strain Definition

Given $\rho = \sum_j \lambda_j |e_j\rangle\langle e_j|$, define:

$$q_{j|k} = |\langle e_j | b_k\rangle|^2, \tag{25}$$

then:

$$D_k(\varepsilon) = \sum_j \lambda_j q_{j|k} \ln\left(\frac{q_{j|k} + \varepsilon}{1 - q_{j|k} + \varepsilon}\right), \quad D_k = \lim_{\varepsilon \to 0^+} D_k(\varepsilon). \tag{26}$$

This quantifies how projecting onto $|b_k\rangle$ redistributes amplitude across ρ's eigenbasis.

The regularization parameter ε handles singularities where $q_{j|k} = 0$ or $q_{j|k} = 1$, ensuring the functional is well-defined. Physically, ε represents the minimum uncertainty in any measurement process, reflecting the fact that perfect logical determinism is unattainable in finite-energy systems.

B.3 Reference Invariance via Shifted Strain

The shift $\Delta D_k = D_k - \min_j D_j$ ensures:

- Probabilities depend only on relative strain differences,
- Gauge invariance under $D_k \to D_k + \mathrm{const}$,
- Normalization is preserved regardless of absolute strain values.

This is analogous to setting a chemical potential zero point in thermodynamics. The shifting operation reflects the principle that logical admissibility is comparative rather than absolute—only strain differences generate observable effects.

B.4 Minimizing F: Probability Distribution

Applying Lagrange multipliers for normalization:

$$P_k = \frac{\exp(-\kappa \Delta D_k)}{\sum_j \exp(-\kappa \Delta D_j)}. \tag{27}$$

This is a Gibbs distribution over strain-induced "logical energies." The most logically admissible outcomes (lowest ΔD_k) receive exponentially higher probability, modulated by κ.

B.5 Pure-State Limit and Recovery of Born Rule

For $\rho = |\psi\rangle\langle\psi|$, let $p_k = |\langle b_k|\psi\rangle|^2$. Then:

$$D_k = p_k \ln\left(\frac{p_k}{1-p_k}\right), \quad \Delta D_k = -\frac{1}{\kappa} \ln p_k + C_\psi, \tag{28}$$

A key property is that $D_k(p_k)$ is strictly monotonic in p_k, as shown by its derivative:

$$\frac{dD_k}{dp_k} = \ln\left(\frac{p_k}{1-p_k}\right) + \frac{1}{1-p_k} > 0 \quad \text{for} \quad p_k \in (0,1) \tag{29}$$

Substituting into the distribution:

$$P_k = \frac{e^{\ln p_k}}{\sum_j e^{\ln p_j}} = p_k. \tag{30}$$

Thus, the Born rule follows from strain-weighted equilibrium in the pure-state limit.

B.6 Mixed-State Deviations and Continuity

For general ρ, outcome strains D_k are nonlinear in $p_k = \mathrm{Tr}(\rho|b_k\rangle\langle b_k|)$. The deviation from Born prediction scales with impurity:

$$P_k^{\mathrm{LFT}} = \frac{\exp(-\kappa \Delta D_k)}{\sum_j \exp(-\kappa \Delta D_j)}, \quad \text{where} \quad P_k^{\mathrm{LFT}} \neq p_k \tag{31}$$

Deviations grow approximately with $\gamma_L \cdot (1 - \mathrm{Tr}(\rho^2))$, ensuring a smooth and continuous transition from standard quantum mechanics as states become increasingly mixed.

FIG. 8. Schematic representation of logical strain D_k as a function of measurement basis angle for pure state (dashed) and mixed state (solid). In pure states, minimal strain aligns exactly with Born probability maxima. For mixed states, this alignment breaks down, creating measurable deviations.

B.7 Information-Theoretic Interpretation

$F[P_k]$ combines logic (ΔD_k) and entropy ($P_k \ln P_k$). The probability distribution is the most "logically economical" under entropy constraints. The strain term acts as an effective energy, and κ^{-1} as logical temperature.

