

## Bayesian inference on impedance spectra

**Introduction:** AC resistance, also known as impedance, is the opposition to the flow of alternating current and is a complex quantity that is affected by frequency, material, and dimensions of the conductor. DC resistance is the opposition to the flow of direct current and is measured in ohms. AC impedance spectroscopy is an important method for evaluating ionic, electronic, and dielectric properties of materials. In conventional analysis of AC impedance spectra, the selection of an equivalent circuit model and the initial parameters are determined visually based on a Nyquist plot; this visual determination can be both inefficient and inaccurate. Therefore, analysis based on a rigorous mathematical method is highly desirable.

**Measurements setup:** The aim, therefore, is to use Bayesian statistics to decide between two alternative electrical circuits . At the same time the parameters should be estimated. The data is collected from two circuits. Circuits A and B have been prepared according to the layout in the red rectangles below . In this assignment you will receive data. Data are attached in two different folders: Circuit1 and Circuit2 . For both circuits data collected in the frequency range between 0 and 12501 Hz with a step size of 250. The first item of each list is the measured impedance.

It is your task to determine which circuit layout is most appropriate using Bayesian inference on the impedance spectra for the two circuits. Note that, shunt is a device created to offer a low-resistance path for electrical current within a circuit. Its main purpose is to redirect current away from a particular system or component, helping to prevent overcurrent.

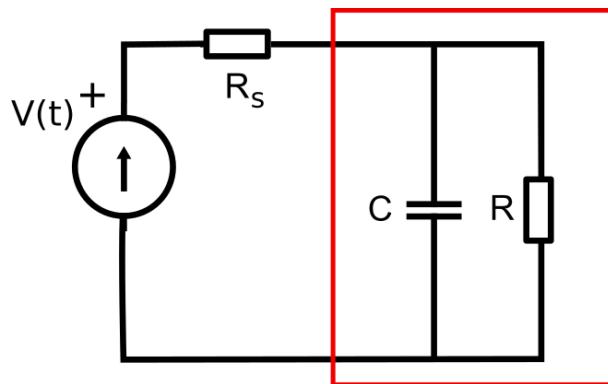


Fig.1 : Model Circuit A, the area in the red rectangle constitutes the first model circuit, is the shunt resistance and is part of the measuring apparatus.

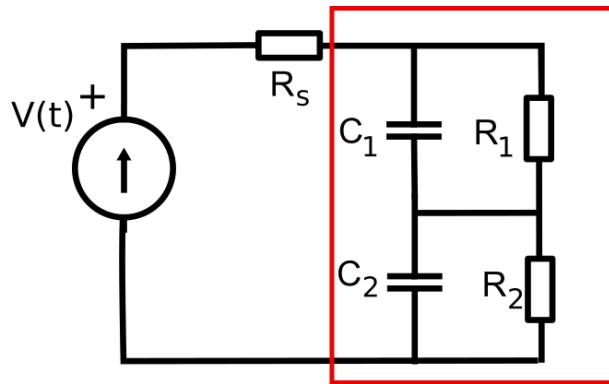


Fig.2 : Model Circuit B, the area in the red rectangle constitutes the second model circuit, is the shunt resistance and is part of the measuring apparatus.

The measurement setup used to collect data is shown in Fig.3. Thin lines indicate breadboard connections, thick lines indicate wires. Wires of the same color are at the same electrical potential. Blue is ground, red is DAC output voltage, green is the potential between the shunt resistor and the circuit of interest. For clarity the USB-connector is left out of the picture. For your prior you can assume that the resistors have values between 100 ohm and 10k ohm and the capacitors have values between 10 nano F and 10 micro F.

