

Artificial Intelligence

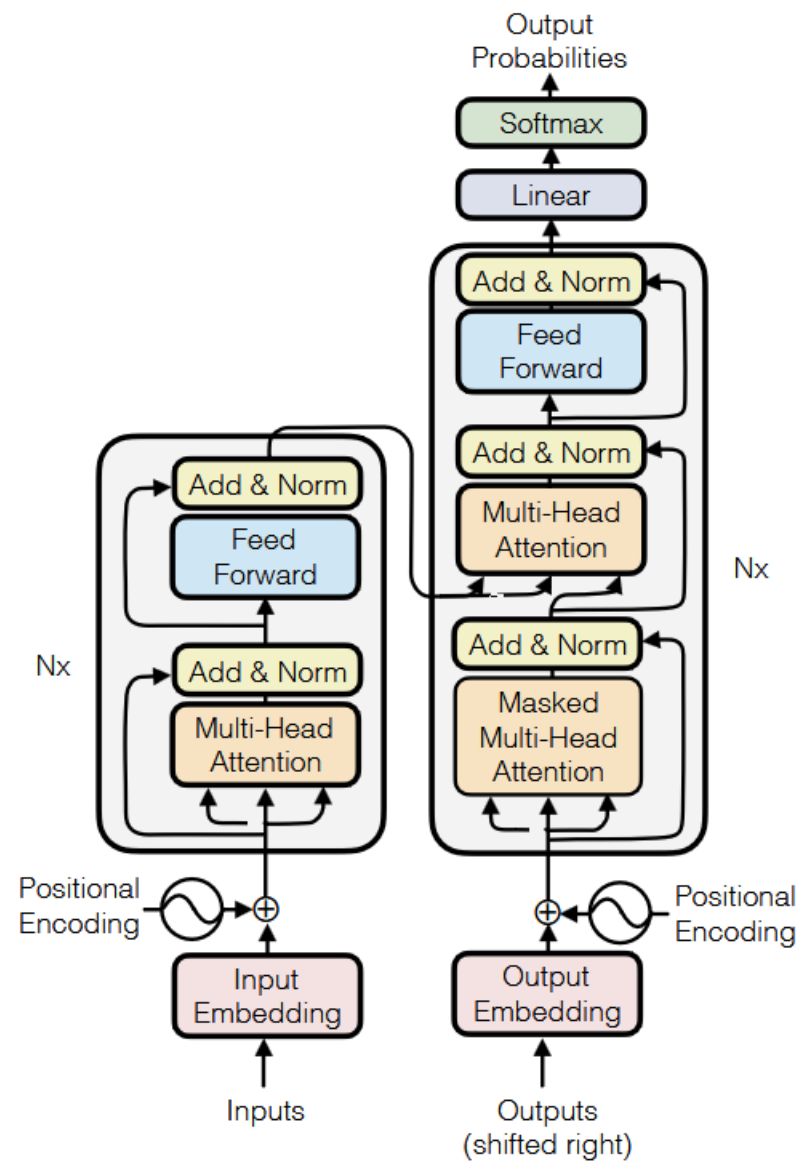
Lecture06a – Attention is
all you need



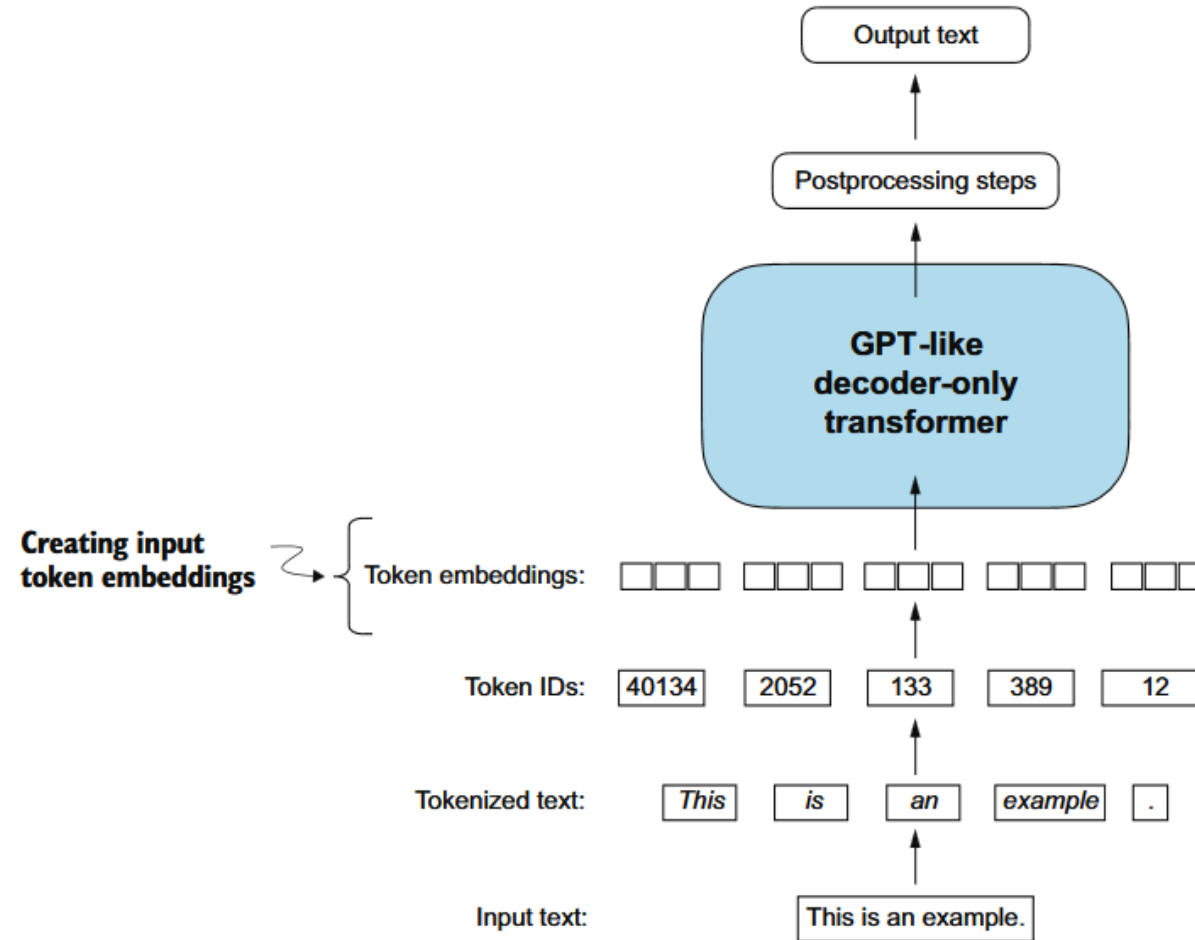
Contenido

1. Token embeddings
2. Positional encodings
3. Sequence modeling
4. Recurrent neural networks
5. Attention





Creating token embeddings



Creating token embeddings

Embedding layers: An efficient way of performing matrix multiplication when working with one-hot encoded vectors.

```
import torch

torch.manual_seed(123);

idx = torch.tensor([2, 3, 1]) # 3 training examples

num_idx = max(idx)+1
out_dim = 5
```

Suppose we want embeddings of size 5

Input dimension of a one-hot encoded vector is the number of indices (the highest index + 1)



Creating token embeddings

```
import torch
```

```
torch.manual_seed(123);
```

```
idx = torch.tensor([2, 3, 1]) # 3 training examples
```

```
num_idx = max(idx)+1
```

```
out_dim = 5
```

```
embedding = torch.nn.Embedding(num_idx, out_dim)
```

```
embedding(idx)
```

```
tensor([[ 0.6957, -1.8061, -1.1589,  0.3255, -0.6315],  
        [-2.8400, -0.7849, -1.4096, -0.4076,  0.7953],  
        [ 1.3010,  1.2753, -0.2010, -0.1606, -0.4015]],  
        grad_fn=<EmbeddingBackward0>)
```

Each training example has
5 feature values



Creating token embeddings

One-hot encoded ("**sparse**") representation of "S U N N Y"

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
S	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
U	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
N	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
N	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
Y	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0



Embedded ("**dense**")
representation of
"S U N N Y"

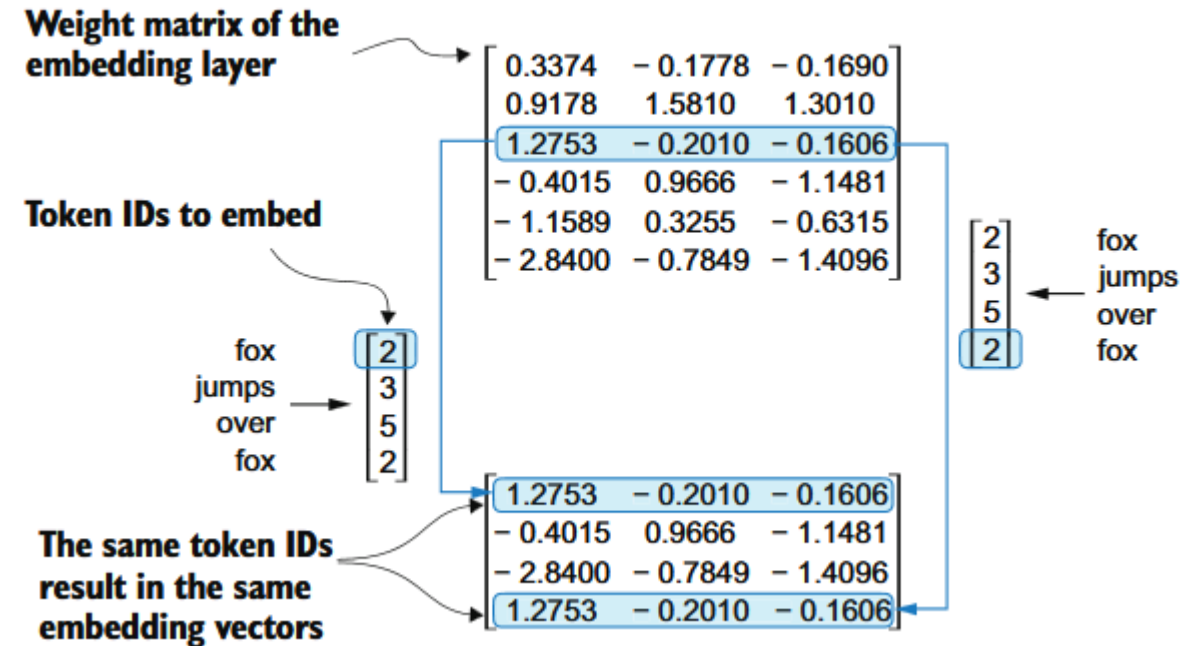
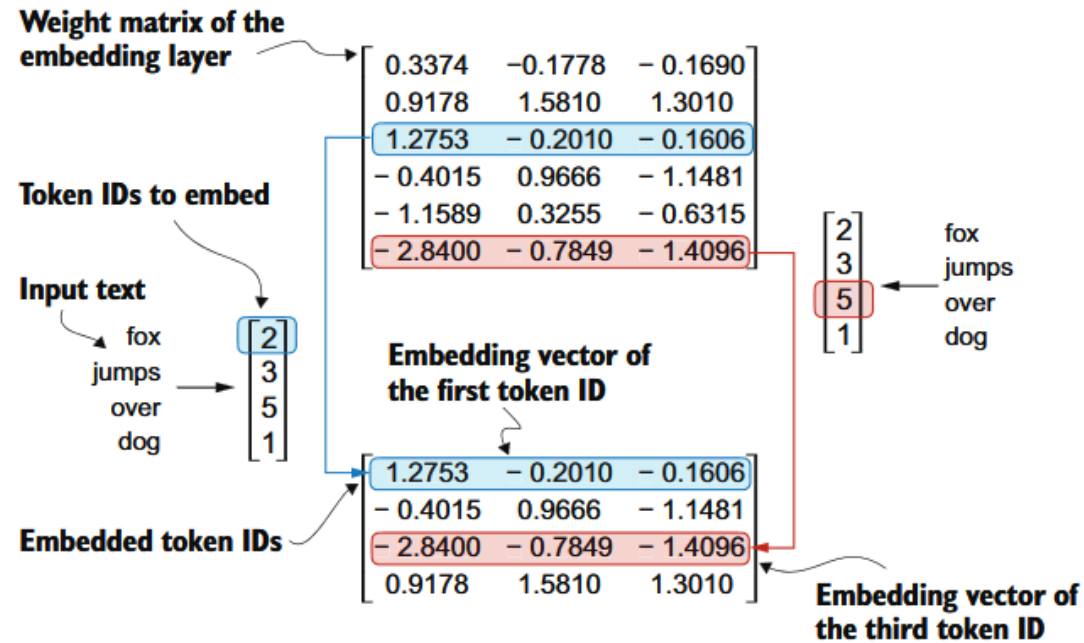
[[0.9816, 0.7363, 0.5899],
[0.2605, 0.3766, 0.3502],
[0.7382, 0.9807, 0.4762],
[0.6231, 0.8825, 0.8836]]

Embedding layer

[[0.6912, 0.8765, 0.4939],
[0.6342, 0.7481, 0.7717],
[0.8395, 0.2128, 0.3696],
[0.4900, 0.1509, 0.0689],
[0.2587, 0.9171, 0.8670],
[0.7213, 0.9922, 0.5701],
[0.7598, 0.5231, 0.3666],
[0.5150, 0.5216, 0.9682],
[0.2248, 0.0261, 0.4427],
[0.1818, 0.6863, 0.8713],
[0.4192, 0.1566, 0.9004],
[0.8102, 0.5741, 0.4241],
[0.1116, 0.0466, 0.2786],
S [0.9816, 0.7363, 0.5899],
[0.9224, 0.3672, 0.6972],
[0.1207, 0.3372, 0.2128],
[0.0660, 0.1524, 0.8440],
[0.2162, 0.5640, 0.0988],
U [0.2605, 0.3766, 0.3502],
[0.7334, 0.4757, 0.7581],
N [0.7382, 0.9807, 0.4762],
[0.2369, 0.8102, 0.8798],
[0.6932, 0.2671, 0.8018],
[0.9593, 0.5302, 0.4290],
Y [0.6231, 0.8825, 0.8836],
[0.4623, 0.8503, 0.7279]]

