

Data Science 1

STAT/CS 287

Jim Bagrow, UVM Dept of Math and Statistics

LECTURE 18

Today

More on **predictive models** (*supervised learning*)

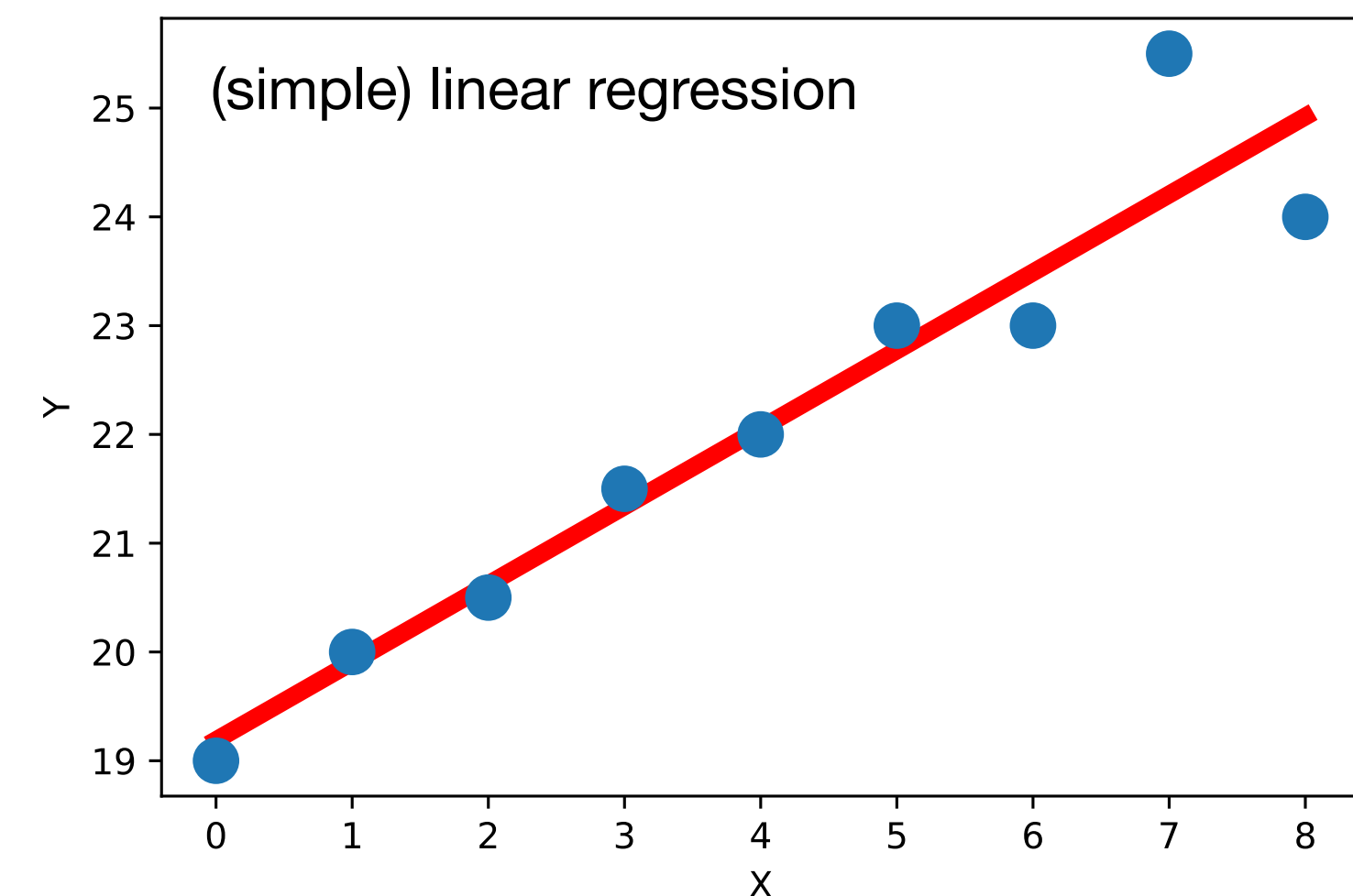
Today

More on **predictive models** (*supervised learning*)

Prediction vs. Inference → **Linear Regression**

Prediction

new x comes in, predict y using $y = f(x) = \beta_0 + \beta_1 x$



Today

More on **predictive models** (*supervised learning*)

Prediction vs. Inference → **Linear Regression**

Prediction

new x comes in, predict y using $y = f(x) = \beta_0 + \beta_1 x$

Inference

learn how changing x changes y by examining β 's

```
=====
                        OLS Regression Results
=====
Dep. Variable:          y      R-squared:                0.999
Model:                  OLS    Adj. R-squared:            0.999
Method:                 Least Squares    F-statistic:          5.849e+04
Date:                  Thu, 31 Oct 2019    Prob (F-statistic):    3.44e-150
Time:                  10:25:51    Log-Likelihood:        153.19
No. Observations:      100    AIC:                   -300.4
Df Residuals:           97    BIC:                   -292.6
Df Model:                2
Covariance Type:        nonrobust
=====
                        coef      std err          t      P>|t|      [0.025      0.975]
-----
const                0.9926      0.016     60.925      0.000      0.960      1.025
x1                   0.0562      2.517      0.022      0.982     -4.939      5.051
x2                   0.5034      0.198      2.539      0.013      0.110      0.897
=====
Omnibus:                4.957    Durbin-Watson:          1.903
Prob(Omnibus):           0.084    Jarque-Bera (JB):        2.704
Skew:                    0.153    Prob(JB):                 0.259
Kurtosis:                2.255    Cond. No.                 3.53e+03
=====
```

Today

More on **predictive models** (*supervised learning*)

Prediction vs. Inference → **Linear Regression**

Prediction

new x comes in, predict y using $y = f(x) = \beta_0 + \beta_1 x$

Inference

learn how changing x changes y by examining β 's

OLS Regression Results

| | | | |
|-------------------|------------------|---------------------|-----------|
| Dep. Variable: | y | R-squared: | 0.999 |
| Model: | OLS | Adj. R-squared: | 0.999 |
| Method: | Least Squares | F-statistic: | 5.849e+04 |
| Date: | Thu, 31 Oct 2019 | Prob (F-statistic): | 3.44e-150 |
| Time: | 10:25:51 | Log-Likelihood: | 153.19 |
| No. Observations: | 100 | AIC: | -300.4 |
| Df Residuals: | 97 | BIC: | -292.6 |
| Df Model: | 2 | | |
| Covariance Type: | nonrobust | | |

| | coef | std err | t | P> t | [0.025 | 0.975] |
|-------|--------|---------|--------|-------|--------|--------|
| const | 0.9926 | 0.016 | 60.925 | 0.000 | 0.960 | 1.025 |
| x1 | 0.0562 | 2.517 | 0.022 | 0.982 | -4.939 | 5.051 |
| x2 | 0.5034 | 0.198 | 2.539 | 0.013 | 0.110 | 0.897 |

| | | | |
|----------------|-------|-------------------|----------|
| Omnibus: | 4.957 | Durbin-Watson: | 1.903 |
| Prob(Omnibus): | 0.084 | Jarque-Bera (JB): | 2.704 |
| Skew: | 0.153 | Prob(JB): | 0.259 |
| Kurtosis: | 2.255 | Cond. No. | 3.53e+03 |

Recall

Natural Language Processing Tasks & Semantic Similarity

Supervised Learning — Classifiers

Documents and labels:

—# unique words (types)—

—# documents—

$$\begin{bmatrix} 0 & 0 & 0 & 7 & 3 & 0 & 0 & 1 & \cdots & 0 & 5 & 0 & 0 & 0 \\ \vdots & \ddots & & & & & & & & & & & \vdots \end{bmatrix}$$

X

$$\begin{bmatrix} \text{spam} \\ \text{spam} \\ \vdots \\ \text{spam} \\ \text{ham} \\ \vdots \\ \text{ham} \end{bmatrix}$$

— Labels vector

y

— Training data

$$y = f(X)$$

A new, unlabeled document comes in:

built a machine \hat{f} to predict label given
count vector —

$$\hat{f}([0 \ 0 \ 0 \ 2 \ 6 \ 3 \ 0 \ 5 \ \cdots \ 0 \ 0 \ 1 \ 1 \ 0 \ 0]) =$$

$$\arg \max [\text{Pr}(\text{spam}), \text{Pr}(\text{ham})] =$$

$$\arg \max [0.6, 0.4] = \text{(for example)}$$

spam

$$y = \hat{f}(X)$$

Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

Examples

X

y

KIDS dataset:

features of hospitalizations

outcome of hospital stay

Image classification:

(raw) image features

label of subject of image

Finance:

values of stocks, bonds, foreign
exchange

tomorrow's stock price of \$IBM

Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

Examples



literal predictions

KIDS dataset:

features of hospitalizations

outcome of hospital stay

Image classification:

(raw) image features

label of subject of image

Finance:

values of stocks, bonds, foreign exchange

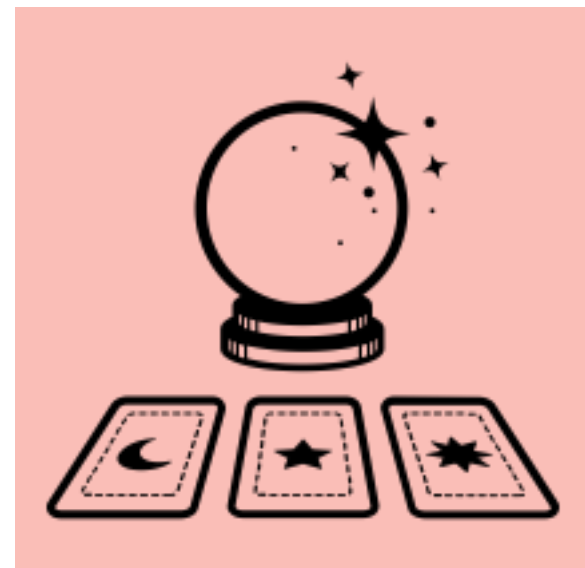
tomorrow's stock price of \$IBM

Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

Examples



literal predictions

KIDS dataset:

features of hospitalizations

outcome of hospital stay

Image classification:

(raw) image features

label of subject of image

Finance:

values of stocks, bonds, foreign exchange

tomorrow's stock price of \$IBM

Why build a predictive model?

X - readily available

y - very hard to come by

so approximate y with $\hat{y} = \hat{f}(X)$

Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

How to build a predictive model?

