Matching in Multi-Agent Pathfinding using M*

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Abstract

Multi-agent pathfinding (*MAPF*) can be extended by assigning agents to teams. Agents in a team need to find the best assignment of goals to minimise the *sum of individual costs*. Such an assignment is called a 'matching'. The algorithm *M** is a complete and optimal algorithm to solve *MAPF* without matching. In this paper, a modification was made to *M** to allow it to solve problems which involve matching. To do this, two strategies are proposed called *inmatching* and *prematching*. In this paper, it is shown that *prematching* is generally preferable to *inmatching*, it compares the benefits of different optimisations for *M**, and it shows how well *M** stacks up against other *MAPF* solving algorithms extended to perform matching.

1. Introduction

A large number of real-world situations require the planning of collisionless routes for multiple agents [1]. For example, the routing of trains over a rail network [1], directing robots in warehouses [2] or making sure autonomous cars do not collide on the road [3]. Problems of this nature are called *Multi-agent pathfinding* problems, which in this paper will often be abbreviated to *MAPF*.

In *MAPF*, each agent has a starting position and a goal position. A route needs to be found for every agent, from their start to their goal. Finding these routes without collisions, has been proven to be *NP-hard* [4].

One algorithm to solve *MAPF* is called *M** [5]. A standard *A** algorithm as described by Standley [6] plans agents together. This means that in each timestep, the number of possible next states grows exponentially with the number of agents. Contrasting that, in *M**, when possible, agents follow an individually optimal path, and in each timestep, only the subset of agents which is part of a collision is jointly planned.

The *MAPF* problem can be extended by grouping agents into teams. A team consists of one or more agents. In contrast to *MAPF*, an agent does not have a single goal here. Instead, agents can use any goal associated with the team it is in, but every goal can only be used once. An algorithm solving this problem must assign each agent a goal. Such an assignment is called a matching. This problem is therefore named multi-agent pathfinding with matching (here abbreviated to *MAPFM*).

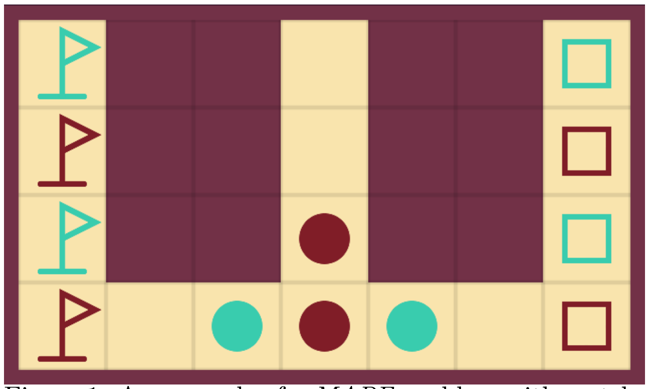


Figure 1: An example of a *MAPF* problem with matchings. Two teams (of different colours) of agents (circles) moving from their starts (squares) to their goals (flags).

In this paper, first *MAPFM* will be defined, and after that possibilities will be discussed to extend *M** to solve *MAPFM* problems as well. To do this, two methods will be proposed. These two methods will be compared, both to each other, and to a number of other algorithms solving *MAPFM*. Apart from this comparison, a number of extensions to *M** will be considered which improve *M**'s performance. Some of these extensions can also improve the performance of *M** on plain *MAPF* problems.

[1]An agent is a collective name for a unit for which a path is calculated. The word agent could refer to a robot, a train or a car, but can also refer to an abstract or simulated entity.

2. Prior work

Before this research, a separate study has been performed [7] on a problem called target-assignment and path-finding (*TAPF*). The difference between *TAPF* and *MAPFM* is that in *TAPF*, the *makespan* (the cost of the longest path) instead of the *sum of individual costs* (the cost of all paths combined) is minimised. To solve *TAPF*, [7] uses conflict based search [8], using a min-cost flow based algorithm for the lower level to solve matchings within one team. It is yet unclear if it is possible to use such a min-cost flow based algorithm for *MAPFM*. However this is not a question that will be answered in this paper. Parallel research to this will however attempt to find an answer to this question [9].

