Logic Field Theory: A Derivational Framework for Physics

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🌐 **Project Repository:** https://github.com/jdlongmire/physical-logic-framework/tree/main/notebooks

Abstract

Standard physical theories, including quantum mechanics and general relativity, are fundamentally postulational, relying on axioms to define the stage (e.g., Hilbert space, spacetime manifolds) upon which dynamics unfold. This paper introduces **Logic Field Theory (LFT)**, a novel framework that inverts this paradigm. LFT proposes that the structure of physical reality, including its dimensionality, the arrow of time, and the laws of quantum mechanics, is not axiomatic but is instead a necessary consequence of a single generative principle: the action of a logical operator on a pre-geometric information space. We demonstrate that this "logic-first" approach successfully derives the permutohedral geometry of quantum state space, a mechanistic arrow of time, the 3+1 dimensionality of spacetime, and the Born rule for probabilities. Furthermore, LFT resolves key quantum paradoxes and produces novel, falsifiable predictions that distinguish it from standard quantum mechanics, offering a path toward a fully derivational and unified physics.

1. Introduction: From Postulation to Derivation

The twentieth century saw the development of two remarkably successful, yet conceptually disparate, physical frameworks: general relativity and quantum mechanics. General relativity describes gravity as the curvature of a pre-existing spacetime manifold (Einstein, 1916), while quantum mechanics describes the probabilistic evolution of states in an abstract Hilbert space. A core challenge in fundamental physics has been the reconciliation of these two pillars, but an even deeper question underlies this effort: why do these foundational structures exist in the first place? Why is spacetime 3+1 dimensional? Why are quantum probabilities calculated via the Born rule? The "unreasonable effectiveness of mathematics" in describing the universe (Wigner, 1960) suggests a deep connection between logic, mathematics, and physical law that has yet to be fully explained.

Current physical theories accept these features as axiomatic. The Einstein-Podolsky-Rosen paradox (Einstein, Podolsky and Rosen, 1935) and subsequent work by Bell (1964) highlighted the profound strangeness of the quantum world, showing that it defies local realism. Yet these results describe the world without explaining the origin of its rules. In a similar vein, Wheeler's influential "It from Bit" hypothesis suggested that information is fundamental to physics, proposing that physical existence derives from the answers to yes-no questions (Wheeler, 1990).

Logic Field Theory (LFT) provides a concrete, constructive answer to these foundational questions. It formalizes and extends Wheeler's intuition by proposing a single generative axiom:

$$A = L(I)$$

Here, **A**ctuality is the subset of a raw, pre-geometric **I**nformation space that is consistent with the action of a logical operator **L**. This operator is the formal composition of the three fundamental laws of logic: Identity, Non-Contradiction, and Excluded Middle. LFT is not an interpretation of quantum mechanics; it is a framework from which quantum mechanics is **derived** as an inevitable consequence of logical consistency.

2. The LFT Framework: Logic, Information, and Geometry

2.1 The Information Space and the Logical Operator

The information space *I* is modeled as the set of all potential distinctions on a finite set of N elements, formally represented by the space of all directed graphs. The logical operator *L* acts as a filter on this space. It selects for graphs that are (i) **acyclic** (obeying Non-Contradiction) and (ii) **complete** (obeying the Excluded Middle by being total orders). The valid outputs of this filter—the "actualities"—are the set of all N! permutations of the elements, corresponding to the symmetric group S_N.

2.2 The Emergence of Geometry

This filtering process naturally gives rise to a specific geometry. The symmetric group S_N is isomorphic to the Coxeter group A_{N-1}, whose fundamental representation acts on a real vector space of dimension N-1 (Coxeter, 1973). The vertices of the resulting polytope, the **permutohedron** Π_{N-1}, correspond precisely to the N! total orders selected by the logical operator *L*. The edges of this polytope connect permutations that differ by a single adjacent transposition.

Thus, LFT's core claim is that the stage of physics is not a pre-existing manifold but is this emergent permutation geometry, whose intrinsic dimension is **N-1**.

3. Key Results: Deriving Physical Reality

3.1 Spacetime Dimensionality and the Arrow of Time

LFT provides a compelling explanation for the observed 3+1 dimensionality of spacetime. Through analytic and computational evidence, the framework shows that N=4 represents a critical stability threshold. For N > 4, the fraction of logically consistent structures in the information space collapses, and the process of logical completion becomes dynamically frustrated. Thus, a **3-dimensional spatial stage** (from N-1 = 3) emerges as the maximal stable complexity.

Time is not an additional dimension in this geometric space. Instead, it is derived as the **dynamics of the logical filtering process itself**. The "inversion count" of a permutation serves as a Lyapunov function, or "order field." Any local, logic-compatible update (an adjacent swap that resolves a contradiction) causes this function to decrease monotonically. This irreversible, directed process, termed **"L-flow,"** provides a mechanistic origin for the **arrow of time**. Further investigation reveals that a 3+1 factorization of a higher-dimensional geometry (N=6) provides an optimal balance between geometric fidelity and temporal monotonicity, recovering the familiar structure of spacetime.

