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# 1 Introduction

Cellular automata is based upon simple rules that are able to exhibit powerful behaviours such as artificial life, self-organization, and more. The implications and applications of which are wide-ranging. Recent research into cellular automata has focused on applying recent advances in graph theory, machine learning, and computer science. These advances include, but are not limited to: Lenia which applies continous dimensions of space, time, and states; neural cellular automata which apply convolutional neural networks to develop rulesets; and graph cellular automata. All of these techniques apply recent discoveries in several fields. These cellular automata are able to exhibit complex behavior and emergence which has implications across several other fields of research.

# 2 Lenia: Biology of Artificial Life[1]

## 2.a Introduction

Lenia is a system of continuous cellular automata. It was derived from Conway's Game of Life by making everything smooth, continuous and generalized. It is a two-dimensional cellular automaton with continuous space-time-state and generalized local rule. It can support a greater diversity of complex autonomous patterns, "life forms", that differ from other cellular automata patterns in being geometric, matemeric, fuzzy, resilient, adaptive, and rule-generic ([10]).

The colab tutorial showing the progression from Conway's Game of Life to Lenia is available online ([1]) [2].

## 2.b Path to Lenia

This tutorial begins by implementing the original Game of Life (GoL) cellular automata. Progress toward Lenia is then made by generalizing GoL by using convolution and a growth function. This way, the cellular automata uses a convolution with a kernel instead of counting the neighbors. The growth function replaces the conditional update in the original automata. Once the Game of Life has been generalized in this manner, it is ready to be extended to continuous cases. Larger-than-Life extends it to a continuous space ([5]). Primordia extends it to continuous states, and Lenia combines these and extends it to continuous states-space-time ([3]).

Larger-than-Life enlarges the kernel we created above to a given radius creating a continuous space as the size of the radius approaches infinity.

Primordia allows for multiple states. The growth function is scaled up for the states and the neighbor sum and initial conditions both take these states into account. It helps to nor-

[1] Much of the information presented in this chapter is paraphrased from ([3])

[2] https://colab.research.google.com/github/OpenLenia/Lenia-Tutorial/blob/main/Tutorial_From_Conway_to_Lenia.ipynb