To the best of the author's knowledge, solving *TAPF* and *MAPFM* with other algorithms than *CBS* has not yet been explored.

In contrast, a lot of research has been done finding and improving algorithms which solve *MAPF* [5, 6, 8, 10–13]. [8] compares *Conflict base search* (*CBS*), *Increasing cost tree search* (*ICTS*) and a derivative of *A**. Which algorithm performs best, heavily depends on the layout of obstacles in maps which are used for benchmarks. [8] concludes that *CBS* performs well when maps used for benchmarks have many bottlenecks and corridors (created by obstacles).

*A** is an algorithm commonly used as a base for algorithms solving *MAPF* [5, 6, 11, 14]. *M** is such a derivative of *A**. In [5], *M** is only compared to *A**, and they demonstrate that *M** is superior to *A** in just one type of map. The performance of *M** on maps with many bottlenecks or corridors is, however, not shown.

Leaving aside which algorithm is fast at solving *MAPF* problems, it is yet unknown how these algorithms compare when extended to also solve *MAPFM* problems.

3. Definition of *MAPF* and *MAPFM*

The definition of multi-agent pathfinding used in this paper is based on Stern's definition given in [15]. The definition is as follows:

A *MAPF* problem consists of the following elements:

$$\langle G, s, g \rangle$$

- G is an undirected graph $\langle V, E \rangle$
  - V is a set of vertices
  - E is a set of unweighted edges between vertices
- s is a list of k vertices where every s_i is a starting position for an agent a_i
- g is a list of k vertices where every g_i is a target position for an agent a_i

The goal of *MAPF* is to find a path for all agents a_i from s_i to g_i, without edge and vertex conflicts. This means that in a timestep t, two agents may not be on the same vertex, and that in a transition from timestep t to $t+1$, two agents may not use the same edge.

The cost c_i of a path π_i is the number of timesteps until the last time that agent a_i arrives at its goal g_i. To find the cost c of a solution to *MAPF*, two criteria are presented in [15]. The *sum of individual costs* is simply the sum of the cost of all paths ($c = \sum_{n=0}^{k} c_n$). Alternatively, the *makespan* is equal the maximum cost of all paths ($c = \max_{n=0}^{k} c_n$). An optimal solution to *MAPF* is a set of paths π with the smallest possible cost c.

Although the algorithms presented in this paper would work on any graph G, in examples and experiments, G is simplified to a 4-connected grid. In this paper, this grid is sometimes called a 'map', on which agents move from their starts to their goals.

MAPFM

This definition is now expanded to a definition of multi-agent pathfinding with matching, called *MAPFM*. The definition of this new problem is as follows:

$$\langle G, s, g, sc, gc \rangle$$

- sc is an array of colours. One colour sc_i for each starting vertex s_i
- gc is an array of colours. One colour gc_i for each target vertex g_i

This definition divides all agents into teams. An agent a_i's team colour is the colour of its starting location sc_i. In total there are K teams $k_1 \dots k_n$. K is equal to the number of different colours in sc and gc.

In *MAPFM*, a goal state is a state in which each agent is on a goal location in g, such that the colour of every agent is equal to the colour of the goal they are on.

MAPFM uses the *sum of individual costs* as an optimisation criterion, and **not** the *makespan*, which is what *TAPF* [16] does.

4. A description of *M**

*M** [5] is a pathfinding algorithms which searches a search space, similar to *A**, but is specifically designed to work with multiple agents. In *M**, the search space consists of the combination of positions of all the agents on a grid. To use *A** for multiple agents, the planning of paths for all agents needs to be coupled to make sure no states are expanded in which collisions occur.

Unlike *A**, in *M** the planning of agents is initially not coupled. Instead, *M** assumes that the optimal path for an agent to their goal does not contain any collisions. As long as this is true, the planning of agents is separated.

However, the assumption that all optimal paths are collisionless obviously does not always hold. Therefore, each state in *M** also holds, apart from the positions of each agent, two sets called the *collision set* and the *backpropagation set* as described in [5]. When *M** detects that a state contains collisions, the algorithm does not continue expanding this state. It instead uses information stored in these two sets to backtrack and find the shortest route around these collisions. Agents associated in the collision temporarily plan routes in a coupled fashion.

