TYPE ABSTRACTION USING A SIGNATURE (20/20 points)

Encapsulate the type and values given in the template in a module named `Exp`.

To make `e` abstract, assign a signature to the module `Exp` that makes the type `e` abstract and publish the functions `int`, `mul` and `add`.

Given that interface, the only way to build a value of type `e` is to use the functions `int`, `mul` `add` and `to_string`. Such functions are called *smart constructors* because they perform some smart operations when they build values.

These smart constructors enforce the invariant that an expression represented by a value of type `e` is always simplified, i.e. it does not contain a subexpression of the form `e * 1`, `1 * e`, `e * 0`, `0 * e`, `0 + e` or `e + 0`.

- The following expression should be accepted.

```
Exp.mul (Exp.int 0) (Exp.add (Exp.int 1) (Exp.int 2))
```

- The following expression should be rejected.

```
Exp.EMul (Exp.EInt 0) (Exp.EAdd (Exp.EInt 1) (Exp.EInt 2))
```

Unfortunately, turning `e` into an abstract data type prevents the user from pattern matching over values of type `e`. To allow pattern matching while forbidding the direct application of data constructors, OCaml provides a mechanism called `private types`. The interested student can get more information about this advanced (off-topic) feature here.

YOUR OCAML ENVIRONMENT

```
1  module Exp : sig
2    type e
3    val int : int -> e
4    val mul : e -> e-> e
5    val add : e -> e-> e
6    val to_string : e -> string
7  end = struct
8    type e = EInt of int | EMul of e * e | EAdd of e * e
9
10   let int x = EInt x
11
12   let mul a b =
13     match a, b with
14     | EInt 0, _ | _, EInt 0 -> EInt 0
15     | EInt 1, e | e, EInt 1 -> e
16     | a, b -> EMul (a, b)
17
18   let add a b =
19     match a, b with
20     | EInt 0, e | e, EInt 0 -> e
21     | a, b -> EAdd (a, b)
22
23   let rec to_string = function
24     | EInt i -> string_of_int i
25     | EMul (l, r) -> "(" ^ to_string l ^ " * " ^ to_string r ^ ")"
26     | EAdd (l, r) -> "(" ^ to_string l ^ " + " ^ to_string r ^ ")"
27 end;;
28
```

Evaluate >

Switch >>

Typecheck

Reset Templ

Full-screen

Check & Sa

Exercise complete (click for details) 20 pts

Found Exp with compatible type.

Your module Exp is compatible. 5 pts

Type Exp.e is abstract as expected. 5 pts

Now I will check that you didn't change the behaviour.

v Testing `Exp.to_string` Completed, 10 pts

Computing and printing

```
(Exp.mul
   (Exp.mul
      (Exp.add
         (Exp.mul (Exp.int 3)
```

```
            (Exp.mul (Exp.mul (Exp.int 3) (Exp.int 4)) (Exp.int 0)))))
      (Exp.add (Exp.int 3)
         (Exp.add
            (Exp.add (Exp.add (Exp.int 3) (Exp.mul (Exp.int 3) (Exp.int 2)))
               (Exp.mul (Exp.add (Exp.int 4) (Exp.int 1))
                  (Exp.mul (Exp.int 1) (Exp.int 4)))) (Exp.int 2))))
```

Correct value 1 pt

```
"((((3 * (4 + 1)) + 4) * (4 + 3)) * (3 + (((3 + (3 * 2)) + ((4 + 1) * 4)) + 2)))"
```

Computing and printing

```
  (Exp.int 4)
```

Correct value "4" 1 pt

Computing and printing

```
  (Exp.int 1)
```

Correct value "1" 1 pt

Computing and printing

```
  (Exp.mul
     (Exp.mul
        (Exp.mul (Exp.int 2)
           (Exp.mul (Exp.mul (Exp.int 0) (Exp.int 3))
              (Exp.add (Exp.int 4) (Exp.mul (Exp.int 3) (Exp.int 3)))))
        (Exp.add
           (Exp.mul (Exp.mul (Exp.int 1) (Exp.add (Exp.int 3) (Exp.int 0)))
              (Exp.mul (Exp.int 2) (Exp.int 4)))
           (Exp.mul (Exp.add (Exp.int 0) (Exp.int 2)) (Exp.int 3))))
     (Exp.add (Exp.add (Exp.mul (Exp.int 3) (Exp.int 2)) (Exp.int 1))
        (Exp.add (Exp.int 0)
           (Exp.add (Exp.int 2)
              (Exp.mul (Exp.add (Exp.int 1) (Exp.int 0))
                 (Exp.mul (Exp.int 0) (Exp.int 2)))))))
```