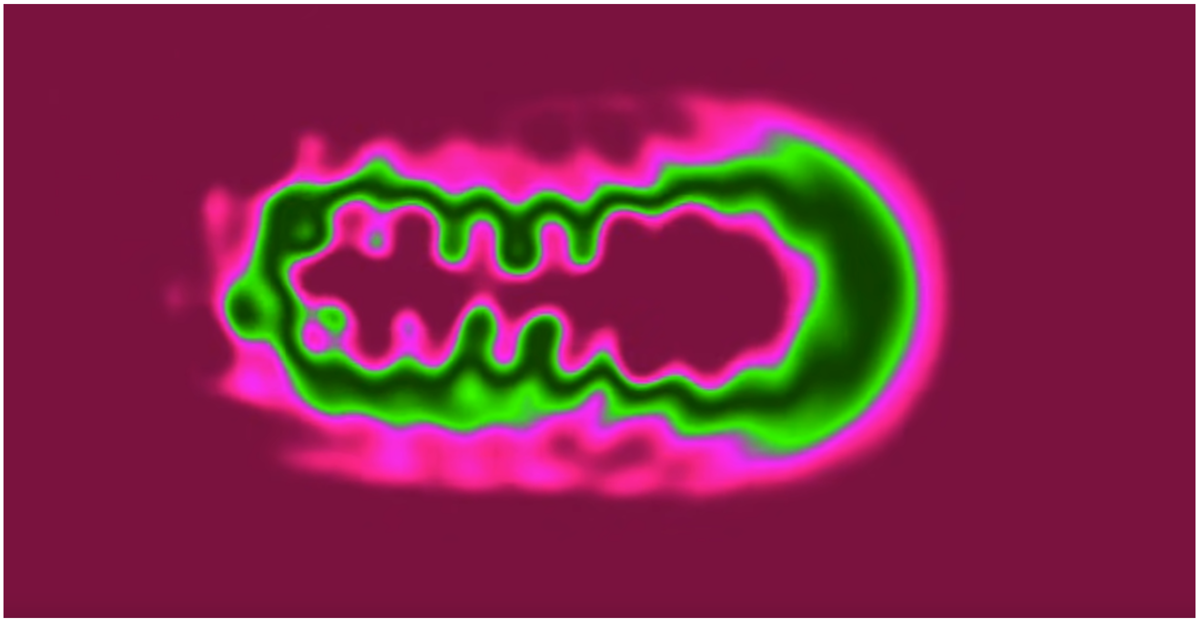


Figure 1: Hydrogeminium organism in Lenia

malize the various properties, states, kernel, and growth ranges to restrict their values making further generalizations much simpler. This also effectively makes the states continuous.

We can add to this continuous state cellular automata, Primordia, by defining an update frequency that scales the updates making time continuous as well.

Once we have continuous states and time, we can add in the continuous space from Larger-than-Life. This is where Lenia starts to form. Once the kernel is smoothed using a bell-shaped function and the growth function is also smoothed by the same type of function, we finally arrive at Lenia.

## 2.c Extensions and Variants

Since the creation of Lenia, many extensions have been discovered. For example, for more efficient calculations, the naive convolution calculation can be replaced by fast Fourier transform. Another extension involves extending the kernel into multiple rings instead of just a single ring. An even more ambitious extension was to extend it to multiple kernels and multiple growth functions making the kernels, neighbor sums, and growth values into lists and exhibiting even more complex behavior. Another extension was to extend the world into multiple channels, such as RGB channels which allows for kernels and the resulting organisms to operate on the full color spectrum.

Extensions aren't the only way that Lenia has been experimented with. Variants of Lenia have also come about. One such variant involves an asymptotic update changing the growth function into a target function which calculates a target state that the cell should approach and is also similar to the neural cellular automata mentioned in the next section. In multi-

