BALANCED BINARY TREES (22/22 points)

A binary tree `t`, of the `'a bt` type given in the prelude, is either an empty tree, or the root of a tree with a value and two children subtrees.

1. Write a function `height : 'a bt -> int` that computes the height of a tree.
2. A tree is *balanced* if, for all internal node `n`, its two subtrees have the same height. Write a function `balanced : 'a bt -> bool` that tells if a tree is balanced.

THE GIVEN PRELUDE

```
type 'a bt =
  | Empty
  | Node of 'a bt * 'a * 'a bt ;;
```

YOUR OCAML ENVIRONMENT

```
let rec height t = match t with
  | Empty -> 0
  | Node (l, _, r) ->
      let l1 = height l in
      let l2 = height r in
      if l1 < l2 then l2 + 1 else l1 + 1
;;

let rec balanced t  = match t with
  | Empty -> true
  | Node (l, _, r) ->
      if height l = height r then
        true && balanced l && balanced r
      else false
;;

```

Evaluate >
Switch >>
Typecheck
Reset Templ
Full-screen
Check & Sa

Exercise complete (click for details) 22 pts

v Exercise 1: `height` Completed, 11 pts

Found `height` with compatible type.

Computing
```
height
  (Node
    (Node (Node (Node (Empty, 4, Empty), -5, Node (Empty, 0, Empty)), 2,
      Node (Empty, 3, Empty)),
    2, Node (Empty, -5, Node (Empty, 4, Node (Empty, -5, Empty)))))
```
Correct value 4 1 pt

Computing `height (Node (Empty, 4, Empty))`

Correct value 1 1 pt

Computing
```
height
  (Node (Node (Empty, 3, Node (Empty, 0, Empty)), -4,
    Node (Empty, -4, Empty)))
```
Correct value 3 1 pt

Computing
```
height
  (Node
    (Node
      (Node (Node (Node (Empty, 3, Empty), -2, Node (Empty, -1, Empty)), -2,
        Node (Node (Empty, 2, Empty), -3, Node (Empty, -4, Empty))),
      -4,
      Node (Node (Node (Empty, -1, Empty), -4, Node (Empty, -1, Empty)), 1,
       Node (Node (Empty, -1, Empty), 1, Node (Empty, -3, Empty)))),
    -5,
    Node
     (Node (Node (Node (Empty, 3, Empty), 0, Node (Empty, -5, Empty)), 1,
```

Correct value 5 1 pt

```
Computing
  height
    (Node (Empty, -2,
      Node
       (Node (Node (Empty, 1, Empty), 1,
         Node (Node (Empty, 4, Empty), 4, Node (Empty, 2, Empty))),
       -3, Empty)))
```

Correct value 5 1 pt

Found `height` with compatible type.

Computing `height Empty`

Correct value 0 1 pt

```
Computing
  height
    (Node
      (Node
        (Node (Node (Node (Empty, 'n', Empty), 's', Node (Empty, 'w', Empty)),
          'o', Node (Node (Empty, 'l', Empty), 'r', Node (Empty, 'q', Empty))),
        'a',
        Node (Node (Node (Empty, 'd', Empty), 'v', Node (Empty, 'z', Empty)),
         'f', Node (Node (Empty, 'z', Empty), 'n', Node (Empty, 'p', Empty)))),
      'w',
      Node
       (Node (Node (Node (Empty, 'c', Empty), 'y', Node (Empty, 'n', Empty)),
         'g', Node (Node (Empty, 'm', Empty), 'p', Node (Empty, 'm', Empty))),
       'u',
       Node (Node (Node (Empty, 'b', Empty), 'm', Node (Empty, 'v', Empty)),
        'w', Node (Node (Empty, 'r', Empty), 'g', Node (Empty, 'r', Empty))))))
```

Correct value 5 1 pt

```
Computing
  height
    (Node
      (Node (Node (Empty, 'u', Empty), 'r',
        Node (Node (Empty, 'x', Empty), 'u',
         Node (Node (Empty, 'k', Empty), 'q', Node (Empty, 'u', Empty)))),
      'k',
      Node (Node (Empty, 't', Empty), 'k',
       Node (Empty, 'q', Node (Empty, 'k', Empty)))))
```

Correct value 5 1 pt

Computing `height (Node (Node (Empty, 'g', Empty), 'p', Node (Empty, 'l', Empty)))`