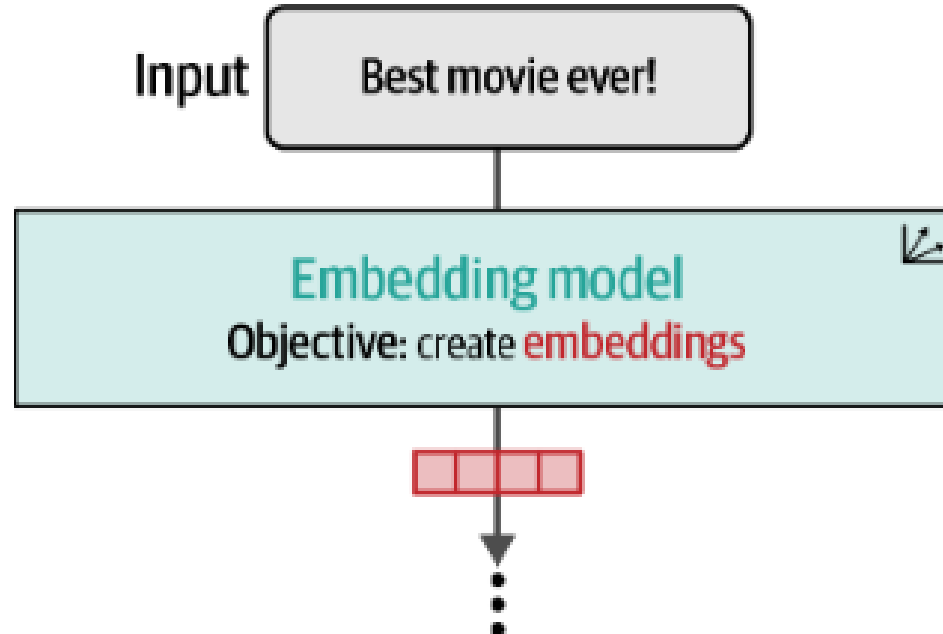


Creating token embeddings

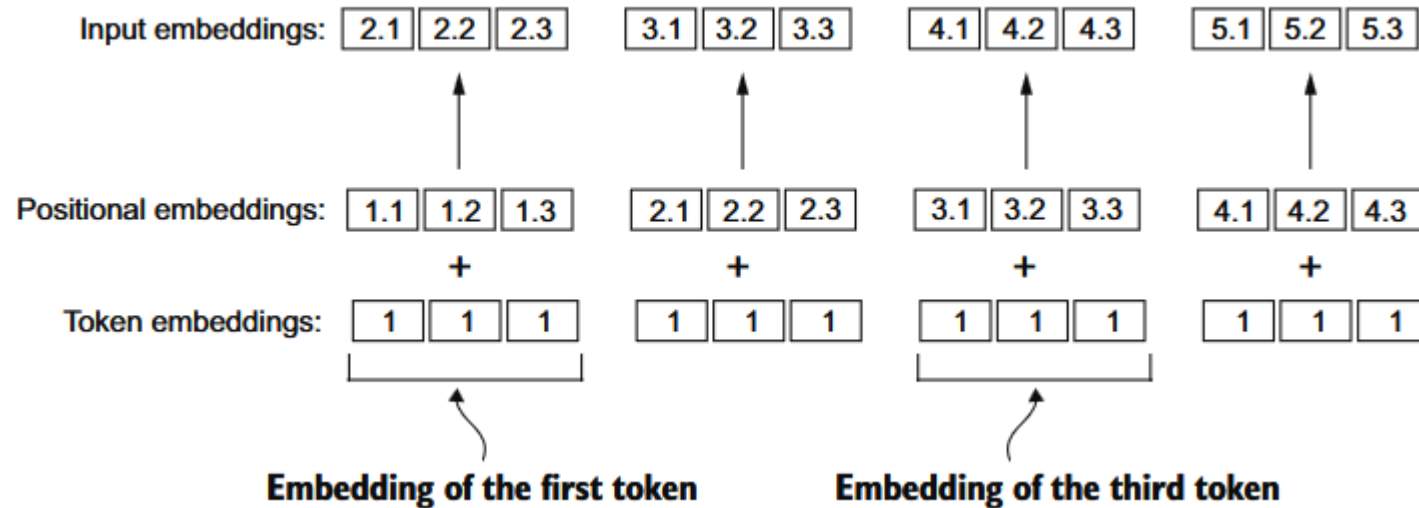


Sentence embeddings

Task: Aggregate token embeddings in a sentence to form a **unique sentence representation**.



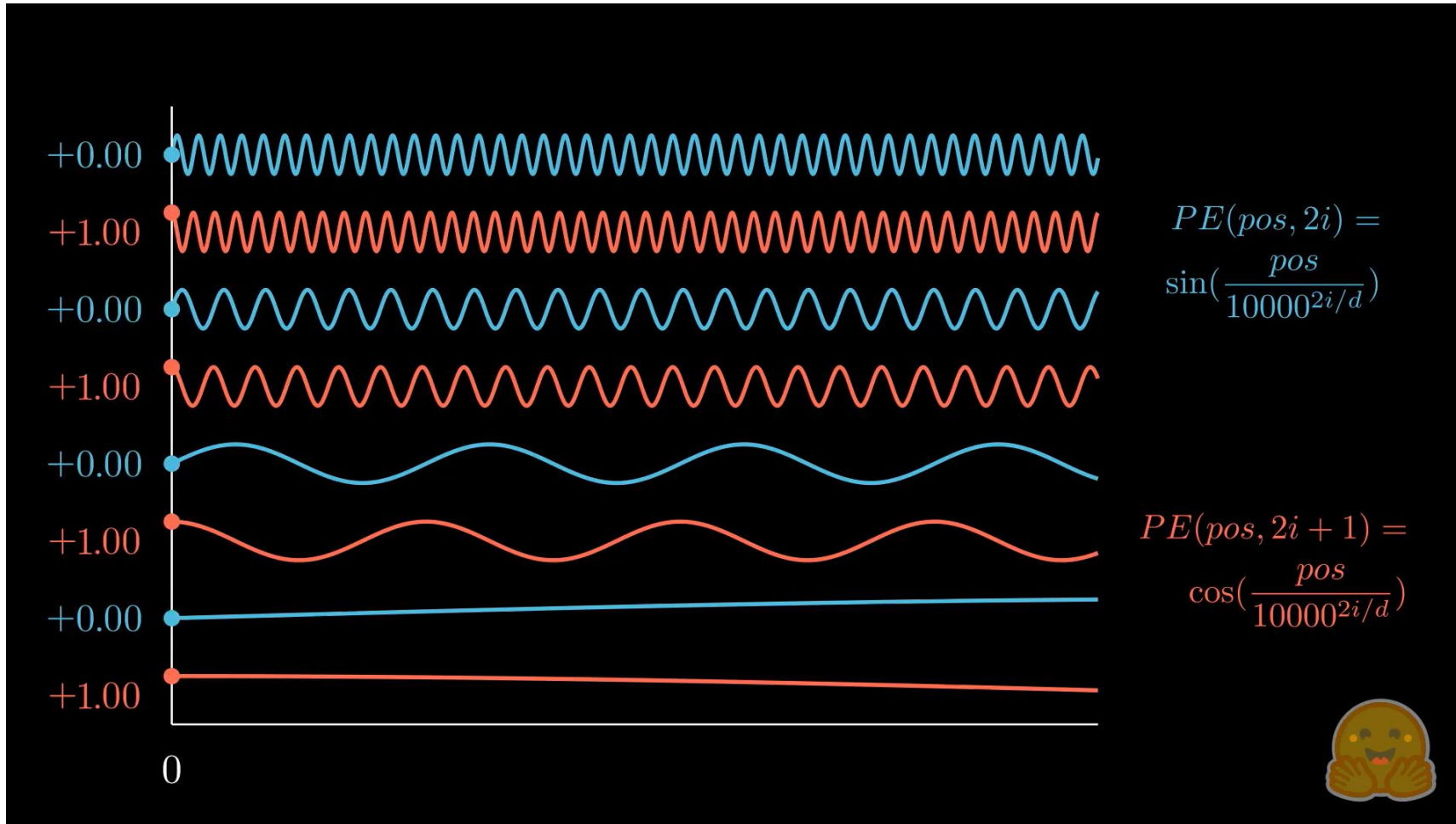
Encoding word positions



OpenAI: Positional embeddings are learnable parameters.
Llama, DeepSeek: Rotational Positional Embeddings – RoPE
Bert: Sinusoidal Positional Embeddings



Sinusoidal positional embeddings



$$PE(pos, 2i) = \sin\left(\frac{pos}{10000^{\frac{2i}{d}}}\right)$$

$$PE(pos, 2i + 1) = \cos\left(\frac{pos}{10000^{\frac{2i}{d}}}\right)$$



Rotary Positional Embeddings (RoPE)

Fundamental idea: we want the dot product of embeddings to result in a function of relative position:

$$f_q(\mathbf{x}_m, m) \cdot f_k(\mathbf{x}_n, n) = g(\mathbf{x}_m, \mathbf{x}_n, m - n)$$

In summary, RoPE uses trigonometry to come up with a function that satisfies this property

RoFORMER: ENHANCED TRANSFORMER WITH ROTARY
POSITION EMBEDDING

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m is the token's position in the sequence.

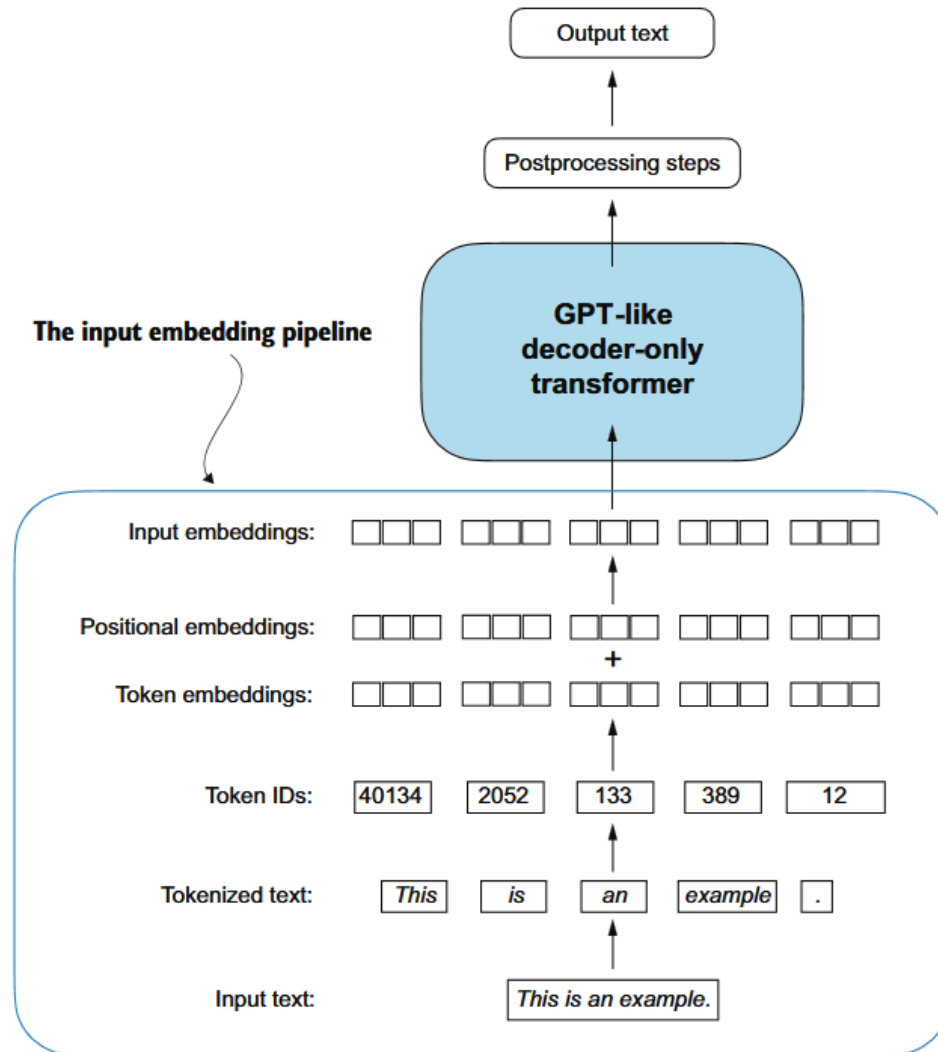
$\Theta = (\theta_1, \theta_2, \dots, \theta_{d/2})$ with:

$$\theta_i = 10000^{-2(i-1)/d}$$

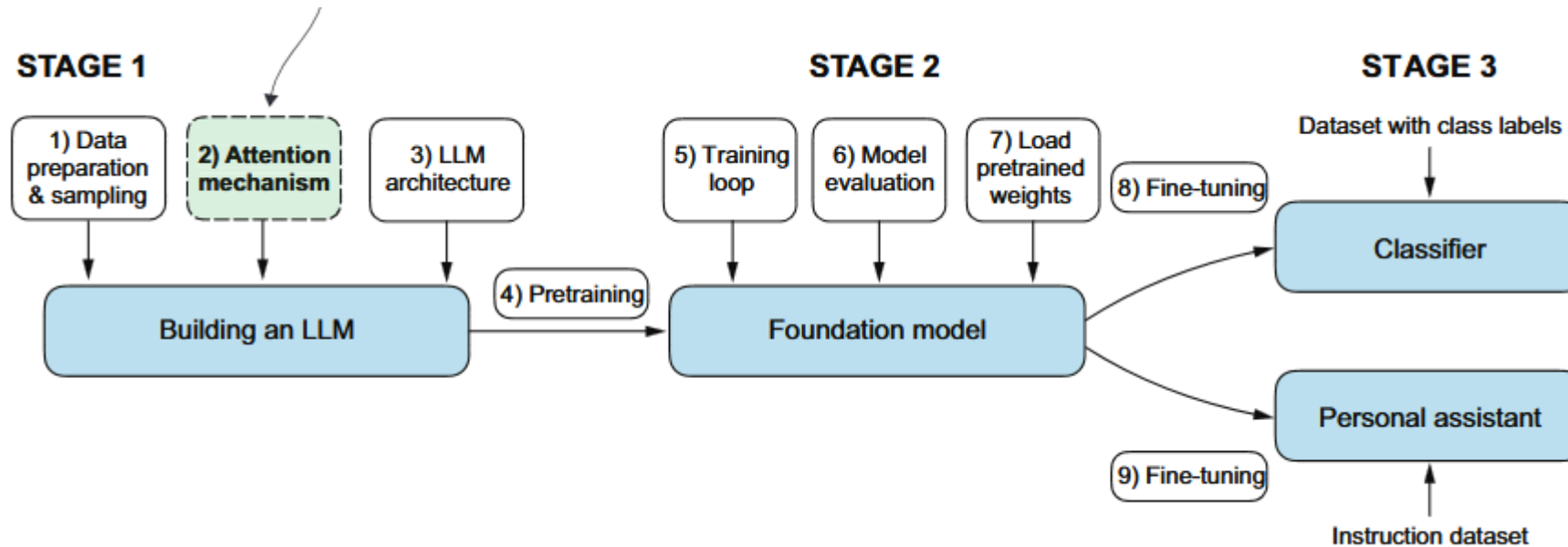
$$R_{\Theta, m}^d \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{d-1} \\ x_d \end{pmatrix} \otimes \begin{pmatrix} \cos m\theta_1 \\ \cos m\theta_1 \\ \cos m\theta_2 \\ \cos m\theta_2 \\ \vdots \\ \cos m\theta_{\frac{d}{2}} \\ \cos m\theta_{\frac{d}{2}} \end{pmatrix} + \begin{pmatrix} -x_2 \\ x_1 \\ -x_4 \\ x_3 \\ \vdots \\ -x_d \\ x_{d-1} \end{pmatrix} \otimes \begin{pmatrix} \sin m\theta_1 \\ \sin m\theta_1 \\ \sin m\theta_2 \\ \sin m\theta_2 \\ \vdots \\ \sin m\theta_{\frac{d}{2}} \\ \sin m\theta_{\frac{d}{2}} \end{pmatrix}$$



