Lecture 28 - BNet Inference

https://www.cs.ubc.ca/~jordon/teaching/cpsc322/2019w2/lectures/lecture28.pdf

Goals

- Define factors. Derive new factors from existing factors. Apply operations to factors, including assigning, summing out and multiplying factors.
- Carry out variable elimination by using factor representation and using the factor operations. Use techniques to simplify variable elimination.

BNet Inference

Our goal in BNet inference is to compute the probabilities of the variables in a belief network:

- What is the posterior distribution over one or more variables, conditioned on one or more observed variables?

BNets in General

Suppose we have that the variables in the belief network are $\{X_1, \ldots, X_n\}$. We also have a variable Z which we call the "query" variable.

We have variables $Y_1 = v_1, \ldots, Y_j = v_j$ which are the **observed varg1ariables**, and their corresponding observed values. $Z_1, \ldots, Z_k$ are the remaining variables.

We wish to compute:

$$P(Z|Y_1 = v_1, \ldots, Y_j = v_j)$$

We bring back the example of the fire alarm, where we had the following belief network:

In this case we may wish to compute: $P(L|S = t, R = f)$. From marginalization we know that:

$$P(L|S = t, R = f) = \frac{P(L, S = t, R = f)}{P(S = t, R = f)}$$

An we are given the necessary data to do so:

L	S	R	P(L, S=t, R=f)
t	t	f	.3
f	t	f	.2

.3 + .2 = .5

L	S	R	P(L / S=t, R=f)
t	t	f	.6
f	t	f	.4

In general, we have

$$\begin{aligned} P(Z|Y_1 = v_1, \dots, Y_j = v_j) &= \frac{P(Z, Y_1 = v_1, \dots, Y_j = v_j)}{P(Y_1 = v_1, \dots, Y_j = v_j)} \\ &= \frac{P(Z, Y_1 = v_1, \dots, Y_j = v_j)}{\sum_Z P(Z, Y_1 = v_1, \dots, Y_j = v_j)} \end{aligned}$$

However we only need to **compute the numerator**, and then **normalize**. This can be framed in terms of operations between factors (that satisfy the semantics of probability)