Assignment 3: Tunably Rugged Landscapes

In our assignment last week we got our first hillclimber up and running, while in class week started to talk about fitness landscapes to begin thinking about search spaces, and population-based evolutionary algorithms to start complexifying how we traverse these search spaces. In this week's assignment, we'll start to put these two things together and begin toying around with the pandora's box of algorithmic experimentation.

In particular, we'll explore the idea of generating parameterized fitness functions to being to explore the relationship between the type of problem we're trying to solve, and what features our evolutionary algorithm should have to solve it.

Note: I know this looks like a lot of coding! While we are building valuable infastructure here, much of the solutions here are modifications on prior work (from eariler in this assignment or the last one), and can largely be copy-and-pasted here, or written once as a function to call again later. Despite this, it's still always a good idea to start in on assignments early (even if just reading through all the questions to estimate how long it might take you to complete)

In [1]:

```
# imports
import numpy as np
import copy
import matplotlib.pyplot as plt
plt.style.use('seaborn')

import scikits.bootstrap as bootstrap
import warnings
warnings.filterwarnings('ignore') # Danger, Will Robinson! (not a scalable hack, and may s

import scipy.stats # for finding statistical significance
import random
import pandas as pd
```

N-K Landscape

In general, you'll be more likely to have a problem provided to you, rather than have to design a fitness funciton by hand. So in this week's assignment, I'll provide the full fitness-landscape-generating function for you. The below function implements Kaffman's N-K Landscape. While it's not entirely necessary for you to understand every implementaton detail below, the N-K landscape idea is chosen because it's a particularly interesting toy problem -- and more reading on it can be found via many online resources (e.g. Kauffman and Weinberger's *The NK model of rugged fitness landscapes and its application to maturation of the immune response* -- inlcuded in the assigment zip folder as it is firewalled online)

The main things to know about the NK model are that: It is a model of a tunably rugged fitness landscape, that means we have parameters that can affect the shape and ruggedness of the fitness landscape produced by this model. While there are many variations, here we follow the original (simplest) model that includes just two parameters: **N** defines the length of the binary bit string genome, while **K** defines the ruggedness of the landscape (in particular how the fitness of each allele depends on other loci (nearby genes) in the genotype.

Note: This is fully implemented and no action is needed from you, besides running the code block.

In [2]:

```
class Landscape:
    """ N-K Fitness Landscape
```

```
    """


    def __init__(self, n=10, k=2):
        self.n = n # genome length
        self.k = k # number of other loci interacting with each gene
        self.gene_contribution_weight_matrix = np.random.rand(n,2**(k+1)) # for each gene,

    # find values of interacting loci
    def get_contributing_gene_values(self, genome, gene_num):
        contributing_gene_values = ""
        for i in range(self.k+1): # for each interacing loci (including the location of th
            contributing_gene_values += str(genome[(gene_num+i)%self.n]) # for simplicity
        return contributing_gene_values # return the string containing the values of all i

    # find the value of a partiuclar genome
    def get_fitness(self, genome):
        gene_values = np.zeros(self.n) # the value of each gene in the genome
        for gene_num in range(len(genome)): # for each gene
            contributing_gene_values = self.get_contributing_gene_values(genome, gene_num)
            gene_values[gene_num] = self.gene_contribution_weight_matrix[gene_num,int(cont
        return np.mean(gene_values) # define the fitness of the full genome as the average
```

Hillclimber

Based on the hillclimber function from you last assignment (and informed by the posted solution, if you wish), copy an slightly modify the hillclimber to use this fitnes function. For sake of running multiple trials, also please modify the record keeping to reutrn the solutions after the completion of the algorithm rather than printing them out during evolution.

Hint: In python, functions can be treated as objects (e.g. passed as an argument to another function)

In [3]:

```
def hillclimber(total_generations = 100, bit_string_length = 10, num_elements_to_mutate= 1
    """ Basic hillclimber, copied from last assignment

        parameters:
        total_generations: (int) number of total iterations for stopping condition
        bit_string_length: (int) length of bit string genome to be evoluved
        num_elements_to_mutate: (int) number of alleles to modify during mutation
        fitness_funciton: (callable function) that return the fitness of a genome
                          given the genome as an input parameter (e.g. as defined in Land

        returns:
        solution: (numpy array) best solution found
        solution_fitness: (float) fitness of returned solution
        solution_generation: (int) generaton at which most fit solution was first discover
    """

    # the initialization proceedure
    parent = []
    for i in range(bit_string_length):
        parent.append(random.randint(0,1))
    fitness_parent = fitness_function(parent)


    # initialize record keeping
    solution = np.zeros(bit_string_length)
    solution_fitness = 0
    solution_generation = 0

    for i in range(total_generations): # repeat
```

```
        # the modification procedure
        child = parent.copy()
        mutate_elements = []
        for j in range(num_elements_to_mutate):
            mutate_elements.append(random.randint(0,bit_string_length-1))
        for j in range(len(mutate_elements)):
            bit = child[mutate_elements[j]]
            if bit==1:
                child[mutate_elements[j]] = 0
            else:
                child[mutate_elements[j]] = 1

        # the assessement procedure
        fitness_child = fitness_function(child)
        if fitness_child > fitness_parent:

            # selection procedure
            parent = child
            fitness_parent = fitness_child

            # record keeping
            solution = np.array(parent).astype(np.float)
            solution_fitness = fitness_parent
            solution_generation = i

    return solution, solution_fitness, solution_generation
```

Q1: Landscape Ruggedness's effect on Hillclimbing

In class we discussed the potential for the fitness landscape to greatly affect a given search algorithm. Let't start by generating varyingly rugged landscapes, and investigating how this impacts the effectiveness of a standard hillclimber.

For each value of `k = 0..14` and a genome legnth of `15` please generate 100 unique fitness landscapes, and record the fitness value and time to convergence (when the most fit solution was found) for the hillclimber algorithm above on that landscape. Print out the mean results for each `k` as you go to keep track of progress. This output may look something like this:

In [4]:

```
# hyperparameters
n=15; max_k=15; repetitions = 100

# initialize array to record results over different settings of k and repeated trials
solutions_found = np.zeros((max_k,repetitions,n))
fitness_found = np.zeros((max_k,repetitions))
generation_found = np.zeros((max_k,repetitions))

# initilize output
print(' k  mean fitness  mean generation found')
print('--  ------------  ---------------------')

# for many values of k
for k in range (max_k):
    # for many repeated (independent -- make sure your results differ each run!) trials
    for j in range (repetitions):
        l = Landscape(n, k) # generate a random fitness landscape with this level of rugge

        # run a hillclimber and record outputs
        solutions_found[k][j], fitness_found[k][j], generation_found[k][j] = hillclimber(l
```

```
    # print average results for all repitions of this k
#     print(k)
#     print(np.mean(fitness_found[k]))
#     print(np.mean)
    print('{}\t{}\t\t{}'.format(k, np.round(np.mean(fitness_found[k]), 3), np.round(np.mea
```

```
 k  mean fitness  mean generation found
--  ------------  ---------------------
0       0.659          39.77
1       0.697          40.57
2       0.703          36.67
3       0.701          34.88
4       0.699          31.41
5       0.686          30.88
6       0.697          27.51
7       0.692          27.33
8       0.666          24.47
9       0.666          19.21
10      0.658          23.48
11      0.645          17.48
12      0.623          19.02
13      0.61           15.74
14      0.6            15.7
```

Let's also record this result in a nested dictionary to be able to recall it later (for comparison to other results). There is an implementation given below, but you're welcome to use `pandas` if you're more comforatable with that library for data manipulation and visualization.

In [5]:
```
experiment_results = {}
experiment_results["hillclimber"] = {"solutions_found":solutions_found, "fitness_found":fi
```

Q2: Plotting Results

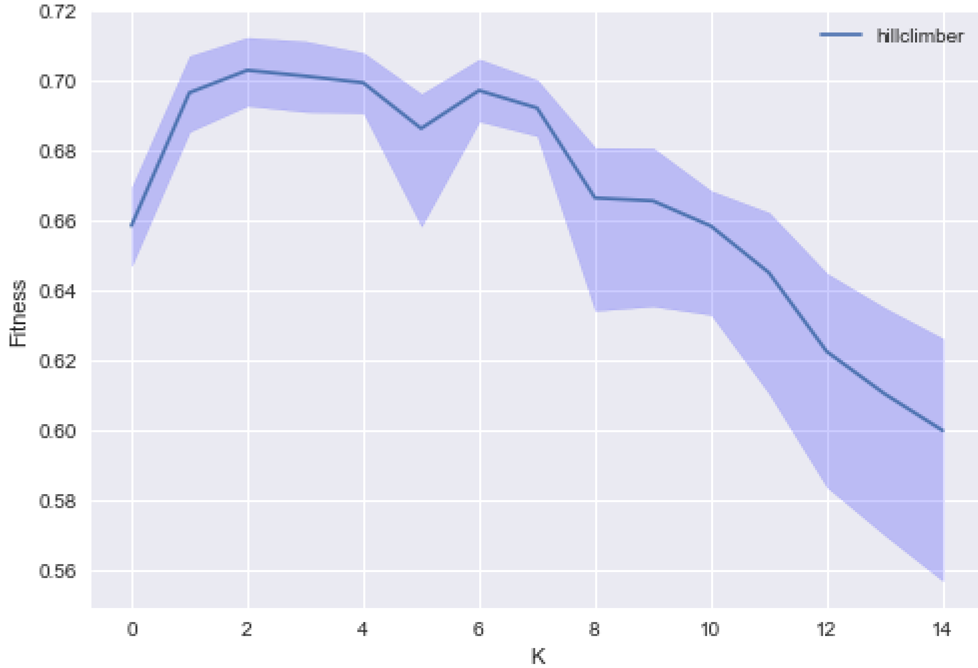
Please visualize the above terminal output in a figure (feel free to recycle code from previous assignments). You'll be generating this same plot many time (and even comparing multiple runs on a single figure), so you may want to invest in implementing this as a function at some point during this assigment -- but that is not strictly necessary now, and fell free to ignore the code stub below.