After collisions, *M** plans agents independently again according to their individually optimal path. It does, however, record information about the previous collisions (in these *backtracking*- and *collision sets*). This is to make sure that when in the future another collision occurs, it can either resolve this locally or backtrack back to the previous collision to plan it differently to potentially avoid this new collision altogether.

[5] proves that *M** provides optimal solutions to *MAPF*.

5. *M** AND MATCHING

To add matching to *M**, this paper proposes two options which are called "inmatching" and "prematching". In this section, both are explained and their advantages and disadvantages are discussed.

5.1. INMATCHING

Inmatching is the process of performing matching as a part of the pathfinding alorithm that is used. To understand it, it is useful to first look at inmatching in *A**. With *A**, the expansion of a state consists of all possible moves for all agents. *A** searches through the search space, until the goal state has been removed from the frontier. An admissible heuristic *A** will guarantee that following the children of this first goal state gives a shortest path between the start and the goal.

With *inmatching*, there is not one goal state. Instead, any state in which all agents stand on a goal of their own colour is considered a goal state. This means there is more than one goal state. To keep the heuristic admissible, the distance to the nearest goal state is used as a heuristic.

Inmatching applied to *M**

Inmatching can similarly be used with *M**. However, there is something that makes it much less efficient in *M** than in *A**, the implications of which will be evaluated in section 8.1.

One of the core ideas of *M** is that agents, when not colliding, follow an individually optimal path to their goal. With *inmatching*, every goal in the team of the agent is a valid goal. Therefore, to preserve optimality, agents should consider optimal paths to each of the goals in the team.

To understand the implication of this, a concept needs to be explained first: the *expansion size*. In *A**, a priority queue is used with states in it. Each time a state is removed from the queue it is expanded into a number of child states which go back into the queue. The number of child states which are expanded is the *expansion size*.

In regular *M**, when there are no collisions, the *expansion size* is 1. Each agent simply follows their individually optimal path to their goal. There is only one such optimal path so there's only one next state. Only when there are collisions, the *expansion size* is bigger than 1 which is less efficient. But with *inmatching*, there is not one optimal path. Because of the teams, an agent can have multiple goal locations, which may all have a different individually optimal path. Thus, the expansion size is often bigger than 1 even when there are no collisions. In fact, the expansion size grows exponentially with the number of goals in a team. The following equation gives an upper bound for this expansion size (where the *goals* function gives the number of goals in team k_n):

$$\prod_{n=1}^{K} \text{goals}(k_n)$$

*M** using *inmatching* will from now on be abbreviated with *imM**

5.2. PREMATCHING

Alternatively, there is *prematching*. With *prematching* the *MAPFM* problem is transformed in a number of *MAPF* problems. Each possible matching is calculated in advance, and normal *M** as described by [5] is performed on each matching. Algorithm 1 shows how this is done.

In section 6.3 an extension to *prematching M** is proposed which uses a heuristic to prune some of the matchings which need to be evaluated.

*M** using *prematching* will from now on be abbreviated with *pmM**

```
Algorithm 1: prematch M*
  Result: Find the matching with smallest cost
  ________________________________________
  foreach $m \leftarrow$ find all matchings(starts, goals) do
    | $S(i) \leftarrow$ mstar(starts, m) {Evaluate with M*}
  end
  return min(S) {by calculated cost}
```

6. Extensions to *M**

*M** can be improved upon in various ways. Some of these extensions are only applicable to matching *M**, while others improve *M** itself.

In this section these extensions are described in detail.

6.1. Precomputed paths and heuristics

A part of solving *M** is finding the individually optimal path for each agent. This can be done with conventional pathfinding algorithms such as *A**. But to avoid repeated recalculation, it is important to first create a lookup table of the distance to each goal, for every open square on the map.

When using *prematching*, the same goals and starts are (although in a different permutation) reused multiple times. The lookup table can remain the same every time.

Using this lookup table, the heuristic *M** can be improved as well. Instead of using the *euclidean-* or *Manhattan distance* to the goal, the lookup table can be used to find the exact distance. The heuristic found with this approach also takes the obstacles on the map into account.

