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Vagueness: Supervaluationism

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Abstract

This piece gives an overview of the supervaluationist theory of vagueness. According to that theory, a sentence is true if and only if it is true on all ways of making it precise. This yields borderline case predications that are neither true nor false, but yet classical logic is preserved almost entirely. The article presents the view and some of its merits and briefly compares it with other theories of vagueness. It raises issues about higher-order vagueness and the definitely operator and about the supervaluationist's accounts of truth and validity.

Vague predicates, such as 'tall', 'bald', 'heap', 'rich', 'baby', typically have borderline cases, apparently lack sharp boundaries and are susceptible to sorites paradoxes. Consider, for example, the question whether my young child is still a baby (rather than, for example, a toddler). There is not an instant at which it suddenly becomes false to say that she is a baby, and there are stages at which she is a borderline case of a baby – neither clearly a baby nor clearly not one. A sorites paradox can be formulated when we formulate the compelling premise 'if a child is a baby at some instant, then it is still a baby one second later'. For we can use this to argue from the true premise that a three-day old child is a baby to the false conclusion that it is still a baby at 5 years old.[1] What is the correct treatment of vague language with these distinctive features? What is its logic and semantics and how is the sorites paradox to be solved?[2]

In this paper I will discuss the *supervaluationist theory of vagueness*. A popular way to present this theory is in terms of semantic indecision. No definite extension is settled to be the extension of a vague predicate such as 'rich' or 'tall', rather there is a range of possible extensions, and it is semantically unsettled which is the extension. The various possible extensions correspond to different ways of making the predicate precise and the supervaluationist idea is that truth conditions involve quantifying over all those ways of making language precise. The semantics of our language may not settle whether Tek is tall if he's in that tricky borderline area, but it does settle that it's true that Bruno is tall when he's way above that area at 6 foot 8. For the supervaluationist, this corresponds to the fact that on all the ways of making 'tall' precise, Bruno will fall on the tall side and thus count as tall. Even though nothing settles the choice between a range of precise extensions for 'tall', those extensions all agree that Bruno is tall which is enough for him to count as tall simpliciter. The supervaluationist's truth-conditions are thus given as follows:

> (SV) a sentence is true if and only if it is true on all ways of making it precise, false iff it is false on all those ways and neither true nor false otherwise.

The supervaluationist theory of vagueness is explained and defended in detail in Fine. Earlier expositions of the view appeared in Mehlberg, Lewis and Dummett (where it is rejected), while the same technique had been developed for other applications – empty singular terms and the liar paradox – in van Fraassen ('Singular Terms, Truth-Value Gaps, and Free Logic'; 'Presupposition, Implication, and Self-Reference'). I defend a supervaluationist theory of vagueness in *Theories of Vagueness* and a broadly supervaluationist framework has been employed in McGee and McLaughlin ('Distinctions without a Difference'), Shapiro and elsewhere.

The view is committed to truth-value gaps, since borderline case predications will be true on some precisifications and false on others, so count as neither true nor false. But although the principle of bivalence is thereby rejected, classical logic is nonetheless preserved (at least in the most part: see below). Consider 'either Miranda is a baby or she isn't'. However we make 'is a baby' precise, this disjunction will be true, so it counts as true simpliciter according to the supervaluationist, despite the fact that neither disjunct is true simpliciter. This is a major advantage of the view. But supervaluationists' opponents have sometimes objected to the idea of a disjunction which is true even though neither disjunct is true and a corresponding feature of existential quantifications (which can be true without a true instance). The supervaluationist's semantic anomalies are defended in *Theories of Vagueness* (181–8).

The supervaluationist responds to the sorites paradox by showing how the main premise comes out false. Consider the main premise of the form 'for all i, if Fx_i then Fx_{i+1}', where the x_i form an appropriate sorites series (e.g. for tall, a series of people one-hundredth of an inch shorter than the previous one, where the first is 7 foot tall). For any way of making F precise, there will be a pair in the series such that one is F and the next is not-F, so that the quantified premise comes out false. Since it is false for each precisification, it counts as false simpliciter, even though there is no instance that is false simpliciter, because different precisifications draw different boundaries and thus have different false instances. Given that there is no false instance of the premise – no pair of which it is true to say they straddle the boundary – the supervaluationist can accommodate the intuition that vague F has no sharp boundary.