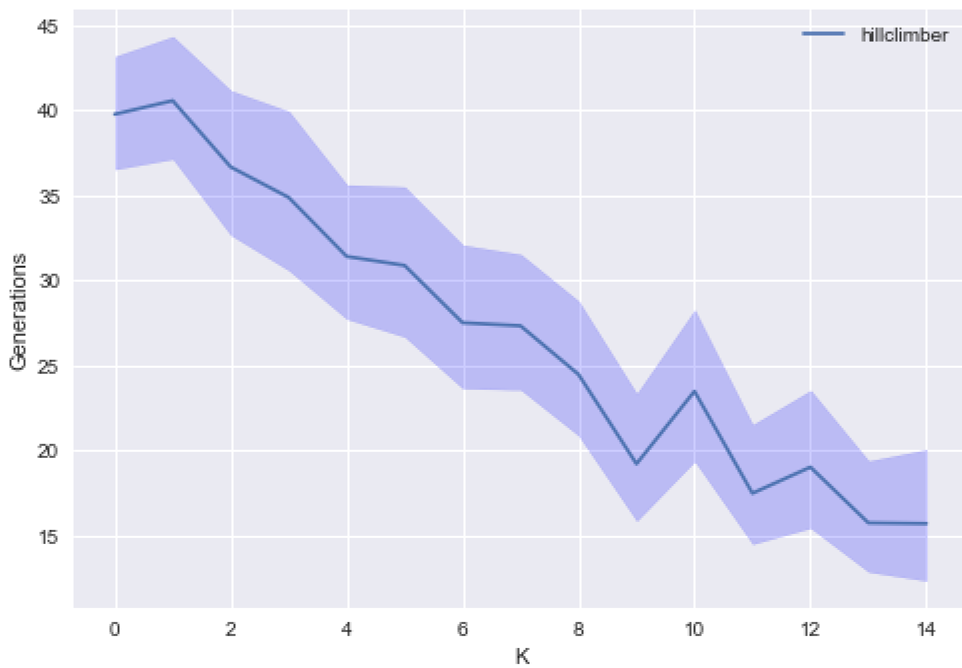
In particular, please plot the `Time to Convergence (Generations)` and `Fitness` values (as you vary `K`) as two separate figures, as a single figure with multiple y-axes is messy and confusing. Please include 95% boostrapped confidence intervals over your 100 repitions for eack `K`. Please also inlcude the title of each experiment as a legend (for now just `hillclimber` is sufficient for this baseline case, and titles will make more sense in follow up experimental conditions).

In [6]:
```
def plot_mean_and_bootstrapped_ci(input_data = None, title=None, name = "change me", x_lab
    """

    parameters:
    input_data: (numpy array of shape (max_k, num_repitions)) solution metric to plot
    name: (string) name for legend
    x_label: (string) x axis label
    y_label: (string) y axis label

    returns:
    None
    """
```

```python
    max_k = input_data.shape[0]

#     plt.title(name)
#     plt.xlabel(x_label)
#     plt.ylabel(y_label)
#     plt.legend()
#     plt.show()

    CIs = []
    mean_values = []
    for i in range(max_k):
        mean_values.append(np.mean(input_data[i]))
        CIs.append(bootstrap.ci(input_data[i], statfunction=np.mean))
    mean_values=np.array(mean_values)

    print(CIs)
    high = []
    low = []
    for i in range(len(CIs)):
        low.append(CIs[i][0])
        high.append(CIs[i][1])

    low = np.array(low)
    high = np.array(high)
    fig, ax = plt.subplots()
    y = range(0, max_k)
    ax.plot(y, mean_values, label=name)
    ax.fill_between(y, high, low, color='b', alpha=.2)
    ax.set_xlabel(x_label)
    ax.set_ylabel(y_label)
    ax.legend()
    if (title) and len(title>0):
        ax.set_title(name)


plot_mean_and_bootstrapped_ci(input_data=fitness_found, name='hillclimber', y_label="Fitne
plot_mean_and_bootstrapped_ci(input_data=generation_found, name='hillclimber', y_label="Ge
```

```
[array([0.64712366, 0.66957644]), array([0.68550348, 0.7071013 ]), array([0.69288909, 0.71
237307]), array([0.69114486, 0.71128359]), array([0.6908174 , 0.70798604]), array([0.65848
358, 0.69622547]), array([0.68844425, 0.70616464]), array([0.68427795, 0.7002047 ]), array
([0.63432824, 0.68085733]), array([0.63559576, 0.68073574]), array([0.6331735 , 0.6685185
6]), array([0.61065792, 0.66232258]), array([0.58387324, 0.64489887]), array([0.57011367,
0.63505752]), array([0.55712137, 0.62635412])]
[array([36.55, 43.18]), array([37.16, 44.35]), array([32.68, 41.17]), array([30.61, 39.9
5]), array([27.75, 35.63]), array([26.7 , 35.51]), array([23.67, 32.09]), array([23.59, 3
1.54]), array([20.89, 28.78]), array([15.88, 23.34]), array([19.39, 28.27]), array([14.52,
21.52]), array([15.46, 23.54]), array([12.88, 19.39]), array([12.36, 20.06])]
```





Q3: Analysis of Hillclimber on Varying Ruggedness

What do you notice about the trend line? Is this what you expected? Why or why not?

The trend lines for both of these plots look similar. They both initially trend upward, then roughly start to fall (with some bouncing up and down). This definitely isn't what I initially expected from these trend lines. I expected the trend line for fitness to initially start high and drop off as the ruggedness increased since it would be a harder fitness landscape to fit to, but as I think about it more this trend line starts to make sense. While ruggedness may make it much tougher to find the global optima, it could make it easier to find a local optima. As for the generational convergence trend line, I initially expected it would be lower and as the ruggedness of the landscape went up, it would also go up. The trend line begins to make more sense when thinking more deeply about a rugged landscape. It may be hard to get off of the local optima that we find with the step size we have. I am almost definitely missing some considerations here. This is a very interesting topic to think about.

Q4: Random Restarts

One of the methods we talked about as a potential approach to escaping local optima in highly rugged fitness landscapes was to randomly restart search. Using the same number of total generations (`100`), please implement a function which restarts search to a new random initialization every `20` generations (passing this value as an additoinal parameter to your hillclimber function). Feel free to just copy and paste the hillclimber code block here to modify, for the sake of simplicity and easy gradability.

In [7]:

```
def hillclimber(total_generations = 100, bit_string_length = 10, num_elements_to_mutate= 1
    """ Basic hillclimber, copied from last assignment

        parameters:
        total_generations: (int) number of total iterations for stopping condition
        bit_string_length: (int) length of bit string genome to be evoloved
        num_elements_to_mutate: (int) number of alleles to modify during mutation
        fitness_funciton: (callable function) that return the fitness of a genome
                          given the genome as an input parameter (e.g. as defined in Lanc
        restart_every: (int) how frequently to randomly restart the hillclimber

        returns:
        solution: (numpy array) best solution found
        solution_fitness: (float) fitness of returned solution
        solution_generation: (int) generaton at which most fit solution was first discover
    """

    # the initialization proceedure
    parent = []
    best = []
    for i in range(bit_string_length):
        parent.append(random.randint(0,1))
    fitness_parent = fitness_function(parent)


    # initialize record keeping
    solution = np.zeros(bit_string_length)
    solution_fitness = 0
    solution_generation = 0

    for i in range(total_generations): # repeat

        if (total_generations % restart_every)==0:
            parent=[]
            for j in range(bit_string_length):
                parent.append(random.randint(0,1))
            fitness_parent = fitness_function(parent)

        # the modification procedure
        child = parent.copy()
        mutate_elements = []
        for j in range(num_elements_to_mutate):
            mutate_elements.append(random.randint(0,bit_string_length-1))
        for j in range(len(mutate_elements)):
            bit = child[mutate_elements[j]]
            if bit==1:
                child[mutate_elements[j]] = 0
            else:
                child[mutate_elements[j]] = 1

        # the assessement procedure
        fitness_child = fitness_function(child)
        if fitness_child > fitness_parent:

            # selection procedure
            parent = child.copy()
            fitness_parent = fitness_child
```

```
        if fitness_parent > fitness_function(best):
            # record keeping
            best = parent.copy()
            solution = np.array(parent).astype(np.float)
            solution_fitness = fitness_parent
            solution_generation = i

    return solution, solution_fitness, solution_generation
```

Q4b: Run Experiment

Slightly modify (feel free to copy and paste here) your experiment running code black above to analyze the effect of modifying `K` on `Time to Convergence (Generations)` and `Fitness`, again print progress and plotting results. Please also save these results (and subsequent new ones) to your `experimental_results` dictionary for later use.

In [8]:

```
name = 'hillclimber_random_restarts'
# hyperparameters
n=15; max_k=15; repetitions = 100; restart_every=20

# initialize array to record results over different settings of k and repeated trials
solutions_found = np.zeros((max_k,repetitions,n))
fitness_found = np.zeros((max_k,repetitions))
generation_found = np.zeros((max_k,repetitions))

# initilize output
print(' k  mean fitness  mean generation found')
print('--  ------------  ---------------------')

# for many values of k
for k in range (max_k):
    # for many repeated (independent -- make sure your results differ each run!) trials
    for j in range (repetitions):
        l = Landscape(n, k) # generate a random fitness landscape with this level of rugge
        
        # run a hillclimber and record outputs
        solutions_found[k][j], fitness_found[k][j], generation_found[k][j] = hillclimber(k



    # print average results for all repitions of this k
#     print(k)
#     print(np.mean(fitness_found[k]))
#     print(np.mean)
    print('{}\t{}\t\t{}'.format(k, np.round(np.mean(fitness_found[k]), 3), np.round(np.mea

experiment_results[name] = {"solutions_found":solutions_found, "fitness_found":fitness_fou
```