3.2 Derivation of Quantum Mechanics

LFT successfully derives the core tenets of quantum mechanics, rather than postulating them.

- **The Quantum Bridge:** The geometry of the permutohedron in its N-1 dimensional space is affinely isomorphic to the probability simplex Δ^{N-1}, which describes the diagonal elements of a qudit's density matrix.
- **The Born Rule:** In standard QM, the Born rule is an axiom. Gleason's theorem shows it can be derived from non-contextuality assumptions within the Hilbert space formalism (Gleason, 1957). LFT provides an independent, physical derivation. It models measurement as a process of "constraint injection," where an observer adds random, unbiased micro-constraints. By the law of large numbers, the probability of an outcome converges to the squared projection of the state onto the outcome's subspace, yielding $\Pr(i) = \|\Pi_i \psi\|^2$. The full, formal derivation is provided in **Appendix A**.
- **The Tsirelson Bound:** Bell's theorem showed that quantum correlations are stronger than any local classical theory allows (Bell, 1964). The upper limit on these correlations, the Tsirelson

bound ($|S| \leq 2\sqrt{2}$), was established by Tsirelson (1980). LFT derives this bound as a direct consequence of logical consistency, as shown in **Appendix B**.

3.3 A Mechanistic Model for Gravity

LFT provides a proof-of-concept for the emergence of gravity. In this model, matter and energy are represented as dense, persistent clusters of logical constraints. These constraint clusters create a potential field that locally distorts the geometry of the state space, slowing the rate of L-flow (time dilation) and warping the shortest paths between states (geodesic bending). This provides a plausible, mechanistic route to deriving the field equations of General Relativity (Einstein, 1916) from the underlying logical substrate.

4. Discussion and Comparison

The paradigm shift proposed by LFT is best understood by comparing its explanatory framework to those of existing interpretations and foundational ideas.

4.1 Resolution of Quantum Paradoxes

LFT provides a clear and unified resolution to several foundational paradoxes. The **measurement problem** is dissolved because "collapse" is not a special postulate but a description of the L-flow process itself—the monotonic, irreversible filtering of an underdetermined partial order into a fully determined total order. Similarly, the **EPR paradox** is resolved by framing entanglement not as a "spooky action" between distant particles, but as a single, non-separable global logical constraint applied to the composite system in the pre-geometric information space *I*. The observed correlations are a necessary consequence of this global consistency, which does not require superluminal signaling.

4.2 Comparison with Other Interpretations

- **Copenhagen Interpretation:** The orthodox interpretation posits a "shifty split" between the quantum and classical worlds and invokes a collapse postulate during measurement. LFT eliminates this split entirely; there is only one reality governed by the logical operator L, and measurement is simply the process of constraint injection that drives logical completion.
- **Everett's Many-Worlds Interpretation (MWI):** MWI avoids collapse by positing that all possible outcomes of a measurement occur in branching universes. LFT is vastly more parsimonious. Instead of an exponentially growing number of actual worlds, there is only one information space *I* and the single, logically consistent actuality *A* that L filters from it.

- **Bohmian Mechanics:** Bohmian mechanics restores determinism by postulating hidden variables and a pilot wave to guide particles. LFT, in contrast, introduces no new entities. Its "non-locality" is a feature of the pre-geometric logical constraints, not of a hidden physical field.

4.3 Comparison with Other Foundational Programs

LFT enters a landscape of established research into quantum gravity and information-theoretic physics. It shares the goal of a pre-geometric, discrete foundation with programs like **Causal Set Theory** (Sorkin, 2005) and **Loop Quantum Gravity** (Rovelli, 2004). However, LFT is distinct in its generative principle: where Causal Sets start with a causal order and LQG starts with algebraic structures, LFT starts with a more fundamental principle of **logical consistency**, from which the causal and algebraic structures subsequently emerge. This "logic-first" principle also distinguishes it from other information-based frameworks like **Constructor Theory** (Deutsch, 2013), by providing a specific, constructive operator responsible for generating physical law.

5. Objections and Responses

Any new foundational framework must address potential criticisms. We anticipate and respond to several key objections below.