To illustrate some of the attractions of the supervaluationist view, I will briefly summarise some other major theories of vagueness with which to contrast it. First, take a view that preserves classical logic without rejecting the principle of bivalence. According to the *epistemic view of vagueness* (e.g. Williamson, *Vagueness*), vague predicates have unknown sharp boundaries and borderline case predications are true or false, we just do not know which. This view denies that there is semantic indecision: something – boundaries in the world or, more likely, our use – somehow fixes a unique sharp extension to each vague predicate. Supervaluationists avoid the difficult question facing this view – what determines these sharp extensions and boundaries – and they maintain the attractive position of denying that any such unique extensions are determined.

Other views that refuse to associate a vague predicate with a unique classical extension, employ many-valued logics. Here, a borderline case predication is neither true nor false, and is treated as taking some intermediate truth-value. On some such theories there is a single intermediate value, whereas on others there is a whole range of degrees of truth, represented by the real numbers between 0 and 1. There is disagreement over which many-valued logic to employ, but whichever is chosen, the divergence from classical logic is more extreme than for the supervaluationist.

The treatment of the connectives is typically truth-functional for the many-valued theorist, which marks another central divergence from supervaluationism.[3] Insisting on truth-functionality leads to some highly unintuitive classifications. For example, a conjunction whose conjuncts both take intermediate values will come out with some intermediate value, as would be expected if we consider 'Tek is tall and Fred is fat' when Tek is borderline tall and Fred is borderline fat. But some conjunctions of this form should intuitively be false – e.g. 'Tek is tall and Todd is not tall', where Todd is also borderline tall, but taller than Tek. Similar examples could be run with other connectives, e.g. 'If Tek is tall then Todd is tall' should be true and 'If Tek is tall then Todd is not tall' should be false, but both will come out neither true nor false on the many-valued theory.

Cases such as these exhibit what Fine calls *penumbral connections* – 'logical relations [that] hold among indefinite sentences', such as the fact that 'red' and 'pink' are exclusive, so 'the blob is red and pink' is false even if it is a borderline case of both colours. Any way of making 'red' and 'pink' precise together will ensure that a blob does not count as both red all over and pink all over, so on all precisifications it will be false that 'the blob is red and pink', thereby rendering that sentence false simpliciter. Similarly, any way of making 'tall' precise will make 'If Tek is tall then Todd is tall' true, when Todd is taller than Tek, so this sentence is true on all precisifications and so, appropriately, true simpliciter.

So, supervaluationism can accommodate penumbral connections and preserve classical logic, while still accommodating the intuitions that nothing determines unique classical extensions for our vague terms and that borderline case predications are neither true nor false.

Supervaluationism also provides a relatively popular way to solve Unger's 'Problem of the Many'. Suppose you look up on a reasonably clear day and report that there is one cloud in the sky. The cloud you are referring to does not have sharp boundaries – there will be droplets of which it is unclear whether or not they are part of the cloud. The problem is that there seem to be too many things in the sky on that apparently one-clouded day, that each looks to be perfect candidates for being clouds (they have the right composition etc.). For, you could compose candidate clouds over and over again by including different sub-sets of those penumbral droplets. The supervaluationist response can recognise all these objects as candidate clouds, leaving it indeterminate which is to count as the cloud. According to different precisifications, different objects will count as the relevant one, but according to all precisifications it will be true that there is just one cloud there. And if you name the cloud, say 'Clara', it will be definitely true that Clara is a cloud because on any precisification, the object counting as the cloud is the same object as that which counts as Clara – there is a penumbral connection here.[4]

Supervaluationist Models (in Brief)

The supervaluationist semantics are built on assignments of truth-values to sentences of the language, which Fine calls *specifications*. A set of such specifications can form a *specification space*, which, roughly, correspond to the framework of all possible ways of making the language precise. Different specifications differ over the assignments to vague sentences; for example when 'Miranda is a baby' is intuitively a borderline case, this will be reflected in its being true at some specifications and false at others. Similarly, they assign different extensions to vague predicates (for 'baby' perhaps it is the set of all children under 1 on one specification and those under 1 and a half on another). *Complete* specifications are those on which all sentences are either true or false – all vague expressions have been made completely precise and classical logic holds. Other specifications correspond to merely partial precisifications of at least some of the vague expressions of the language – so some expressions are not made completely precise, resulting in some sentences remaining neither true nor false on the specification. Among these specifications is the *base point*, where the actual values of sentences are assigned prior to any precisifying, including all neither true nor false borderline case predications.