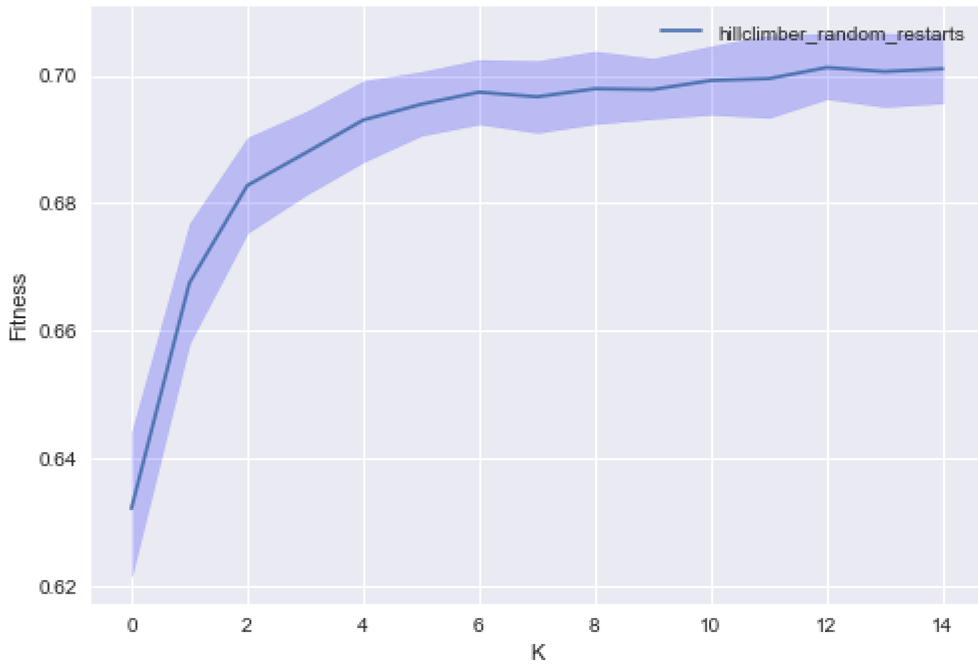
```
 k  mean fitness  mean generation found
--  ------------  ---------------------
0       0.632         50.58
1       0.668         50.71
2       0.683         44.33
3       0.688         51.38
4       0.693         48.52
5       0.696         46.81
6       0.697         48.91
7       0.697         50.33
8       0.698         51.07
9       0.698         51.38
10      0.699         48.06
```

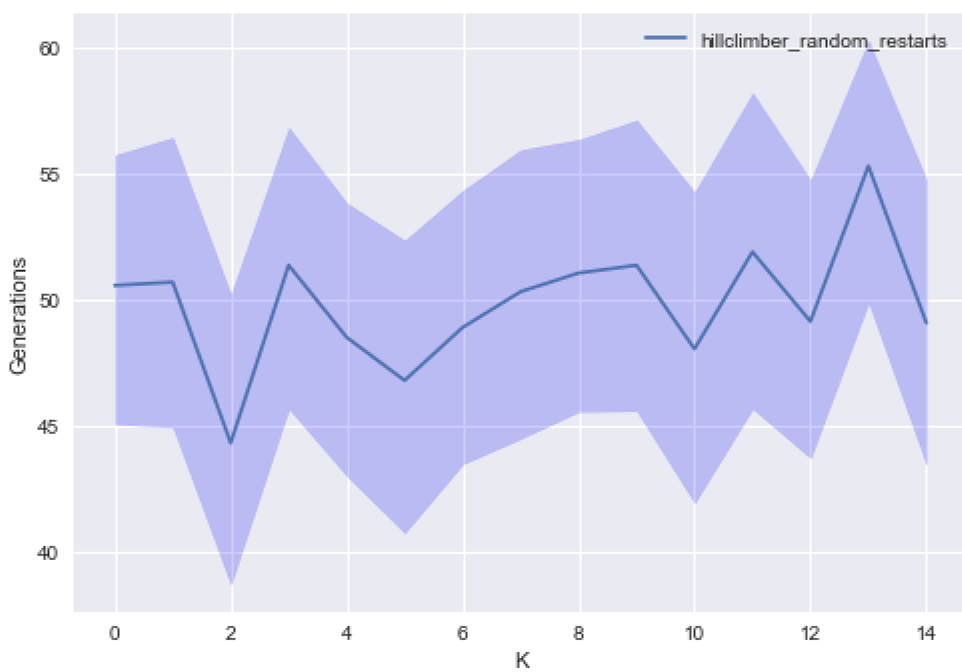
```
11      0.7              51.91
12      0.701            49.15
13      0.701            55.32
14      0.701            49.09
```

In [9]:

```python
#plotting
plot_mean_and_bootstrapped_ci(input_data=fitness_found, name=name, y_label="Fitness")
plot_mean_and_bootstrapped_ci(input_data=generation_found, name=name, y_label="Generations
```

```
[array([0.62162242, 0.64421172]), array([0.6581965 , 0.67689326]), array([0.67543573, 0.69
031684]), array([0.68131534, 0.69433178]), array([0.68651645, 0.69916596]), array([0.69065
111, 0.70061929]), array([0.69243161, 0.70250994]), array([0.69108863, 0.70232075]), array
([0.69247229, 0.70381026]), array([0.69326335, 0.70268836]), array([0.69392391, 0.7046256
6]), array([0.69342622, 0.70634033]), array([0.69637019, 0.70657602]), array([0.69512634,
0.70660221]), array([0.6957118 , 0.70645015])]
[array([45.09, 55.76]), array([44.96, 56.45]), array([38.72, 50.22]), array([45.7 , 56.8
5]), array([43.03, 53.85]), array([40.77, 52.37]), array([43.5 , 54.35]), array([44.51, 5
5.96]), array([45.58, 56.37]), array([45.61, 57.14]), array([41.96, 54.28]), array([45.7 ,
58.22]), array([43.74, 54.76]), array([49.91, 60.3 ]), array([43.47, 54.77])]
```





Q5: Analysis of Random Restarts

What trends do you see? Is this what you were expecting? How does this compare to the original hillclimber algorithm without random resets (please not any y-axis differences when comparing values/shapes of the curves)?

The trend line for fitness with respect to ruggedness shows a steady increase until it plateaus at around 70%. Early on as ruggedness increases, it appears so does fitness. This suggests that the random restarts help the hill climber find steeper hills (maxima) especially as the landscape becomes more rugged which is potentially increasing the number of these hills. The generational convergence to this fitness shows that the best candidates are being found at roughly the same generation regardless of ruggedness of the landscape. All of the best candidates are found in the 40-60 generational range. This differs largely from hillclimber without these restarts since it does not improve in fitness as ruggedness increases. This is probably due to the hillclimber being stuck on a lower local optima and not being able to find its way out. This would also explain why the generational convergence was so quick for these more rugged landscapes in the hillclimber without restarts.

Q6: Modifying mutation size

We've talked about a number of other potential modifications/complexifications to the original hillclimber aglorithm in class, so let's experiment with some of them here. Here, please modying your above a hillclimber (again please just copy and paste the code block here) to mutate multiple loci when generating the child from a parent.

Hint: Be careful of the difference between modifying multiple genes and modifying the same gene multiple times

In [10]:

```
def hillclimber(total_generations = 100, bit_string_length = 10, num_elements_to_mutate= 1
    """ Basic hillclimber, copied from last assignment

        parameters:
        total_generations: (int) number of total iterations for stopping condition
        bit_string_length: (int) length of bit string genome to be evoloved
        num_elements_to_mutate: (int) number of alleles to modify during mutation
        fitness_funciton: (callable function) that return the fitness of a genome
                          given the genome as an input parameter (e.g. as defined in Lanc
        restart_every: (int) how frequently to randomly restart the hillclimber

        returns:
        solution: (numpy array) best solution found
        solution_fitness: (float) fitness of returned solution
        solution_generation: (int) generaton at which most fit solution was first discover
    """

    # the initialization proceedure
    parent = []
    best = []
    for i in range(bit_string_length):
        parent.append(random.randint(0,1))
    fitness_parent = fitness_function(parent)


    # initialize record keeping
    solution = np.zeros(bit_string_length)
    solution_fitness = 0
    solution_generation = 0

    for i in range(total_generations): # repeat

        if (total_generations % restart_every)==0:
```

```
            parent=[]
            for j in range(bit_string_length):
                parent.append(random.randint(0,1))
            fitness_parent = fitness_function(parent)

        # the modification procedure
        child = parent.copy()
        mutate_elements = []
        mutate_elements = random.sample(range(0, bit_string_length-1), num_elements_to_mut
        for j in range(len(mutate_elements)):
            bit = child[mutate_elements[j]]
            if bit==1:
                child[mutate_elements[j]] = 0
            else:
                child[mutate_elements[j]] = 1

        # the assessement procedure
        fitness_child = fitness_function(child)
        if fitness_child > fitness_parent:

            # selection procedure
            parent = child.copy()
            fitness_parent = fitness_child

        if fitness_parent > fitness_function(best):
            # record keeping
            best = parent.copy()
            solution = np.array(parent).astype(np.float)
            solution_fitness = fitness_parent
            solution_generation = i

    return solution, solution_fitness, solution_generation
```

Q6b: Expectations

In this experiment, let's set the number of elements to be mutated to `5` when generating a new child.

Before running the code, what do (did) you expect the result to be based on the results of the original hillclimber, the random restart condition, and the implications that a larger mutatoin rate may have?

Depending on whether or not we are running this with or without restarts, my expectations may differ some. I'm going to run it without restarts to exemplify the difference it makes on the original hillclimber. In this situation, I expect that a larger mutation will allow for more exploration, but it may also bounce around the fitness landscape making it difficult for it to converge to very high values of fitness without getting lucky. As for the generational time it will take, I suspect that it will be a bit more random without a definite trend since the changes being made each iteration will be larger thus causing this generational convergence to bounce around drastically.

Q7: Run experiment

Run the experiment and visualize (similar to **Q4b**, and feel free to copy a paste here again) to analyze the effect of a larger mutation size on the realationship between `K` and `Time to Convergence (Generations)` / `Fitness`.