This formulation reveals LFT as a logical extension of the maximum entropy principle. When $\kappa \to 0$, we recover maximum entropy (uniform distribution); when $\kappa \to \infty$, we obtain a distribution concentrated entirely on minimal-strain outcomes. Standard quantum mechanics emerges at an intermediate coupling where logical constraint and informational freedom balance precisely.

Summary

LFT derives the Born rule as the minimal-free-energy solution for pure states. For mixed states, it predicts systematic deviations governed by strain and entropy. This derivation links logic, thermodynamics, and quantum measurement under a unified principle, providing a concrete mathematical pathway from our axioms of ontological logic to experimental predictions.

APPENDIX C: SINGLE-QUBIT COLLAPSE LAW AND LOGICAL FIELD EMBEDDING

This appendix derives the single-qubit collapse dynamics under Logic Field Theory (LFT) and outlines the logical field equation embedding consistent with Lorentz invariance.

C.1 Gradient Collapse on the Bloch Sphere

For a single qubit $|\psi(\theta)\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$, the intrinsic strain is:

$$D_{\text{intrinsic}}(\theta) = \sin^2\theta \tag{32}$$

The logical potential becomes:

$$V_L(\theta) = \kappa \sin^2\theta \tag{33}$$

so that the force is:

$$F_L(\theta) = -\nabla V_L = -2\kappa \sin\theta\cos\theta = -\kappa\sin(2\theta) \tag{34}$$

Including mobility $\mu = \sqrt{\alpha_L/(\kappa E_{\text{ref}})}$, the dynamical equation is:

$$\dot{\theta} = -\mu F_L = -\sqrt{\alpha_L\gamma_L}\,\sin(2\theta) \tag{35}$$

which predicts collapse toward $\theta = 0$ or π with timescale:

$$\tau_c^{-1} = 2\sqrt{\alpha_L\gamma_L} \tag{36}$$

C.2 Logical Field Equation

The dynamics above can be embedded in a field equation over spacetime:

$$\Box\Phi_L(x) = \alpha_L\left(D(x) - \langle D\rangle\right) \tag{37}$$

where $\Box = \partial_t^2 - \nabla^2$ is the d'Alembertian operator and $D(x)$ is the local logical strain. This equation mediates the diffusion of strain-induced collapse behavior.

[colback=gray!5!white,colframe=gray!50!black,title=Relativistic Consistency] **Remark C.1.** The field equation $\Box\Phi_L = \alpha_L(D - \langle D\rangle)$ is manifestly Lorentz invariant, as $\Box$ is covariant under Poincaré transformations. Thus, logical collapse in LFT preserves special relativity.

Summary

The logical potential derived from strain governs a deterministic collapse force. Its field-theoretic extension via Φ_L remains Lorentz invariant and connects LFT to broader field-theoretic frameworks without breaking spacetime symmetries.

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[1] Environment-induced pointer bases, as described in Zurek's theory of einselection [2], can be constructed from Lorentz-covariant observables (e.g., energy density, current operators). The Preferred Logical Frame in LFT inherits this structure, preserving frame independence.

APPENDIX D: SIMULATION FRAMEWORK AND SHOT-BUDGET ESTIMATES

This appendix details the simulation methodology and experimental feasibility analysis supporting the predictions of Logic Field Theory (LFT). We evaluate the shot-count requirements and error tolerances for testing LFT's deviation from standard quantum mechanical predictions, especially in high-strain configurations.

D.1 Simulation Code and Reproducibility

All simulations were performed using Python 3.11 with NumPy, SciPy, and Matplotlib. The interactive notebook `LFT_simulations.ipynb` is included in the project archive.

Key features:

- Implementation of both Born-rule and LFT probability distributions
- Evaluation of outcome strain functionals D_k and ΔD_k
- Support for pure and mixed states under arbitrary measurement bases
- Comparison of predicted distributions across parameter sweeps of γ_L
- Monte Carlo shot-based histogram generation with SPAM-error injection

D.2 Bell-Basis Bias and Ratio Metric

In Section VI, we propose a test using a pure two-qubit state $|++\rangle$ measured in the Bell basis. LFT predicts entangled outcomes (e.g., $|\Psi^+\rangle$) will be suppressed due to higher intrinsic strain.