6.2. Operator decomposition

In [6], Standley describes a technique called operator decomposition (*OD*). With operator decomposition the number of expanded nodes is divided by n (where n is the number of agents), while the search depth is multiplied by n.

To do this, expansions can be partial or complete. Each time a state is expanded, only the moves of one agent are expanded and put back in the queue. Only when this partially expanded node comes to the top of the queue, the moves of another agent are considered. Once all agents in a state have expanded, the state is said to be complete again.

The queue now contains both partial and complete states. This provides the search algorithm with more granularity in prioritising moves, which in turn can mean a considerable improvement in runtime.

[17] showed that *M** can also benefit from operator decomposition when solving *MAPF* problems.

6.3. Pruning of matchings

*pmM**, as previously described, usually has to evaluate every matching of every team. However, it is possible to discard some matchings without evaluating them with *M** at all, by using heuristics.

To do this, first calculate the heuristic (i.e. the sum of distances between starts and goals) of the initial state of every matching. Because this heuristic is admissible, it represents a lower bound for the cost of this matching.

Now, before every evaluation of a matching m, its heuristic is first checked against the best matching m_{best} found so far. If heuristic$(m) \geq cost(m_{best})$, there is no need to evaluate m and m can be pruned.

6.3.1. Sorting

To take maximum advantage of pruning, matchings can be sorted based on their heuristic. Making sure the matching with the lowest heuristic is evaluated first can increase chances that later matchings are pruned. Algorithm 2 shows how pruning with sorting works.

Algorithm 2: prematch *M** with pruning

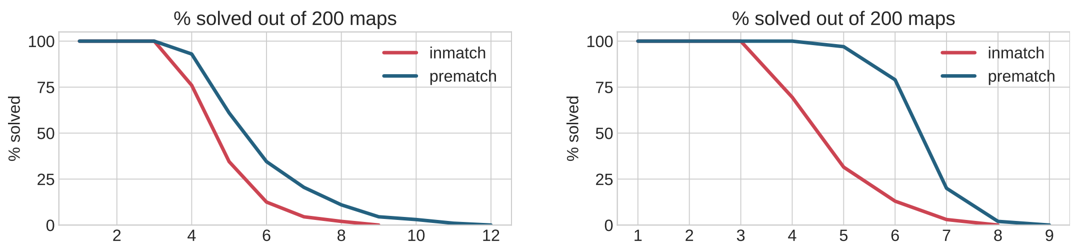
```
Result: Find the matching with smallest cost
        without evaluating every matching.
$m_{best} \leftarrow \varnothing$
---------------------------------------------
$M \leftarrow$ find all matchings$(starts, goals)$;
sort$(M)$ {on heuristic; ascending}
foreach $m \leftarrow M$ do
    if $heuristic(m) < cost(m_{best})$ then
        $s \leftarrow$ mstar$(starts, m)$;
        $m_{best} \leftarrow min(m_{best}, s)$;
    end
end
```

7. Experimental setup

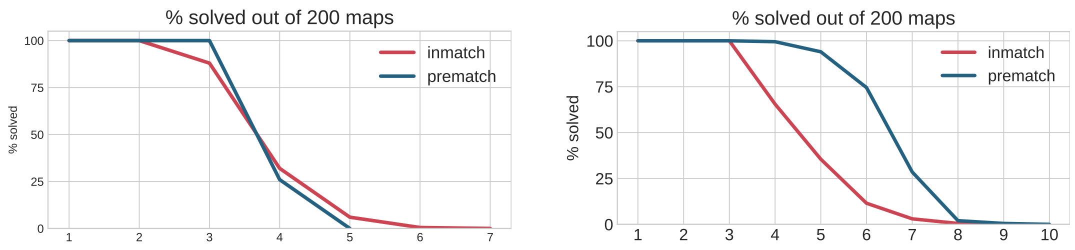
To evaluate the performance of matching *M**, a number of experiments were performed. For each of these experiments, the algorithm used was written in **python 3.9** and benchmarks were run on a virtualized system with a 12 core 12 thread Intel Xeon E5-2683 running at 2GHz, which has 8Gib of RAM.