In [11]:

```
name = 'hillclimber_larger_mutations'
# hyperparameters
n=15; max_k=15; repetitions = 100; restart_every=100; num_elements_to_mutate=5

# initialize array to record results over different settings of k and repeated trials
```

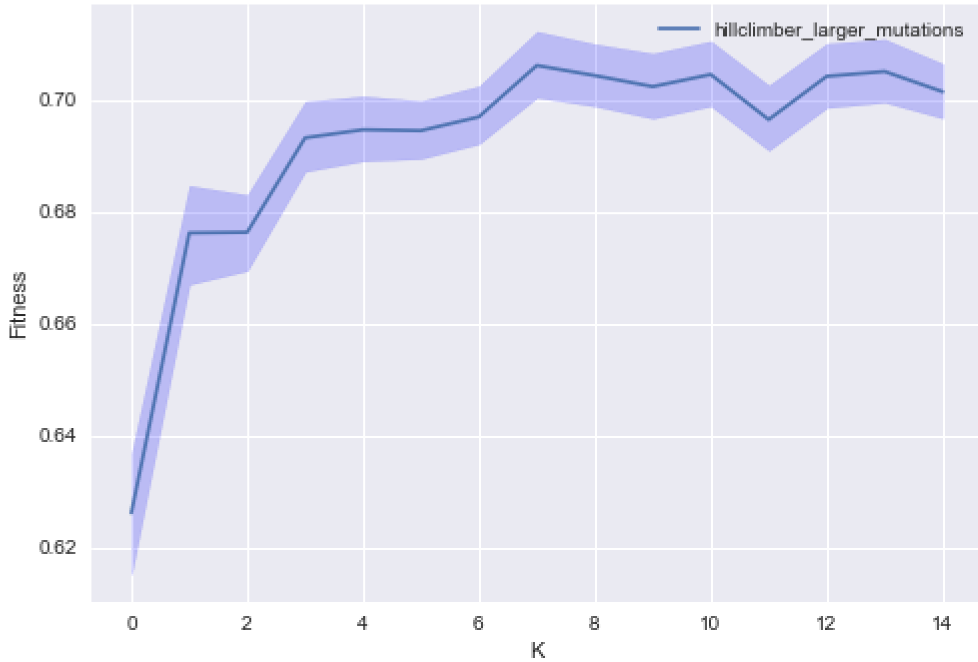
```
    solutions_found = np.zeros((max_k,repetitions,n))
    fitness_found = np.zeros((max_k,repetitions))
    generation_found = np.zeros((max_k,repetitions))

    # initilize output
    print(' k  mean fitness  mean generation found')
    print('--  ------------  ---------------------')

    # for many values of k
    for k in range (max_k):
        # for many repeated (independent -- make sure your results differ each run!) trials
        for j in range (repetitions):
            l = Landscape(n, k) # generate a random fitness landscape with this level of rugge
            
            # run a hillclimber and record outputs
            solutions_found[k][j], fitness_found[k][j], generation_found[k][j] = hillclimber(l



        # print average results for all repitions of this k
    #     print(k)
    #     print(np.mean(fitness_found[k]))
    #     print(np.mean)
        print('{}\t{}\t\t{}'.format(k, np.round(np.mean(fitness_found[k]), 3), np.round(np.mea
    experiment_results[name] = {"solutions_found":solutions_found, "fitness_found":fitness_fou
```

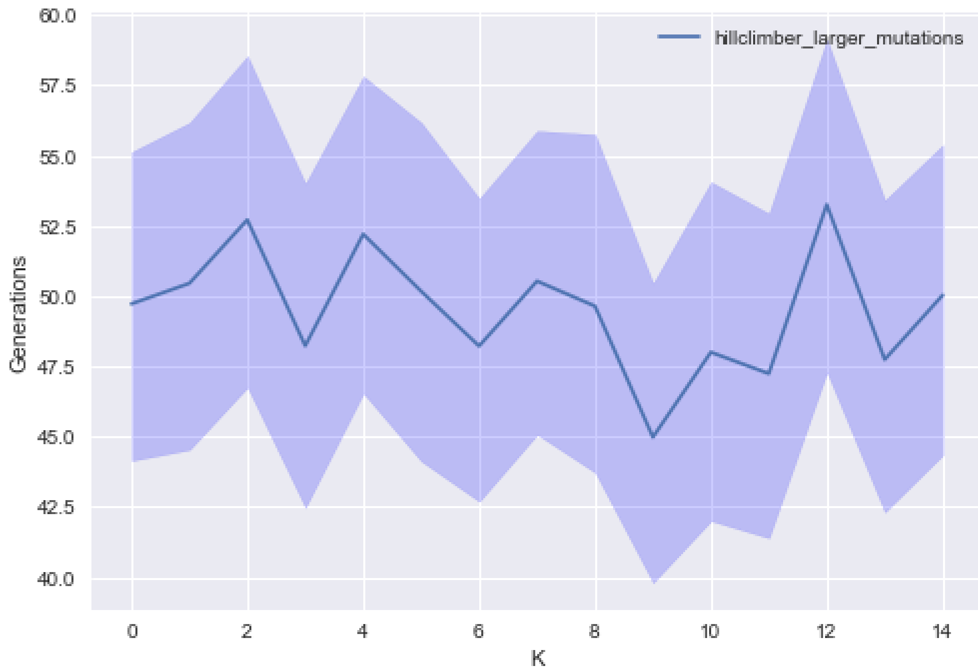
```
 k  mean fitness  mean generation found
--  ------------  ---------------------
0       0.626           49.72
1       0.676           50.45
2       0.676           52.71
3       0.693           48.23
4       0.695           52.2
5       0.694           50.17
6       0.697           48.22
7       0.706           50.53
8       0.704           49.64
9       0.702           44.98
10      0.704           48.0
11      0.696           47.24
12      0.704           53.25
13      0.705           47.74
14      0.701           50.03
```

In [12]:

```
# plotting
plot_mean_and_bootstrapped_ci(input_data=fitness_found, name=name, y_label="Fitness")
plot_mean_and_bootstrapped_ci(input_data=generation_found, name=name, y_label="Generations
```

```
[array([0.61512971, 0.63672451]), array([0.66694815, 0.6846246 ]), array([0.66939682, 0.68
296712]), array([0.68714236, 0.69965411]), array([0.68901521, 0.70061588]), array([0.68943
109, 0.69977206]), array([0.69206174, 0.70240164]), array([0.7003688 , 0.71218118]), array
([0.6988217, 0.7099389]), array([0.69663412, 0.7082608 ]), array([0.6988288 , 0.7104457
6]), array([0.69089742, 0.70254227]), array([0.69854671, 0.71002126]), array([0.69949341,
0.71078659]), array([0.69667925, 0.70641324])]
[array([44.15, 55.13]), array([44.54, 56.17]), array([46.76, 58.54]), array([42.48, 54.
]), array([46.55, 57.82]), array([44.13, 56.17]), array([42.7 , 53.47]), array([45.08, 55.
88]), array([43.73, 55.75]), array([39.81, 50.45]), array([42.01, 54.06]), array([41.4 , 5
2.94]), array([47.3 , 59.13]), array([42.32, 53.41]), array([44.35, 55.38])]
```





Q7b: Analysis

Is this what you expected/predicted? If not, what is different and why might that be?

This is slightly different than I expected and looks quite similar to the random restart case. The fitness values are relatively high compared to what I might have expected. They are lower on the less rugged landscape which makes sense with my expecations. I believe the reason they tend to be higher in the more rugged landscapes is due to being able to bounce around and land on a hill since it is more likely when there are more hills. The generational convergence is similar to what I predicted with some vaules being higher and some lower, but they do all fall in the range of 40-60 which surprises me some. I think this could be because the mutations were still able to climb larger hills even with the relatively large mutations.

Q8: Accepting Negative Mutations

Another way we might be able to get out of local optima is by taking steps downhill away from that optima. Add another arguement (`downhill_prob`) to your `hillclimber` function, which accepts a child with a negative mutataion with that given probability.

In [13]:

```
def hillclimber(total_generations = 100, bit_string_length = 10, num_elements_to_mutate= 1
    """ Basic hillclimber, copied from last assignment

        parameters:
        total_generations: (int) number of total iterations for stopping condition
        bit_string_length: (int) length of bit string genome to be evoloved
        num_elements_to_mutate: (int) number of alleles to modify during mutation
        fitness_funciton: (callable function) that return the fitness of a genome
                                   given the genome as an input parameter (e.g. as defined in Lanc
        restart_every: (int) how frequently to randomly restart the hillclimber
        downhill_prob: (float) proportion of times when a downhill mutation is accepted

        returns:
        solution: (numpy array) best solution found
        solution_fitness: (float) fitness of returned solution
        solution_generation: (int) generaton at which most fit solution was first discover
    """

    # the initialization proceedure
    parent = []
    best = []
    for i in range(bit_string_length):
        parent.append(random.randint(0,1))
    fitness_parent = fitness_function(parent)


    # initialize record keeping
    solution = np.zeros(bit_string_length)
    solution_fitness = 0
    solution_generation = 0

    for i in range(total_generations): # repeat

        if (restart_every > 0) and (total_generations % restart_every)==0:
            parent=[]
            for j in range(bit_string_length):
                parent.append(random.randint(0,1))
            fitness_parent = fitness_function(parent)

        # the modification procedure
        child = parent.copy()
        mutate_elements = []
        mutate_elements = random.sample(range(0, bit_string_length-1), num_elements_to_mut
        for j in range(len(mutate_elements)):
            bit = child[mutate_elements[j]]
            if bit==1:
                child[mutate_elements[j]] = 0
            else:
                child[mutate_elements[j]] = 1

        # the assessement procedure
        fitness_child = fitness_function(child)
        if fitness_child > fitness_parent or random.randint(0,99)<(downhill_prob*100):

            # selection procedure
            parent = child.copy()
            fitness_parent = fitness_child

        if fitness_parent > fitness_function(best):
            # record keeping
            best = parent.copy()
            solution = np.array(parent).astype(np.float)
            solution_fitness = fitness_parent
```

```
            solution_generation = i

    return solution, solution_fitness, solution_generation
```

Q8b: Run the experiment

Same as above (run and plot), but now investigating the effect of a `downhill_prob` of `0.1` (10% chance) on relationship between ruggedness and performance

In [14]:

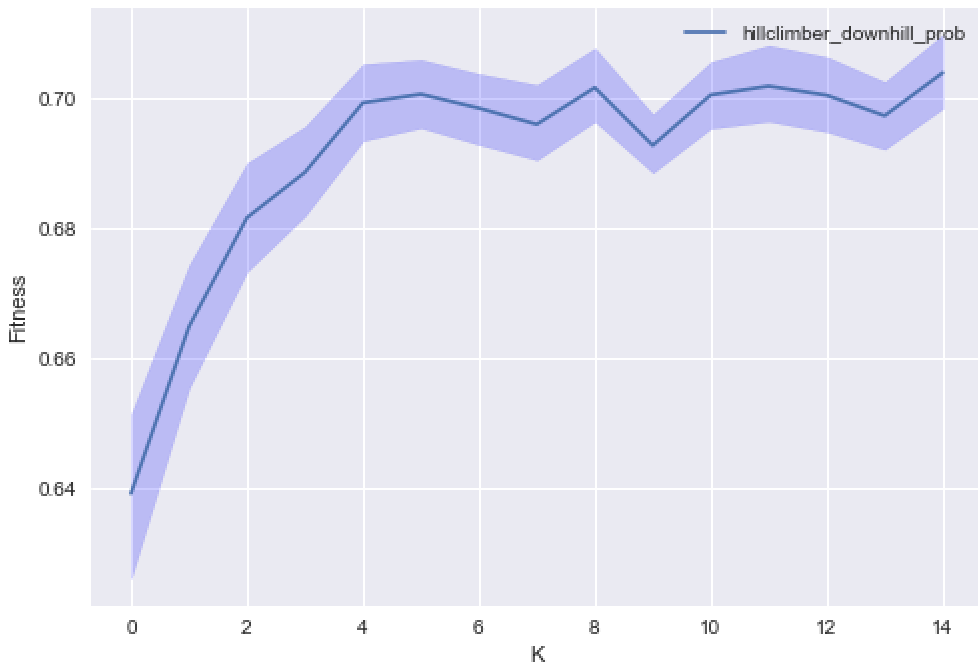
```
name = 'hillclimber_downhill_prob'
# hyperparameters
n=15; max_k=15; repetitions = 100; restart_every=100; num_elements_to_mutate=1; downhill_p

# initialize array to record results over different settings of k and repeated trials
solutions_found = np.zeros((max_k,repetitions,n))
fitness_found = np.zeros((max_k,repetitions))
generation_found = np.zeros((max_k,repetitions))

# initilize output
print(' k  mean fitness  mean generation found')
print('--  ------------  ---------------------')

# for many values of k
for k in range (max_k):
    # for many repeated (independent -- make sure your results differ each run!) trials
    for j in range (repetitions):
        l = Landscape(n, k) # generate a random fitness landscape with this level of rugge

        # run a hillclimber and record outputs
        solutions_found[k][j], fitness_found[k][j], generation_found[k][j] = hillclimber(k



    # print average results for all repitions of this k
#     print(k)
#     print(np.mean(fitness_found[k]))
#     print(np.mean)
    print('{}\t{}\t\t{}'.format(k, np.round(np.mean(fitness_found[k]), 3), np.round(np.mea

experiment_results[name] = {"solutions_found":solutions_found, "fitness_found":fitness_fou
```

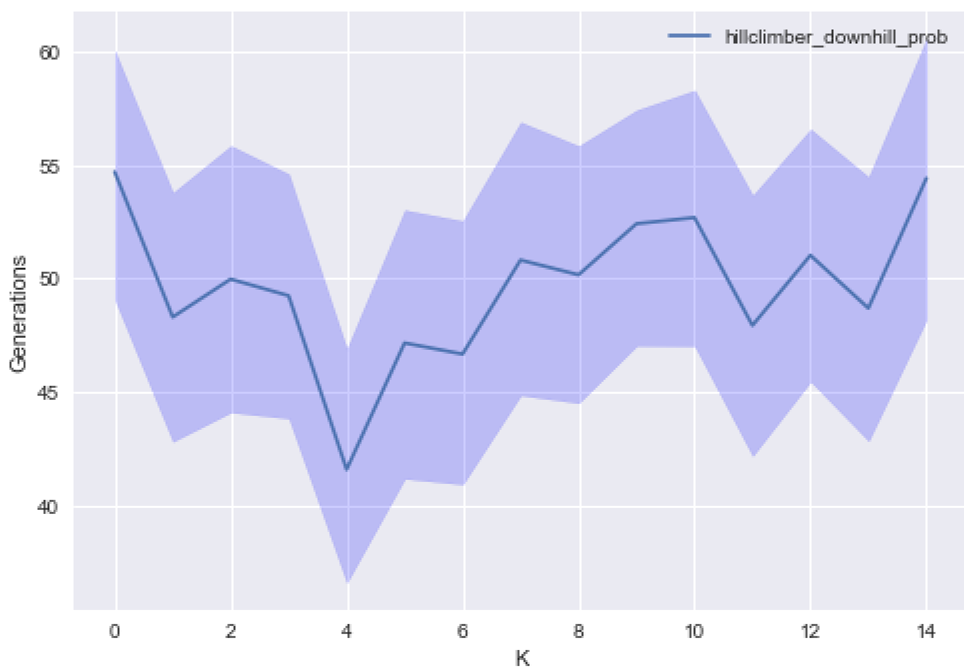
```
 k  mean fitness  mean generation found
--  ------------  ---------------------
0       0.639           54.7
1       0.665           48.31
2       0.682           49.98
3       0.689           49.25
4       0.699           41.6
5       0.701           47.17
6       0.698           46.68
7       0.696           50.82
8       0.702           50.17
9       0.693           52.42
10      0.7             52.69
11      0.702           47.94
12      0.7             51.03
13      0.697           48.7
14      0.704           54.42
```

In [15]:

```
# plotting
plot_mean_and_bootstrapped_ci(input_data=fitness_found, name=name, y_label="Fitness")
plot_mean_and_bootstrapped_ci(input_data=generation_found, name=name, y_label="Generations
```

```
[array([0.62624628, 0.65148346]), array([0.65548201, 0.67438046]), array([0.67335017, 0.69
004882]), array([0.68190586, 0.695625  ]), array([0.69343492, 0.70521201]), array([0.69540
676, 0.70588988]), array([0.69284475, 0.70372422]), array([0.69049487, 0.70203496]), array
([0.69637639, 0.70765177]), array([0.68852892, 0.69749796]), array([0.69533232, 0.7055562
1]), array([0.69641132, 0.70809488]), array([0.69480407, 0.70631244]), array([0.6921315 ,
0.70247939]), array([0.6983862 , 0.70968017])]
[array([49.08, 60.03]), array([42.84, 53.79]), array([44.13, 55.86]), array([43.86, 54.6
1]), array([36.63, 46.91]), array([41.21, 53.03]), array([40.96, 52.55]), array([44.88, 5
6.9 ]), array([44.54, 55.85]), array([47.06, 57.43]), array([47.05, 58.3 ]), array([42.21,
53.67]), array([45.49, 56.59]), array([42.87, 54.48]), array([48.21, 60.58])]
```





Q9: Visualizing Mulitple Runs

On the same plot (which may require modifying or reimplementing your plotting function, if you made one above), please plot the curves for all 4 of our experiments above on a single plot (including bootsrapped confidence intervals for all).

Hint: Legends are especially important here!

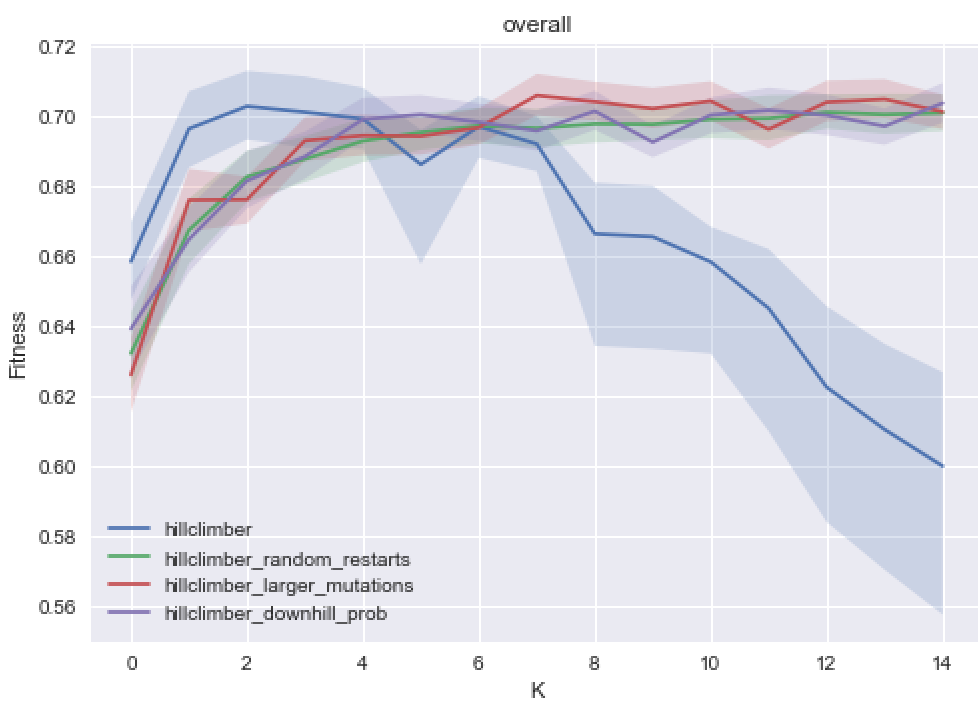
Hint: It may be convenient to iterate over the dictionaries, turning them into lists before plotting (depending on your plotting script)