One specification, s, is said to *extend* another, t, iff, intuitively, s corresponds to a way of making t at least as precise. If a sentence is true at some specification, then it is true at all specifications that extend it. This is Fine's *Stability* requirement: if a sentence is already true, then making it more precise cannot alter that. According to Fine's *Completability* requirement, every specification can be extended to a complete specification: i.e. there is always some way of making the language *completely* precise, eliminating all truth-value gaps. A specification space consists of all the admissible specifications and a sentence is super-true iff it is true on all complete specifications in the space. This amounts to the same as being true at the privileged base point given the various requirements Fine imposes on the space.

I take a supervaluationist theory of vagueness to be one that employs this kind of framework, including the identification of truth with super-truth. But some theorists have been tempted by aspects of the framework, without adopting the whole package. (Whether these, or any of these, should also be called 'supervaluationism' is merely a terminological question.) Take, for example, Fine's completability requirement stating that every specification can be extended to a complete specification. Such specifications, or sharpenings, of our language need not be practically accessible to us in any way – they are merely a technical device that delivers a powerful and plausible account of the truth-value of vague sentences.[5] Nonetheless, Shapiro rejects completability in the light of his views on tolerance principles – principles which correspond to the main premises of sorites paradoxes and state that being *F* is tolerant to small changes of the relevant kind, e.g. something remains a heap if it loses a single grain. These tolerance principles should count as expressing penumbral connections, Shapiro argues, which would render completely sharp specifications inadmissible since they violate such principles. He explores how a supervaluationist framework can be used without relying on completely sharp specifications, where he is prepared to abandon the classical logic that results from the completability requirement (see e.g. Shapiro 71).[6]

Higher-Order Vagueness and the D Operator

Not only do most of our predicates have borderline cases, but those borderline cases are not sharply bounded. This observation leads to discussion of *higher-order vagueness* – the possibility of borderline borderline cases, borderline borderline borderline cases and so on and the iteration of the phenomena of vagueness above the first level. Let us, as is standard, introduce a 'definitely' operator, D, where Dp, means that it is definitely the case that p (which, for the supervaluationist, is true iff p is true on all precisifications), and a borderline case is one in which $\neg Dp \ \& \ \neg D\neg p$. Recognising higher-order vagueness is then naturally equated with sentences for which it is true that $\neg DDp \ \& \ \neg D\neg Dp$ or that $\neg D\neg D\neg p \ \& \ \neg DD\neg p$ – so that there are borderline cases between the definite cases and the borderline ones and between the borderline cases and the definitely-not cases.

Any theory of vagueness must recognise and accommodate this phenomenon and not simply avoid problems with the boundary between the Fs and the not-Fs by postulating a precise category of in-between cases. And many theories of vagueness have been thought to fall at this hurdle. In particular, it may look as if, for any vague predicate F, a supervaluationist theory divides things into three precise categories – those that are F (because F on all precisifications), those that are not-F (because F on no precisifications) and those for which it is neither true nor false that they are F (because they are F on some precisifications and not on others). In any specification space there will be a determinate set of admissible precisifications and a corresponding determinate set of neither true nor false sentences. How can the supervaluationist accommodate higher-order vagueness while employing this framework?

In *Theories of Vagueness* (ch. 8), I attempt to accommodate higher-order vagueness by appealing to a vague metalanguage. 'Admissible specification' is itself vague as there is no sharp boundary to the acceptable ways of making, say, 'bald' precise. We cannot settle for a story that fixes the admissible specifications once and for all, and this thought seems to require us to iterate the supervaluationary technique to higher-orders. There will be no single intended specification space, but a range of admissible ones over which we need to supervaluate.