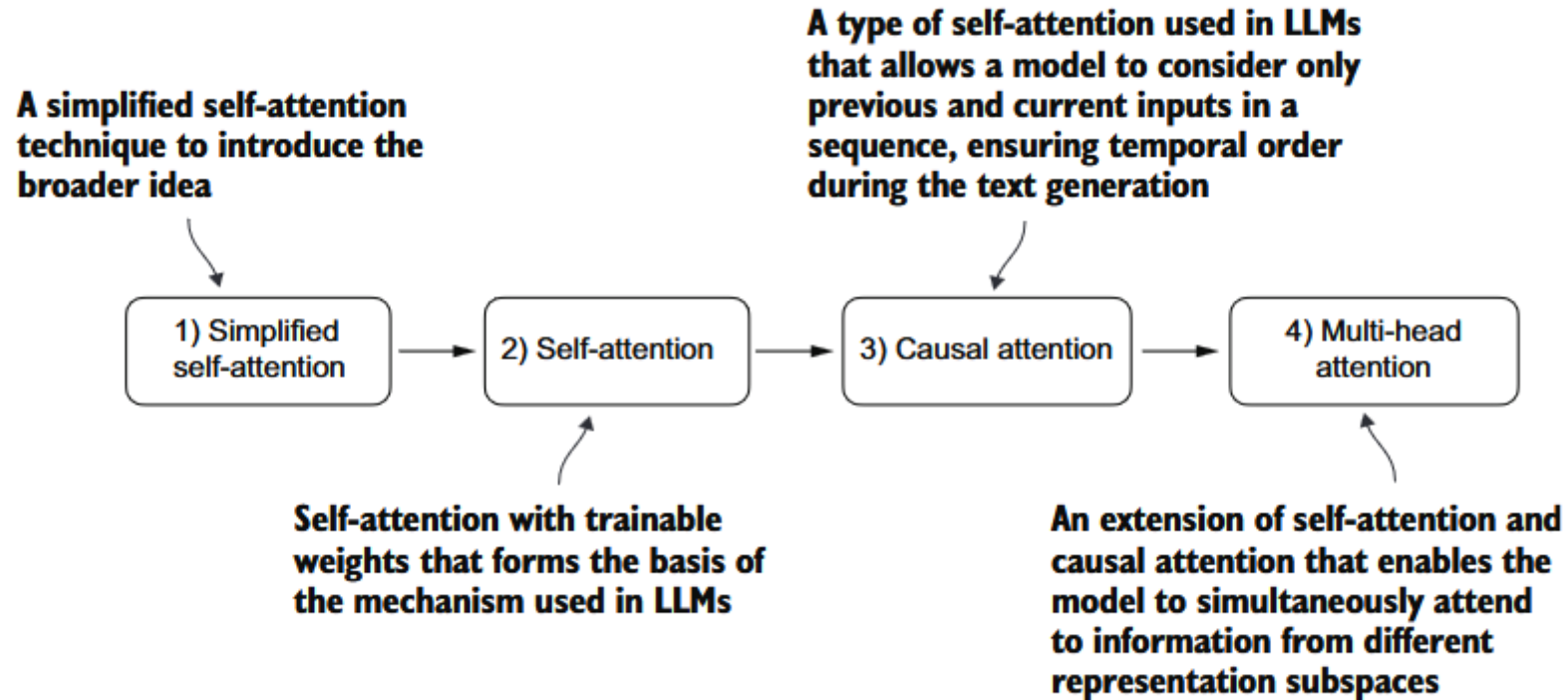
Encoding word positions



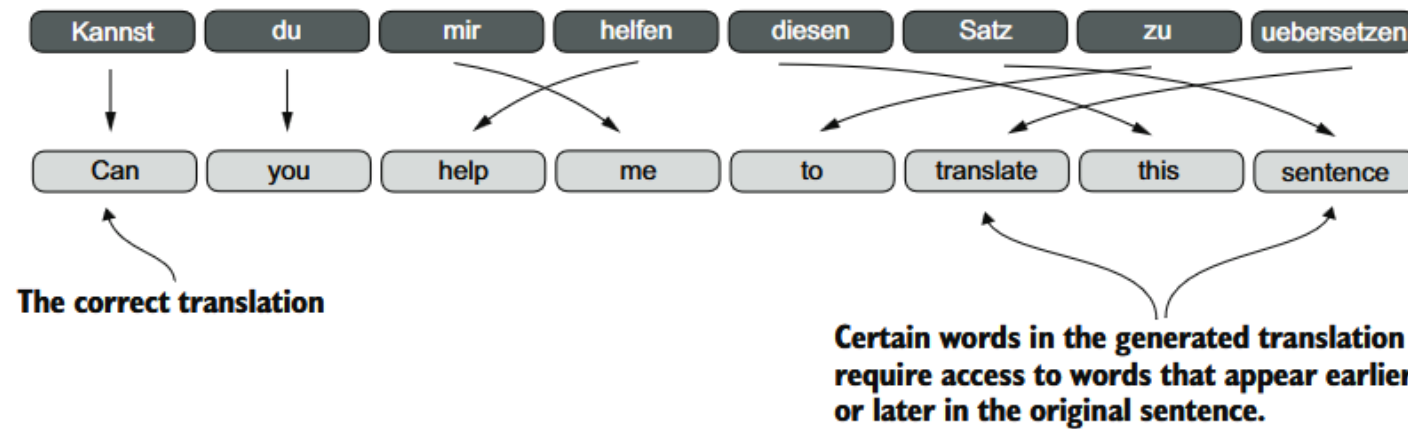
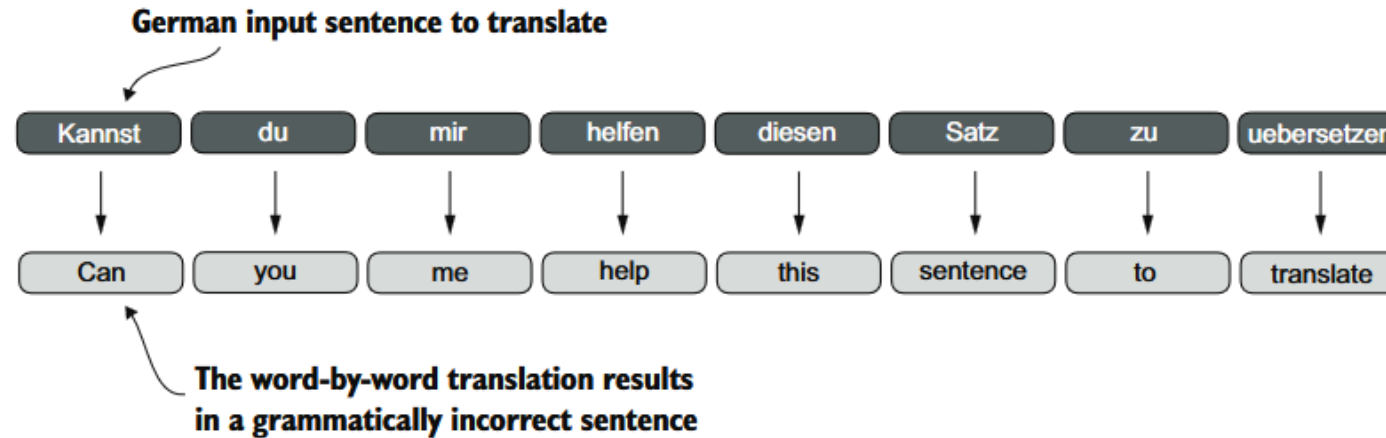
Attention mechanism



Attention mechanism



Problem with modeling long sequences



Modeling sequences

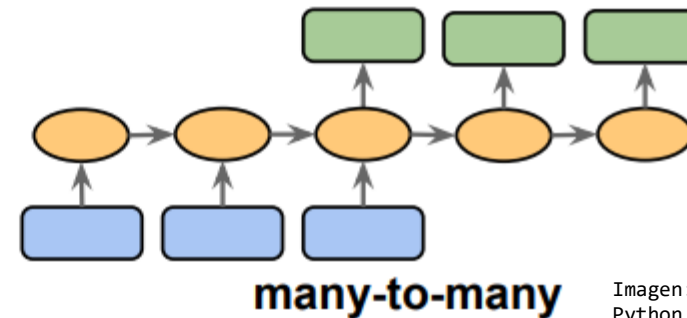
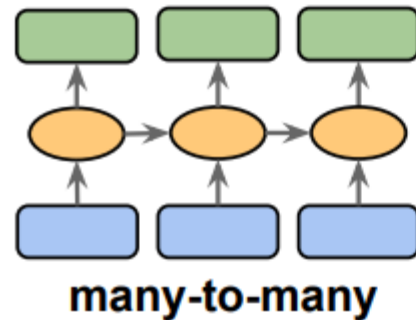
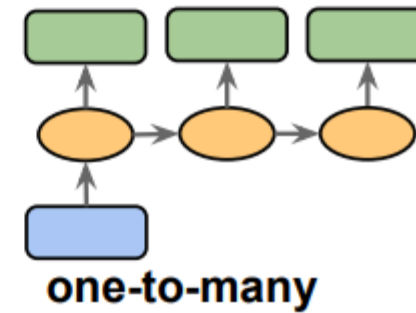
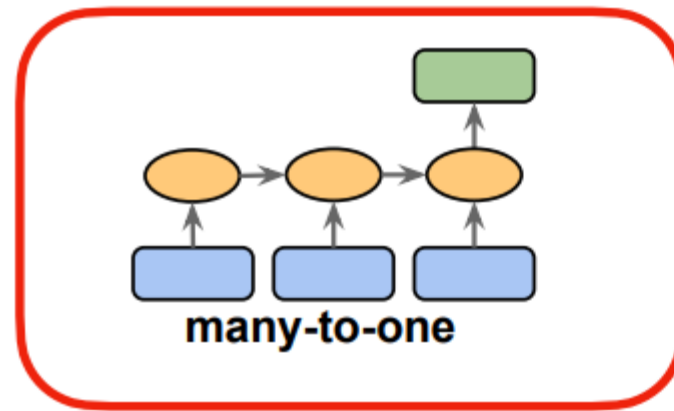
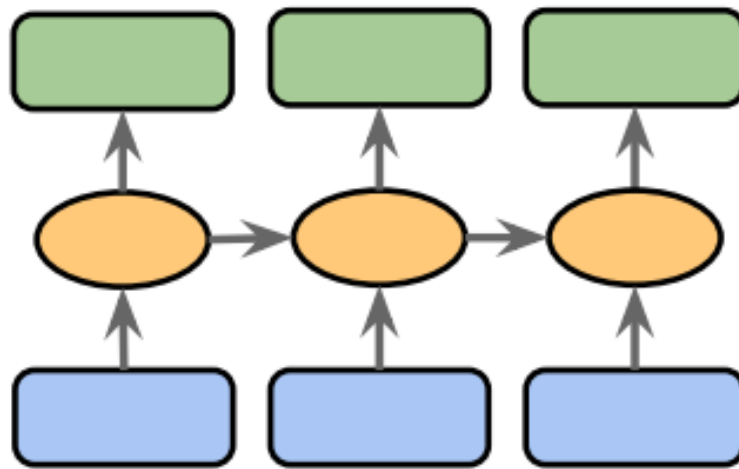


Imagen: Sebastian Raschka, Vahid Mirjalili.
Python Machine Learning. 3rd Edition.
Birmingham, UK: Packt Publishing, 2019

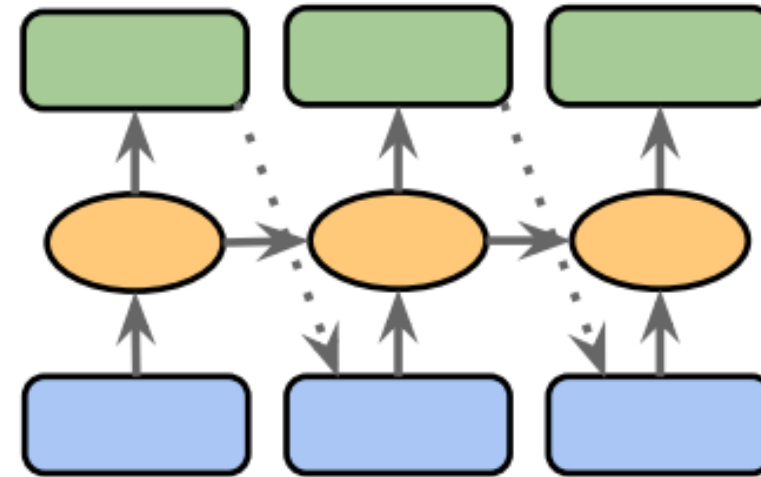


Modeling sequences



many-to-many

"training"

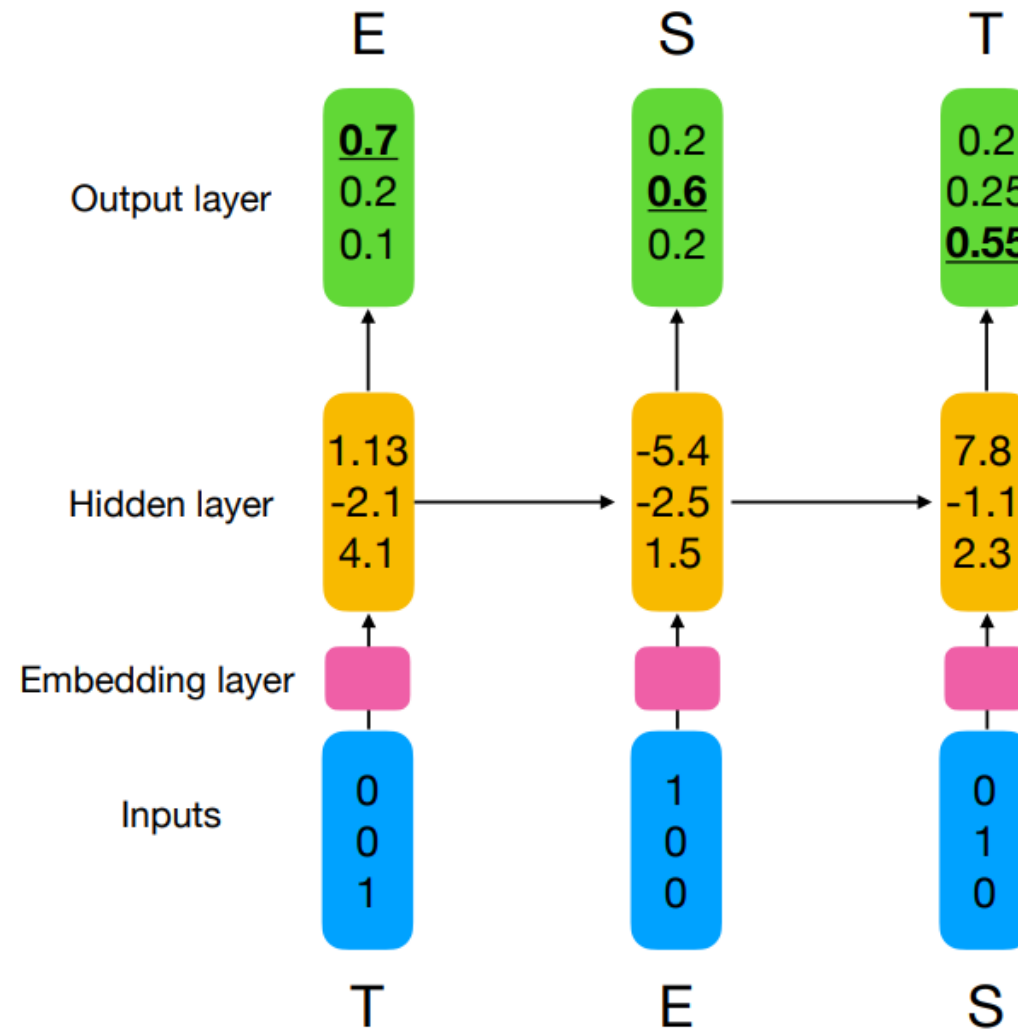


~~many-to-many~~
"one"