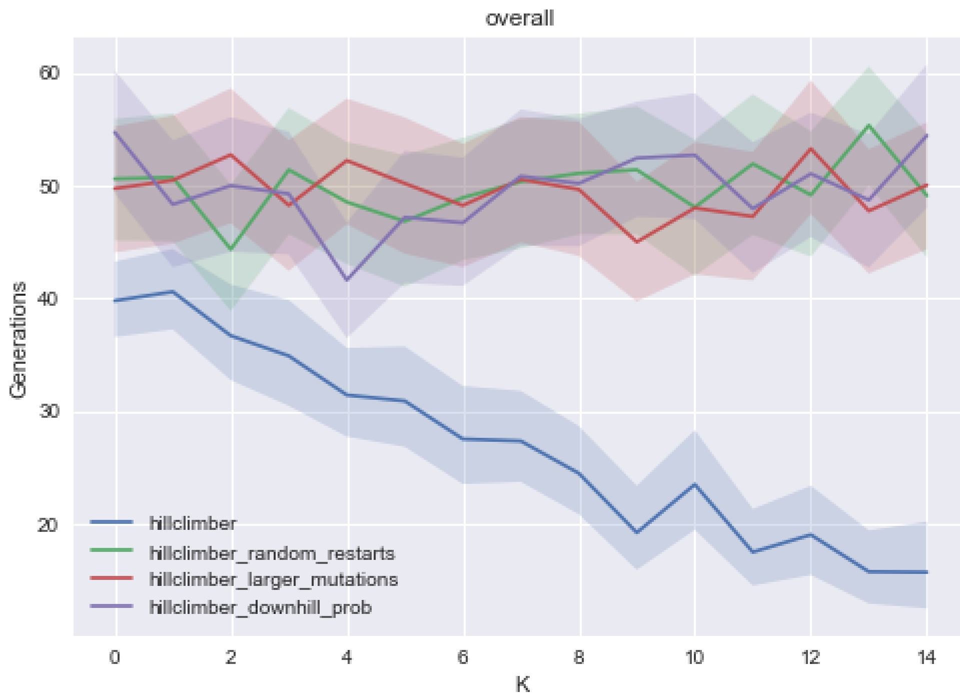
In [16]:

```
# plotting
def plot_mean_and_bootstrapped_ci_multiple(input_data = None, title = 'overall', name = "c
    """

    parameters:
    input_data: (numpy array of numpy arrays of shape (max_k, num_repitions)) solution met
    name: numpy array of string names for legend
    x_label: (string) x axis label
    y_label: (string) y axis label

    returns:
    None
    """

    max_k = len(input_data[0])

#     plt.title(name)
#     plt.xlabel(x_label)
#     plt.ylabel(y_label)
#     plt.legend()
#     plt.show()

    fig, ax = plt.subplots()
    ax.set_xlabel(x_label)
    ax.set_ylabel(y_label)
    ax.set_title(title)
    for i in range(len(input_data)):
        CIs = []
        mean_values = []
        for j in range(max_k):
            mean_values.append(np.mean(input_data[i][j]))
            CIs.append(bootstrap.ci(input_data[i][j], statfunction=np.mean))
        mean_values=np.array(mean_values)

        print(CIs)
        high = []
        low = []
        for j in range(len(CIs)):
            low.append(CIs[j][0])
            high.append(CIs[j][1])

        low = np.array(low)
        high = np.array(high)
#         fig, ax = plt.subplots()
        y = range(0, max_k)
        ax.plot(y, mean_values, label=name[i])
        ax.fill_between(y, high, low, alpha=.2)
        ax.legend()

experimental_results_fitnesses = []
experimental_results_generations = []
for k,v in experiment_results.items():
    for ki,vi in experiment_results[k].items():
        if ki == 'fitness_found':
            experimental_results_fitnesses.append(vi)
        if ki == 'generation_found':
            experimental_results_generations.append(vi)
plot_mean_and_bootstrapped_ci_multiple(input_data=experimental_results_fitnesses, name=[x
plot_mean_and_bootstrapped_ci_multiple(input_data=experimental_results_generations, name=
```

```
[array([0.64749614, 0.66980852]), array([0.68548857, 0.70724938]), array([0.69344789, 0.71
```

```
310237]), array([0.69113922, 0.71153346]), array([0.69077107, 0.7082886 ]), array([0.65780
031, 0.6962561 ]), array([0.6881845 , 0.70584084]), array([0.68439028, 0.70015541]), array
([0.63438159, 0.68110258]), array([0.63356245, 0.68020461]), array([0.63221002, 0.6684649
3]), array([0.60996846, 0.66208028]), array([0.58397693, 0.64570785]), array([0.57031017,
0.634918  ]), array([0.55752656, 0.62686108])]
[array([0.6214698 , 0.64404732]), array([0.65789265, 0.67652096]), array([0.6754943 , 0.69
026958]), array([0.68129691, 0.69441954]), array([0.68697754, 0.69963847]), array([0.69044
266, 0.70072577]), array([0.69239698, 0.70255807]), array([0.69113453, 0.7021919 ]), array
([0.69256898, 0.70380027]), array([0.6932847 , 0.70275309]), array([0.69383692, 0.7046801
7]), array([0.69328435, 0.70613195]), array([0.69655997, 0.70638139]), array([0.69506301,
0.7065391 ]), array([0.69581461, 0.70651202])]
[array([0.61552226, 0.63676658]), array([0.66711666, 0.68504374]), array([0.66936394, 0.68
284742]), array([0.68729069, 0.69981779]), array([0.68895936, 0.7005513 ]), array([0.68913
862, 0.69964657]), array([0.69205844, 0.70231205]), array([0.70036576, 0.71224743]), array
([0.69890592, 0.70996396]), array([0.6967293 , 0.70826578]), array([0.69890592, 0.7101006
5]), array([0.6908776 , 0.70239056]), array([0.6987077 , 0.71031783]), array([0.69959537,
0.71073969]), array([0.69647735, 0.70625536])]
[array([0.62681732, 0.65149493]), array([0.65548878, 0.6742155 ]), array([0.67379745, 0.69
018018]), array([0.68219095, 0.69568424]), array([0.69356029, 0.70540771]), array([0.69542
72 , 0.70613858]), array([0.69305911, 0.703756  ]), array([0.6902787 , 0.70183035]), array
([0.69621567, 0.70740921]), array([0.68837079, 0.69739261]), array([0.6952349 , 0.7054856
7]), array([0.69650001, 0.70828553]), array([0.69472973, 0.70637103]), array([0.69193178,
0.70237204]), array([0.69827935, 0.70961017])]
[array([36.55, 43.24]), array([37.23, 44.35]), array([32.73, 41.2 ]), array([30.45, 39.8
3]), array([27.72, 35.58]), array([26.82, 35.71]), array([23.52, 32.2 ]), array([23.71, 3
1.77]), array([20.84, 28.66]), array([15.91, 23.36]), array([19.46, 28.29]), array([14.51,
21.3 ]), array([15.45, 23.36]), array([12.92, 19.4 ]), array([12.49, 20.18])]
[array([45.11, 55.9 ]), array([45.04, 56.38]), array([38.87, 50.07]), array([45.66, 56.8
7]), array([43.02, 53.86]), array([41.02, 52.69]), array([43.36, 54.21]), array([44.42, 5
5.89]), array([45.64, 56.34]), array([45.64, 57.01]), array([41.98, 54.05]), array([45.61,
58.08]), array([43.67, 54.79]), array([50.  , 60.54]), array([43.6 , 54.62])]
[array([44.04, 55.23]), array([44.83, 56.17]), array([46.64, 58.6 ]), array([42.42, 54.
]), array([46.55, 57.69]), array([43.94, 56.02]), array([42.75, 53.68]), array([44.93, 56.
06]), array([43.74, 55.66]), array([39.7 , 50.32]), array([42.11, 53.84]), array([41.56, 5
2.97]), array([47.45, 59.32]), array([42.18, 53.2 ]), array([44.32, 55.58])]
[array([49.29, 60.19]), array([42.75, 53.99]), array([44.11, 56.04]), array([43.88, 54.7
6]), array([36.46, 46.71]), array([41.34, 53.07]), array([41.08, 52.45]), array([44.64, 5
6.73]), array([44.6, 55.9]), array([47.21, 57.42]), array([47.02, 58.17]), array([42.23, 5
3.82]), array([45.41, 56.45]), array([42.72, 54.55]), array([48.  , 60.73])]
```





Q9b: Analyzing Mulitple Runs

Do any new relationships or questions occur to you as you view these?

All of the edited versions of the hill climber that we played with look more similar to one another than to the original. They all have a similar effect on the outcome of the fitness values and the generational convergence. The original hill climber does extremely well on very simple fitness landscapes with a low generational convergence and high fitness. If the landscape is more rugged, these other versions tend to do better with higher fitnesses, but also with higher generational convergences. They continue looking for solutions where the unmodified hillclimber gives up.

Q10: Statistical Significance

Using the `ranksums` test for significance, please compare the values for each algorithm at `K=14` using your saved `experiment_results`, reporting the p-value for each combination of the 4 experiments. Please do this for both the resulting fitness values, and the generation for which that solution was found. The output may look something like this:

In [17]:

```
# test for statistical significance across treatments
k = 14

experimental_results_fitnesses_k = []
experimental_results_generations_k = []
for i in range(len(experiment_results.keys())):
    experimental_results_fitnesses_k.append(experimental_results_fitnesses[i][k])
    experimental_results_generations_k.append(experimental_results_generations[i][k])

print('fitness_found')
print('----------------')

for i in range(len(experimental_results_fitnesses_k)):
    for j in range(len(experimental_results_fitnesses_k)):
        if i >= j:
            continue
        else:
            print('{}: ({}) \t <-> \t {}: ({}) \t p-val={}'.format(
```

```
                         list(experiment_results.keys())[i],
                         np.round(np.mean(experimental_results_fitnesses_k[i]),3),
                         list(experiment_results.keys())[j],
                         np.round(np.mean(experimental_results_fitnesses_k[j]),3),
                         np.round(scipy.stats.ranksums(experimental_results_fitnesses_k[i], exp

print()
print('generation_found')
print('----------------')
for i in range(len(experimental_results_generations_k)):
    for j in range(len(experimental_results_generations_k)):
        if i >= j:
            continue
        else:
            print('{}: ({}) \t <-> \t {}: ({}) \t p-val={}'.format(
                         list(experiment_results.keys())[i],
                         np.round(np.mean(experimental_results_generations_k[i]),3),
                         list(experiment_results.keys())[j],
                         np.round(np.mean(experimental_results_generations_k[j]),3),
                         np.round(scipy.stats.ranksums(experimental_results_generations_k[i], e



...
```

```
fitness_found
----------------
hillclimber: (0.6)       <->     hillclimber_random_restarts: (0.701)    p-val=0.0
hillclimber: (0.6)       <->     hillclimber_larger_mutations: (0.701)   p-val=0.0
hillclimber: (0.6)       <->     hillclimber_downhill_prob: (0.704)      p-val=0.0
hillclimber_random_restarts: (0.701)     <->     hillclimber_larger_mutations: (0.701)    p
-val=0.895
hillclimber_random_restarts: (0.701)     <->     hillclimber_downhill_prob: (0.704)       p
-val=0.45
hillclimber_larger_mutations: (0.701)    <->     hillclimber_downhill_prob: (0.704)       p
-val=0.517

generation_found
----------------
hillclimber: (15.7)      <->     hillclimber_random_restarts: (49.09)    p-val=0.0
hillclimber: (15.7)      <->     hillclimber_larger_mutations: (50.03)   p-val=0.0
hillclimber: (15.7)      <->     hillclimber_downhill_prob: (54.42)      p-val=0.0
hillclimber_random_restarts: (49.09)     <->     hillclimber_larger_mutations: (50.03)    p
-val=0.765
hillclimber_random_restarts: (49.09)     <->     hillclimber_downhill_prob: (54.42)       p
-val=0.241
hillclimber_larger_mutations: (50.03)    <->     hillclimber_downhill_prob: (54.42)       p
-val=0.226
```

Out[17]: Ellipsis

Q11: Hyperparameter Search

Its cool to see the differences that these approaches have over the baseline hillcimber, but the values for each parameter that we've asked you to investigate are totally arbitrarily chosen. For example, who's to say that doing random resets every `20` generations is ideal? So let's find out!