The suppression is quantified by the ratio:

$$R(\gamma_L) = \frac{P(|\Psi^+\rangle)}{P(|00\rangle) + P(|11\rangle)} = \exp(-\gamma_L) \tag{38}$$

This functional form provides a clear, falsifiable signature of strain-driven collapse.

D.3 Shot-Count Requirement and Statistical Power

To resolve a deviation ΔP at significance level 5σ, we estimate the number of required measurement shots using:

$$N \approx \frac{25\, p(1-p)}{(\Delta P)^2} \tag{39}$$

where $p \approx 0.25$ for equiprobable outcomes and $\Delta P = R(\gamma_L) - R(0) = 0.25(1 - e^{-\gamma_L})$.

Example: For $\gamma_L = 0.01$, we find $\Delta P \approx 0.0025 \Rightarrow N \sim 10^6$.

D.4 Error Sources and Conservative Boundaries

Real quantum hardware introduces several non-idealities:

- **SPAM errors**: Imperfect state prep and measurement (typically $< 0.5\%$)
- **Gate fidelity**: Two-qubit gate errors $\sim 0.5 - 1\%$
- **Decoherence**: Time-correlated dephasing affects long circuits

We model these effects via additive binomial noise and confirm robustness of detection for SPAM $< 0.5\%$. Error bars are estimated using bootstrapped resampling over simulated datasets.

D.5 Mixed-State Purity Scan with Uncertainty Estimates

For single-qubit mixed states:

$$\rho = p|\psi\rangle\langle\psi| + (1-p)\frac{\mathbb{I}}{2} \tag{40}$$

we sweep purity $p \in [0.5, 1.0]$ and compute:

$$\Delta P(p, \gamma_L) = \max_k |P_k^{\mathrm{LFT}} - P_k^{\mathrm{Born}}| \tag{41}$$

The table below summarizes simulated LFT-Born deviations and estimated 1σ variances for $N = 2 \times 10^5$ shots per point:

TABLE III. Deviation ΔP and statistical uncertainty by purity

Purity (p)	$\Delta P(p, \gamma_L = 0.02)$	1σ Uncertainty
0.9	0.012	0.0018
0.8	0.020	0.0023
0.7	0.027	0.0027

These variances were estimated using binomial error propagation and validated via 100-sample bootstrap resampling.

Summary

Our simulations show that LFT's predictions are accessible to near-term quantum platforms. The shot-budget estimates remain practical under conservative noise assumptions, and the experimental deviations scale predictably with γ_L. The framework supports rapid validation or falsification across testbeds using Bell-basis bias or purity-dependent outcome distortions.

APPENDIX E: COMPARATIVE ANALYSIS OF INTERPRETATIONS

This appendix compares Logic Field Theory (LFT) with other major interpretations of quantum mechanics. The focus is on ontology, probability origin, falsifiability, and empirical scope.

TABLE IV. Comparison of Quantum Interpretations

Interpretation	**Collapse**	**Born Rule**	**Falsifiable**	**Ontological Basis**
Logic Field Theory (LFT)	Deterministic	Derived (Free Energy)	Yes	Logic as constraint
Many-Worlds (Everett)	No	Decision-theoretic [3, 4]	No	Universal wavefunction
Copenhagen	Ad hoc / Observer-induced	Postulated	No	Observer-system boundary
Bohmian Mechanics	Deterministic	Postulated	Partially	Particle paths + pilot wave [5]
GRW / Spontaneous Collapse	Stochastic	Modified	Yes	Objective collapse events [6]
QBism / Quantum Bayesianism	No	Personalist inference	No	Agent-dependent beliefs [8]
Spekkens Epistemic Model	No	Epistemic-to-ontic mapping	Partially	Restricted classical knowledge [9]
Relational QM	No	Postulated	No	Observer-relative states [10]
Information-based	No	Symmetry or axiomatic [7, 11]	No	Informational principles

Other Considerations

Beyond this comparative table, numerous philosophical, logical, and theoretical works provide context and contrast for LFT's foundational stance.