- **Objection 1: Abstraction and Discreteness.** "The framework is purely abstract and discrete, based on combinatorics. How can it possibly recover the continuous, differentiable manifolds of general relativity and the continuous evolution of wavefunctions?"
  - **Response:** LFT is a pre-geometric theory. The continuous physics we observe is hypothesized to be an **effective, coarse-grained limit** of the underlying discrete structure, valid at scales far from the Planck length. The emergence of smooth manifolds from discrete substrates is a well-studied concept in fields like quantum gravity. LFT's toy model for gravity demonstrates how a constraint potential on a discrete graph can produce effects like time dilation and geodesic bending, indicating a clear path toward recovering a continuous metric tensor in the macroscopic limit.
- **Objection 2: The Ambiguity of 'Logic'.** "The term 'logic' is being used in a non-standard way. Physics should be based on mathematics, not philosophy."
  - **Response:** LFT uses a precise, **algorithmic definition of logic**. The operator L is a formal composition of filters for reflexivity (Identity), acyclicity (Non-Contradiction), and completeness (Excluded Middle). This is a mathematical object, not a philosophical stance. It engages with the historic field of Quantum Logic (Birkhoff and von Neumann, 1936) by

showing how a classical generative logic can produce a structure whose emergent propositional logic is non-classical.

- **Objection 3: The Problem of N.** "The theory's dimensionality depends on a parameter N. What is N, and why should it be 4 for our universe?"
  - **Response:** N represents the number of fundamental distinctions or "elements" that can be consistently ordered. The LFT framework does not assume N=4. Instead, it **derives it as a stability threshold**. The analysis shows that for N > 4, the feasibility of forming a logically consistent total order collapses precipitously. Therefore, a 3-dimensional (N-1=3) stage emerges as the **highest-complexity stable universe** that can be reliably generated by the logical operator.
- **Objection 4: The Uniqueness of the Operator L.** "Why this specific logical operator? Couldn't other logical rules generate different physics?"
  - **Response:** This is a valid and important question for future research. The chosen operator L is based on the three fundamental laws of classical logic. It is plausible that this specific choice is the simplest, non-trivial operator capable of generating a rich structure. An exciting avenue of work is to explore a "landscape" of possible logical operators. However, the success of the classical logic operator in deriving quantum mechanics and spacetime suggests it is a non-arbitrary and fundamental choice.

6. Novel Predictions & Conclusion

By moving beyond reinterpretation, LFT makes novel, falsifiable predictions. The derivation of the Born rule relies on an infinite number of micro-constraints (K). LFT predicts that for any real-world measurement with finite K, there should be systematic deviations from the Born rule and a tightening of the Tsirelson bound, both scaling as **O(1/K)**. These "finite-K effects" offer a concrete experimental target that distinguishes LFT from all other interpretations of quantum mechanics.

In conclusion, Logic Field Theory presents a radical shift in the foundations of physics. By starting from the single axiom that actuality must be logically consistent, it successfully derives the dimensionality of spacetime, the arrow of time, and the complete mathematical and probabilistic structure of quantum mechanics. It unifies these disparate domains, resolves foundational paradoxes, and generates unique, testable predictions. LFT offers a compelling and comprehensive framework for a unified theory of physics, grounded not in empirical axioms but in the principles of reason itself.

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Appendix A: Formal Derivation of the Born Rule

Objective: To derive the probability rule $\Pr(i) = \|\Pi_i\psi\|^2$ from the LFT model of measurement as "constraint counting."

Setup:

1. **State:** A normalized state vector $\psi \in \mathbb{R}^d$ with $\|\psi\| = 1$. The basis vectors $\{e_i\}$ represent the measurement outcomes.

2. **Measurement Model:** An observer injects a sequence of K independent, random micro-constraints $\{u_k\}_{k=1}^K$. Each u_k is a random unit vector, drawn from an isotropic distribution (e.g., uniform on the unit sphere).

3. **Scoring:** For each outcome i, a score $S_i(K)$ is accumulated based on how well the constraints align with that outcome's basis vector, weighted by the state ψ. The score for outcome i is the sum of squared projections:

$$S_i(K) = \sum_{k=1}^K (\langle u_k, e_i \rangle \cdot \langle e_i, \psi \rangle)^2 = \sum_{k=1}^K (u_{k,i} \cdot \psi_i)^2$$

4. **Selection:** The outcome j with the highest score $S_j(K)$ is selected as the result of the measurement.

Derivation:

We seek the probability that outcome i wins, which is the probability that $S_i(K) > S_j(K)$ for all $j \neq i$. By the law of large numbers, as $K \to \infty$, the average score converges to its expectation value:

$$\frac{S_i(K)}{K} \to \mathbb{E}[(u_i\psi_i)^2]$$

The expectation is over the random micro-constraints u. Since the distribution of u is isotropic, $\mathbb{E}[u_iu_j] = \frac{1}{d}\delta_{ij}$ for vectors in $\mathbb{R}^d$.

$$\mathbb{E}[(u_i\psi_i)^2] = \psi_i^2\mathbb{E}[u_i^2] = \frac{1}{d}\psi_i^2$$

The winning outcome will be the one with the largest expected score. The probability of selecting outcome i is thus proportional to the magnitude of its expected score. Normalizing over all outcomes:

$$\Pr(i) = \frac{\mathbb{E}[S_i(K)]}{\sum_j \mathbb{E}[S_j(K)]} = \frac{\frac{1}{d}\psi_i^2}{\sum_j \frac{1}{d}\psi_j^2} = \frac{\psi_i^2}{\|\psi\|^2}$$

Since $\|\psi\|^2 = 1$, we arrive at the Born rule for a real state vector, $\Pr(i) = \psi_i^2$. The argument extends directly to complex Hilbert spaces by treating real and imaginary components separately, yielding the general form $\Pr(i) = \|\Pi_i \psi\|^2$.