Correct value "0" 1 pt

Computing and printing

```
  (Exp.mul
     (Exp.mul (Exp.add (Exp.int 1) (Exp.int 3))
        (Exp.add
           (Exp.add (Exp.mul (Exp.int 0) (Exp.int 1))
              (Exp.add (Exp.int 4) (Exp.int 2)))
           (Exp.mul (Exp.int 4) (Exp.int 3))))
     (Exp.add
        (Exp.mul (Exp.int 2)
           (Exp.add (Exp.int 3)
              (Exp.mul (Exp.int 2) (Exp.add (Exp.int 3) (Exp.int 0)))))
        (Exp.int 4)))
```

Correct value "(((1 + 3) * ((4 + 2) + (4 * 3))) * ((2 * (3 + (2 * 3))) + 4))" 1 pt

Computing and printing

```
  (Exp.mul
     (Exp.add
        (Exp.add
           (Exp.add (Exp.int 1)
              (Exp.add (Exp.int 0) (Exp.mul (Exp.int 1) (Exp.int 0))))
           (Exp.mul (Exp.int 3) (Exp.int 2)))
        (Exp.add
           (Exp.mul (Exp.add (Exp.add (Exp.int 2) (Exp.int 2)) (Exp.int 3))
              (Exp.mul (Exp.add (Exp.int 4) (Exp.int 1))
                 (Exp.add (Exp.int 3) (Exp.int 1))))
           (Exp.add (Exp.int 0)
              (Exp.mul (Exp.mul (Exp.int 4) (Exp.int 0)) (Exp.int 1)))))
     (Exp.int 4))
```

Correct value "(((1 + (3 * 2)) + (((2 + 2) + 3) * ((4 + 1) * (3 + 1)))) * 4)" 1 pt

Computing and printing

```
  (Exp.add
     (Exp.mul
        (Exp.mul
           (Exp.mul
              (Exp.mul (Exp.mul (Exp.int 3) (Exp.int 1))
                 (Exp.mul (Exp.int 0) (Exp.int 2)))
              (Exp.add (Exp.int 0) (Exp.int 4)))
           (Exp.mul (Exp.add (Exp.int 2) (Exp.add (Exp.int 0) (Exp.int 0)))
              (Exp.add (Exp.mul (Exp.int 2) (Exp.int 1)) (Exp.int 2))))
        (Exp.add
           (Exp.add
              (Exp.add (Exp.add (Exp.int 1) (Exp.int 0))
                 (Exp.mul (Exp.int 1) (Exp.int 1)))
              (Exp.mul (Exp.mul (Exp.int 0) (Exp.int 4))
                 (Exp.mul (Exp.int 4) (Exp.int 3))))
           (Exp.add (Exp.add (Exp.int 2) (Exp.mul (Exp.int 2) (Exp.int 3)))
              (Exp.int 2))))
     (Exp.add
        (Exp.add (Exp.int 3)
           (Exp.add (Exp.int 0) (Exp.mul (Exp.int 2) (Exp.int 4))))
        (Exp.add
           (Exp.mul (Exp.add (Exp.mul (Exp.int 4) (Exp.int 3)) (Exp.int 2))
              (Exp.mul (Exp.int 4) (Exp.int 3)))
           (Exp.add (Exp.mul (Exp.int 0) (Exp.add (Exp.int 1) (Exp.int 3)))
              (Exp.mul (Exp.int 1) (Exp.add (Exp.int 3) (Exp.int 4)))))))
```

```
(Exp.add (Exp.add (Exp.int 2) (Exp.int 0))
   (Exp.add (Exp.int 0)
      (Exp.mul
         (Exp.mul (Exp.mul (Exp.int 0) (Exp.int 4))
            (Exp.add (Exp.int 0) (Exp.int 3))) (Exp.int 1))))
(Exp.mul
   (Exp.add (Exp.int 2)
      (Exp.mul (Exp.mul (Exp.int 2) (Exp.int 3))
         (Exp.add (Exp.int 3) (Exp.int 2)))) (Exp.int 1)))
```

Correct value "(2 * (2 + ((2 * 3) * (3 + 2))))" 1 pt

Computing and printing

```
(Exp.int 1)
```

Correct value "1" 1 pt

Computing and printing

```
(Exp.mul
   (Exp.mul
      (Exp.mul (Exp.add (Exp.int 1) (Exp.int 3))
         (Exp.add (Exp.int 1)
            (Exp.mul (Exp.add (Exp.int 1) (Exp.int 3))
               (Exp.mul (Exp.int 2) (Exp.int 0))))) (Exp.int 1))
   (Exp.mul (Exp.add (Exp.int 2) (Exp.int 3)) (Exp.int 0)))
```

Correct value "0" 1 pt