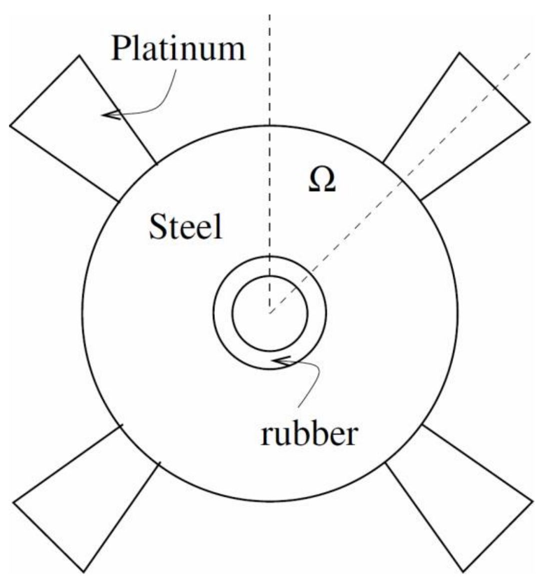
DEPT. OF CIVIL ENGINEERING & ENGINEERING MECHANICS

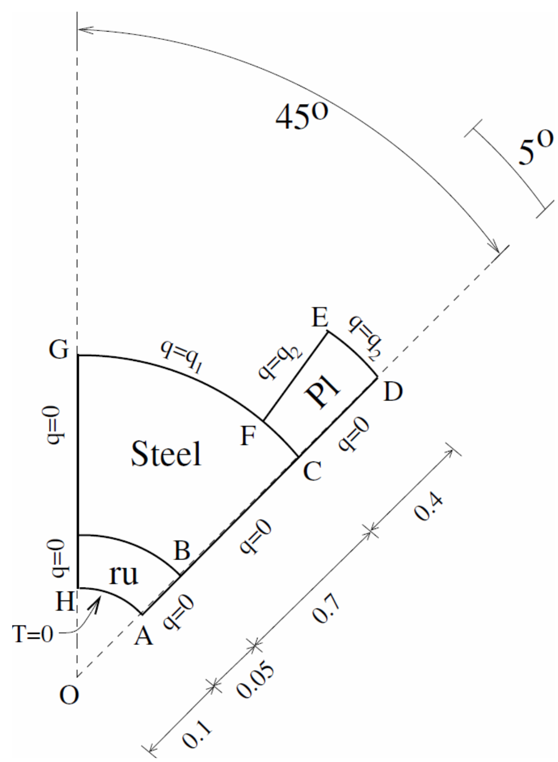
Final Finite Element Programming Project (Optional):

Heat Conduction in a Turbine Blade

Due by the end of Wednesday 12/19/2018 in Prof. Waisman's mailbox

The objective of this project is to analyze the heat conduction in a turbine blade using the Finite Element Method. Consider the turbine disk shown in the left figure with 4 blades and a center hole for the rotor. The disk itself is made of steel and the blades of platinum. A narrow rubber band is added around the hole for insulation purposes of the rotor and the disk. Due to symmetry, it is sufficient to consider only $\frac{1}{8}$ of the turbine in the finite element analysis, also taking into account symmetry conditions on the heat flux (as shown). A segment of 45^0 of the computational domain Ω is shown in the right figure. The symmetric Boundary Conditions on the flux are applied to edges GH and ABCD, for which the flux is $q = 0$.





The lower boundary AH (hole boundary) is set to a relative temperature of $T = 0^0$. The heat flux through the external boundaries are $q_1 = 1300 \left[\frac{W}{m^2}\right]$ and $q_2 = 1800 \left[\frac{W}{m^2}\right]$ and the dimensions of the blade are given in [m]. In addition, the coefficients of thermal expansion of rubber, steel and platinum are $\kappa_{rubber} = 1 \left[\frac{W}{m\ {}^0C}\right]$, $\kappa_{steel} = 55 \left[\frac{W}{m\ {}^0C}\right]$ and $\kappa_{platinum} = 71 \left[\frac{W}{m\ {}^0C}\right]$, respectively.

Assignment:

Modify the heat conduction MATLAB code (or alternatively use other coding language that you prefer), posted on the coursework website, to analyze the problem using 3-node linear triangular elements. The analysis should include a coarse mesh with about 300-500 elements and a fine mesh with 1500-5000 elements. To generate the mesh, either write your own mesh generator or simply use ABAQUS meshing tools and extract the required element and nodal information (the *.inp file generated by ABAQUS in your working directory).

The final report (submitted as a hardcopy and also uploaded to courseworks) should include:

1. (10%) Theoretical (physics) Background: problem description, governing equations and boundary conditions, etc.
2. (10%) Numerical (FEM) Background: e.g. derivation of FEM weak forms, Galerkin's method, discretization by triangular elements, postprocessing, etc.
3. (10%) Plots of both meshes and the appropriate boundary conditions. Node numbers should be printed <u>only in the figure of the coarse mesh</u>.
4. (20%) Color plots of the temperature field distribution in Ω, including a scale. If a color printer is not available, B&W plots will also be accepted.
5. (20%) Plots of the temperature variation along the edges ABCD and the arc CFG (two separate figures). Include the results of both meshes on the same figures for comparison. Make sure to use legends, labels, grids and titles on the graph.

6. (10%) Plot the normal heat fluxes on the boundaries GF, EF and DE as obtained from the finite element solution and compare it with the prescribed natural Boundary Conditions q_1 and q_2. Are the two matching? If not, explain why.
7. (5%) The code should also output the Average Band Width (ABW) of the global stiffness matrix (for both meshes). Add it to your report. Is the mesh numbering ideal or can it be improved to get sparser matrices?
8. (5%) A short discussion on physical and numerical implications of the study, and some concluding remarks and suggestions (if any) on improving the project.
9. (10%) Appendix A: Brief description of the code structure, code flow diagram, and its subroutines. Don't include in the description mesh connectivity and nodal coordinates data. The full code should be uploaded as a zipped file to coarse works.

Bonus Question 1 (5%):

Analyze the problem using Abaqus, and repeat questions 4-6 by adding the Abaqus results and comparing the results to your own code. Do you have good agreement with Abaqus ?

Bonus Question 2 (10%):

Implement an isoparametric triangular element and use a gauss quadrature rule for triangles to analyze the problem. Show that the results are in good agreement (should be identical) with the exact integration triangle implemented in the project.

NOTE: Once you are ready to submit the project, zip your MATLAB code (call the file My_Name_UNI.zip) and upload it to the course website under the instructor's directory (please do not email the code to me directly). Include a Readme file (Readme.txt file) with the name of your team member (if any), and a brief instruction on how to run the code. Your code will be tested, therefore make sure that your code is running before you zip it, otherwise your project will be considered as incomplete and only partial credit will be given. In addition include with your zipped code a copy of your report in PDF or Word formats. Make sure to use legends, labels and titles in all plots required for the report. However, please don't include in the report mesh connectivity and coordinates (these will be part of your code files).