Invest in the effort to generate training data, many X, y pairs

Figure out a good \hat{f} by **comparing $\hat{f}(X)$ and y when both are known - training or learning**

Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

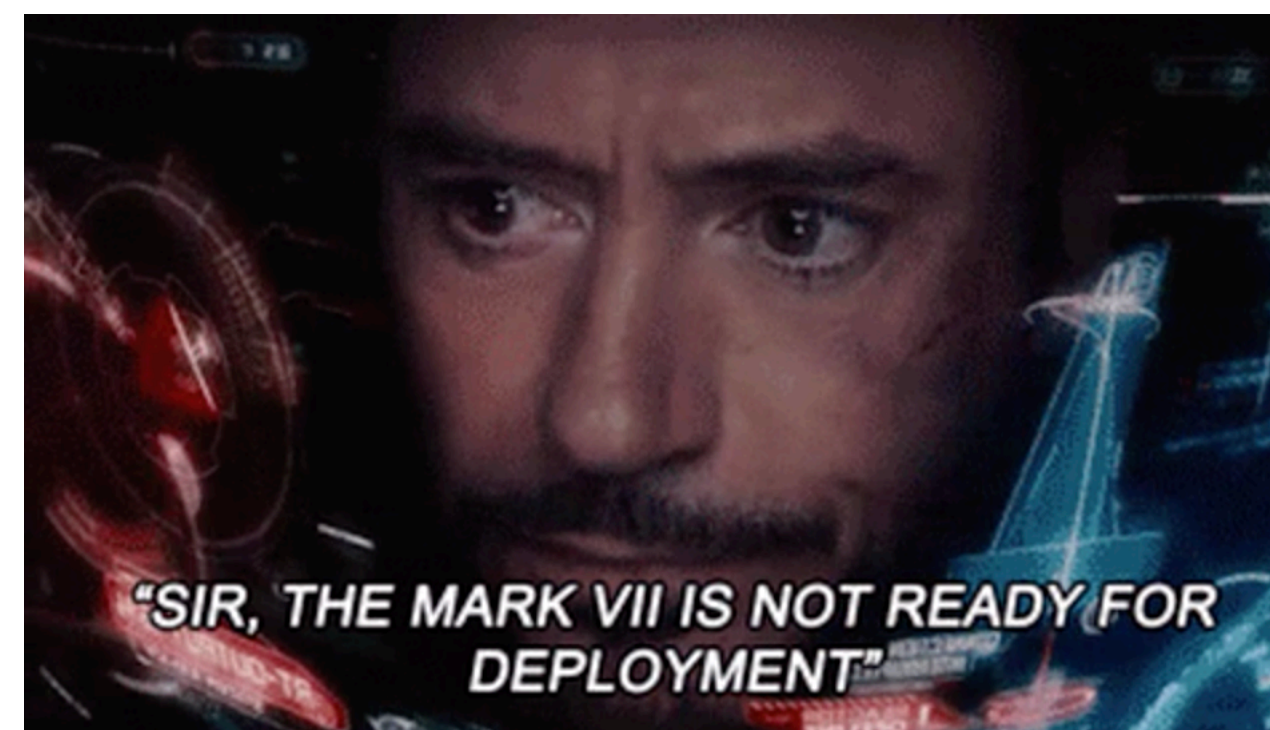
Never forget the **guiding principle – deployment**

Invest in the effort to generate training
data, many examples X, y pairs

Figure out a good \hat{f} by comparing $\hat{f}(X)$ and y
when both are known - training or learning

X - readily available

y - very hard to come by
so approximate y with $\hat{y} = \hat{f}(X)$



Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

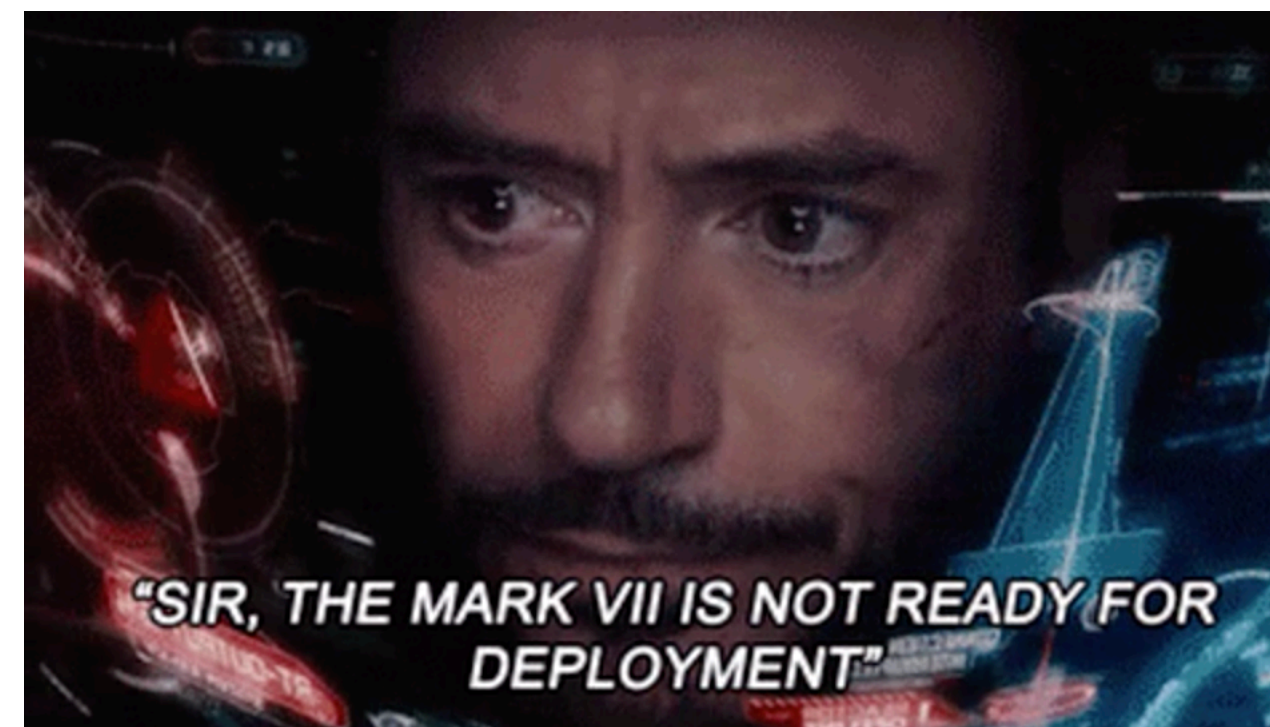
Never forget the **guiding principle – deployment**

Invest in the effort to generate training
data, many examples X, y pairs

Figure out a good \hat{f} by comparing $\hat{f}(X)$ and y
when both are known - training or learning

X - readily available

y - very hard to come by
so approximate y with $\hat{y} = \hat{f}(X)$



Predictive Models

$$y \approx \hat{f}(X)$$

Use function \hat{f} to predict an unknown y given a known X

Never forget the **guiding principle – deployment**

Invest in the effort to generate training
data, many examples X, y pairs

Figure out a good \hat{f} by comparing $\hat{f}(X)$ and y
when both are known - training or learning

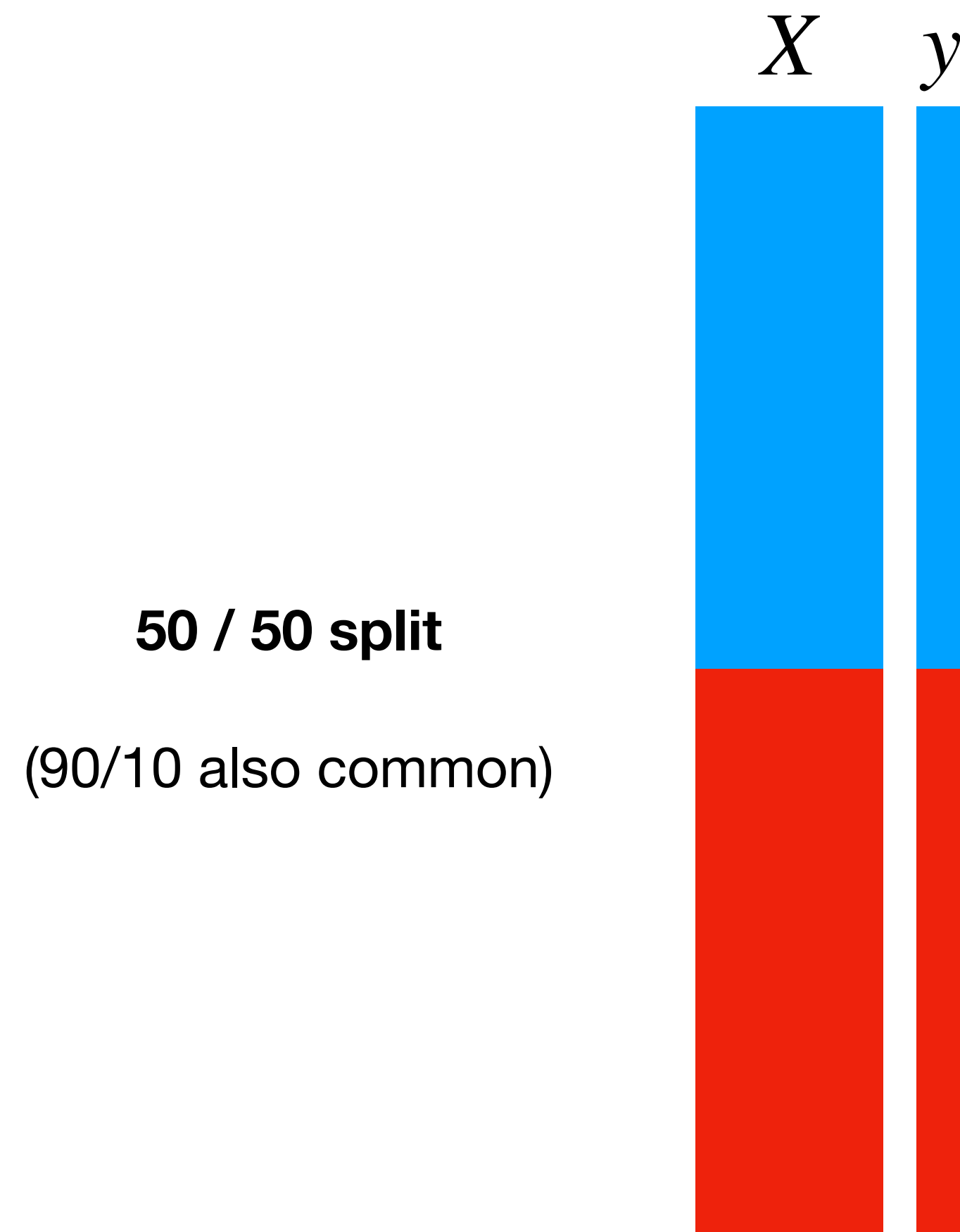
X - readily available

y - very hard to come by

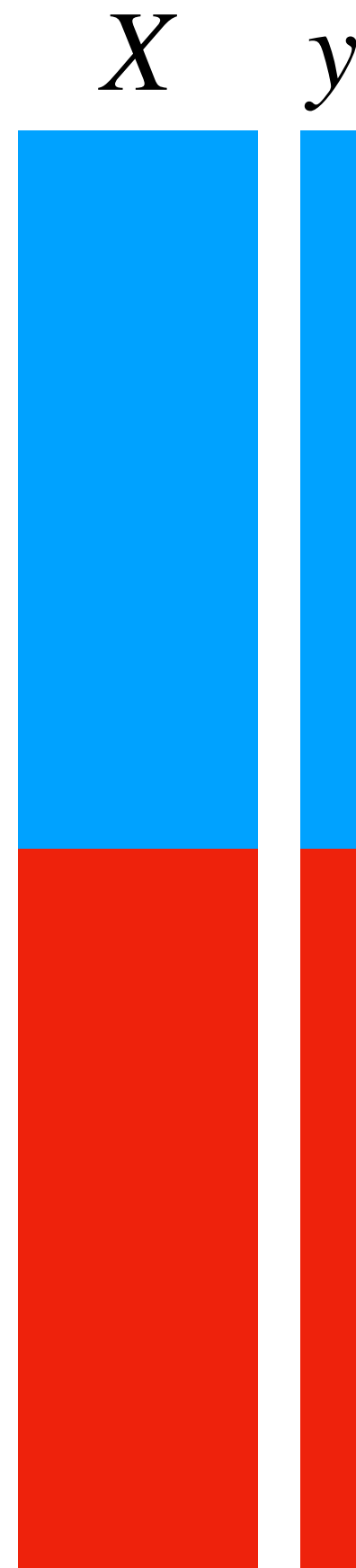
so approximate y with $\hat{y} = \hat{f}(X)$

Very easy, especially for beginners, to get **lost** fitting \hat{f} to data (error metrics, cross-validation, hyperparameters) but **remember: you are building a system that works without knowing y**