To accommodate higher-order vagueness, we might also employ the idea of relative admissibility. Within a specification space, the specifications that count as admissible vary relative to different specifications. Here we can utilise modal logics by interpreting the necessity operator as 'definitely' or D. If Dp is true at a specification when p is true at all specifications that are admissible relative to it, then Dp can then be true at some specifications within a space at not at others, so indefinite overall (see e.g. Williamson, 'On the Structure of Higher-Order Vagueness' on this framework). For recent discussion of problems facing the supervaluationist in connection with higher-order vagueness, see Greenough and Fara.

The introduction of a D operator into the supervaluationist language also threatens the claim that supervaluationary logic is classical. For it seems that the argument 'A therefore DA' must be valid, for if A is true (i.e. true on all specifications) then so is DA. But we cannot then apply conditional proof to conclude that 'A ⊃ DA' is valid, for, given borderline A, DA is super-false and 'A ⊃ DA' is false on those specifications where A is true. So conditional proof appears to fail in the presence of the D operator. Other classical rules of inference also come under attack. Take contraposition and, again, take the argument 'A therefore DA'. By contraposition, we should be able to infer the validity of '¬DA therefore ¬A'. But this latter argument is invalid: for borderline A, ¬DA is super-true but ¬A is not super-true. Machina (51–3) and Williamson (*Vagueness* 151–2) argued that these and other classical rules of inference (e.g. *reductio ad absurdum* and argument by cases) fail on the supervaluationist framework with D.

Do these cases show that supervaluationist logic is not classical after all? And, if so, is that a damaging objection to the view? The apparent demonstration of the failure of the rules above and in Machina and Williamson assumes a particular account of validity in the supervaluationist framework. One response is then to challenge this account of validity: see below on alternative characterisations of validity, which, for example, block the counterexamples by invalidating the inference from A to DA. A different response is to accept the counterexamples, but argue that the failure of these rules in the presence of the D operator is a desirable feature of the view, not a problematic one, given that D is a non-classical notion (see Keefe, *Theories of Vagueness* 176–81).[7]

Truth and Validity

Fine and Keefe (*Theories of Vagueness*) both maintain that truth is super-truth. One consequence is that the (T) schema – 'p' is true iff p – is not true for borderline p, where there are precisifications where the right-hand-side is true, but the left-hand-side is not. Keefe (*Theories of Vagueness* ch. 8) defends this consequence, arguing, for example, that it is enough to preserve the validity of the inference rules 'p can be inferred from "p" is true' and vice versa.

There are, however, uses of the supervaluationist framework that do not endorse this claim. McGee and McLaughlin, for example, distinguish between two notions of truth – definite truth, which is a matter of correspondence and truth as fits the disquotational conception. Definite truth is equated with super-truth, while truth is classical.[8] Such a position raises questions about how definite truth differs from known truth, and so how the position differs from an epistemic view, questions that do not face the standard identification of truth with super-truth.

If truth is equated with super-truth, the natural account of validity will be to take it as necessary preservation of super-truth. This is known as *global validity*: an argument is globally valid iff necessarily, whenever the premises are all true (i.e. super-true), the conclusion is also true (i.e. super-true). There are alternative characterisations of validity within the framework, however. In particular, there is a notion of *local validity*: an argument is locally valid iff necessarily, in every complete and admissible specification, if the premises are true in that specification, the conclusion is true as well. These notions of validity deliver different verdicts regarding certain arguments involving the D operator. For example, the argument from A to DA is globally valid but not locally valid since, for vague A, there are complete and admissible specifications where A is true and DA is not.

Alternatives to global validity may be defensible if truth is not identified with super-truth (e.g. Shapiro calls upon local validity). But even with this identification, there may be a role for one or more of these alternatives. Keefe ('Supervaluation and Validity') suggests a pluralism about notions of validity within the framework: there may be nothing about our pre-theoretic notion of validity that determines which notion of validity is right and there may be different roles for the different notions.[9] Cobreros details an alternative version of validity, *regional validity*, where the property that is necessarily preserved is defined relative to specifications.