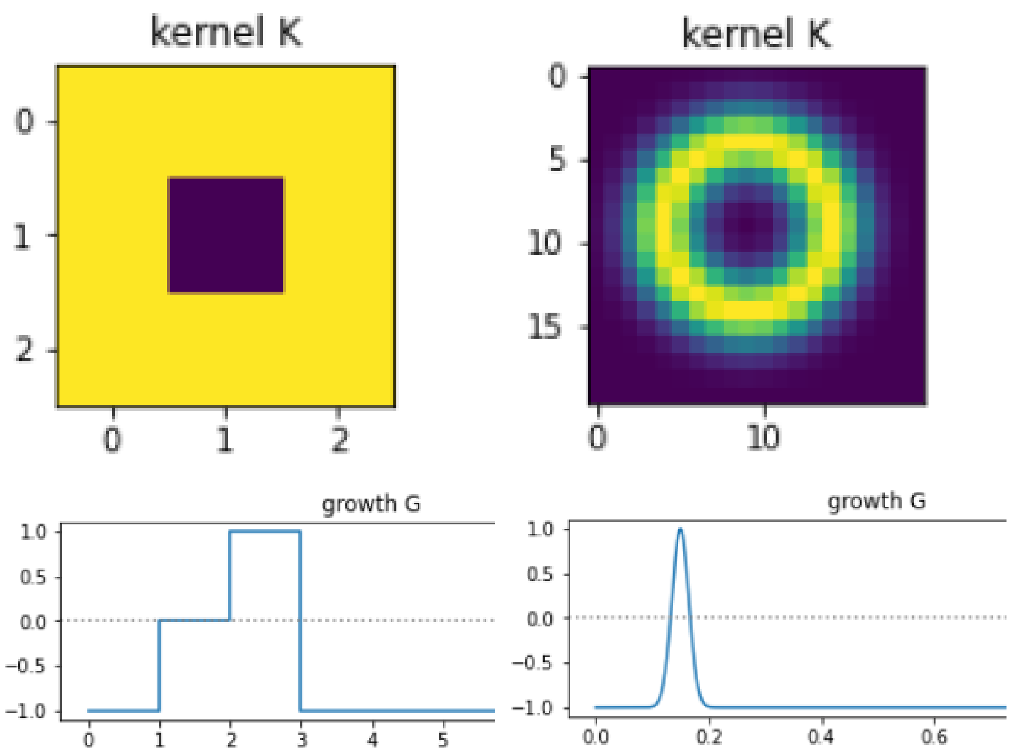



Figure 2: Left: kernel and growth function in Conway's Game of Life (GoL). Right: kernel and growth function in Lenia. Lenia has a smooth kernal and growth function when compared to GoL

channel Lenia, the growth or target function can be used for different channels, creating even more complex, interesting organisms. The last extension mentioned here involves changing the soft clip function to hard clip which creates smoother patterns and can exhibit long-range sensing.

## 2.d Observations from Author

Some initial experimentation with initial patterns and kernels in Lenia has led to the following observations.

Multiple initial patterns have been discovered in the Lenia framework. When these patterns are paired with a specific kernel, they perform interesting, life-like actions. These initial patterns and kernels are very sensitive to one another. If a stable pattern and kernel are found, deviating slightly from the initialization or kernel can cause the cellular automata to exhibit random noise or crash completely to nothing. There seem to be very few of these stable states compared to unstable ones. An interesting direction for future research could be experimenting with the discovery of the stable, versus unstable, conditions of Lenia. So far this experimentation has involved evolutionary algorithms searching for these patterns ([3]).

Several kernels have been applied to the Lenia cellular automata. All of the kernels experimented with so far involve between one and four concentric rings. If one applies a square ring kernel, the resulting cellular automaton behaves significantly differently. This work was unable to discover a stable state for the square ring kernel experimented with here, but it is worth noting that the only technique applied was manual perturbation of the initial state. The square kernel, however, also goes against the "smooth" nature of Lenia, so this could also be a contributing factor. An interesting research direction is hinted at here and described further in the next section.

## 2.e Discussion and Future Directions

Lenia is a recent extension of cellular automata that has an immense amount of potential for experimentation and discovery and has already started to see attention from researchers. There is still much to be discovered in regards to almost every aspect of Lenia's composition and theoretical implications. This work only begins to present the possibilities that Lenia offers.

Lenia has implications in the field of artificial life as it is a less-explored type of cellular automata in the area. It is useful for studying several properties of artificial life including emergence, self-organization, morphogenesis, and more.

Lenia is useful for studying with regards to artificial intelligence since the explode in complexity that is seen within it could be an aspect of generating artificial intelligence. The extended architecture of Lenia can incorporate convolutional networks much like the neural cellular automata. Pattern discover and generation within Lenia can utilize techniques from artifical intelligence or suggest new discoveries within this area.