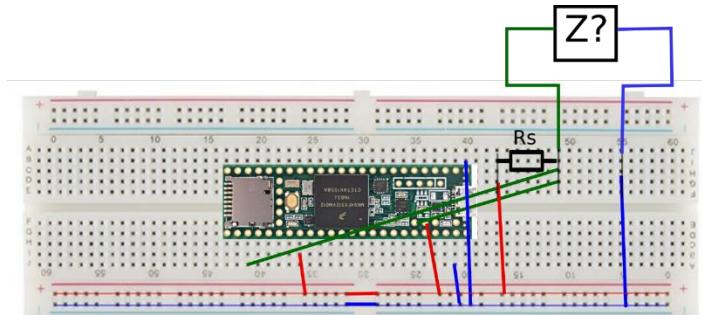


Fig.3 : Measurement setup with  $Z$  as a free parameter and an unknown circuit with unknown impedance connected.

The analysis should include the following:

- A detailed write-up explaining the approach, methodology, and reasoning for choosing the most appropriate circuit.
- Explanation and Implementing Bayesian inference for choosing between Circuit A and B.
- Sensor uncertainties from experiments
- Fusion of data to improve precision
- Applicable sensor fusion architectures
- A comparison of both circuits based on the impedance spectra and Bayesian model results.

- Critical analysis of the assumptions (e.g., prior knowledge on resistor and capacitor values) and how sensitive the results are to these assumptions.
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#### **References:**

1. Miyazaki, Yu & Nakayama, Ryo & Yasuo, Nobuaki & Watanabe, Yuki & Shimizu, Ryota & Packwood, Daniel & Nishio, Kazunori & Ando, Yasunobu & Sekijima, Masakazu & Hitosugi, Taro. (2020). Bayesian statistics-based analysis of AC impedance spectra. *AIP Advances*. 10. 045231. 10.1063/1.5143082. <https://doi.org/10.1063/1.5143082>.
2. Electrochemical Impedance Spectroscopy System Based on a Teensy Board, Leila Es Sebar, Leonardo Iannucci, Emma Angelini, Sabrina Grassini, and Marco Parvis, *IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT*, VOL. 70, 2021, <https://ieeexplore.ieee.org/document/9259014>.

## Assessment matrix

Assessment Aspect	Weight	Excellent (10)	Good (8)	Fair (5.5)	Insufficient (3)
<b>Apply Bayesian inference to fuse sensor data.</b>	<b>0.4</b>	A plausible mathematical model for Bayesian inference is proposed with correct motivation containing reasonable priors, likelihoods and posteriors. The possible use of conjugate priors is explicitly assessed.	A plausible mathematical model for Bayesian inference is proposed with correct motivation containing reasonable priors, likelihoods and posteriors.	A plausible mathematical model for Bayesian inference is provided with reasonable priors, likelihoods and posteriors.	Creating a mathematical model for Bayesian inference is attempted but visibly incomplete or containing visible errors.
<b>Create software implementing Bayesian inference in a sensor system.</b>	<b>0.3</b>	Bayesian inference is correctly implemented in code. The numerical method is motivated from a theoretical and resource perspective. A critical reflection on the achieved result is present with either a comparison with the state of the art, a ground truth or simulated data.	Bayesian inference is correctly implemented in code. The numerical method is motivated from a theoretical and resource perspective.	Bayesian inference is correctly implemented in code.	Bayesian inference in software is attempted but visibly incomplete or containing visible errors.
<b>Analyse system requirements to identify applicable sensor fusion architectures.</b>	<b>0.15</b>	Different architecture categorizations are used and applied correctly to describe proposed designs. The categorization are compared regarding their applicability for sensor fusion problem at hand.	Different architecture categorizations are used and applied correctly to describe proposed designs.	Different architecture categorizations are used and applied correctly to describe proposed designs .	Different architecture categorizations are used and applied but there are visible errors in the application .
<b>Apply error propagation techniques.</b>	<b>0.15</b>	A classical error analysis is provided showing how model assumptions influence the final outcome. The classical error is compared with the spread found in the posterior. The error analysis is related back to the implementation and possible improvements to the implementation are discussed.	A classical error analysis is provided showing how model assumptions influence the final outcome. The classical error is compared with the spread found in the posterior.	A classical error analysis is provided showing how model assumptions influence the final outcome.	An error analysis is provided but visibly incomplete or it contains visible errors.