"generating new text"

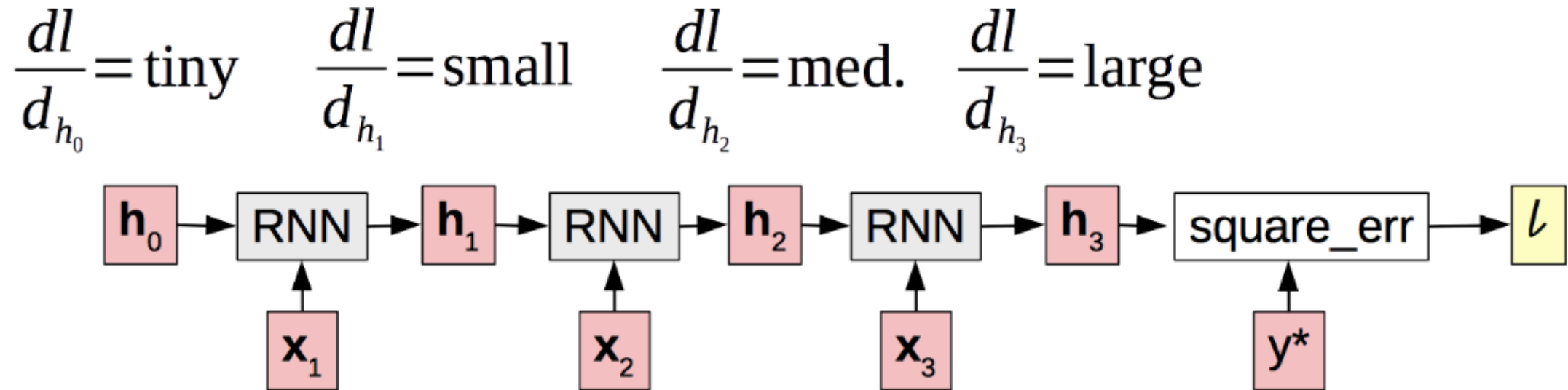


Recurrent Neural Networks



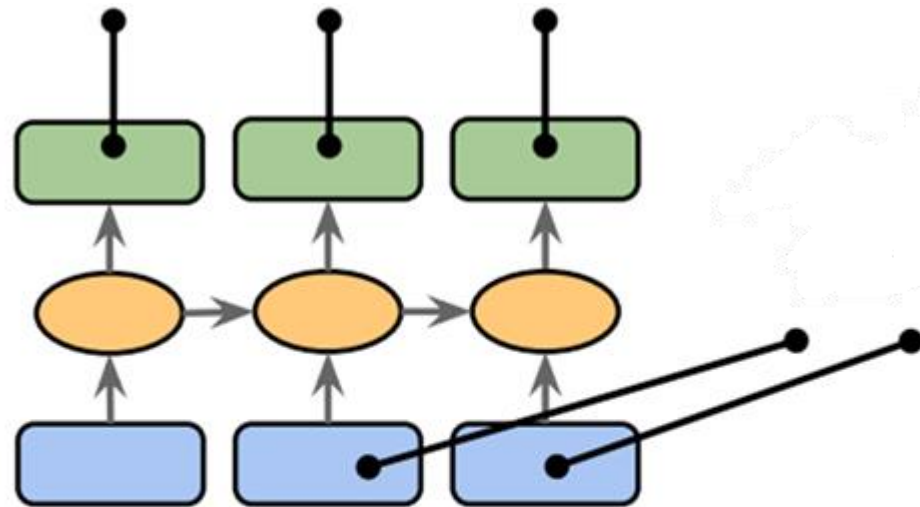
Recurrent Neural Networks

In RNNs, we asked about how backdrop through a network causes gradients can vanish or explode



Recurrent Neural Networks

At each time step, Softmax
output (probability) for each
possible 'next letter'



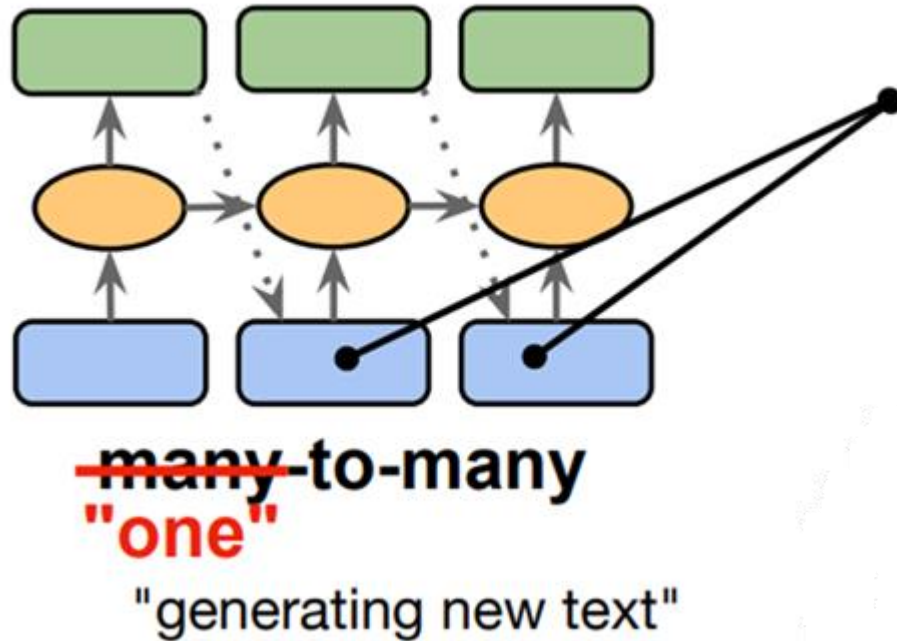
For the next input, ignore the
prediction but use the 'correct' next
letter from the dataset.

many-to-many

"training"



Recurrent Neural Networks



To generate new text, now display the softmax outputs and provide the letter as input for the next time step.



Long Short Term Memory Networks

<https://pytorch.org/docs/stable/generated/torch.nn.LSTM.html>

Parameters

- **input_size** – The number of expected features in the input x
- **hidden_size** – The number of features in the hidden state h
- **num_layers** – Number of recurrent layers. E.g., setting `num_layers=2` would mean stacking two LSTMs together to form a *stacked LSTM*, with the second LSTM taking in outputs of the first LSTM and computing the final results. Default: 1
- **bias** – If `False`, then the layer does not use bias weights b_{ih} and b_{hh} . Default: `True`
- **batch_first** – If `True`, then the input and output tensors are provided as (batch, seq, feature). Default: `False`
- **dropout** – If non-zero, introduces a *Dropout* layer on the outputs of each LSTM layer except the last layer, with dropout probability equal to `dropout`. Default: 0
- **bidirectional** – If `True`, becomes a bidirectional LSTM. Default: `False`
- **proj_size** – If > 0 , will use LSTM with projections of corresponding size. Default: 0

Examples:

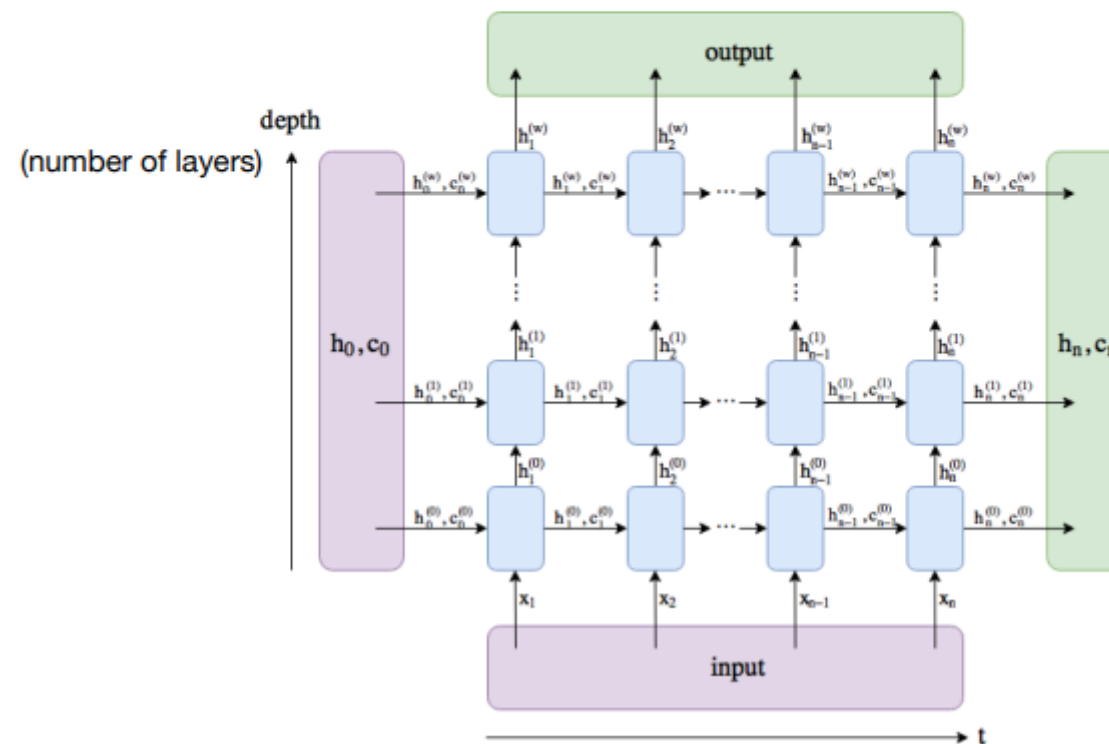
```
>>> rnn = nn.LSTM(10, 20, 2)
>>> input = torch.randn(5, 3, 10)
>>> h0 = torch.randn(2, 3, 20)
>>> c0 = torch.randn(2, 3, 20)
>>> output, (hn, cn) = rnn(input, (h0, c0))
```



LSTM

Examples:

```
>>> rnn = nn.LSTM(10, 20, 2)
>>> input = torch.randn(5, 3, 10)
>>> h0 = torch.randn(2, 3, 20)
>>> c0 = torch.randn(2, 3, 20)
>>> output, (hn, cn) = rnn(input, (h0, c0))
```



<https://pytorch.org/docs/stable/generated/torch.nn.LSTMCell.html>

Inputs: input, (h₀, c₀)

- **input** of shape (batch, input_size): tensor containing input features
 - **h₀** of shape (batch, hidden_size): tensor containing the initial hidden state for each element in the batch.
 - **c₀** of shape (batch, hidden_size): tensor containing the initial cell state for each element in the batch.
- If (h₀, c₀) is not provided, both **h₀** and **c₀** default to zero.

Outputs: (h₁, c₁)

- **h₁** of shape (batch, hidden_size): tensor containing the next hidden state for each element in the batch
- **c₁** of shape (batch, hidden_size): tensor containing the next cell state for each element in the batch

Examples:

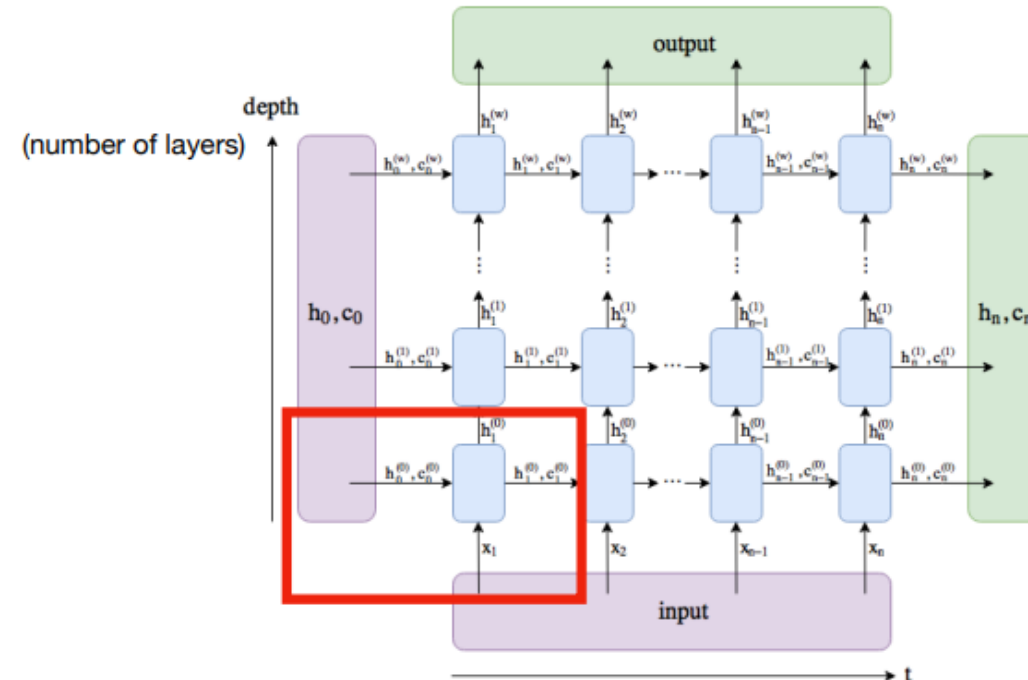
```
>>> rnn = nn.LSTMCell(10, 20) # (input_size, hidden_size)
>>> input = torch.randn(2, 3, 10) # (time_steps, batch, input_size)
>>> hx = torch.randn(3, 20) # (batch, hidden_size)
>>> cx = torch.randn(3, 20)
>>> output = []
>>> for i in range(input.size()[0]):
>>>     hx, cx = rnn(input[i], (hx, cx))
>>>     output.append(hx)
>>> output = torch.stack(output, dim=0)
```