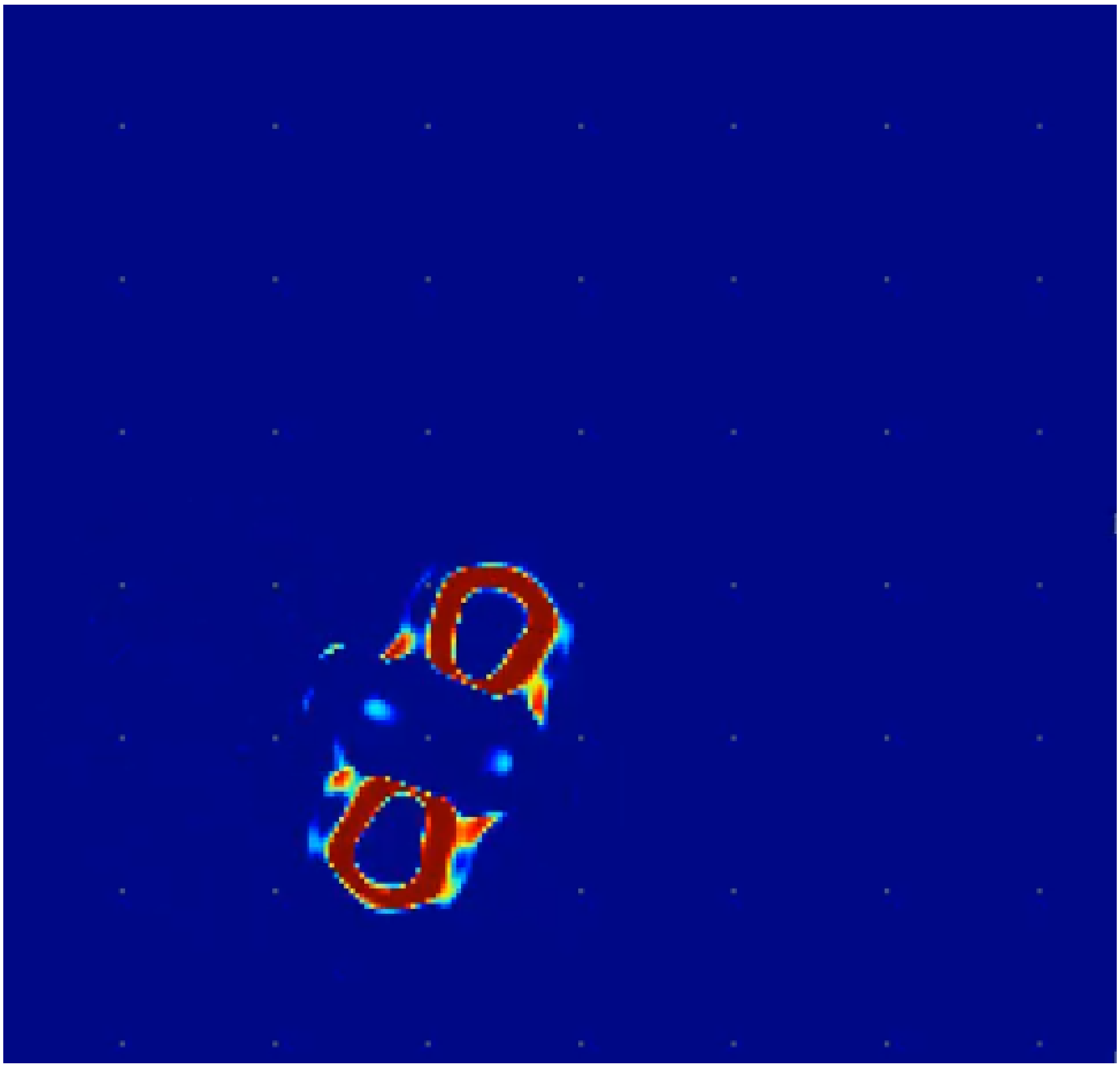


Figure 3: Moment of self-replication within the multi-kernel extension of Lenia.

It has implications for theoretical biology since forming purely digital lifeforms with capabilities similar to those of biological lifeforms can suggest areas for discovery in biology.

It has future potential work in the field of computer science. It has already been discovered that fast fourier transforms speed the computation within Lenia, but it is still undiscovered if Lenia is Turing complete or not. It could also be studied with regards to information dynamics for pattern discovery therein.

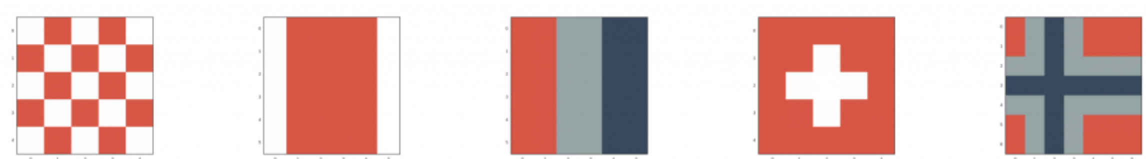
Lenia has a potential connection with mathematics and physics as it has seen attention with regard to the geometry of patterns discovered within it as well as the dynamics of these organisms across time. The self-propagation within Lenia may also be linked with soliton theory.