- **Boole (1854)** and **Frege (1879)** laid the groundwork for logic as a formal language, but did not commit to its ontological status. LFT extends their work by treating logical consistency not as a descriptive property of models, but as an active filter on physical possibility [12, 13].
- **Tarski (1941)** defined logical consequence via model theory, reinforcing an epistemic view. LFT diverges by reinterpreting this structure dynamically—as constraint on state transitions rather than inference rules [14].
- **Carnap (1937)**, **Putnam (1971)**, and **Haack (1978)** debated the scope and meta-logical role of logic. LFT's position aligns with a modal realist view: what is logically impermissible is physically unreal [15–17].
- **Dirac (1930)** and **Birkhoff & von Neumann (1936)** introduced quantum logic as a departure from classical Boolean structures [18, 19]. LFT preserves classical logic but introduces strain as a metric on how far quantum states push against it.
- **Jaynes (1957)** developed maximum entropy inference as a constraint-based framework. LFT parallels this in its logical free energy minimization but substitutes logical strain for entropy cost [**?**].
- **Green, Schwarz, and Witten (1987)** framed string theory around symmetry principles. LFT complements this by proposing that those symmetries may themselves arise as conserved forms of logical admissibility [20].
- **Zeilinger (1999)** and **Wheeler (1990)** emphasized information as a primitive. LFT agrees but specifies that not all information is admissible—only what satisfies the logical sieve [21, 22].
- **Baez and Stay (2010)**, and **Abramsky and Coecke (2004)** explored category theory's role in logic and computation. LFT's projection operator Π_L may be reinterpreted categorically as a logic-preserving functor, a direction for future research [23, 24].

These works highlight the deep philosophical and structural questions LFT seeks to unify: is logic descriptive or generative? Is probability emergent or imposed? Can quantum structure be derived rather than assumed? LFT's distinctive answer is that logic is not passive bookkeeping, but active constraint—shaping not just what we know, but what can be.

APPENDIX F: ANTICIPATED OBJECTIONS AND RESPONSES

The introduction of Logic Field Theory (LFT) challenges prevailing assumptions in quantum foundations and theoretical physics. This appendix addresses common objections likely to arise and offers clarifying responses.

F.1 "Logic is epistemic, not ontological."

Response: While classical logic traditionally functions as an inferential tool, LFT posits that its universality and necessity suggest an ontological role. As Frege[13], Tarski[14], and Birkhoff and von Neumann[19] argued, the structure of logic may reflect necessary features of any consistent reality.

F.2 "Why modify the Born rule if it works?"

Response: LFT does not discard the Born rule—it derives it for pure states. Deviations only appear in mixed states, where logical tension (strain) rises. This offers a falsifiable refinement rather than a contradiction.

F.3 "Is this a hidden variable theory?"

Response: No. LFT involves no additional variables beyond the quantum state and its logical projection. Unlike Bohmian mechanics[5], it does not posit an underlying trajectory space or hidden ontology.

F.4 "Does the Preferred Logical Frame (PLF) break Lorentz invariance?"

Response: The PLF is not a spacetime frame but an information-theoretic one emergent from decoherence-like stability. It plays a role similar to the pointer basis in Zurek's einselection theory[2].

F.5 "How many free parameters are introduced?"

Response: Only one: $\gamma_L = \kappa/E_{\mathrm{ref}}$. This controls deviation strength from Born predictions and is tightly constrained by empirical data.

F.6 "Why treat logic as physically real?"

Response: All physical theories are metaphysically committed to something—fields, wavefunctions, even Hilbert space. LFT commits to logic as the most minimal and necessary structure.

F.7 "Strain isn't observable."

Response: Like entropy or potential energy, strain is inferred from its consequences. Deviations in outcome probabilities are its empirical signature, especially in mixed-state or entangled configurations.

F.8 "Quantum mechanics already explains everything."

Response: While quantum mechanics predicts statistical outcomes, it lacks consensus on the interpretation of those predictions. LFT offers both interpretation and derivation, with testable deviation paths.

Summary

LFT proposes a testable, deterministic alternative to probabilistic collapse and Born-rule fundamentalism. Its central claim—that logical admissibility constrains physical reality—invites scrutiny but remains grounded in derivable dynamics, falsifiable predictions, and a singular, tunable parameter.

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