CS225 Midterm Review

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Section 1.7

1. Use a direct proof to show that the sum of two odd integers is even.
$n_1 + n_2 = odd$ when n is an integer.
$n_1 = 2k_1 + 1, n_2 = 2k_2 + 1$
$2k_1 + 1 + 2k_2 + 1$
$2 * (k_1 + k_2 + 1)$ def of even number.

3. Show that the square of an even number is an even number using a direct proof.
n^2 is even.
$n = 2k$
$(2k)^2 = 2 * (2k^2)$ definition of even number

6. Use a direct proof to show that the product of two odd numbers is odd.
$(2k_1 + 1) * (2k_2 + 1)$
$4 * k_1k_2 + 2k_1 + 2k_2 + 1$
$2(2k_1k_2 + k_1 + k_2) + 1$ definition of odd number.

14. Prove that if x is rational and x = 0, then 1/x is rational.
x is rational so $x = \frac{a}{b}$, where a and b are integers.
$\frac{1}{\frac{a}{b}} = \frac{b}{a}$ which is also rational.

18. Prove that if n is an integer and 3n + 2 is even, then n is even using
a) a proof by contraposition.
If n is odd, then $3n + 2$ is odd.
n = 2k+1
$3 * (2k + 1) + 2 = 6k + 5 = 2 * (3k + 2) + 1$

b) a proof by contradiction.
Assume for contradiction that n is odd, $n = 2k + 1$.
$3*(2k+1)+2 = 6k+5 = 2*(3k+2)+1$ which says that $3*n+2$ is odd which is a contradiction.

8. Prove that if n is a perfect square, then n + 2 is not a perfect square.
Using proof by contradicion. Assume that n+2 is a perfect square.
$n + 2 = k * k$
$n = kk + 2$ so n is not a perfect square. This is a contradiction to our assumption.

Set Notation

Sets are unordered collections of objects.

O is the set of all odd positive integers less than 10.
$O = \{x \in Z^+ | x \, is \, odd \, and \, x < 10\}$

Two sets are equal if and only if they have the same elements. Therefore, if A and B are sets, then A and B are equal if and only if $\forall x (x \in A \leftrightarrow x \in B)$. We write A = B if A and B are equal sets.

The set A is a subset of B if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of the set B.

Theorem 1 shows that every nonempty set S is guaranteed to have at least two subsets, the empty set and the set S itself, that is, $\emptyset \subseteq S$ and $S \subseteq S$

Set Operations

16. Let a and b be sets. show:
a. $(A \cap B) \subseteq A$
suppose $x \in (A \cap B)$ then $x \in A, x \in B$ since $x \in A$ we can conclude that it's true.

b. $A \subseteq (A \cup B)$
Suppose that $x \in A$ then it follows that $x \in A$ or $x \in B$ and so it's true.
c. $A - B \subseteq A$
suppose that $x \in (A - B)$ then it follows that $x \in A$ and $x \notin B$
d. $A \cap (B - A) = \emptyset$
Proof by contradiction. Assume that $A \cap (B - A) \neq \emptyset$ and $x \in A \cap (B - A)$ it follows that $x \in A$ and $x \in B, x \notin A$ which is a contradiction.
e. $A \cup (B - A) = A \cup B$

A	B	$\neg A$	(B-A)	$A \cup (B - A)$	$A \cup B$
1	1	0	0	1	1
1	0	0	0	1	1
0	1	1	1	1	1
0	0	1	0	0	0

18. a. $(A \cup B) \subseteq (A \cup B \cup C)$
These are all pretty much the same.
1. assume x is an element of the left hand side. Break it down into each element and come to a conclusion.
2. if it proving something is NULL then use contradiciton.
3. if something = something else just use a table.

Quiz 4 questions

4. Let A: the set of all red cars
B: the set of all fast cars.
The set of all caars that are neither red nor fast.
$\neg A \cap \neg B$

5. Prove: $A - B \subseteq \overline{B}$
Suppose that $x \in (A - B)$. Then $x \in A$ and $x \notin B$. Since $x \notin B$ it follows that $A - B \subseteq \overline{B}$ is true.

Sums and Sequences

29.
$\sum_{k=1}^{5}(k+1) = 1+1+2+1+3+1+4+1+5+1 = 20$

$\sum_{j=0}^{4}(-2)^j = \frac{(-2)^{4+1}-1}{(-2)-1}$

$\sum_{i=1}^{10} 3 = 3 * 10 = 30$

$\sum_{j=0}^{8}(2^{j+1} - 2^j) = \sum_{j=0}^{8} 2^j = \frac{(2)^{8+1}-1}{(2)-1}$

31.
$\sum_{j=0}^{8} 3 * 2^j = 3 * \frac{(2)^{8+1}-1}{(2)-1}$

$\sum_{j=1}^{8} 2^j = \frac{(2)^{8+1}-1}{(2)-1} - 1$

$\sum_{j=2}^{8}(-3)^j = \frac{(-3)^{8+1}-1}{(-3)-1} - \frac{(-3)^{1+1}-1}{(-3)-1}$

$\sum_{j=0}^{8} 2 \cdot (-3)^j = 2 * \frac{(-3)^{8+1}-1}{(-3)-1}$

32.
$\sum_{j=0}^{8} (1 + (-1)^j) = \sum_{j=0}^{8} 1 + \sum_{j=0}^{8} (-1)^j$

Weak Induction

General method to complete proofs:
Step 1: Show that the basis P(0) or P(1) is true. Sometimes you need to read the problem to see what the basis is. It is not always 0 or 1.
Step 2: Substitute k in for n or whatever variable there is. On both sides of the equation.
Step 3: add k+1 to the right side.
Step 4: Do math and show that it is equal to step 2.

Ex:
$1 * 1! + 2 * 2!....n * n! = (n + 1)! - 1$
Basis: 0*0 = 1-1 = 0
Inductive:
$k * k! = (k + 1)! - 1$
$k * k! + (k + 1) * (k + 1)! = (k + 1 + 1)! - 1 + (k + 1) * (k + 1)!$