Lastly Lenia has become a source of digital art and is therefore linked to this area. It may be implemented in virtual reality as well for exploration within that space.

There has been much research in regards to Lenia, but there is still much to be discovered about this exciting, novel version of cellular automata.

# 3 Neural Cellular Automata

Neural Cellular Automata(NCA) are CA with neural networks representing the transition function that evolves the states of the CA. Using evolutionary algorithms, we are able to find NCAs that can successfully grow certain types of patterns(4) simulating the process of Morphogenesis. Tuning a system to grow slightly more complex target structures (such as the Nordic flag in the figure below) has proven difficult due to the possibility of getting stuck in a local optima during the evolutionary search using the developmental system.[8]



(a) 5x5 "Mosaic" 2 states
(b) 6x6 "Border" 2 states
(c) 6x6 "Tricolor" 4 states
(d) 5x5 "Swiss" 2 states
(e) 7x7 "Nordic" 4 states

Figure 4: Nordic Flag

A recent approach by Mordvintsev et al. (2020)[7] showed that, if a 2D target pattern is given, NCAs can be trained efficiently by supervised gradient descent to grow those patterns. One of the interesting phenomena discovered was that the NCA is able to re-grow the pattern after removal of certain cells from the target shape showing repair capabilities when the system was not explicitly trained for such a behavior.

Let's investigate this NCA work as done by Mordvintsev et al can help model self-repair and self-organization. A colab tutorial accompanies the article can be found here. [3].

[3]`https://colab.research.google.com/github/google-research/self-organising-systems/blob/`

## 3.a Problem Setup

The CA is defined on a two dimensional discrete(space) grid with states representing color space of the pattern plus some hidden channels that can be used for “cell-communication”. The states are continuous with RGBA in range (0,1). The alpha channel has a special meaning in that it represents the growth rate of the structure with 1 being fully grown. The updates are performed with discrete steps.

## 3.b Modeling the update step of the CA with a neural network

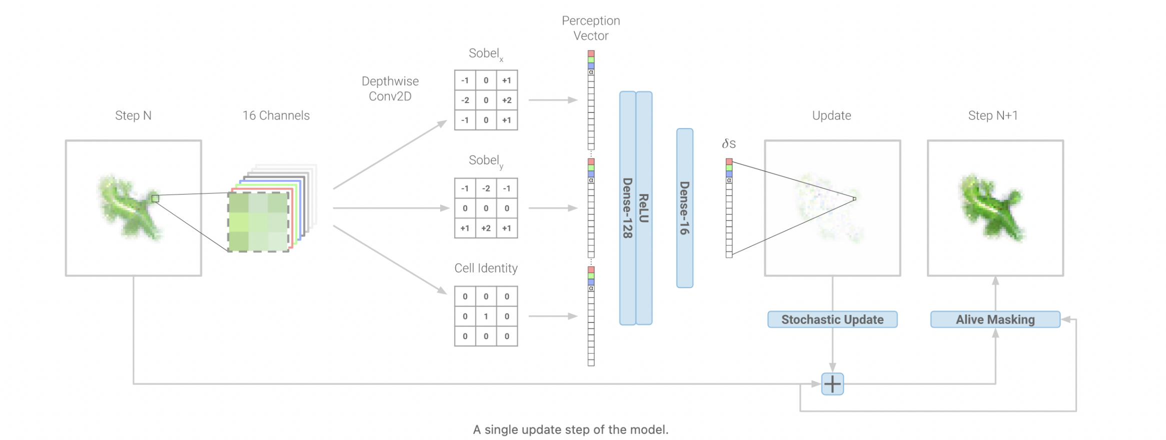



Figure 5: Update with NCA

Here we see that the update step is carried out in steps with first using sobel filters to determine where to grow, followed by a series of fully-connected layers with non-linear activations. The convolutions are applied with periodic boundary conditions.