Correct value 2 1 pt

```
Computing
  height
    (Node
      (Node (Node (Node (Empty, 'v', Node (Empty, 'c', Empty)), 'v', Empty),
        'c', Node (Node (Node (Empty, 'f', Empty), 'f', Empty), 'd', Empty)),
      'g',
      Node (Empty, 's',
       Node (Node (Node (Empty, 't', Empty), 'i', Empty), 'v', Empty))))
```

Correct value 5 1 pt

Computing `height (Node (Empty, 'c', Empty))`

Correct value 1 1 pt

v Exercise 2: `balanced` Completed, 11 pts

Found `balanced` with compatible type.

```
Computing
  balanced
    (Node (Node (Node (Empty, 1, Empty), 1, Node (Empty, 0, Empty)), -3,
      Node (Node (Empty, 4, Empty), -3, Node (Empty, -1, Empty))))
```

Correct value `true` 1 pt

```
Computing
  balanced
    (Node
      (Node
        (Node (Node (Node (Empty, 1, Empty), -2, Node (Empty, 1, Empty)), 2,
          Node (Empty, 1, Node (Empty, -3, Empty))),
        -5, Empty),
      2,
      Node (Empty, 4,
       Node (Empty, -1, Node (Empty, 3, Node (Empty, 2, Empty))))))
```

Correct value `false` 1 pt

```
Computing
  balanced
    (Node (Node (Empty, -3, Empty), 1,
      Node (Empty, 3,
       Node (Node (Empty, 0, Empty), 1,
        Node (Node (Empty, 2, Empty), 0, Empty)))))
```

Correct value `false` 1 pt

Computing

```
Computing
  balanced
    (Node (Empty, 1,
      Node (Node (Empty, -1, Node (Empty, 1, Empty)), -5,
       Node (Empty, 4, Node (Node (Empty, -1, Empty), -5, Empty)))))
```

Correct value `false` 1 pt

Found `balanced` with compatible type.

Computing `balanced Empty`

Correct value `true` 1 pt

```
Computing
  balanced
    (Node
      (Node
        (Node (Node (Empty, 'f', Node (Empty, 'e', Empty)), 'o',
          Node (Node (Empty, 'w', Empty), 'p', Node (Empty, 'd', Empty))),
        'q', Empty),
      'f',
      Node
       (Node (Node (Empty, 'b', Node (Empty, 'w', Empty)), 'k',
         Node (Empty, 'v', Empty)),
       'r',
       Node (Node (Node (Empty, 'g', Empty), 'l', Node (Empty, 'z', Empty)),
        'k', Node (Node (Empty, 'z', Empty), 'z', Node (Empty, 'r', Empty))))))
```

Correct value `false` 1 pt

```
Computing
  balanced
    (Node
      (Node (Node (Node (Empty, 'v', Empty), 'z', Node (Empty, 'f', Empty)),
        'o', Node (Node (Empty, 'i', Empty), 'y', Node (Empty, 'c', Empty))),
      'o',
      Node (Node (Node (Empty, 'g', Empty), 'j', Node (Empty, 'p', Empty)),
       'p', Node (Node (Empty, 'v', Empty), 'i', Node (Empty, 'x', Empty)))))
```

Correct value `true` 1 pt

```
Computing
  balanced
    (Node
      (Node (Node (Node (Empty, 'c', Empty), 'z', Node (Empty, 'a', Empty)),
        'y', Node (Node (Empty, 'v', Empty), 'l', Node (Empty, 'i', Empty))),
      'b',
      Node (Node (Node (Empty, 'd', Empty), 'g', Node (Empty, 'v', Empty)),
       'g', Node (Node (Empty, 'x', Empty), 'h', Node (Empty, 'm', Empty)))))
```

Correct value `true` 1 pt

```
Computing
  balanced
    (Node
      (Node (Node (Node (Empty, 'd', Node (Empty, 'b', Empty)), 'r', Empty),
        'v', Node (Empty, 's', Node (Empty, 'r', Empty))),
      'x',
      Node (Node (Node (Node (Empty, 'u', Empty), 't', Empty), 'e', Empty),
       'n', Node (Node (Empty, 'g', Node (Empty, 'y', Empty)), 'r', Empty))))
```

Correct value `false` 1 pt

```
Computing
  balanced
    (Node
      (Node
        (Node (Node (Node (Empty, 'i', Empty), 't', Empty), 'q',
          Node (Empty, 'x', Node (Empty, 'r', Empty))),
        'e', Node (Empty, 'p', Node (Empty, 'c', Node (Empty, 'e', Empty)))),
      'm', Empty))
```

Correct value `false` 1 pt