There are various factors deciding how an algorithm or variant of an algorithm performs in benchmarks. Three important factors are the total number of agents for which paths need to be found, the number of teams over which the agents are distributed, and the layout of the maps on which benchmarks are run.

To show the differences in performance as these parameters vary, all experimental are grouped into sets of four *scenarios*. Performance is assessed on maps where either 25% or 75% of the grid is an obstacle, and in situations where agents are grouped into either 1 or 3 teams.



(a) Agents split over 3 teams. Left: 75% wall, right: 25%wall



(b) All agents in 1 team. Left: 75% wall, right: 25%wall

Figure 2: Percentage of maps solved in 2 minutes out of 200 random 20×20 maps.

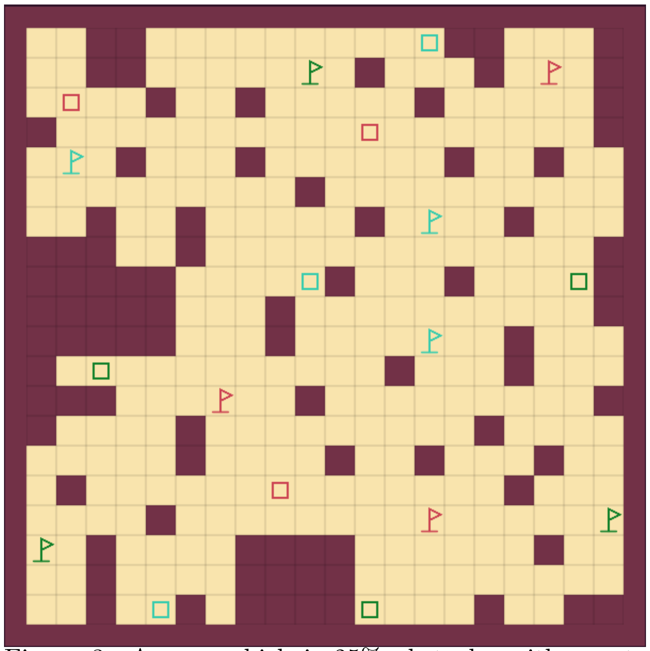


Figure 3: A map which is 25% obstacle, with agents distributed in three teams. This map was one of many used in the experiments.

For these benchmarks, maps with a size of 20×20 units are randomly generated using an algorithm similar to maze generation algorithms. Four sets of maps were generated for each experiment with the previously discussed parameters. Agents and goals are uniformly distributed on this map and teams are assigned randomly. When the team size does not divide the number of agents, there will be one smaller team. Only maps in which each individual agent is capable of reaching their goal are used to greatly reduce the chance that maps are impossible to solve. An example of such a randomly generated map can be found in figure 3.

Because *MAPFM* is an *NP-Hard* problem, it is possible that finding the solution to an instance of the problem takes an unreasonable amount of time. Therefore, in each experiment, a timeout of 2 minutes is used. Experimental results do not show the time it takes for an algorithm to find a solution. Instead they show the percentage of maps (out of a set of 200) which the the algorithm manages to solve within this timeout of 2 minutes.

8. Results and Discussion

8.1. Matching strategies

In section 5, two strategies were proposed to add matching to *M**. Both strategies were tested following the experimental setup described in section 7. In figure 2 the results of this experiment are shown.

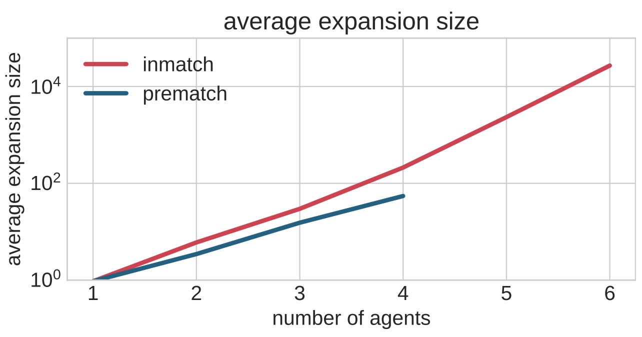
The figure (2) shows that *prematching* is generally superior to *inmatching*. In all but one of the scenarios, *prematching* performs better. In section 5.1, it was hypothesised that this could happen, because of the larger expansion size.

