Skepticism & Contextualism

November 6th, 2018

A Modal Logic for Knowledge

Symbolization

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with

$$\mathcal{K}_S\phi$$

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For any propositions ϕ and ψ,

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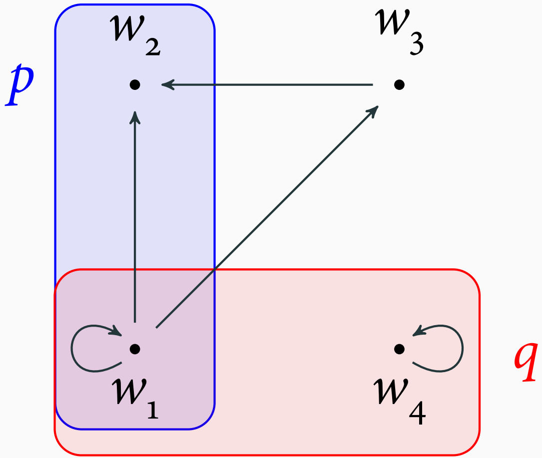

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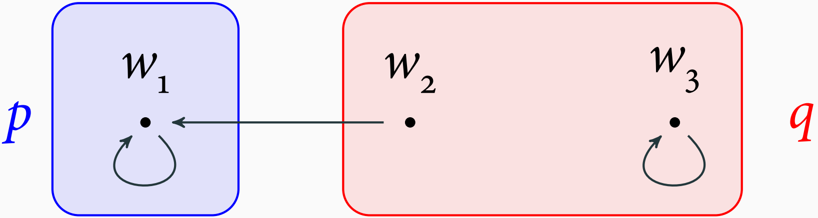

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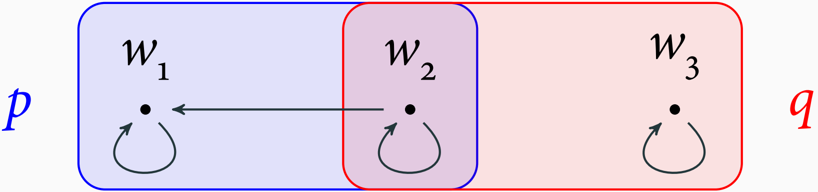

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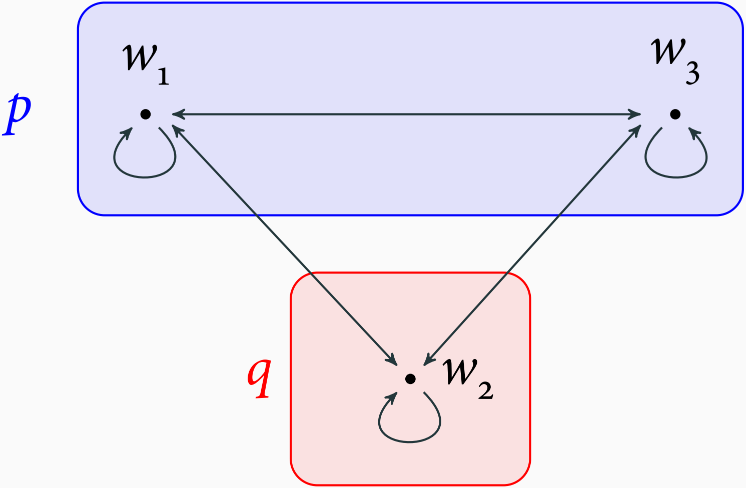

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Propositions

- For any sentence ϕ,

$$\langle\phi\rangle \stackrel{\text{def}}{=} \{w \in \mathcal{W} \mid [\![\phi]\!]^w = 1\}$$

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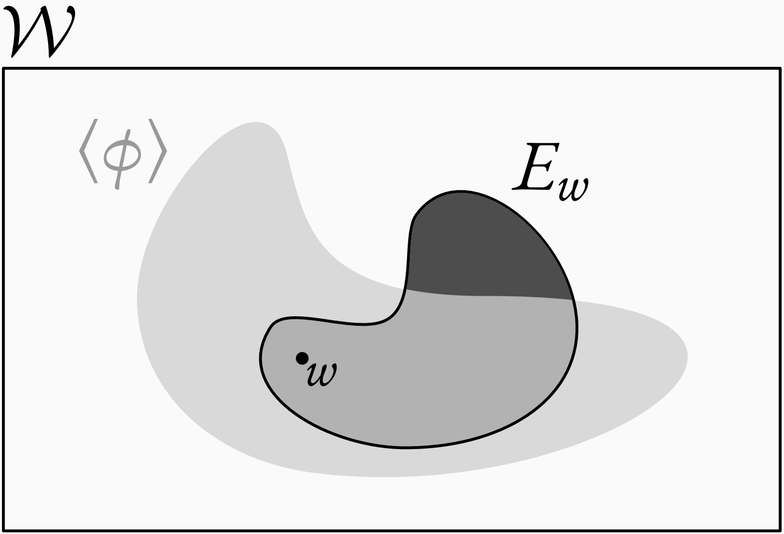

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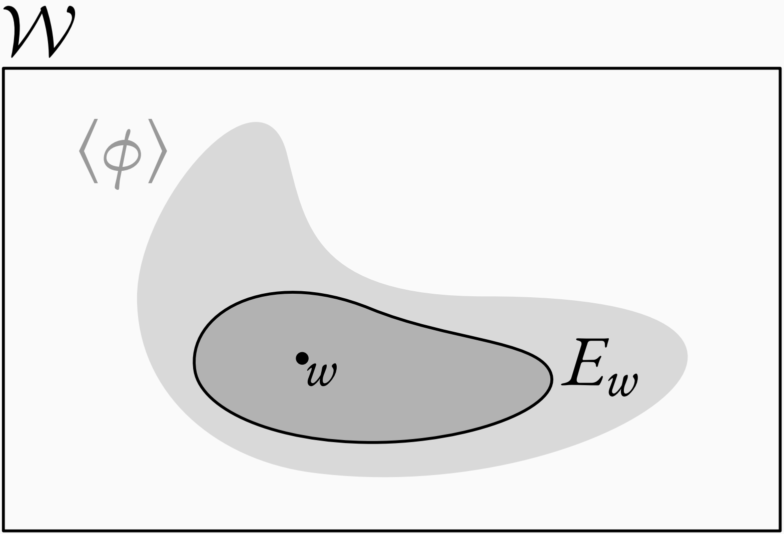

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Transitivity

For any w, w^*, w^{**}, if Rww^* and Rw^*w^{**}, then Rww^{**}.