Simulating Deployment: Training / Testing



Simulating Deployment: Training / Testing

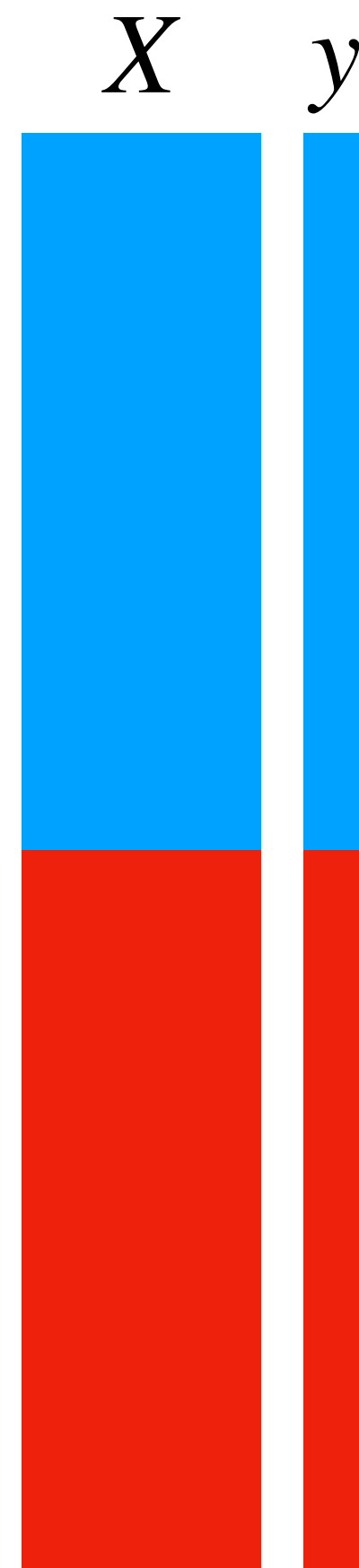


50 / 50 split

(90/10 also common)

Predictive model is given X_{tr} and y_{tr} to **learn**
 $\hat{f}(X) \approx f(X)$

Simulating Deployment: Training / Testing



50 / 50 split

(90/10 also common)

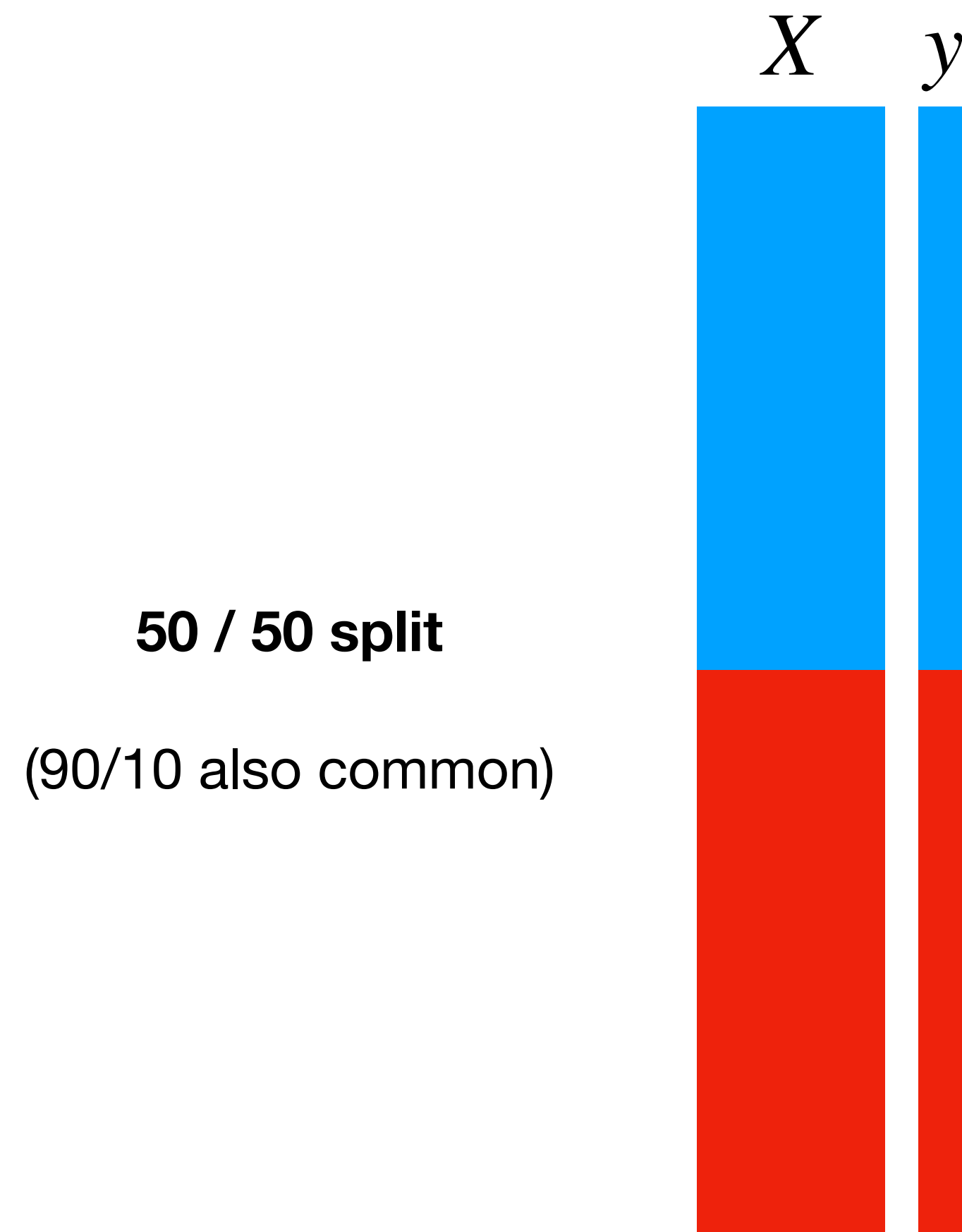
Predictive model is given X_{tr} and y_{tr} to **learn**

$$\hat{f}(X) \approx f(X)$$

Predictive model is **tested** with X_{te} and y_{te} by **comparing** $\hat{f}(X_{\text{te}})$ against y_{te}

- Example *error metric* (sum of squared errors): $\|\hat{f}(X) - y\|_2^2$

Simulating Deployment: Training / Testing



Predictive model is given X_{tr} and y_{tr} to **learn**
 $\hat{f}(X) \approx f(X)$

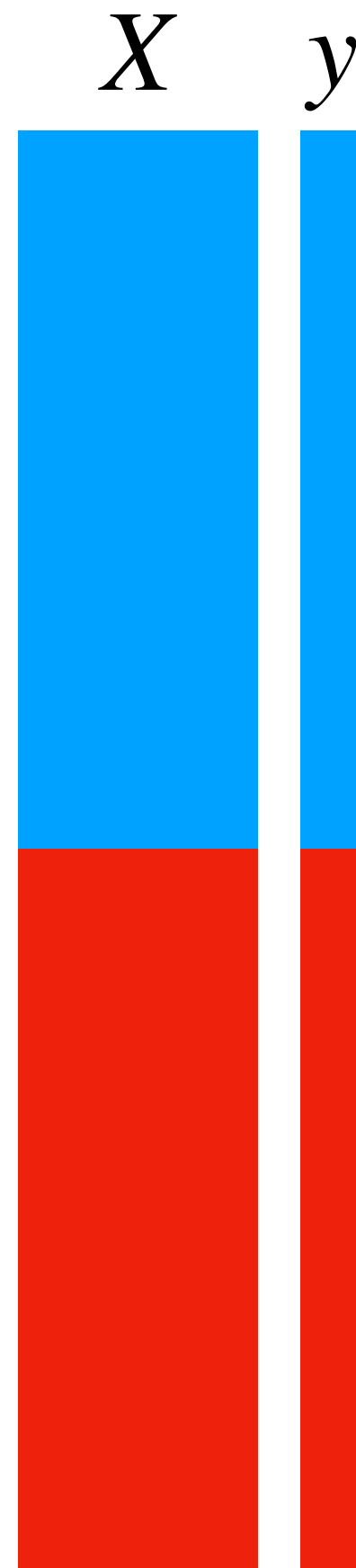
Predictive model is **tested** with X_{te} and y_{te} by **comparing** $\hat{f}(X_{\text{te}})$
against y_{te}

- Example *error metric* (sum of squared errors): $\|\hat{f}(X) - y\|_2^2$

Why not use all the data to learn,
maximize the information we have?

Simulating Deployment: Training / Testing

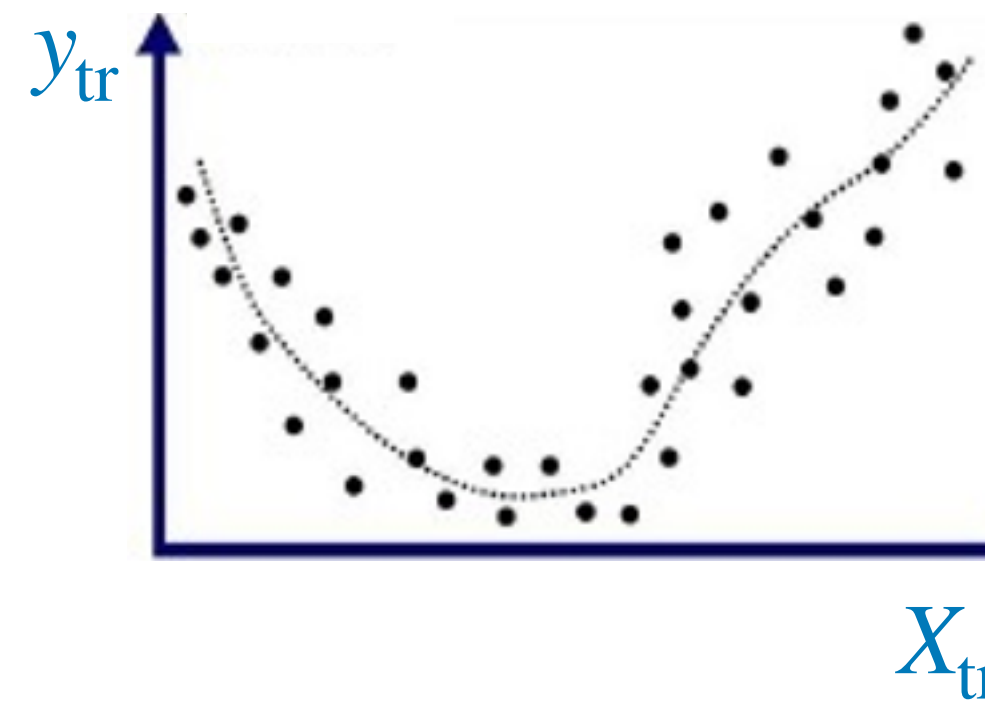
50 / 50 split
(90/10 also common)



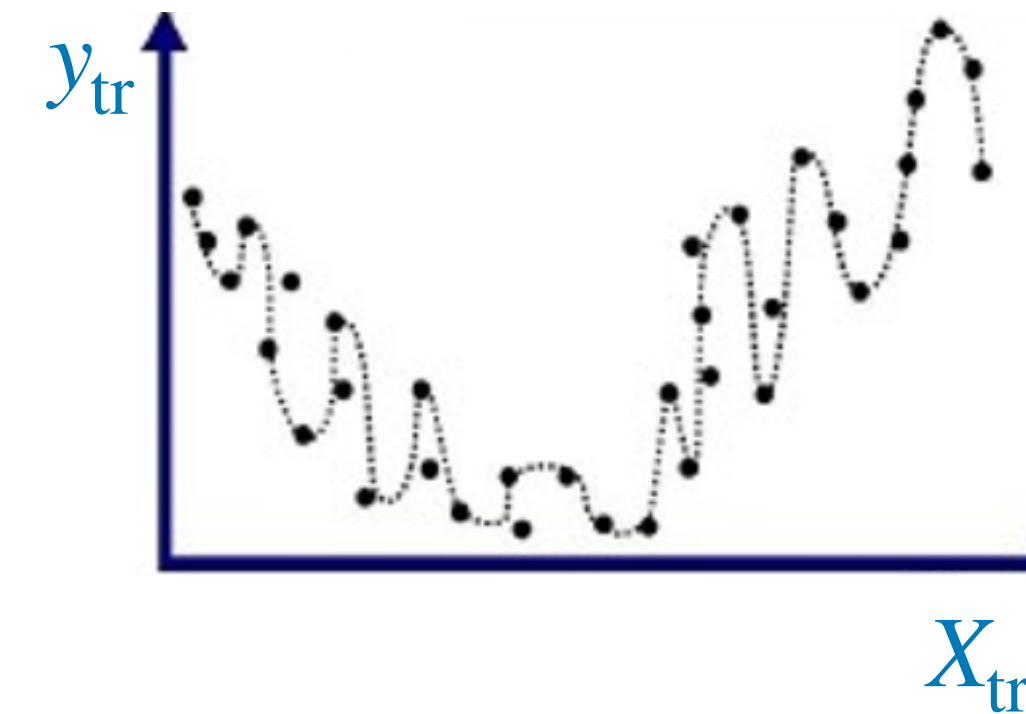
Why not use all the data to learn,
maximize the information we have?