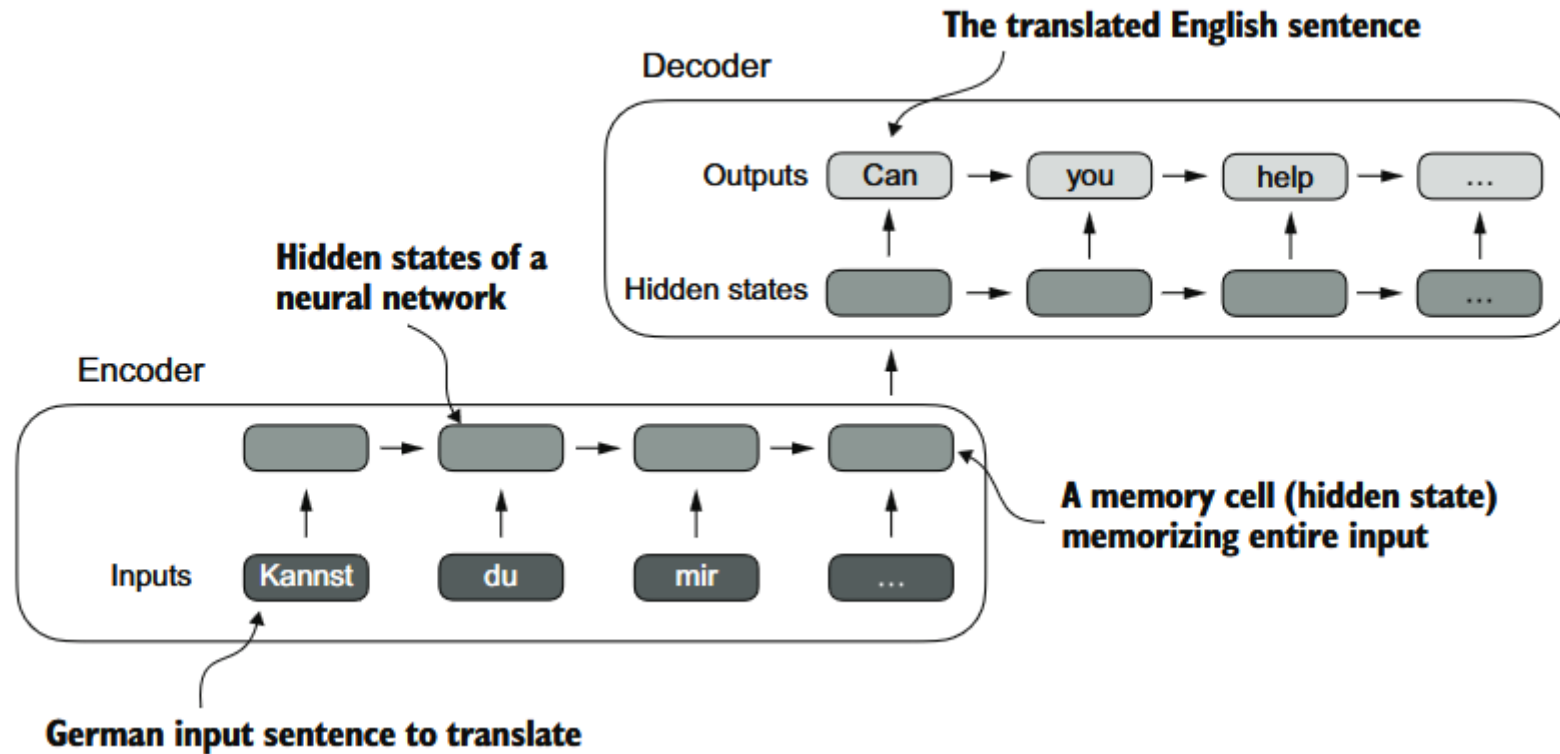
Clase LSTM

Examples:

```
>>> rnn = nn.LSTMCell(10, 20) # (input_size, hidden_size)
>>> input = torch.randn(2, 3, 10) # (time_steps, batch, input_size)
>>> hx = torch.randn(3, 20) # (batch, hidden_size)
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>>>     output.append(hx)
>>> output = torch.stack(output, dim=0)
```



Seq2seq

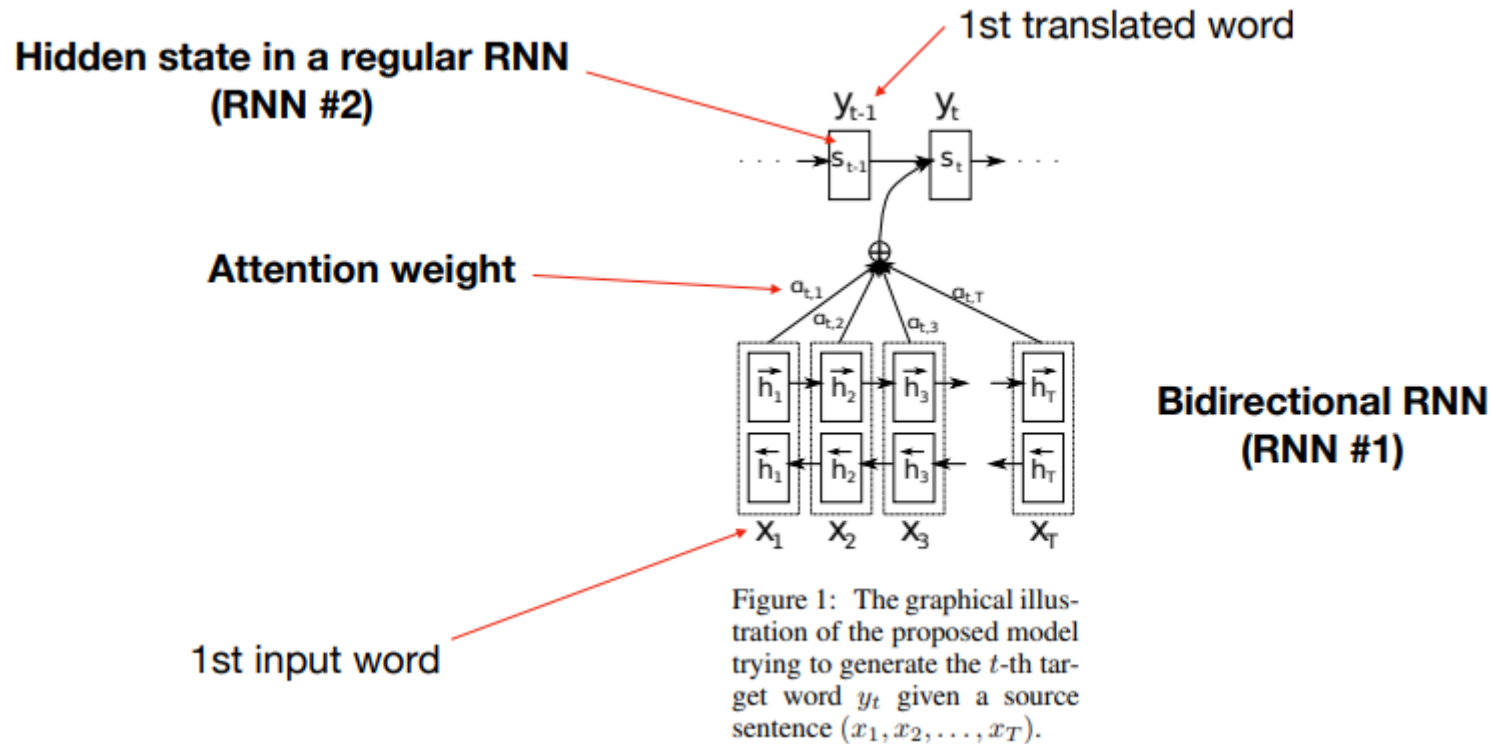


Attention mechanism

Assign an attention weight to each word to determine how much 'attention' the model should pay to each word (that is, for each word, the network learns a 'context').



Attention mechanism



NEURAL MACHINE TRANSLATION BY JOINTLY LEARNING TO ALIGN AND TRANSLATE

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ABSTRACT



Attention is all you need

Attention Is All You Need

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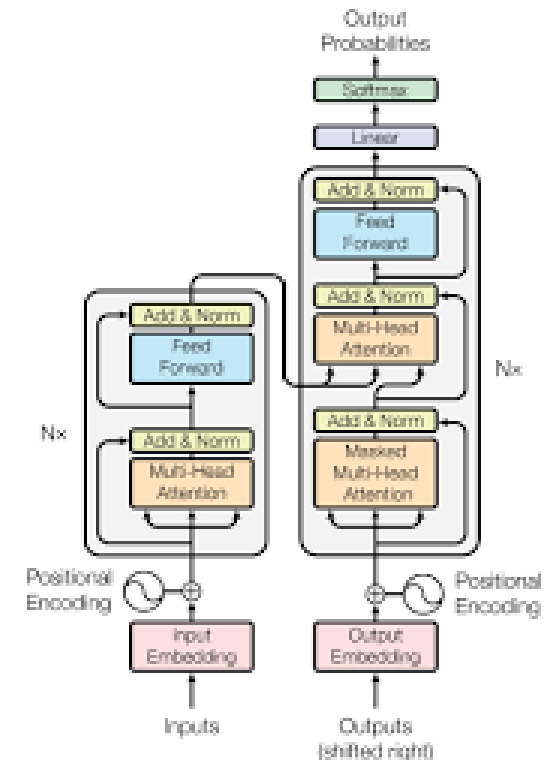
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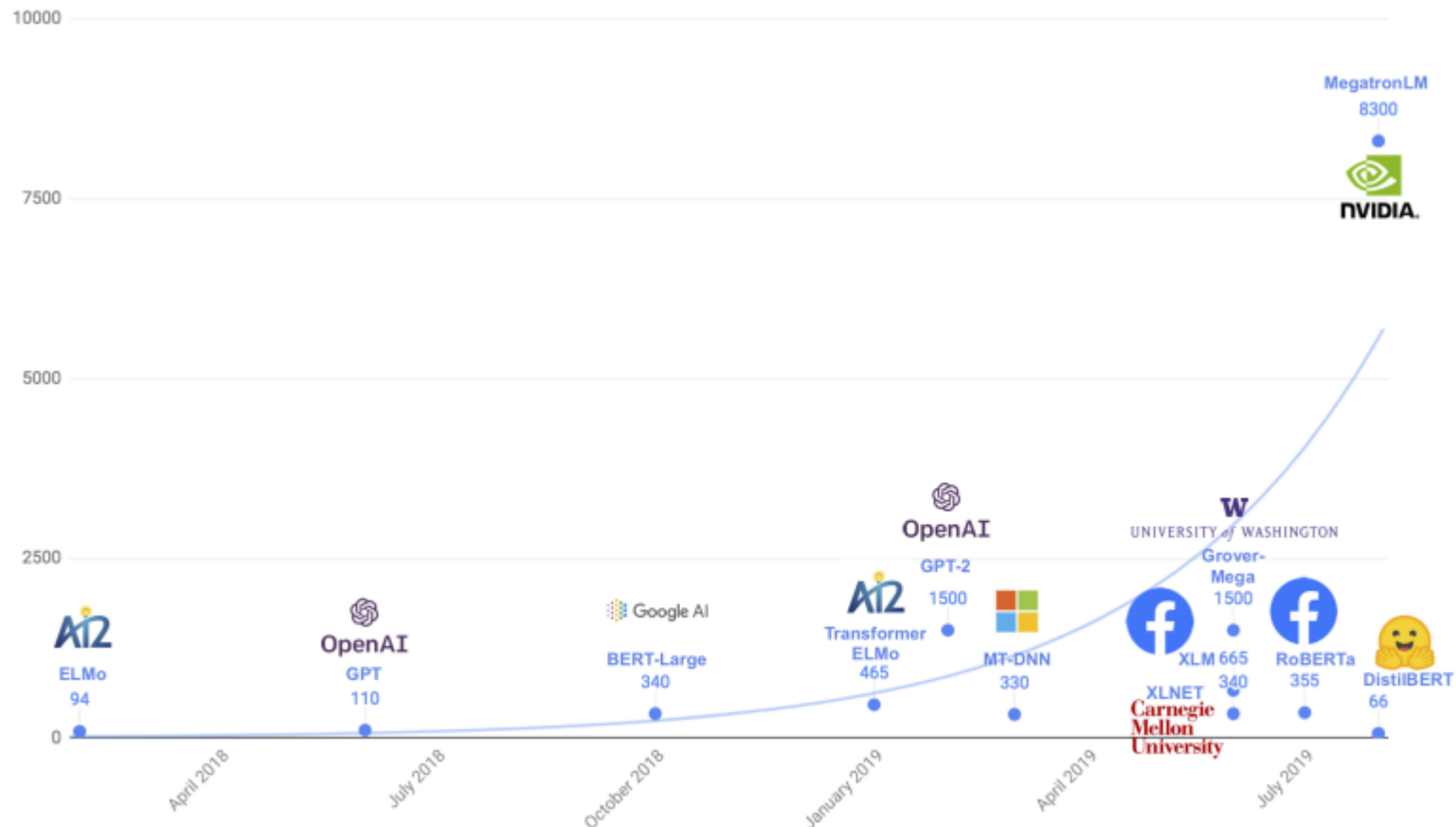
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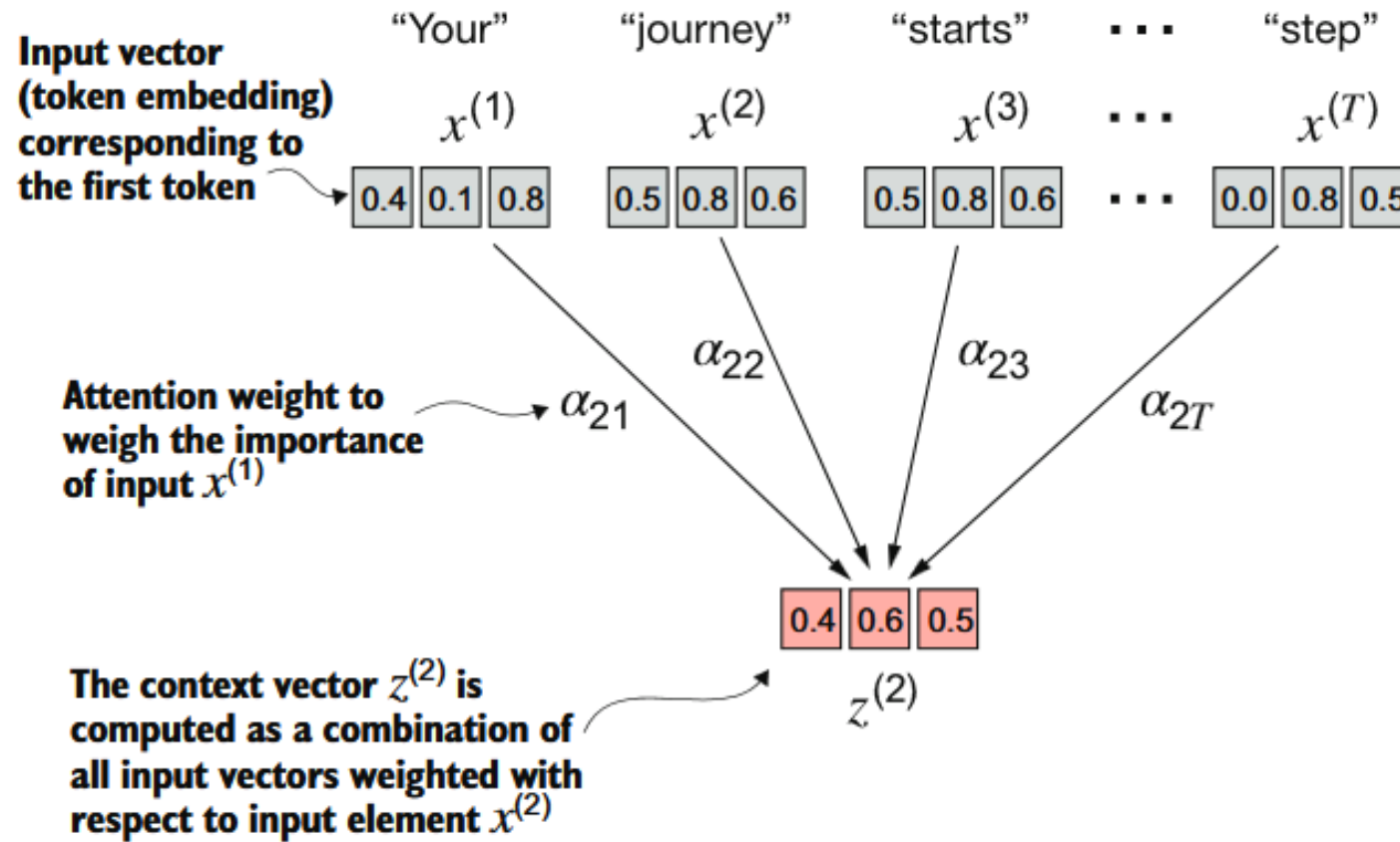
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Since ~2018, transformers have been growing in popularity... and size



A simple self-attention mechanism without trainable weights



A simple self-attention mechanism without trainable weights

Self-attention as weighted sum:

$$A_i = \sum_{j=0}^T a_{ij} \mathbf{x}_j$$

output corresponding to the i-th input

weight based on similarity between current input \mathbf{x}_i and all other inputs

How to compute the attention weights?