The correct treatment of validity within the supervaluationist framework is still, it seems, an open question. And it is bound up with questions about the treatment of higher-order vagueness and the acceptability of rejecting classical rules of inference.

Conclusion

Supervaluationism is, by any reckoning, a leading theory of vagueness. Given the threat that vagueness appears to pose to classical logic, super-valuationism can seem to offer the ideal solution – a consequence relation that matches the classical ones in all standard cases and yet a way to avoid the implausible sharpness in our language imposed by classical semantics. The idea that nothing in our language chooses between a range of possible precise extensions to our vague predicates can seem compelling and the theory puts this idea to good use. It still faces many objections, however, which have been tackled to varying extents in my *Theories of Vagueness* and elsewhere in the literature. There may be more work to be done on higher-order vagueness, for example, but the prospects look at least as good as the prospects for any other theory of vagueness.

Short Biography

Rosanna Keefe works in the Philosophy of Logic and Language. She has published a monograph, *Theories of Vagueness* (Cambridge UP, 2000) and co-edited *Vagueness: A Reader* (MIT Press, 1996). She has published articles in, among other places, *Mind*, *Australasian Journal of Philosophy*, *Proceedings of the Aristotelian Society*, *Analysis* and *Philosophical Topics*. Many of her publications have been on vagueness: she defends a supervaluationist theory in her monograph. She is currently a senior lecturer at the Department of Philosophy at Sheffield University and was previously a research fellow and student at Cambridge University.

Notes

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[1] If you think, say, that a child suddenly becomes a toddler at 1 year old, then substitute 'young baby' in the example.

[2] I have not given necessary and sufficient conditions for a predicate to be vague, as the criteria are controversial. Rather, I hope that a sketch of these typical features and a list of examples are enough to identify the phenomenon.

[3] Edgington, however, proposes a many-valued theory that abandons truth-functionality. For discussion, see Keefe, *Theories of Vagueness* 98–100.

[4] Recent discussion of the supervaluationist response to the Problem of the Many can be found in McGee and McLaughlin ('Lessons of the Many'), McKinnon, Weatherson and Williams. Weatherson sees Schiffer's discussion of supervaluationism and indirect speech reports as an example of an objection to this response to the Problem of the Many (see Schiffer), though it isn't clear that this correctly captures the scope of the problem. To illustrate Schiffer's objection, suppose Alice points and says 'there is where Harry and I first danced the rumba' and I report 'there is where Alice said Harry and she first danced the rumba'. The worry is that when supervaluationists come to assess this latter statement, they will require that it is true for all precisifications of 'there'. But that seems to require that Alice said something about a huge array of precise places, and surely she didn't make any of those precise utterances, let alone all of them. Some of Schiffer's (implicit) assumptions are questionable in this argument, however; see Keefe, 'Supervaluationism, Indirect Speech Reports and Demonstratives'.

[5] See Keefe (*Theories of Vagueness* 188–93) in defence of the role of precisifications against objections from Sanford, Fodor and Lepore and others.

[6] Varzi details a number of different approaches to constructing supervaluations, asking, for example, whether a precisification of our vague language should be seen as a precise language of its own or a precise interpretation of the vague language.

[7] Fara has objected that our ordinary inferential practices are inappropriately challenged not only in the presence of the D operator, but when we consider not just logical consequence but 'our everyday context-dependent conception of what follows from what' (207). She offers the supervaluationist a treatment of 'relativised consequence relations' where what matters is preservation of truth in a particular class of models. But whether the supervaluationist should accept her treatment is controvertible.

[8] Similarly, in Shapiro, super-truth does not play a central role.

[9] In that paper, a further global option is distinguished amounting to 'necessarily if the premises are all true, the conclusion is not false'. There are other notions of validity available when one also considers multiple conclusion arguments, for which a natural global characterisation will state than an argument is valid iff necessarily whenever the premises are super-true, *some* conclusion is super-true. Varzi considers such arguments and distinguishes four global notions, two local ones and also two collective ones, for which we ask, e.g. whether the disjunction of the conclusions is true if the conjunction of the premises is.

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