To prevent
overfitting

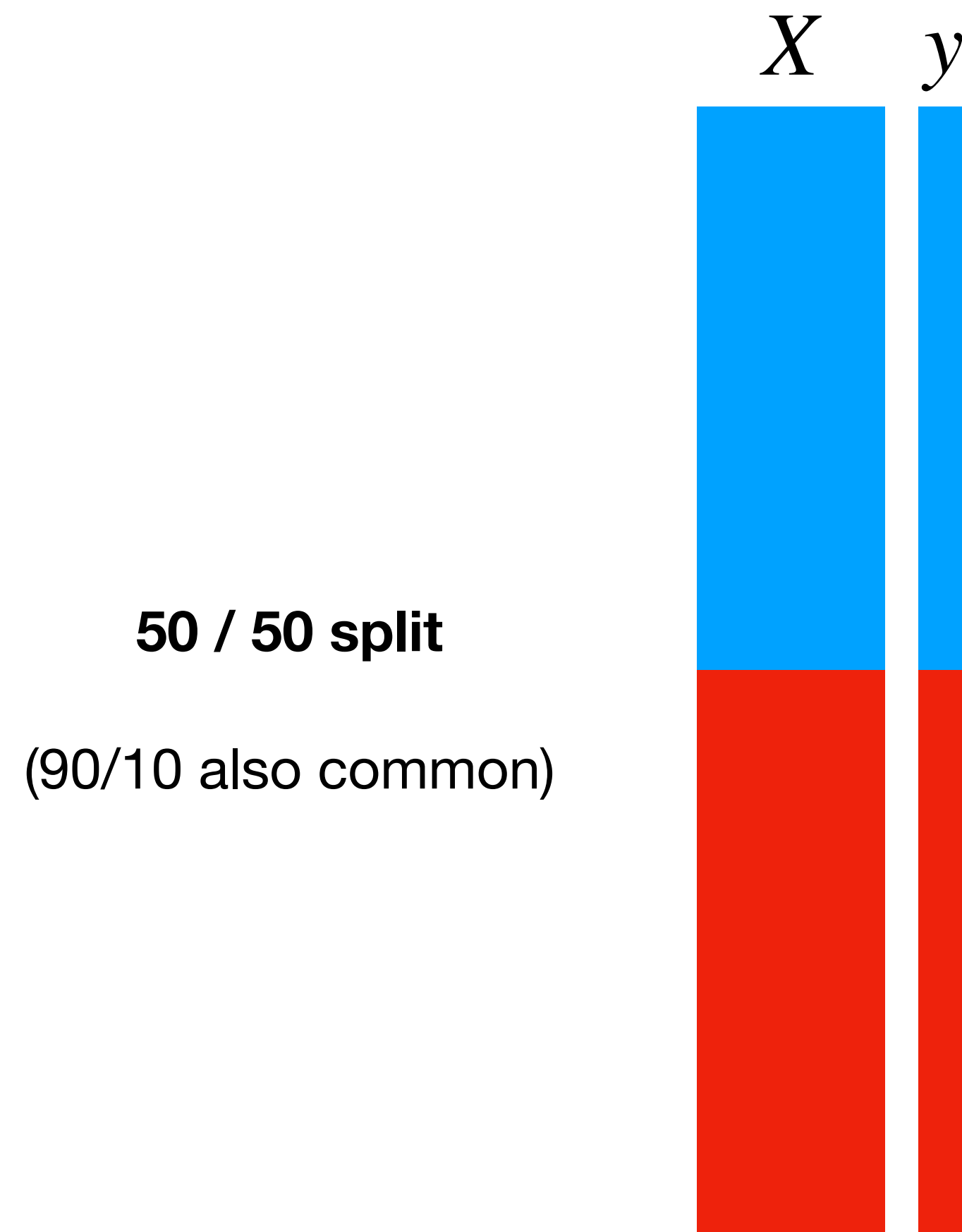
Good fit (?)



Overfit!

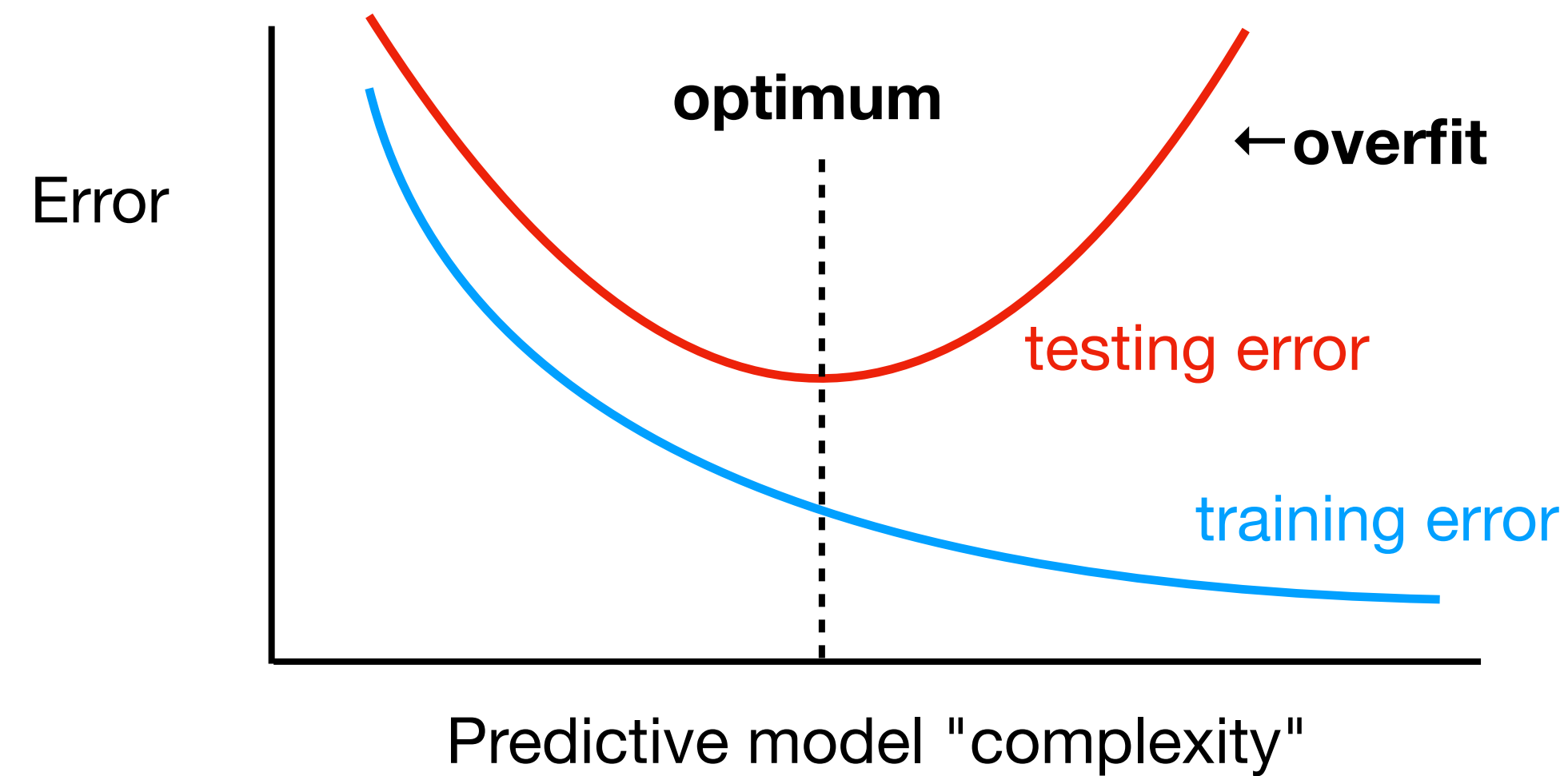


Simulating Deployment: Training / Testing

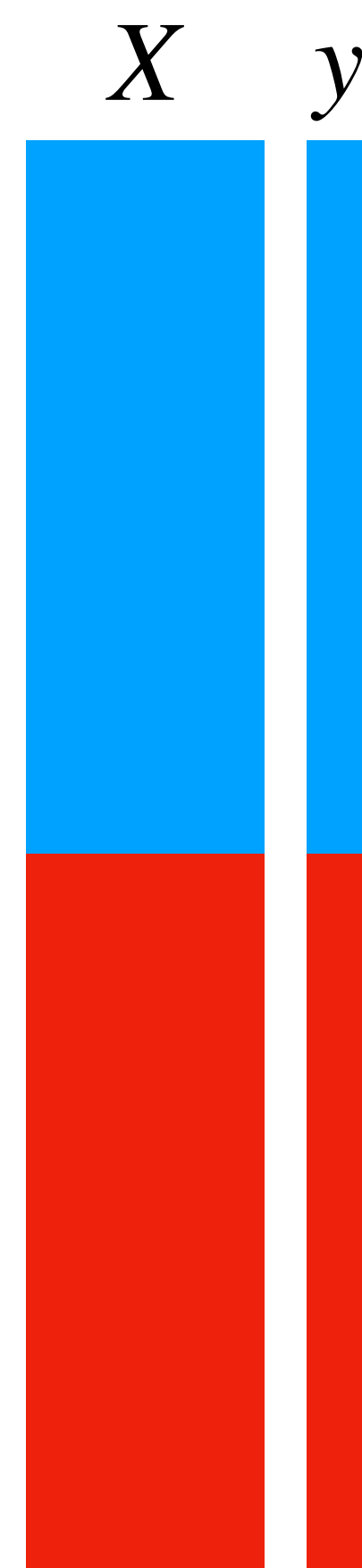


Why not use all the data to learn,
maximize the information we have?

To prevent
overfitting



Training / Validation / Testing



Many predictive models have both *parameters* and *hyperparameters*

Parameters: changed during/due to training

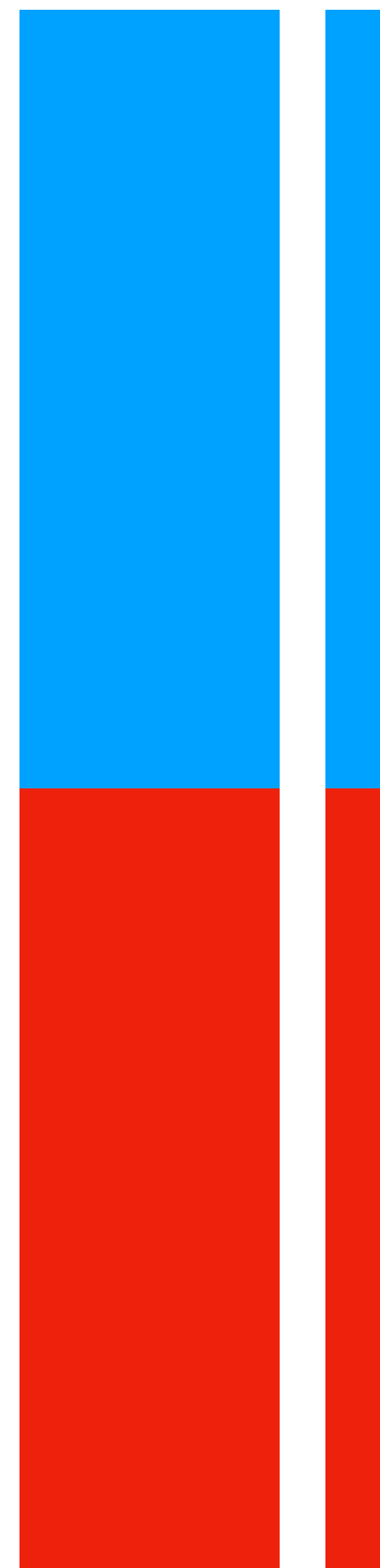
Hyperparameters: chosen before training

50 / 50 split

(90/10 also common)

Training / Validation / Testing

X y



50 / 50 split

(90/10 also common)

