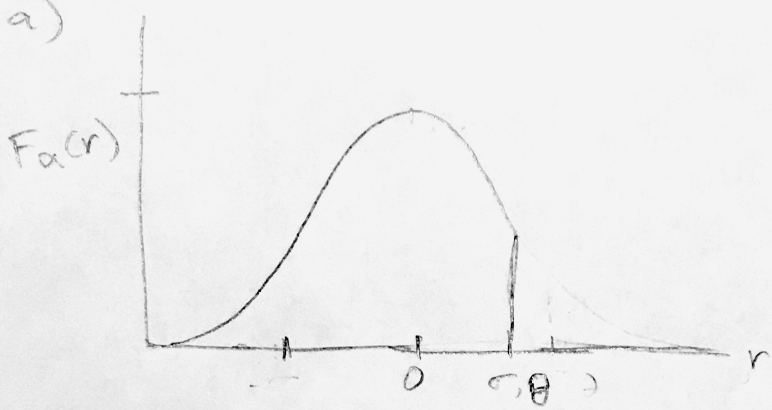
1. a)



b) Every PDF must, by definition, integrate to 1. The normal distribution PDF did, but then we lobbed off a corner of it, so we include a normalization factor to ensure that the PDF $F_\alpha(r)$ still integrates to 1.

c) The mode of α is 0 (most likely value).

d) $\langle\alpha\rangle = \int_{-\infty}^{\infty} r F_\alpha(r)\,dr$ ← mean eqn (16)

$$= \int_{-\infty}^{\theta} r \frac{p(r)}{D(\theta)}\,dr$$

$$p(r) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(r - \cancel{\langle x\rangle}^{\,0})^2}{2\sigma^2}\right)$$ ← normal dist (27)

$$\langle\alpha\rangle = \frac{1}{D(\theta)\sigma\sqrt{2\pi}} \int_{-\infty}^{\theta} r^* \exp\left(\frac{-r^2}{2\sigma^2}\right)$$

$$\int x e^{-ax}\,dx = -\frac{1}{2a} e^{-ax^2}$$

$$\langle\alpha\rangle = \frac{1}{D(\theta)\sigma\sqrt{2\pi}} \left[-\frac{1}{2\left(\frac{1}{2\sigma^2}\right)} e^{-r^2/2\sigma^2}\right]_{r=-\infty}^{r=\theta}$$

$$= \frac{-\sigma^2}{D(\theta)\sigma\sqrt{2\pi}} \left[e^{-\theta^2/2\sigma^2} - \cancel{e^{-(-\infty)^2/2\sigma^2}}^{\,0}\right]$$

$$\langle\alpha\rangle = \frac{-\sigma}{D(\theta)\sqrt{2\pi}} e^{-\theta^2/2\sigma^2}$$

$$= \left(\frac{-1}{D(\theta)}\right)(\sigma^2)\left(\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\theta - \langle x\rangle^2)}{2\sigma^2}\right)\right)$$

$$\boxed{\langle\alpha\rangle = -\sigma^2 \frac{p(\theta)}{D(\theta)}}$$

The sign of α is negative, which agrees with my drawing. By lobbing off the top part of the PDF, the mean is ensured to be below the previous mean of 0, making the mean value be negative.