Complete 0/1-Matrices

Consider the polytope $P = \{Ax = b, 0 \leq x \leq 1\}$ with A being the complete 0/1 matrix with m rows and $2^m - 1$ columns. Consider following questions:

1. Let $\mathbf{1} \cdot 2^{m-3} \leq b \leq \mathbf{1} \cdot 2^{m-2}$. Assume that b is feasible. How often do we have to randomly select a subset of columns (select a random solution x) until we find a feasible solution?
2. Let $\mathbf{1} \cdot 2^{m/c} \leq b \leq \mathbf{1} \cdot 2^{m-2}$ for some constant c. Assume that b is feasible. How often do we have to randomly select a subset of columns of A (i.e., select a random solution x) until we find a feasible solution?
3. Let $\mathbf{1} \cdot 2^{m/\log(m)} \leq b \leq \mathbf{1} \cdot 2^{m-2}$. Assume that b is feasible. How often do we have to randomly select a subset of columns (select a random solution x) until we find a feasible solution?
4. Can we define an ordering of feasible right-hand sides such that the number of solutions is non-decreasing?

For the points 1. - 3., preferably, we would like to get something strictly smaller than 2^{m^2}