Many predictive models have both *parameters* and *hyperparameters*

Parameters: changed during/due to training

Hyperparameters: chosen before training

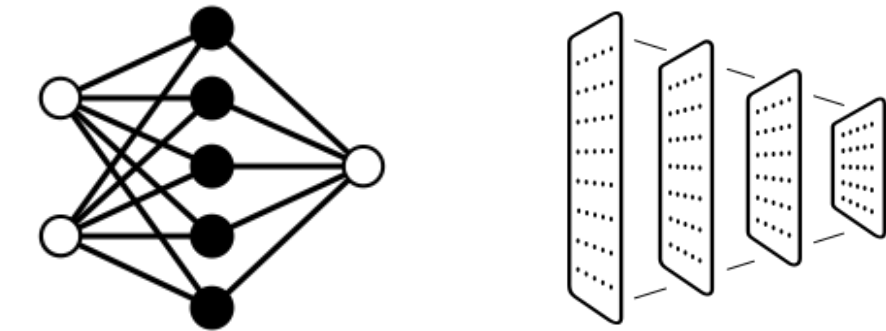
Polynomial regression

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \dots + \beta_d x^d$$

Parameters: coefficients β_i

Hyperparameter: polynomial order d

Neural networks



Parameters: weights on links

Hyperparameters:

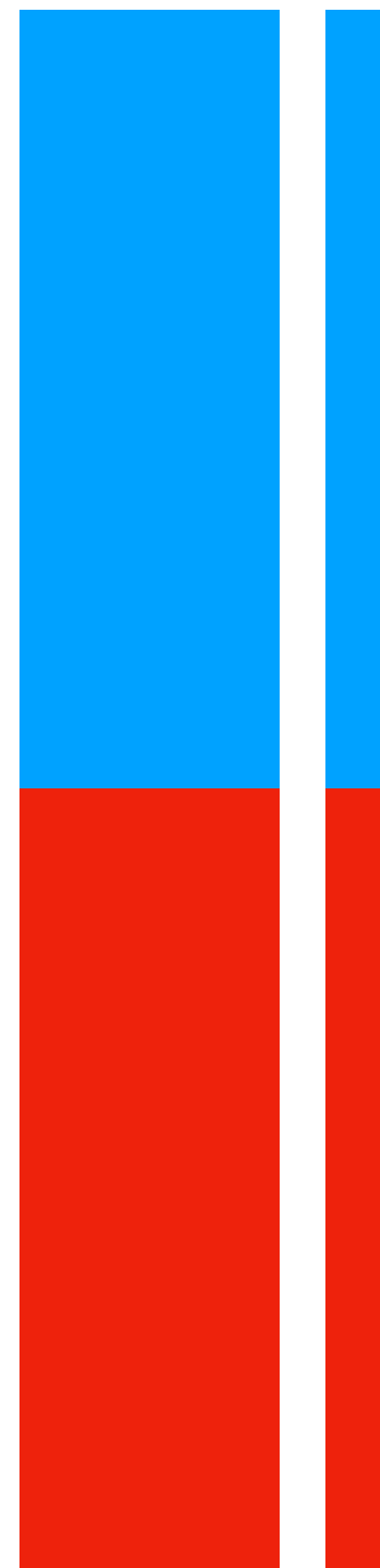
Network architecture

Choice of activation function

...

Training / Validation / Testing

X y



50 / 50 split

(90/10 also common)

Many predictive models have both *parameters* and *hyperparameters*

Parameters: changed during/due to training

Hyperparameters: chosen before training

You could:

1. Use **training data** to fit *parameters*
2. Use **testing data** to compare different *hyperparameters*

But:

- **Risk overfitting again**—all your data went into the model, nothing is left over for testing

Training / Validation / Testing



Many predictive models have both *parameters* and *hyperparameters*

Parameters: changed during/due to training

Hyperparameters: chosen before training

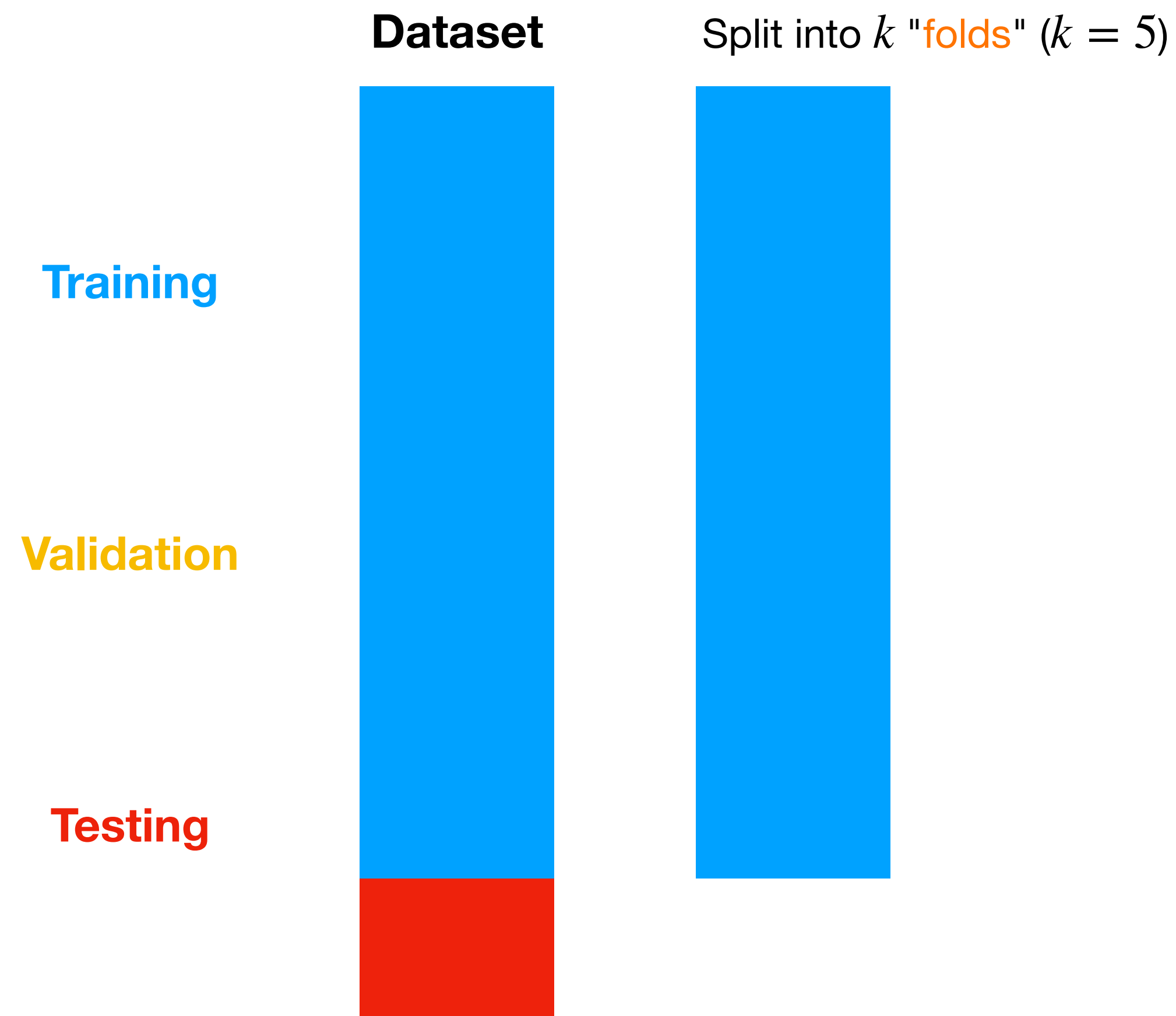
Instead:

1. Use **training data** to fit *parameters*
2. Use **validation data** to compare different *hyperparameters*
3. Use **testing data** to pick best model

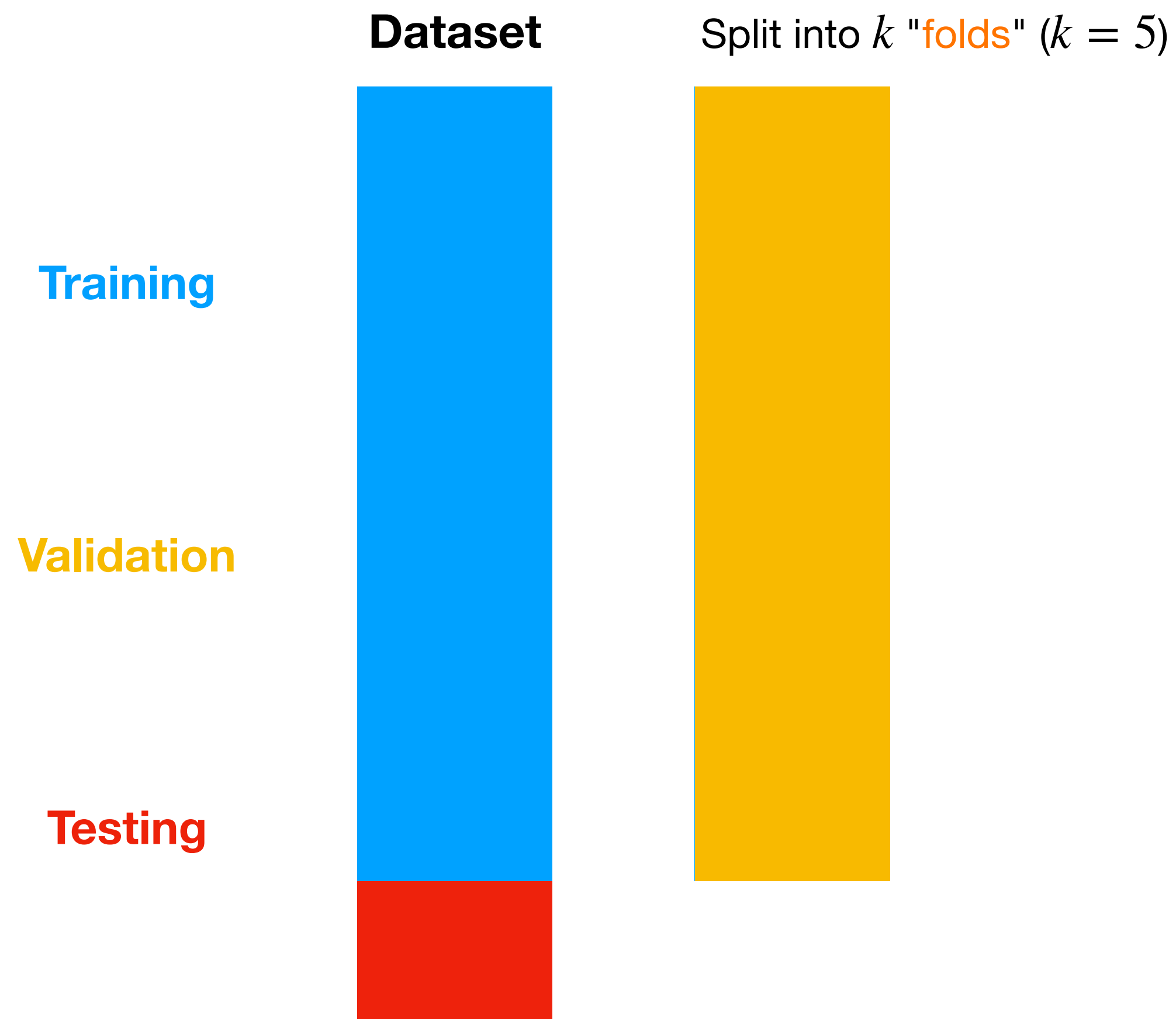
One issue:

May need **multiple** training/validation splits, to get enough statistics (repetitions) to make good comparisons

