Jason Dorweiler
Assignment 5.1 CS225
Assignment: Section 5.1: 8, 12, 16, 24, 26, 42, 46, 60 (6th edition))

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8. How many different three-letter initials with none of the letters repeated can people have?

With no repeated letters there will be $26 * 25 * 24 = 15600$ different three letter initials.

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12. How many bit strings are there of length six or less?

There will be 6 separate cases where each case is 2^n. The total will be $\sum_{n=0}^{6} 2^n = 127$

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16. How many strings are there of four lowercase letters that have the letter x in the term?

If there were no X's then the highest number would be $26 * 26 * 26 * 26 = 456976$. Since there needs to be an at least one X there will be 4 cases:

Case 1 (one x): With one X there are 4 combinations of where the letter can be placed in the string. That makes the total $4 * 25 * 25 * 25 = 62500$

Case 2 (two x): There will now be 6 combinations of placements for the letter in the string but it also takes up two positions. That makes the total $6 * 25 * 25 = 3750$

Case 3 (three x): There are 4 combinations of placements for three x's. That makes the total $2 * 25 = 100$

Case 4 (four x): There is only one string with 4 x's.

Adding all of the cases up gives a total of 66351 four letter strings.

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24. How many strings of four decimal digits
a) do not contain the same digit twice?
b) end with an even digit?
c) have exactly three digits that are 9s?

A: Since the string can't contain the same digit twice it can have 10 choices for the first digit, 9 for the second, 8 for the third, and 7 for the last. Giving a total of $10 * 9 * 8 * 7 = 5040$

B: Half of the strings will end in an even digit. The total of all four digit strings is 10^4 and the even strings will be $\frac{10^4}{2} = 5000$

C: There are 4 combinations of three digits (9) in a 4 digit string. That makes the total $4 * 9 = 36$

26. How many license plates can be made using either three digits followed by three letters or three letters followed by three digits?

There are $10 * 10 * 10$ number combinations and $26 * 26 * 26$ letter combinations. The total for one type of plate is $10^3 * 26^3$. The other plate will give the same number so the overall total is $2 * (10^3 * 26^3) = 35152000$

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42. How many bit strings of length seven either begin with two 0s or end with three 1s?

There will be 2^5 strings that begin with two 0s and 2^4 strings that end with three 1s. The overlap between the two sets is 2^2 so the total is $(2^5 + 2^4) - 2^2 = 44$.

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46. Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors(including joint majors), 23 mathematics majors(including joint majors), and 7 joint majors?

There are $38 compsci + 23 math - 7 joint = 54$ students in the class.

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60. Use mathematical induction to prove the product rule for m tasks from the product rule for two tasks.

$P(m)$ is the product rule for m tasks.
Basis step:
m = 2, $P(2)$ is true since there are n_1 ways to do the first task and n_2 ways to do the second one. The total being $n_1 n_2$.

Inductive Step:
Prove P(k) is true for $k \geq 2$.
For a k+1 tasks we have $T_1 T_2 \cdot \cdot T_{k+1}$. Each of which can be done $n_1 n_2 \cdot \cdot n_{k+1}$ ways. That means that for the next k+1 task we will have $(n_1 n_2 \cdot \cdot n_k) * n_{k+1}$ ways which proves $P(k+1)$.