The reason for this difference is the same as was hypothesised in section 5.1. *inmatching* leads to a large expansion size.

Expansion size

To demonstrate this is indeed the case, another, separate, experiment was performed. Again on 200 random maps as described in section 7, but this time showing the average number of states expanded each expansion.

In figure 4, on a logarithmic scale, the outcomes of this experiment are shown. Indeed, on average, inmatching expands many more states.



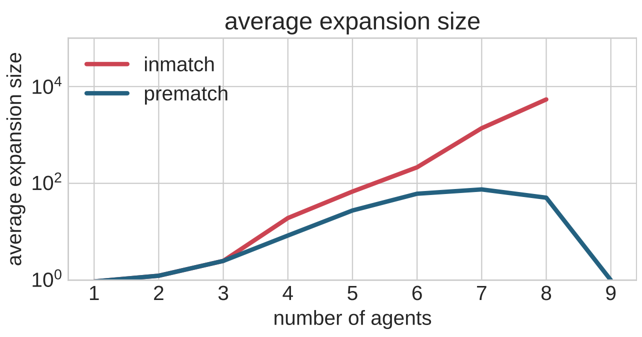


Figure 4: Average expansion size on the same maps as used in figure 2b (i.e. all agents in one team). Left: 75% wall, right: 25%wall.

When *inmatching* is better

However, in one of the scenarios in figure 2b *inmatching* does seem to perform slightly better than *prematching*. In this benchmark, all agents are in a single team, and the map is 75% filled with obstacles.

When agents are all in one team, there are more matchings than when agents are separated into multiple teams. With *prematching*, each matching needs to be exhaustively searched which is slow.

In contrast, the problem of *inmatching* expanding into many states is reduced in this scenario. There are fewer directions for agents to go because of the large number of obstacles.

So in general, when there are many matchings, but few options for agents to move in, *inmatching* can perform better.

8.2. Extensions to *M**

Various extensions have been proposed in section 6. To show the benefits of each extension, they were compared following the experimental setup described in section 7.

In the graphs of figure 5, the performance of these extensions is shown. Each line in the graph shows a new extension added to the algorithm, ordered following the legend.

Because in section 8.1, *prematching* was observed to generally perform better, only the *prematching* matching strategy is used in this experiment.

Pruning and sorting

In the graphs where 25% the maps is filled with obstacles, pruning and sorting have a large impact on performance. However, in graphs where 75% is a wall there is barely a difference at all. The heuristic used to prune, is the Manhattan distance between the agent start locations and the closest goal. This does not take into account the obstacles in the map.

Pruning is only possible when the heuristic is larger than the best matching found so far (as described in section 6.3). This does not happen often with this inaccurate heuristic. Therefore, using the precomputed heuristic (also shown in figure 5) makes such a large difference.

When there are less obstacles in the map, the probability of pruning is much higher explaining the difference in performance of pruning and sorting between these two scenarios.

Memory usage

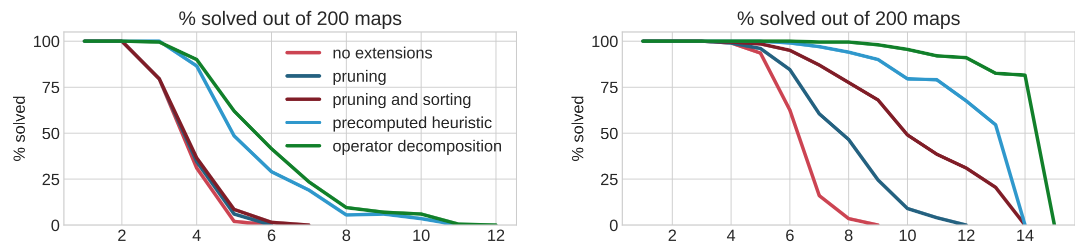
The best variant of *M** can be seen solving maps with 14 agents in figure 5a. After this, the percentage abruptly drops to 0. This is not caused by the complexity of the problems, but rather by the memory that *M** uses here.

Attempts with maps with 15 agents saw memory usages of over 8GiB to solve a single instance due to which attempts were stopped by the operating system.