here as simple dot product:

$$e_{ij} = \mathbf{x}_i^\top \mathbf{x}_j$$

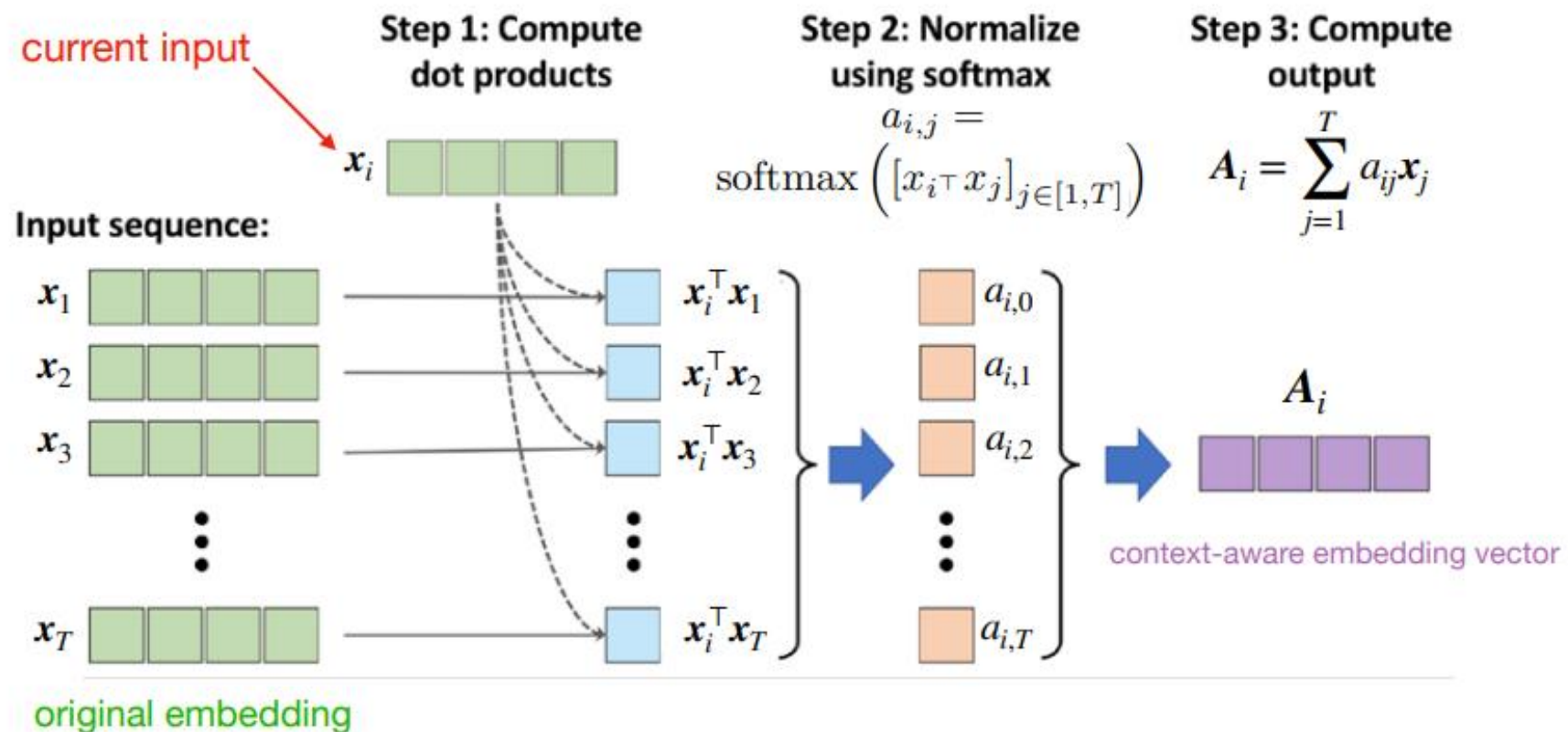
repeat this for all inputs $j \in \{1 \dots T\}$, then normalize

$$a_{ij} = \frac{\exp(e_{ij})}{\sum_{j=1}^T \exp(e_{ij})} = \text{softmax} \left(\left[e_{ij} \right]_{j=1 \dots T} \right)$$



A simple self-attention mechanism without trainable weights

Self-attention: Relating different positions within a single sequence (vs. between input and output sequences)



Self-attention mechanism

- Previous basic version did not involve any learnable parameters, so not very useful for learning a language model
- We are now adding 3 trainable weight matrices that are multiplied with the input sequence embeddings

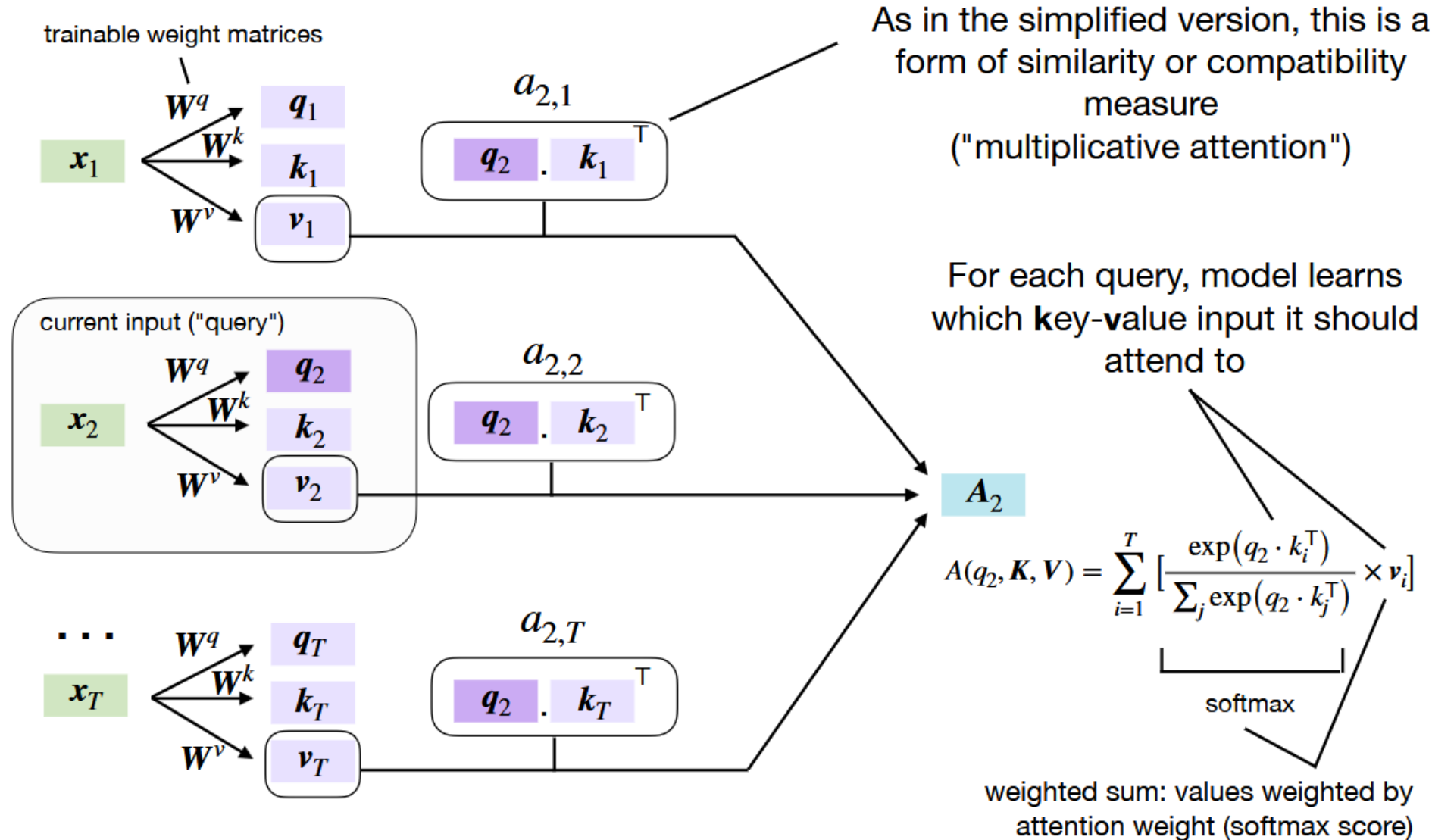
$$\text{query} = \mathbf{W}^q \mathbf{x}_i$$

$$\text{key} = \mathbf{W}^k \mathbf{x}_i$$

$$\text{value} = \mathbf{W}^v \mathbf{x}_i$$



Self-attention mechanism

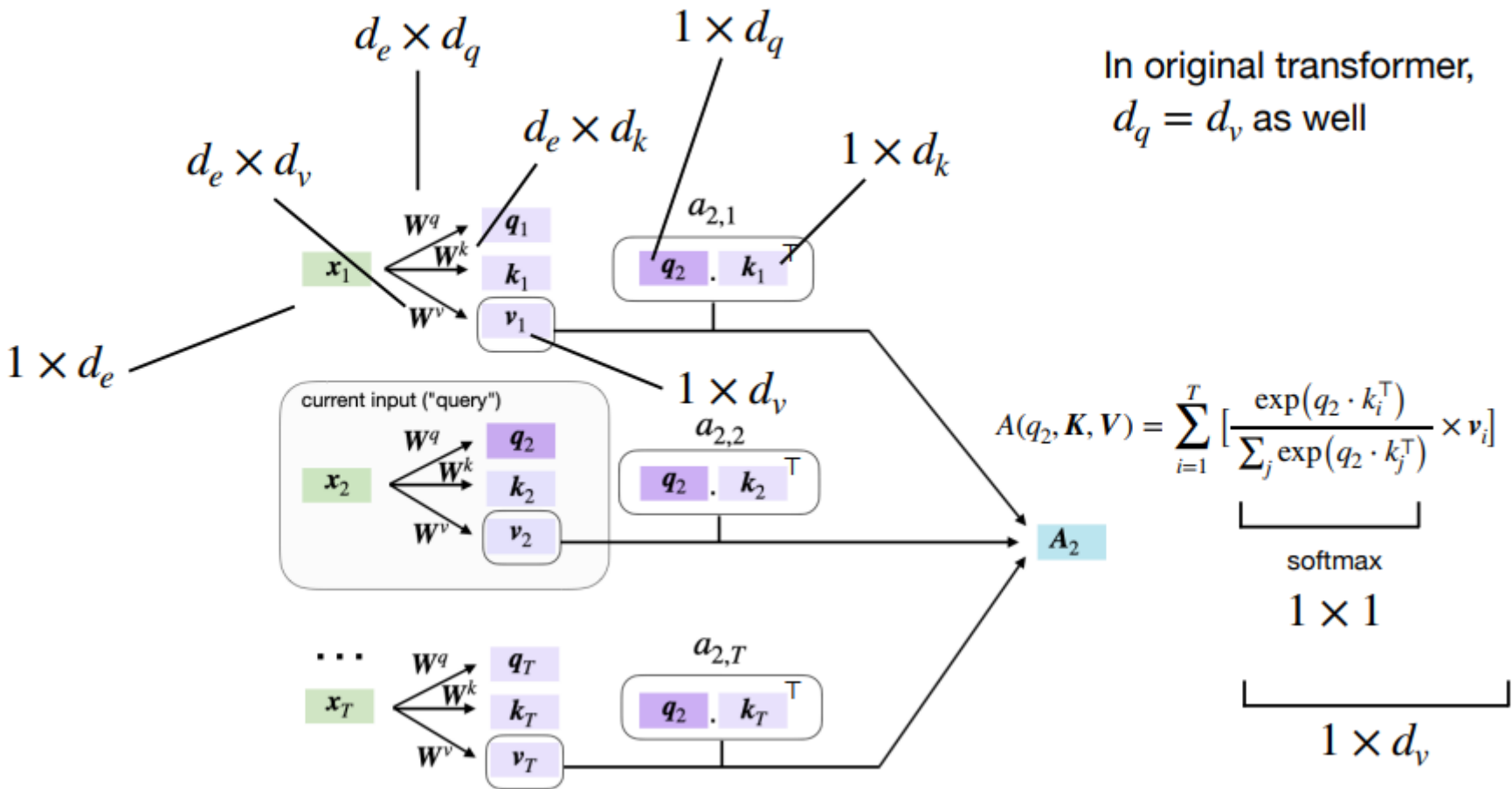


Self-attention mechanism

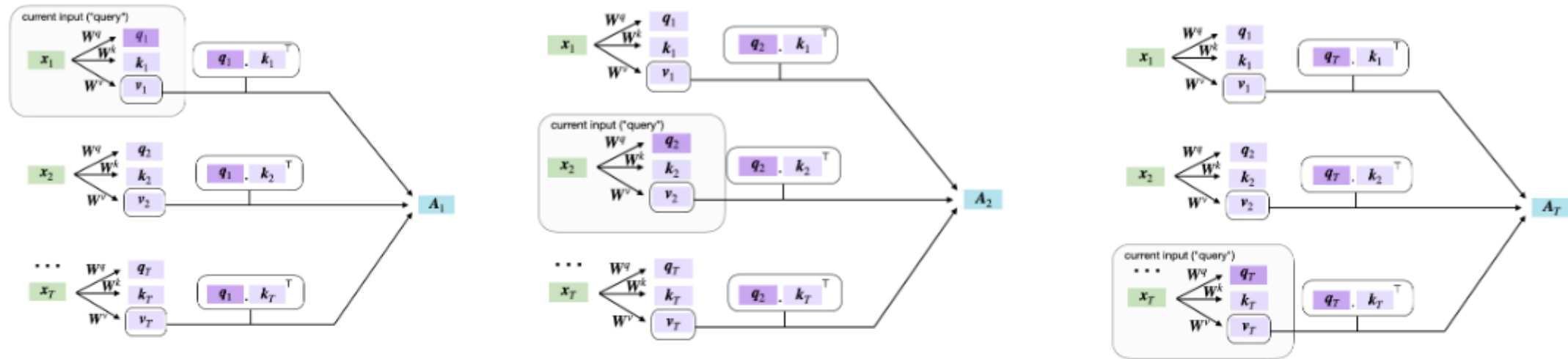
d_e = embedding size (original transformer = 512)

where $d_q = d_k$

In original transformer,
 $d_q = d_v$ as well



Self-attention mechanism



Attention score matrix: $A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}$