## 3.c Choice of Network

They are using Sobel filters to estimate the partial derivatives of cell state channels in the X and Y directions, forming a 2D gradient vector in each direction, for each state channel. The update rule is designed to exhibit “do-nothing” initial behaviour - implemented by initializing the weights of the final convolutional layer in the update rule with zero. They also do not add ReLU(Rectified Linear Unit) activation in the final layer so that incremental updates to the cell state must necessarily be able to both add or subtract from the state.

`master/notebooks/growing_ca.ipynb`

## 3.d Stochastic Cell Update and Living Cell Mask

The CA is updated synchronously with cell states updated all at once. The authors introduce stochasticity in cell state updates to not rely on a global synchronizing mechanism which is not expected from a self-organizing system.

To not include empty cells to participate in the computation, they introduce a living cell mask that includes cells with Alpha channel values greater than 0.3 to only participate in the update step.

## 3.e Training the Neural Network

They define the loss to be the L2 norm of the pixel wise difference of states produced by the NCA not including the hidden states and the target pattern. They run the NCA for 50 steps where they expect the target pattern to be reached. During initial experiments, through gradient based optimizers like ADAM, they train the weights of the network to successfully match the target pattern. Using the optimized weights, they are able to grow to the desired pattern while the pattern dissipates after running the CA further than the steps used in training. In order to achieve static stability of the states, in addition to training from seed state, they use a pool of NCA states at various iteration times that are maintained and randomly sampled as starting states to simulate loss being applied after a time period longer than the NCAs iteration period. The author used intentional seed states that resembled damage to develop repair capabilities. This translates to increasing the basin of attraction for the target pattern by increasing the space of cell states that naturally gravitate towards the target pattern.

## 3.f Discussion and Future Work

Using Differentiable Programming (optimization) to learn agent-level policies represented by neural networks to satisfy system-level objectives, presents a new framework to generate and understand self-organization.

Since the local interaction rules are learned through gradient optimization, the loss function and initial seed states shape steady state dynamics of the CA beyond training. One drawback of this framework is that they can only generate (and regenerate) the single artifact they are trained on with no scope for a diverse set of generating self-organizing behavior.

To conclude, NCA's offers the toolsets of developments from Neural Networks to learn the transition function in a CA that simulates morphogenesis to a target shape. It would be interesting to see how this framework could help understand self-organization and explore in the continuous state space using CA modeling framework that uses local rules to affect global behavior.

# 4 Applications

This section will discuss some applications and implications of the above cellular automata, Lenia and NCAs as well as a couple of other recently discovered cellular automata.

## 4.a Morphogenesis

Morphogenesis is the process in which order is developed in a growing organism. This organization begins at the cellular level, in which cells organize themselves and differentiate into tissues, organs, and eventually an entire organism. The mechanisms behind morphogenesis have been investigated since the 1920's, when studies were performed to investigate regions of the embryo responsible for creating body structures. As more discoveries were made, focus shifted to applications such as limb development, limb regeneration, and growth coordination, however many questions still remain today on the mechanisms behind morphogenesis and how they can lead to the formation of an organism ([2]).

Rasolonjanahary et al. utilized von Neumann cellular automata to study the formation of morphogenic patterns due to short-range interactions. By investigating local interactions in the formation of periodic patterns, the mechanisms behind biological pattern formation can be better understood. The authors utilized one-dimensional cellular automata to discover what rules result in the formation of stationary periodic patterns that: allow for recovery conservation of periodic patterns, allow for local recovery of periodic patterns perturbed by noise, and allow for the formation of two-periodic patterns from random initial conditions. The two-state model utilized in this study is representative of cells either expressing a gene or not expressing a gene. Out of the 256 rules of elementary cellular automata, it was determined that 64 rules allow for existence of two-periodic patterns, 33 allow for recovery of two-periodic patterns, and only 6 allow for the formation of two-periodic patterns from initial conditions. While the growth of tissues, organs, and other biological development take place in three-dimensional space, pattern formation can accurately be represented as a one-dimensional process. The authors state that further study is warranted to model the formation of patterns in a growing medium and to perform a similar analysis on a four-state CA system ([9]).