[For any w, w^*, w^{**}, if $w^* \in E_w$ and $w^{**} \in E_{w^*}$, then $w^{**} \in E_w$]

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Factivity

$$\mathcal{K}\phi \rightarrow \phi \qquad [(\mathrm{T})]$$

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$\mathcal{K}\phi \to \mathcal{K}\mathcal{K}\phi$ [(S4)]

Negative Introspection

$\neg\mathcal{K}\phi \to \mathcal{K}\neg\mathcal{K}\phi$ [(S5)]

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A Skeptical Argument

$$\begin{array}{ll} \text{P1.} & \mathcal{K}p \rightarrow \mathcal{K}\neg s \\ \text{P2.} & \neg\mathcal{K}\neg s \\ \hline \text{C1.} & \neg\mathcal{K}p \end{array}$$

- Deny P1
- Deny P2

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P2. $\neg\mathcal{K}\neg s$

C1. $\neg\mathcal{K}p$

- Deny P1
- Deny P2
- Accept C1

Accept C1?

- Knowledge plays important normative roles.

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$\neg\mathcal{K}p$

- Knowledge plays important normative roles.
    - Excuses

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C4. *S* doesn't know that they're not a handless brain-in-a-vat.

Deny P1?

$\mathcal{K}p \wedge \neg\mathcal{K}\neg s$

Deny P1?

Closure

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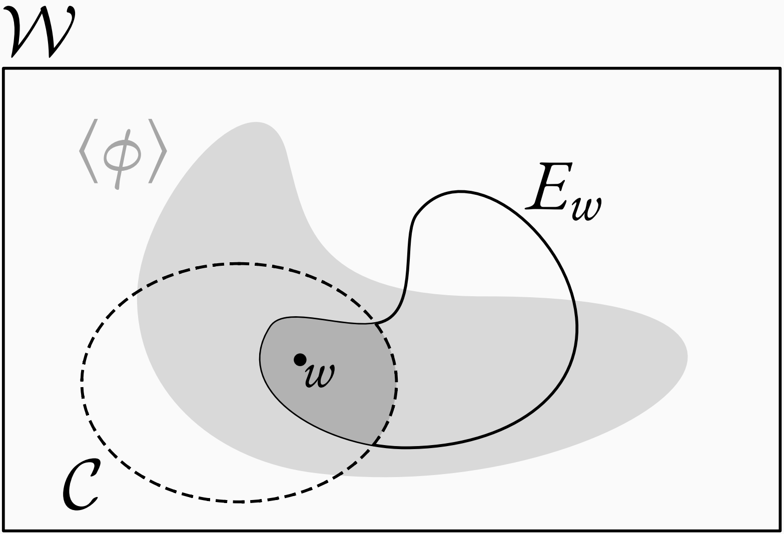

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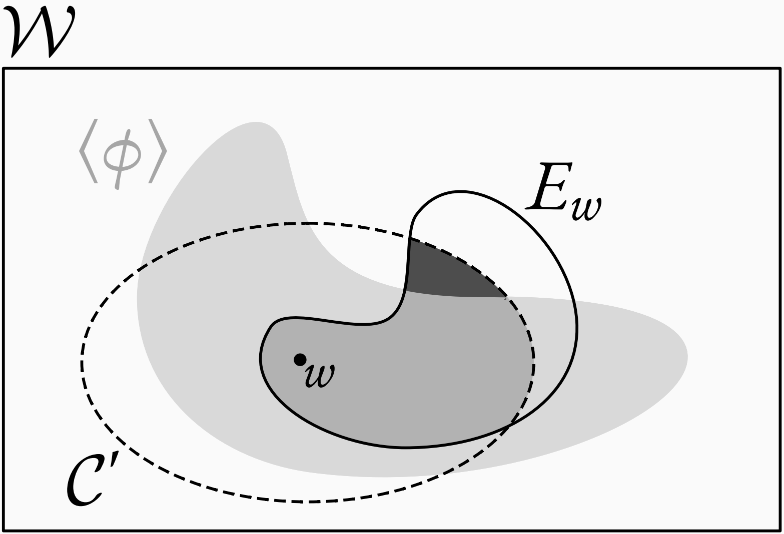

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- Lewis characterizes C by way of a series of *rules* about which possibilities may and may not be properly ignored.

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Rule of Resemblance

Do not ignore worlds that saliently resemble worlds you have already not ignored—those worlds must be included in $\mathcal{C}$.

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Rule of Method

So long as doing so doesn't violate the preceeding rules, possibilities in which inductive inference or inference to the best explanation fail may be properly ignored—they may be excluded from $\mathcal{C}$.

Rules of Permission

Rule of Conservativism

If it is common knowledge in our linguistic community that we standardly *do* ignore possibilities, then these possibilities may be ignored—they may be excluded from $\mathcal{C}$.

Rule of Attention

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A possibility which is not being ignored is not being properly ignored.