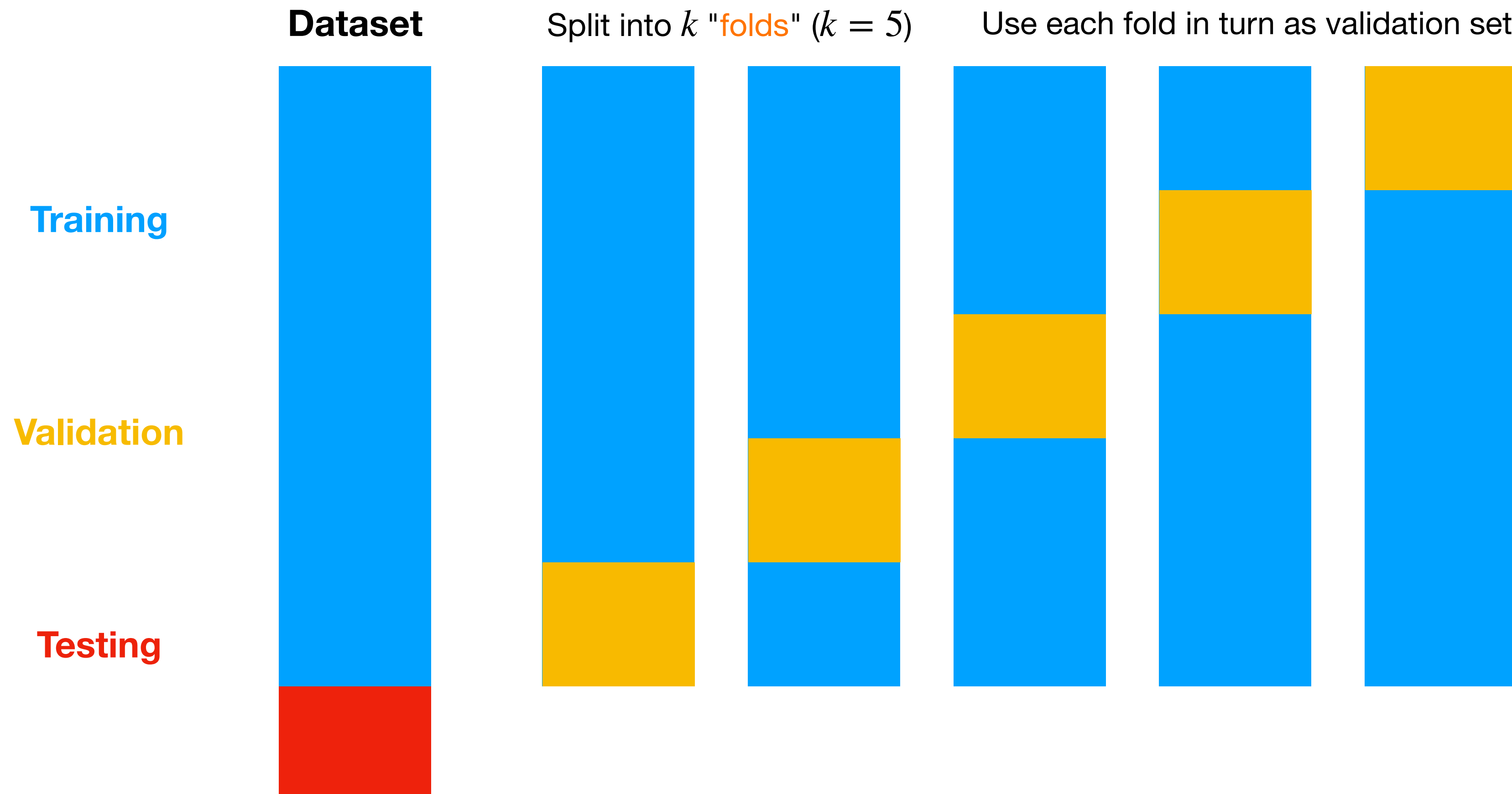
(k-Fold) Cross-validation



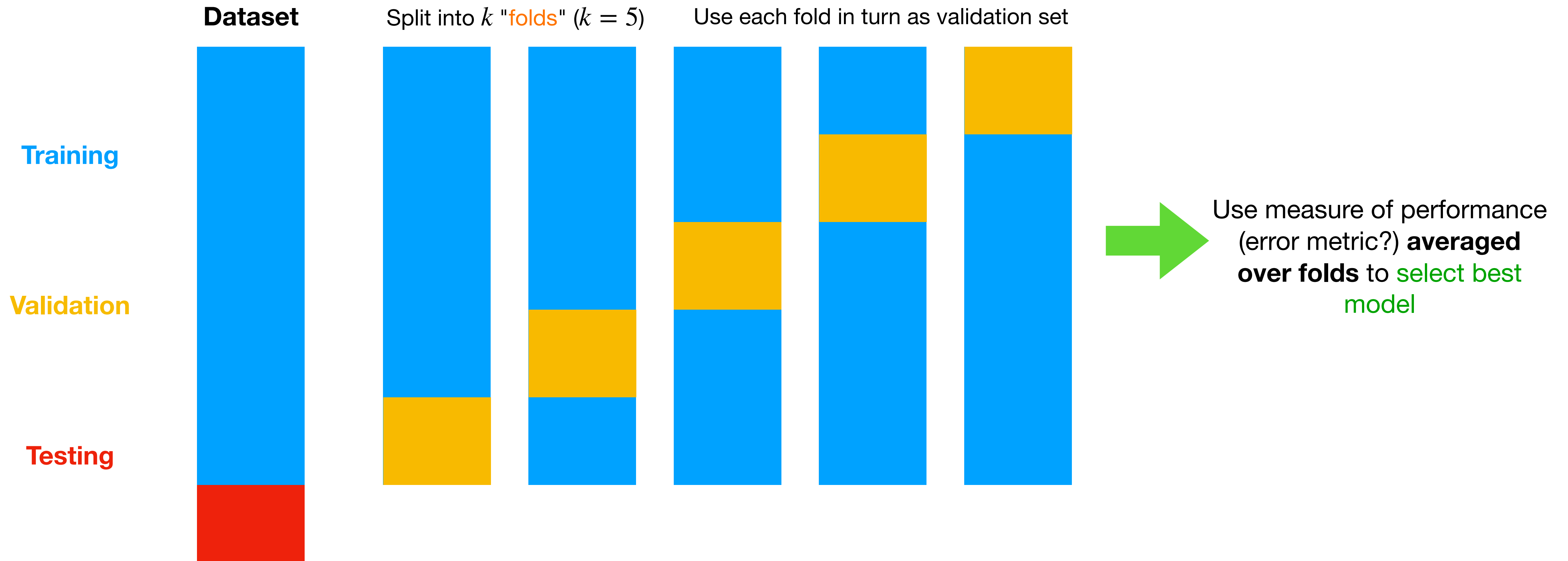
(k-Fold) Cross-validation



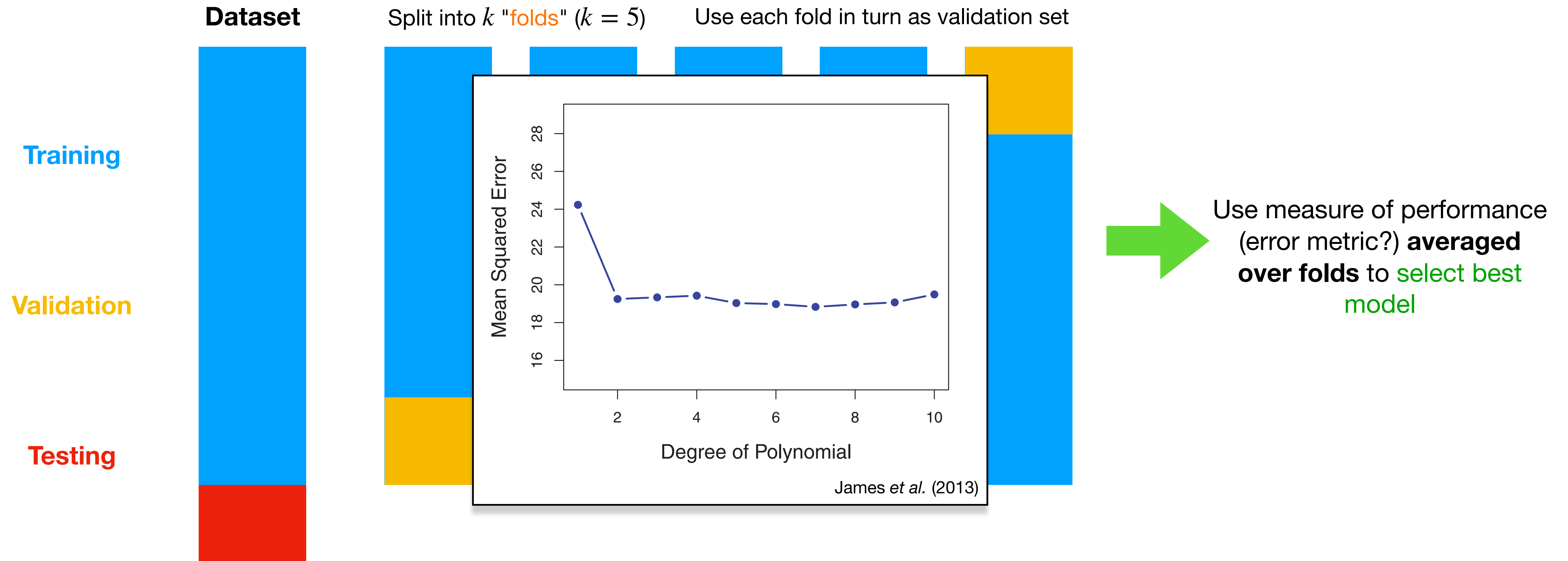
(k-Fold) Cross-validation



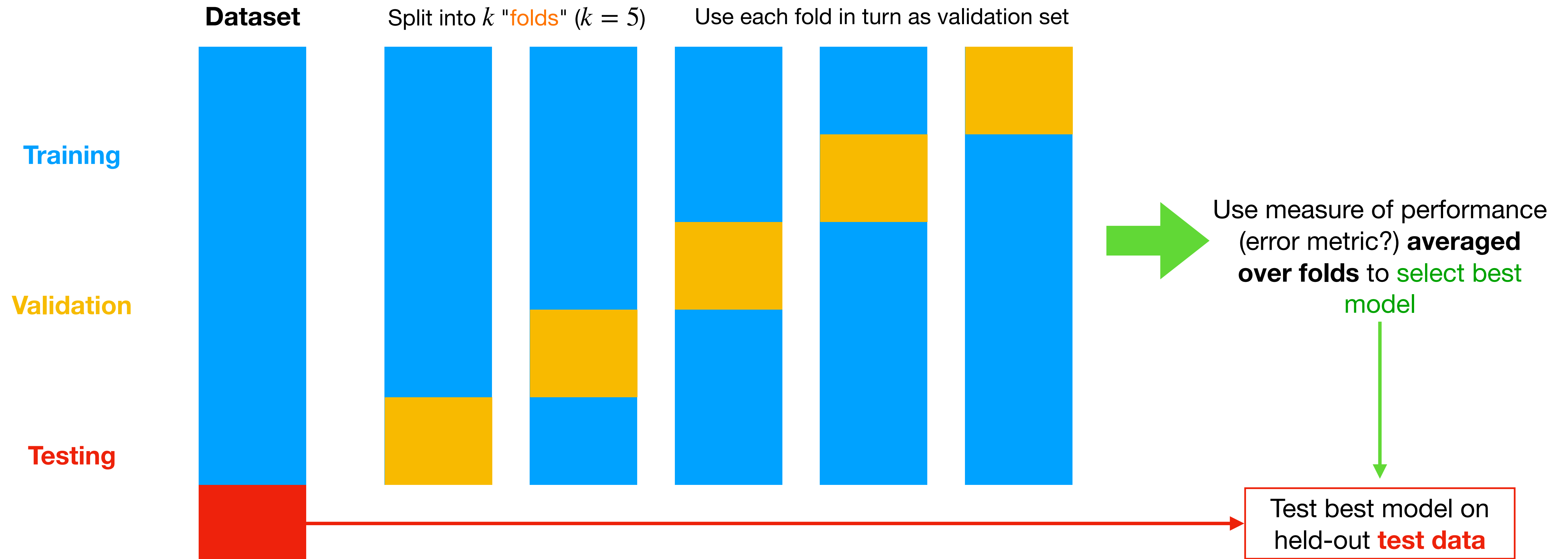
(k-Fold) Cross-validation



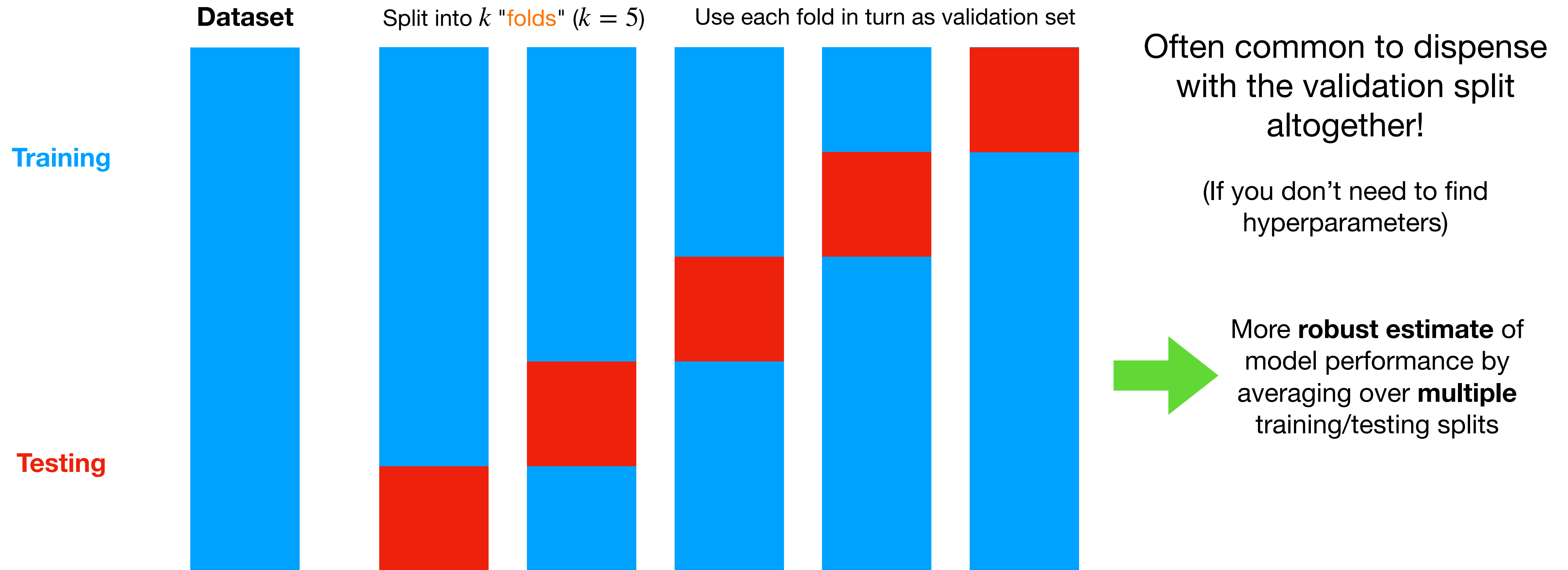
(k-Fold) Cross-validation



(k-Fold) Cross-validation



(k-Fold) Cross-validation



Splits/Cross-Validation **are biased**

Compare

A) Take a single dataset and split it into two datasets

B) Generate a dataset and, later, generate a second dataset

Splits/Cross-Validation **are biased**

Compare

A) Take a single dataset and split it into two datasets

B) Generate a dataset and, later, generate a second dataset

It's likely there is **unaccounted variation(s)** that is changing the two datasets in **option B** that is not present in **option A**

→ Larger difference between the two B datasets than the two A datasets