Please modify the code above for which you varied `K` to see the effect on `Fitness` and `Time to Convergence (Generations)`, to now keep a constant `K=14` and vary how frequently do you random resets within the fixed `100` generations of evolution. Explore this relationship for values of resets ranging from never (`0`) up to every `29` generations.

In [18]:

```python
name = 'hillclimber_restart_interval_k14'
# hyperparameters
n=15; max_k=14; repetitions = 100; max_restart_interval=29; num_elements_to_mutate=1; dowr

# initialize array to record results over different settings of k and repeated trials
solutions_found = np.zeros((max_restart_interval,repetitions,n))
fitness_found = np.zeros((max_restart_interval,repetitions))
generation_found = np.zeros((max_restart_interval,repetitions))

# initilize output
print(' i  mean fitness  mean generation found')
print('--  ------------  ---------------------')

# for many values of k
for i in range (max_restart_interval):
    # for many repeated (independent -- make sure your results differ each run!) trials
    for j in range (repetitions):
        l = Landscape(n, max_k) # generate a random fitness landscape with this level of i

        # run a hillclimber and record outputs
        solutions_found[i][j], fitness_found[i][j], generation_found[i][j] = hillclimber(k



    # print average results for all repitions of this k
#     print(k)
#     print(np.mean(fitness_found[k]))
#     print(np.mean)
    print('{}\t{}\t\t{}'.format(i, np.round(np.mean(fitness_found[i]), 3), np.round(np.mea

experiment_results2 = {}
experiment_results2[name] = {"solutions_found":solutions_found, "fitness_found":fitness_fo
```

```
 i  mean fitness  mean generation found
--  ------------  ---------------------
0       0.647           14.59
1       0.704           50.54
2       0.702           59.92
3       0.639           15.19
4       0.701           49.2
5       0.701           50.9
6       0.64            12.22
7       0.641           14.79
8       0.642           17.8
9       0.64            16.44
10      0.7             51.17
11      0.643           14.19
12      0.646           14.42
13      0.642           14.42
14      0.633           12.65
15      0.642           13.33
16      0.643           15.57
17      0.645           15.93
18      0.638           15.17
19      0.639           16.11
20      0.7             56.16
21      0.649           14.34
22      0.646           16.14
23      0.646           12.84
24      0.638           15.88
25      0.697           45.28
26      0.639           14.41
27      0.647           17.73
28      0.644           14.37
```

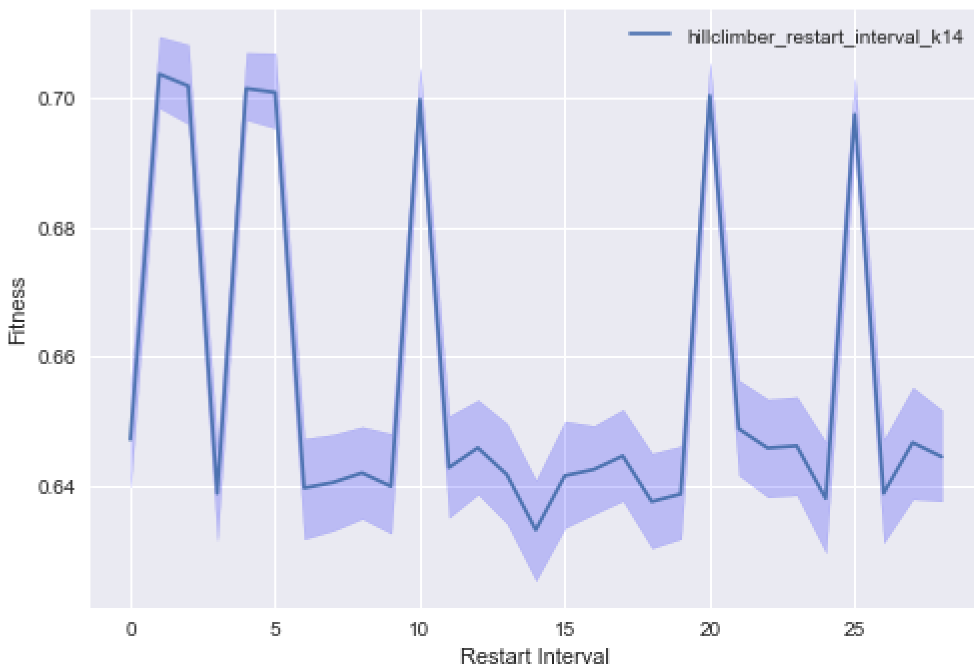
Q11b: Visualization

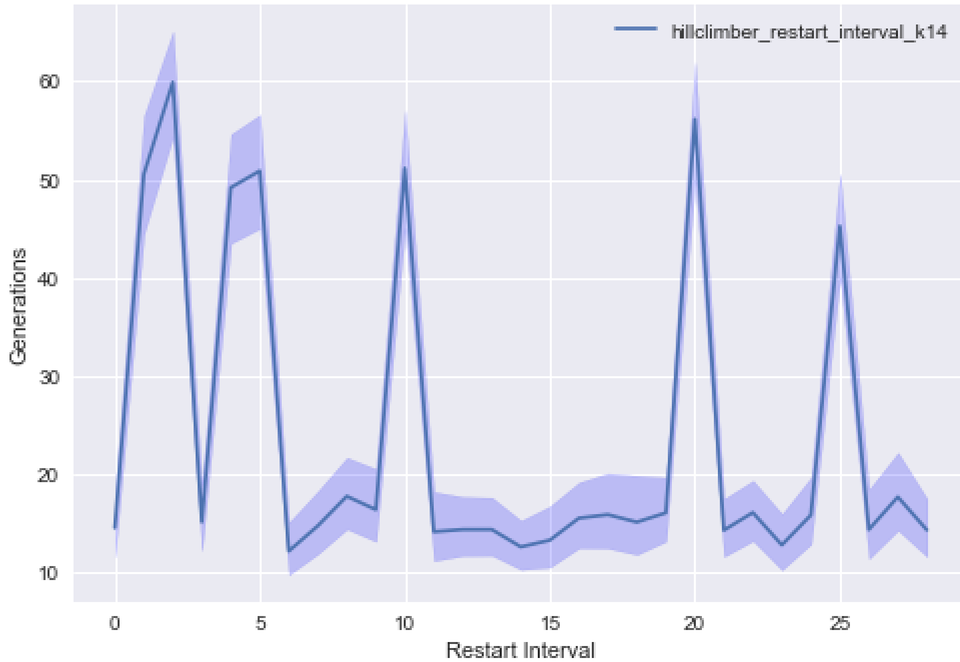
Similar to before (with `K`), please plot `Fitness and Time to Convergence (Generations)` as a funciton of how frequently we apply random restarts (`Restart Every`)

In [19]:

```python
# plotting
plot_mean_and_bootstrapped_ci(input_data=fitness_found, name=name, y_label="Fitness", x_la
plot_mean_and_bootstrapped_ci(input_data=generation_found, name=name, y_label="Generations
```

```
[array([0.63979479, 0.6558371 ]), array([0.69843942, 0.70947372]), array([0.69595968, 0.70
823474]), array([0.63157139, 0.64608332]), array([0.69660902, 0.70700913]), array([0.69531
145, 0.70691646]), array([0.63179894, 0.64733601]), array([0.63308079, 0.6479798 ]), array
([0.63493384, 0.64916017]), array([0.63266098, 0.64808684]), array([0.69510403, 0.7045904
5]), array([0.63511436, 0.65083932]), array([0.63872278, 0.65329287]), array([0.63419281,
0.64965998]), array([0.62534978, 0.64087261]), array([0.63353357, 0.65003406]), array([0.6
3565789, 0.64931292]), array([0.63768991, 0.65182113]), array([0.63037617, 0.64504405]), a
rray([0.63182594, 0.64617832]), array([0.6955379 , 0.70540429]), array([0.64163529, 0.6563
3073]), array([0.63828364, 0.65342869]), array([0.63859287, 0.65379062]), array([0.6296891
2, 0.64675029]), array([0.69232875, 0.70303473]), array([0.63123205, 0.64719684]), array
([0.63797326, 0.65525104]), array([0.63766244, 0.65174747])]
[array([11.64, 18.17]), array([44.5 , 56.51]), array([54.41, 65.1 ]), array([12.28, 18.9
9]), array([43.55, 54.65]), array([45.06, 56.63]), array([ 9.84, 15.16]), array([11.95, 1
8.35]), array([14.48, 21.75]), array([13.22, 20.6 ]), array([45.09, 57.05]), array([11.25,
18.26]), array([11.77, 17.76]), array([11.79, 17.69]), array([10.36, 15.34]), array([10.5
9, 16.81]), array([12.55, 19.23]), array([12.53, 20.15]), array([11.88, 19.9 ]), array([1
3.23, 19.74]), array([50.04, 61.93]), array([11.68, 17.55]), array([13.28, 19.41]), array
([10.32, 16.01]), array([12.95, 19.73]), array([40.1 , 50.59]), array([11.46, 18.51]), arr
ay([14.35, 22.25]), array([11.67, 17.57])]
```





Q11c: The effect of ruggedness

The above plots are for a single value of `K` =14. Repeat this same experiment below, just changing the value of `K` to `0`, to see what this experiment looksl like on a less-rugged landscape.

