12.
Basis Step:
$f_1^2 = f_1 * f_2$
$f_1 = 1$ and $f_2 = 1$
$f_1^2 = 1 * 1 = f_1 * f_2$ so the basis step checks out.

Recursive Step:
Assume that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n * f_{n+1}$
Prove: $f_1^2 + f_2^2 + \cdots + f_n^2 + f_{n+1} = f_n * f_{n+1} * f_{n+2}$
Rewriting the assumption:
$(f_1^2 + f_2^2 + \cdots + f_n^2) + f_{n+1}^2$ where $(f_1^2 + f_2^2 + \cdots + f_n^2)$ is our hypothesis.
We can rewrite as $f_n * f_{n+1} + f_{n+1}^2$ and factor to get $f_{n+1} * (f_n + f_{n+1})$ which is the definition of a Fibonacci number $f_{n+1}f_{n+2}$ and proves our hypothesis.

26 c)
Basis step:
The basis $\{0, 0\}$ is still valid since $5|\{0, 0\} = 0$

Recursive Step: Suppose that $a + b = 5k$. Then $5|\{a + 2, b + 3\}$ which is $a + b + 5 = 5k + 5 = 5(k + 1)$ where $k + 1$ is also an integer.
The same is true for $5|\{a + 3, b + 2\}$ which is $a + b + 5 = 5k + 5 = 5(k + 1)$
This completes the proof.

43.
Base Case:
$n(T) = 1$ then the height is $h(T) = 0$ and $1 \geq 2 * 0 + 1$
Assume that the hypothesis holds for trees smaller then T. We will make a new tree from subtrees T_1 and T_2. Since the hypothesis is still true we know that $n(T_1) \geq 2 * h(T_1) + 1$ and the same applies for T_2. The new tree is then $n(T_n) = 1 + n(T_1) + (T_2)$ accounting for the top node.
Rewriting is equal to:
$n(T_n) = 1 + 2 * h(T_1) + 1 + 2 * h(T_2) + 1$
$1 + 2(h(T_1) + h(T_2) + 1)$
$1 + 2 * h(T)$ the height of both trees+1 is the height of the larger tree.
This completes the proof.

44.
Base Case:
$n = 1$ so $l(T) = 1$ since a single node tree has no internal vertices the base case is true.
Recursive Case:
Assuming that the hypothesis holds for smaller trees we will make a new tree out of two smaller trees T_1 and T_2. The number of leaves in the new tree is: $l(T) = l(T_1) + l(T_2)$.
Substituting the definition of $i(T)$ gives:
$l(T) = i(T_1) + 1 + i(T_2) + 1$
i(T)+1 which is the definition of a full binary tree. This completes the proof.

58.
Basis Case: The empty string λ has the same number of left and right parentheses so the base case is true.

Recursive Case:
Assume: $x, y \in B$
Let the number of left parentheses for x,y be l_x and l_y, and the number of right parentheses be r_x and r_y. The number of left and right parentheses for $(x) \in B$ will be one greater on each side. $l_x + 1$ and $r_x + 1$. Since both l_x and r_x are still equal the hypothesis is true for this case.
For the $xy \in B$ case: the number of left parentheses will be $l_x + l_y$ and the number of right parentheses will be $r_x + r_y$ by the hypothesis we know that $l_x + l_y = r_x + r_y$ and so there will be the same number of parentheses.
This completes the proof.