Splits/Cross-Validation **are biased**

Compare

A) Take a single dataset and split it into two datasets

B) Generate a dataset and, later, generate a second dataset

It's likely there is **unaccounted variation(s)** that is changing the two datasets in **option B** that is not present in **option A**

→ Larger difference between the two B datasets than the two A datasets

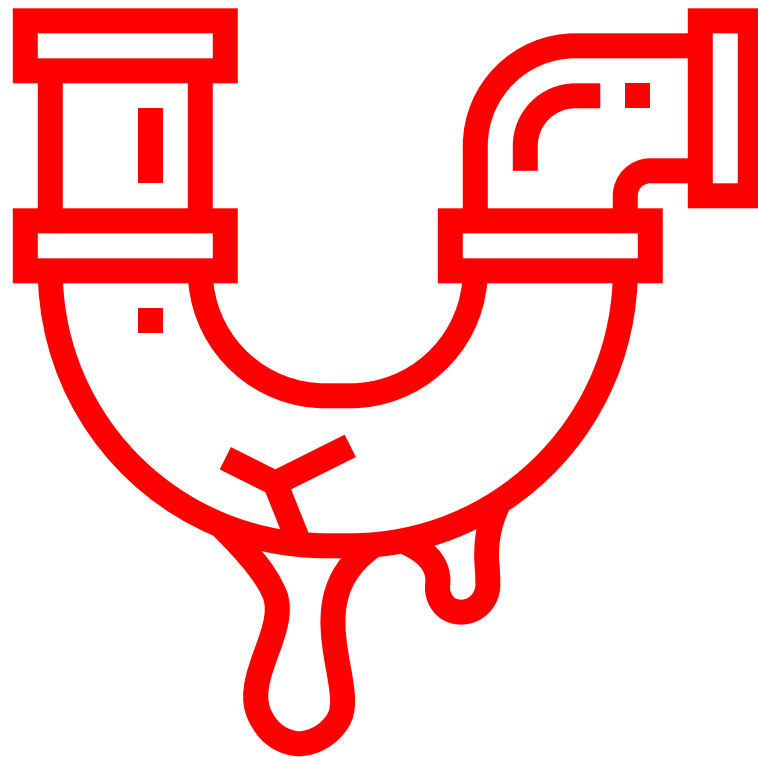
Therefore, training/testing or cross-validation will likely **overestimate** the true performance of a predictive model

Underestimates the error of the model

Data Leakage

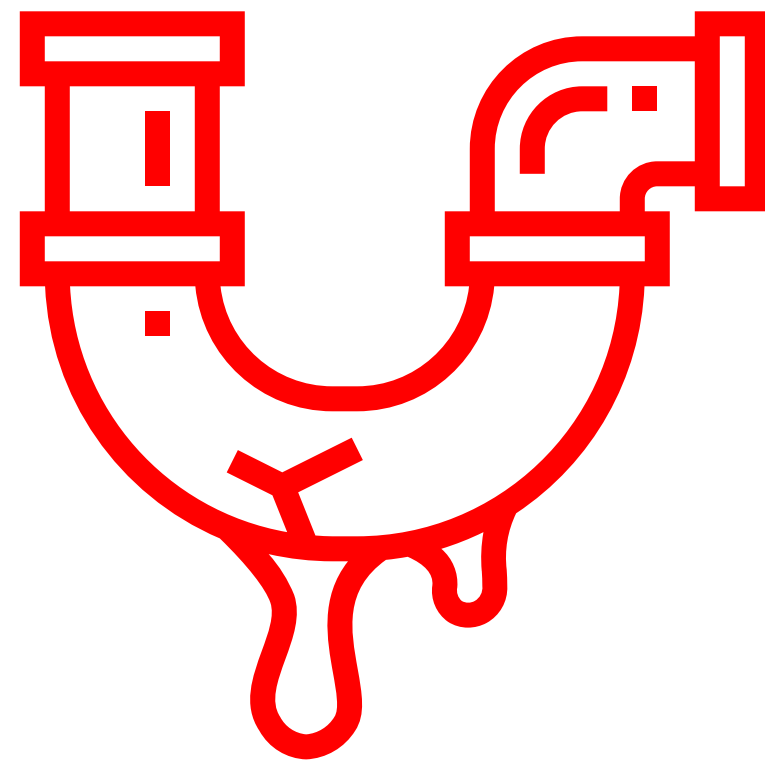
Failure to simulate deployment

Information from the held out test set was used during training



Data Leakage

Failure to simulate deployment



Information from the held out test set was used during training

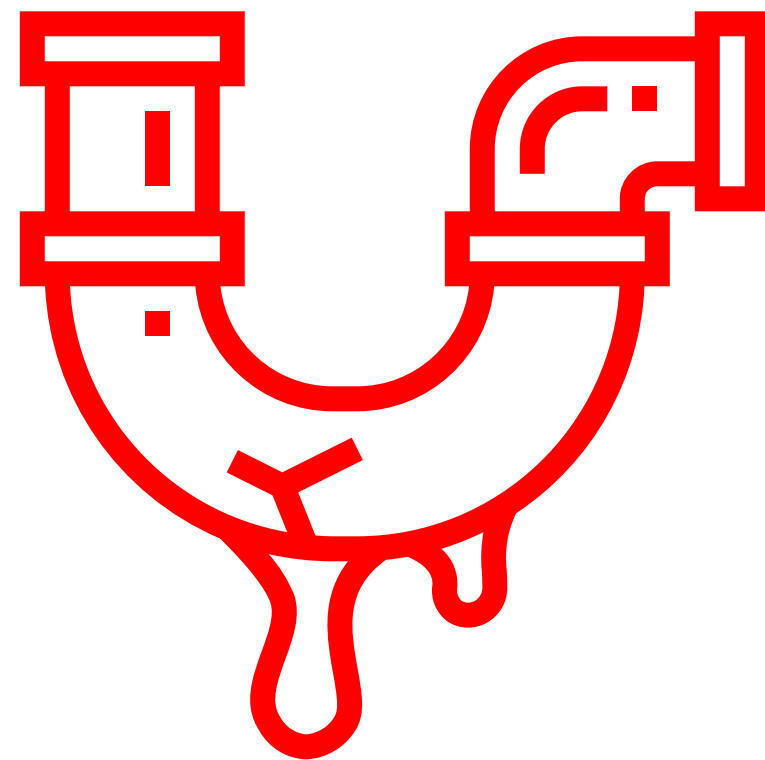
Example

Suppose I want to **rescale** (one of) my predictive features:

$$X_i \text{ becomes } \frac{X_i - \bar{X}_i}{\sigma_{X_i}}$$

Data Leakage

Failure to simulate deployment

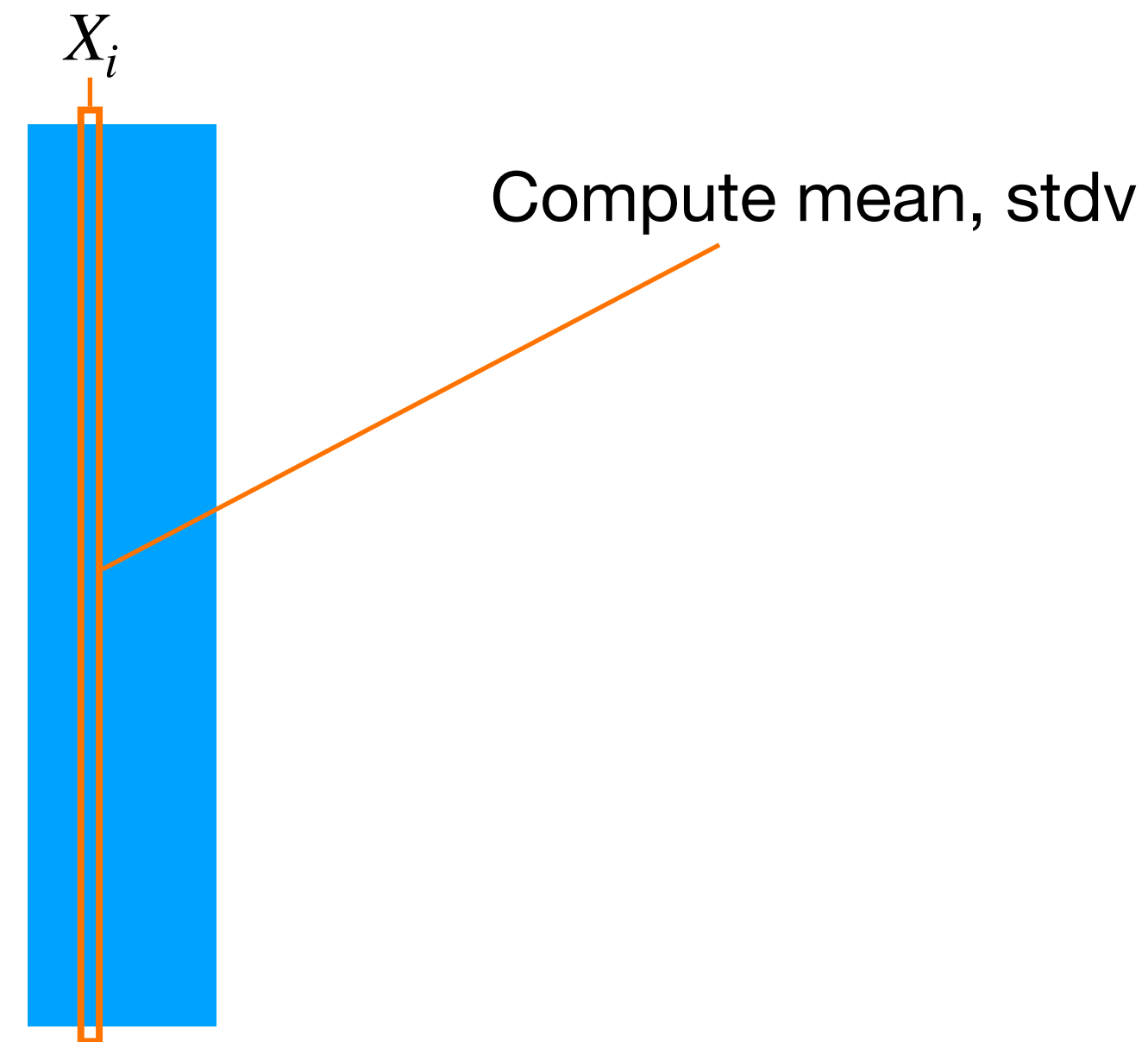


Information from the held out test set was used during training

Example

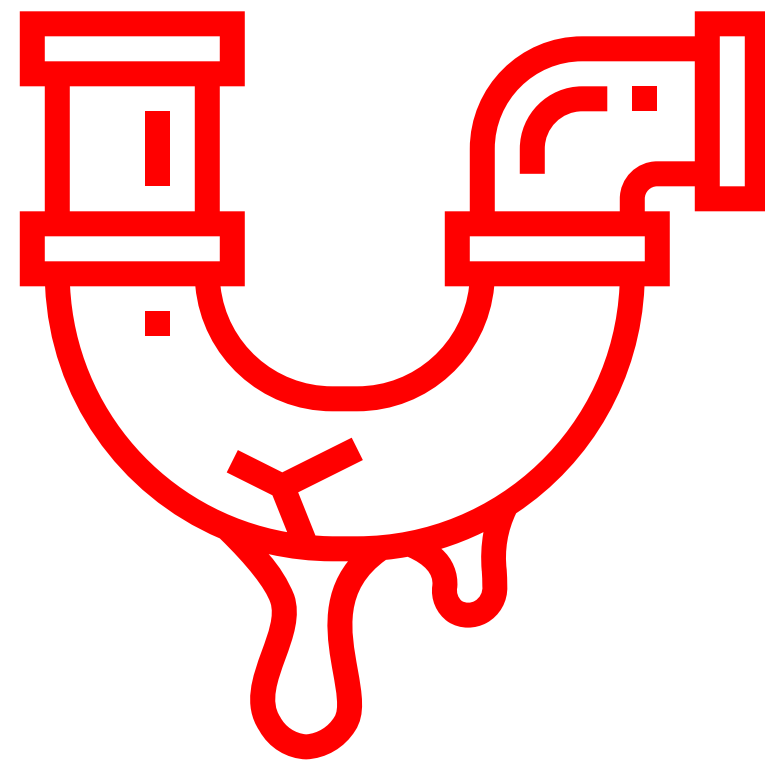
Suppose I want to **rescale** (one of) my predictive features:

$$X_i \text{ becomes } \frac{X_i - \bar{X}_i}{\sigma_{X_i}}$$



Data Leakage

Failure to simulate deployment

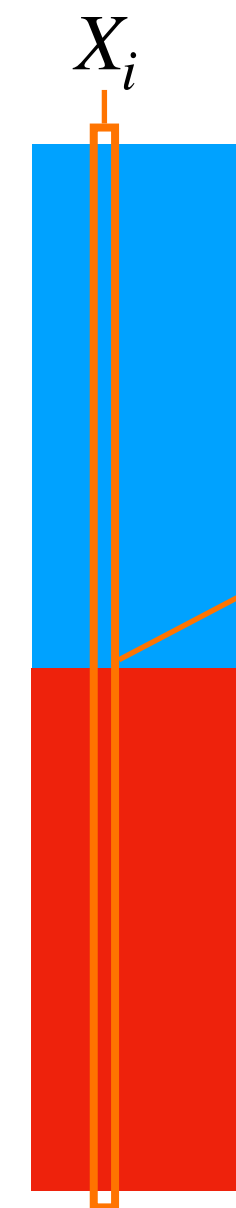


Information from the held out test set was used during training

Example

Suppose I want to **rescale** (one of) my predictive features:

$$X_i \text{ becomes } \frac{X_i - \bar{X}_i}{\sigma_{X_i}}$$



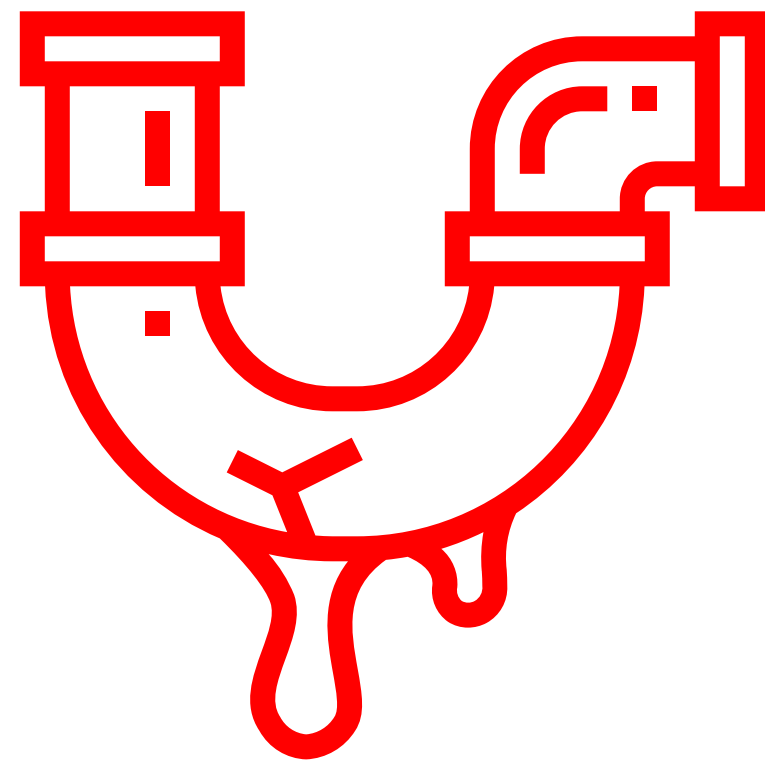
Compute mean, stdv

I haven't split training and testing yet

→ \bar{X}_i, σ_{X_i} used **test points**. *Leakage!*

Data Leakage

Failure to simulate deployment

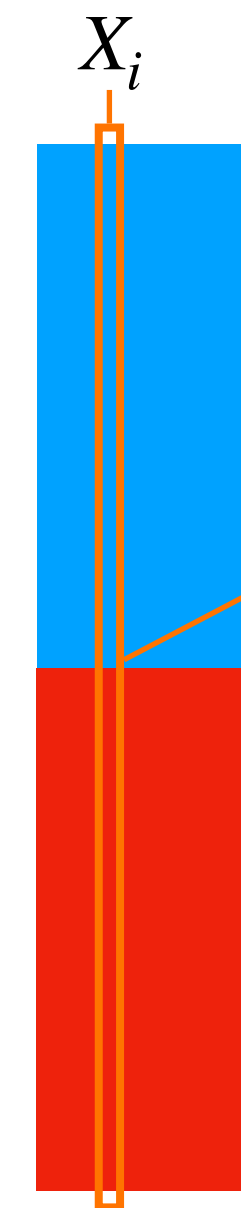


Information from the held out test set was used during training

Example

Suppose I want to **rescale** (one of) my predictive features:

$$X_i \text{ becomes } \frac{X_i - \bar{X}_i}{\sigma_{X_i}}$$



Compute mean, stdv

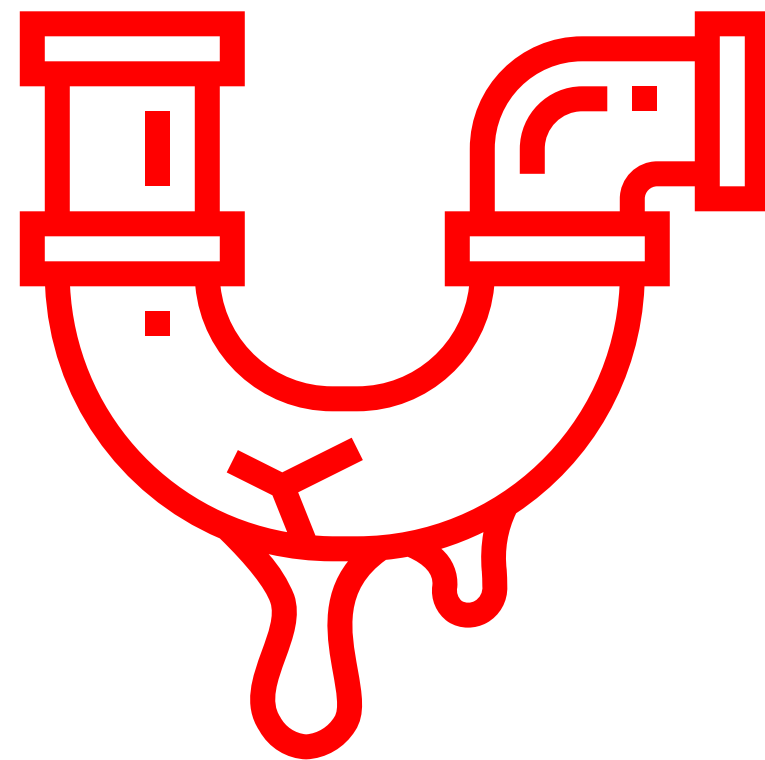
I haven't split training and testing yet

→ \bar{X}_i, σ_{X_i} used **test points**. *Leakage!*

Sometimes the data already come rescaled (pre-leaked!)

Data Leakage

Failure to simulate deployment



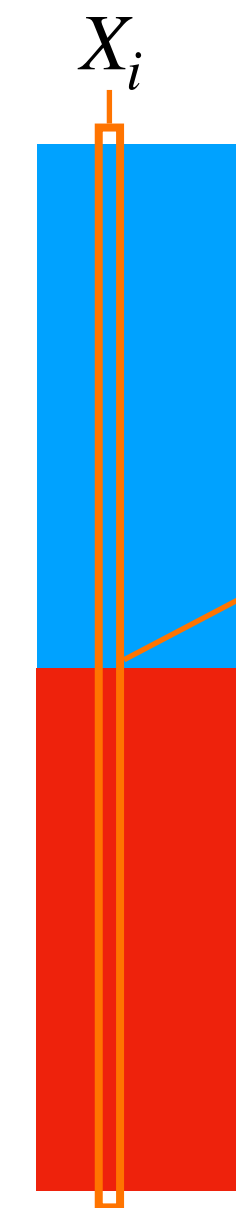
Amazing model performance?
Too-good-to-be-true performance?
Might be leakage!

Information from the held out test set was used during training

Example

Suppose I want to **rescale** (one of) my predictive features:

$$X_i \text{ becomes } \frac{X_i - \bar{X}_i}{\sigma_{X_i}}$$



Compute mean, stdv

I haven't split training and testing yet

→ \bar{X}_i, σ_{X_i} used **test points**. **Leakage!**

Sometimes the data already come rescaled (pre-leaked!)

Summary — Predictive Models

- Prediction vs inference
- Prediction with supervised learning $y = \hat{f}(X)$
 - dominant form of machine learning
 - use pre-existing X, y (*training data*) to figure out \hat{f}
- Why build predictive models?
 - X is cheap, y expensive, so use $\hat{f}(X)$ instead of y
- Deployment - how will model work when there is no y ?
 - Simulate deployment with cross-validation
 - Take care—data leakage!