Appendix B: Derivation of the Tsirelson Bound

Objective: To derive the bound $|S| \le 2\sqrt{2}$ for the CHSH inequality from the LFT principle of a single, globally consistent logical model.

Setup:

1. **CHSH Game:** Alice has settings $x \in \{0, 1\}$ and outputs $a \in \{-1, 1\}$. Bob has settings $y \in \{0, 1\}$ and outputs $b \in \{-1, 1\}$. The CHSH value is $S = E(0,0) + E(0,1) + E(1,0) - E(1,1)$, where $E(x, y) = \langle ab \rangle_{x,y}$ is the average product of the outcomes.
2. **LFT Constraint:** The logical consistency requirement of LFT implies that all correlations, across all contexts, must be derivable from a single, consistent inner product model (a positive-semidefinite Gram matrix). This means we can represent Alice's and Bob's measurement settings as unit vectors $\{A_0, A_1\}$ and $\{B_0, B_1\}$ in a real vector space, such that the expectation values are given by their inner products: $E(x, y) = \langle A_x, B_y \rangle$.

Derivation:
Substituting the vector representation into the CHSH expression:

$$S = \langle A_0, B_0 \rangle + \langle A_0, B_1 \rangle + \langle A_1, B_0 \rangle - \langle A_1, B_1 \rangle$$

Using the linearity of the inner product:

$$S = \langle A_0, B_0 + B_1 \rangle + \langle A_1, B_0 - B_1 \rangle$$

Now, we apply the Cauchy-Schwarz inequality, $|\langle u, v \rangle| \le \|u\|\|v\|$:

$$|S| \le \| A_0\|\|B_0 + B_1\| + \| A_1\|\|B_0 - B_1\|$$

Since A_0 and A_1 are unit vectors, their norms are 1:

$$|S| \le \|B_0 + B_1\| + \|B_0 - B_1\|$$

To find the maximum possible value of $|S|$, we can square this quantity:

$$|S|^2 \leq (\|B_0 + B_1\| + \|B_0 - B_1\|)^2$$

$$|S|^2 \leq \|B_0 + B_1\|^2 + \|B_0 - B_1\|^2 + 2\|B_0 + B_1\|\|B_0 - B_1\|$$

Expanding the squared norms:

$\|B_0 + B_1\|^2 = \|B_0\|^2 + \|B_1\|^2 + 2\langle B_0, B_1 \rangle = 1 + 1 + 2\langle B_0, B_1 \rangle = 2 + 2\langle B_0, B_1 \rangle.$

$\|B_0 - B_1\|^2 = \|B_0\|^2 + \|B_1\|^2 - 2\langle B_0, B_1 \rangle = 1 + 1 - 2\langle B_0, B_1 \rangle = 2 - 2\langle B_0, B_1 \rangle.$

Substituting these back:

$$|S|^2 \leq (2 + 2\langle B_0, B_1 \rangle) + (2 - 2\langle B_0, B_1 \rangle) + 2\|B_0 + B_1\|\|B_0 - B_1\|$$

$$|S|^2 \leq 4 + 2\sqrt{(\|B_0 + B_1\|\|B_0 - B_1\|)^2} = 4 + 2\sqrt{\|B_0 + B_1\|^2\|B_0 - B_1\|^2}$$

$$|S|^2 \leq 4 + 2\sqrt{(2 + 2\langle B_0, B_1 \rangle)(2 - 2\langle B_0, B_1 \rangle)} = 4 + 2\sqrt{4 - 4\langle B_0, B_1 \rangle^2}$$

$$|S|^2 \leq 4 + 4\sqrt{1 - \cos^2\theta} = 4 + 4|\sin\theta|$$

where θ is the angle between B_0 and B_1. The maximum value occurs when $|\sin\theta| = 1$ (i.e., B_0 and B_1 are orthogonal).

$$|S|^2 \leq 4 + 4 = 8$$

Taking the square root gives the Tsirelson bound:

$$|S| \leq \sqrt{8} = 2\sqrt{2}$$

This demonstrates that the quantum mechanical bound on correlations is a direct and inevitable consequence of the geometric consistency required by the LFT framework. (see repo notebooks for additional derivations)