In [20]:

```
name = 'hillclimber_restart_interval_k0'
# hyperparameters
n=15; max_k=0; repetitions = 100; max_restart_interval=29; num_elements_to_mutate=1; downh

# initialize array to record results over different settings of k and repeated trials
solutions_found = np.zeros((max_restart_interval,repetitions,n))
fitness_found = np.zeros((max_restart_interval,repetitions))
generation_found = np.zeros((max_restart_interval,repetitions))

# initilize output
print(' i  mean fitness  mean generation found')
print('--  ------------  ---------------------')

# for many values of k
for i in range (max_restart_interval):
    # for many repeated (independent -- make sure your results differ each run!) trials
    for j in range (repetitions):
        l = Landscape(n, max_k) # generate a random fitness landscape with this level of r

        # run a hillclimber and record outputs
        solutions_found[i][j], fitness_found[i][j], generation_found[i][j] = hillclimber(k



    # print average results for all repitions of this k
#     print(k)
#     print(np.mean(fitness_found[k]))
#     print(np.mean)
    print('{}\t{}\t\t{}'.format(i, np.round(np.mean(fitness_found[i]), 3), np.round(np.mea

experiment_results2 = {}
experiment_results2[name] = {"solutions_found":solutions_found, "fitness_found":fitness_fo
```

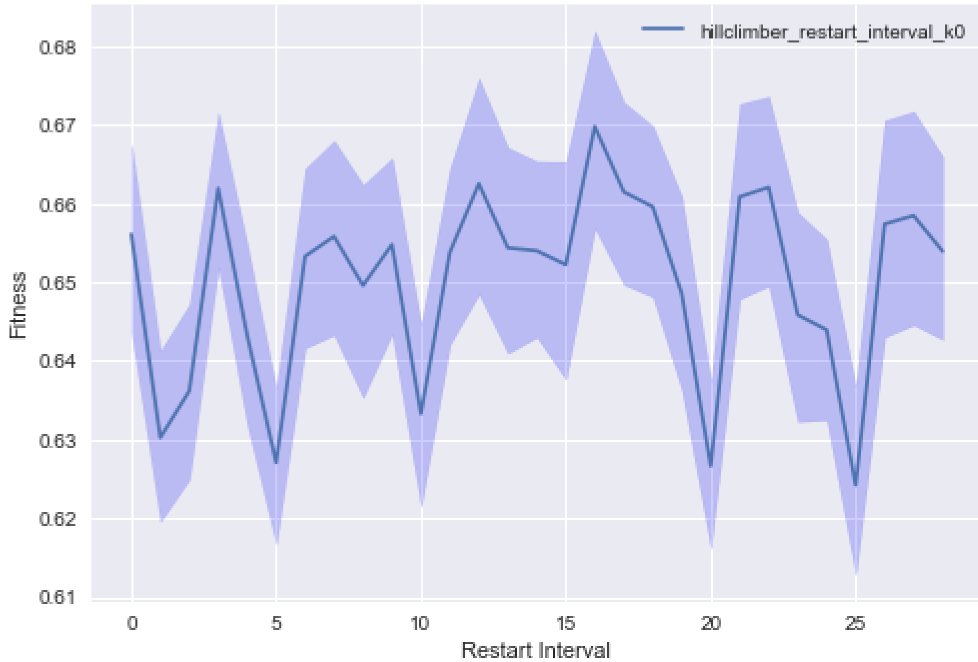
```
 i  mean fitness  mean generation found
--  ------------  ---------------------
0       0.656          34.04
```

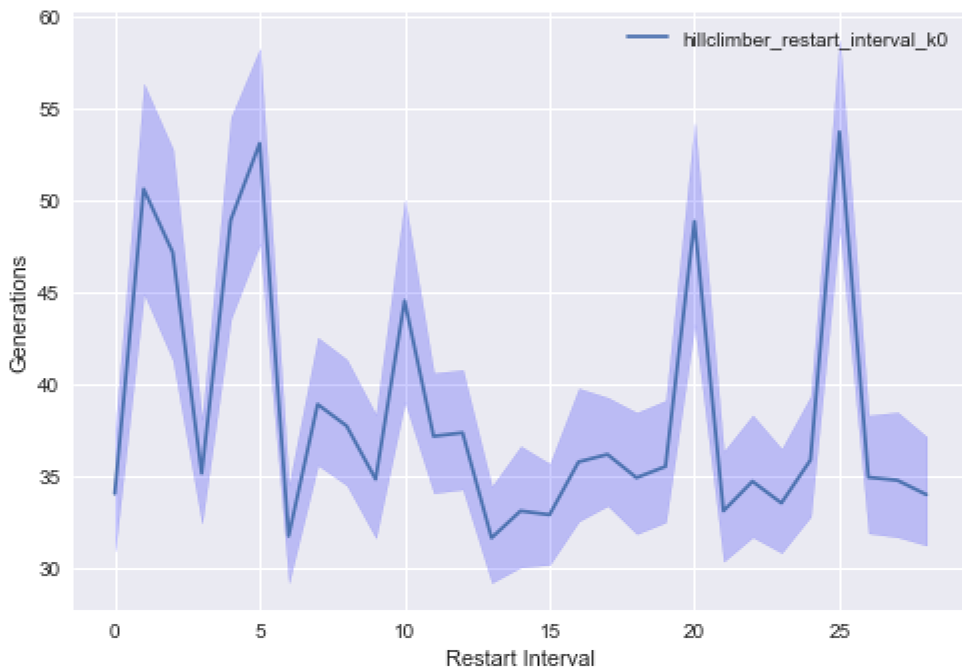
```
1       0.63            50.64
2       0.636           47.19
3       0.662           35.15
4       0.643           48.94
5       0.627           53.13
6       0.653           31.73
7       0.656           38.91
8       0.65            37.73
9       0.655           34.83
10      0.633           44.55
11      0.654           37.17
12      0.663           37.36
13      0.654           31.63
14      0.654           33.1
15      0.652           32.89
16      0.67            35.77
17      0.661           36.18
18      0.66            34.91
19      0.649           35.52
20      0.627           48.88
21      0.661           33.1
22      0.662           34.71
23      0.646           33.53
24      0.644           35.87
25      0.624           53.76
26      0.657           34.93
27      0.658           34.77
28      0.654           33.99
```

In [21]:

```python
# plotting
plot_mean_and_bootstrapped_ci(input_data=fitness_found, name=name, y_label="Fitness", x_la
plot_mean_and_bootstrapped_ci(input_data=generation_found, name=name, y_label="Generations
```

```
[array([0.643642  , 0.66755491]), array([0.6196107 , 0.64129808]), array([0.62494654, 0.64
720644]), array([0.65195199, 0.67155997]), array([0.63188279, 0.65477705]), array([0.61671
817, 0.63651475]), array([0.64165012, 0.66448782]), array([0.64329542, 0.66803215]), array
([0.63535543, 0.66239608]), array([0.64344285, 0.66580606]), array([0.62154342, 0.6446638
9]), array([0.64211887, 0.66450489]), array([0.64850856, 0.67593909]), array([0.64098849,
0.66713914]), array([0.64302219, 0.66540637]), array([0.63765888, 0.66536976]), array([0.6
5687672, 0.6819247 ]), array([0.64971   , 0.67291191]), array([0.64814036, 0.66987981]), a
rray([0.63630912, 0.66088625]), array([0.61622078, 0.63735828]), array([0.64788456, 0.6727
2813]), array([0.64954208, 0.67364355]), array([0.63230653, 0.65888825]), array([0.6324931
9, 0.65545135]), array([0.6128782 , 0.63674768]), array([0.64303697, 0.67062448]), array
([0.64457683, 0.67174531]), array([0.64272239, 0.66597975])]
[array([30.91, 37.52]), array([44.96, 56.38]), array([41.32, 52.81]), array([32.45, 38.1
]), array([43.59, 54.56]), array([47.67, 58.31]), array([29.2 , 34.53]), array([35.61, 42.
56]), array([34.51, 41.36]), array([31.64, 38.36]), array([39.05, 50.06]), array([34.09, 4
0.63]), array([34.29, 40.8 ]), array([29.2 , 34.44]), array([30.05, 36.64]), array([30.19,
35.68]), array([32.55, 39.79]), array([33.4 , 39.29]), array([31.86, 38.47]), array([32.5
, 39.13]), array([43.26, 54.26]), array([30.36, 36.35]), array([31.69, 38.33]), array([30.
83, 36.49]), array([32.81, 39.36]), array([48.54, 58.8 ]), array([31.9 , 38.32]), array([3
1.69, 38.49]), array([31.24, 37.16])]
```





Q12: Analysis

What trends to you see from the figs for `K=14` vs. `K=0` ? Are you surprised by this? What does it imply about the relationship between ruggedness and random restarts? Does it make you want to try and other experiments (what would be the next thing you'd investigate)?

These charts are hard to dissect. I question whether I generated them correctly or not. It seems that when the landscape is very rugged then there is a positive correlation between generational convergence and fitness values across the restart interval range. On the other hand, there is a negative correlation between these two metrics when the landscape is less rugged. This would mean that with more rugged landscapes if some optima is found late in the game, it has a good probability of being a good one. It's a rugged landscape so we need more exploration with the restarts. If on the other hand the landscape is less rugged, then we aren't giving the hillclimber enough time to climb a hill and when it restarts it is unlikely to land on top of one. This could be wrong, but I'm typing this as a stream of consciousness at the moment. These are just some thoughts and speculations.

Congratulations, you made it to the end!

Wow that was a bit of a long one. Hopefully you enjoyed the open-ended experimentation though

Please save this file as a .ipynb, and also download it as a .pdf, uploading **both** to blackboard to complete this assignment.

For your submission, please make sure that you have renamed this file (and that the resulting pdf follows suit) to replce `[netid]` with your UVM netid. This will greatly simplify our grading pipeline, and make sure that you receive credit for your work.

Academic Integrity Attribution

During this assignment I collaborated with:

Just me again.