## 4.b Neural Models of Self-Organization in Cellular Automata

Elmenrich et al. evaluated an evolutionary design process to generate self-organizing cellular automata system utilizing a neural network to control cell behavior. Utilizing a genotypical controller can allow for better control of state-transition logic, allowing cells to develop into a complex system more representative of that in natural evolution. An artificial neural network was used to control each cell after supplying the model with a reference image to replicate. The network consisted of 9 input neurons, 5 output neurons, and 6 hidden neurons; each of which were connected and assigned a weight, and each neuron was assigned a bias value. The artificial neural network can sense the colors of the neighboring cells, with information propagation occurring between inter-cell connections. The authors utilized a Frevo Java framework to develop the evolutionary programming and evolve the weights of the network. Frevo is used to generate distributed self-organizing systems and allows for combinations of different representations with optimization methods and a problem to be solved. The artificial neural network implemented mutation and recombination on the weights and biases of the artificial neurons. Pixels of different colors in their neighborhood had higher weightings to better simulate the perception of human vision. The network was fed reference images to which it attempted to replicate. The authors discovered that flags were the simplest patterns the network was able to recreate, while natural patterns, such as zebra stripes, were not recreated successfully. This was due to the parameters set in the model which focused on reproducing pixels rather than fully replicating an intricate pattern. Through the use of a genotypical template for each cell, the authors were able to reduce the search for a compatible algorithm, introducing a method to overcome complexity limits when developing an evolutionary algorithm. The authors state that further studies are necessary to optimize model configurations in order to better represent morphogenesis and natural patterning mechanisms ([4]).

Grattarola et al. studied the applications of graph neural networks (GNN) to learn graph cellular automata (GCA) transition rules, in a structure that allows cells to have various numbers of neighbors instead of a fixed neighborhood as in a typical lattice structure. Through the use of GNN's in the modeling of GCA systems, unknown rules can be uncovered that allow for the computation and discovery of complex solutions to both local and global problems. The authors discovered that the GNN can model both isotropic and anisotropic transition rules, and for CA with large neighborhoods, such as Lenia, the CA rules can be implemented in the Graph Neural Cellular Automata (GNCA) by redefining the neighborhood function to include higher-order neighbors. Experiments were performed to test the capabilities of the GNCA, including using supervised learning to discover a desired GCA transition rule, and learning an unknown transition rule by training the GNCA to converge to a desired state. The GNCA was trained for a given number of steps, applying backpropagation to update the weights, encouraging the model to converge to the target state in the given number of steps. A cache was also utilized for memory to train the GNCA on batches of states, while also training on states resulting from a repeated application of the transition function. The authors discovered that at t = 10, the GNCA oscillated between high and low-error states, while at t = 20, the GNCA was able to converge to a stable attractor. The use of GNCA can allow for the computation of transition rules in applications such as social, epidemiological, and technology networks, and can ultimately lead to a deeper understanding of the mechanisms behind these systems, such as how a virus spreads ([6]).

# 5 Conclusion

Cellular automata has existed for several decades and has seen a great amount of attention of that time. However, as new advances across several closely linked fields of research emerge, cellular automata can be utilized and updated to continue to present meaningful discoveries. It is a simple, yet powerful, tool to explore numerous phenomena including self-organization, complexity, morphogenesis and more. This work presents a few of the recent advances with regards to cellular automata including Lenia, Neural Cellular Automata, and Graph Cellular Automata and discusses future research and implications of these. There is still much to discover in this area and the authors do not foresee a drop in attention or research within it any time soon.

# References

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