

Using the three cases for the ratio:

$$\left(\frac{a}{a^d}\right)$$

from DPV pg. 60:

Case 1: $\left(\frac{a}{a^d}\right) < 1$ Then the series is decreasing. It's sum is given by $O(n^d)$

Case 2: $\left(\frac{a}{a^d}\right) > 1$ Then the series is decreasing. It's sum is given by $n^{\log_b a}$

Case 3: $\left(\frac{a}{a^d}\right) = 1$ Then the series is decreasing. It's sum is given by $n^d \log(n)$

(a) $T(n) = 2T\left(\frac{n}{3}\right) + 1$

case 1: $\Theta(n)$

(b) $T(n) = 5T\left(\frac{n}{4}\right) + n$

case 2: $\Theta(n^{\log_4 5})$

(c) $T(n) = 7T\left(\frac{n}{7}\right) + n$

case 3 with $d = 1$: $\Theta(n \log(n))$

(d) $T(n) = 9T\left(\frac{n}{3}\right) + n^2$

case 3 with $d = 2$: $\Theta(n^2 \log(n))$

(e) $T(n) = 8T\left(\frac{n}{2}\right) + n^3$

case 3 with $d = 3$: $\Theta(n^3 \log(n))$

(f) $T(n) = 49T\left(\frac{n}{25}\right) + n^{\frac{3}{2}} \log(n)$

case 2 but the non-recursive portion dominates so: $\Theta(n^{\frac{3}{2}} \log(n))$

(g) $T(n) = T(n-1) + 2$

The depth of the recursion tree will be n so it's easy to see that this one is $\Theta(n)$

(h) $T(n) = T(n-1) + n^c$

In this case the depth of the recursion tree is still n but each level of the tree takes n^c so: $\Theta(n * n^c)$

(i) $T(n) = T(n-1) + c^n$

In this case the depth of the recursion tree is still n but c^n for $c > 1$ is going to dominate so: $\Theta(c^n)$

(j) $T(n) = 2T(n-1) + 1$

The depth of the tree will be n and time 2 at each level: $\Theta(2^n)$

(k) $T(n) = T(\sqrt{n}) + 1$ Stuck on this one. There's going to be a $\log(n)$ in there somewhere since n decreases by \sqrt{n} at each level. : $\Theta(?)$