question 3.

$$A - B \subseteq A$$

Assume that $x \in A - B$. Then it follows that $x \in A$ and $x \notin B$. Since $x \in A$ the statement $A - B \subseteq A$ holds.

Question 4

a. All good weight lifters are strong.

$$(W(x) \to S(x))$$

b) Good weight lifters are not a good basketball player

$$W(x) \rightarrow \neg B(x)$$

c) If someone is strong then they are a good basketball player or a good weight lifter. $\forall x (S(x) \to (B(x) \lor W(x))$

d) There is someone who is a good basketball player and a good weight lifter. $\exists x (B(x) \land W(x))$

question 5

$\lceil n \rceil$		r	$n \setminus a(2)$	(2) R	$P \wedge \neg Q$ (3)	(3) \/B
p	q	r	$p \rightarrow q \ (z)$	$(2) \rightarrow R$	1 /\ \Q (3)	(3) $\vee n$
1	1	1	1	1	0	1
1	1	0	1	0	0	0
1	0	1	0	1	1	1
0	0	0	0	0	0	0
0	1	1	1	1	0	1
0	1	0	1	0	0	0

Columns 5 and 7 are identical so they are logically equivalent.

question 6

Use a direct proof to show that the sum of two rational numbers is rational. (Recall that a number is rational if and only if it can be expressed as the ratio of integers.)

Assume that a and b are rational numbers. so $a = \frac{x}{z}$, $andb = \frac{j}{k}$ and x,z,j, and k are also integers. Then the sum of $a + b = \frac{x+j}{z+k}$ which is also a rational number.

Question 7.

Assume for contradiction that n and m have the same parity. If m and n are even:

$$n = 2k_1, m = 2k_2$$

 $2k_1 - 5(2k_2) = 2(k_1 - 5k_2)$ which is an even number and contradicts our assumption that n - 5m is odd.

For the case when they are odd.

$$n = 2k_1 + 1, m = 2k_2 + 1$$

 $2k_1 + 1 - 5(2k_2 + 1) = 2(k_1 - 5k_2 + 3)$ which is also even and contradicts our assumption.

Question 8.
$$\sum_{j=3}^{6} (5j^2 + (-1)^j) = \sum_{j=3}^{6} (5j^2) + \sum_{j=3}^{6} (-1)^j = (15^2 + 20^2 + 30^2) + \sum_{j=0}^{6} (-1)^j - \sum_{j=0}^{2} (-1)^j = (15^2 + 20^2 + 30^2) + (\frac{(-1)^7 - 1}{-1 - 1}) - (\frac{(-1)^3 - 1}{-1 - 1})$$

$$\sum_{i=0}^{8} (2^{i+1} - 3) = \sum_{i=0}^{8} 2^{i+1} - 8 * 3 = 2 * \sum_{i=0}^{8} 2^{i} - 8 * 3$$
$$= 2 * \frac{2^{9} - 1}{2 - 1} - 24$$

question 9: Base case: $P(0) 3^0 = 1, \frac{3^0 - 1}{2} = 1$

inductive case:
$$3^n = \frac{3^{n+1}-1}{2}$$

for n=k:

$$3^k = \frac{3^{k+1}-1}{2}$$

and for
$$n=k+1$$

 $3^{k+1} = \frac{3^{k+2}-1}{2}$

assuming that
$$3^k = \frac{3^{k+1}-1}{2}$$
 is true then:

assuming that
$$3^k = \frac{3^{k+1}-1}{2}$$
 is true then:
$$\frac{3^{k+1}-1}{2} + 3^{k+1} = \frac{3^{k+1}-1}{2} + \frac{2*3^{k+1}}{2} = \frac{3^{k+1}-1+2*3^{k+1}}{2} = \frac{3^{k+1}*(1+2)-1}{2} = \frac{3^{k+2}-1}{2}$$
 so the hypothesis holds.

question 10:

Use strong induction to prove that every amount of postage of 12 cents or more can be formed using just 4-cent and 5-cent stamps

Base case:

$$P(12) = 4*4*4 = 12$$

$$p(13) = 4*4*5 = 13$$

$$p(14) = 5*5*4 = 14$$

This proves the first three base cases. n, n+1, and n+2

Inductive case:

Assume a new stamp k such that $12 \le k \le j$. We know p(n)...p(n+2) is true. The next stamp P(n+3) has to be true since P(n+2) was proven in our base case.