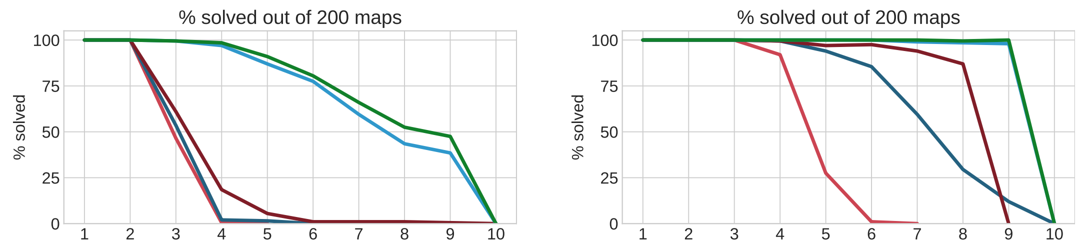
8.3. Comparison with other algorithms

This research is part of a set of parallel studies on how to extend a collection of *MAPF* algorithms to give them the ability to solve matchings:

- Extended partial expansion *A** (*EPEA**) [11] (implementation and research by [18])
- *A** with operator decomposition and independence detection (*A*+OD+ID*) [6] (implementation and research by [19])
- Increasing cost tree search (ICTS) [13] (implementation and research by [20])
- Conflict-Based Min-Cost-Flow (CBM) [7] (implementation and research by [9])



(a) 3 teams. Left: 75% wall, right: 25%wall



(b) 1 team. Left: 75% wall, right: 25%wall

Figure 5: Percentage of maps solved in 2 minutes out of 200 random 20×20 maps. Each line and extension to *M** is graphed in combination with all previous extensions. For example, the line displaying *operator composition* also uses precomputed heuristics and all other previous extensions.

Experiments were performed following the experimental setup described in section 7. All algorithms were benchmarked on the same computer to ensure a fair comparison. The results of this comparison can be found in figure 6.

In figure 6b, it may look like an error was made. *CBM* solves 100% of the maps up to maps with 25 agents. In fact, *CBM* is able to solve maps with many more agents. *CBM* makes use of min-cost flow to solve *MAPFM* for single teams in polynomial time. Solutions for single teams are combined and modified when conflicts are found between agents in different teams.

But, this means that in cases where there *is* only one team, *CBM* is able to solve *MAPFM* in polynomal time, scaling linearly with the number of agents for which a route needs to be found.

In the experiments where multiple teams were used (as shown in figure 6a), *CBM* performs more comparably with other algorithms. Still, when there are few obstacles, *CBM* excels.

Apart from *CBM*, all other algorithms (including *M**) show very similar performance characteristics. All these algorithms use a form of prematching, where all matchings are exhaustively searched. This seems to be a factor which limits the capabilities of all these algorithms.

Only *A*+OD+ID* performs exceptionally well on maps 25% filled with obstacles, and with 3 teams. The reason for this is likely the TODO.

It must however be said, that this comparison is **not** complete. For example, testing with agents split over more teams, on larger maps, or with specifically crafted obstacles such as long corridors may show very different results.

9. Conclusion

In this paper, a new addition to multi agent pathfinding (*MAPF*) was introduced, called matching. This new problem, is called *MAPFM M**, an algorithm for *MAPF*, was modified to solve these *MAPFM* problems. To do this, two strategies were explored called *inmatching* and *prematchings*. Experimental results showed that in many cases, *prematching* is superior to *inmatching*.

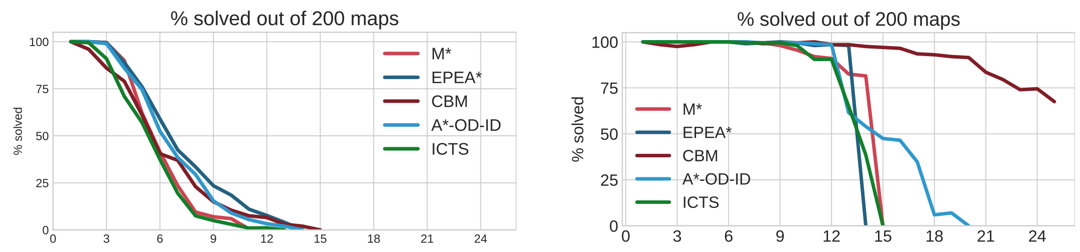
After that, several improvements to *prematching M** were considered and their benefits were experimentally evaluated. Pruning and sorting of matchings, using precomputed heuristics, and independence detection were shown to make a large effect on performance. Experiments showed that in one scenario, *M** without extensions could solve maps with up to 8 agents. With all extensions combined, maps with up to 14 agents could be reliably solved.