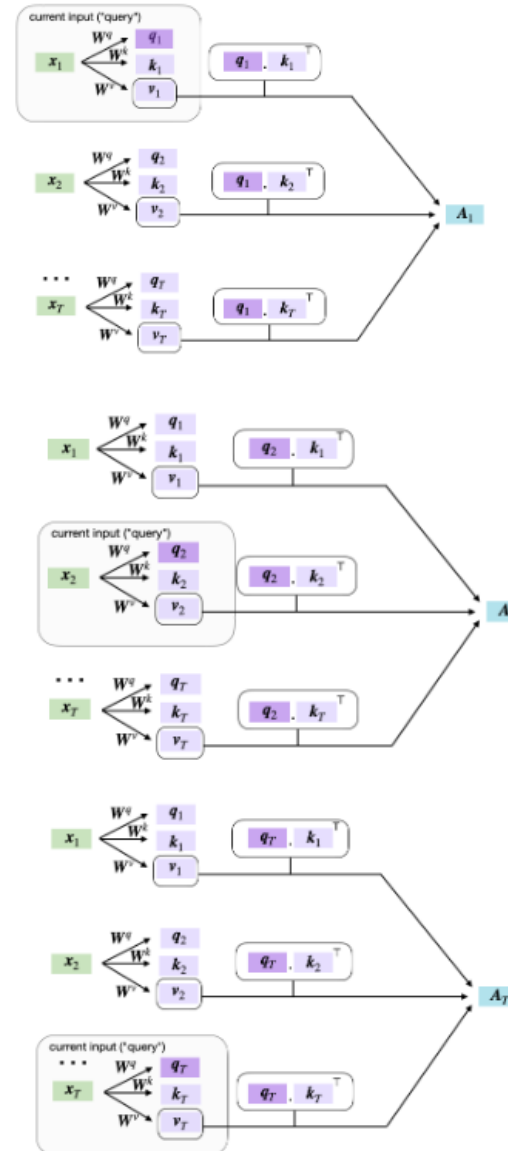


Self attention mechanism – Scaled dot product attention

d_e = embedding size

T = input sequence size

$$\mathbf{x} \in \mathbb{R}^{T \times d_e}$$



$$\mathbf{Q} \in \mathbb{R}^{T \times d_q}$$

$$\mathbf{K} \in \mathbb{R}^{T \times d_k}$$

$$\mathbf{V} \in \mathbb{R}^{T \times d_v}$$

"attention matrix"
 $T \times T$

$$A(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \left[\text{softmax} \left(\frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d_k}} \right) \mathbf{V} \right]$$

$T \times d_v$

"attention-based
embedding"

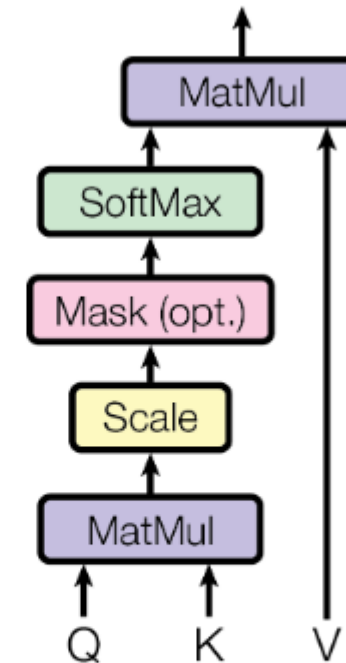


Scaled dot product attention

$$A(Q, K, V) = \text{softmax} \left(\frac{QK^T}{\sqrt{d_k}} \right) V$$

To ensure that the dot-products
between query and key don't
grow too large (and softmax
gradient become too small) for
large d_k

Scaled Dot-Product Attention



Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L. and Polosukhin, I., 2017. Attention Is All You Need.



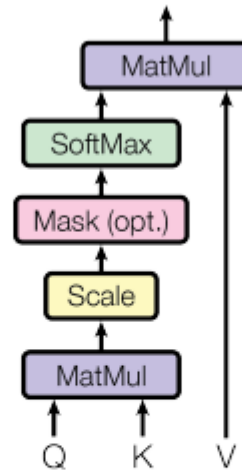
Multi-Head attention

- Apply self-attention multiple times in parallel (similar to multiple kernels for channels in CNNs)
- For each head (self-attention layer), use different $\mathbf{W}^q, \mathbf{W}^k, \mathbf{W}^v$ then concatenate the results $\mathbf{A}_{(i)}$.
- 8 attention heads in the original transformer.
- Allows attending to different parts in the sequence differently.



Multi-Head attention

Scaled Dot-Product Attention



Multi-Head Attention

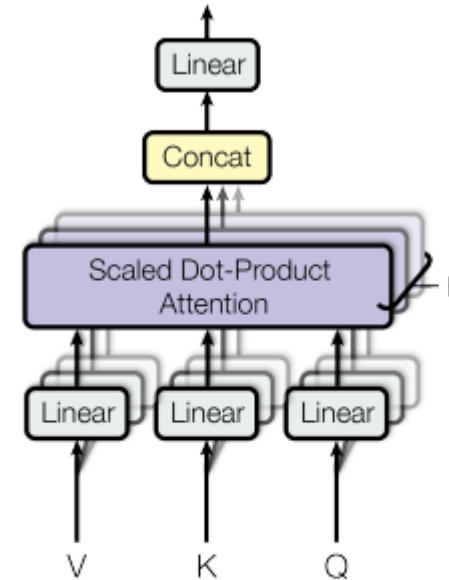
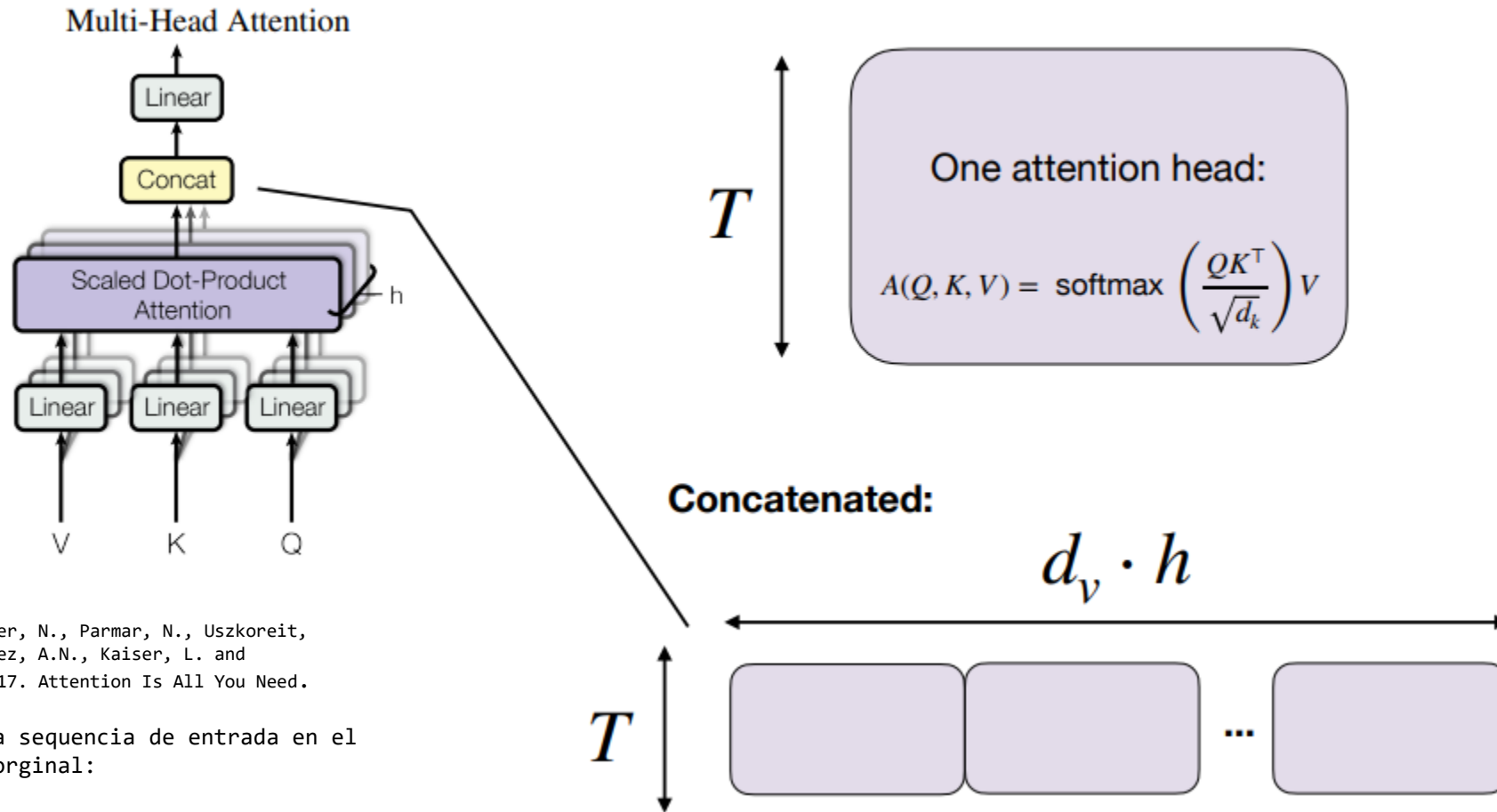


Figure 2: (left) Scaled Dot-Product Attention. (right) Multi-Head Attention consists of several attention layers running in parallel.

Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L. and Polosukhin, I., 2017. Attention Is All You Need.



Multi-Head attention



Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L. and Polosukhin, I., 2017. Attention Is All You Need.

Dimensión de la secuencia de entrada en el transformador original:

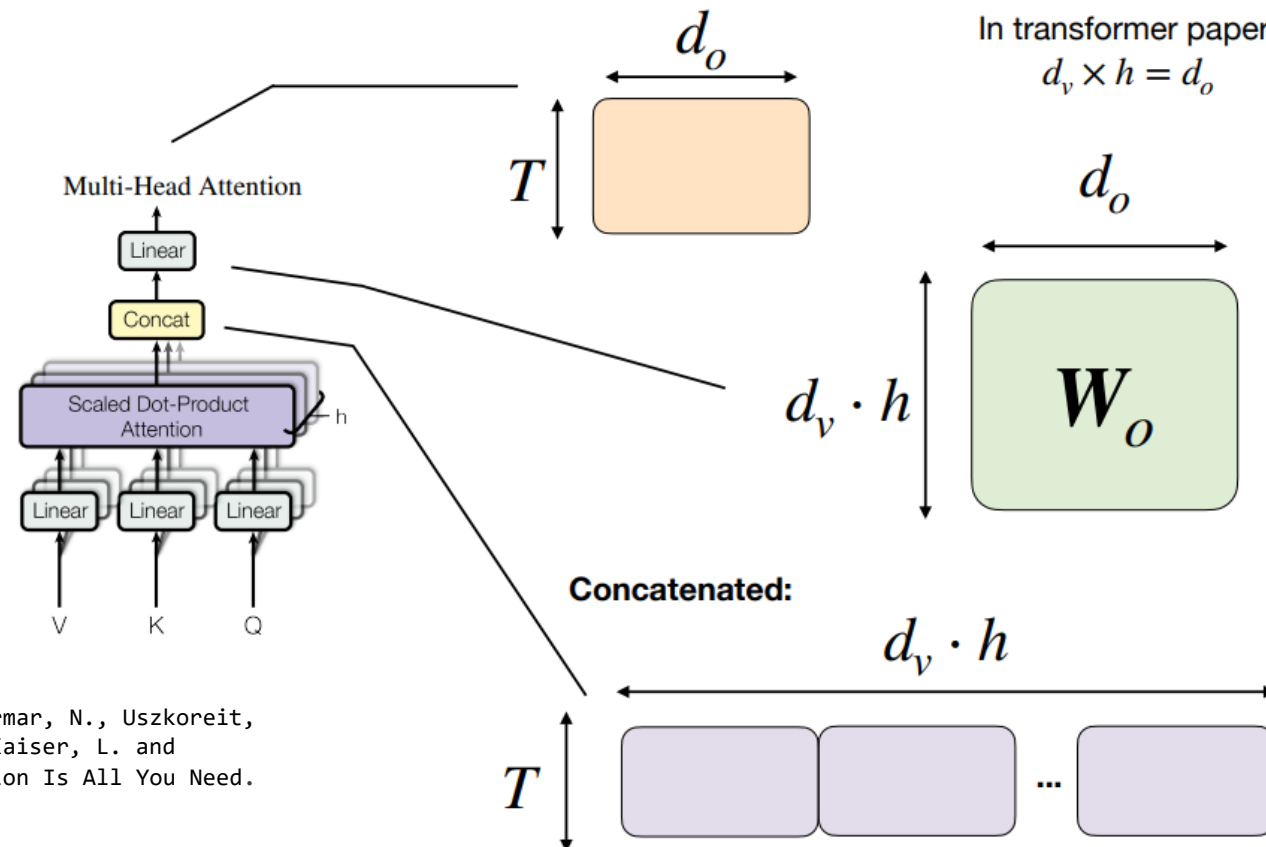
$$T \times d_e = T \times 512$$

y

$$d_v = 512/h = 64$$



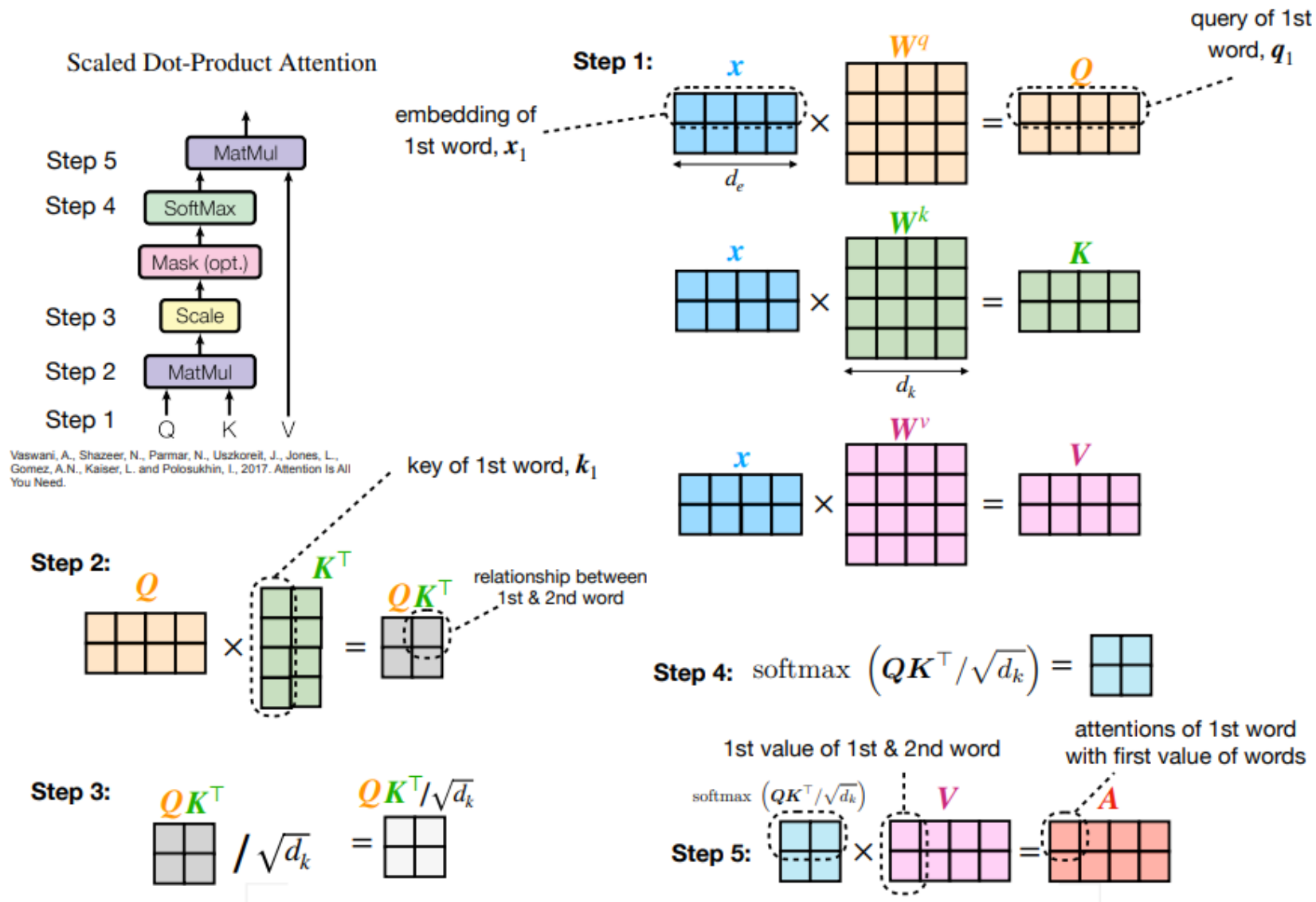
Multi-Head attention



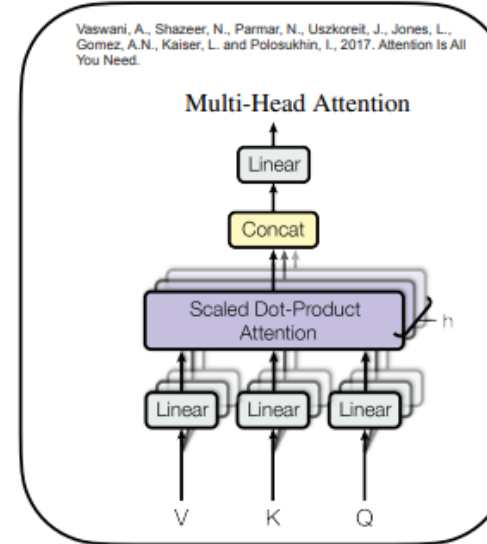
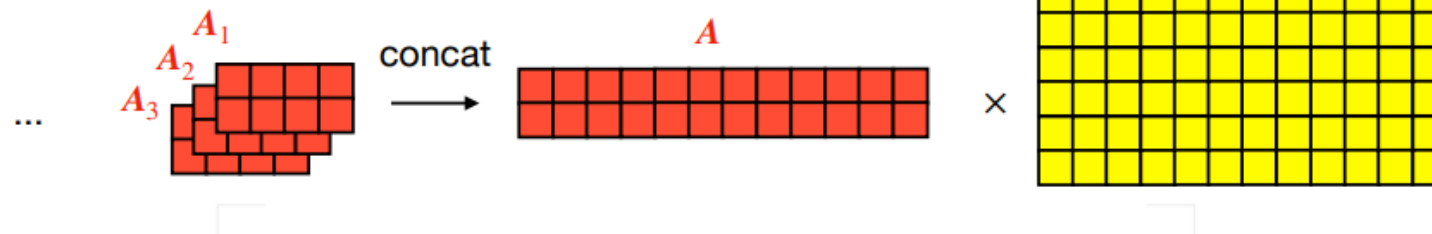
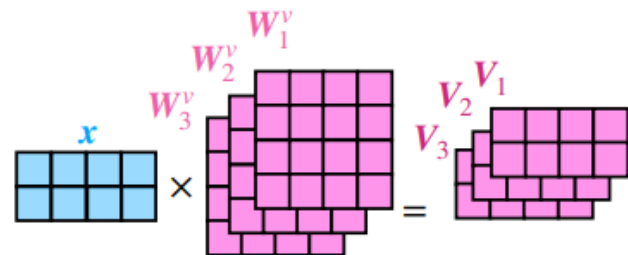
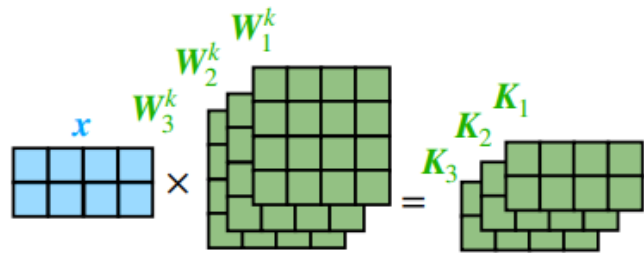
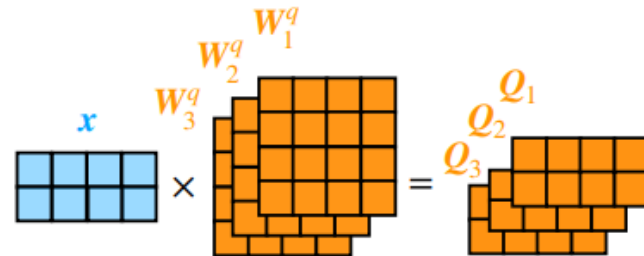
Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L. and Polosukhin, I., 2017. Attention Is All You Need.



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Scaled Dot-Product Attention Recap



Effects of Multi-Heads

- Example from Vaswani et al

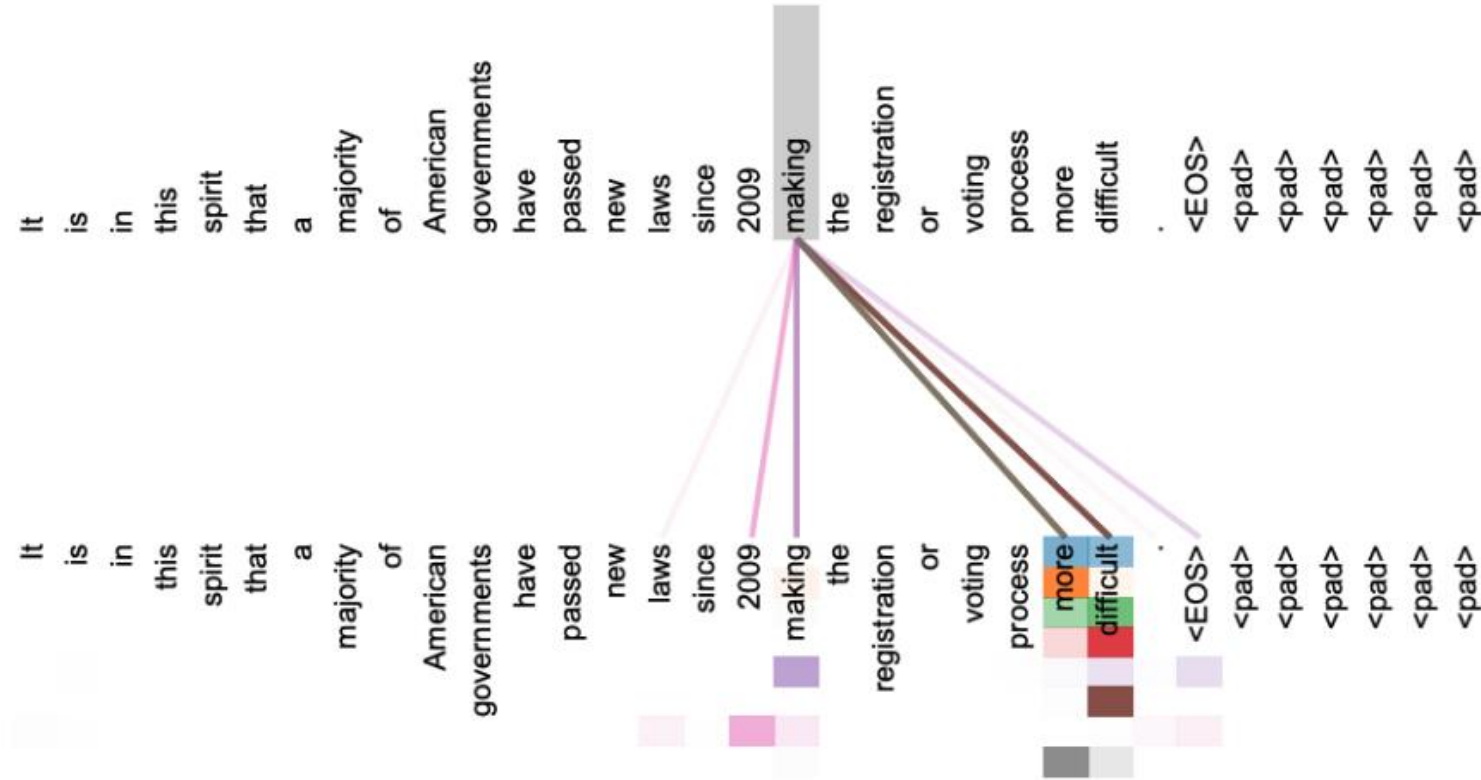


Figure 3: An example of the attention mechanism following long-distance dependencies in the encoder self-attention in layer 5 of 6. Many of the attention heads attend to a distant dependency of the verb 'making', completing the phrase 'making...more difficult'. Attentions here shown only for the word 'making'. Different colors represent different heads. Best viewed in color.



Recommended reading

<https://jalammar.github.io/illustrated-transformer/>



Layer Normalization

- **Internal Covariate Shift** occurs when the activation distributions of neurons change across training steps, forcing frequent weight adjustments.
- This phenomenon slows down training by causing instability in optimization.
- It happens when earlier layers undergo significant updates, leading to drastic changes in the inputs of subsequent layers.

	f1	f2	f3	μ	σ^2		f1	f2	f3
Item 1	a_1	a_2	a_3	μ_1	σ_1^2	Item 1	a'_1	a'_2	a'_3
Item 2						Item 2			
Item 3						Item 3			
Item 10						Item 10			

X =
(10, 3)

X' =

$$y = \frac{x - \mathbb{E}[x]}{\sqrt{\text{Var}[x] + \epsilon}} \cdot \gamma + \beta$$

- **Batch Normalization** normalizes across **columns (features)**.
- **Layer Normalization** normalizes across **rows (data items)**.



Root Mean Square Layer Normalization

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4 RMSNorm

A well-known explanation of the success of LayerNorm is its re-centering and re-scaling invariance property. The former enables the model to be insensitive to shift noises on both inputs and weights, and the latter keeps the output representations intact when both inputs and weights are randomly scaled. In this paper, we hypothesize that the re-scaling invariance is the reason for success of LayerNorm, rather than re-centering invariance.

We propose RMSNorm which only focuses on re-scaling invariance and regularizes the summed inputs simply according to the root mean square (RMS) statistic:

$$\bar{a}_i = \frac{a_i}{\text{RMS}(\mathbf{a})} g_i, \quad \text{where } \text{RMS}(\mathbf{a}) = \sqrt{\frac{1}{n} \sum_{i=1}^n a_i^2}. \quad (4)$$

Intuitively, RMSNorm simplifies LayerNorm by totally removing the mean statistic in Eq. (3) at the cost of sacrificing the invariance that mean normalization affords. When the mean of summed inputs is zero, RMSNorm is exactly equal to LayerNorm. Although RMSNorm does not re-center



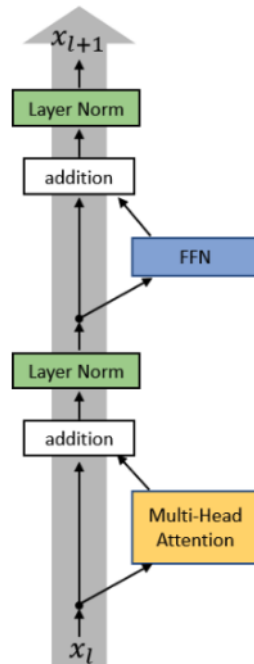
Residual Connections

- Add an additive connection between the input and output

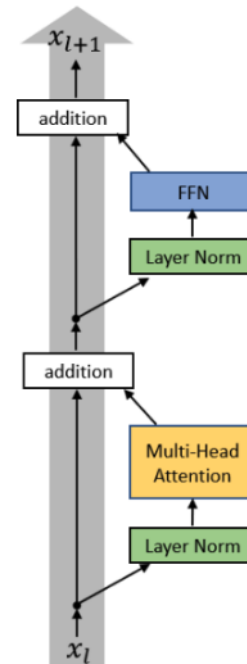
$$\text{Residual}(\mathbf{x}, f) = f(\mathbf{x}) + \mathbf{x}$$

- Prevents vanishing gradients and allows f to learn the difference from the input
- Pre-layer-norm is better for gradient propagation

- Post-layer normalization



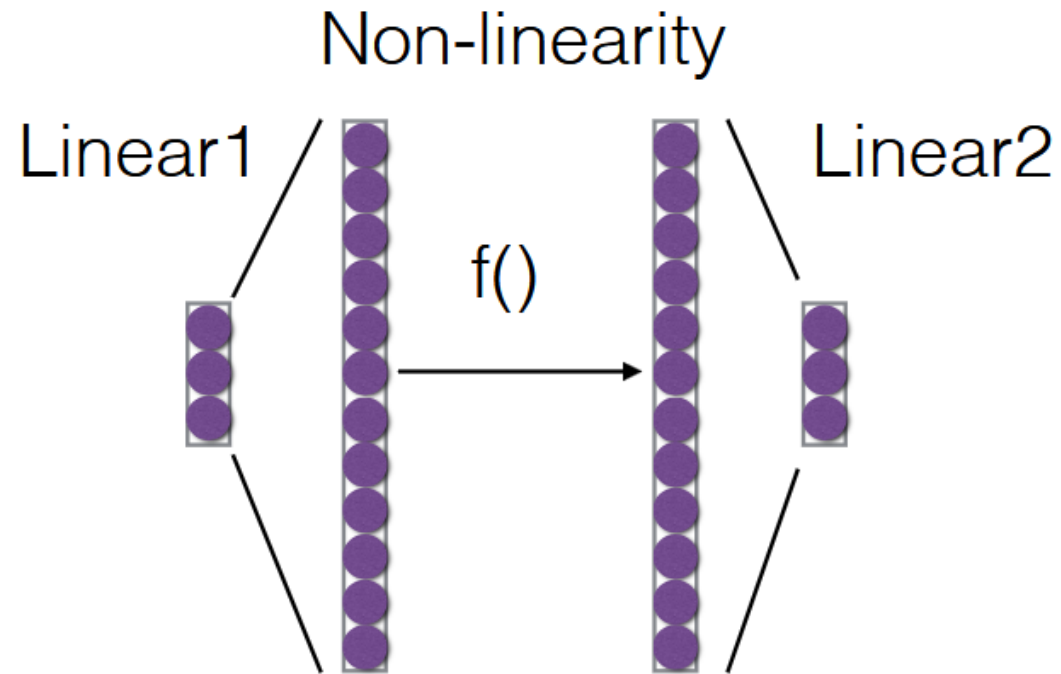
- Pre-layer normalization



Feed Forward Layers

- Extract combination features from the attended outputs

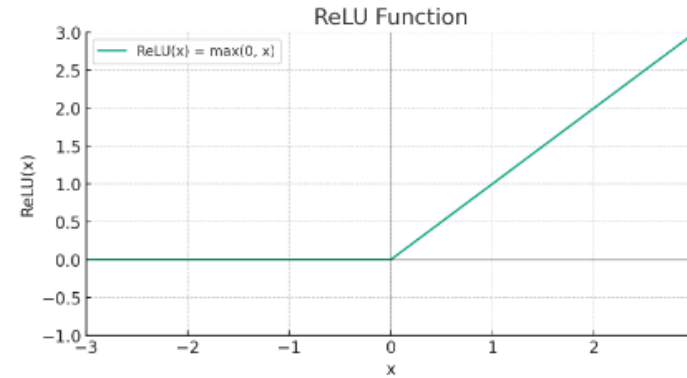
$$\text{FFN}(x; W_1, \mathbf{b}_1, W_2, \mathbf{b}_2) = f(\mathbf{x}W_1 + \mathbf{b}_1)W_2 + \mathbf{b}_2$$



Feed Forward Layers

- Vaswani et al.: ReLU

$$\text{ReLU}(\mathbf{x}) = \max(0, \mathbf{x})$$



- LLaMa: Swish/SiLU (Hendricks and Gimpel 2016)

$$\text{Swish}(\mathbf{x}; \beta) = \mathbf{x} \odot \sigma(\beta \mathbf{x})$$

