

Extra Credit Quiz - Jason Dorweiler

1. Using the method of contradiction proof that the product of a nonzero rational number and an irrational number is irrational.

Y is a rational number greater than zero and A is a irrational number. Using proof by contradiction $y*a = \frac{x}{z}$ where x,z are both integers making this a rational number. Since Y is also rational we can substitute $\frac{j}{k}$ where j,k are also integers. $\frac{j}{k} * a = \frac{x}{z}$. Then $a = \frac{x * k}{z * j}$ a ratio of integers which is a rational number. This contradicts our assumption that A was a irrational number.

2. Use proof by induction to show that $5^{2k} - 1$ is divisible by 4 for all k that belongs to N .

Basis: P(1): $5^{2*1} - 1 = 24$, $24/4 = 6$, basis step is divisible by 4.

Inductive step: $5^{2(k+1)} - 1 = 5^{2k+2} - 1 = 25 * 5^{2k} - 1$.

Subtracting $5^{2k} - 1$ gives:

$$25 * 5^{2k} - 1 - 5^{2k} - 1 = 24 * 25^k$$

and adding $5^{2k} - 1$ gives:

$$25 * 5^{2k} - 1 = 24 * 25^k + 5^{2k} - 1.$$

The right hand side gives us $24 * 25^k$ which is divisible by 4 for all $k \geq 1$ and $5^{2k} - 1$ which is divisible 4 from our hypothesis. Both right side terms are divisible by 4 so the left term is also divisible by 4. This proves the hypothesis.

3. Answer the following questions : 1) One state's lottery tickets consists of choosing six numbers out of fifty-four. How many different lottery tickets are there? 2) A committee of 8 people wishes to choose a chairperson, a vice-chairperson, and a treasurer. How many lists of such officers are possible?

a. I'm assuming order doesn't matter so $C(54,6) = \frac{54!}{6! * 48!} = 2582765$

b. $8*7*6 = 336$

4. Every Graph that has an Euler circuit is connected. - True

5. An Euler circuit covers each vertex once but not more than once. - False

6. Given a recursive denition of the sequence an, n = 1, 2, 3, . . . if $a_n = n^2 + 1$
 $a_{n+1} = (n + 1)^2 + 1 = n^2 + 2n + 1 + 1 = a_n + 2n + 1$

7. Compute the following sum: $\sum_2^5 4 * 3^j$

$$4 * \sum_2^5 3^j = 4 * [\sum_0^5 3^j - \sum_0^1 3^j] = 4 * [\frac{3^6 - 1}{2} - \frac{3^2 - 1}{2}] = 1440$$

8. Describe a graph model that represents a transit system in a large city. Should edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

It would be a directed graph where each station is a node and each edge is the route connecting two stations. This graph could have multiple edges because a station could have many connections to another. There is no use for loops in this graph.