Finally, it was shown that *pmM** has very similar performance characteristics as *ICTS*, *EPEA** and *A*+OD+ID*. *CBM* performed notably better than *M** in experiments performed in this paper.

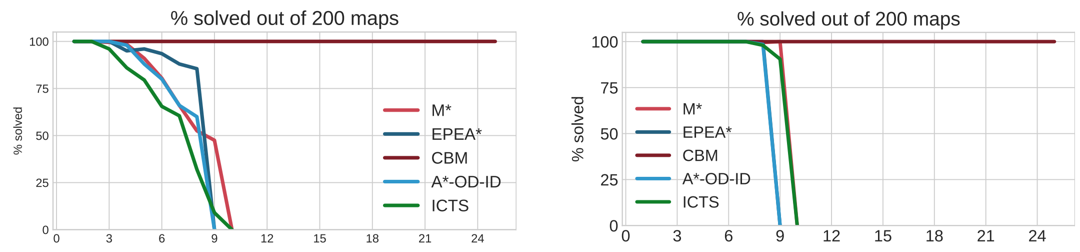
10. Future work

Partial Expansion

One problem with the *inmatching* strategy presented in section 5.1 is that the expansion size is very large.



(a) 3 teams. Left: 75% wall, right: 25%wall



(b) 1 team. Left: 75% wall, right: 25%wall. In the right graph, the lines for *EPEA** and *A*+OD+ID* overlap, and so do the lines for *ICTS* and *M**.

Figure 6: Percentage of maps solved in 2 minutes out of 200 random 20×20 maps.

However, a lot of the expanded states are never needed or accessed again. But even with *prematching* this was a problem. For example, while creating the graphs in figure 5, the data points where 14 or 15 agents were involved could use as much as 8GiB of memory.

partial- or *enhanced partial expansion* as presented in [11] and [12] only expands states when they are needed to reduce memory usage. It is likely that partial expansion can also be applied to *M** to improve its memory performance.

Conflict based search

In this paper, matching *M** was compared to several other algorithms. One of those was CBS [8]. However, this implementation used a min-cost flow based approach similar to the one described in [7], but altered to find the *sum of individual costs* instead of the *makespan*.

CBS, without matching and these min-cost flow extensions, is one of the most studied algorithms for *MAPF* ([8, 21–24]). Comparing *M** with *inmatching* and *prematching* to *CBS* using similar approaches to add matching to it, could provide useful data.

Waypoints

In previous research [25–28], *MAPF* was extended with waypoints instead of matching. It may be possible to combine these two extensions. For example, each team may have a set of waypoints. It doesn't matter which agent visits which waypoint as long as all waypoints are visited. Or alternatively, each agent has their own waypoints, but the goals are still shared with a team of other agents.

Agents and goals

In the definition of matching presented in this paper, in each team the number of goals was always equal to the number of agents. However, in real world scenarios this may not always be the case.

For example, there may be a number of robots who have a number of tasks to do. Some robots cannot perform certain tasks so robots are teamed. But there may be more tasks than robots in each team, and robots will need to prioritise.

Some research has already been done in this area, [29], but this uses *TAPF* as a basis.

Recursive M*

In [5], an extension to *M** is described called recursive *M**. It could increase the performance of *M** on regular *MAPF* problems. Whether recursive *M** makes a difference combined with matching, has not yet been verified.

11. Reproducibility

Results in this paper have been generated using an implementation of *M** made in python specifically for this research. The code for this is publicly available on github at https://github.com/jonay2000/research-project, doubly licensed under the Apache 2.0 and MIT licenses. Reports of bugs and new additions to this repository are always welcomed.

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